SSC MATHEMATICS

Covers the entire syllabus of the following Entrance Tests:

- SSC CGL TIER I and TIER II
- SSC CPO for SI and ASI posts in: CRPF, ITBP, CBI, CISF, BSF, DP
- SSC CHSL
- SSC MTS
- SSC Constable (GD)
- Section Officer (Audit)
- Section Officer (Commercial Audit)
- FCI
- DMRC

The book comprises 2400+ questions with detailed explanations.

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BOOK COMPANY
From the house of
Dr. R.S. Aggarwal

RADIAN BOOK COMPANY

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ALTHOUGH THIS BOOK IS COMPLETE IN ITSELF FOR **SSC** ASPIRANTS, WE RECOMMEND THAT YOU CHECK OUR ONLINE RESOURCES THAT HAVE ALSO BEEN DESIGNED TO HELP YOU CLEAR THE TEST.

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PREFACE

I feel delighted to write the foreword of Radian Book Company's latest title, Mathematics for SSC (Staff Selection Commission).

When my team and I were planning this book, we decided to present to learners a ready reckoner, a test-practice tool, and last-minute-revision notes. In this endeavour, we were helped a lot by Dr RS Aggarwal who counselled us whenever required. Dr Aggarwal's vast experience–spanning over five decades in which he has authored 175+ academic bestsellers–is one of Radian Book Company's most invaluable assets.

Salient features of the book:

1. The entire syllabus is fully covered;
2. The theory required for each topic is presented in a crisp manner along with all important formulae;
3. All questions asked in the SSC entrance test in the past five years have been included;
4. It has been made sure that all new questions that have been created require application of theory (to ensure that learners can solve any variation that is asked in the test);
5. The book includes a comprehensive question set (2,400+ questions); and
6. The book contains detailed and easy-to-understand explanations and solutions of all questions.

While this book focuses on the SSC entrance test, it also covers the syllabi of CPO, CHSL, MTS, Police Constable, Section Officer (Audit), FCI, DMRC, along with other similar entrance tests.

I look forward to receiving your comments and critiques about this book and wish you a successful career ahead.

Preeti Aggarwal,

Founder & CEO,

Radian Book Company.

M: +91 98113 41569

E: preetiaggarwal@radianbooks.in

W: https://www.radianlearning.in

SYLLABUS

SSC CGL EXAMINATION PATTERN:

There are four levels in this examination.

- Tier I and Tier II—Computer Based Exams
- Tier III—Pen and Paper
- Tier IV—Computer Proficiency Test/Skill Test/Document Verification

SSC CGL TIER I

Tier I is an objective exam that will be held online.

Paper No	Subject	No of Questions	Max Marks
1.	General Awareness	25	50
2.	Quantitative Aptitude	25	50
3.	English Comprehension	25	50
4.	General Intelligence & Reasoning	25	50
	TOTAL	100	200

1. **DURATION OF EXAM: 60** minutes (100 minutes for visually handicapped).
2. **MARKING SCHEME: 2** marks for each right answer.
3. **NEGATIVE MARKING: 0.50** for each wrong answer will be deducted.

SSC TIER I SUBECT WISE SYLLABUS

General Intelligence	General Awareness	Quantitative Aptitude	English Comprehension
Classification	Static General Knowledge (Indian History, Culture etc.)	Simplification	Reading Comprehension
Analogy	General Science	Interest	Fill in the Blanks
Coding-Decoding	Current Affairs	Averages	Spellings
Puzzle	Sports	Percentage	Phrases and Idioms
Matrix	Books and Authors	Ratio and Proportion	One word Substitution
Word Formation	Important Schemes	Problem on Ages	Sentence Correction
Venn Diagram	Portfolios	Speed, Distance and Time	Error Spotting
Direction and Distance	People in News	Number System	
Blood Relations		Mensuration	

General Intelligence	General Awareness	Quantitative Aptitude	English Comprehension
Series		Data Interpretation	
Verbal reasoning		Time and Work	
Non-Verbal Reasoning		Algebra	
	Trigonometry		
	Geometry		

SSC CGL TIER II

Tier II exam is an objective exam which will be also conducted online.

Paper No.	Subject	No. of Questions	Max Marks	Exam Duration	Negative Marking
1.	Quantitative Aptitude	100	200	2 hours	0.5 for each wrong question
2.	English Language & Comprehension	200	200	2 hours	0.25 for each wrong question
3.	Statistics	100	200	2 hours	0.5 for each wrong question
4.	General Studies (Finance and Economics)	100	200	2 hours	0.5 for each wrong question

- **Paper 1 and 2** are compulsory for all posts.
- **Paper 3** is only for the Statistical Investigator Gr. II and Compiler.
- **Paper 4** is only for the post of "Assistant Audit Officer".
- Level for Paper 1-10th to Standard 12th, Paper-2 of 10 + 2 level and Papers-3 & 4-Graduation Level.

SSC CGL TIER II SUBJECT WISE SYLLABUS:

Quantitative Aptitude	English Language	Statistics	General Awareness
Simplification	Reading Comprehension	Collection and Representation	Finance and of Data Accounting
Interest	Spelling	Measure of Dispersion	Fundamental Principles
Averages	Fill in the Blanks	Measure of Central Tendency	Financial Accounting
Percentage	Phrases and Idioms	Moments, Skewness and Kurtosis	Basic Concepts of Accounting
Ratio and Proportion	One Word Substitution	Correlation and Regression	Self-Balancing Ledger
Speed, Distance and Time	Sentence Correction	Random Variables	Error Spotting and Correction
Number System	Error Spotting	Random Variables	Economics and Governance
Mensuration	Cloze Test	Sampling Theory	Comptroller and Auditor General of India

Quantitative Aptitude	English Language	Statistics	General Awareness
Data Interpretation	Para Jumbles	Analysis and Variance	Finance Commission
Time and Work	Synonyms-Antonyms	Time Series Analysis	Theory of Demand and Supply
Algebra	Active-Passive Voice	Index Number	
Trigonometry			
Geometry			
Data Sufficiency			

TIER III EXAM

This is a Descriptive Exam which tests candidates writing skills in English or Hindi. It is held offline. Candidates are required to write an Essay/letter/application.

- **Duration of exam**-60 minutes (80 minutes for visually handicapped)
- Carries 100 marks.
- 10 + 2 level questions.

SSC CGL TIER IV (SKILL TEST)

Tier IV exam checks couple of skill tests which are necessary for certain posts.

1. DEST (data entry speed test): It is a qualifying test for all the posts except Compiler. It is held to test the typing speed of the candidate. In this test the candidates are given printed material in English which they should type accurately on a computer with a minimum of 2000 key depressions. The duration of this test is 15 minutes.
2. CPT (Computer proficiency Test): SSC will hold CPT, in three modules –spread sheet, word processing and generation of slides. It will be applicable for applicants of Asst. Section Officer in CSS, MEA, SFIO, AFHQ posts. It is a qualifying test.

1

NUMBER SYSTEM

THEORY AND FORMULAE (TF)

TF 1. Numbers:

In Hindu – Arabic System, we use ten digits, namely 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9.

A number is denoted by a group of digits, called **numeral.**

The place-value chart for a Hindu – Arabic Numeral is as follows:

Ten Crores	Crores	Ten Lakhs	Lakhs	Ten Thousands	Thousands	Hundreds	Tens	Ones

TF 2. Face Value and Place Value (or Local Value) of a Digit in a Numeral:

(*i*) The face value of a digit in a numeral is its own value, at whatever place it may be.

(*ii*) The place value of a digit is its value by virtue of its place in the numeral. The place value of a digit is obtained by multiplying its face value to the value of its place in the numeral.

Ex. The place value of 5 in 275328 is $5 \times 1000 = 5000$.

Note: The place value of 0 in any numeral is 0, at whatever place it may be.

TF 3. Types of Numbers:

(*i*) **Natural Numbers:** Counting numbers namely 1, 2, 3, 4, ... are called natural numbers.

(*ii*) **Whole Numbers:** All natural numbers together with 0, form the set of whole number. Thus, 0, 1, 2, 3, 4, ... are whole numbers.

(*iii*) **Integers:** All counting numbers, zero and negatives of counting numbers, together form the set of integers. Thus, ..., – 3, – 2, – 1, 0, 1, 2, 3, ... are all integers.

(*iv*) **Rational Numbers:** The numbers which can be expressed in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$ are called rational numbers.

Ex. $\frac{-3}{-5}, \frac{-4}{7}, \frac{1}{-3}, \frac{11}{8}, 0, -1, 7\frac{2}{3}$ etc. are all rational numbers.

Note: Rational numbers when expressed in decimals are either in terminating or repeating form.

(*v*) **Irrational Numbers:** A number which can neither be expressed as a terminating decimal nor as a repeating decimal, is called an irrational number.

Ex. $\sqrt{2}, \sqrt{3}, \sqrt{6}, \pi, e, 2+\sqrt{3}, 10.2376134$ ... etc. are all irrational numbers.

(*vi*) **Real Numbers:** A number whose square is non-negative, is called a real number. All rational and irrational numbers form the collection of all real numbers.

(*vii*) **Complex (or Imaginary) Numbers:** A number whose square is negative is called a complex number.

TF 4. Various Types of Natural Numbers:

(*i*) **Even Numbers:** Counting numbers which are divisible by 2 are called even numbers. Thus, 0, 2, 4, 6, 8, 10, 12, ... are all even numbers.

(*ii*) **Odd Numbers:** Counting numbers which are not divisible 2 are called odd numbers. Thus, 1, 3, 5, 7, 9, 11, ... are all odd numbers.

(*iii*) **Prime Numbers:** A natural number is said to be prime, if it has exactly two factors namely 1 and the number itself.

Ex. All prime numbers less than 100 are
2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89 and 97.

(*iv*) **Composite Numbers:** A natural number is said to be composite, if it has at least one factor other than 1 and the number itself. Thus, a composite number has more than 2 factors.

(*v*) **Square Numbers:** A number which is obtained by multiplying a natural number by itself, is called a square number.

Ex. 1, 4, 9, 16, 25, ... are all square numbers.
All square numbers end in either a 0, 1, 4, 5, 6 or 9.

(*vi*) **Co-primes (or Relative Primes):** Two numbers whose H.C.F. is 1 are called co-prime numbers.

Ex. (2, 5), (4, 11), (9, 10) etc. are pairs of co-primes.

(*vii*) **Perfect Numbers:** A number, the sum of whose factors (except the number itself) is equal to the number, is called a perfect number.

Ex. (*a*) The factors of 6 are 1, 2 and 3. And, 1 + 2 + 3 = 6.
(*b*) The factors of 28 are 1, 2, 4, 7, 14. And, 1 + 2 + 4 + 7 + 14 = 28.
(*c*) The factors of 496 are 1, 2, 4, 8, 16, 31, 62, 124 and 248. And, 1 + 2 + 4 + 8 + 16 + 31 + 62 + 124 + 248 = 496.
Thus, 6, 28, 496 etc. are perfect numbers.

(*viii*) **Triangular Number:** Triangular numbers are those which show a pattern of dots forming equilateral triangles.

The triangular numbers form a sequence 1, 3, 6, 10, 15, ... whose *n*th term $T_n = \frac{n(n+1)}{2}$.

(*ix*) **Pythagorean Triples:** A pythagorean triple consists of three positive integers *a*, *b*, and *c*, such that $a^2 + b^2 = c^2$. A right triangle whose sides form a Pythagorean triple is called a **Pythagorean Triangle.** In a Pythagorean triple (*a*, *b*, *c*), if *a*, *b* and *c* are coprime, then it is called a **Primitive Pythagorean Triple.**

Euclid's formula for generating Pythagorean triples: For arbitrary integers *m* and *n* s.t. $m > n > 0$; the Pythagorean triple (*a*, *b*, *c*) is given by $a = m^2 - n^2$, $b = 2mn$, $c = m^2 + n^2$.

Here the triple will be primitive if *m* and *n* are coprime and not both odd.

TF 5. **Twin Primes:** A pair of prime numbers which differ by 2.

Ex. Pairs of twin primes are
(3, 5), (5, 7), (11, 13), (17, 19), (29, 31), (41, 43), (59, 61), (71, 73) etc.

Note: Every twin prime pair except (3, 5) is of the form $(6k - 1, 6k + 1)$ where $k \in N$.

TF 6. **Mersenne Prime:** A prime number of the form $2^p - 1$ where *p* is a prime number.

Ex. 3, 7, 31, 127, ... are Mersenne Primes.

In general, numbers of the form $M_n = 2^n - 1$ where $n \in N$ are all called **Mersenne numbers.**

If *n* is a composite number, then $M_n = 2^n - 1$ is always composite.

If however *n* is a prime number, then numbers $M_n = 2^n - 1$ are called **Pernicious.**

Mersenne numbers: The smallest Prime number *n* for which $M_n = 2^n - 1$ is composite is 11.

Thus, smallest composite Pernicious Mersenne number is $M_n = 2^{11} - 1 = 2047 = 23 \times 89$.

Next is for *n* = 23. The Mersenne Primes are obtained for *n* = 2, 3, 5, 7, 13, 17, ...

TF 7. **Primality Test (Test for a number to be Prime):** To check whether a given natural number *n* is prime or not, we divide it by each prime number *m* from 2 to $\sqrt{n}$. If *n* is completely divisible by any *m*, then it is not prime.

TF 8. **Tests of divisibility**

(*i*) **Divisibility by 2:** A number is divisible by 2 if its units digit is any of 0, 2, 4, 6, 8.

(*ii*) **Divisibility by 3:** A number is divisible by 3 if the sum of its digits is divisible by 3.

(*iii*) **Divisibility by 4:** A number is divisible by 4 if the number formed by its last two digits is divisible by 4.

(*iv*) **Divisibility by 5:** A number is divisible by 5 if its unit digit is either 0 or 5.

(*v*) **Divisibility by 7:** Divide the number into groups of 3 digits (starting from right) and find the difference between the sum of the numbers in odd and even places. If the difference is 0 or divisible by 7, then the given number is divisible by 7.

Ex. 61870368 $\rightarrow$ 61 / 870 / 368

We have: 870 – (61 + 368) = 870 – 429 = 441 Which is divisible by 7 and so 61870368 is divisible by 7.

Another Test: Take the last digit off the given number, double it and subtract the double *d* number from the remaining number. If the result is either 0 or divisible by 7, then the given number is divisible by 7.

Ex. 2401 is divisible by 7

if (240 – 2) i.e. 238 is divisible by 7

i.e. if (23 – 16) i.e. 7 is divisible by 7 and that is true.

$\therefore$ 2401 is divisible by 7.

(*vi*) **Divisibility by 8:** A number is divisible by 8 if the number formed by its last three digits is divisible by 8.

(*vii*) **Divisibility by 9:** A number is divisible by 9 if the sum of its digits is divisible by 9.

(*viii*) **Divisibility by 10:** A number is divisible by 10 if its last digit is 0.

(*ix*) **Divisibility by 11:** A number is divisible by 11 if the difference between the sum of its digits at odd places and the sum of its digits at even places is either 0 or divisible by 11.

Ex. Let the given number be 34716. We have:

(Sum of digits at odd places) – (Sum of digits at even places)

= (3 + 7 + 6) – (4 + 1) = 16 – 5 = 11 which is divisible by 11.

$\therefore$ 34716 is divisible by 11.

(*x*) **Divisibility by 13:** Divide the given number into groups of 3 digits (starting from right) and find the difference between the sum of the numbers in odd and even places. If the difference is either 0 or divisible by 13, then the given number is divisible by 13.

Ex. 4061772 $\rightarrow$ 4 / 061 / 772

(4 + 772) – 061 = 776 – 061 = 715 which is divisible by 13.

$\therefore$ 4061772 is divisible by 13.

(*xi*) **Divisibility by 16:** A number is divisible by 16, if the number formed by its last 4 digits is divisible by 16.

(*xii*) **Divisibility by 25:** A number is divisible by 25 if the number formed by its last two digits is either 00 or divisible by 25.

(*xiii*) A number is divisible

(*a*) by 6: if it is divisible by both 2 and 3.

(*b*) by 12: if it is divisible by both 3 and 4.

(*c*) by 14: if it is divisible by both 2 and 7.

(*d*) by 15: if it is divisible by both 3 and 5.

(*e*) by 18: if it is divisible by both 2 and 9.

(*f*) by 24: if it is divisible by both 3 and 8.

(*g*) by 40: if it is divisible by both 5 and 8.

(*h*) by 80: if it is divisible by both 5 and 16.

Rule: If a number is divisible by p as well as q where p and q are coprimes, then the given number is divisible by (pq).

TF 9. Some Important Theorems:

(*i*) Product of n consecutive positive integers is always divisible by $n!$.

Also, $n!$ is the largest number by which the product of n consecutive positive integers is always divisible.

(*ii*) **Fundamental Theorem of Arithmetic:**

Every natural number greater than 1 is uniquely expressible as the product of prime numbers.

Ex. The Prime Factorisation of 5040 gives $5040 = 2^4 \times 3^2 \times 5 \times 7$.

This is also called **Unique Prime Factorisation Theorem.**

(*iii*) **Number of Divisors of a Number:**

The number of divisors of a number $N = p_1^{a_1} \cdot p_2^{a_2} \cdots p_r^{a_r}$ is given by $(a_1 + 1)(a_2 + 1) \ldots (a_r + 1)$ where $p_1, p_2, \ldots p_r$ are distinct prime numbers and $a_1, a_2, \ldots, a_r$ are the positive integral powers of $p_1, p_2, \ldots, p_r$ respectively.

Ex. We have: $96 = 2^5 \times 3^1$ [By Prime Factorisation]

$\therefore$ Number of divisors of 96 is $(5 + 1) \times (1 + 1) = 6 \times 2 = 12$.

Note: A square number always has an odd number of divisors.

(*iv*) **Division Algorithm or Euclidean Algorithm:**

Dividend = (Divisor × Quotient) + Remainder.

(*v*) **Division of Algebraic Expressions:**

(*a*) $x^n - y^n$ is divisible by $x - y$ for any natural number n.

(*b*) $x^n - y^n$ is divisible by $x + y$ if n is an even natural number.

(*c*) $x^n + y^n$ is divisible by $x + y$ if n is an odd natural number.

(*d*) $x^n + y^n$ is never divisible by $x - y$ and $x + y$ if n is an even natural number.

(*vi*) Highest power of p in $n! = \left[\frac{n}{p}\right] + \left[\frac{n}{p^2}\right] + \left[\frac{n}{p^3}\right] + \ldots + \left[\frac{n}{p^r}\right]$, where $p^r \le n \le p^{r+1}$ and $[x]$ is the greatest integer function.

TF 10. Some Important Formulae:

(*i*) $(a + b)^2 = a^2 + b^2 + 2ab$

(*ii*) $(a - b)^2 = a^2 + b^2 - 2ab$

(*iii*) $(a + b)^2 + (a - b)^2 = 2(a^2 + b^2)$

(*iv*) $(a + b)^2 - (a - b)^2 = 4ab$

(*v*) $a^2 - b^2 = (a + b)(a - b)$

(*vi*) $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

(*vii*) $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$

(*viii*) $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$

(*ix*) $a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$

(*x*) $a^3 - b^3 = (a - b)(a^2 + b^2 + ab)$

TF 11. Congruence of Numbers and Congruent Modulo:

If an integer 'n' when divided by any positive integer 'a' gives quotient q and remainder r, then

We have: $n = aq + r$ [By Euclidean Algorithm]

The integer 'a' is called the modulus and we write $n \equiv r \pmod{a}$.

If $m \equiv r \pmod{a}$ and $n \equiv r \pmod{a}$; then $(m - n) \equiv 0 \pmod{a}$

[**Note:** This means both m and n leave remainder r when divided by a]

and we say that m and n are congruent with respect to the same modulus 'a'.

Thus, for a positive integer 'a', two integers m and n are said to be congruent modulo a; written as $m \equiv n \pmod{a}$, if both m and n leave the same remainder when divided by 'a' or equivalently, if the difference of m and n is a multiple of a. The number 'a' is called the modulus of the congruence.

Ex. $313 \equiv 65 \pmod 4$ since both 313 and 65 leave a remainder 1 when divided by 4 and $313 - 65 = 248$ is divisible by 4.

EXERCISE

1. How many numbers are there from 700 to 950 (including both) which are neither divisible by 3 nor by 7? **[SSC CGL 2017]**

(*a*) 107 (*b*) 141
(*c*) 144 (*d*) 145

2. What least value which should be added to 1812 to make it divisible by 7, 11 and 14? **[SSC CGL 2017]**

(*a*) 12 (*b*) 36
(*c*) 72 (*d*) 154

3. The sum of four numbers is 48. When 5 and 1 are added to the first two and 3 and 7 are subtracted from the 3rd and 4th, the numbers will be equal. The numbers are **[SSC CGL 2015]**

(*a*) 2, 12, 14, 20 (*b*) 5, 11, 13, 19
(*c*) 6, 10, 14, 18 (*d*) 9, 7, 15, 17

4. In a division sum, the divisor is 3 times the quotient and 6 times the remainder. If the remainder is 2, then the dividend is **[SSC 2013]**

(*a*) 28 (*b*) 36
(*c*) 48 (*d*) 50

5. A number when divided by 2736 leaves the remainder 75. If the same number is divided by 24, then the remainder is **[SSC CGL 2014 & 2015]**

(*a*) 0 (*b*) 3
(*c*) 12 (*d*) 23

6. If a number is as much greater than 31 as it is less than 75, then the number is **[SSC 2013]**
(*a*) 47 (*b*) 53
(*c*) 74 (*d*) 106

7. Which one of the numbers is divisible by 25? **[SSC CGL 2013]**
(*a*) 22040 (*b*) 303310
(*c*) 303375 (*d*) 373355

8. When 335 is added to $5A7$, the result is $8B2$. $8B2$ is divisible by 3. What is the largest possible value of A? **[SSC CGL T-II 2013]**
(*a*) 1 (*b*) 2
(*c*) 4 (*d*) 8

9. The maximum value of F in the following equation $5E9 + 2F8 + 3G7 = 1114$ is (where E, F, G each stands for any digit) **[SSC CPO 2015]**
(*a*) 5 (*b*) 7
(*c*) 8 (*d*) 9

10. When an integer K is divided by 3, the remainder is 1 and when $K + 1$ is divided by 5, the remainder is 0. Of the following, a possible value of K is **[SSC 2011]**
(*a*) 65 (*b*) 64
(*c*) 63 (*d*) 62

11. The least number of five digits which has 123 as a factor is **[SSC DP 2012]**
(*a*) 10037 (*b*) 10063
(*c*) 10081 (*d*) 10086

12. The difference between the greatest and the least four-digit numbers that begin with 3 and end with 5 is **[SSC 2015]**
(*a*) 900 (*b*) 909
(*c*) 990 (*d*) 999

13. The difference between the greatest and the least prime numbers which are less than 100 is **[SSC 2015]**
(*a*) 94 (*b*) 95
(*c*) 96 (*d*) 97

14. A number x when divided by 289 leaves 18 as the remainder. The same number when divided by 17 leaves y as a remainder. The value of y is **[SSC CGL 2013]**
(*a*) 1 (*b*) 2
(*c*) 3 (*d*) 5

15. The number which can be written in the form of $n(n + 1)(n + 2)$, where n is a natural number is **[SSC CGL 2015]**
(*a*) 3 (*b*) 5
(*c*) 6 (*d*) 7

16. If two numbers are each divided by the same divisor, then the remainders are respectively 3 and 4. If the sum of the two numbers be divided by the same divisor, then the remainder is 2. The divisor is
(*a*) 3 (*b*) 5
(*c*) 7 (*d*) 9

17. When two numbers are separately divided by 33, the remainders are 21 and 28, respectively. If the sum of the two numbers is divided by 33, then the remainder will be **[SSC 2010]**
(*a*) 16 (*b*) 14
(*c*) 12 (*d*) 10

18. A number when divided by 361 gives a remainder 47. If the same number is divided by 19, then the remainder obtained is **[SSC CGL 2015]**
(*a*) 1 (*b*) 3
(*c*) 8 (*d*) 9

19. 5349 is added to 3957. Then 7062 is subtracted from the sum. The result is not divisible by **[SSC 2014]**
(*a*) 3 (*b*) 4
(*c*) 7 (*d*) 11

20. The least number which must be added to the greatest number of 4 digits in order that the sum may be exactly divisible by 307 is **[SSC CGL 2013 & 2014]**
(*a*) 32 (*b*) 43
(*c*) 175 (*d*) 132

21. How many numbers between 400 and 800 are divisible by 4, 5 and 6? **[SSC GD 2013]**
(*a*) 10 (*b*) 9
(*c*) 8 (*d*) 7

22. If m and n are positive integers and $(m - n)$ is an even number, then $(m^2 - n^2)$ will be always divisible by **[SSC CGL 2012]**
(*a*) 12 (*b*) 8
(*c*) 6 (*d*) 4

23. Two numbers 11284 and 7655, when divided by a certain number of three digits, leave the same remainder. The sum of digits of such a three-digit number is **[SSC 2013]**
(*a*) 11 (*b*) 10
(*c*) 9 (*d*) 8

24. The number 323 has **[SSC CGL 2013]**
(*a*) No prime factors
(*b*) Two prime factors
(*c*) Three prime factors
(*d*) Five prime factors

25. Among the following statements, the statement which is not correct is **[SSC 2015]**
(*a*) Every natural number is an integer.
(*b*) Every natural number is a real number.
(*c*) Every real number is a rational number.
(*d*) Every integer is a rational number.

