DC PANDEY'S

Physics QUICK BOOK

Last Minute Prep for

JEE, NEET, Class 11/12

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PREFACE

Dear students, it gives me immense pleasure to present this Physics Quick book for daily revision of concepts and formulae. For a long time I was planning to write this book but due to my extremely busy schedule I could not get time. But this lockdown period was an opportunity for me to completed this book.

I have seen that students want to make short notes for quick revision but due to lack of experience, they can't make the short notes properly. Sometimes they miss the important concepts/formulae or they note down those things which are not very useful. This book is an effort from my side on behalf of such students. I am sure, it will definitely solve your purpose. Try to revise at least one chapter per day till the final exam.

I am extremely thankful to Mr. Anoop Dhyani for their special contribution in this book.

Please feel free to share your suggestions and mistakes (if any in the book) to improve its revised editions from next year.

mail id. arihantcorrections@gmail.com

Thanks

DC Pandey

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CHAPTER 01

General Physics

Rules for Counting Significant Figures

***Rule* 1** All non-zero digits are significant. For example, 126.28 has five significant figures.

***Rule* 2** The zeros appearing between two non-zero digits are significant. For example, 6.025 has four significant figures.

***Rule* 3** Trailing zeros after decimal places are significant. Measurement l = 6.400 cm has four significant figures.

Let us take an example in its support.

Measurement	**Accuracy**	***l* lies between** (in cm)	**Significant figures**	**Remarks**
$l = 6.4$ cm	0.1 cm	6.3-6.5	Two	
$l = 6.40$ cm	0.01 cm	6.39-6.41	Three	closer
$l = 6.400$ cm	0.001 cm	6.399-6.401	Four	more closer

Thus, the significant figures depend on the accuracy of measurement. More the number of significant figures, more accurate is the measurement.

***Rule* 4** The powers of ten are not counted as significant figures.

For example, 1.4×10^{-7} has only two significant figures 1 and 4.

***Rule* 5** If a measurement is less than one, then all zeros occurring to the left of last non-zero digit are not significant. For example, 0.0042 has two significant figures 4 and 2.

***Rule* 6** Change in units of measurement of a quantity does not change the number of significant figures. Suppose a measurement was done using mm scale and we get l = 72 mm (two significant figures).

We can write this measurement in other units also (without changing the number of significant figures) :

7.2 cm $\rightarrow$ Two significant figures

0.072 m $\rightarrow$ Two significant figures

0.000072 km $\rightarrow$ Two significant figures

7.2×10^{7} nm $\rightarrow$ Two significant figures

***Rule* 7** The terminal or trailing zeros in a number without a decimal point are not significant. This also sometimes arises due to change of unit.

For example, 264 m = 26400 cm = 264000 mm

All have only three significant figures 2, 6 and 4.

Zeros at the end of a number are significant only, if they are behind a decimal point as in Rule 3. Otherwise, it is impossible to tell if they are significant.

For example, in the number 8200, it is not clear, if the zeros are significant or not. The number of significant digits in 8200 is at least two, but could be three or four.
To avoid uncertainty, use scientific notation to place significant zeros behind a decimal point

8.200×10^3 has four significant digits

8.20×10^3 has three significant digits

8.2×10^3 has two significant digits

Therefore, if it is not expressed in scientific notations, then write least number of significant digits. Hence, in the number 8200, take significant digits as two.
***Rule* 8** Exact measurements have infinite number of significant figures. For example,

10 bananas in a basket

46 students in a class

Speed of light in vacuum = 299,792,458 m/s (exact)

$$\pi = \frac{22}{7} \text{ (exact)}$$

All these measurements have infinite number of significant figures.

Rounding off a Digit

Following are the rules for rounding off a measurement

***Rule* 1** If the number lying to the right of cut off digit is less than 5, then the cut off digit is retained as such. However, if it is more than 5, then the cut off digit is increased by 1.

For example, $x = 6.24$ is rounded off to 6.2 to two significant digits and $x = 5.328$ is rounded off to 5.33 to three significant digits.

***Rule* 2** If the digit to be dropped is 5 followed by digits other than zero, then the preceding digit is increased by 1.

For example, $x = 14.252$ is rounded off to $x = 14.3$ to three significant digits.

***Rule* 3** If the digit to be dropped is simply 5 or 5 followed by zeros, then the preceding digit is left unchanged if it is even.
For example, $x = 6.250$ or $x = 6.25$ becomes $x = 6.2$ after rounding off to two significant digits.

***Rule* 4** If the digit to be dropped is 5 or 5 followed by zeros, then the preceding digit is raised by one if it is odd.
For example, $x = 6.350$ or $x = 6.35$ becomes $x = 6.4$ after rounding off to two significant digits.

Algebraic Operations with Significant Figures

Addition or Subtraction

Suppose in the measured values to be added or subtracted, the least number of digits after the decimal is n. Then, in the sum or difference, the number of significant digits after the decimal should also be n.

For example $1.2 + 3.45 + 6.789 = 11.439 \approx 11.4$

Here, the least number of significant digits after the decimal is one. Hence, the result will be 11.4 (when rounded off to smallest number of decimal places).

Multiplication or Division

Suppose in the measured values to be multiplied or divided, the least number of significant digits be n. Then, in the product or quotient, the number of significant digits should also be n.

For example $1.2 \times 36.72 = 44.064 \approx 44$

The least number of significant digits in the measured values are two. Hence, the result when rounded off to two significant digits will be 44. Therefore, the answer is 44.

Error Analysis

- **Least count**

Instrument	Its least count
mm scale	1 mm
Vernier callipers	0.1 mm
Screw gauge	0.01 mm
Stop watch	0.1 s
Temperature thermometer	1°C

- **True value**

  Usually the mean value a_m is taken as the true value. So,

$$a_m = \frac{a_1 + a_2 + \ldots + a_n}{n}$$

- **Absolute error**

$$\Delta a_1 = a_m - a_1$$
$$\Delta a_2 = a_m - a_2$$
$$\ldots \quad \ldots \quad \ldots$$
$$\Delta a_n = a_m - a_n$$

  Absolute error may be positive or negative.

- **Mean absolute error**

$$\Delta a_{\text{mean}} = \frac{|\Delta a_1| + |\Delta a_2| + \ldots + |\Delta a_n|}{n}$$

  The final result of measurement can be written as, $a = a_m \pm \Delta a_{\text{mean}}$.