26. If the sum of two numbers be multiplied by each number separately, the products so obtained are 247 and 114. The sum of the numbers is **[SSC CGL 2011]**
(*a*) 23 (*b*) 21
(*c*) 20 (*d*) 19

27. If doubling a number and adding 20 to the result gives the same answer as multiplying the number by 8 and taking away 4 from the product, the number is
(*a*) 6 (*b*) 4
(*c*) 3 (*d*) 2

28. If the sum of five consecutive integers is S, then the largest of those integers in terms of S is **[SSC 2011]**
(*a*) $\frac{S+4}{4}$ (*b*) $\frac{S+5}{4}$
(*c*) $\frac{S+10}{5}$ (*d*) $\frac{S-10}{5}$

29. If n is even, $(6^n - 1)$ is divisible by **[SSC 2014]**
(*a*) 6 (*b*) 30
(*c*) 35 (*d*) 37

30. When n is divided by 6, the remainder is 4. When $2n$ is divided by 6, the remainder is **[SSC 2013]**
(*a*) 0 (*b*) 1
(*c*) 2 (*d*) 4

31. If the sum of the digits of any integer lying between 100 and 1000 is subtracted from the number, then the result is always **[SSC 2013]**
(*a*) Divisible by 2 (*b*) Divisible by 5
(*c*) Divisible by 6 (*d*) Divisible by 9

32. Which one of the following will completely divide $5^{71} + 5^{72} + 5^{73}$? **[SSC CGL 2011]**
(*a*) 30 (*b*) 150
(*c*) 155 (*d*) 160

33. $2^{16} - 1$ is divisible by **[SSC CGL 2011]**
(*a*) 19 (*b*) 17
(*c*) 13 (*d*) 11

34. Which of the following numbers will always divide a six-digit number of the form *xyxyxy* (where $1 \le x \le 9$, $1 \le y \le 9$)? **[SSC 2011]**
(*a*) 1010 (*b*) 10101
(*c*) 11010 (*d*) 11011

35. Find the largest number, which exactly divides every number of the form $(n^3 - n)(n - 2)$ where n is a natural number greater than 2.
(*a*) 6 (*b*) 12
(*c*) 24 (*d*) 48

36. If n is an integer, then $(n^3 - n)$ is always divisible by **[SSC 2010]**
(*a*) 4 (*b*) 5
(*c*) 6 (*d*) 7

37. Both the end digits of a 99-digit number N are 2. N is divisible by 11, then all the middle digits are **[SSC FCI 2012]**
(*a*) 4 (*b*) 3
(*c*) 2 (*d*) 1

38. The expression $2^{6n} - 4^{2n}$, where n is a natural number is always divisible by **[SSC 2011]**
(*a*) 48 (*b*) 36
(*c*) 18 (*d*) 15

39. When 2^{31} is divided by 5, the remainder is **[SSC CGL 2011]**
(*a*) 1 (*b*) 2
(*c*) 3 (*d*) 4

40. It is given that $(2^{32} + 1)$ is exactly divisible by a certain number, which one of the following is also definitely divisible by the same number?
(*a*) $2^{16} + 1$ (*b*) $2^{16} - 1$
(*c*) $2^{96} + 1$ (*d*) 7×2^{33}

41. The unit digit in the sum of $(124)^{372} + (124)^{373}$ is **[SSC CGL 2011]**
(*a*) 0 (*b*) 2
(*c*) 4 (*d*) 5

42. The last digit of 3^{40} is **[SSC 2012]**
(*a*) 1 (*b*) 3
(*c*) 7 (*d*) 9

43. The digit in units place of the product $49237 \times 3995 \times 738 \times 83 \times 9$ is **[SSC 2014]**
(*a*) 0 (*b*) 5
(*c*) 6 (*d*) 7

44. The digit in the unit's place of $[(251)^{98} + (21)^{29} - (106)^{100} + (705)^{35} - 16^4 + 259]$ is
(*a*) 6 (*b*) 5
(*c*) 4 (*d*) 1

45. The unit digit in the product $(122)^{173}$ is **[SSC CGL 2011]**
(*a*) 8 (*b*) 6
(*c*) 4 (*d*) 2

46. The digit in unit's place of the product $81 \times 82 \times 83 \times ... \times 89$ is
(*a*) 8 (*b*) 6
(*c*) 2 (*d*) 0

47. The units digit in the product $7^{71} \times 6^{63} \times 3^{65}$ is **[SSC MTS 2011]**
(*a*) 4 (*b*) 3
(*c*) 2 (*d*) 1

48. $(3^{25} + 3^{26} + 3^{27} + 3^{28})$ is divisible by
(*a*) 30 (*b*) 25
(*c*) 16 (*d*) 11

49. If n is a whole number greater than 1, then $n^2(n^2 - 1)$ is always divisible by
(*a*) 8 (*b*) 10
(*c*) 12 (*d*) 16

50. If $(7^{19} + 2)$ is divided by 6, then the remainder is
(*a*) 1 (*b*) 2
(*c*) 3 (*d*) 5

51. The number 64329 is divided by a certain number. While dividing, the numbers, 175,114 and 213 appear as three successive remainders. The divisor is

(*a*) 296 (*b*) 234
(*c*) 224 (*d*) 184

52. A number when divided successively by 4 and 5 leaves remainders 1 and 4 respectively. When it is successively divided by 5 and 4, the respective remainders will be
(*a*) 1, 2 (*b*) 2, 3
(*c*) 3, 2 (*d*) 4, 1

53. If 17^{200} is divided by 18, then the remainder is
(*a*) 1 (*b*) 2
(*c*) 16 (*d*) 17

54. The sum of first 60 numbers from 1 to 60 is divisible by
(*a*) 61 (*b*) 60
(*c*) 59 (*d*) 13

55. The least number, which must be added to 6709 to make it exactly divisible by 9 is
(*a*) 2 (*b*) 4
(*c*) 5 (*d*) 7

56. A six-digit number is formed by repeating a three-digit number; for example, 256, 256 or 678, 678, etc. Any number of this form is always exactly divisible by
(*a*) 7 only (*b*) 11 only
(*c*) 13 only (*d*) 1001

57. The sum of all those prime numbers which are not greater than 17 is **[SSC GD 2012]**
(*a*) 41 (*b*) 42
(*c*) 58 (*d*) 59

58. '*a*' divides 228 leaving a remainder 18. The biggest two-digit value of '*a*' is **[SSC 2013]**
(*a*) 21 (*b*) 30
(*c*) 35 (*d*) 70

59. If *a* and *b* are odd numbers, then which of the following is even? **[SSC CGL 2011]**
(*a*) $a + b + 1$ (*b*) $a + b - 1$
(*c*) $a + b + ab$ (*d*) $a + b + 2ab$

60. Find the maximum number of trees which can be planted, 20 metres apart, on the two sides of a straight road which is 1760 metres long. **[SSC CGL 2013]**
(*a*) 174 (*b*) 176
(*c*) 178 (*d*) 180

61. The numbers 1, 3, 5, 7, ..., 99 and 128 are multiplied together. The number of zeros at the end of the product must be
(*a*) 7 (*b*) 19
(*c*) 22 (*d*) Nil

62. In a two-digit number if it is known that its units digit exceeds its tens digit by 2 and that the product of the given number and the sum of its digits is equal to 144, then the number is
(*a*) 24 (*b*) 26
(*c*) 42 (*d*) 46

63. In a three-digit number, the digit at the hundred's place is two times the digit at the unit's place and the sum of the digits is 18. If the digits are reversed, the number is reduced by 396. The difference of hundred's and ten's digit of the number is **[SSC 2011]**
(*a*) 5 (*b*) 3
(*c*) 2 (*d*) 1

64. A number consists of two digits and the digit in the ten's place exceeds that in the unit's place by 5. If 5 times the sum of the digits be subtracted from the number, then the digits of the number are reversed. Then the sum of digits of the number is **[SSC 2011]**
(*a*) 7 (*b*) 9
(*c*) 11 (*d*) 13

65. If the digits in the unit and the ten's places of a three-digit number are interchanged, a new number is formed, which is greater than the original number by 63. Suppose the digit in the unit place of the original number be *x*. Then, all the possible values of *x* are **[SSC 2011]**
(*a*) 0, 1, 2 (*b*) 1, 2, 8
(*c*) 2, 7, 9 (*d*) 7, 8, 9

66. Which one of the following is a factor of the sum of first 25 natural numbers? **[SSC CPO 2010]**
(*a*) 12 (*b*) 13
(*c*) 24 (*d*) 26

67. A number divided by 68 gives the quotient 269 and remainder 0. If the same number is divided by 67, then the remainder is
(*a*) 0 (*b*) 1
(*c*) 2 (*d*) 3

ANSWERS

1. (*c*)	**2.** (*b*)	**3.** (*c*)	**4.** (*d*)	**5.** (*b*)	**6.** (*b*)	**7.** (*c*)	**8.** (*c*)	**9.** (*d*)	**10.** (*b*)
11. (*d*)	**12.** (*c*)	**13.** (*d*)	**14.** (*a*)	**15.** (*c*)	**16.** (*b*)	**17.** (*a*)	**18.** (*d*)	**19.** (*c*)	**20.** (*d*)
21. (*d*)	**22.** (*d*)	**23.** (*a*)	**24.** (*b*)	**25.** (*c*)	**26.** (*d*)	**27.** (*b*)	**28.** (*c*)	**29.** (*c*)	**30.** (*c*)
31. (*d*)	**32.** (*c*)	**33.** (*b*)	**34.** (*b*)	**35.** (*c*)	**36.** (*c*)	**37.** (*a*)	**38.** (*a*)	**39.** (*c*)	**40.** (*c*)
41. (*a*)	**42.** (*a*)	**43.** (*a*)	**44.** (*c*)	**45.** (*d*)	**46.** (*d*)	**47.** (*a*)	**48.** (*a*)	**49.** (*c*)	**50.** (*c*)
51. (*b*)	**52.** (*b*)	**53.** (*a*)	**54.** (*a*)	**55.** (*c*)	**56.** (*d*)	**57.** (*c*)	**58.** (*d*)	**59.** (*d*)	**60.** (*c*)
61. (*a*)	**62.** (*a*)	**63.** (*c*)	**64.** (*b*)	**65.** (*d*)	**66.** (*b*)	**67.** (*b*)			

SOLUTIONS

1. Numbers divisible by 3 are 702, 705, 708, ... , 948.

Let 948 be (n_1) th term of the A.P.

Then, $948 = 702 + (n_1 - 1)3 \Rightarrow n_1 - 1 = 82 \Rightarrow n_1 = 83$.

Numbers divisible by 7 are 700, 707, 714, ... , 945.

Let 945 be (n_2) th term of the A.P.

Then, $945 = 700 + (n_2 - 1)7 \Rightarrow n_2 - 1 = 35 \Rightarrow n_2 = 36$.

Numbers divisible by LCM (3,7) i.e. by 21 are 714, 735, 756, ... , 945. Let 945 be (n_3) th term of this A.P.

Then, $945 = 714 + (n_3 - 1)21 \Rightarrow n_3 - 1 = 11 \Rightarrow n_3 = 12$.

∴ Number of numbers from 700 to 950 which are divisible by 3 or 7 $= n_1 + n_2 - n_3 = (83 + 36 - 12) = 107$.

Hence, required numbers = (950 – 700 + 1) – 107 = 144.

2. LCM (7,11,14) = 154.

$$\begin{array}{r|l} 154 & 1812 \,(11 \\ & 154 \\ \hline & 272 \\ & 154 \\ \hline & 118 \end{array}$$

∴ Required number to be added

= (154 – 118) = 36.

3. Let the four numbers be a, b, c and d.

Then, $a + 5 = b + 1 = c - 3 = d - 7 = m$ [Say]

$\Rightarrow a = m - 5,\ b = m - 1,\ c = m + 3,\ d = m + 7$.

Now, $a + b + c + d = 48$

$\Rightarrow m - 5 + m - 1 + m + 3 + m + 7 = 48$

$\Rightarrow 4m = 44 \Rightarrow m = 11$.

∴ $a = m - 5 = 11 - 5 = 6$, $b = m - 1 = 11 - 1 = 10$

$c = m + 3 = 11 + 3 = 14$, $d = m + 7 = 11 + 7 = 18$.

Hence, the numbers are 6, 10, 14 and 18.

4. Remainder = 2, Divisor = 6 × 2 = 12.

Divisor = 3 × Quotient $\Rightarrow$ Quotient $= \dfrac{\text{Divisor}}{3} = \dfrac{12}{3} = 4$.

∴ Dividend = Divisor × Quotient + Remainder

= 12 × 4 + 2 = 50.

5. Let the number be x.

On dividing x by 2736, let the quotient be k and the remainder be 75.

Then, $x = 2736k + 75 = 24 \times 114k + 24 \times 3 + 3 = 24\,(114k + 3) + 3$.

∴ When x is divided by 24, we get remainder = 3.

6. Let the number be x.

Then, $x - 31 = 75 - x \Rightarrow 2x = 106 \Rightarrow x = \dfrac{106}{2} = 53$.

7. Clearly, 303375 is divisible by 25.

[∵ A number is divisible by 25 if the last two digits of the number are 00 or 25 or 50 or 75.]

8. Since $8B2$ is divisible by 3, we have:

$8 + B + 2 = 12$ or $8 + B + 2 = 15$ or $8 + B + 2 = 18$

[∵ A number is divisible by 3 only when the sum of its digits is divisible by 3]

$\Rightarrow B = 2$ or $B = 5$ or $B = 8$.

∴ $1 + A + 3 = 2$ or $1 + A + 3 = 5$ or $1 + A + 3 = 8$

$\Rightarrow A = -2$ or $A = 1$ or $A = 4$.

Thus the largest possible value of $A = 4$.

$$\begin{array}{r} (1) \\ 5\,A\,7 \\ +\ 3\,3\,5 \\ \hline 8\,B\,2 \\ \hline \end{array}$$

9.

$$\begin{array}{rccc} & ① & ② & \\ & 5 & E & 9 \\ & 2 & F & 8 \\ + & 3 & G & 7 \\ \hline 1 & 1 & 1 & 4 \\ \hline \end{array}$$

Clearly, $2 + E + F + G = 11$.

∴ The maximum value of F can be (11 – 2) i.e. 9 (when $E = G = 0$).

10. On dividing 64 by 3 and 65 by 5, we get 1 and 0 as remainders respectively.

∴ The possible value of k is 64.

11. Least number of five digits = 10000.

On dividing 10000 by 123, the remainder is 37.

∴ Required number = 10000 + (123 – 37) = 10086.

[**Note :** This question is of the form: What should be added to 10000 to make it divisible by 123?]

12. The greatest four-digits number that begins with 3 and ends with 5 = 3995.

The least four-digits number that begin with 3 and ends with 5 = 3005.

∴ Required difference = 3995 – 3005 = 990.

13. The smallest prime number which is less than 100 = 2.

The greatest prime number which is less than 100 = 97.

∴ Required difference = 97 – 2 = 95.

14. On dividing x by 289, let the quotient be k.

Then, $x = 289k + 18$ [∵ remainder = 18]

$\Rightarrow x = 17 \times 17k + 17 + 1 = 17\,(17k + 1) + 1$.

∴ When x is divided by 17, the remainder = 1.

Hence, the value of $y = 1$.

15. We need to find a number which can be written in the form of $n(n + 1)(n + 2)$ (i.e. the product of three consecutive natural numbers).

Clearly, 6 = 1 × 2 × 3 (i.e. the product of first three natural numbers).

16. The first number gives a remainder 3 and the second number gives a remainder 4 when divided by the same divisor say x. So, the sum of those two numbers leaves a reaminder (3 + 4) i.e 7 when divided by x. But it is given that the remainder is 2.

So, (7 – 2) i.e. 5 must be divisible by x.

Clearly , $x = 5$.

17. Let the numbers be x and y.

On dividing x and y by 33, let the quotients be m and n respectively and their respective remainders 21 and 28.

Then, $x = 33m + 21$ and $y = 33n + 28$

$\Rightarrow x + y = 33m + 21 + 33n + 28 = 33(m + n) + 49$

$= 33(m + n) + 33 + 16 = 33\ (m + n + 1) + 16.$

$\therefore$ On dividing $(x + y)$ by 33, the remainder = 16.

18. Let the number be x.

On dividing x by 361, let k be the quotient and 47 be the remainder.

Then, $x = 361k + 47 = 19 \times 19k + 19 \times 2 + 9 = 19(19k + 2) + 9.$

$\therefore$ When x is divided by 19, we get remainder = 9.

19. Required result = (3957 + 5349) – 7062 = 2244.

2244 is divisible by 3. [$\because$ 2 + 2 + 4 + 4 = 12 is divisible by 3]

2244 is divisible by 4.

[$\because$Number formed from last two digits i.e. 44 is divisible by 4]

2244 is divisible by 11. [$\because$ (2 + 4) – (2 + 4) = 0]

2244 is not divisible by 7. [$\because$ 244 – 2 = 242 is not divisible by 7.]

20. The greatest number of four digits = 9999.

On dividing 9999 by 307, we get 175 as remainder.

$\therefore$ The least number to be added = 307 – 175 = 132.

21. If a number is divisible by 4, 5 and 6, then it would also be divisible by the LCM (4, 5, 6) i.e. by 60.

On dividing 800 by 60, the quotient = 13.

On dividing 400 by 60, the quotient = 6.

$\therefore$ Required numbers between 400 and 800 that are divisible by 60 are (60 × 7), (60 × 8),...., (60 × 13) i.e. There are 7 such numbers.

22. $(m - n)$ is an even number [given]

$\Rightarrow$ m and n are either both odd or both even

$\Rightarrow (m + n)$ is also even.

Let $m - n = 2x$ and $m + n = 2y$.

Then, $m^2 - n^2 = (m - n)(m + n) = (2x)(2y) = 4xy.$

Clearly, $(m^2 - n^2)$ is always divisible by 4.

23. Let the remainder be R.

Then, the numbers $(11284 - R)$ and $(7655 - R)$ are exactly divisible by the given three-digits number and so their difference $(11284 - R) - (7655 - R) = 3629$ is also divisible by that three-digits number.

Now, 3629 = 19 × 191.

So, the three-digits number is 191.

$\therefore$ Required sum = 1 + 9 + 1 = 11.

24. All the factors of 323 are 1,17,19 and 323.

$\therefore$ 323 has two prime factors i.e. 17 and 19.

25. Every real number is not a rational number.

$\left[\because \sqrt{2} \text{ is a real number, but } \sqrt{2} \text{ is not a rational number}\right]$

26. Let the numbers be x and y.

Then, $x(x + y) = 247 \quad \Rightarrow x^2 + xy = 247$ ... (i)

And $y(x + y) = 114 \quad \Rightarrow yx + y^2 = 114$ ... (ii)

Addding (i) and (ii) we get :

$x^2 + xy + yx + y^2 = 247 + 114 = 361$

$\Rightarrow (x + y)^2 = 361 \quad \Rightarrow x + y = \sqrt{361} = 19.$

Thus, the sum of the numbers is 19.

27. Let the number be x.

Then, $2x + 20 = 8x - 4$

$\Rightarrow 6x = 24 \quad \Rightarrow x = \frac{24}{6} = 4.$

28. Let the five consecutive integers be $x - 2, x - 1, x, x + 1, x + 2$.

Then, $x - 2 + x - 1 + x + x + 1 + x + 2 = S$

$\Rightarrow 5x = S \quad \Rightarrow x = \frac{S}{5}.$

$\therefore$ The largest integer $= x + 2 = \frac{S}{5} + 2 = \frac{S + 10}{5}.$

29. When n is even, $a^n - b^n$ is always divisible by both $(a - b)$ and $(a + b)$ and therefore by $a^2 - b^2$.

Hence, $6^n - 1$ is divisible by $(6^2 - 1^2)$ i.e. by 35.

30. On dividing n by 6, let the quotient be k.

Then, $n = 6k + 4$. [$\because$ 4 is the remainder]

$\Rightarrow 2n = 2(6k + 4) = 12k + 8 = 6 \times 2k + 6 + 2 = 6(2k + 1) + 2.$

$\therefore$ When $2n$ is divided by 6, the remainder = 2.

31. Let the number be $100x + 10y + z$.

Then, the sum of its digits $= x + y + z$.

$\therefore$ The required difference $= 100x + 10y + z - (x + y + z)$

$= 99x + 9y = 9(11x + y).$

Clearly, the requried difference is divisible by 9.

32. $5^{71} + 5^{72} + 5^{73} = 5^{71}(1 + 5 + 5^2) = 31 \times 5^{71}.$

Clearly, the given number is divisible by (31 × 5) i.e. by 155.

33. $2^{16} - 1 = (2^8 - 1)(2^8 + 1)$

$= (2^4 - 1)(2^4 + 1)(2^8 + 1)$

$= (2^2 - 1)(2^2 + 1)(2^4 + 1)(2^8 + 1)$

$= (2 - 1)(2 + 1)(2^2 + 1)(2^4 + 1)(2^8 + 1)$

$= 1 \times 3 \times 5 \times 17 \times 257.$

Clearly, the last expression is divisible by 17.

34. $xyxyxy = xy \times 10000 + xy \times 100 + xy$

$= xy\,(10000 + 100 + 1) = xy \times 10101.$

$\therefore$ $xyxyxy$ is always divisible by 10101.

35. $(n^3 - n)(n - 2) = n(n^2 - 1)(n - 2) = n(n - 1)(n + 1)(n - 2)$

$= (n - 2)(n - 1)(n)(n + 1)$

$\Rightarrow (n^3 - n)(n - 2)$ is the product of four consecutive integers.

Clearly, one of these four integers must surely be divisible by 2, one must be divisible by 3 and one must be divisible by 4.

$\therefore$ $(n^3 - n)(n - 2)$ is definitely divisible by 2 × 3 × 4 i.e. 24.

For the smallest value of n i.e. 3 we have:

$(n^3 - n)(n - 2) = (3^3 - 3)(3 - 2) = 24 \times 1 = 24.$

$\therefore$ 24 is the largest number by which $(n^3 - n)(n - 2)$ is always divisible.

36. $n^3 - n = n(n^2 - 1) = n(n - 1)(n + 1) = (n - 1)(n)(n + 1)$

$\Rightarrow n^3 - n$ is the product of three consective integers.

Clearly, one of the three integers must surely be divisible by 2 and one must be divisible by 3.

$\therefore$ $(n^3 - n)$ is definitely divisible by 2 × 3 i.e. 6.

For the smallest value of n i.e. 2, we have:

$n^3 - n = 2^3 - 2 = 6.$

$\therefore$ 6 is the largest number by which $(n^3 - n)$ is always divisible.

37. Let all the middle digits of N be x.

Then $N = 2\underbrace{x\,x\,x\,.....\,x\,x}_{97 \text{ times}}2.$

The digits at odd places are $2, \underbrace{x, x, ..., x}_{48 \text{ times}}, 2.$

The digits at even places are $\underbrace{x, x, ..., x}_{49 \text{ times}}$

$\therefore \left(2 + \underbrace{x + x\,....., + x}_{48 \text{ times}} + 2\right) - \left(\underbrace{x + x + ... + x}_{49 \text{ times}}\right) = 0.$

[$\because$ N is divisible by 11]

$\Rightarrow 4 - x = 0 \quad \Rightarrow x = 4.$

38. $2^{6n} - 4^{2n} = (2^6)^n - (4^2)^n = 64^n - 16^n.$

$a^n - b^n$ is always divisible by $a - b$.

$\therefore$ $64^n - 16^n$ is always divisible by (64 – 16) i.e by 48.

39. $2 \equiv 2 \pmod 5$

$\Rightarrow 2^4 \equiv 16 \equiv 1 \pmod 5.$

$\therefore (2^4)^7 \equiv 1^7 \pmod 5 \equiv 1 \pmod 5.$

Also $2^3 = 8 \equiv 3 \pmod 5.$

And so, $2^{31} = 2^{28} \times 2^3 = (2^4)^7 \times 2^3$

$\equiv (1 \times 3) \pmod 5 \equiv 3 \pmod 5.$

$\therefore$ When 2^{31} is divided by 5, the remainder will be 3.

40. $2^{96} + 1 = (2^{32})^3 + (1)^3 = (2^{32} + 1)(2^{64} - 2^{32} + 1)$

[$\because a^3 + b^3 = (a + b)\,a^2 - ab + b^2)$]

$\therefore$ $2^{32} + 1$ is a factor of $2^{96} + 1$.

Hence, $2^{96} + 1$ is definitely divisible by the same number by which $2^{32} + 1$ is divisible.

41. Unit's digit in $(124)^{372} + (124)^{373}$

= Units digit in $(4)^{372} + (4)^{373}$ = Unit's digit in (6 + 4) = 0.

$\left[\because \text{Unit's digit in } 4^n = \begin{cases} 4, \text{ when } n \text{ is odd} \\ 6, \text{when } n \text{ is even} \end{cases}\right]$

42. Units digit in $3^4 = 1$ [$\because 3 \times 3 \times 3 \times 3 = 81$]

$\Rightarrow$ Units digit in $(3^4)^n = 1.$

Units digit in 3^{40}

= Units digit in $(3^4)^{10} = 1.$

43. Clearly, units digit in the given product

= Units digit in $7 \times 5 \times 8 \times 3 \times 9 = 0.$ [$\because 8 \times 5 = 40$]

44. Unit's digit in $[(251)^{98} + (21)^{29} - (106)^{100} + (705)^{35} - 16^4 + 259]$

= Unit's digit in $[(1)^{98} + (1)^{29} - (6)^{100} + (5)^{35} - 6^4 + 9]$

= Unit's digit in [1 + 1 – 6 + 5 – 6 + 9] = 4.

45. Units digit in $2^4 = 6$

$\Rightarrow$ Units digit in $2^{4n} = 6.$ [$\because$ 2 has cyclicity 4]

Units digit in $(2^4)^{43} = 6$

Units digit in 2^{173} = Units digit in $(2^4)^{43}$ × Units digit in 2

= Units digit of 6 × 2 = 2.

$\therefore$ Units digit in $(122)^{173}$ = Units digit in $(2)^{173} = 2.$

46. Unit's digit in the product 81 × 82 × 83 × ...... × 89

= Unit's digit in 1 × 2 × 3 × ...... × 9 = 0. [$\because 2 \times 5 = 10$]

47. Unit's digit in $7^4 = 1$

$\Rightarrow$ Unit's digit in $(7^4)^{17} = 1.$ [$\because$ 7 has cyclicity 4]

Units digit in 7^{71} = Unit's digit in $(7^4)^{17}$ × Unit's digit in 7^3

= 1 × 3 = 3.

Unit's digit in $6^{63} = 6.$ [$\because$ 6 has cyclcity 1]

Units digit in $3^4 = 1$

$\Rightarrow$ Unit's digit in $(3^4)^{16} = 1.$ [$\because$ 3 has cyclcity 4]

Unit's digit in 3^{65} = Unit's digit in $(3^4)^{16}$ × Unit's digit in 3

= (1 × 3) = 3.

$\therefore$ Unit's digit in $7^{71} \times 6^{63} \times 3^{65}$ = Unit's digit in (3 × 6 × 3) = 4.

48. $3^{25} + 3^{26} + 3^{27} + 3^{28} = 3^{25}(1 + 3 + 3^2 + 3^3) = 2^{25} \times 40$

$= 3^{25} \times 2 \times 2 \times 2 \times 5.$

Clearly, the given expression is divisible by (3 × 2 × 5) i.e. 30.

49. $n^2(n^2 - 1) = n^2(n - 1)(n + 1) = (n - 1)(n^2)(n + 1).$

Since the given expression includes the product of three consecutive integers so it is divisible by 3 atleast once.

Case 1 : When n is odd, then $(n - 1)$ and $(n + 1)$ are both even and so each has 2 as a factor, therefore, the number is divisible by 2 × 2 i.e. by 4.

Case 2 : When n is even, then $n^2 = n \times n$ is clearly divisible by 4 as each n has 2 as a factor.

Clearly, the given number is divisible by 3 × 4 i.e. 12 for all values of n.

50. $7^{19} + 2 = 7^{19} - 1 + 3 = (7)^{19} - (1)^{19} + 3.$

$(7)^{19} - (1)^{19}$ is exactly divisible by 6.

[$\because a^n - b^n$ is always divisible by $a - b$]

$\therefore$ When $7^{19} + 2$ is divided by 6, we get 3 as the remainder.

51. Since, the biggest remainder is 213, therefore the divisor must be greater than 213

```
. . .)64329
      – . . .   (i)
       1752
      – . . .   (ii)
       1149
      – . . .   (iii)
        213
```

$\therefore$ First number (i) = 643 – 175 = 468.

Second number (ii) = 1752 – 114 = 1638.

Third number (iii) = 1149 – 213 = 936.

Hence, the divisor = HCF (468, 1638, 936) = 234.

52.

4	x
5	$y - 1$
	1 – 4

$\therefore y = 5 \times 1 + 4 = 9$; $x = 4y + 1 = 4 \times 9 + 1 = 37.$

Now,

5	37
4	7 – 2
	1 – 3

$\therefore$ The respective remainders are 2, 3.

53. $17^2 = 289 \equiv 1 \pmod{18}$

$\therefore\ 17^{200} = (17^2)^{100} \equiv 1^{100} \pmod{18} \equiv 1 \pmod{18}$.

Thus, the remainder is 1 when 17^{200} is divided by 18.

54. The sum of first 60 numbers $= \dfrac{60 \times 61}{2} = 30 \times 61$.

$\left[\because \text{ The sum of first } n \text{ natural numbers} = \dfrac{n(n+1)}{2}\right]$

Clearly, (30×61) is divisible by 61 and so, the sum is divisible by 61.

55. A number is divisible by 9 if the sum of its digits is divisible by 9.

Sum of digits of 6709 = 6 + 7 + 0 + 9 = 22.

The number to be added to make 6709 exactly divisible by 9 = 27 – 22 = 5. [$\because$ 27 is divisible by 9 and greater than 22]

56. Let the six-digit number be *xyzxyz*

$\Rightarrow xyzxyz = 100000x + 10000y + 1000z + 100x + 10y + z$

$= 100100x + 10010y + 1001z$

$= 1001(100x + 10y + z)$.

Clearly, any number of the given form is always divisible by 1001.

57. The Prime numbers not greater than 17 are 2, 3, 5, 7, 11, 13 and 17.

$\therefore$ Required sum = 2 + 3 + 5 + 7 + 11 + 13 + 17 = 58.

58. If '*a*' divides 228 leaving a remainder 18,

then, '*a*' exactly divides (228 – 18) i.e. 210

$\Rightarrow$ '*a*' is a factor of 210.

All the factors of 210 are 1,2,3,5,6,7,10,14,15,21,30,35,42,70,105, and 210.

Clearly, the biggest two-digits factor of 210 is 70.

Hence, the biggest two-digits value of '*a*' = 70.

59. We know that the sum of two odd numbers is always even, product of two odd number is always odd and sum of an even and a odd number is always odd.

So, $a + b$ is an even number.

And $2ab$ is also an even number.

$\therefore\ a + b + 2ab$ is an even number.

[$\because$ Sum of two even numbers is always an even number]

60. Number of trees planted on one side of the road $= \dfrac{1760}{20} + 1 = 89$.

$\therefore$ Total number of trees planted on both sides of the road

$= 89 \times 2 = 178$.

61. Let $N = 1 \times 3 \times 5 \times 7 \times .... \times 99 \times 128$.

Clearly, only the multiples of 2 and 5 gives a zero on multiplication.

In the given product, the highest power of 2 is less than the highest power of 5. Highest power of 2 in $N = 7$. [$\because 2^7 = 128$]

$\therefore$ The required number of zeros = 7.

62. Let the digit in unit's place be x.

Then, the digit in ten's place $= x - 2$.

$\therefore$ The original number $= 10(x - 2) + x = 11x - 20$.

Sum of its digit $= x + x - 2 = 2x - 2$.

$(11x - 20)(2x - 2) = 144$

$\Rightarrow 22x^2 - 22x - 40x + 40 = 144$

$\Rightarrow 22x^2 - 62x - 104 = 0 \quad \Rightarrow 11x^2 - 31x - 52 = 0$

$\Rightarrow 11x^2 - 44x - 13x - 52 = 0$

$\Rightarrow 11x(x - 4) + 13(x - 4) = 0 \quad \Rightarrow (x - 4)(11x + 13) = 0$

$\Rightarrow x = 4$. $\left[\because x = \dfrac{-13}{11} \text{ can not be the digit}\right]$

$\therefore$ The original number = 11x – 20 = 11 × 4 – 20 = 24.

63. Let the digits in unit and ten's places be x and y respectively.

Then, the digit in hundred's place $= 2x$.

Sum of the digits $= 2x + y + x = 18 \quad \Rightarrow 3x + y = 18$. ... (*i*)

Original number $= 100(2x) + 10y + x = 201x + 10y$.

After reversing the digits, new number formed

$= 100x + 10y + 2x = 102x + 10y$.

$\therefore\ 201x + 10y - 102x - 10y = 396 \quad \Rightarrow 99x = 396 \quad \Rightarrow x = 4$.

Putting the value of x in (*i*) we get: $3 \times 4 + y = 18 \quad \Rightarrow y = 6$.

$\therefore$ The difference of hundred's and ten's digit $= 2x - y$

$= 2 \times 4 - 6 = 2$.

64. Let the unit's digit be x.

Then, the digit in ten's place $= x + 5$.

The original number $= 10(x + 5) + x = 11x + 50$.

New number formed by reversing the digits $= 10x + x + 5 = 11x + 5$.

Sum of digits of the number $= x + x + 5 = 2x + 5$.

$\therefore\ 11x + 50 - 5(2x + 5) = 11x + 5$

$\Rightarrow 11x + 50 - 10x - 25 = 11x + 5 \quad \Rightarrow 10x = 20 \quad \Rightarrow x = 2$.

$\therefore$ Sum of digits of the number $= 2x + 5 = 2 \times 2 + 5 = 9$.

65. Let the digits in the unit, ten and hundred's places be x, y and z respectively.

Then, the original number $= 100z + 10y + x$.

New number formed by interchanging the digits in the unit and the ten's place $= 100z + 10x + y$.

$\therefore\ 100z + 10x + y - 100z - 10y - x = 63$

$\Rightarrow 9x - 9y = 63 \quad \Rightarrow x - y = 7 \quad \Rightarrow x = 7 + y$.

$\therefore$ The possible values of x are 7, 8 and 9. [if y = 0, 1 and 2]

66. The sum of first 25 natural numbers $= \dfrac{25 \times 26}{2} = 25 \times 13$.

$\left[\because \text{ Sum of first } n \text{ natural numbers} = \dfrac{n(n+1)}{2}\right]$

So, cleraly 13 is a factor of sum of first 25 natural numbers.

67. Divisor = 68, Quotient = 269, Remainder = 0.

$\therefore$ The number = 68 × 269 + 0

[$\because$ Dividend = Divisor × Quotient + Remainder]

= 269 × (67 + 1) = 269 × 67 + 269 = 269 × 67 + 4 × 67 + 1

= 67 (269 + 4) + 1 = 67 × 273 + 1.

Clearly, when the number is divided by 67, we get 1 as a remainder.

❑❑❑

WEIGHTAGE ANALYSIS

Topics	*4 June (I) 2018 (Tier I)*	*12 June (III) 2018 (Tier I)*	*17 Feb 2018 (Tier II)*	*11 Sept 2019 (Tier II)*	*13 Sept (I) 2019 (Tier II)*	*12th Sept (I) 2019 (Tier II)*
Number System	1	1	3	3	4	4
H.C.F. and L.C.M.			1	2	2	2
Decimal Fractions			2	2	5	4
Simplification	1	1	4	5	2	2
Square Roots and Cube Roots			1		1	1
Surds and Indices	1		2	3	2	2
Average	1	1	4	2	2	2
Ratio and Proportion	1	1	10	6	7	7
Percentage	1	1	4	5	5	5
Profit and Loss	2	2	8	7	7	7
Simple Interest and Compound Interest	1	1	4	4	4	4
Time and Work	1	1	4	4	4	4
Time and Distance	1	1	4	4	4	4
Algebra	2	3	9	6	5	6
Trigonometry	3	3	7	10	11	10
Heights and Distances			3	1	1	1
Geometry	4	4	10	12	12	12
Mensuration	1	1	15	12	13	14
Data Interpretation	4	4	5	7	7	7
Coordinate Geometry				1	2	2
Total	25	25	100	100	100	100

2 H.C.F. AND L.C.M.

THEORY AND FORMULAE (TF)

TF 1. **Factors and Multiples:** A number A that divides another number B exactly is called a ***factor*** of B and B is called a ***multiple*** of A.

Example. (i) Each one of 1, 2, 3, 4, 6 and 12 is a factor of 12.

(ii) 3, 6, 9, 12, ... are all multiples of 3.

Note: 0 is a multiple of every number except 0 itself.

TF 2. **Fundamental Theorem of Arithmetic (or the Unique Factorization Theorem):** Every integer greater than 1 can be uniquely expressed as a product of prime numbers except for the order in which these prime factors occur.

Example. $90 = 2 \times 3^2 \times 5$.

TF 3. **H.C.F. (Highest Common Factor) or G.C.D. (Greatest Common Divisor):** The H.C.F. (or G.C.D.) of two or more numbers is the greatest number which is a factor of each one of the given numbers. The H.C.F. is determined either by ***Factorization Method*** or by ***Division Method.***

Note:

(i) H.C.F. of two or more numbers cannot be greater than any of them.

(ii) If a number A is a factor of another number B, then the H.C.F. of A and B is equal to A. i.e. H.C.F. $(A, B) = A$.

(iii) H.C.F. of two or more numbers is the product of the common prime factors of these numbers.

(iv) Two numbers are said to be co-primes if their H.C.F. is 1.

TF 4. **L.C.M. (Least Common Multiple):** The L.C.M. of two or more numbers is the least number which is divisible by each of the given numbers.

The L.C.M. is determined either by ***Factorization Method*** or by ***Common Division Method***.

Note:

(i) L.C.M. of two or more numbers cannot be less than any of them.

(ii) If a number A is a factor of another number B, then the L.C.M. of A and B is equal to B. i.e. L.C.M. $(A, B) = B$.

(iii) If we resolve each of the given numbers into a product of prime numbers (by Prime Factorization), then L.C.M. is of the given numbers is the product of highest powers of all the different factors.

Example. $90 = 2 \times 3^2 \times 5$

$84 = 2^2 \times 3 \times 7$.

$\therefore$ L.C.M. $= 2^2 \times 3^2 \times 5 \times 7 = 1260$.

(iv) The L.C.M. of two co-primes is equal to their product. And so, the L.C.M. of two consecutive numbers is equal to their product.

TF 5. Product of two numbers = H.C.F. × L.C.M.

TF 6. (i) H.C.F. of Fractions $= \dfrac{\text{H.C.F. of Numerators}}{\text{L.C.M. of Denominators}}$. ($ii$) L.C.M. of Fractions $= \dfrac{\text{L.C.M. of Numerators}}{\text{H.C.F. of Denominators}}$.

TF 7. **H.C.F. and L.C.M. of Decimal Fractions:** Convert the given numbers into Like Decimals (by annexing zeros on the RHS so that all have the same number of decimal places). Suppose now they have n decimal places each. Find the H.C.F. or L.C.M. by neglecting the decimal point. In the result, mark off n decimal places to get the H.C.F. or L.C.M. of the given numbers.

EXERCISE

1. There are 24 peaches, 36 apricots and 60 bananas and they have to be arranged in several rows in such a way that every row contains the same number of fruits of only one type. What is the minimum number of rows required for this to happen? **[SSC 2014]**
(*a*) 6 (*b*) 9
(*c*) 10 (*d*) 12

2. Three men step off together from the same spot. Their steps measure 63 cm, 70 cm and 77 cm respectively. The minimum distance each should cover so that all can cover the distance in complete steps is **[SSC CGL 2014]**
(*a*) 6930 cm (*b*) 6950 cm
(*c*) 9360 cm (*d*) 9630 cm

3. The smallest number, which when increased by 5 is divisible by each of 24, 32, 36 and 54 is
(*a*) 427 (*b*) 859
(*c*) 869 (*d*) 4320

4. The least number which when divided by 5, 6, 7 and 8 leaves a remainder 3, but when divided by 9 leaves no remainder is
(*a*) 3363 (*b*) 2523
(*c*) 1683 (*d*) 1677

5. 4 bells ring at intervals of 30 minutes, 1 hour, $1\frac{1}{2}$ hour and 1 hour 45 minutes respectively. All the bells ring simultaneously at 12 noon. They will again ring simultaneously at
(*a*) 9 a.m. (*b*) 6 a.m.
(*c*) 3 a.m. (*d*) 12 midnight

6. Three tankers contain 403 litres, 434 litres and 465 litres of diesel respectively. Then the maximum capacity of a container that can measure the diesel of the three containers the exact number of times is **[SSC 2014]**
(*a*) 31 litres (*b*) 41 litres
(*c*) 62 litres (*d*) 84 litres

7. The H.C.F. and L.C.M. of two numbers are 44 and 264 respectively. If the first number is divided by 2, the quotient is 44. The other number is **[SSC 2014]**
(*a*) 132 (*b*) 147
(*c*) 264 (*d*) 528

8. The H.C.F. of two numbers is 8. Which one of the following can never be the L.C.M.?
(*a*) 60 (*b*) 56
(*c*) 48 (*d*) 24

9. The L.C.M. of three different numbers is 120. Which of the following cannot be their H.C.F.? **[SSC CGL 2011]**
(*a*) 35 (*b*) 24
(*c*) 12 (*d*) 8

10. Three numbers which are co-prime to one another are such that the product of the first two is 551 and that of the last two is 1073. The sum of the three numbers is
(*a*) 89 (*b*) 85
(*c*) 81 (*d*) 75

11. The L.C.M. of two numbers is 120 and their H.C.F. is 10. Which of the following can be the sum of those two numbers? **[SSC CGL 2011]**
(*a*) 60 (*b*) 70
(*c*) 80 (*d*) 140

12. The L.C.M. of two multiples of 12 is 1056. If one of the numbers is 132, then the other number is
(*a*) 132 (*b*) 96
(*c*) 72 (*d*) 12

13. The L.C.M. of two numbers is 2079 and their H.C.F. is 27. If one of the numbers is 189, the other number is **[SSC 2013]**
(*a*) 189 (*b*) 216
(*c*) 297 (*d*) 584

14. The product of two numbers is 216. If the H.C.F. is 6, then their L.C.M. is **[SSC 2010]**
(*a*) 36 (*b*) 48
(*c*) 60 (*d*) 72

15. L.C.M. of (2/3), (4/9), (5/6) is **[SSC CGL 2013]**
(*a*) 8/27 (*b*) 10/3
(*c*) 20/3 (*d*) 20/27

16. A number which when divided by 10 leaves a remainder of 9, when divided by 9 leaves a remainder of 8 and when divided by 8 leaves a remainder of 7 is
(*a*) 359 (*b*) 539
(*c*) 1359 (*d*) 1539

17. Find the greatest number which will exactly divide 200 and 320. **[SSC CGL 2014]**
(*a*) 10 (*b*) 16
(*c*) 20 (*d*) 40

18. The smallest number, which, when divided by 12 or 10 or 8, leaves remainder 6 in each case, is **[SSC 2010]**
(*a*) 66 (*b*) 126
(*c*) 186 (*d*) 246

19. The L.C.M. of two numbers is 48. The numbers are in the ratio 2 : 3. The sum of the numbers is **[SSC MTS 2011]**
(*a*) 64 (*b*) 40
(*c*) 32 (*d*) 28

20. H.C.F. of (2/3), (4/5) and (6/7) is **[SSC 2012]**
(*a*) 1/105 (*b*) 2/105
(*c*) 24/105 (*d*) 48/105

21. The L.C.M. of two numbers is 44 times of their H.C.F.. The sum of the L.C.M. and H.C.F. is 1125. If one number is 25, then the other number is **[SSC 2010]**
(*a*) 800 (*b*) 900
(*c*) 975 (*d*) 1100

22. Let x be the smallest number, which when added to 2000 makes the resulting number divisible by 12, 16, 18 and 21. The sum of the digits of x is **[SSC CGL 2015]**

(*a*) 4 (*b*) 5
(*c*) 6 (*d*) 7

23. The product of two numbers is 2028 and their H.C.F. is 13. The number of such pairs is **[SSC CGL 2011]**

(*a*) 4 (*b*) 3
(*c*) 2 (*d*) 1

24. 84 Maths books, 90 Physics books and 120 Chemistry books have to be stacked topic-wise. How many books will be there in each stack so that each stack will have the same height too? **[SSC 2014]**

(*a*) 6 (*b*) 12
(*c*) 18 (*d*) 21

25. If the students of a class can be grouped exactly into 6 or 8 or 10, then the minimum number of students in the class must be

(*a*) 240 (*b*) 180
(*c*) 120 (*d*) 60

26. The traffic lights at three different road crossings change after 24 seconds, 36 seconds and 54 seconds respectively. If they all change simultaneously at 10.15 a.m., then at what time will they again change simultaneously? **[SSC CGL 2011]**

(*a*) 10 : 22 : 12 a.m. (*b*) 10 : 18 : 36 a.m.
(*c*) 10 : 17 : 02 a.m. (*d*) 10 : 16 : 54 a.m.

27. The ratio of two numbers is 3 : 4 and their H.C.F. is 5. Their L.C.M. is **[SSC 2013]**

(*a*) 60 (*b*) 15
(*c*) 12 (*d*) 10

28. The maximum number of students among whom 1001 pens and 910 pencils can be distributed in such a way that each student gets same number of pens and same number of pencils is

(*a*) 1911 (*b*) 1001
(*c*) 910 (*d*) 91

29. The H.C.F. of two numbers, each having three digits is 17 and their L.C.M. is 714. The sum of the numbers will be

(*a*) 221 (*b*) 289
(*c*) 391 (*d*) 731

30. Let x be the least number, which when divided by 5, 6, 7 and 8 leaves a remainder 3 in each case but when divided by 9 leaves no remainder. The sum of digits of x is **[SSC CGL 2015]**

(*a*) 18 (*b*) 21
(*c*) 22 (*d*) 24

31. The product of two numbers is 2160 and their H.C.F. is 12. The number of such possible pairs is **[SSC 2013]**

(*a*) 4 (*b*) 3
(*c*) 2 (*d*) 1

32. The product of two numbers is 4107. If the H.C.F. of the numbers is 37, then a greater number is

(*a*) 101 (*b*) 107
(*c*) 111 (*d*) 185

33. The smallest perfect square divisible by each of 6, 12 and 18 is **[SSC 2010]**

(*a*) 36 (*b*) 108
(*c*) 144 (*d*) 196

34. The number nearest to 43582 divisible by each of 25, 50 and 75 is

(*a*) 43500 (*b*) 43550
(*c*) 43600 (*d*) 43650

35. Which is the least number which when doubled will be exactly divisible by 12, 18, 21 and 30?

(*a*) 196 (*b*) 630
(*c*) 1260 (*d*) 2520

36. If $x : y$ be the ratio of two whole numbers and z be their H.C.F., then the L.C.M. of those two numbers is **[SSC 2014]**

(*a*) xyz (*b*) xy/z
(*c*) xz/y (*d*) yz

37. A number x is divisible by 7. When this number is divided by 8, 12 and 16 it leaves a remainder 3 in each case. The least value of x is **[SSC 2015]**

(*a*) 147 (*b*) 148
(*c*) 149 (*d*) 150

38. The sum of two numbers is 36 and their H.C.F. and L.C.M. are 3 and 105 respectively. The sum of the reciprocals of the two numbers is **[SSC CGL 2010]**

(*a*) 2/25 (*b*) 2/45
(*c*) 3/35 (*d*) 4/35

39. The L.C.M. of two numbers is 4 times their H.C.F.. The sum of L.C.M. and H.C.F. is 125. If one of the numbers is 100, then the other number is **[SSC 2011]**

(*a*) 125 (*b*) 100
(*c*) 25 (*d*) 5

40. The H.C.F. and L.C.M. of two numbers are 13 and 455 respectively. If one of the number lies between 75 and 125, then that number is

(*a*) 117 (*b*) 104
(*c*) 91 (*d*) 78

41. The product of two co-prime numbers is 117. Then their L.C.M. is **[SSC CGL 2013]**

(*a*) 9 (*b*) 13
(*c*) 39 (*d*) 117

42. The bells begin to toll together and they toll respectively at intervals of 6, 7, 8, 9 and 12 seconds. After how many seconds will they toll again? **[SSC 2013]**

(*a*) 72 seconds (*b*) 318 seconds
(*c*) 504 seconds (*d*) 612 seconds

43. Three numbers are in the ratio 1 : 2 : 3 and their H.C.F. is 12. The numbers are **[SSC CGL 2014]**

(*a*) 4, 8, 12 (*b*) 5, 10, 15
(*c*) 10, 20, 30 (*d*) 12, 24, 36

44. Three numbers are in the ratio 2 : 3 : 4 and their H.C.F. is 12. The L.C.M. of the numbers is
(*a*) 72 (*b*) 96
(*c*) 144 (*d*) 192

45. Sum of two numbers is 384. The H.C.F. of the numbers is 48. The difference of the numbers is
(*a*) 336 (*b*) 288
(*c*) 192 (*d*) 100

46. The H.C.F. and L.C.M. of two numbers are 12 and 924 respectively. Then the number of such pairs is **[SSC CGL 2011]**
(*a*) 3 (*b*) 2
(*c*) 1 (*d*) 0

47. The H.C.F. and L.C.M. of two 2-digit numbers are 16 and 480 respectively. The numbers are
(*a*) 80 and 96 (*b*) 64 and 80
(*c*) 60 and 72 (*d*) 40 and 48

48. The greatest 4-digit number exactly divisible by 10, 15, 20 is **[SSC 2013]**
(*a*) 9960 (*b*) 9980
(*c*) 9990 (*d*) 9995

49. Two numbers are in the ratio 3 : 4. The product of their H.C.F. and L.C.M. is 2028. The sum of the numbers is
(*a*) 91 (*b*) 86
(*c*) 72 (*d*) 68

50. *A*, *B* and *C* start running at the same time and at the same point in the same direction in a circular stadium. *A* completes a round in 252 seconds, *B* in 308 seconds and *C* in 198 seconds. After what time will they meet again at the starting point? **[SSC 2012]**
(*a*) 46 minutes 12 seconds
(*b*) 45 minutes
(*c*) 42 minutes 36 seconds
(*d*)26 minutes 18 seconds

51. The H.C.F. and L.C.M. of two numbers are 21 and 84 respectively. If the ratio of the two numbers is 1 : 4, then the largest of the two numbers is **[SSC CGL 2015]**
(*a*) 12 (*b*) 48
(*c*) 84 (*d*) 108

52. The L.C.M. of two positive integers is twice the larger number. The difference of the smaller number and the GCD of the two numbers is 4. The smaller number is **[SSC 2012]**
(*a*) 6 (*b*) 8
(*c*) 10 (*d*) 12

53. The H.C.F. (GCD) of *a*, *b* is 12 as *a* and *b* are positive integers and $a > b > 12$. The smallest values of (*a* and *b*) are respectively **[SSC 2015]**
(*a*) 36 and 24 (*b*) 24 and 36
(*c*) 24 and 12 (*d*) 12 and 24

54. The H.C.F. of two numbers is 23 and the other two factors of their L.C.M. are 13 and 14. The larger of the two numbers is
(*a*) 345 (*b*) 322
(*c*) 299 (*d*) 276

55. The least number which when divided by 16, 18, 20 and 25 leaves 4 as remainder in each case but when divided by 7 leaves no remainder is **[SSC 2011]**
(*a*) 18004 (*b*) 18002
(*c*) 18000 (*d*) 17004

56. Find the least number which when divided separately by 15, 20, 36 and 48 leaves 3 as remainder in each case. **[SSC CGL 2014]**
(*a*) 723 (*b*) 483
(*c*) 243 (*d*) 183

57. The greatest number by which 2300 and 3500 are divided leaving the remainders of 32 and 56 respectively is **[SSC 2015]**
(*a*) 42 (*b*) 84
(*c*) 136 (*d*) 168

58. Let the least number of six digits which when divided by 4, 6, 10, 15 leaves in each case the same remainder 2 be *N*. The sum of digits *N* is
(*a*) 3 (*b*) 4
(*c*) 5 (*d*) 6

59. Which is the least number of square tiles required to pave the floor of a room 15 m 17 cm long and 9 m 2 cm broad?
(*a*) 814 (*b*) 820
(*c*) 840 (*d*) 841

60. The number between 4000 and 5000 that is divisible by each of 12, 18, 21 and 32 is **[SSC 2015]**
(*a*) 4023 (*b*) 4032
(*c*) 4203 (*d*) 4302

61. The greatest number that divides 411, 684, 821 and leaves 3, 4 and 5 as remainders respectively is **[SSC 2013]**
(*a*) 136 (*b*) 146
(*c*) 204 (*d*) 254

62. The largest number, which divides 25, 73 and 97 to leave the same remainder in each case is
(*a*) 6 (*b*) 21
(*c*) 23 (*d*) 24

63. If *A* and *B* are the H.C.F. and L.C.M. respectively of two algebraic expressions *x* and *y*, and $A + B = x + y$, then the value of $A^3 + B^3$ is **[SSC 2013]**
(*a*) x^3 (*b*) y^3
(*c*) $x^3 + y^3$ (*d*) $x^3 - y^3$

64. The H.C.F. and L.C.M. of two numbers are 7 and 140 respectively. If the numbers are between 20 and 45, then the sum of the numbers is
(*a*) 56 (*b*) 63
(*c*) 70 (*d*) 77

65. If $P = 2^3.3^{10}.5$; $Q = 2^5.3.7$, then H.C.F. of P and Q is **[SSC CGL 2011]**

(*a*) 3.2^3 (*b*) $2^5.3^{10}.5.7$
(*c*) $2^2.3^7$ (*d*) 2.3.5.7

66. Find the least multiple of 23, which when divided by 18, 21 and 24 leaves the remainder 7, 10 and 13 respectively.

(*a*) 3002 (*b*) 3013
(*c*) 3024 (*d*) 3036

67. The greatest number, which when subtracted from 5834, gives a number exactly divisible by each of 20, 28, 32 and 35 is **[SSC CGL 2010]**

(*a*) 5600 (*b*) 5200
(*c*) 4714 (*d*) 1120

68. The greatest number of four digits which when divided by 3, 5, 7, 9 leave remainders 1, 3, 5, 7 respectively is **[SSC 2012]**

(*a*) 9766 (*b*) 9765
(*c*) 9764 (*d*) 9763

69. The largest number of five digits which, when divided by 16, 24, 30 or 36 leaves the same remainder 10 in each case is

(*a*) 99269 (*b*) 99279
(*c*) 99350 (*d*) 99370

70. A farmer has 945 cows and 2475 sheep. He farms them into flocks, keeping the cows and sheep separately and having the same number of animals in each flock. If these flocks are as large as possible, then the maximum number of animals in each flock and the total number of flocks required for the purpose are respectively **[SSC 2011]**

(*a*) 9 and 380 (*b*) 15 and 228
(*c*) 45 and 76 (*d*) 46 and 75

71. A number between 1000 and 2000 which when divided by 30, 36 and 80 gives a remainder 11 in each case is **[SSC 2015]**

(*a*) 1451 (*b*) 1523
(*c*) 1641 (*d*) 1712

72. Let N be the greatest number that will divide 1305, 4665 and 6905 leaving the same remainder in each case. Then, the sum of the digits in N is

(*a*) 8 (*b*) 6
(*c*) 5 (*d*) 4

73. If the H.C.F. and L.C.M. of two consecutive (positive) even numbers be 2 and 84 respectively, then the sum of the numbers is **[SSC 2011]**

(*a*) 14 (*b*) 26
(*c*) 30 (*d*) 34

74. A fraction becomes (1/6) when 4 is subtracted from its numerator and 1 is added to its denominator. If 2 and 1 are respectively added to its numerator and the denominator, it becomes (1/3). Then, the L.C.M. of the numerator and denominator of the said fraction, must be **[SSC CGL 2011]**

(*a*) 5 (*b*) 14
(*c*) 70 (*d*) 350

ANSWERS

1. (*c*)	**2.** (*a*)	**3.** (*b*)	**4.** (*c*)	**5.** (*a*)	**6.** (*a*)	**7.** (*a*)	**8.** (*a*)	**9.** (*a*)	**10.** (*b*)
11. (*b*)	**12.** (*b*)	**13.** (*c*)	**14.** (*a*)	**15.** (*c*)	**16.** (*a*)	**17.** (*d*)	**18.** (*b*)	**19.** (*b*)	**20.** (*b*)
21. (*d*)	**22.** (*d*)	**23.** (*c*)	**24.** (*a*)	**25.** (*c*)	**26.** (*b*)	**27.** (*a*)	**28.** (*d*)	**29.** (*a*)	**30.** (*a*)
31. (*c*)	**32.** (*c*)	**33.** (*a*)	**34.** (*d*)	**35.** (*b*)	**36.** (*a*)	**37.** (*a*)	**38.** (*d*)	**39.** (*c*)	**40.** (*c*)
41. (*d*)	**42.** (*c*)	**43.** (*d*)	**44.** (*c*)	**45.** (*b*)	**46.** (*b*)	**47.** (*a*)	**48.** (*a*)	**49.** (*a*)	**50.** (*a*)
51. (*c*)	**52.** (*b*)	**53.** (*a*)	**54.** (*b*)	**55.** (*a*)	**56.** (*a*)	**57.** (*b*)	**58.** (*c*)	**59.** (*a*)	**60.** (*b*)
61. (*a*)	**62.** (*d*)	**63.** (*c*)	**64.** (*b*)	**65.** (*a*)	**66.** (*b*)	**67.** (*c*)	**68.** (*d*)	**69.** (*d*)	**70.** (*c*)
71. (*a*)	**72.** (*d*)	**73.** (*b*)	**74.** (*c*)						

SOLUTIONS

1. For the minimum number of rows, we have to arrange maximum number of fruits in each row.

Maximum number of fruits in each row = H.C.F. (24, 36, 60) = 12.

$\therefore$ Required number of rows = $\dfrac{24 + 36 + 60}{12}$ = 10.

2. Required minimum distance = L.C.M. (63, 70, 77) = 6930 cm.

3. Required number = L.C.M. (24, 32, 36, 54) – 5 = 864 – 5 = 859.

4. L.C.M. (5, 6, 7, 8) = 840

$\Rightarrow$ Required number = $840k + 3$, where k is a positive integer.

Least value of k for which $(840k + 3)$ is divisible by 9 is ($k = 2$).

$\therefore$ Required number = (840 × 2 + 3) = 1683.

5. Clearly 1 hour = 60 minutes, $1\frac{1}{2}$ hours = 90 minutes and 1 hour 45 minutes = 105 minutes.

Interval after which the bells will ring simultaneously

= L.C.M. (30, 60, 90, 105) = 1260 minutes = $\dfrac{1260}{60}$ = 21 hours.

$\therefore$ They will ring simultaneously at (12 noon + 21 hours) = 9 a.m.

6. Maximum capacity of container = H.C.F. (403, 434, 465)

= 31 litres.

7. Let the other number be x.

First number = 2 × 44 = 88.

H.C.F. × L.C.M. = Product of numbers

$\Rightarrow 44 \times 264 = 88x \quad \Rightarrow x = \dfrac{44 \times 264}{88} = 132.$

So, the other number is 132.

8. H.C.F. of two numbers completely divides their L.C.M..

Clearly, 8 does not divide 60 completely. So, L.C.M. $\neq$ 60.

9. H.C.F. always completely divides L.C.M.

Clearly, 35 does not completely divide 120. So, H.C.F. $\neq$ 35.

10. Let x, y and z be the three co-prime numbers.

Then, $xy = 551 = 19 \times 29$ and $yz = 1073 = 29 \times 37$

$\Rightarrow x = 19, y = 29$ and $z = 37$.