- **Relative (or fractional) and percentage error**

$$\text{Relative error} = \frac{\Delta a_{\text{mean}}}{a_m}$$

$$\text{Percentage error} = \frac{\Delta a_{\text{mean}}}{a_m} \times 100$$

- **Error in sum or difference**

$$\text{Let} \quad x = a \pm b$$

$$\text{Then, } \Delta x = \pm (\Delta a + \Delta b)$$

- **Error in product**

$$\text{Let} \quad x = ab$$

$$\text{Then, } \frac{\Delta x}{x} = \pm \left(\frac{\Delta a}{a} + \frac{\Delta b}{b} \right)$$

- **Error in division**

$$\text{Let} \quad x = \frac{a}{b}$$

$$\text{Then, } \frac{\Delta x}{x} = \pm \left(\frac{\Delta a}{a} + \frac{\Delta b}{b} \right)$$

- **Error in quantity raised to some power**

$$\text{Let} \quad x = \frac{a^n}{b^m}$$

$$\text{Then, } \frac{\Delta x}{x} = \pm \left(n \frac{\Delta a}{a} + m \frac{\Delta b}{b} \right)$$

Experiments

1. Vernier Callipers

(i) $\text{VC} = \text{LC} = \dfrac{1 \text{ MSD}}{n} = \dfrac{\text{smallest division on main scale}}{\text{number of divisions on vernier scale}} = 1 \text{ MSD} - 1 \text{ VSD}$

(ii) In ordinary vernier callipers, 1 MSD = 1 mm and $n = 10$

$\therefore$ $$\text{VC or LC} = \frac{1}{10} \text{ mm} = 0.01 \text{ cm}$$

(iii) Total reading $= (N + n \times \text{VC})$ (N = main scale reading)

(iv) Zero correction = – Zero error

(v) Zero error is algebraically subtracted while the zero correction is algebraically added.

(vi) If zero of vernier scale lies to the right of zero of main scale, the error is positive. The actual length in this case is less than observed length.

(vii) If zero of vernier scale lies to the left of zero of main scale, the error is negative and the actual length is more than the observed length.

(viii) Positive zero error $= (N + x \times \text{VC})$

(ix) In negative zero error, suppose 8th vernier scale division coincides with the main scale division, then

Negative zero error = $-$ [0.00 + 8 $\times$ VC] = $-$ [0.00 + 8 $\times$ 0.01cm] = $-$ 0.08 cm

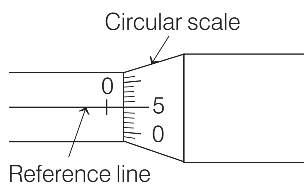
2. Screw Gauge

$$\text{Least count} = \frac{\text{pitch}}{\text{number of divisions on circular scale}}$$

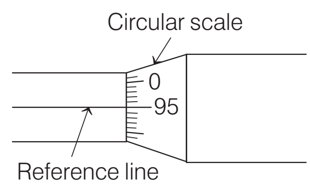
Total reading = $N + n \times \text{LC}$

If the zero of the circular scale advances beyond the reference line, the zero error is negative and zero correction is positive. If it is left behind the reference line, the zero error is positive and zero correction is negative.

For example, if zero of circular scale advances beyond the reference line by 5 divisions, zero correction = + 5 $\times$ (LC) and if the zero of circular scale is left behind the reference line by 5 divisions, zero correction = $-$ 5 $\times$ (LC).



(a) Positive zero error



(b) Negative zero error

Note *In negative zero error, 95th divisions of the circular scale is coinciding with the reference line. Hence, there are 5 divisions between zero mark on the circular scale and the reference line.*

3. Speed of Sound using Resonance Tube

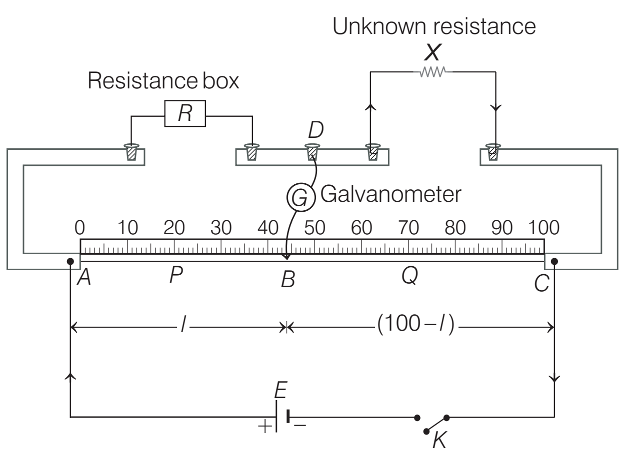
(i) Result is independent of end correction

(ii) $v = 2f(l_2 - l_1)$, where f = frequency of tuning fork,
l_1 = first resonance length and l_2 = second resonance length.

(iii) End correction, $e = \dfrac{l_2 - 3l_1}{2}$

4. Meter Bridge Experiment

Meter bridge experiment is based on the principle of Wheatstone's bridge.



When current through galvanometer is zero or bridge is balanced, then

$$\frac{P}{Q} = \frac{R}{X}$$

$$\therefore \qquad X = R\left(\frac{Q}{P}\right) = \left(\frac{100 - l}{l}\right) R$$

End Corrections

In meter bridge, some extra length (under the metallic strips) comes at points A and C. Therefore, some additional length (α and β) should be included at the ends.

Here, α and β are called the end corrections. Hence, in place of l, we use $l + \alpha$ and in place of $100 - l$, we use $100 - l + \beta$.

To find α and β, use known resistors R_1 and R_2 in place of R and X and suppose we get null point length equal to l_1. Then,

$$\frac{R_1}{R_2} = \frac{l_1 + \alpha}{100 - l_1 + \beta} \qquad \text{...(i)}$$

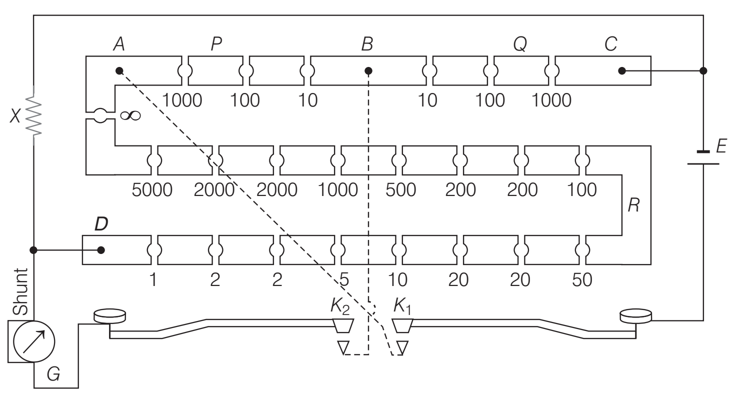
Now, we interchange the positions of R_1 and R_2 and suppose the new null point length is l_2. Then,

$$\frac{R_2}{R_1} = \frac{l_2 + \alpha}{100 - l_2 + \beta} \qquad \text{...(ii)}$$

Solving Eqs. (i) and (ii), we can find α and β.

5. Post Office Box

Post office box also works on the principle of Wheatstone's bridge.



In a Wheatstone's bridge circuit, if $\frac{P}{Q} = \frac{R}{X}$, then the bridge is balanced. So, unknown resistance $X = \frac{Q}{P} R$.

P and Q are set in arms AB and BC, where we can have $10\,\Omega$, $100\,\Omega$ or $1000\,\Omega$ resistances to set any ratio $\frac{Q}{P}$.

These arms are called ratio arms, initially we take $Q = 10\ \Omega$ and $P = 10\ \Omega$ to set $\frac{Q}{P} = 1$.

The unknown resistance (X) is connected between C and D and battery is connected across A and C.

Now, put resistance in part A to D such that the bridge gets balanced.