$\therefore x + y + z = 19 + 29 + 37 = 85.$

11. Let the numbers be $10x$ and $10y$, where x and y are co-primes.

Then, $10x \times 10y = 120 \times 10 \Rightarrow xy = \dfrac{120 \times 10}{10 \times 10} = 12$

$\Rightarrow$ The possible values of (x, y) can be (1, 12) and (3, 4).

$\therefore$ The possible sum of numbers can be 130 and 70.

12. Let the other number be $12x$.

Then, L.C.M. $(12x, 132) = 132x$

$\Rightarrow 132x = 1056 \quad \Rightarrow x = \dfrac{1056}{132} = 8.$

$\therefore$ The other number is 96.

13. The other number = $\dfrac{\text{H.C.F.} \times \text{L.C.M.}}{\text{One of the numbers}} = \dfrac{2079 \times 27}{189} = 297.$

14. H.C.F. × L.C.M. = Product of two numbers

$\Rightarrow 6 \times \text{L.C.M.} = 216 \quad \Rightarrow \text{L.C.M.} = \dfrac{216}{6} = 36.$

15. L.C.M. $\left(\dfrac{2}{3}, \dfrac{4}{9}, \dfrac{5}{6}\right) = \dfrac{\text{L.C.M.}(2, 4, 5)}{\text{H.C.F.}(3, 9, 6)} = \dfrac{20}{3}.$

16. We have: (10 – 9) = 1, (9 – 8) = 1 and (8 – 7) = 1

$\therefore$ Required number = L.C.M. (10, 9, 8) – 1 = 360 – 1 = 359.

17. Required greatest number = H.C.F. (200, 320) = 40.

18. L.C.M. (12, 10, 8) = 120.

$\therefore$ Required least number = 120 + 6 = 126.

19. Let the numbers be $2x$ and $3x$.

Then, L.C.M. $(2x, 3x) = 6x \quad \Rightarrow 6x = 48 \quad \Rightarrow x = 8.$

$\therefore$ The numbers are 16 and 24.

So, the sum of the numbers = 16 + 24 = 40.

20. H.C.F. $\left(\dfrac{2}{3}, \dfrac{4}{5}, \dfrac{6}{7}\right) = \dfrac{\text{H.C.F.}(2, 4, 6)}{\text{L.C.M.}(3, 5, 7)} = \dfrac{2}{105}.$

21. Let the H.C.F. of two numbers be x.

Then, L.C.M. $= 44x \quad \Rightarrow x + 44x = 1125 \quad \Rightarrow 45x = 1125$

$\Rightarrow x = 25$

$\Rightarrow$ H.C.F. = 25 and L.C.M. = 1100.

$\therefore$ The other number = $\dfrac{\text{H.C.F.} \times \text{L.C.M.}}{\text{Given Number}} = \dfrac{25 \times 1100}{25} = 1100.$

22. L.C.M. (12, 16, 18, 21) = 1008

$\Rightarrow 1008 \times 2 = 2016$ is the nearest number to 2000 which is divisible by 12, 16, 18 and 21 exactly

$\Rightarrow x = 2016 - 2000 = 16.$

$\therefore$ The sum of the digits of x is 7.

23. Let the numbers be $13x$ and $13y$, where x and y are co-primes.

Then, $13x \times 13y = 2028 \quad \Rightarrow xy = \dfrac{2028}{13 \times 13} = 12$

The possible pairs of (x, y) can be (1, 12) and (3, 4)

$\therefore$ The number of such possible pairs is 2 i.e.

(13, 156) and (39, 52).

24. Number of books in each stack = H.C.F. (84, 90, 120) = 6.

25. Minimum number of students = L.C.M. (6, 8, 10) = 120.

26. Interval after which they will change again simultaneously

= L.C.M. (24, 36, 54) = 216 seconds = 3 minutes 36 seconds

$\therefore$ Required time = 10 : 18 : 36 a.m.

27. Let the numbers be $3x$ and $4x$.

Then, H.C.F. $(3x, 4x) = x \quad \Rightarrow x = 5.$

$\therefore$ L.C.M. (15, 20) = 60.

28. Maximum number of students required = H.C.F. (1001, 910) = 91.

29. Let the numbers be $17x$ and $17y$.

Then, $17x \times 17y = 17 \times 714 \quad \Rightarrow xy = \dfrac{17 \times 714}{17 \times 17} = 42.$

The possible values of (x, y) can be (1, 42), (2, 21), (3, 14) and (6, 7).

But, we will be getting both numbers of three digit when $x = 6$ and $y = 7$.

So, the numbers are 102 and 119.

$\therefore$ The sum of the numbers = 102 + 119 = 221.

30. L.C.M. (5, 6, 7, 8) = 840.

So, x is of the form $840k + 3$.

The least value of k for which $(840k + 3)$ is divisible by 9 is $(k = 2)$

$\Rightarrow x = 840 \times 2 + 3 = 1683.$

$\therefore$ The sum of digits of $x = 1 + 6 + 8 + 3 = 18.$

31. Let the two numbers be $12x$ and $12y$, where x and y are co-primes

Then, $12x \times 12y = 2160 \quad \Rightarrow xy = \dfrac{2160}{144} = 15.$

$\Rightarrow$ The possible values of (x, y) can be (1, 15) and (3, 5).

$\therefore$ The number of such possible pairs are two i.e. (12, 180) and (36, 60).

32. Let the numbers be $37x$ and $37y$, where x and y are co-primes.

H.C.F. × L.C.M. = Product of numbers

$\Rightarrow 37x \times 37y = 4107 \quad \Rightarrow xy = \dfrac{4107}{37 \times 37} = 3$

The possible values of (x, y) can be (1, 3) or (3, 1).

$\therefore$ The greater number = 37 × 3 = 111.

33. L.C.M. (6, 12, 18) = 36

$\Rightarrow$ Required number is of the form $36k$

Least value of k for which $18k$ is a perfect square is $(k = 1)$.

∴ Required number = 36 × 1 = 36.

34. L.C.M. (25, 50, 75) = 150

On dividing 43582 by 150, the remainder is 82

Clearly (150 – 82 = 68) is smaller than 82

∴ Required number = 43582 + 68 = 43650.

35. L.C.M. (12, 18, 21, 30) = 1260.

∴ Required number = $\frac{1260}{2}$ = 630.

36. Let the numbers be *xm* and *ym*.

Then, H.C.F. (*xm*, *ym*) = *m* ⇒ *m* = *z*.

∴ L.C.M. (*xz*, *yz*) = *xyz*.

37. L.C.M. (8, 12, 16) = 48

⇒ Required number is of the form 48*k* + 3.

The least value of *k* for which (48*k* + 3) is divisible by 7 is (*k* = 3).

∴ Required number = (48 × 3 + 3) = 147.

38. Let the numbers be *x* and 36 – *x*.

Then, $x(36 - x) = 3 \times 105 \Rightarrow 36x - x^2 = 315$

$\Rightarrow x^2 - 36x + 315 = 0$

$\Rightarrow x^2 - 21x - 15x + 315 = 0$

$\Rightarrow (x - 21)(x - 15) = 0 \Rightarrow x = 21$ or $x = 15$.

So, the numbers are 21 and 15.

∴ The sum of their reciprocals = $\frac{1}{21} + \frac{1}{15} = \frac{12}{105} = \frac{4}{35}$.

39. Let the H.C.F. of two numbers be *x*.

Then, their L.C.M. = 4*x*

∴ $x + 4x = 125 \Rightarrow 5x = 125 \Rightarrow x = 25$

⇒ H.C.F. = 25 and L.C.M. = 100.

∴ The other number = $\frac{\text{H.C.F.} \times \text{L.C.M.}}{\text{One of the numbers}} = \frac{25 \times 100}{100} = 25$.

40. Let the numbers be 13*x* and 13*y*, where *x* and *y* are co-primes.

Then, Product of the numbers = H.C.F. × L.C.M.

$\Rightarrow 13x \times 13y = 13 \times 455 \Rightarrow xy = \frac{13 \times 455}{13 \times 13} = 35$

The values of (*x*, *y*) can be (1, 35) or (5, 7)

∴ For *y* = 7, 13*y* = 91.

41. H.C.F. of two co-prime numbers is 1.

Now, H.C.F. × L.C.M. = Product of two numbers

⇒ 1 × L.C.M. = 117 ⇒ L.C.M. = 117.

42. Interval after which they will toll again = L.C.M. (6, 7, 8, 9, 12)

= 504 seconds.

43. Let the numbers be *x*, 2*x* and 3*x*.

Then, H.C.F. (*x*, 2*x*, 3*x*) = *x* ⇒ *x* = 12.

∴ The numbers are 12, 24, 36.

44. Let the numbers be 2*x*, 3*x* and 4*x*.

Then, H.C.F. (2*x*, 3*x*, 4*x*) = *x* ⇒ *x* = 12.

⇒ The numbers are 24, 36 and 48.

∴ L.C.M. (24, 36, 48) = 144.

45. Let the numbers be 48*x* and 48*y*, where *x* and *y* are co-primes.

Then, 48*x* + 48*y* = 384 ⇒ *x* + *y* = 8.

The possible values of (*x*, *y*) can be (1, 7) and (3, 5).

So, the numbers are (48, 336) and (144, 240).

When the numbers are 48 and 336, then the difference between them = 336 – 48 = 288.

[But when we take the numbers to be 144 and 240, then difference = 240 – 144 = 96.]

46. Let the two numbers be 12*x* and 12*y*, where *x* and *y* are co-primes.

Then, $12x \times 12y = 12 \times 924 \Rightarrow xy = \frac{12 \times 924}{144} = 77$.

The only possible values of (*x*, *y*) can be (1, 77) and (11, 7).

∴ The number of such possible pairs is 2 i.e. (12, 924) and (132, 84).

47. Let the numbers be 16*x* and 16*y*, where *x* and *y* are co-primes.

Product of numbers = H.C.F. × L.C.M.

$16x \times 16y = 16 \times 480 \Rightarrow xy = \frac{16 \times 480}{16 \times 16} = 30$.

The possible values of (*x*, *y*) can be (1, 30), (2, 15), (3, 10) and (5, 6).

Clearly, when *x* = 5 and *y* = 6, we get:

80 and 96 as the required numbers (which are given as options).

48. Greatest 4-digit number = 9999.

L.C.M. (10, 15, 20) = 60.

On dividing 9999 by 60, the remainder is 39.

∴ Required number = 9999 – 39 = 9960.

49. Let the numbers be 3*x* and 4*x*.

Then, $3x \times 4x = 2028 \Rightarrow x^2 = \frac{2028}{3 \times 4} = 169 \Rightarrow x = 13$.

∴ The numbers are 39 and 52.

So, the sum of the numbers is 91.

50. Time after which they will meet again at the starting point

= L.C.M. (252, 308, 198) = 2772 seconds = 46 minutes 12 seconds.

51. Let the numbers be *x* and 4*x*.

Then, $21 \times 84 = x \times 4x \Rightarrow x^2 = \frac{21 \times 84}{4} = (21)^2 \Rightarrow x = 21$.

∴ The larger number = 4 × 21 = 84.

52. Let the larger and smaller number be *x* and *y* respectively

Then, L.C.M. = 2*x* and H.C.F. = *y* – 4.

∴ $xy = 2x(y - 4) \Rightarrow xy = 2xy - 8x \Rightarrow xy = 8x \Rightarrow y = 8$.

So, the smaller number is 8.

53. Let *a* = 12*x* and *b* = 12*y*, where *x* and *y* are co-primes.

The least value of *x* and *y* for which *a* > *b* > 12 is *x* = 3 and *y* = 2.

∴ The smallest values of *a* and *b* are 36 and 24 respectively.

54. H.C.F. = 23 (given)

Since the other two factors of L.C.M. are 13 and 14

∴ L.C.M. = 23 × 13 × 14

So, there are two possible pairs of number i.e.

(23, 23 × 13 × 14) and (23 × 13, 23 × 14)

$\therefore$ The larger of the two numbers = 23 × 14 = 322.

[**Note:** (23 × 13 × 14) is not an option]

55. L.C.M. (16, 18, 20, 25) = 3600

$\Rightarrow$ Required number = 3600 k + 4, where k is a positive integer.

Least value of k for which (3600k + 4) is divisible by 7 is (k = 5).

$\therefore$ Required number = (3600 × 5 + 4) = 18004.

56. Required least number = L.C.M. (15, 20, 36, 48) + 3

= 720 + 3 = 723.

57. 2300 – 32 = 2268 and 3500 – 56 = 3444

$\therefore$ Required number = H.C.F. (2268, 3444) = 84.

58. Smallest six-digit number = 100000.

L.C.M. (4, 6, 10, 15) = 60.

On dividing 100000 by 60, the remainder is 40.

$\Rightarrow$ Least number (N) = (100000 – 40 + 60 + 2) = 100020.

$\therefore$ The sum of digits of N = 1 + 2 + 2 = 5.

59. Clearly, 15 m 17 cm = 1517 cm and 9 m 2 cm = 902 cm.

For the least number of tiles, the size of the tile must be maximum.

Maximum size of the tile = H.C.F. (1517, 902) = 41 cm.

$\therefore$ Number of tiles required

$$= \frac{\text{Area of floor}}{\text{Area of each tile}} = \frac{1517 \times 902}{41 \times 41} = 814.$$

60. L.C.M. (12, 18, 21, 32) = 2016

$\Rightarrow$ Required number is of the form 2016k and for (k = 2), the number lies between 4000 and 5000.

$\therefore$ Required number = 2016 × 2 = 4032.

61. We have: (411 – 3) = 408, (684 – 4) = 680 and (821 – 5) = 816.

$\therefore$ Required greatest number = H.C.F. (408, 680, 816) = 136.

62. Required number = H.C.F. $\left[(73-25),(97-73) \text{ and } (97-25)\right]$

= H.C.F. (24, 24, 48) = 24.

63. $A + B = x + y$ (given)

H.C.F. × L.C.M. = Product of numbers $\Rightarrow AB = xy.$

$A^3 + B^3 = (A + B)^3 - 3AB(A + B) = (x + y)^3 - 3xy(x + y)$

$= x^3 + y^3$ $[\because (a + b)^3 = a^3 + b^3 + 3ab(a + b)]$

64. Let the numbers be 7x and 7y, where x and y are co-primes.

$\therefore$ $7x \times 7y = 7 \times 140 \quad \Rightarrow 8\ xy = \frac{7 \times 140}{7 \times 7} = 20.$

The possible value of (x, y) can be (1, 20) and (4, 5).

But both the numbers will lie between 20 and 45, when x = 4 and y = 5.

So, the numbers are 28 and 35.

$\therefore$ The sum of the numbers = 28 + 35 = 63.

65. $P = 2^3.3^{10}.5$

$Q = 2^5.\ 3.7$

$\Rightarrow$ H.C.F. = $2^3.3$.

66. We have: (18 – 7) = 11, (21 – 10) = 11 and (24 – 13) = 11.

L.C.M. (18, 21, 24) = 504.

So, the required number is of the form 504k – 11.

The least value of k for which (504k – 11) is divisible by 23 is (k = 6).

$\therefore$ The required number = 504 × 6 – 11 = 3013.

67. L.C.M. (20, 28, 32, 35) = 1120.

$\therefore$ Required number = (5834 – 1120) = 4714.

68. Greatest four digits number = 9999.

L.C.M. (3, 5, 7, 9) = 315.

On dividing 9999 by 315, the remainder is 234.

Now, (3 – 1) = 2, (5 – 3) = 2, (7 – 5) = 2 and (9 – 7) = 2.

$\therefore$ Required number = (9999 – 234 – 2) = 9763.

69. Largest five digit number = 99999.

L.C.M. (16, 24, 30, 36) = 720.

On dividing 99999 by 720, the remainder is 639.

$\therefore$ Required number = (99999 – 639 + 10) = 99370.

70. Total number of animals = (945 + 2475) = 3420.

Maximum number of animals in each flock

= H.C.F. (945, 2475) = 45.

Number of flocks required = $\frac{3420}{45}$ = 76.

71. L.C.M. (30, 36, 80) = 720

$\Rightarrow$ Required number is of the form 720k + 11 and for (k = 2), the number lies between 1000 and 2000.

$\therefore$ Required number = (720 × 2 + 11) = 1451.

72. Greatest number (N)

= H.C.F. $\{(4665-1305),(6905-4665) \text{ and } (6905-1305)\}$

= H.C.F. (3360, 2240, 5600) = 1120.

$\therefore$ The sum of digits of N is 4.

73. Let the numbers be x and x + 2.

Then, $x(x + 2) = 2 \times 84 = 168$

$\Rightarrow x(x + 2) = 12 \times 14 \quad \Rightarrow x = 12$ and $x + 2 = 14$.

So, the numbers are 12 and 14.

$\therefore$ The sum of the numbers = 12 + 14 = 26.

74. Let the fraction be $\frac{x}{y}$.

When 4 is subtracted from its numerator and 1 is added to its denominator, then fraction becomes $\frac{1}{6}$.

$\therefore$ $\frac{x-4}{y+1} = \frac{1}{6} \quad \Rightarrow 6x - 24 = y + 1 \quad \Rightarrow 6x - y = 25$ ... (i)

When 2 and 1 are respectively added to its numerator and denominator, it becomes $\frac{1}{3}$.

$\therefore$ $\frac{x+2}{y+1} = \frac{1}{3} \quad \Rightarrow 3x + 6 = y + 1 \quad \Rightarrow 3x - y = -5$ ... (ii)

On solving (i) and (ii), we get x = 10 and y = 35.

So, the fraction is $\frac{10}{35}$.

Now, L.C.M. (10, 35) = 70.

❑❑❑

3 DECIMAL FRACTIONS

THEORY AND FORMULAE (TF)

TF 1. **Decimal Fractions:** Fractions in which denominators are powers of 10 are known as **decimal fractions or decimals.**

Ex. (*a*) $\frac{1}{10} = 0.1$ (*b*) $\frac{1}{100} = 0.01$ (*c*) $\frac{17}{100} = 0.17$ (*d*) $\frac{99}{10000} = 0.0099$

TF 2. **Conversion of a Decimal into Vulgar Fraction:** Put 1 in the denominator under the decimal point and annex with it as many zeros as is the number of decimal places. Now, remove the decimal point and reduce the fraction to its lowest terms.

Ex. (*a*) $0.35 = \frac{35}{100} = \frac{7}{20}$ (*b*) $3.012 = \frac{3012}{1000} = \frac{753}{250} = 3\frac{3}{250}$.

TF 3. **Like and Unlike Decimals**

(*i*) **Like Decimals:** Decimals having the same number of decimal places are called **like decimals**.

e.g. 2.01, 17.98, 108.50 are like decimals.

(*ii*) **Unlike Decimals:** Decimals having different number of decimal places are called unlike decimals.

e.g. 0.91, 1.1, 301.1013 and 0.003 are all unlike decimals.

Note: (*i*) Annexing any number of zeros to the extreme right of a decimal fraction does not change its value.

Ex. (*a*) 0.3 = 0.30 = 0.300 etc (*b*) 61.19 = 61.190 = 61.1900 etc.

(*ii*) If both numerator and denominator of a fraction contain decimals, then we first convert both into like decimals and then we remove the decimal signs from both.

Ex. $\frac{1.725}{0.31} = \frac{1.725}{0.310} = \frac{1725}{310} = \frac{345}{62} = 5\frac{35}{62}$.

TF 4. **Operations on Decimals**

(*i*) **Addition and Subtraction:** We arrange the decimal numbers in a vertical column in such a way that the decimal points in all the numbers lie one above the other. We now add or subtract in the usual way, placing the decimal point at the place where it occurs.

(*ii*) **Multiplication of a decimal number by a power of 10:** We shift the decimal point to the right by as many places as the power of 10.

Ex. (*a*) 3.17 × 10 = 31.7 (*b*) 0.085 × 100 = 8.5

(*iii*) **Multiplication of two or more decimal numbers:** Neglecting the decimal points, the numbers are multiplied as usual. Now, in the product, the decimal point is marked off to obtain as many places of decimal as is the sum of the number of decimal places in the given numbers.

Ex. 0.3 × 0.07 × 0.011 = 0.000231

(*iv*) **Division of a Decimal by a whole Number:** Perform the division by considering the dividend as a whole number. When the division of whole number part of the dividend is complete, put the decimal point in the quotient and proceed with the division as in case of whole numbers.

(*v*) **Division of a Decimal by another Decimal:** Multiply both the dividend and the divisor by a suitable power of 10 to make divisor a whole number. Now, proceed as above.

Ex. $\frac{0.065}{1.3} = \frac{0.065 \times 10}{1.3 \times 10} = \frac{0.65}{13} = 0.05$.

Note: To compare two or more fractions we convert them to decimals and then arrange them in ascending or descending order (as desired).

TF 5. Terminating and Repeating (or Recurring) Decimals

(*i*) **Terminating Decimals:** In converting a fraction into a decimal by the division method, if the remainder is zero after a certain number of steps, then the decimal obtained is a terminating decimal.

Ex. $\frac{12}{25} = 0.48$ is a terminating decimal.

However, if the division process continues indefinitely and zero remainder is never obtained, then the decimal number so obtained is known as **non-terminating decimal.**

Note: In a fraction, if the prime factorization of the denominator gives only powers of 2 and 5, then the decimal will be **terminating.**

(*ii*) **Repeating (or Recurring) Decimals:** If in a decimal, a digit or a set of digits is repeated continuously, then such a number is called a repeating (or recurring) decimal.

In a recurring decimal, if a single digit is repeated, then it is expressed by putting a dot on it. If however, a set of digits is repeated, it is expressed by putting a bar on the set.

Ex. (*a*) $\frac{2}{3} = 0.666 \ldots.. = 0.\dot{6}$ (*b*) $\frac{1}{7} = 0.142857142857\ldots.. = 0.\overline{142857}$

(*iii*) **Pure Recurring Decimal:** A decimal number in which all the digits after the decimal point are repeating is called a pure recurring decimal.

(*iv*) **Converting a Pure Recurring Decimal into Vulgar Fraction:** Write the repeated digits only once in the numerator and take as many nines in the denominator as is the number of repeating digits.

Ex. (*a*) $0.\overline{3} = \frac{3}{9} = \frac{1}{3}$ (*b*) $0.\overline{013} = \frac{13}{999}$ (*c*) $7.\overline{07} = 7 + 0.\overline{07} = 7 + \frac{7}{99} = 7\frac{7}{99}$.

(*v*) **Mixed Recurring Decimal:** A decimal number in which some digits are repeating while some digits are not repeating is called a mixed recurring decimal.

(*vi*) **Converting a Mixed Recurring Decimal into Vulgar Fraction:** The numerator is obtained by subtracting the number formed by non-repeating digits from the number formed by all the digits after the decimal point (taking the repeating digits only once). The denominator is obtained by taking the number formed by as many nines as is the number of repeating digits followed by as many zeros as is the number of non-repeating digits.

Ex. (*a*) $0.1\overline{7} = \frac{17-1}{90} = \frac{16}{90} = \frac{8}{45}$ (*b*) $0.34\overline{67} = \frac{3467-34}{9900} = \frac{3433}{9900}$.

TF 6. Fractions with prime denominators: A fraction in the lowest terms with a prime denominator other than 2 or 5 (i.e. coprime to 10) always produces a pure repeating decimal.

The period of the repeating decimal (i.e. the length of the repetend) is $(p - 1)$ if p does not divide any number of the form 999 .... 99.

Ex. (*a*) $\frac{1}{7} = 0.\overline{142857}$ (6 repeating digits) (*b*) $\frac{1}{17} = 0.\overline{0588235294117647}$ (16 repeating digits)

(*c*) $\frac{1}{19} = 0.\overline{052631578947368421}$ (18 repeating digits) etc.

If however, p divides a number of the form 999 .... 99, then the repetend length is a factor of $(p - 1)$. In such cases, the repetend length is equal to the number of 9's in the smallest number of the form 999 ... 99 which is divisible by p.

Ex. (*a*) $\frac{1}{3} = 0.\dot{3}$ (1 repeating digits since 3 divides 9)

(*b*) $\frac{1}{11} = 0.\overline{09}$ (2 repeating digits since 11 divides 99)

(*c*) $\frac{1}{13} = 0.\overline{076923}$ (6 repeating digits since 13 divides 999999)

(*d*) $\frac{1}{37} = 0.\overline{027}$ (3 repeating digits since 37 divides 999)

(*e*) $\frac{1}{41} = 0.\overline{02439}$ (5 repeating digits since 41 divides 99999) etc.

EXERCISE

1. $\dfrac{0.3555 \times 0.5555 \times 2.025}{0.225 \times 1.7775 \times 0.2222}$ is equal to **[SSC 2012]**

(*a*) 4.5 (*b*) 4.58
(*c*) 5.4 (*d*) 5.45

2. If $\dfrac{547.527}{0.0082} = x$, then the value of $\dfrac{547527}{82}$ is **[SSC 2012]**

(*a*) $\dfrac{x}{10}$ (*b*) $\dfrac{x}{100}$
(*c*) $10x$ (*d*) $100x$

3. $\dfrac{(2.3)^3 + 0.027}{(2.3)^2 - 0.69 + 0.09}$ is equal to

(*a*) 2.00 (*b*) 2.33
(*c*) 2.60 (*d*) 2.80

4. On simplification 3034 – 3(1002 ÷ 20.04) is equal to

(*a*) 2543 (*b*) 2884
(*c*) 2993 (*d*) 3029

5. $\left\{\dfrac{(0.1)^2 - (0.01)^2}{0.0001} + 1\right\}$ is equal to **[SSC CGL 2010]**

(*a*) 100 (*b*) 101
(*c*) 110 (*d*) 1010

6. The value of $\left[\dfrac{(0.337 + 0.126)^2 - (0.337 - 0.126)^2}{0.337 \times 0.126}\right]$ is

(*a*) 0.211 (*b*) 0.4246
(*c*) 0.463 (*d*) 4

7. The simplification of $(0.\overline{63} + 0.\overline{37} + 0.\overline{80})$ yields the result

(*a*) $1.\overline{81}$ (*b*) $1.\overline{80}$
(*c*) 1.80 (*d*) $1.\overline{79}$

8. Simplify : $\dfrac{0.0347 \times 0.0347 \times 0.0347 + (0.9653)^3}{(0.0347)^2 - (0.347)(0.09653) + (0.9653)^2}$ **[SSC CGL 2011]**

(*a*) 0.9306 (*b*) 1
(*c*) 1.0009 (*d*) 1.0050

9. The value of $(0.98)^3 + (0.02)^3 + 3 \times 0.98 \times 0.02 - 1$ is

(*a*) 0 (*b*) 1
(*c*) 1.09 (*d*) 1.98

10. Out of the numbers 0.3, 0.03, 0.9, 0.09 the number that is nearest to the value of $\sqrt{0.9}$ is **[SS 2013]**

(*a*) 0.03 (*b*) 0.09
(*c*) 0.3 (*d*) 0.9

11. The value of $\left(\sqrt[3]{3.5} + \sqrt[3]{2.5}\right)\left\{\left(\sqrt[3]{3.5}\right)^2 - \sqrt[3]{8.75} + \left(\sqrt[3]{2.5}\right)^2\right\}$ is

(*a*) 1 (*b*) 5
(*c*) 5.375 (*d*) 6

12. $8.7 - \left[7.6 - \left\{6.5 - \left(5.4 - \overline{4.3 - 2}\right)\right\}\right]$ is simplified to

(*a*) 5.5 (*b*) 4.5
(*c*) 3.5 (*d*) 2.5

13. Simplify : $\dfrac{5.32 \times 56 + 5.32 \times 44}{(7.66)^2 - (2.34)^2}$

(*a*) 12 (*b*) 10
(*c*) 8.5 (*d*) 7.2

14. $\dfrac{4.41 \times 0.16}{2.1 \times 1.6 \times 0.21}$ is simplified to **[SSC CGL 2010]**

(*a*) 0.01 (*b*) 0.1
(*c*) 1 (*d*) 10

15. Find the value of $(1.28)^3 + (0.72)^3 + 6 \times 1.28 \times 0.72 - 8$.

(*a*) – 1.24 (*b*) 0
(*c*) 1.24 (*d*) 2.56

16. The simplification of $3.\overline{36} - 2.\overline{05} + 1.\overline{33}$ equals

(*a*) $2.\overline{64}$ (*b*) 2.64
(*c*) $2.\overline{61}$ (*d*) 2.60

17. Simplify : [0.9 – (2.3 – 3.2 – (7.1 – 5.4 – 3.5))]

(*a*) 0 (*b*) 0.18
(*c*) 1.8 (*d*) 2

18. $\dfrac{8(3.75)^3 + 1}{(7.5)^2 - 6.5}$ is equal to

(*a*) $\dfrac{9}{5}$ (*b*) 8.5
(*c*) 4.75 (*d*) 2.75

19. The ascending order of $(2.89)^{0.5}$, $2 - (0.5)^2$, $\sqrt{3}$ and $\sqrt[3]{0.008}$ is **[SSC CGL 2013]**

(*a*) $\sqrt[3]{0.008}$, $\sqrt{3}$, $(2.89)^{0.5}$, $2 - (0.5)^2$
(*b*) $\sqrt{3}$, $\sqrt[3]{0.008}$, $2 - (0.5)^2$, $(2.89)^{0.5}$
(*c*) $2 - (0.5)^2$, $\sqrt{3}$, $\sqrt[3]{0.008}$, $(2.89)^{0.5}$
(*d*) $(2.89)^{0.5}$, $\sqrt{3}$, $2 - (0.5)^2$, $\sqrt[3]{0.008}$

20. The simplified value of $[(0.111)^3 + (0.222)^3 - (0.333)^3 + (0.333)^2 (0.222)]^3$ is

(*a*) 0 (*b*) 0.111
(*c*) 0.888 (*d*) 0.999

21. The value of $\dfrac{(3.2)^3 - 0.008}{(3.2)^2 + 0.64 + 0.04}$ is **[SSC CGL 2011]**

(*a*) 0 (*b*) 2.994
(*c*) 3 (*d*) 3.208

22. The value of $\dfrac{0.1\times 0.1\times 0.1+0.02\times 0.02\times 0.02}{0.2\times 0.2\times 0.2+0.04\times 0.04\times 0.04}$ is
(*a*) 0.5 (*b*) 0.25
(*c*) 0.125 (*d*) 0.0125

23. $(0.2 \times 0.2 + 0.01)\ (0.1 \times 0.1 + 0.02)^{-1}$ equal to
(*a*) $\dfrac{5}{3}$ (*b*) $\dfrac{9}{5}$
(*c*) $\dfrac{41}{4}$ (*d*) $\dfrac{41}{12}$

24. $\dfrac{0.8\overline{3}\div 7.5}{2.3\overline{21}-0.\overline{098}}$ is equal to
(*a*) 0.05 (*b*) 0.06
(*c*) 0.1 (*d*) 0.6

25. $\dfrac{10.3\times 10.3\times 10.3+1}{10.3\times 10.3-10.3+1}$ is equal to
(*a*) 12.3 (*b*) 11.3
(*c*) 10.3 (*d*) 9.3

26. The value of $\dfrac{0.125+0.027}{0.25-0.15+0.09}$ is **[SSC CGL 2008 & 2010]**
(*a*) 0.8 (*b*) 0.3
(*c*) 0.25 (*d*) 0.2

27. The largest among the numbers 0.9, $(0.9)^2$, $\sqrt{0.9}$, $0.\overline{9}$ is **[SSC 2010]**
(*a*) $\sqrt{0.9}$ (*b*) $0.\overline{9}$
(*c*) 0.9 (*d*) $(0.9)^2$

28. The value of $2+\sqrt{0.09}-\sqrt[3]{0.008}-75\%\text{ of }2.80$ is
(*a*) 0.001 (*b*) 0.01
(*c*) 0 (*d*) – 1

29. $(0.9 \times 0.9 \times 0.9 + 0.1 \times 0.1 \times 0.1)$ is equal to **[SSC CPO 2010]**
(*a*) 1.00 (*b*) 0.91
(*c*) 0.82 (*d*) 0.73

30. The value of $\sqrt[3]{\dfrac{0.2\times 0.2\times 0.2+0.04\times 0.04\times 0.04}{0.4\times 0.4\times 0.4+0.08\times 0.08\times 0.08}}$ is
(*a*) 0.25 (*b*) 0.5
(*c*) 0.75 (*d*) 0.125

31. Simplify : $\dfrac{3.25\times 3.25+1.75\times 1.75-2\times 3.25\times 1.75}{3.25\times 3.25-1.75\times 1.75}$ **[SSC CPO 2010]**
(*a*) 0.2 (*b*) 0.3
(*c*) 0.4 (*d*) 0.5

32. The value of
$\dfrac{0.9\times 0.9\times 0.9+0.2\times 0.2\times 0.2+0.3\times 0.3\times 0.3-3\times 0.9\times 0.2\times 0.3}{0.9\times 0.9+0.2\times 0.2+0.3\times 0.3-0.9\times 0.2-0.2\times 0.3-0.3\times 0.9}$ is
(*a*) 0.0054 (*b*) 0.8
(*c*) 1.0 (*d*) 1.4

33. $(0.05 \times 5 - 0.005 \times 5)$ equals
(*a*) 0.0225 (*b*) 0.225
(*c*) 0.275 (*d*) 2.250

34. Simplify : $\left(0.\overline{1}\right)\left\{1-9\left(0.1\overline{6}\right)^2\right\}$
(*a*) $\dfrac{1}{108}$ (*b*) $\dfrac{1}{109}$
(*c*) $-\dfrac{1}{162}$ (*d*) $\dfrac{7696}{10^6}$

35. If $\sqrt[2]{0.014\times 0.14\,x} = 0.014\times 0.14\sqrt[2]{y}$, then find the value of $\dfrac{x}{y}$.
(*a*) 0.196 (*b*) 0.0196
(*c*) 0.00196 (*d*) 0.000196

36. Simplify : $\dfrac{(0.05)^2+(0.41)^2+(0.073)^2}{(0.005)^2+(0.041)^2+(0.0073)^2}$ **[SSC CGL 2011]**
(*a*) 10 (*b*) 100
(*c*) 1000 (*d*) None of these

37. $\dfrac{3.25\times 3.20-3.20\times 3.05}{0.064}$ is equal to **[SSC CGL 2010]**
(*a*) $\dfrac{1}{2}$ (*b*) $\dfrac{1}{10}$
(*c*) 1 (*d*) 10

38. $\dfrac{(4.53-3.07)^2}{(3.07-2.15)(2.15-4.53)}+\dfrac{(3.07-2.15)^2}{(2.15-4.53)(4.53-3.07)}$ $+\dfrac{(2.15-4.53)^2}{(4.53-3.07)(3.07-2.15)}$ simplified to
(*a*) 3 (*b*) 2
(*c*) 1 (*d*) 0

39. The value of $0.008 \times 0.01 \times 0.072 \div (0.12 \times 0.0004)$ is
(*a*) 0.012 (*b*) 0.12
(*c*) 1.02 (*d*) 1.2

40. Which one is the largest among the fractions (5/113), (7/120), (13/145) and (17/160)? **[SSC CGL 2017]**
(*a*) 5/113 (*b*) 7/120
(*c*) 13/145 (*d*) 17/160

41. If $p/q = r/s = t/u = \sqrt{5}$, then what is the value of $[(3p^2 + 4r^2 + 5t^2)/(3q^2 + 4s^2 + 5u^2)]$? **[SSC CGL 2017]**
(*a*) 1/5 (*b*) 5
(*c*) 25 (*d*) 60

42. Which of the following is the smallest fraction?
$$\frac{8}{25}, \frac{7}{23}, \frac{11}{23}, \frac{14}{53}$$
(*a*) $\dfrac{7}{23}$ (*b*) $\dfrac{8}{25}$
(*c*) $\dfrac{11}{23}$ (*d*) $\dfrac{14}{53}$

43. Arrangement of the fractions $\frac{4}{3}, -\frac{2}{9}, -\frac{7}{8}, \frac{5}{12}$, into ascending order is **[SSC 2015]**

(*a*) $-\frac{7}{8}, -\frac{2}{9}, \frac{5}{12}, \frac{4}{3}$ (*b*) $-\frac{7}{8}, -\frac{2}{9}, \frac{4}{3}, \frac{5}{12}$
(*c*) $-\frac{2}{9}, -\frac{7}{8}, \frac{5}{12}, \frac{4}{3}$ (*d*) $-\frac{2}{9}, -\frac{7}{8}, \frac{5}{12}, \frac{4}{3}$

44. Sum of three fractions is $2\frac{11}{24}$.On dividing the largest fraction by the smallest fraction, $\frac{7}{6}$ is obtained which is $\frac{1}{3}$ greater than the middle fraction. The smallest fraction is **[SSC CGL 2014 & 2015]**

(*a*) $\frac{3}{4}$ (*b*) $\frac{5}{6}$
(*c*) $\frac{5}{8}$ (*d*) $\frac{7}{3}$

45. Divide 50 into two parts so that the sum of their reciprocals is $\frac{1}{12}$. **[SSC 2013]**

(*a*) 20, 30 (*b*) 24, 36
(*c*) 28, 22 (*d*) 35, 15

46. In an office, there are 108 tables and 132 chairs. If $\frac{1}{6}$ of the tables and $\frac{1}{4}$ of the chairs are broken. How many people can work in the office if each person requires one table and one chair? **[SSC MTS 2013]**

(*a*) 99 (*b*) 92
(*c*) 90 (*d*) 86

47. A school group charters three identical buses and occupies $\frac{4}{5}$ of the seats. After $\frac{1}{4}$ of the passengers leave, the remaining passengers use only two of the buses. The fraction of the seats on the two buses that are now occupied is **[SSC CGL 2015]**

(*a*) $\frac{8}{9}$ (*b*) $\frac{9}{10}$
(*c*) $\frac{7}{9}$ (*d*) $\frac{7}{10}$

48. Neeraj left $\frac{1}{3}$ of his property to his wife and $\frac{3}{5}$ of the remainder to his daughter. He gave the rest to his son who received ₹ 6400. How much was his original property worth? **[SSC 2014]**

(*a*) ₹ 1600 (*b*) ₹ 16000
(*c*) ₹ 24000 (*d*) ₹ 32000

49. If the sum of two numbers, one of which is $\frac{2}{5}$ times the other is 50, then the numbers are **[SSC CGL 2015]**

(*a*) $\frac{115}{7}$ and $\frac{235}{7}$ (*b*) $\frac{150}{7}$ and $\frac{200}{7}$
(*c*) $\frac{240}{7}$ and $\frac{110}{7}$ (*d*) $\frac{250}{7}$ and $\frac{100}{7}$

50. A number whose one-fifth part increased by 4 is equal to its one-fourth part diminished by 10, find the number. **[SSC 2011]**

(*a*) 240 (*b*) 260
(*c*) 270 (*d*) 280

51. The numerator of a fraction is 4 less than its denominator. If the numerator is decreased by 2 and the denominator is increased by 1, then the denominator becomes eight times the numerator. Find the fraction. **[SSC CGL 2013]**

(*a*) $\frac{2}{7}$ (*b*) $\frac{3}{7}$
(*c*) $\frac{3}{8}$ (*d*) $\frac{4}{8}$

52. A fraction becomes $\frac{9}{11}$, if 2 is added to both the numerator and the denominator. If 3 is added to both the numerator and the denominator, then it becomes $\frac{5}{6}$. What is the fraction? **[SSC CGL 2013]**

(*a*) $\frac{3}{7}$ (*b*) $\frac{5}{9}$
(*c*) $\frac{7}{9}$ (*d*) $\frac{7}{10}$

53. A tree increases annually by $\frac{1}{8}$th of its height. By how much will it increase after 2 years, if it stands 64 cm high today? **[SSC FCI 2012]**

(*a*) 81 cm (*b*) 75 cm
(*c*) 74 cm (*d*) 72 cm

54. The denominator of a fraction is 3 more than its numerator. If the numerator is increased by 7 and the denominator is decreased by 2, we obtain 2. The sum of numerator and denominator of the fraction is **[SSC 2011]**

(*a*) 19 (*b*) 17
(*c*) 13 (*d*) 5

55. A boy was asked to find $\frac{3}{5}$ of a fraction. Instead, he divided the fraction by $\frac{3}{5}$ and got an answer which exceeded the correct answer by $\frac{32}{75}$. The correct answer is

(*a*) $\frac{2}{15}$ (*b*) $\frac{2}{25}$
(*c*) $\frac{3}{25}$ (*d*) $\frac{6}{25}$

56. How many $\frac{1}{6}$ all together make $41\frac{2}{3}$? **[SSC 2010]**

(*a*) 350 (*b*) 250
(*c*) 150 (*d*) 125

57. A fraction having denominator 30 and lying between $\frac{5}{8}$ and $\frac{7}{11}$ is **[SSC 2010]**

(*a*) $\frac{21}{30}$ (*b*) $\frac{20}{30}$
(*c*) $\frac{19}{30}$ (*d*) $\frac{18}{30}$

58. A tin of oil was $\frac{4}{5}$ full. When 6 bottles of oil were taken out and 4 bottles of oil was poured into it, it was $\frac{3}{4}$ full. How many bottles of oil can the tin contain?

(*a*) 40 (*b*) 30
(*c*) 20 (*d*) 10

59. A runner runs $1\frac{1}{4}$ laps of a 5 lap race. What fractional part of the race remains to be run?

(*a*) $\frac{2}{3}$ (*b*) $\frac{3}{4}$
(*c*) $\frac{4}{5}$ (*d*) $\frac{15}{4}$

60. In a class, $\frac{3}{5}$ of the students are girls and rest are boys. If $\frac{2}{9}$ of the girls and $\frac{1}{4}$ of the boys are absent. What part of the total number of students are present?

(*a*) $\frac{17}{25}$ (*b*) $\frac{18}{49}$
(*c*) $\frac{23}{30}$ (*d*) $\frac{23}{36}$

61. Express 45 minutes as the fraction of the day.

(*a*) $\frac{1}{24}$ (*b*) $\frac{1}{32}$
(*c*) $\frac{1}{40}$ (*d*) $\frac{1}{60}$

62. If one-third of one-fourth of a number is 15, then three-tenths of the number is

(*a*) 54 (*b*) 45
(*c*) 36 (*d*) 35

63. A student was asked to find $\frac{5}{16}$ of a number. By mistake he found $\frac{5}{6}$ of that number. His answer was 250 more than the correct answer. Find the given number.

(*a*) 300 (*b*) 450
(*c*) 480 (*d*) 500

64. By how much does $\frac{6}{7/8}$ exceed $\frac{6/7}{8}$?

(*a*) $6\frac{1}{8}$ (*b*) $6\frac{3}{4}$
(*c*) $7\frac{3}{4}$ (*d*) $7\frac{5}{6}$

65. The sum of the numerator and denominator of a positive fraction is 11. If 2 is added to both numerator and denominator, the fraction is increased by $\frac{1}{24}$. The difference of numerator and denominator of the fraction is **[SSC 2011]**

(*a*) 1 (*b*) 3
(*c*) 5 (*d*) 9

66. The number $2.\overline{52}$, when written as a fraction and reduced to its lowest terms, the sum of the numerator and denominator is **[SSC FCI 2012]**

(*a*) 349 (*b*) 141
(*c*) 29 (*d*) 7

67. In a school $\frac{1}{10}$ of the boys are same in number as $\frac{1}{4}$ of the girls and $\frac{5}{8}$ of the girls are same in number as $\frac{1}{4}$ of the boys. The ratio of the boys to girls in that school is **[SSC GD 2013]**

(*a*) 2 : 1 (*b*) 3 : 2
(*c*) 4 : 3 (*d*) 5 : 2

68. *A*, *B*, *C* and *D* purchase a gift worth ₹ 60. *A* pays $\frac{1}{2}$ of what others are paying, *B* pays $\frac{1}{3}$ of what others are paying and *C* pays $\frac{1}{4}$ of what others are paying. What is the amount paid by *D*? **[SSC CGL 2013]**

(*a*) 13 (*b*) 14
(*c*) 15 (*d*) 16

69. If $\frac{3}{4}$ of a number is 7 more than $\frac{1}{6}$ of the number, then $\frac{5}{3}$ of the number is **[SSC CGL 2015]**

(*a*) 12 (*b*) 15
(*c*) 18 (*d*) 20

70. A student was asked to multiply a given number by $\frac{8}{17}$. Instead, he divided the number by $\frac{8}{17}$. His answer was 225 more than the correct answer. The given number was **[SSC 2011]**

(*a*) 64 (*b*) 136
(*c*) 225 (*d*) 289

71. The product of two fractions is $\frac{14}{15}$ and their quotient is $\frac{35}{24}$. The greater of the fractions is

(*a*) $\frac{4}{5}$ (*b*) $\frac{7}{3}$

(*c*) $\frac{7}{4}$ (*d*) $\frac{7}{6}$

72. $\frac{1}{10}$ of a rod is coloured red, $\frac{1}{20}$ orange, $\frac{1}{30}$ yellow, $\frac{1}{40}$ green, $\frac{1}{50}$ blue, $\frac{1}{60}$ black and the rest in violet. If the length of the violet portion of the rod is 12.08 metres, then the length of the rod is

(*a*) 30 m (*b*) 20 m
(*c*) 18 m (*d*) 16 m

73. A man spends $\frac{1}{3}$ of his income on food, $\frac{2}{5}$ of his income on house rent and $\frac{1}{5}$ of his income on clothes. If he still has ₹ 400 left with him, his income is

(*a*) ₹ 7000 (*b*) ₹ 6000
(*c*) ₹ 5000 (*d*) ₹ 4000

74. $\frac{1}{2}$ of $\frac{3}{4}$ of a number is $2\frac{1}{2}$ of 10. What is the number?

(*a*) 50 (*b*) 56
(*c*) 60 (*d*) $66\frac{2}{3}$

ANSWERS

1. (*a*)	**2.** (*a*)	**3.** (*c*)	**4.** (*b*)	**5.** (*a*)	**6.** (*d*)	**7.** (*a*)	**8.** (*b*)	**9.** (*a*)	**10.** (*d*)
11. (*d*)	**12.** (*b*)	**13.** (*b*)	**14.** (*c*)	**15.** (*a*)	**16.** (*a*)	**17.** (*a*)	**18.** (*b*)	**19.** (*d*)	**20.** (*a*)
21. (*c*)	**22.** (*c*)	**23.** (*a*)	**24.** (*a*)	**25.** (*b*)	**26.** (*a*)	**27.** (*b*)	**28.** (*c*)	**29.** (*d*)	**30.** (*b*)
31. (*b*)	**32.** (*d*)	**33.** (*b*)	**34.** (*a*)	**35.** (*c*)	**36.** (*b*)	**37.** (*d*)	**38.** (*a*)	**39.** (*b*)	**40.** (*d*)
41. (*b*)	**42.** (*d*)	**43.** (*a*)	**44.** (*b*)	**45.** (*a*)	**46.** (*c*)	**47.** (*b*)	**48.** (*c*)	**49.** (*d*)	**50.** (*d*)
51. (*b*)	**52.** (*c*)	**53.** (*a*)	**54.** (*c*)	**55.** (*d*)	**56.** (*b*)	**57.** (*c*)	**58.** (*a*)	**59.** (*b*)	**60.** (*c*)
61. (*b*)	**62.** (*a*)	**63.** (*c*)	**64.** (*b*)	**65.** (*a*)	**66.** (*a*)	**67.** (*d*)	**68.** (*a*)	**69.** (*d*)	**70.** (*b*)
71. (*d*)	**72.** (*d*)	**73.** (*b*)	**74.** (*d*)						

SOLUTIONS

1. $\frac{0.3555 \times 0.5555 \times 2.025}{0.225 \times 1.7775 \times 0.2222} = \frac{3555 \times 5555 \times 2025}{225 \times 17775 \times 2222} = 4.5.$

2. $\frac{547.527}{0.0082} = x \Rightarrow \frac{547527}{1000} \times \frac{10000}{82} = x$

$\Rightarrow \frac{547527}{82} \times 10 = x \Rightarrow \frac{547527}{82} = \frac{x}{10}.$

3. $\frac{(2.3)^3 + 0.027}{(2.3)^2 - 0.69 + 0.09} = \frac{(2.3)^3 + (0.3)^3}{(2.3)^2 - 2.3 \times 0.3 + (0.3)^2}$

$= \frac{(2.3 + 0.3)\left[(2.3)^2 - 2.3 \times 0.3 + (0.3)^2\right]}{(2.3)^2 - 2.3 \times 0.3 + (0.3)^2}$

$\left[\because a^3 + b^3 = (a+b)\left(a^2 - ab + b^2\right)\right]$

$= 2.3 + 0.3 = 2.60.$

4. $3034 - 3\,(1002 \div 20.04)$

$= 3034 - 3\left(\frac{1002}{20.04}\right) = 3034 - 3\left(\frac{1002 \times 100}{2004}\right) = 3034 - 3 \times 50$

$= 3034 - 150 = 2884.$

5. $\left\{\frac{(0.1)^2 - (0.01)^2}{0.0001} + 1\right\} = \frac{0.01 - 0.0001}{0.0001} + 1$

$= \frac{0.0099}{0.0001} + 1 = 99 + 1 = 100.$

6. $\frac{(0.337 + 0.126)^2 - (0.337 - 0.126)^2}{0.337 \times 0.126}$

$= \frac{4 \times 0.337 \times 0.126}{0.337 \times 0.126} = 4.$ $\left[\because (a+b)^2 - (a-b)^2 = 4ab\right]$

7. $\left(0.\overline{63} + 0.\overline{37} + 0.\overline{80}\right)$

$= \frac{63}{99} + \frac{37}{99} + \frac{80}{99} = \frac{180}{99} = \frac{20}{11} = 1\frac{9}{11} = 1.\overline{81}.$

8. $\frac{0.0347 \times 0.0347 \times 0.0347 + (0.9653)^3}{(0.0347)^2 - (0.347)(0.09653) + (0.9653)^2}$

$= \frac{(0.0347)^3 + (0.9653)^3}{(0.0347)^2 - (0.0347)(0.9653) + (0.9653)^2}$

$= \frac{a^3 + b^3}{a^2 - ab + b^2}$ where $a = 0.0347$ and $b = 0.9653$

$= \frac{(a+b)\left(a^2 - ab + b^2\right)}{\left(a^2 - ab + b^2\right)} = (a+b) = 0.0347 + 0.9653 = 1.$

9. $(0.98)^3 + (0.02)^3 + 3 \times 0.98 \times 0.02 - 1$

$= (0.98)^3 + (0.02)^3 + (-1)^3 - 3 \times 0.98 \times 0.02 \times (-1)$

$= a^3 + b^3 + c^3 - 3abc$ where $a = 0.98$, $b = 0.02$, $c = -1$

$= 0.$ $\left[\because a^3 + b^3 + c^3 - 3abc = 0 \text{ when } a + b + c = 0 \text{ and we have : } a + b + c = 0.98 + 0.02 - 1 = 0\right]$

10. $(0.3)^2 = 0.09$, $(0.03)^2 = 0.0009$, $(0.9)^2 = 0.81$ and $(0.09)^2 = 0.0081$.

Clearly, 0.81 is the nearest one to 0.9

$\therefore \sqrt{0.81}$ i.e. 0.9 is the nearest one to $\sqrt{0.9}$.

11. Let $x = \sqrt[3]{3.5}$ and $y = \sqrt[3]{2.5}$

$$\left(\sqrt[3]{3.5}+\sqrt[3]{2.5}\right)\left\{\left(\sqrt[3]{3.5}\right)^2-\sqrt[3]{8.75}+\left(\sqrt[3]{2.5}\right)^2\right\}$$

$$= (x+y)(x^2-xy+y^2) = x^3+y^3$$

$$= \left(\sqrt[3]{3.5}\right)^3+\left(\sqrt[3]{2.5}\right)^3 = 3.5+2.5 = 6.$$

12. $8.7-\left[7.6-\left\{6.5-\left(5.4-\overline{4.3-2}\right)\right\}\right]$

$= 8.7 - [7.6 - \{6.5 - (5.4 - 2.3)\}]$

$= 8.7 - [7.6 - \{6.5 - 3.1\}] = 8.7 - [7.6 - 3.4]$

$= 8.7 - 4.2 = 4.5.$

13. $$\frac{5.32\times 56+5.32\times 44}{(7.66)^2-(2.34)^2} = \frac{5.32\,(56+44)}{(7.66+2.34)(7.66-2.34)}$$

$$\left[\because a^2-b^2 = (a+b)(a-b)\right]$$

$$= \frac{5.32\times 100}{10\times 5.32} = 10.$$

14. $$\frac{4.41\times 0.16}{2.1\times 1.6\times 0.21} = \frac{(2.1)^2\times(0.4)^2}{(2.1)\times(0.16\times 10)\times\left(2.1\times\frac{1}{10}\right)}$$

$$= \frac{(2.1)^2\times(0.4)^2}{(2.1)^2\times(0.4)^2} = 1.$$

15. Let $x = 1.28$, $y = 0.72$ and $z = -2$

then $x + y + z = 1.28 + 0.72 - 2 = 0$

Now, $(1.28)^3 + (0.72)^3 + 6 \times 1.28 \times 0.72 - 8$

$= (1.28)^3 + (0.72)^3 + (-2)^3 - 3 \times (-2) \times 1.28 \times 0.72$

$= x^3 + y^3 + z^3 - 3xyz = 0.$

16. $3.\overline{36}-2.\overline{05}+1.\overline{33} = 3+0.\overline{36}-2-0.\overline{05}+1+0.\overline{33}$

$$= 3+\frac{36}{99}-2-\frac{5}{99}+1+\frac{33}{99} = (3-2+1)+\left(\frac{36-5+33}{99}\right)$$

$$= 2+\frac{64}{99} = 2+0.\overline{64} = 2.\overline{64}.$$

17. $[0.9 - (2.3 - 3.2 - (7.1 - 5.4 - 3.5))]$

$= [0.9 - (2.3 - 3.2 - (-1.8))] = [0.9 - (2.3 - 3.2 + 1.8)]$

$= [0.9 - 0.9] = 0.$

18. $$\frac{8(3.75)^3+1}{(7.5)^2-6.5} = \frac{(2\times 3.75)^3+1^3}{(7.5)^2-7.5+1} = \frac{(7.5)^3+1^3}{(7.5)^2-(7.5\times 1)+1^2}$$

$$= \frac{a^3+b^3}{a^2-ab+b^2}\text{, where } a = 7.5 \text{ and } b = 1$$

$$= \frac{(a+b)(a^2-ab+b^2)}{(a^2-ab+b^2)} = a+b = 7.5+1 = 8.5.$$

19. $(2.89)^{0.5} = (2.89)^{\frac{1}{2}} = 1.7$

$2 - (0.5)^2 = 2 - 0.25 = 1.75$

$\sqrt{3} = 1.732$

$\sqrt[3]{0.008} = \sqrt[3]{(0.2)^3} = 0.2.$

Clearly, $0.2 < 1.7 < 1.732 < 1.75$

$\therefore \sqrt[3]{0.008} < (2.89)^{0.5} < \sqrt{3} < 2 - (0.5)^2.$

20. $[(0.111)^3 + (0.222)^3 - (0.333)^3 + (0.333)^2 (0.222)]^3$

$= [(0.111 \times 1)^3 + (0.111 \times 2)^3 - (0.111 \times 3)^3 + (0.111 \times 3)^2 (0.111 \times 2)]^3$

$= (0.111)^3 [1^3 + 2^3 - 3^3 + (3^2 \times 2)]^3$

$= (0.111)^3 [1 + 8 - 27 + 18]^3 = (0.111)^3 \times 0 = 0.$

21. $$\frac{(3.2)^3-0.008}{(3.2)^2+0.64+0.04} = \frac{(3.2)^3-(0.2)^3}{(3.2)^2+(3.2)(0.2)+(0.2)^2}$$

$$= \frac{a^3-b^3}{a^2+ab+b^2} \text{ where } a = 3.2 \text{ and } b = 0.2$$

$$= \frac{(a-b)(a^2+ab+b^2)}{(a^2+ab+b^2)} = a-b = 3.2-0.2 = 3.$$

22. $$\frac{0.1\times 0.1\times 0.1+0.02\times 0.02\times 0.02}{0.2\times 0.2\times 0.2+0.04\times 0.04\times 0.04} = \frac{(0.1)^3+(0.02)^3}{(0.2)^3+(0.04)^3}$$

$$= \frac{(0.1)^3+(0.02)^3}{(2\times 0.1)^3+(2\times 0.02)^3} = \frac{(0.1)^3+(0.02)^3}{8(0.1)^3+8(0.02)^3}$$

$$= \frac{(0.1)^3+(0.02)^3}{8\left\{(0.1)^3+(0.02)^3\right\}} = \frac{1}{8} = 0.125.$$

23. $(0.2\times 0.2+0.01)(0.1\times 0.1+0.02)^{-1}$

$$= \frac{(0.2\times 0.2+0.01)}{(0.1\times 0.1+0.02)} = \frac{2\times 0.1\times 2\times 0.1+0.01}{0.1\times 0.1+2\times 0.01}$$

$$= \frac{4\times 0.01+0.01}{0.01+2\times 0.01} = \frac{(0.01)(4+1)}{(0.01)(1+2)} = \frac{5}{3}.$$

24. $$\frac{0.8\overline{3}\div 7.5}{2.\overline{321}-0.\overline{098}} = \frac{0.8\overline{3}\div 7.5}{\left(2+0.\overline{321}\right)-0.\overline{098}}$$

$$= \frac{\left(\frac{83-8}{90}\right)\div\frac{75}{10}}{2+\left(\frac{321-3}{990}\right)-\left(\frac{98}{990}\right)} = \frac{\frac{75}{90}\times\frac{10}{75}}{2+\left(\frac{318-98}{990}\right)}$$

$$= \frac{\frac{1}{9}}{\left(\frac{1980+220}{990}\right)} = \frac{1}{9}\times\frac{990}{2200} = \frac{1}{20} = 0.05.$$

25. Suppose $x = 10.3$ and $y = 1$. Then,

$$\frac{10.3\times 10.3\times 10.3+1}{10.3\times 10.3-10.3+1} = \frac{(10.3)^3+(1)^3}{(10.3)^2-10.3\times 1+(1)^2}$$

$$= \frac{x^3+y^3}{x^2-xy+y^2} = \frac{(x+y)(x^2-xy+y^2)}{x^2-xy+y^2}$$

$$= x+y = 10.3+1 = 11.3.$$

26. $$\frac{0.125+0.027}{0.25-0.15+0.09} = \frac{(0.5)^3+(0.3)^3}{(0.5)^2-(0.5)(0.3)+(0.3)^2}$$

$$= \frac{a^3+b^3}{a^2-ab+b^2} \text{ where } a = 0.5,\ b = 0.3$$

$$= \frac{(a+b)(a^2-ab+b^2)}{(a^2-ab+b^2)} = a+b = 0.5+0.3 = 0.8.$$

27. $(0.9)^2 = 0.81$, $\sqrt{0.9} = 0.95$ and $0.\overline{9} = 0.999....$

Clearly, $0.\overline{9} > \sqrt{0.9} > 0.9 > (0.9)^2$

$\therefore$ The largest one is $0.\overline{9}$.