For this, keep on increasing the resistance with $1\ \Omega$ interval, check the deflection in galvanometer by first pressing key K_1, then galvanometer key K_2.

Suppose at $R = 4\Omega$, we get deflection towards left and at $R = 5\Omega$, we get deflection towards right.

Then, we can say that for balanced condition, R should lies between $4\ \Omega$ to $5\ \Omega$.

Now,
$$X = \frac{Q}{P} R = \frac{10}{10} R$$
$$= R = 4\ \Omega \text{ to } 5\ \Omega$$

To get closer value of X, in the second observation, let us choose $\frac{Q}{P} = \frac{1}{10}$, i.e. $\left(\frac{P = 100}{Q = 10}\right)$

Suppose now at $R = 42\Omega$, we get deflection towards left and at $R = 43\Omega$, deflection is towards right.

So, $R \in (42, 43)$.

Now, $X = \frac{Q}{P} R = \frac{10}{100} R = \frac{1}{10} R$, where $R \in (42, 43\ \Omega)$.

Now, to get further closer value, take $\frac{Q}{P} = \frac{1}{100}$ and so on.

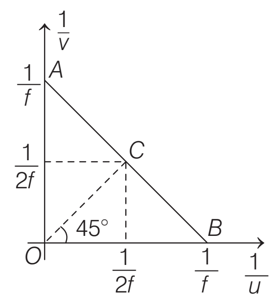
The observation table is shown below

Resistance in the ratio arms		**Resistance in arm** *AD* (*R*) (ohm)	**Direction of deflection**	**Unknown resistance** $X = \frac{Q}{P} \times R$ (ohm)
AB (*P*) (ohm)	*BC* (*Q*) (ohm)			
10	10	4	Left	4 to 5
		5	Right	
100	10	40	Left (large)	(4.2 to 4.3)
		50	Right (large)	
		42	Left	
		43	Right	
1000	10	420	Left	4.25
		424	Left	
		425	No deflection	
		426	Right	

So, the correct value of X is $4.25\ \Omega$.

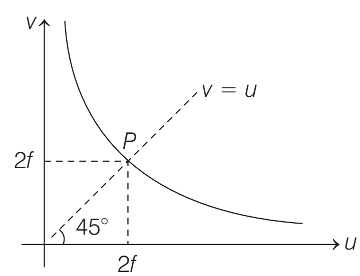
6. Focal Length of Concave Mirror

(i) $\frac{1}{v}$ ***versus*** $\frac{1}{u}$ **graph**



The coordinates of point C are $\left(\frac{1}{2f}, \frac{1}{2f}\right)$. The focal length of the concave mirror can be calculated by measuring the coordinates of either of the points A, B or C.

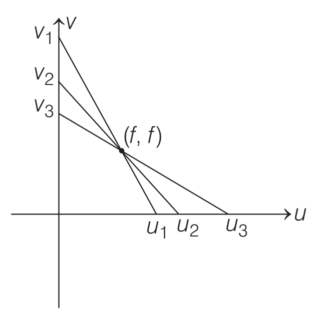
(ii) ***v versus u*** **graph**



From u-v data, plot v *versus* u curve and draw a line bisecting the axis. Find the intersection point and equate them to $(2f, 2f)$.

(iii) **By joining v_n and u_n**

All lines intersect at a common point (f, f).

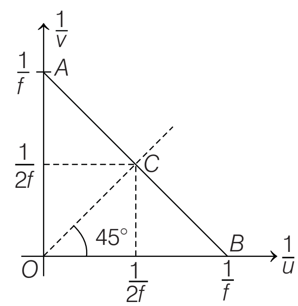


Find common intersection point and equate it to (f, f).

7. Focal Length of Convex Lens

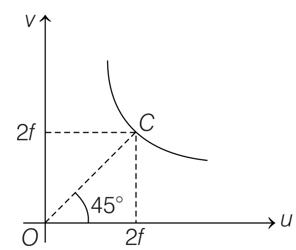
(i) $\frac{1}{v}$ ***versus*** $\frac{1}{u}$ **graph**

The focal length of convex lens can be calculated by measuring the coordinates of either of the points *A, B* or *C*.



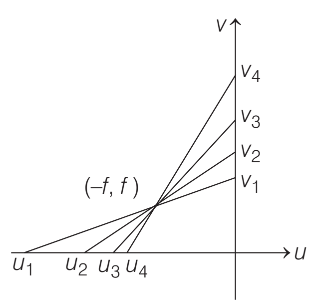
(ii) ***v versus u*** **graph**

By measuring the coordinates of point *C*, whose coordinates are $(2f, 2f)$, we can calculate the focal length of the lens.



(iii) **By joining** v_n **and** u_n

All lines intersect at a common point $(-f, f)$.



Find common intersection point and equate it to $(-f, f)$.

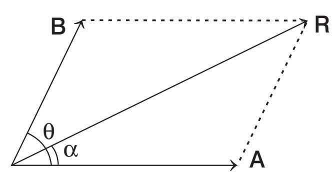
Note *All graphs are for real images.*

Physical quantities having the same dimensions

Physical Quantities or Combination of Physical Quantities	Dimensions
Angle, strain, $\sin\theta$, π, e^x	$[M^0 L^0 T^0]$
Work, energy, torque, Rhc	$[ML^2 T^{-2}]$
Time, $\frac{L}{R}$, CR, $\sqrt{LC}$	$[M^0 L^0 T]$
Frequency, ω, $\frac{R}{L}$, $\frac{1}{CR}$, $\frac{1}{\sqrt{LC}}$, velocity gradient, decay constant, activity of a radioactive substance	$[M^0 L^0 T^{-1}]$
Pressure, stress, modulus of elasticity, energy density (energy per unit volume), $\varepsilon_0 E^2$, $\frac{B^2}{\mu_0}$	$[ML^{-1} T^{-2}]$
Angular impulse, angular momentum, Planck's constant	$[ML^2 T^{-1}]$
Linear momentum, linear impulse	$[MLT^{-1}]$
Wavelength, radius of gyration, light year	$[M^0 LT^0]$
Velocity, $\frac{1}{\sqrt{\varepsilon_0\mu_0}}$, $\sqrt{\frac{GM}{R}}$, $\frac{E}{B}$	$[M^0 LT^{-1}]$

Vectors

- $R = |\mathbf{A} + \mathbf{B}| = \sqrt{A^2 + B^2 + 2AB\cos\theta} = |\mathbf{R}|$
- Angle of $\mathbf{R}$ from $\mathbf{A}$ towards $\mathbf{B}$ is given by, $\tan\alpha = \dfrac{B\sin\theta}{A + B\cos\theta}$



- If $|\mathbf{B}| = |\mathbf{A}| = A$(say), then $R = 2A\cos\dfrac{\theta}{2}$ and $\mathbf{R}$ passes along the bisector line of $\mathbf{A}$ and $\mathbf{B}$.