28. $2+\sqrt{0.09}-\sqrt[3]{0.008} - 75\%$ of 2.80

$$= 2+\sqrt{0.3\times 0.3}-\sqrt[3]{0.2\times 0.2\times 0.2}-\left(\frac{75}{100}\times 2.80\right)$$

$= 2 + 0.3 - 0.2 - 2.10 = 0.$

29. $(0.9 \times 0.9 \times 0.9 + 0.1 \times 0.1 \times 0.1) = (0.9)^3 + (0.1)^3$

$= 0.729 + 0.001 = 0.73.$

30. $$\sqrt[3]{\frac{0.2\times 0.2\times 0.2+0.04\times 0.04\times 0.04}{0.4\times 0.4\times 0.4+0.08\times 0.08\times 0.08}}$$

$$= \sqrt[3]{\frac{(0.2)^3+(0.04)^3}{(0.4)^3+(0.08)^3}} = \sqrt[3]{\frac{(0.2)^3+(0.04)^3}{(2\times 0.2)^3+(2\times 0.04)^3}}$$

$$= \sqrt[3]{\frac{(0.2)^3+(0.04)^3}{8(0.2)^3+8(0.04)^3}} = \sqrt[3]{\frac{(0.2)^3+(0.04)^3}{8\{(0.2)^3+(0.04)^3\}}}$$

$$= \sqrt[3]{\frac{1}{8}} = \frac{1}{\sqrt[3]{8}} = \frac{1}{2} = 0.5.$$

31. Let $x = 3.25$ and $y = 1.75$

$$\frac{3.25\times 3.25+1.75\times 1.75-2\times 3.25\times 1.75}{3.25\times 3.25-1.75\times 1.75} = \frac{x^2+y^2-2xy}{x^2-y^2}$$

$$= \frac{(x-y)^2}{(x+y)(x-y)} \qquad [\because a^2-b^2 = (a+b)(a-b)]$$

$$= \frac{x-y}{x+y} = \frac{3.25-1.75}{3.25+1.75} = \frac{1.5}{5} = 0.3.$$

32. $$\frac{0.9\times 0.9\times 0.9+0.2\times 0.2\times 0.2+0.3\times 0.3\times 0.3-3\times 0.9\times 0.2\times 0.3}{0.9\times 0.9+0.2\times 0.2+0.3\times 0.3-0.9\times 0.2-0.2\times 0.3-0.3\times 0.9}$$

$$= \frac{(0.9)^3+(0.2)^3+(0.3)^3-3(0.9)(0.2)(0.3)}{(0.9)^2+(0.2)^2+(0.3)^2-(0.9\times 0.2)-(0.2\times 0.3)-(0.3\times 0.9)}$$

$$= \frac{a^3+b^3+c^3-3abc}{a^2+b^2+c^2-ab-bc-ca}$$

where $a = 0.9,\ b = 0.2,\ c = 0.3$

$$= \frac{(a+b+c)(a^2+b^2+c^2-ab-bc-ca)}{(a^2+b^2+c^2-ab-bc-ca)} = a+b+c$$

$= 0.9 + 0.2 + 0.3 = 1.4.$

33. $(0.05 \times 5 - 0.005 \times 5) = 5\,(0.05 - 0.005)$

$= 5 \times 0.045 = 0.225.$

34. $$(0.\overline{1})^2\left\{1-9(0.1\overline{6})^2\right\} = \left(\frac{1}{9}\right)^2\left\{1-9\left(\frac{16-1}{90}\right)^2\right\}$$

$$= \left(\frac{1}{81}\right)\left\{1-9\cdot\left(\frac{15}{90}\right)^2\right\} = \left(\frac{1}{81}\right)\left\{1-9\times\left(\frac{1}{6}\right)^2\right\}$$

$$= \frac{1}{81}\left\{1-9\times\frac{1}{36}\right\} = \frac{1}{81}\left\{1-\frac{1}{4}\right\} = \frac{1}{81}\times\frac{3}{4} = \frac{1}{108}.$$

35. $\sqrt[2]{0.014\times 0.14\,x} = 0.014\times 0.14\sqrt[2]{y}$

Squaring both sides, we get :

$0.014 \times 0.14 \times x = (0.014)^2 \times (0.14)^2 \times y$

$$\Rightarrow \frac{x}{y} = \frac{(0.014)^2\times(0.14)^2}{(0.014)\times(0.14)} = (0.014)\times(0.14) = 0.00196.$$

36. $$\frac{(0.05)^2+(0.41)^2+(0.073)^2}{(0.005)^2+(0.041)^2+(0.0073)^2}$$

$$= \frac{(10\times 0.005)^2+(10\times 0.041)^2+(10\times 0.0073)^2}{(0.005)^2+(0.041)^2+(0.0073)^2}$$

$$= \frac{100\,(0.005)^2+100\,(0.041)^2+100\,(0.0073)^2}{(0.005)^2+(0.041)^2+(0.0073)^2}$$

$$= \frac{100\left[(0.005)^2+(0.041)^2+(0.0073)^2\right]}{(0.005)^2+(0.041)^2+(0.0073)^2} = 100.$$

37. $$\frac{3.25\times 3.20-3.20\times 3.05}{0.064} = \frac{3.20\times(3.25-3.05)}{0.064}$$

$$= \frac{3.20\times 0.2}{0.064} = \frac{0.64}{0.064} = \frac{64}{100}\times\frac{1000}{64} = 10.$$

38. $$\frac{(4.53-3.07)^2}{(3.07-2.15)(2.15-4.53)} + \frac{(3.07-2.15)^2}{(2.15-4.53)(4.53-3.07)} + \frac{(2.15-4.53)^2}{(4.53-3.07)(3.07-2.15)}$$

$$= \frac{(a-b)^2}{(b-c)(c-a)} + \frac{(b-c)^2}{(c-a)(a-b)} + \frac{(c-a)^2}{(a-b)(b-c)}$$

where $a = 4.53,\ b = 3.07$ and $c = 2.15$

$$= \frac{(a-b)^3}{(a-b)(b-c)(c-a)} + \frac{(b-c)^3}{(a-b)(b-c)(c-a)} + \frac{(c-a)^3}{(a-b)(b-c)(c-a)}$$

$$= \frac{(a-b)^3+(b-c)^3+(c-a)^3}{(a-b)(b-c)(c-a)}$$

$$= \frac{x^3+y^3+z^3}{xyz} \text{ where } x = a-b,\ y = b-c,\ z = c-a$$

$$= \frac{3xyz}{xyz} = 3.$$

$$\left[\begin{array}{l}\because x+y+z = (a-b)+(b-c)+(c-a) = 0\\ \Rightarrow x^3+y^3+z^3-3xyz = 0 \Rightarrow x^3+y^3+z^3 = 3xyz\end{array}\right]$$

39. $0.008 \times 0.01 \times 0.072 \div (0.12 \times 0.0004)$

$= \dfrac{0.008 \times 0.01 \times 0.072}{0.12 \times 0.0004} = \dfrac{8 \times 1 \times 72 \times 100 \times 10000}{12 \times 4 \times 1000 \times 100 \times 1000}$

$= \dfrac{12}{100} = 0.12$.

40. Converting the given fractions into decimals, we get :

$\dfrac{5}{113} \approx 0.044, \dfrac{7}{120} \approx 0.058$.

$\dfrac{13}{145} \approx 0.089, \dfrac{17}{160} \approx 0.106$.

Clearly, $\dfrac{17}{160}$ is the largest.

41. Given : $\dfrac{p}{q} = \dfrac{r}{s} = \dfrac{t}{u} = \sqrt{5}$

$\Rightarrow p = \sqrt{5}q, r = \sqrt{5}s, t = \sqrt{5}u$.

$\Rightarrow p^2 = 5q^2, r^2 = 5s^2, t^2 = 5u^2$.

$\dfrac{3p^2 + 4r^2 + 5t^2}{3q^2 + 4s^2 + 5u^2} = \dfrac{3(5q^2) + 4(5s^2) + 5(5u^2)}{3q^2 + 4s^2 + 5u^2} = 5.$

42. Converting each of the given fractions into decimal form, we get:

$\dfrac{8}{25} = 0.32$, $\dfrac{7}{23} \approx 0.30, \dfrac{11}{23} \approx 0.47, \dfrac{14}{53} \approx 0.26$.

Clearly, 0.26 is the smallest decimal number .

$\therefore$ The smallest fraction is $\dfrac{14}{53}$.

43. Converting each of the given fractions into decimal form, we get:

$\dfrac{4}{3} = 1.3\overline{3}$, $-\dfrac{2}{9} = -0.2\overline{2}$, $-\dfrac{7}{8} = -0.875$, $\dfrac{5}{12} = 0.41\overline{6}$.

Clearly, $-0.875 < -0.2\overline{2} < 0.41\overline{6} < 1.3\overline{3}$.

$\therefore -\dfrac{7}{8} < -\dfrac{2}{9} < \dfrac{5}{12} < \dfrac{4}{3}$.

44. Let the three fractions be x, y and z such that $x < y < z$.

Then, $x + y + z = 2\dfrac{11}{24} = \dfrac{59}{24}$. ... ($i$)

$\dfrac{z}{x} = \dfrac{7}{6} \quad \Rightarrow z = \dfrac{7}{6}x$.

And $y + \dfrac{1}{3} = \dfrac{7}{6} \quad \Rightarrow y = \dfrac{7}{6} - \dfrac{1}{3} = \dfrac{5}{6}$.

Putting $y = \dfrac{5}{6}$ and $z = \dfrac{7}{6}x$ in (i), we get:

$x + \dfrac{5}{6} + \dfrac{7}{6}x = \dfrac{59}{24} \quad \Rightarrow \dfrac{13}{6}x = \dfrac{59}{24} - \dfrac{5}{6} = \dfrac{39}{24}$

$\Rightarrow x = \dfrac{39 \times 6}{13 \times 24} = \dfrac{3}{4}$.

Thus, the smallest fraction is x i.e. $\dfrac{3}{4}$.

45. Let the first part be x.

Then, the second part = $50 - x$.

$\therefore \dfrac{1}{x} + \dfrac{1}{50 - x} = \dfrac{1}{12}$ [$\because$ Sum of their reciprocals = $\dfrac{1}{12}$]

$\Rightarrow \dfrac{50 - x + x}{x(50 - x)} = \dfrac{1}{12} \quad \Rightarrow 600 = 50x - x^2 \quad \Rightarrow x^2 - 50x + 600 = 0$

$\Rightarrow (x - 20)(x - 30) = 0 \quad \Rightarrow x = 20$ or $x = 30$.

Hence, the required two parts are 20, 30.

46. Number of broken tables = $\dfrac{1}{6} \times 108 = 18$

$\Rightarrow$ Number of unbroken tables = 108 – 18 = 90.

Number of broken chairs = $\dfrac{1}{4} \times 132 = 33$

$\Rightarrow$ Number of unbroken chairs = 132 – 33 = 99.

$\therefore$ Total number of unbroken pairs = 90.

Thus, the required number of people = 90.

47. Let each bus contains x seats.

Then, total number of passengers = $\dfrac{4}{5}$ of $3x = \dfrac{4}{5} \times 3x$.

Number of passengers left = $\dfrac{1}{4}$ of $\dfrac{12x}{5} = \dfrac{1}{4} \times \dfrac{12x}{5} = \dfrac{3x}{5}$.

Remaining passengers = $\dfrac{12x}{5} - \dfrac{3x}{5} = \dfrac{9x}{5}$.

Total seats in 2 buses = $2x$.

$\therefore$ Fraction of the seats occupied in two buses = $\dfrac{9x}{5} \times \dfrac{1}{2x} = \dfrac{9}{10}$.

48. Let Neeraj's original property be worth ₹ x.

Then, his wife's share = ₹ $\dfrac{x}{3}$.

Remaining worth = ₹ $\left(x - \dfrac{x}{3}\right)$ = ₹ $\left(\dfrac{2x}{3}\right)$.

His daughter's share = ₹ $\left(\dfrac{3}{5} \times \dfrac{2x}{3}\right)$ = ₹ $\left(\dfrac{2x}{5}\right)$.

His son's share = $\dfrac{2x}{3} - \dfrac{2x}{5}$ = ₹ $\dfrac{4x}{15}$.

$\therefore \dfrac{4x}{15} = 6400 \quad \Rightarrow x = \dfrac{6400 \times 15}{4} = 24000$.

Thus, his original property was worth ₹ 24000.

49. Let the first number be x.

Then, the second number = $\dfrac{2x}{5}$.

$\therefore x + \dfrac{2x}{5} = 50$ [$\because$ Sum of two numbers = 50]

$\Rightarrow \dfrac{7x}{5} = 50 \quad \Rightarrow x = \dfrac{50 \times 5}{7} = \dfrac{250}{7}$.

Now, $\dfrac{2x}{5} = \dfrac{2 \times 250}{5 \times 7} = \dfrac{100}{7}$.

Thus, the numbers are $\dfrac{250}{7}$ and $\dfrac{100}{7}$.

50. Let the number be x.

Then, $\dfrac{x}{5} + 4 = \dfrac{x}{4} - 10$

$\Rightarrow 4(x + 20) = 5(x - 40) \quad \Rightarrow 4x + 80 = 5x - 200 \quad \Rightarrow x = 280$.

51. Let the denominator of the fraction be x.

Then, its numerator = $x - 4$.

If the numerator is decreased by 2 and the denominator is increased by 1, then the denominator becomes eight times the numerator

$\therefore x + 1 = 8(x - 4 - 2) \Rightarrow x + 1 = 8x - 48 \Rightarrow x = 7.$

Thus, the original fraction $= \frac{x-4}{x} = \frac{7-4}{7} = \frac{3}{7}$.

52. Let the numerator and denominator of the fractions be x and y respectively.

If 2 is added to both the numerator and denominator, the fraction becomes $\frac{9}{11}$.

$\therefore \frac{x+2}{y+2} = \frac{9}{11} \Rightarrow 11x + 22 = 9y + 18 \Rightarrow 11x - 9y = -4$ ... (*i*)

If 3 is added to both the numerator and denominator, then it becomes $\frac{5}{6}$.

$\therefore \frac{x+3}{y+3} = \frac{5}{6} \Rightarrow 6x + 18 = 5y + 15 \Rightarrow 6x - 5y = -3.$ ... (*ii*)

On solving (*i*) and (*ii*), we get $x = 7$ and $y = 9$.

53. Present height of the tree = 64 cm.

Height of the tree after one year $= \left(64 + \frac{1}{8} \times 64\right)$ cm = 72 cm.

Height of the tree after the second year $= \left(72 + \frac{1}{8} \times 72\right)$ cm

= 81 cm.

54. Let the numerator of the fraction be x.

Then, its denominator $= x + 3$.

Original fraction $= \frac{x}{x+3}$.

If the numerator is increased by 7 and the denominator is decreased by 2, then we obtain 2.

$\therefore \frac{x+7}{x+3-2} = 2 \Rightarrow x + 7 = 2x + 2 \Rightarrow x = 5.$

So, the numerator $= x = 5$ and denominator $= x + 3 = 5 + 3 = 8$.

$\therefore$ The required sum = 5 + 8 = 13.

55. Let the given fraction be x.

Then, the correct answer $= \frac{3x}{5}$.

Wrong answer $= \frac{x}{\frac{3}{5}} = \frac{5x}{3}$.

$\therefore \frac{5x}{3} = \frac{3x}{5} + \frac{32}{75} \Rightarrow \frac{5x}{3} - \frac{3x}{5} = \frac{32}{75}$.

$\Rightarrow \frac{25x - 9x}{15} = \frac{32}{75} \Rightarrow \frac{16x}{15} = \frac{32}{75} \Rightarrow x = \frac{32 \times 15}{16 \times 75} = \frac{2}{5}$.

$\therefore$ The correct answer $= \frac{3x}{5} = \frac{3}{5} \times \frac{2}{5} = \frac{6}{25}$.

56. $41\frac{2}{3} = \frac{125}{3}$

$\therefore \frac{\frac{125}{3}}{\frac{1}{6}} = \frac{125 \times 6}{3} = 250 \cdot$

Hence, 250 times $\frac{1}{6} = \frac{125}{3} = 41\frac{2}{3}$.

57. Converting the given fraction into the fraction whose denominator is 30

$\frac{5}{8} = \frac{5 \times 30}{8 \times 30} = \frac{5 \times 30}{8} \times \frac{1}{30} = \frac{18.75}{30}$.

$\frac{7}{11} = \frac{7 \times 30}{11 \times 30} = \frac{7 \times 30}{11} \times \frac{1}{30} \approx \frac{19.09}{30}$.

Clearly, 19 lies between 18.75 and 19.09

$\therefore \frac{19}{30}$ lies between $\frac{5}{8}$ and $\frac{7}{11}$.

58. Let the tin contains B bottles of oil.

Then, $\frac{4}{5}B - 6 + 4 = \frac{3}{4}B$

$\Rightarrow \frac{4}{5}B - \frac{3}{4}B = 2 \Rightarrow \frac{B}{20} = 2 \Rightarrow B = 40$.

Thus, the tin can contains 40 bottles of oil.

59. Total number of laps = 5.

The runner runs $= 1\frac{1}{4}$ laps $= \frac{5}{4}$ laps.

Remaining laps $= 5 - \frac{5}{4} = \frac{15}{4}$.

$\therefore$ Required fraction $= \frac{15/4}{5} = \frac{15}{4 \times 5} = \frac{3}{4}$.

60. Let the total number of students be x.

Then, number of girls $= \frac{3x}{5}$

$\Rightarrow$ Number of boys $= x - \frac{3x}{5} = \frac{2x}{5}$.

Number of absent girls $= \frac{2}{9} \times \frac{3x}{5} = \frac{2x}{15}$.

Number of absent boys $= \frac{1}{4} \times \frac{2x}{5} = \frac{x}{10}$.

$\therefore$ Number of students who are present in the class

$= x - \left(\frac{2x}{15} + \frac{x}{10}\right) = x - \frac{7x}{30} = \frac{23x}{30}$.

Thus, $\frac{23}{30}$ part of the total number of students are present.

61. Clearly, 1 day = 24 × 60 minutes

$\therefore$ Required fraction $= \frac{45}{24 \times 60} = \frac{1}{32}$.

62. Let the number be x.

Then, one-third of one-fourth of $x = 15$

$\Rightarrow \frac{1}{3} \times \frac{1}{4} \times x = 15 \Rightarrow x = 15 \times 4 \times 3 = 180.$

$\therefore$ Three-tenth of 180 $= \frac{3}{10} \times 180 = 54$.

63. Let the given number be x.

Then, the correct answer $= \frac{5x}{16}$.

Wrong answer $= \frac{5x}{6}$.

$\therefore \frac{5x}{6} = \frac{5x}{16} + 250 \Rightarrow \frac{5x}{6} - \frac{5x}{16} = 250$

$\Rightarrow \frac{40x - 15x}{48} = 250 \quad \Rightarrow 25x = 250 \times 48 \quad \Rightarrow x = \frac{250 \times 48}{25}$
$= 480.$

Thus, the given number is 480.

64. $\frac{6}{7/8} - \frac{6/7}{8} = \frac{6 \times 8}{7} - \frac{6}{7 \times 8} = \frac{6 \times 8 \times 8 - 6}{7 \times 8}$

$= \frac{378}{56} = \frac{27}{4} = 6\frac{3}{4}.$

$\therefore \frac{6}{7/8}$ exceed $\frac{6/7}{8}$ by $6\frac{3}{4}$.

65. Let the numerator of the fraction be x.

Then, its denominator $= 11 - x$.

So, the original fraction $= \frac{x}{11 - x}$.

$\therefore \frac{x + 2}{11 - x + 2} = \frac{x}{11 - x} + \frac{1}{24}$

$\Rightarrow \frac{x + 2}{13 - x} - \frac{x}{11 - x} = \frac{1}{24}$

$\Rightarrow \frac{11x + 22 - x^2 - 2x - 13x + x^2}{143 - 11x - 13x + x^2} = \frac{1}{24}$

$\Rightarrow 24(-4x + 22) = x^2 - 24x + 143$

$\Rightarrow x^2 - 24x + 143 + 96x - 528 = 0 \quad \Rightarrow x^2 + 72x - 385 = 0$

$\Rightarrow (x + 77)(x - 5) = 0 \quad \Rightarrow x = 5.$ [$\because$ The fraction is positive]

$\therefore$ The difference of the numerator and denominator $= 11 - x - x$
$= 11 - 2x = 11 - 2 \times 5 = 1.$

66. $2.\overline{52} = 2 + \frac{52}{99} = \frac{250}{99}$

$\therefore$ The sum of the numerator and denominator $= 250 + 99 = 349.$

67. Let the number of boys and girls be x and y respectively.

Then, $\frac{1}{10}$ of $x = \frac{1}{4}$ of $y \quad \Rightarrow \frac{x}{10} = \frac{y}{4} \quad \Rightarrow \frac{x}{y} = \frac{10}{4} = \frac{5}{2} = 5:2.$

Thus, the ratio of the number of boys to girls = 5 : 2.

68. $A + B + C + D = 60$ ... (i)

$A = \frac{B + C + D}{2}$ [$\because$ A pays $\frac{1}{2}$ of what others are paying]

$\Rightarrow 2A = B + C + D \quad \Rightarrow 2A = 60 - A \quad \Rightarrow 3A = 60 \quad \Rightarrow A = 20.$

$B = \frac{A + C + D}{3}$ [$\because$ B pays $\frac{1}{3}$ of what others are paying]

$\Rightarrow 3B = A + C + D \quad \Rightarrow 3B = 60 - B \quad \Rightarrow 4B = 60 \quad \Rightarrow B = 15.$

$C = \frac{A + B + D}{4}$ [$\because$ C pays $\frac{1}{4}$ of what others are paying]

$\Rightarrow 4C = A + B + D \quad \Rightarrow 4C = 60 - C \quad \Rightarrow 5C = 60 \quad \Rightarrow C = 12.$

$\therefore D = 60 - (A + B + C) = 60 - (20 + 15 + 12) = 13.$ [Using (i)]

Thus, the amount paid by D = ₹ 13.

69. Let the number be x.

Then, $\frac{3}{4}$ of $x = \frac{1}{6}$ of $x + 7 \quad \Rightarrow \frac{3}{4}x = \frac{1}{6}x + 7$

$\Rightarrow \frac{3}{4}x - \frac{1}{6}x = 7 \quad \Rightarrow \frac{7x}{12} = 7 \quad \Rightarrow x = \frac{12 \times 7}{7} = 12.$

$\therefore \frac{5}{3}$ of $x = \frac{5}{3}$ of $12 = \frac{5}{3} \times 12 = 20.$

70. Let the number be x.

Then, the correct answer $= \frac{8x}{17}$.

Wrong answer $= \frac{x}{8/17} = \frac{17x}{8}$.

$\therefore \frac{17x}{8} = \frac{8x}{17} + 225 \quad \Rightarrow \frac{17x}{8} - \frac{8x}{17} = 225.$

$\Rightarrow \frac{289x - 64x}{136} = 225 \quad \Rightarrow 225x = 225 \times 136 \quad \Rightarrow x = 136.$

Thus, the given number was 136.

71. Let the two fractions be x and y, where $x > y$.

Then, $xy = \frac{14}{15}$ [$\because$ Product of two fractions $= \frac{14}{15}$]

$\frac{x}{y} = \frac{35}{24}$ [$\because$ Their quotient $= \frac{35}{24}$]

$xy \times \frac{x}{y} = \frac{14}{15} \times \frac{35}{24} \quad \Rightarrow x^2 = \frac{49}{36} \quad \Rightarrow x = \sqrt{\frac{49}{36}} = \frac{7}{6}.$

Thus, the greater fraction is x i.e. $\frac{7}{6}$.

72. Let the length of the rod be L m.

Then, $\frac{L}{10} + \frac{L}{20} + \frac{L}{30} + \frac{L}{40} + \frac{L}{50} + \frac{L}{60} + 12.08 = L$

$\Rightarrow L - \left(\frac{L}{10} + \frac{L}{20} + \frac{L}{30} + \frac{L}{40} + \frac{L}{50} + \frac{L}{60}\right) = 12.08$

$\Rightarrow L - \left(\frac{60L + 30L + 20L + 15L + 12L + 10L}{600}\right) = 12.08$

$\Rightarrow 600L - 147L = 12.08 \times 600 \quad \Rightarrow 453L = 1208 \times 6$

$\Rightarrow L = \frac{1208 \times 6}{453} = 16$ m.

Thus, the length of the rod is 16 m.

73. Let the income of the man be ₹ x.

Then, amount spent on the food = ₹ $\frac{x}{3}$.

Amount spent on house rent = ₹ $\frac{2x}{5}$.

Amount spent on clothes = ₹ $\frac{x}{5}$.

Remaining amount $= x - \left(\frac{x}{3} + \frac{2x}{5} + \frac{x}{5}\right) = x - \frac{14x}{15} = \frac{x}{15}.$

$\therefore \frac{x}{15} = 400 \quad \Rightarrow x = 6000.$

Thus, his income is ₹ 6000.

74. Let the number be x.

Then, $\frac{1}{2}$ of $\frac{3}{4}$ of $x = 2\frac{1}{2}$ of 10

$\Rightarrow \frac{1}{2} \times \frac{3}{4} \times x = \frac{5}{2} \times 10 \quad \Rightarrow x = \frac{5 \times 10 \times 2 \times 4}{3 \times 2} = \frac{200}{3} = 66\frac{2}{3}.$

4 SIMPLIFICATION

THEORY AND FORMULAE (TF)

TF 1. **Associative Law:** For any real numbers a, b, c; we have:

(i) $(a + b) + c = a + (b + c)$

(ii) $(a \times b) \times c = a \times (b \times c)$

TF 2. **Distributive Law of Multiplication Over Addition:** For any real numbers a, b, c; we have:

(i) $a \times (b + c) = (a \times b) + (a \times c)$

(ii) $(a + b) \times c = (a \times c) + (b \times c)$

TF 3. **BODMAS Rule:** When we have to find the value of an expression involving more than one mathematical operations, then the sequence in which various operations are executed is given by

$B \rightarrow$ Bracket
$O \rightarrow$ Of
$D \rightarrow$ Division
$M \rightarrow$ Multiplication
$A \rightarrow$ Addition
$S \rightarrow$ Subtraction

Note: (i) The brackets are removed in the order (), {} and [].

(ii) While executing 'of', it is replaced by '×' and solved as multiplication.