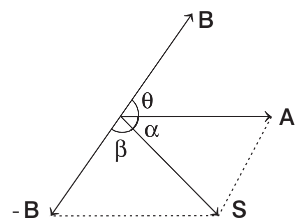
  In this case, if

  $\theta = 0°$, $R = 2A$
  $\theta = 60°$, $R = \sqrt{3}\,A$
  $\theta = 90°$, $R = \sqrt{2}\,A$
  $\theta = 120°$, $R = A$
  and $\theta = 180°$, $R = 0$

- $S = |\mathbf{A} - \mathbf{B}| = \sqrt{A^2 + B^2 - 2AB\cos\theta} = |\mathbf{S}|$

  Here, θ is the angle between $\mathbf{A}$ and $\mathbf{B}$, not the angle between $\mathbf{A}$ and $-\mathbf{B}$.

- Angle of **S** from **A** towards **–B** is given by, $\tan\alpha = \dfrac{B\sin\theta}{A - B\cos\theta}$



or angle of **S** from **–B** towards **A** is given by

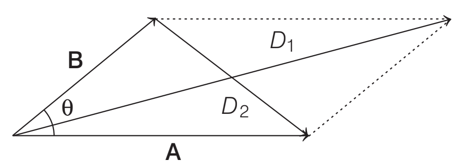
$$\tan\beta = \frac{A\sin\theta}{B - A\cos\theta}$$

- If $|\mathbf{B}| = |\mathbf{A}| = A$ (say), then $S = 2A\sin\dfrac{\theta}{2}$ and **S** passes through the bisector line of **A** and **–B**.

In this case, if

$$\begin{aligned} \theta &= 0^\circ, & S &= 0 \\ \theta &= 60^\circ, & S &= A \\ \theta &= 90^\circ, & S &= \sqrt{2}\,A \\ \theta &= 120^\circ, & S &= \sqrt{3}\,A \\ \text{and}\quad \theta &= 180^\circ, & S &= 2A \end{aligned}$$

- In the figure shown,



$$\text{diagonal, } D_1 = |\mathbf{A} + \mathbf{B} \text{ or } \mathbf{R}| = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

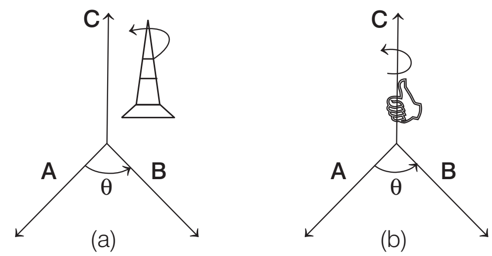
$$\text{diagonal, } D_2 = |\mathbf{A} - \mathbf{B} \text{ or } \mathbf{S}| = \sqrt{A^2 + B^2 - 2AB\cos\theta}$$

$$D_1 = D_2 = \sqrt{A^2 + B^2}, \text{ if } \theta = 90^\circ$$

- $\mathbf{A}\cdot\mathbf{B} = AB\cos\theta$. Here, A and B are always positive as these are the magnitudes of **A** and **B**. Hence,

$$\begin{aligned} 0^\circ \le \theta < 90^\circ, &\quad \text{if } \mathbf{A}\cdot\mathbf{B} \text{ is positive} \\ 90^\circ < \theta \le 180^\circ, &\quad \text{if } \mathbf{A}\cdot\mathbf{B} \text{ is negative.} \\ \text{and } \theta = 90^\circ, &\quad \text{if } \mathbf{A}\cdot\mathbf{B} \text{ is zero.} \end{aligned}$$

- $|\mathbf{A}\times\mathbf{B}| = AB\sin\theta$
- $\mathbf{A}\times\mathbf{B} = -\mathbf{B}\times\mathbf{A}$
- **Direction of Vector Cross Product** When $\mathbf{C} = \mathbf{A}\times\mathbf{B}$, then the direction of **C** is at right angles to the plane containing the vectors **A** and **B**. The direction of **C** is determined by the right hand screw rule and right hand thumb rule.



(a) **Right Hand Screw Rule** Rotate a right handed screw from first vector (**A**) towards second vector (**B**) through the smaller angle between them. The direction in which the right handed screw moves gives the direction of vector (**C**).

(b) **Right Hand Thumb Rule** Curl the fingers of your right hand from **A** to **B** through the smaller angle between them. Then, the direction of the erect thumb will point in the direction of $\mathbf{A}\times\mathbf{B}$ or **C**.

- **Direction Cosines of a Vector** If any vector **A** subtend angles α, β and γ with x-axis, y-axis and z-axis respectively and its components along these axes are A_x, A_y and A_z, then

$$\cos\alpha = \frac{A_x}{A}, \quad \cos\beta = \frac{A_y}{A}, \quad \cos\gamma = \frac{A_z}{A}$$

and $$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

Here, $\cos\alpha$, $\cos\beta$ and $\cos\gamma$ are called the direction cosines of **A** along x, y and z-axis.

- If we have to prove two vectors mutually perpendicular, then show their dot product equal to zero.
- To prove two vectors mutually parallel or antiparallel, we have two methods :

First Show their cross product equal to zero.

Second Show that the ratio of coefficients of $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$ and $\hat{\mathbf{k}}$ of two vectors is constant. If this constant is positive, vectors are parallel and if this constant is negative, vectors are antiparallel.

- **Angle between two vectors** In some cases, angle between two vectors can be obtained just by observation as given in following table :

A	**B**	**θ between A and B**
$2\hat{\mathbf{i}}$	$6\hat{\mathbf{i}}$	0°
$3\hat{\mathbf{j}}$	$-5\hat{\mathbf{j}}$	180°
$2\hat{\mathbf{i}}$	$3\hat{\mathbf{j}} - 4\hat{\mathbf{k}}$	90°
$6\hat{\mathbf{i}}$	$2\hat{\mathbf{i}} + 2\hat{\mathbf{j}}$	45°
$8\hat{\mathbf{i}}$	$-4\hat{\mathbf{i}} + 4\hat{\mathbf{j}}$	135°

In general, angle between **A** and **B** can be obtained by the following relation,

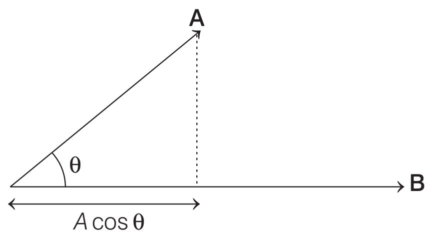
$$\theta = \cos^{-1}\left(\frac{\mathbf{A}\cdot\mathbf{B}}{AB}\right)$$

It is not always, $\sin^{-1}\left\{\frac{|\mathbf{A}\times\mathbf{B}|}{AB}\right\}$

Let's Practice Explain the reason why θ is not always given by the following relation?

$$\theta = \sin^{-1}\left\{\frac{|\mathbf{A}\times\mathbf{B}|}{AB}\right\}$$

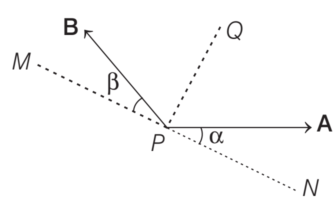
- Component of **A** along **B** $= A\cos\theta = \dfrac{\mathbf{A}\cdot\mathbf{B}}{B}$



Similarly, component of **B** along **A** $= B\cos\theta = \dfrac{\mathbf{A}\cdot\mathbf{B}}{A}$

Component of **A** along **B** = component of **B** along **A**, if $|\mathbf{A}| = |\mathbf{B}|$ or $A = B$. Otherwise they are not equal.

- If resultant of n vectors is zero, of which $(n-1)$ vectors are known and only one vector is unknown, then this last unknown vector is equal and opposite to the resultant of $(n-1)$ known vectors.
- Vector sum of n vectors of same magnitudes is always zero if angle between two successive vectors is always $\left(\dfrac{360}{n}\right)^\circ$.
- If resultant of **A** and **B** is along PQ, then components of **A** and **B** perpendicular to PQ or along MN should be equal and opposite.



$$\Rightarrow \qquad A\cos\alpha = B\cos\beta$$

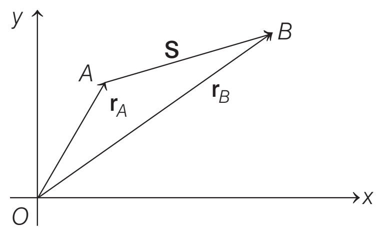
and the resultant along PQ is, $\qquad R = A\sin\alpha + B\sin\beta$

- A unit vector perpendicular to both **A** and **B**

$$\hat{\mathbf{C}} = \pm\frac{\mathbf{A}\times\mathbf{B}}{|\mathbf{A}\times\mathbf{B}|}$$

- $\hat{\mathbf{i}}\cdot\hat{\mathbf{i}} = \hat{\mathbf{j}}\cdot\hat{\mathbf{j}} = \hat{\mathbf{k}}\cdot\hat{\mathbf{k}} = 1,\ \hat{\mathbf{i}}\cdot\hat{\mathbf{j}} = \hat{\mathbf{j}}\cdot\hat{\mathbf{k}} = \hat{\mathbf{i}}\cdot\hat{\mathbf{k}} = 0$
- If coordinates of point A are (x_1, y_1, z_1) and coordinates of point B are (x_2, y_2, z_2), then

$$\mathbf{r}_A = \text{position vector of } A = x_1\hat{\mathbf{i}} + y_1\hat{\mathbf{j}} + z_1\hat{\mathbf{k}}$$

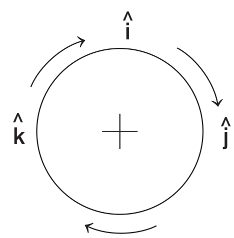


$$\mathbf{r}_B = x_2\,\hat{\mathbf{i}} + y_2\,\hat{\mathbf{j}} + z_2\,\hat{\mathbf{k}}$$

$$\mathbf{S} = \mathbf{r}_B - \mathbf{r}_A = (x_2 - x_1)\,\hat{\mathbf{i}} + (y_2 - y_1)\,\hat{\mathbf{j}} + (z_2 - z_1)\,\hat{\mathbf{k}}$$

$$= \text{displacement vector from } A \text{ to } B$$

- $\hat{\mathbf{i}} \times \hat{\mathbf{j}} = \hat{\mathbf{k}}$, $\hat{\mathbf{j}} \times \hat{\mathbf{k}} = \hat{\mathbf{i}}$, $\hat{\mathbf{k}} \times \hat{\mathbf{i}} = \hat{\mathbf{j}}$, $\hat{\mathbf{i}} \times \hat{\mathbf{k}} = -\,\hat{\mathbf{j}}$, $\hat{\mathbf{k}} \times \hat{\mathbf{j}} = -\,\hat{\mathbf{i}}$
  $\hat{\mathbf{j}} \times \hat{\mathbf{i}} = -\,\hat{\mathbf{k}}$, $\hat{\mathbf{i}} \times \hat{\mathbf{i}} = \hat{\mathbf{j}} \times \hat{\mathbf{j}} = \hat{\mathbf{k}} \times \hat{\mathbf{k}} =$ a null vector



- If vectors are given in terms of $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$ and $\hat{\mathbf{k}}$,
  let $\mathbf{A} = a_1\hat{\mathbf{i}} + a_2\hat{\mathbf{j}} + a_3\hat{\mathbf{k}}$ and $\mathbf{B} = b_1\hat{\mathbf{i}} + b_2\hat{\mathbf{j}} + b_3\hat{\mathbf{k}}$, then

  (i) $|\mathbf{A}| = A = \sqrt{a_1^2 + a_2^2 + a_3^2}$ and $|\mathbf{B}| = B = \sqrt{b_1^2 + b_2^2 + b_3^2}$

  (ii) $\mathbf{A} + \mathbf{B} = (a_1 + b_1)\,\hat{\mathbf{i}} + (a_2 + b_2)\,\hat{\mathbf{j}} + (a_3 + b_3)\,\hat{\mathbf{k}}$

  (iii) $\mathbf{A} - \mathbf{B} = (a_1 - b_1)\,\hat{\mathbf{i}} + (a_2 - b_2)\,\hat{\mathbf{j}} + (a_3 - b_3)\,\hat{\mathbf{k}}$

  (iv) $\mathbf{A} \cdot \mathbf{B} = a_1b_1 + a_2b_2 + a_3b_3$

  (v) $|\mathbf{A} \times \mathbf{B}| = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = (a_2b_3 - b_2a_3)\,\hat{\mathbf{i}} + (b_1a_3 - b_3a_1)\,\hat{\mathbf{j}} + (a_1b_2 - b_1a_2)\,\hat{\mathbf{k}}$

  (vi) Component of **A** along **B**

  $$= A\cos\theta = \frac{\mathbf{A} \cdot \mathbf{B}}{B} = \frac{a_1b_1 + a_2b_2 + a_3b_3}{\sqrt{b_1^2 + b_2^2 + b_3^2}}$$

  (vii) Unit vector parallel to **A**

  $$= \hat{\mathbf{A}} = \frac{\mathbf{A}}{A} = \frac{a_1\hat{\mathbf{i}} + a_2\hat{\mathbf{j}} + a_3\hat{\mathbf{k}}}{\sqrt{a_1^2 + a_2^2 + a_3^2}}$$

(viii) Angle between **A** and **B**,

$$\theta = \cos^{-1}\left(\frac{\mathbf{A}\cdot\mathbf{B}}{AB}\right)$$

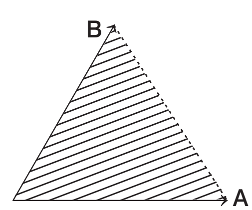
$$\therefore \quad \theta = \cos^{-1}\left(\frac{a_1 b_1 + a_2 b_2 + a_3 b_3}{\sqrt{a_1^2 + a_2^2 + a_3^2} \times \sqrt{b_1^2 + b_2^2 + b_3^2}}\right)$$

- $(\mathbf{A} + \mathbf{B})$ is perpendicular to $(\mathbf{A} - \mathbf{B})$, if $A = B$.
- $(\mathbf{A} \times \mathbf{B})$ is perpendicular to both **A** and **B** separately, or it is perpendicular to the plane formed by **A** and **B**.
- $\hat{\mathbf{i}} \times \hat{\mathbf{j}}$ should always be in the direction of $\hat{\mathbf{k}}$.
- Pressure is a scalar quantity, not a vector quantity. It has magnitude but no direction sense associated with it. Pressure acts in all directions at a point inside a fluid.
- Surface tension is scalar quantity because it has no specific direction. Current is also a scalar quantity.
- Stress and moment of inertia are tensor quantities.
- To qualify as a vector, a physical quantity must not only possess magnitude and direction but must also satisfy the parallelogram law of vector addition.