(iii) If there is a Virnaculum or Bar, it is executed even before applying BODMAS rule to the remaining part of the expression.

TF 4. **Some Important Formulae:** For three real numbers a, b, c, we have:

(i) $(a + b)^2 = a^2 + b^2 + 2ab = (a - b)^2 + 4ab$

(ii) $(a - b)^2 = a^2 + b^2 - 2ab = (a + b)^2 - 4ab$

(iii) $a^2 - b^2 = (a + b)(a - b)$

(iv) $\frac{1}{2}\left\{(a+b)^2 + (a-b)^2\right\} = a^2 + b^2$

(v) $(a + b)^2 - (a - b)^2 = 4ab$

(vi) $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$ or $a^3 + b^3 = (a + b)^3 - 3ab(a + b)$

(vii) $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$ or $a^3 - b^3 = (a - b)^3 + 3ab(a - b)$

($viii$) $a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$

(ix) $a^3 - b^3 = (a - b)(a^2 + b^2 + ab)$

(x) $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

(xi) $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca) = \frac{1}{2}(a+b+c)\left\{(a-b)^2 + (b-c)^2 + (c-a)^2\right\}$

And so, $a + b + c = 0 \Rightarrow a^3 + b^3 + c^3 = 3abc$.

TF 5. Componendo and Dividendo: For any numbers a, b, c and d where b and d are non-zero and $\frac{a}{b} = \frac{c}{d}$, then each of the following holds:

(*i*) $\frac{a+b}{b} = \frac{c+d}{d}$ [Componendo]

(*ii*) $\frac{a-b}{b} = \frac{c-d}{d}$ [Dividendo]

(*iii*) For $k \neq \frac{a}{b}$, $\frac{a+kb}{a-kb} = \frac{c+kd}{c-kd}$

For $k = 1$, $\frac{a+b}{a-b} = \frac{c+d}{c-d}$ [This is commonly referred to as Componendo and Dividendo]

EXERCISE

1. If $2x + (9/x) = 9$, then what is the minimum value of $x^2 + (1/x^2)$? **[SSC CGL 2017]**
(*a*) 95/36 (*b*) 97/36
(*c*) 86/25 (*d*) 62/27

2. If $(x + y)^2 = xy + 1$ and $x^3 - y^3 = 1$, then what is the value of $x - y$? **[SSC CGL 2017]**
(*a*) – 1 (*b*) 0
(*c*) 1 (*d*) 2

3. If $x + (1/x) = 2$, then what is the value of $x^{64} + x^{121}$? **[SSC CGL 2017]**
(*a*) 0 (*b*) 1
(*c*) 2 (*d*) –2

4. If $x = 6 + 2\sqrt{6}$, then what is the value of

$$\sqrt{x-1} + \frac{1}{\sqrt{x-1}}?$$ **[SSC CGL 2017]**

(*a*) $2\sqrt{2}$ (*b*) $3\sqrt{2}$
(*c*) $2\sqrt{3}$ (*d*) $3\sqrt{3}$

5. If $a + b + c = 27$, then what is the value of $(a - 7)^3 + (b - 9)^3 + (c - 11)^3 - 3(a - 7)(b - 9)(c - 11)$? **[SSC CGL 2017]**
(*a*) 0 (*b*) 3
(*c*) 9 (*d*) 27

6. If $x = \frac{2\sqrt{15}}{\sqrt{3}+\sqrt{5}}$, then what is the value of

$$\frac{x+\sqrt{5}}{x-\sqrt{5}} + \frac{x+\sqrt{3}}{x-\sqrt{3}}?$$ **[SSC CGL 2017]**

(*a*) 1 (*b*) 2
(*c*) $\sqrt{5} - \sqrt{3}$ (*d*) $\sqrt{15}$

7. If $(x - y) = 7$, then what is the value of $(x - 15)^3 - (y - 8)^3$? **[SSC CGL 2017]**
(*a*) 0 (*b*) 343
(*c*) 392 (*d*) 2863

8. If $x - y - \sqrt{18} = -1$ and $x + y - 3\sqrt{2} = 1$, then what is the value of $12xy\,(x^2 - y^2)$? **[SSC CGL 2017]**
(*a*) 0 (*b*) 1
(*c*) $512\sqrt{2}$ (*d*) $612\sqrt{2}$

9. If $x^{1/4} + x^{-1/4} = 2$, then what is the value of $x^{81} + (1/x^{81})$? **[SSC CGL 2017]**
(*a*) $-\frac{4}{81}$ (*b*) 1
(*c*) 2 (*d*) $\frac{2}{9}$

10. If $x^2 + (1/x^2) = 1$, then what is the value of $x^{48} + x^{42} + x^{36} + x^{30} + x^{24} + x^{18} + x^{12} + x^6 + 1$? **[SSC CGL 2017]**
(*a*) – 9 (*b*) 0
(*c*) 1 (*d*) 9

11. $(53 \times 87 + 159 \times 21 + 106 \times 25)$ is equal to
(*a*) 1060 (*b*) 10600
(*c*) 16000 (*d*) 60100

12. $3\frac{3}{5} \times 3\frac{3}{5} + 2 \times 3\frac{3}{5} \times \frac{2}{5} + \frac{2}{5} \times \frac{2}{5} = ?$ **[SSC GD 2013]**
(*a*) 18 (*b*) 17
(*c*) 16 (*d*) 15

13. $\frac{(5+5+5+5) \div 5}{3+3+3+3 \div 3}$ is equal to **[SSC CHSL 2010]**
(*a*) $\frac{2}{5}$ (*b*) $\frac{3}{10}$
(*c*) $\frac{4}{9}$ (*d*) 1

14. The simplified value of $\frac{4}{15}$ of $\frac{5}{8} \times 6 + 15 - 10$ is **[SSC CPO 2016]**
(*a*) 3 (*b*) 4
(*c*) 5 (*d*) 6

15. The value of $3\frac{1}{2}-\left[2\frac{1}{4}+\left\{1\frac{1}{4}-\frac{1}{2}\left(1\frac{1}{2}-\frac{1}{3}-\frac{1}{6}\right)\right\}\right]$ is **[SSC CHSL 2013]**

(*a*) $9\frac{1}{2}$ (*b*) $3\frac{1}{2}$

(*c*) $2\frac{1}{2}$ (*d*) $\frac{1}{2}$

16. The value of $4-\cfrac{1}{1+\cfrac{1}{3+\cfrac{1}{2+\cfrac{1}{4}}}}$ is **[SSC CGL 2015]**

(*a*) $\frac{1}{8}$ (*b*) $\frac{1}{16}$

(*c*) $\frac{1}{32}$ (*d*) $\frac{1}{64}$

17. The value of $1-\cfrac{a}{1-\cfrac{1}{1+\cfrac{a}{1-a}}}$ is **[SSC CGL 2014]**

(*a*) 0 (*b*) 1

(*c*) *a* (*d*) $1-a$

18. The simplified value of **[SSC CGL 2015]**

$$\left[\left(1+\cfrac{1}{10+\cfrac{1}{10}}\right)\times\left(1+\cfrac{1}{10+\cfrac{1}{10}}\right)-\left(1-\cfrac{1}{10+\cfrac{1}{10}}\right)\times\left(1-\cfrac{1}{10+\cfrac{1}{10}}\right)\right]$$
$$\div\left[\left(1+\cfrac{1}{10+\cfrac{1}{10}}\right)+\left(1-\cfrac{1}{10+\cfrac{1}{10}}\right)\right]$$

(*a*) 2 (*b*) $\frac{20}{101}$

(*c*) $\frac{90}{101}$ (*d*) $\frac{100}{101}$

19. The value of $\cfrac{2\frac{1}{3}-1\frac{2}{11}}{3+\cfrac{1}{3+\cfrac{1}{3+\cfrac{1}{3}}}}$ is **[SSC CGL 2011]**

(*a*) 1 (*b*) $\frac{38}{109}$

(*c*) $\frac{109}{38}$ (*d*) $\frac{116}{109}$

20. If $\left[4-\cfrac{5}{1+\cfrac{1}{3+\cfrac{1}{2+\cfrac{1}{4}}}}\right]$ the part of a journey takes 10 minutes, then to complete $\frac{3}{5}$th of that journey, it will take **[SSC CHSL 2013]**

(*a*) 36 minutes (*b*) 40 minutes

(*c*) 45 minutes (*d*) 48 minutes

21. The value of $1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{2}{3}}}}}$ is **[SSC CGL 2011]**

(*a*) $\frac{8}{5}$ (*b*) $\frac{17}{3}$

(*c*) $\frac{21}{13}$ (*d*) $\frac{34}{21}$

22. Simplify : $\frac{19}{43}\div\cfrac{1}{2+\cfrac{1}{3+\cfrac{1}{1+\cfrac{1}{4}}}}$

(*a*) 1 (*b*) $\frac{19}{22}$

(*c*) $\frac{22}{43}$ (*d*) $\frac{38}{43}$

23. $\cfrac{2}{2+\cfrac{2}{3+\cfrac{2}{3+\cfrac{2}{3}}}\times 0.39}$ is simplified to

(*a*) $\frac{2}{39}$ (*b*) $\frac{3}{37}$

(*c*) $\frac{50}{117}$ (*d*) $\frac{100}{111}$

24. If $2 = x+\cfrac{1}{1+\cfrac{1}{3+\cfrac{1}{4}}}$, then the value of *x* is

(*a*) $\frac{12}{17}$ (*b*) $\frac{13}{17}$

(*c*) $\frac{18}{17}$ (*d*) $\frac{21}{17}$

25. Find the sum of

$$\left(1-\frac{1}{n+1}\right)+\left(1-\frac{2}{n+1}\right)+\left(1-\frac{3}{n+1}\right)+...+\left(1-\frac{n}{n+1}\right)$$

[SSC CGL 2013]

(*a*) $\frac{1}{2}(n+1)$ (*b*) $\frac{1}{2}n$

(*c*) $(n+1)$ (*d*) n

26. $\left(\frac{1}{3.5}+\frac{1}{5.7}+\frac{1}{7.9}+\frac{1}{9.11}+\frac{1}{11.3}+\frac{1}{13.15}\right)$ is equal to

(*a*) $\frac{2}{15}$ (*b*) $\frac{2}{45}$

(*c*) $\frac{4}{45}$ (*d*) $\frac{7}{45}$

27. Simplify : $\dfrac{\frac{5}{3}\times\frac{7}{51}\text{ of }\frac{17}{5}-\frac{1}{3}}{\frac{2}{9}\times\frac{5}{7}\text{ of }\frac{28}{5}-\frac{2}{3}}$

(*a*) $\frac{1}{2}$ (*b*) $\frac{1}{4}$

(*c*) 2 (*d*) 4

28. Evaluate : $\dfrac{9|3-5|-5|4|\div 10}{-3(5)-2\times 4\div 2}$

(*a*) $\frac{4}{7}$ (*b*) $-\frac{8}{17}$

(*c*) $\frac{9}{10}$ (*d*) $-\frac{16}{19}$

29. When $\left(\frac{1}{2}-\frac{1}{4}+\frac{1}{5}-\frac{1}{6}\right)$ is divided by $\left(\frac{2}{5}-\frac{5}{9}+\frac{3}{5}-\frac{7}{18}\right)$, the result is

(*a*) $2\frac{1}{18}$ (*b*) $3\frac{1}{6}$

(*c*) $3\frac{3}{10}$ (*d*) $5\frac{1}{10}$

30. For what value of $*$, the statement $\left[\frac{(*)}{21}\times\frac{(*)}{189}\right]=1$ is correct?

(*a*) 21 (*b*) 63

(*c*) 147 (*d*) 3969

31. $9-1\frac{2}{9}\text{ of }3\frac{3}{11}\div 5\frac{1}{7}\text{ of }\frac{7}{9}$ is equal to

(*a*) $\frac{3}{4}$ (*b*) $8\frac{32}{81}$

(*c*) 8 (*d*) 9

32. Find the sum of following :

$\frac{1}{9}+\frac{1}{6}+\frac{1}{12}+\frac{1}{20}+\frac{1}{30}+\frac{1}{42}+\frac{1}{56}+\frac{1}{72}$

(*a*) 0 (*b*) $\frac{1}{2}$

(*c*) $\frac{1}{9}$ (*d*) $\frac{1}{2520}$

33. $\frac{1}{30}+\frac{1}{42}+\frac{1}{56}+\frac{1}{72}+\frac{1}{90}+\frac{1}{110}=?$

(*a*) $\frac{1}{9}$ (*b*) $\frac{2}{27}$

(*c*) $\frac{5}{27}$ (*d*) $\frac{6}{55}$

34. Simplify : $\left[3\frac{1}{4}\div\left\{1\frac{1}{4}-\frac{1}{2}\left(2\frac{1}{2}-\overline{\frac{1}{4}-\frac{1}{6}}\right)\right\}\right]\div\left(\frac{1}{2}\text{ of }4\frac{1}{3}\right)$

(*a*) 78 (*b*) 39

(*c*) 36 (*d*) 18

35. $\text{I}=\frac{3}{4}\div\frac{5}{6}$, $\text{II}=3\div[(4\div 5)\div 6]$, then

$\text{III}=[3\div(4\div 5)]\div 6$, $\text{IV}=3\div 4(5\div 6)$

(*a*) I and II are equal (*b*) I and IV are equal

(*c*) I and III are equal (*d*) All are equal

36. The value of $1\div[1+1\div\{1+1\div(1+1\div 2)\}]$ is

(*a*) $\frac{1}{2}$ (*b*) $\frac{5}{8}$

(*c*) 1 (*d*) 2

37. $\dfrac{-\frac{1}{2}-\frac{2}{3}+\frac{4}{5}-\frac{1}{3}+\frac{1}{5}+\frac{3}{4}}{\frac{1}{2}+\frac{2}{3}-\frac{4}{3}+\frac{1}{3}-\frac{1}{5}-\frac{4}{5}}$ is simplified to

(*a*) $-\frac{3}{10}$ (*b*) $-\frac{10}{3}$

(*c*) -2 (*d*) 1

38. $\frac{17}{15}\times\frac{17}{15}+\frac{2}{15}\times\frac{2}{15}-\frac{17}{15}\times\frac{4}{15}$ is equal to

(*a*) 11 (*b*) 10

(*c*) 1 (*d*) 0

39. Find the value of $*$ in the following :

$$1\frac{2}{3}\div\frac{2}{7}\times\frac{*}{7}=1\frac{1}{4}\times\frac{2}{3}\div\frac{1}{6}$$

(*a*) 0.006 (*b*) 0.6

(*c*) $\frac{1}{6}$ (*d*) 6

40. Simplify: $\dfrac{2\frac{3}{4}}{1\frac{5}{6}}\div\frac{7}{8}\times\left(\frac{1}{3}+\frac{1}{4}\right)+\frac{5}{7}\div\frac{3}{4}\text{ of }\frac{3}{7}$

(*a*) $\frac{2}{3}$ (*b*) $3\frac{2}{9}$

(*c*) $\frac{49}{80}$ (*d*) $\frac{56}{77}$

41. $\dfrac{(998)^2-(997)^2-45}{(98)^2-(97)^2}$ is equal to

(*a*) 10 (*b*) 95

(*c*) 195 (*d*) 1995

42. When simplified, the product

$$\left(2-\frac{1}{3}\right)\left(2-\frac{3}{5}\right)\left(2-\frac{5}{7}\right)\ldots\left(2-\frac{997}{999}\right) \text{ equals}$$

[SSC CAPF & CISF 2015]

(*a*) $\frac{5}{3}$ (*b*) $\frac{5}{999}$

(*c*) $\frac{1001}{3}$ (*d*) $\frac{1001}{999}$

43. $\frac{3^0+3^{-1}}{3^{-1}-3^0}$ is simplified to

(*a*) 1 (*b*) – 1

(*c*) 2 (*d*) – 2

44. $\left[3-4(3-4)^{-1}\right]^{-1}$ is equal to

(*a*) – 7 (*b*) $-\frac{1}{7}$

(*c*) $\frac{1}{7}$ (*d*) 7

45. $\left(\frac{1}{1.4}+\frac{1}{4.7}+\frac{1}{7.10}+\frac{1}{10.13}+\frac{1}{13.16}\right)$ is equal to

(*a*) $\frac{1}{3}$ (*b*) $\frac{3}{8}$

(*c*) $\frac{5}{16}$ (*d*) $\frac{41}{7280}$

46. The value of $\frac{5}{1\frac{7}{8}\text{ of }1\frac{1}{3}}\times\frac{2\frac{1}{10}}{3\frac{1}{2}}\text{ of }1\frac{1}{4}$

(*a*) 0.05 (*b*) $1\frac{1}{2}$

(*c*) 1 (*d*) 2

47. Simplify : $\frac{\frac{1}{3}+\frac{1}{4}\left[\frac{2}{5}-\frac{1}{2}\right]}{1\frac{2}{3}\text{ of }\frac{3}{4}-\frac{3}{4}\text{ of }\frac{4}{5}}$ **[SSC MTS 2011]**

(*a*) $\frac{37}{13}$ (*b*) $\frac{37}{39}$

(*c*) $\frac{37}{78}$ (*d*) $\frac{74}{13}$

48. $\frac{0.04}{0.03}\text{ of }\frac{\left(3\frac{1}{3}-2\frac{1}{2}\right)\div\frac{1}{2}\text{ of }1\frac{1}{4}}{\frac{1}{3}+\frac{1}{5}\text{ of }\frac{1}{9}}$ **[SSC MTS 2011]**

(*a*) $\frac{1}{2}$ (*b*) $\frac{1}{5}$

(*c*) 1 (*d*) 5

49. If $x = 1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{2}}}}$, then the value of $2x+\frac{7}{4}$ is

(*a*) 6 (*b*) 5

(*c*) 4 (*d*) 3

50. $\frac{13}{48}$ is equal to

(*a*) $\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{8}}}$ (*b*) $\cfrac{1}{3+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{4}}}}$

(*c*) $\cfrac{1}{3+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{8}}}}$ (*d*) $\cfrac{1}{3+\cfrac{1}{1+\cfrac{1}{16}}}$

51. Find the value of $\cfrac{2}{1+\cfrac{1}{1-\cfrac{1}{2}}}\times\frac{3}{\frac{5}{6}\text{ of }\frac{3}{2}\div1\frac{1}{4}}$

(*a*) 2 (*b*) 4

(*c*) 6 (*d*) 8

52. Simplify : $1+\cfrac{4}{2+\cfrac{3}{5-\cfrac{1}{2}}}-\frac{1}{2}(10\div2)$

(*a*) 0 (*b*) $-\frac{1}{2}$

(*c*) $-\frac{15}{2}$ (*d*) 1

53. On simplification, the expression

$\frac{4\frac{1}{7}-2\frac{1}{7}}{3\frac{1}{2}+1\frac{1}{7}}\div\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{5-\cfrac{1}{5}}}}$ is equal **[SSC CGL 2014 & 2015]**

(*a*) $\frac{14}{65}$ (*b*) $\frac{24}{53}$

(*c*) $\frac{28}{65}$ (*d*) $\frac{56}{53}$

54. $\sqrt{\frac{4\frac{1}{7}-2\frac{1}{4}}{3\frac{1}{2}+1\frac{1}{7}}\div\cfrac{2}{2+\cfrac{1}{2+\cfrac{1}{5-\cfrac{1}{5}}}}}$ is equal to **[SSC CHSL 2013]**

(*a*) $\sqrt{2}$ (*b*) $\sqrt{3}$

(*c*) $\frac{1}{\sqrt{2}}$ (*d*) $\frac{1}{\sqrt{3}}$

ANSWERS

1. (*b*)	**2.** (*c*)	**3.** (*c*)	**4.** (*c*)	**5.** (*a*)	**6.** (*b*)	**7.** (*a*)	**8.** (*d*)	**9.** (*c*)	**10.** (*c*)
11. (*b*)	**12.** (*c*)	**13.** (*a*)	**14.** (*d*)	**15.** (*d*)	**16.** (*a*)	**17.** (*a*)	**18.** (*b*)	**19.** (*b*)	**20.** (*d*)
21. (*d*)	**22.** (*a*)	**23.** (*d*)	**24.** (*d*)	**25.** (*b*)	**26.** (*a*)	**27.** (*c*)	**28.** (*d*)	**29.** (*d*)	**30.** (*b*)
31. (*c*)	**32.** (*b*)	**33.** (*d*)	**34.** (*c*)	**35.** (*b*)	**36.** (*b*)	**37.** (*a*)	**38.** (*c*)	**39.** (*d*)	**40.** (*b*)
41. (*a*)	**42.** (*c*)	**43.** (*d*)	**44.** (*c*)	**45.** (*c*)	**46.** (*b*)	**47.** (*c*)	**48.** (*d*)	**49.** (*b*)	**50.** (*b*)
51. (*a*)	**52.** (*d*)	**53.** (*d*)	**54.** (*c*)						

SOLUTION

1. $2x + \frac{9}{x} = 9 \Rightarrow 2x^2 + 9 = 9x \Rightarrow 2x^2 - 9x + 9 = 0$.

$\Rightarrow 2x^2 - 6x - 3x + 9 = 0 \Rightarrow 2x(x-3) - 3(x-3) = 0$

$\Rightarrow (x-3)(2x-3) = 0 \Rightarrow x = 3$ or $x = \frac{3}{2}$.

For $x = 3$:

Value of $x^2 + \frac{1}{x^2} = 9 + \frac{1}{9} = \frac{82}{9}$.

For $x = \frac{3}{2} : x^2 + \frac{1}{x^2} = \frac{9}{4} + \frac{4}{9} = \frac{97}{36}$.

$\therefore$ Minimum value of $x^2 + \frac{1}{x^2}$ is $\frac{97}{36}$.

2. $(x+y)^2 = xy + 1 \Rightarrow x^2 + y^2 + 2xy = xy + 1$

$\Rightarrow x^2 + y^2 + xy = 1$ ... (*i*)

Now, $x^3 - y^3 = 1$

$\Rightarrow (x-y)(x^2 + y^2 + xy) = 1$

$\Rightarrow (x-y) \times 1 = 1$ [Using (*i*)]

$\Rightarrow x - y = 1$.

3. $x + \frac{1}{x} = 2 \Rightarrow x^2 - 2x + 1 = 0$

$\Rightarrow (x-1)^2 = 0 \Rightarrow x - 1 = 0$

$\Rightarrow x = 1$

$\therefore x^{64} + x^{121} = 1^{64} + 1^{121} = 1 + 1 = 2$

4. $\sqrt{x-1} + \frac{1}{\sqrt{x-1}} = \frac{(x-1)+1}{\sqrt{x-1}} = \frac{x}{\sqrt{x-1}} = \frac{6+2\sqrt{6}}{\sqrt{5+2\sqrt{6}}}$

$= \frac{\sqrt{6}\left(\sqrt{6}+2\right)}{\sqrt{\left(\sqrt{3}\right)^2 + \left(\sqrt{2}\right)^2 + 2\left(\sqrt{3}\right)\left(\sqrt{2}\right)}} = \frac{\sqrt{6}\left(\sqrt{3}\sqrt{2}+\sqrt{2}\sqrt{2}\right)}{\sqrt{\left(\sqrt{3}+\sqrt{2}\right)^2}}$

$= \frac{\sqrt{6}\sqrt{2}\left(\sqrt{3}+\sqrt{2}\right)}{\left(\sqrt{3}+\sqrt{2}\right)} = \sqrt{6}\sqrt{2} = \left(\sqrt{3}\sqrt{2}\right)\left(\sqrt{2}\right) = 2\sqrt{3}$.

5. Let $x = a - 7$, $y = b - 9$, $z = c - 11$ Then,

$x + y + z = (a-7) + (b-9) + (c-11) = (a+b+c) - 27 = 0$.

[$\because a + b + c = 27$]

$\Rightarrow x^3 + y^3 + z^3 - 3xyz = 0$.

$\Rightarrow (a-7)^3 + (b-9)^3 + (c-11)^3 - 3(a-7)(b-9)(c-11) = 0$.

6. $\frac{x+\sqrt{5}}{x-\sqrt{5}} = \frac{\left(x+\sqrt{5}\right)+\left(x-\sqrt{5}\right)}{\left(x+\sqrt{5}\right)-\left(x-\sqrt{5}\right)} = \frac{2x}{2\sqrt{5}}$

$= \frac{x}{\sqrt{5}} = \frac{2\sqrt{15}}{\sqrt{3}+\sqrt{5}} \times \frac{1}{\sqrt{5}}$ [By Componendo-Dividendo]

$= \frac{2\sqrt{3}\sqrt{5}}{\sqrt{5}\left(\sqrt{3}+\sqrt{5}\right)} = \frac{2\sqrt{3}}{\sqrt{3}+\sqrt{5}}$.

And,

$\frac{x+\sqrt{3}}{x-\sqrt{3}} = \frac{\left(x+\sqrt{3}\right)+\left(x-\sqrt{3}\right)}{\left(x+\sqrt{3}\right)-\left(x-\sqrt{3}\right)} = \frac{2x}{2\sqrt{3}} = \frac{x}{\sqrt{3}} = \frac{2\sqrt{15}}{\sqrt{3}+\sqrt{5}} \times \frac{1}{\sqrt{3}}$

[By Componendo-Dividendo]

$= \frac{2\sqrt{3}\sqrt{5}}{\sqrt{3}\left(\sqrt{3}+\sqrt{5}\right)} = \frac{2\sqrt{5}}{\sqrt{3}+\sqrt{5}}$.

$\therefore \frac{x+\sqrt{5}}{x-\sqrt{5}} = \frac{x+\sqrt{3}}{x-\sqrt{3}} = \frac{2\sqrt{3}}{\sqrt{3}+\sqrt{5}} + \frac{2\sqrt{5}}{\sqrt{3}+\sqrt{5}} = \frac{2\left(\sqrt{3}+\sqrt{5}\right)}{\left(\sqrt{3}+\sqrt{5}\right)} = 2$.

7. $x - y = 7$.

$\Rightarrow x - y - 7 = 0$.

$\Rightarrow x - y - 15 + 8 = 0$.

$\Rightarrow (x - 15) - (y - 8) = 0$.

Cubing both sides, we have : [$\because (a-b)^3 = a^3 - b^3 - 3ab(a-b)$]

$(x-15)^3 - (y-8)^3 - 3(x-15)(y-8)[(x-15)-(y-8)] = 0$.

$\Rightarrow (x-15)^3 - (y-8)^3 = 0$. $\left[\begin{array}{l}\because (x-15)-(y-8) \\ = x - y - 7 = 0\end{array}\right]$

8. We have :

$x - y - \sqrt{18} = -1$ and $x + y - 3\sqrt{2} = 1$

$\Rightarrow x - y = 3\sqrt{2} - 1$ and $x + y = 3\sqrt{2} + 1$.

Now, $(x-y)^2 = (x+y)^2 - 4xy$

$\Rightarrow \left(3\sqrt{2}-1\right)^2 = \left(3\sqrt{2}+1\right)^2 - 4xy$

$\Rightarrow 18 + 1 - 6\sqrt{2} = 18 + 1 + 6\sqrt{2} - 4xy$

$\Rightarrow 4xy = 12\sqrt{2}$

$\Rightarrow xy = 3\sqrt{2}$.

$12xy(x^2 - y^2) = 12xy(x-y)(x+y)$

$= 12(3\sqrt{2})[(3\sqrt{2}-1)(3\sqrt{2}+1)] = 36\sqrt{2}(18-1) = 612\sqrt{2}$.

9. $x^{\frac{1}{4}} + \dfrac{1}{x^{\frac{1}{4}}} = 2$. ... (*i*)

Squaring both sides of (*i*); we get :

$x^{\frac{1}{2}} + \dfrac{1}{x^{\frac{1}{2}}} + 2 = 4$.

$\Rightarrow x^{\frac{1}{2}} + \dfrac{1}{x^{\frac{1}{2}}} + 2 = 4$

$\Rightarrow x^{\frac{1}{2}} + \dfrac{1}{x^{\frac{1}{2}}} = 2$ ... (*ii*)

Squaring both sides of (*ii*), we get ;

$x + \dfrac{1}{x} + 2 = 4$.

$\Rightarrow x + \dfrac{1}{x} = 2 \quad \Rightarrow x^2 - 2x + 1 = 0 \quad \Rightarrow (x-1)^2 = 0$

$\Rightarrow x - 1 = 0 \quad \Rightarrow x = 1$

$\therefore x^{81} + \dfrac{1}{x^{81}} = (1)^{81} + \dfrac{1}{(1)^{81}} = 2$.