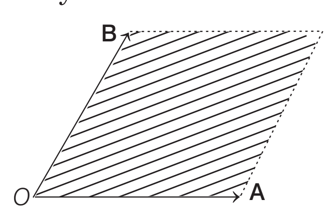
  For example, the finite rotation of a rigid body about a given axis has magnitude (the angle of rotation) and also direction (the direction of the axis) but it is not a vector quantity.

  This is so far the simple reason that the two finite rotations of the body do not add up in accordance with the law of vector addition.

  However, if the rotation be small or infinitesimal, it may be regarded as a vector quantity.
- Area can behave either as a scalar or a vector and how it behaves depends on circumstances.
- Area (vector), dipole moment and current density are defined as vectors with specific direction.
- The area of triangle bounded by vectors **A** and **B** is $\frac{1}{2}|\mathbf{A} \times \mathbf{B}|$.



- Area of parallelogram bounded by vectors **A** and **B** is $|\mathbf{A} \times \mathbf{B}|$.



CHAPTER 02

Kinematics I

General Points

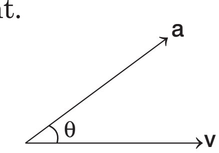
- If a particle is just dropped from a moving body, then just after dropping, velocity of the particle (not acceleration) is equal to the velocity of the moving body at that instant.
- If y (may be velocity, acceleration, etc.) is a function of time or $y = f(t)$ and we want to find the average value of y between a time interval of t_1 and t_2, then

$$\langle y \rangle_{t_1 \text{ to } t_2} = \text{average value of } y \text{ between } t_1 \text{ and } t_2$$
$$= \frac{\int_{t_1}^{t_2} f(t)\, dt}{t_2 - t_1}$$

If $f(t)$ is a linear function of t, then $y_{\text{av}} = \dfrac{y_f + y_i}{2}$

Here, y_f = final value of y and y_i = initial value of y.

- Angle between velocity vector $\mathbf{v}$ and acceleration vector $\mathbf{a}$ decides whether the speed of particle is increasing, decreasing or constant.



Speed increases, if $0° \le \theta < 90°$

Speed decreases, if $90° < \theta \le 180°$

Speed is constant, if $\theta = 90°$

The angle θ between $\mathbf{v}$ and $\mathbf{a}$ can be obtained by the following relation,

$$\theta = \cos^{-1}\left(\frac{\mathbf{v} \cdot \mathbf{a}}{va}\right)$$

- The magnitude of instantaneous velocity is called the instantaneous speed, i.e.

$$v = |\mathbf{v}| = \left|\frac{d\mathbf{r}}{dt}\right|$$

Speed is not equal to $\dfrac{dr}{dt}$, i.e. $v \neq \dfrac{dr}{dt}$

where, r is the modulus of radius vector $\mathbf{r}$ because in general $|d\mathbf{r}| \neq dr$.

For example, when $\mathbf{r}$ changes only in direction, i.e. if a point moves in a circle, then r = constant, $dr = 0$ but $|d\mathbf{r}| \neq 0$.

- The sign of acceleration does not tell us whether the particle's speed is increasing or decreasing. This sign of acceleration depends upon the choice of the positive direction of the axis.

  For example, if the vertically upward direction is chosen to be the positive direction, the acceleration due to gravity is negative.

  If a particle is falling under gravity, this acceleration though negative, results in increase in speed.

  For a particle thrown upward, the same negative acceleration (of gravity) results in decrease in speed.

- The zero velocity of a particle at any instant does not necessarily imply zero acceleration at that instant. A particle may be momentarily at rest and yet have non-zero acceleration.

  For example, a particle thrown up has zero velocity at its highest point but the acceleration at that instant continues to be the acceleration due to gravity.

- **Reaction time** When a situation demands our immediate action, it takes some time before we really respond. Reaction time is the time, a person takes to observe, think and act.

  For example, if a person is driving and suddenly a boy appears on the road, then the time elapsed before he applies the brakes of the car is the reaction time.

Basic Definitions

- Displacement $\mathbf{s} = \mathbf{r}_f - \mathbf{r}_i = (x_f - x_i)\,\hat{\mathbf{i}} + (y_f - y_i)\,\hat{\mathbf{j}} + (z_f - z_i)\,\hat{\mathbf{k}}$
- Distance = actual path length
- Average velocity $= \dfrac{\text{total displacement}}{\text{total time}} = \dfrac{s}{t}$
- Average speed $= \dfrac{\text{total distance}}{\text{total time}} = \dfrac{d}{t}$
- Average acceleration $= \dfrac{\text{change in velocity}}{\text{time}} = \dfrac{\Delta \mathbf{v}}{\Delta t}$

$$= \frac{\mathbf{v}_f - \mathbf{v}_i}{t}$$

- Instantaneous velocity $= \dfrac{d\mathbf{s}}{dt}$ or $\dfrac{d\mathbf{r}}{dt}$
- Instantaneous acceleration = rate of change of velocity

$$= \frac{d\mathbf{v}}{dt} = \frac{d^2\mathbf{s}}{dt^2} = \frac{d^2\mathbf{r}}{dt^2}$$

In one dimensional motion : (say along x-axis)

- Displacement $s = x_f - x_i$
- Instantaneous velocity $v = \frac{dx}{dt}$ or $\frac{ds}{dt}$ = slope of x-t or s-t graph
- Average velocity $= \frac{s}{t} = \frac{x_f - x_i}{t}$
- Instantaneous acceleration $a = \frac{dv}{dt}$ = slope of v-t graph
- Average acceleration $= \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t}$

Note *In the above expressions, all vectors quantities are to be substituted with sign (in 1-D motion).*

Uniform Motion

In a uniform motion, velocity remains constant and acceleration is zero. Since, velocity is constant, so motion will be one dimensional without change in direction with same speed.

Hence, distance (d) and displacement (s) will be same in magnitude.

Due to direction, displacement may be positive or negative but distance is always positive. Equations are as follows

(i) $a = 0$

(ii) $v =$ constant

(iii) $d = |\mathbf{s}| = |\mathbf{v}|\, t$ or, sometimes we simply write, $d = s = vt$

Note *Most of the problems of average speed are based on uniform motion.*

One Dimensional Motion with Uniform Acceleration

- Important equations for one dimensional motion with uniform acceleration are as folows

  (i) $v = u + at$

  (ii) $s = ut + \frac{1}{2}at^2$

  (iii) $s' = s_0 + ut + \frac{1}{2}at^2$

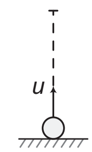
  (iv) $v^2 = u^2 + 2as$

  (v) $s_t = (u + at) - \frac{a}{2}$

- While using above equations, take a sign convention and substitute all vector quantities (v, u, a, s and s_t) with sign.
- In equation $s = ut + \frac{1}{2}at^2$, s is the displacement measured from the starting point ($t = 0$).
- s_t is the displacement (not distance) in t^{th} second or between the time $(t-1)$ and t.
- In the above equations, s and s' both are displacements from time $t = 0$ to $t = t$. But s is measured from the point $t = 0$, while s' is measured from any other point (say P). Further, s_0 is the displacement of starting point ($t = 0$) from point P in that case.

- For small heights, if the motion is taking place under gravity, then acceleration is always constant (= acceleration due to gravity), i.e. 9.8 m/s^2 ($\approx$ 10 m/s^2) in downward direction. According to sign convention, upward direction is positive and downward direction is negative. Therefore, $a = g = -9.8 \text{ m/s}^2 \approx -10 \text{ m/s}^2$
- In most of the problems of time calculations, equation $s = ut + \frac{1}{2}at^2$ is useful. But s has to be measured from the starting point ($t = 0$).

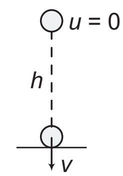
(i) If a particle is projected upwards with velocity u, then



(a) maximum height attained by the particle, $h = \frac{u^2}{2g}$

(b) time of ascent = time of descent = $\frac{u}{g}$ $\Rightarrow$ Total time of flight = $\frac{2u}{g}$

(ii) If a particle is released from rest from a height h (also called free fall), then

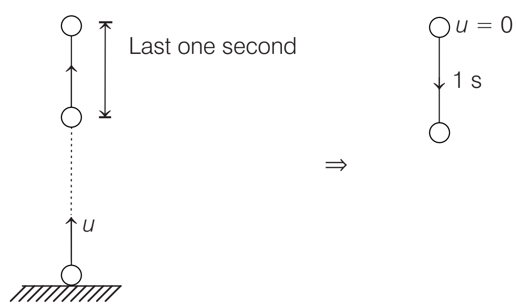


(a) velocity of particle at the time of striking with ground, $v = \sqrt{2gh}$

(b) time of descent (also called free fall time), $t = \sqrt{\frac{2h}{g}}$

Note *In the above results, air resistance has been neglected and we have already substituted the signs of u, g, etc. So, you have to substitute only their magnitudes.*

- A particle is thrown upwards with velocity u. Suppose it takes time t to reach its highest point, then distance travelled in last second is independent of u.



This is because this distance is equal to the distance travelled in first second of a freely falling object. Thus,

$$s = \frac{1}{2} g \times (1)^2 = \frac{1}{2} \times 10 \times 1 = 5 \text{ m}$$

- If a particle is just dropped ($u = 0$) under gravity, then after time t, velocity of particle in downward direction will be gt or $10t$ (if $g = 10 \text{ m/s}^2$) and total distance fallen downward would be $\frac{1}{2}gt^2$ or $5t^2$.

Now in 1 s, it will fall 5 m, in 2 s, it will fall 20 m, in 3 s, it will fall 45 m and in 4 s, it will fall 80 m and so on.

– In Ist second, it will fall 5 m.

– In 2nd second it will fall $(20 - 5)$ m = 15 m

– In 3rd second, it will fall $(45 - 20)$ m = 25 m and so on.

The ratio of these distances would be 5m : 15 m : 25 m or 1 : 3 : 5.

This result can be generalised with every constant acceleration problems for any equal time interval (not necessarily 1s), if $u = 0$.

$s_{0\to t} : s_{t\to 2t} : s_{2t\to 3t} = 1:3:5$ (if $u = 0$ and a = constant)

$g = 10\ \text{m/s}^2$ (downwards) means in every second velocity changes by 10 m/s in downward direction.

For example, if a particle is projected upwards by 35 m/s, then $t_{up} = \frac{35}{10} = 3.5$ s

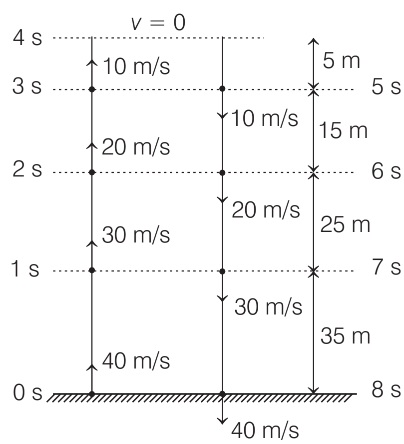
because in every second, velocity is changing by 10 m/s in downward direction. At $t = 0$, $v = 35$ m/s (upwards), at $t = 1$ s, it will remain 25 m/s (upwards). So, it will take total 3.5 s to become zero.

If air resistance is neglected, then $t_{down} = 3.5$ s or $t_{total} = 7$ s

- **Second's diagram** Suppose a particle is projected upwards with 40 m/s, then $t_{up} = 4$ s and t_{down} is also equal to 4s.

$\therefore$ $t_{total} = 8$ s

Now, let us make a diagram showing the position and velocity at every second.



One Dimensional Motion with Non-uniform Acceleration

- s-$t \to v$-$t \to a$-$t \to$ Differentiation
- a-$t \to v$-$t \to s$-$t \to$ Integration
- **Equations of differentiation**

$$v = \frac{ds}{dt} \quad \text{and} \quad a = \frac{dv}{dt} = v \cdot \frac{dv}{ds}$$

- **Equations of integration**

$$\int ds = \int v dt, \int dv = \int a dt, \int v dv = \int a ds$$

In first integration equation, v should be either a constant or a function of t.

In second equation, a should be either a constant or a function of t. Similarly, in third equation, a should be either a constant or a function of s.

Two or Three Dimensional Motion with Uniform Acceleration

(i) $\mathbf{v} = \mathbf{u} + \mathbf{a}t$ (ii) $\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$ (iii) $\mathbf{v} \cdot \mathbf{v} = \mathbf{u} \cdot \mathbf{u} + 2\mathbf{a} \cdot \mathbf{s}$

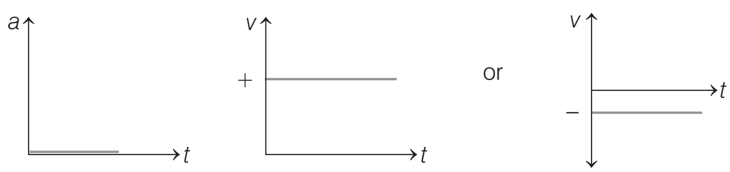
Two or Three Dimensional Motion with Non-uniform Acceleration

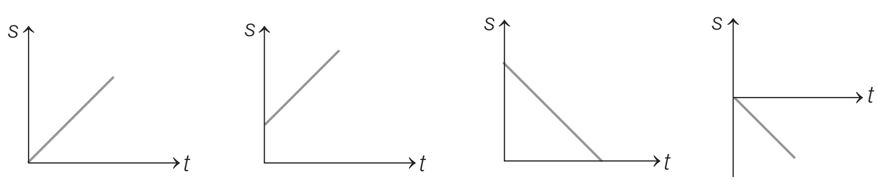
(i) $\mathbf{v} = \frac{d\mathbf{s}}{dt}$ or $\frac{d\mathbf{r}}{dt}$ (ii) $\mathbf{a} = \frac{d\mathbf{v}}{dt}$