10. $x^2 + \dfrac{1}{x^2} = 1$ ... (*i*)

Then, $x + \dfrac{1}{x} = \sqrt{3}$

$\Rightarrow x^3 + \dfrac{1}{x^3} = (\sqrt{3})^3 - (3\sqrt{3}) = 0$.

$\Rightarrow x^6 + 1 = 0$.

Then,

$x^{42}(x^6+1) + x^{30}(x^6+1) + x^{18}(x^6+1) + x^6(x^6+1) + 1$

$= x^{42} \times 0 + x^{30} \times 0 + x^{18} \times 0 + x^6 \times 0 + 1$

$= 1$.

11. $(53 \times 87 + 159 \times 21 + 106 \times 25) = 53(87 + 3 \times 21 + 2 \times 25)$

$= 53 \times (87 + 63 + 50) = 53 \times 200 = 10600$.

12. $3\frac{3}{5} \times 3\frac{3}{5} + 2 \times 3\frac{3}{5} \times \frac{2}{5} + \frac{2}{5} \times \frac{2}{5} = \frac{18}{5} \times \frac{18}{5} + 2 \times \frac{18}{5} \times \frac{2}{5} + \frac{2}{5} \times \frac{2}{5}$

$= \left(\frac{18}{5}\right)^2 + 2\left(\frac{18}{5}\right)\left(\frac{2}{5}\right) + \left(\frac{2}{5}\right)^2$

$= a^2 + 2ab + b^2$ where $a = \frac{18}{5}, b = \frac{2}{5}$

$= (a+b)^2 = \left(\frac{18}{5} + \frac{2}{5}\right)^2 = \left(\frac{20}{5}\right)^2 = 4^2 = 16$.

13. $\dfrac{(5+5+5+5) \div 5}{3+3+3+3 \div 3} = \dfrac{20 \div 5}{3+3+3+\frac{3}{3}} = \dfrac{4}{9+1} = \dfrac{4}{10} = \dfrac{2}{5}$.

14. $\frac{4}{15}$ of $\frac{5}{8} \times 6 + 15 - 10 = \left(\frac{4}{15} \times \frac{5}{8}\right) \times 6 + 15 - 10$

$= \frac{1}{6} \times 6 + 15 - 10 = 1 + 15 - 10 = 16 - 10 = 6$.

15. $3\frac{1}{2} - \left[2\frac{1}{4} + \left\{1\frac{1}{4} - \frac{1}{2}\left(1\frac{1}{2} - \frac{1}{3} - \frac{1}{6}\right)\right\}\right]$

$= \frac{7}{2} - \left[\frac{9}{4} + \left\{\frac{5}{4} - \frac{1}{2}\left(\frac{3}{2} - \frac{1}{3} - \frac{1}{6}\right)\right\}\right]$

$= \frac{7}{2} - \left[\frac{9}{4} + \left\{\frac{5}{4} - \frac{1}{2}\left(\frac{9-2-1}{6}\right)\right\}\right]$

$= \frac{7}{2} - \left[\frac{9}{4} + \left\{\frac{5}{4} - \frac{1}{2} \times \frac{6}{6}\right\}\right] = \frac{7}{2} - \left[\frac{9}{4} + \left\{\frac{5}{4} - \frac{1}{2}\right\}\right]$

$= \frac{7}{2} - \left[\frac{9}{4} + \left\{\frac{5-2}{4}\right\}\right] = \frac{7}{2} - \left[\frac{9}{4} + \frac{3}{4}\right] = \frac{7}{2} - \frac{12}{4} = \frac{7}{2} - 3 = \frac{1}{2}$.

16. $4 - \cfrac{5}{1 + \cfrac{1}{3 + \cfrac{1}{2 + \cfrac{1}{4}}}} = 4 - \cfrac{5}{1 + \cfrac{1}{3 + \cfrac{1}{\frac{9}{4}}}} = 4 - \cfrac{5}{1 + \cfrac{1}{3 + \frac{4}{9}}}$

$= 4 - \cfrac{5}{1 + \frac{9}{31}} = 4 - \dfrac{31 \times 5}{40} = 4 - \dfrac{31}{8} = \dfrac{1}{8}$.

17. $1 - \cfrac{a}{1 - \cfrac{1}{1 + \cfrac{a}{1-a}}} = 1 - \cfrac{a}{1 - \cfrac{1}{\left(\frac{1}{1-a}\right)}} = 1 - \dfrac{a}{1 - (1-a)}$

$= 1 - \dfrac{a}{a} = 1 - 1 = 0$.

18. We have : $\dfrac{1}{10 + \frac{1}{10}} = \dfrac{1}{\left(\frac{101}{10}\right)} = \dfrac{10}{101}$

$\therefore$ Given expression $= \dfrac{\left(1 + \frac{10}{101}\right)^2 - \left(1 - \frac{10}{101}\right)^2}{\left(1 + \frac{10}{101}\right) + \left(1 - \frac{10}{101}\right)}$

$= \left(1 + \frac{10}{101}\right) - \left(1 - \frac{10}{101}\right) = 2 \times \frac{10}{101} = \frac{20}{101}$.

$\left[\because \dfrac{a^2 - b^2}{a+b} = a - b\right]$

19. $\dfrac{2\frac{1}{3} - 1\frac{2}{11}}{3 + \cfrac{1}{3 + \cfrac{1}{3 + \frac{1}{3}}}} = \dfrac{\frac{7}{3} - \frac{13}{11}}{3 + \cfrac{1}{3 + \frac{3}{10}}} = \dfrac{\left(\frac{77-39}{33}\right)}{3 + \frac{10}{33}} = \dfrac{38}{33} \times \dfrac{33}{109} = \dfrac{38}{109}$.

20. $4 - \cfrac{5}{1 + \cfrac{1}{3 + \cfrac{1}{2 + \frac{1}{4}}}} = 4 - \cfrac{5}{1 + \cfrac{1}{3 + \frac{4}{9}}}$

$= 4 - \cfrac{5}{1 + \frac{9}{31}} = 4 - \dfrac{5 \times 31}{40} = 4 - \dfrac{31}{8} = \dfrac{1}{8}$.

Now, $\frac{1}{8}$th of the journey takes = 10 min.

$\therefore$ Complete journey takes = (8 × 10) min = 80 min.

And so, $\frac{3}{5}$th of the journey will take = $\left(\frac{3}{5} \times 80\right)$min = 48 min.

21. $1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\frac{2}{3}}}}} = 1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\frac{3}{5}}}} = 1+\cfrac{1}{1+\cfrac{1}{1+\frac{5}{8}}}$

$= 1+\cfrac{1}{1+\frac{8}{13}} = 1+\frac{13}{21} = \frac{34}{21}.$

22. $\frac{19}{43}\div\cfrac{1}{2+\cfrac{1}{3+\cfrac{1}{1+\frac{1}{4}}}} = \frac{19}{43}\div\cfrac{1}{2+\cfrac{1}{3+\frac{4}{5}}}$

$= \frac{19}{43}\div\cfrac{1}{2+\frac{5}{19}} = \frac{19}{43}\div\frac{19}{43} = 1.$

23. $\cfrac{2}{2+\cfrac{2}{3+\cfrac{2}{3+\frac{2}{3}}}\times 0.39} = \cfrac{2}{2+\cfrac{2}{3+\cfrac{2}{\left(\frac{11}{3}\right)}}\times 0.39} = \cfrac{2}{2+\cfrac{2}{3+\frac{6}{11}}\times 0.39}$

$= \cfrac{2}{2+\frac{2\times 11}{39}\times 0.39} = \cfrac{2}{2+\frac{22}{100}} = \cfrac{2}{2+\frac{11}{50}} = \frac{100}{111}.$

24. $2 = x+\cfrac{1}{1+\cfrac{1}{3+\frac{1}{4}}} = x+\cfrac{1}{1+\frac{4}{13}} = x+\frac{13}{17}$

$\Rightarrow x = 2-\frac{13}{17} = \frac{21}{17}.$

25. $\left(1-\frac{1}{n+1}\right)+\left(1-\frac{2}{n+1}\right)+\left(1-\frac{3}{n+1}\right)+\ldots+\left(1-\frac{n}{n+1}\right)$

$= \left(\frac{n}{n+1}\right)+\left(\frac{n-1}{n+1}\right)+\left(\frac{n-2}{n+1}\right)+\ldots+\left(\frac{1}{n+1}\right)$

$= \frac{n+(n-1)+(n-2)+\ldots+1}{(n+1)}$

$= \frac{1}{(n+1)}\cdot\{1+2+\ldots+(n-2)+(n-1)+n\}$

$= \frac{1}{(n+1)}\cdot\frac{n(n+1)}{2} = \frac{n}{2}.$

$\left[\because \text{ Sum of first } n \text{ natural numbers} = \frac{n(n+1)}{2}\right]$

26. $\left(\frac{1}{3.5}+\frac{1}{5.7}+\frac{1}{7.9}+\frac{1}{9.11}+\frac{1}{11.13}+\frac{1}{13.15}\right)$

$= \frac{1}{2}\left(\frac{1}{3}-\frac{1}{5}\right)+\frac{1}{2}\left(\frac{1}{5}-\frac{1}{7}\right)+\frac{1}{2}\left(\frac{1}{7}-\frac{1}{9}\right)$

$+\frac{1}{2}\left(\frac{1}{9}-\frac{1}{11}\right)+\frac{1}{2}\left(\frac{1}{11}-\frac{1}{13}\right)+\frac{1}{2}\left(\frac{1}{13}-\frac{1}{15}\right)$

$= \left(\frac{1}{2}\right)\left\{\frac{1}{3}-\frac{1}{5}+\frac{1}{5}-\frac{1}{7}+\frac{1}{7}-\frac{1}{9}+\frac{1}{9}-\frac{1}{11}+\frac{1}{11}-\frac{1}{13}+\frac{1}{13}-\frac{1}{15}\right\}$

$= \left(\frac{1}{2}\right)\left\{\frac{1}{3}-\frac{1}{15}\right\} = \frac{1}{2}\times\frac{4}{15} = \frac{2}{15}.$

27. $\dfrac{\frac{5}{3}\times\frac{7}{51}\text{ of }\frac{17}{5}-\frac{1}{3}}{\frac{2}{9}\times\frac{5}{7}\text{ of }\frac{28}{5}-\frac{2}{3}} = \dfrac{\frac{5}{3}\times\frac{7}{51}\times\frac{17}{5}-\frac{1}{3}}{\frac{2}{9}\times\frac{5}{7}\times\frac{28}{5}-\frac{2}{3}}$

$= \dfrac{\frac{7}{9}-\frac{1}{3}}{\frac{8}{9}-\frac{2}{3}} = \dfrac{\left(\frac{4}{9}\right)}{\left(\frac{2}{9}\right)} = \frac{4}{9}\times\frac{9}{2} = 2.$

28. $\frac{9|3-5|-5|4|\div 10}{-3(5)-2\times 4\div 2} = \frac{9|-2|-5(4)\div 10}{-15-2\times 2}$

$= \frac{9(2)-20\div 10}{-15-4} = \frac{18-2}{-19} = \frac{16}{-19} = -\frac{16}{19}.$

29. $\dfrac{\left(\frac{1}{2}-\frac{1}{4}+\frac{1}{5}-\frac{1}{6}\right)}{\left(\frac{2}{5}-\frac{5}{9}+\frac{3}{5}-\frac{7}{18}\right)} = \dfrac{\left(\frac{30-15+12-10}{60}\right)}{\left(\frac{36-50+54-35}{90}\right)}$

$= \dfrac{\left(\frac{17}{60}\right)}{\left(\frac{5}{90}\right)} = \frac{17}{60}\times\frac{90}{5} = \frac{51}{10} = 5\frac{1}{10}.$

Another Method :

$\dfrac{\left(\frac{1}{2}-\frac{1}{4}+\frac{1}{5}-\frac{1}{6}\right)}{\left(\frac{2}{5}-\frac{5}{9}+\frac{3}{5}-\frac{7}{18}\right)} = \dfrac{\left(\frac{1}{2}-\frac{1}{4}\right)+\left(\frac{1}{5}-\frac{1}{6}\right)}{\left(\frac{2}{5}+\frac{3}{5}\right)-\left(\frac{5}{9}+\frac{7}{18}\right)}$

$= \dfrac{\frac{1}{4}+\frac{1}{30}}{1-\frac{17}{18}} = \dfrac{\left(\frac{15+2}{60}\right)}{\left(\frac{1}{18}\right)} = \frac{51}{10} = 5\frac{1}{10}.$

30. Let us replace $*$ by a variable x. Then,

$\left[\frac{x}{21}\times\frac{x}{189}\right] = 1 \Rightarrow x^2 = 21\times 189$

$= 3\times 7\times 3\times 3\times 3\times 7 = (3\times 3\times 7)^2 \Rightarrow x = 3\times 3\times 7 = 63.$

$\therefore$ The value of $*$ is 63.

31. $9-1\frac{2}{9}\text{ of }3\frac{3}{11}\div 5\frac{1}{7}\text{ of }\frac{7}{9}$

$= 9-\frac{11}{9}\text{ of }\frac{36}{11}\div\frac{36}{7}\text{ of }\frac{7}{9} = 9-\left(\frac{11}{9}\times\frac{36}{11}\right)\div\left(\frac{36}{7}\times\frac{7}{9}\right)$

$= 9-4\div 4 = 9-\left(\frac{4}{4}\right) = 9-1 = 8.$

32. $\frac{1}{9}+\frac{1}{6}+\frac{1}{12}+\frac{1}{20}+\frac{1}{30}+\frac{1}{42}+\frac{1}{56}+\frac{1}{72}$

$= \frac{1}{9}+\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)+\left(\frac{1}{4}-\frac{1}{5}\right)+\left(\frac{1}{5}-\frac{1}{6}\right)$

$+\left(\frac{1}{6}-\frac{1}{7}\right)+\left(\frac{1}{7}-\frac{1}{8}\right)+\left(\frac{1}{8}-\frac{1}{9}\right) = \frac{1}{9}+\frac{1}{2}-\frac{1}{9} = \frac{1}{2}.$

33. $\frac{1}{30}+\frac{1}{42}+\frac{1}{56}+\frac{1}{72}+\frac{1}{90}+\frac{1}{110}$

$=\left(\frac{1}{5}-\frac{1}{6}\right)+\left(\frac{1}{6}-\frac{1}{7}\right)+\left(\frac{1}{7}-\frac{1}{8}\right)+\left(\frac{1}{8}-\frac{1}{9}\right)+\left(\frac{1}{9}-\frac{1}{10}\right)+\left(\frac{1}{10}-\frac{1}{11}\right)$

$=\frac{1}{5}-\frac{1}{11}=\frac{11-5}{55}=\frac{6}{55}$

34. $\left[3\frac{1}{4}\div\left\{1\frac{1}{4}-\frac{1}{2}\left(2\frac{1}{2}-\overline{\frac{1}{4}-\frac{1}{6}}\right)\right\}\right]\div\left(\frac{1}{2}\text{ of }4\frac{1}{3}\right)$

$=\left[\frac{13}{4}\div\left\{\frac{5}{4}-\frac{1}{2}\left(\frac{5}{2}-\frac{3-2}{12}\right)\right\}\right]\div\left(\frac{1}{2}\times\frac{13}{3}\right)$

$=\left[\frac{13}{4}\div\left\{\frac{5}{4}-\frac{1}{2}\left(\frac{5}{2}-\frac{1}{12}\right)\right\}\right]\div\left(\frac{13}{6}\right)$

$=\left[\frac{13}{4}\div\left\{\frac{5}{4}-\frac{1}{2}\times\frac{29}{12}\right\}\right]\div\left(\frac{13}{6}\right)=\left[\frac{13}{4}+\left\{\frac{5}{4}-\frac{29}{24}\right\}\right]\div\left(\frac{13}{6}\right)$

$=\left[\frac{13}{4}\div\frac{1}{24}\right]\div\left(\frac{13}{6}\right)=\left(\frac{13}{4}\times 24\right)\div\frac{13}{6}=13\times 6\times\frac{6}{13}=36.$

35. **I** $=\frac{3}{4}\div\frac{5}{6}=\frac{3}{4}\times\frac{6}{5}=\frac{9}{10}$

II $=3\div[(4\div 5)\div 6]=3\div\left(\frac{4}{5}\div 6\right)$

$=3\div\left(\frac{4}{5}\times\frac{1}{6}\right)=3\div\frac{2}{15}=3\times\frac{15}{2}=\frac{45}{2}.$

III $=[3\div(4\div 5)]\div 6=\left[3\div\left(\frac{4}{5}\right)\right]\div 6$

$=\left(3\times\frac{5}{4}\right)\div 6=\frac{15}{4}\times\frac{1}{6}=\frac{5}{8}.$

IV $=3\div 4(5\div 6)=3\div 4\times\frac{5}{6}=3\div\frac{10}{3}=3\times\frac{3}{10}=\frac{9}{10}.$

Clearly, I and IV are equal.

36. $1\div[1+1\div\{1+1\div(1+1\div 2)\}]$

$=1\div\left[1+1\div\left\{1+1\div\left(1+\frac{1}{2}\right)\right\}\right]$

$=1\div\left[1+1\div\left\{1+1\div\frac{3}{2}\right\}\right]=1\div\left[1+1\div\left\{1+\left(1\times\frac{2}{3}\right)\right\}\right]$

$=1\div\left[1+1\div\left\{1+\frac{2}{3}\right\}\right]=1\div\left[1+1\div\frac{5}{3}\right]$

$=1\div\left[1+\left(1\times\frac{3}{5}\right)\right]=1\div\left[1+\frac{3}{5}\right]=1\div\frac{8}{5}=1\times\frac{5}{8}=\frac{5}{8}.$

37. $\dfrac{-\frac{1}{2}-\frac{2}{3}+\frac{4}{5}-\frac{1}{3}+\frac{1}{5}+\frac{3}{4}}{\frac{1}{2}+\frac{2}{3}-\frac{4}{3}+\frac{1}{3}-\frac{1}{5}-\frac{4}{5}}=\dfrac{-\frac{1}{2}-\left(\frac{2}{3}+\frac{1}{3}\right)+\left(\frac{4}{5}+\frac{1}{5}\right)+\frac{3}{4}}{\frac{1}{2}+\left(\frac{2}{3}-\frac{4}{3}+\frac{1}{3}\right)-\left(\frac{1}{5}+\frac{4}{5}\right)}$

$=\dfrac{-\frac{1}{2}-1+1+\frac{3}{4}}{\frac{1}{2}+\left(-\frac{1}{3}\right)-1}=\dfrac{\frac{1}{4}}{\left(\frac{3-2-6}{6}\right)}=\frac{1}{4}\times\frac{6}{(-5)}=-\frac{3}{10}.$

38. $\frac{17}{15}\times\frac{17}{15}+\frac{2}{15}\times\frac{2}{15}-\frac{17}{15}\times\frac{4}{15}$

$=\left(\frac{17}{15}\right)^2+\left(\frac{2}{15}\right)^2-2\left(\frac{17}{15}\right)\left(\frac{2}{15}\right)$

$=a^2+b^2-2ab$ where $a=\frac{17}{15}, b=\frac{2}{15}$

$=(a-b)^2=\left(\frac{17}{15}-\frac{2}{15}\right)^2=\left(\frac{15}{15}\right)^2=1^2=1.$

39. Let us replace * by variable x. Then,

$1\frac{2}{3}\div\frac{2}{7}\times\frac{x}{7}=1\frac{1}{4}\times\frac{2}{3}\div\frac{1}{6}\Rightarrow\frac{5}{3}\div\frac{2}{7}\times\frac{x}{7}=\frac{5}{4}\times\frac{2}{3}\div\frac{1}{6}$

$\Rightarrow\frac{5}{3}\times\frac{7}{2}\times\frac{x}{7}=\frac{5}{4}\times\frac{2}{3}\times 6\Rightarrow\frac{5x}{6}=5\Rightarrow x=6.$

Thus, the value of * is 6.

40. $\dfrac{2\frac{3}{4}}{1\frac{5}{6}}\div\frac{7}{8}\times\left(\frac{1}{3}+\frac{1}{4}\right)+\frac{5}{7}\div\frac{3}{4}\text{ of }\frac{3}{7}$

$=\dfrac{\left(\frac{11}{4}\right)}{\left(\frac{11}{6}\right)}\div\frac{7}{8}\times\left(\frac{7}{12}\right)+\frac{5}{7}\div\left(\frac{3}{4}\times\frac{3}{7}\right)$

$=\frac{3}{2}\div\frac{7}{8}\times\frac{7}{12}+\frac{5}{7}\div\frac{9}{28}=\frac{3}{2}\times\frac{8}{7}\times\frac{7}{12}+\frac{5}{7}\times\frac{28}{9}$

$=1+\frac{20}{9}=\frac{29}{9}=3\frac{2}{9}.$

41. $\dfrac{(998)^2-(997)^2-45}{(98)^2-(97)^2}=\dfrac{(998+997)(998-997)-45}{(98+97)(98-97)}$

$[\because a^2-b^2=(a-b)(a+b)]$

$=\frac{1995-45}{195}=\frac{1950}{195}=10.$

42. $\left(2-\frac{1}{3}\right)\left(2-\frac{3}{5}\right)\left(2-\frac{5}{7}\right)...\left(2-\frac{997}{999}\right)$

$=\left(\frac{5}{3}\right)\left(\frac{7}{5}\right)\left(\frac{9}{7}\right)...\left(\frac{1001}{999}\right)=\frac{1001}{3}.$

43. $\dfrac{3^0+3^{-1}}{3^{-1}-3^0}=\dfrac{1+\frac{1}{3}}{\frac{1}{3}-1}=\dfrac{\frac{4}{3}}{-\frac{2}{3}}$

$=\frac{4\times 3}{3\times(-2)}=-2.$

44. $\left[3-4(3-4)^{-1}\right]^{-1}=\left[3-4(-1)^{-1}\right]^{-1}=\left[3-4\left(\frac{1}{-1}\right)\right]^{-1}$

$=(3+4)^{-1}=\frac{1}{7}.$ $\left[\because a^{-m}=\frac{1}{a^m}\right]$

45. $\frac{1}{1.4}+\frac{1}{4.7}+\frac{1}{7.10}+\frac{1}{10.13}+\frac{1}{13.16}$

$=\frac{1}{3}\left(\frac{3}{1.4}+\frac{3}{4.7}+\frac{3}{7.10}+\frac{3}{10.13}+\frac{3}{13.16}\right)$

$$= \frac{1}{3}\left[\left(\frac{1}{1}-\frac{1}{4}\right)+\left(\frac{1}{4}-\frac{1}{7}\right)+\left(\frac{1}{7}-\frac{1}{10}\right)+\left(\frac{1}{10}-\frac{1}{13}\right)+\left(\frac{1}{13}-\frac{1}{16}\right)\right]$$

$$= \frac{1}{3}\left[1+\left(-\frac{1}{4}+\frac{1}{4}\right)+\left(-\frac{1}{7}+\frac{1}{7}\right)+\left(-\frac{1}{10}+\frac{1}{10}\right)+\left(-\frac{1}{13}+\frac{1}{13}\right)-\frac{1}{16}\right]$$

$$=\frac13\left(1-\frac1{16}\right)=\frac13\left(\frac{15}{16}\right)=\frac{5}{16}.$$

46. $$\frac{5}{1\frac78 \text{ of } 1\frac13}\times\frac{2\frac{1}{10}}{3\frac12}\text{ of }1\frac14 = \frac{5}{\frac{15}{8}\text{ of }\frac43}\times\frac{\frac{21}{10}}{\frac72}\text{ of }\frac54$$

$$=\frac{5}{\left(\frac{15}{8}\times\frac43\right)}\times\frac{21}{10}\times\frac27\times\frac54=\frac{5}{\left(\frac52\right)}\times\frac34=2\times\frac34=\frac32=1\frac12.$$

47. $$\frac{\frac13+\frac14\left[\frac25-\frac12\right]}{1\frac23\text{ of }\frac34-\frac34\text{ of }\frac45}=\frac{\frac13+\frac14\left[\frac{4-5}{10}\right]}{\frac53\text{ of }\frac34-\frac34\text{ of }\frac45}$$

$$=\frac{\frac13+\frac14\times\left(-\frac1{10}\right)}{\left(\frac53\times\frac34\right)-\left(\frac34\times\frac45\right)}=\frac{\frac13-\frac1{40}}{\frac54-\frac35}=\frac{\left(\frac{40-3}{120}\right)}{\left(\frac{25-12}{20}\right)}$$

$$=\frac{37}{120}\times\frac{20}{13}=\frac{37}{78}.$$

48. $$\frac{0.04}{0.03}\text{ of }\frac{\left(3\frac13-2\frac12\right)\div\frac12\text{ of }1\frac14}{\frac13+\frac15\text{ of }\frac19}$$

$$=\frac43\text{ of }\frac{\left(\frac{10}{3}-\frac52\right)\div\left(\frac12\times\frac54\right)}{\frac13+\left(\frac15\times\frac19\right)}=\frac43\text{ of }\frac{\left(\frac{20-15}{6}\right)\div\left(\frac58\right)}{\frac13+\frac1{45}}$$

$$=\frac43\text{ of }\frac{\left(\frac56\right)\times\left(\frac85\right)}{\frac{15+1}{45}}=\frac43\text{ of }\frac{\left(\frac43\right)}{\left(\frac{16}{15}\right)}=\frac43\text{ of }\left(\frac43\times\frac{15}{16}\right)$$

$$=\frac43\times\frac{15}{4}=5.$$

49. $$x=1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\frac12}}}=1+\cfrac{1}{1+\cfrac{1}{1+\frac23}}$$

$$=1+\frac{1}{1+\frac35}=1+\frac58=\frac{13}{8}.$$

$$\therefore\ 2x+\frac74=2\times\frac{13}{8}+\frac74=\frac{13}4+\frac74=\frac{20}4=5.$$

50. $$\frac{13}{48}=\frac{1}{\left(\frac{48}{13}\right)}=\frac{1}{3+\frac{9}{13}}=\cfrac{1}{3+\cfrac{1}{\left(\frac{13}{9}\right)}}$$

$$=\cfrac{1}{3+\cfrac{1}{1+\frac49}}=\cfrac{1}{3+\cfrac{1}{1+\cfrac{1}{\left(\frac94\right)}}}=\cfrac{1}{3+\cfrac{1}{1+\cfrac{1}{2+\frac14}}}.$$

51. $$\frac{2}{1+\cfrac{1}{1-\frac12}}\times\frac{3}{\frac56\text{ of }\frac32\div1\frac14}=\frac{2}{1+2}\times\frac{3}{\frac56\times\frac32\times\frac45}$$

$$=\frac23\times\frac31=2.$$

52. $$1+\cfrac{4}{2+\cfrac{3}{5-\frac12}}-\frac12(10\div2)=1+\cfrac{4}{2+\frac69}\div\frac12\times5$$

$$=1+\frac{4\times9}{24}\div\frac52=\left(1+\frac32\right)\times\frac25=\frac52\times\frac25=1.$$

53. $$\frac{4\frac17-2\frac17}{3\frac12+1\frac17}\div\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{5-\frac15}}}=\frac{\frac{29}7-\frac{15}7}{\frac72+\frac87}\div\cfrac{1}{2+\cfrac{1}{2+\frac5{24}}}$$

$$=\frac{\left(\frac{14}{7}\right)}{\left(\frac{49+16}{14}\right)}\div\cfrac{1}{2+\cfrac{1}{\left(\frac{53}{24}\right)}}=\frac{2}{\left(\frac{65}{14}\right)}\div\frac{1}{2+\frac{24}{53}}$$

$$=\frac{28}{65}\div\frac{53}{130}=\frac{28}{65}\times\frac{130}{53}=\frac{56}{53}.$$

54. $$\frac{4\frac17-2\frac14}{3\frac12+1\frac17}\div\cfrac{2}{2+\cfrac{1}{2+\cfrac{1}{5-\frac15}}}=\frac{\frac{29}7-\frac94}{\frac72+\frac87}\div\cfrac{2}{2+\cfrac{1}{2+\frac5{24}}}$$

$$=\frac{\left(\frac{116-63}{28}\right)}{\left(\frac{49+16}{14}\right)}\div\frac{2}{2+\frac{24}{53}}=\frac{53}{28}\times\frac{14}{65}\div\frac{2\times53}{130}$$

$$=\frac{53}{130}\times\frac{130}{2\times53}=\frac12$$

$$\therefore\ \sqrt{\frac{4\frac17-2\frac14}{3\frac12+1\frac17}\div\cfrac{2}{2+\cfrac{1}{2+\cfrac{1}{5-\frac15}}}}=\frac{1}{\sqrt2}.$$