(iii) $\int d\mathbf{v} = \int \mathbf{a} \cdot dt$ (iv) $\int d\mathbf{s} = \int \mathbf{v} \cdot dt$

Graphs

- Slope of s - t graph = velocity
  Slope of v - t graph = acceleration
  Area under v - t graph = displacement and
  Area under a - t graph = change in velocity.
- **Uniform motion** v = constant, $a = 0, s = vt$

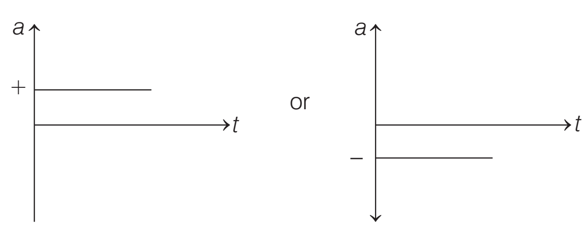


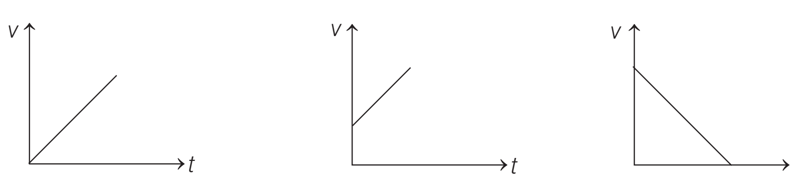


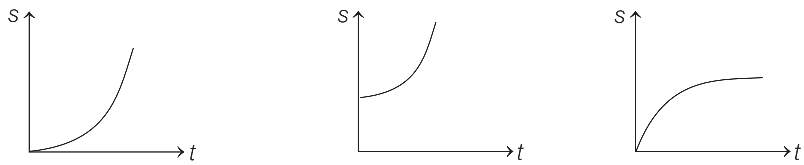
Since $a = 0$, therefore slope of v-t graph = 0
Further v = constant, therefore slope of s - t graph = constant.

- **Uniformly accelerated or retarded motion**

a = constant, $v = u \pm at$, $s = ut \pm \frac{1}{2}at^2$



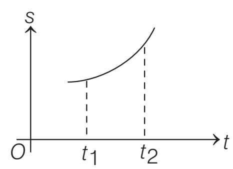




Since a = constant, therefore slope of v - t graph = constant.

Further, v is increasing or decreasing, therefore slope of s - t graph should either increase or decrease.

Note *From the given s-t graph, we can find sign of velocity and acceleration. For example, in the given graph, slope at* t_1 *and* t_2 *both are positive. Therefore,* v_{t_1} *and* v_{t_2} *are positive. Further, slope at* t_2 > *slope at* t_1. *Therefore,* $v_{t_2} > v_{t_1}$. *Hence, acceleration of the particle is also positive.*



Relative Motion

- $\mathbf{v}_{AB}$ = velocity of A with respect to B

  $= \mathbf{v}_A - \mathbf{v}_B$
- $\mathbf{a}_{AB}$ = acceleration of A with respect to B

  $= \mathbf{a}_A - \mathbf{a}_B$

In one dimensional motion take a sign convention. Then,

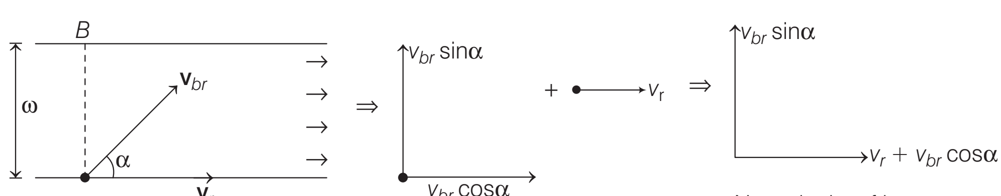
- $v_{AB} = v_A - v_B$
- $a_{AB} = a_A - a_B$

Uses of Relative Motion

- **Minimum distance or collision problems** When two bodies are in motion, the questions like, the minimum distance between them or the time when one body overtakes the other can be solved easily by the principle of relative motion.

  In these types of problems, one body is assumed to be at rest and the relative motion of the other body is considered.

  By assuming so, two body problem is converted into one body problem and the solution becomes easy.
- **River boat problems** In a general case, resolve $\mathbf{v}_{br}$ along the river and perpendicular to river as shown below.



Net velocity of boatman

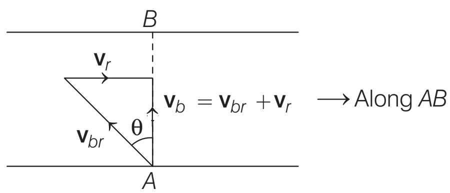
Now, the boatman will cross the river with component of $\mathbf{v}_{br}$ perpendicular to river ($= v_{br} \sin\alpha$ in above case)

$$\therefore \qquad t = \frac{\omega}{v_{br} \sin\alpha}$$

To cross the river in minimum time, why to take help of component of $\mathbf{v}_{br}$ (which is always less than v_{br}), the complete vector $\mathbf{v}_{br}$ should be kept perpendicular to the river current. Due to the other component $v_r + v_{br} \cos\alpha$, boatman will drift along the river by a distance $x = (v_r + v_{br} \cos\alpha)$ (time)

To reach a point B, which is just opposite to the starting point A, net velocity of boatman $\mathbf{v}_b$ or the vector sum of $\mathbf{v}_r$ and $\mathbf{v}_{br}$ should be along AB.

The velocity diagram is as under



From the diagram, we can see that,

$$|\mathbf{v}_b| \text{ or } v_b = \sqrt{v_{br}^2 - v_r^2} \qquad \text{...(i)}$$

Time, $t = \dfrac{\omega}{v_b} = \dfrac{\omega}{\sqrt{v_{br}^2 - v_r^2}}$

Drift $x = 0$ and $\sin\theta = \dfrac{v_r}{v_{br}}$ or $\theta = \sin^{-1}\left(\dfrac{v_r}{v_{br}}\right)$

From Eq. (i), we can see that this case is possible, if $v_{br} > v_r$ otherwise, v_b is either zero or imaginary.

If the boatman rows his boat along the river (downstream), then net velocity of boatman will be $v_{br} + v_r$.

If he rows along the river upstream, then net velocity of boatman will be $v_{br} \sim v_r$.

Note *Aircraft wind problems can also be solved in the similar manner. Velocity of boatman w.r.t. river ($\mathbf{v}_{br}$) can be replaced by velocity of aircraft w.r.t. wind ($\mathbf{v}_{aw}$). Velocity of river ($\mathbf{v}_r$) can be replaced by velocity of wind ($\mathbf{v}_w$) and net velocity of boatman ($\mathbf{v}_b$) can be replaced by net velocity of aircraft ($\mathbf{v}_a$).*

- **Rain umbrella problems** A person should hold his umbrella in the direction of $\mathbf{v}_{rp}$ or $\mathbf{v}_r - \mathbf{v}_p$ or velocity of rain w.r.t. person.