Indian Institute of Technology

IIT/JEE MAIN MATHEMATICS

Objective Chapterwise Solved Papers

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Syllabus for JEE (Main) - 2024

Syllabus for JEE Main Paper-1 (B.E./B.Tech.)

MATHEMATICS

UNIT 1: SETS, RELATIONS, AND FUNCTIONS: Sets and their representation: Union, intersection, and complement of sets and their algebraic properties; Power set; Relation, Type of relations, equivalence relations, functions; one-one, into and onto functions, the composition of functions.

UNIT 2: COMPLEX NUMBERS AND QUADRATIC EQUATIONS: Complex numbers as ordered pairs of reals, Representation of complex numbers in the form a + ib and their representation in a plane, Argand diagram, algebra of complex number, modulus, and argument (or amplitude) of a complex number, Quadratic equations in real and complex number system and their solutions Relations between roots and co-efficient, nature of roots, the formation of quadratic equation with given roots.

UNIT 3: MATRICES AND DETERMINATS: Matrices, algebra of matrices, type of matrices, determinants, and matrices of order two and three, evaluation of determinants, area of triangles using determinants, Adjoint, and evaluation of inverse of a square matrix using determinants and, Test of consistency and solution of simultaneous linear equations in two or three variables using matrices.

UNIT 4: PERMUTATIONS AND COMBINATIONS: The fundamental principle of counting, permutation as an arrangement and combination as section, Meaning of P (n, r) and C(n, r), simple applications.

UNIT 5: BINOMIAL THEOREM AND ITS SIMPLE APPLICATIONS: Binomial theorem for a positive integral index, general term and middle term, and simple applications.

UNIT 6: SEQUENCE AND SERIES: Arithmetic and Geometric progressions, insertion of arithmetic, geometric means between two given numbers, Relation between A.M and G.M.

UNIT 7: LIMIT, CONTINUITY, AND DIFFERENTIABILITY: Real-valued functions, algebra of functions, polynomials, rational, trigonometric, logarithmic, and exponential functions, inverse function. Graphs of simple functions. Limits, continuity, and differentiability. Differentiation of the sum, difference, product, and quotient of two functions. Differentiation of trigonometric, inverse trigonometric, logarithmic, exponential, composite, and implicit functions; derivatives of order up to two, Applications of derivatives: Rate of change of quantities, monotonic-Increasing and decreasing functions, Maxima and minima of functions of one variable.

UNIT 8: INTEGRAL CALCULAS: Integral as an anti-derivative, Fundamental integral involving algebraic, trigonometric, exponential, and logarithmic functions. Integrations by substitution, by parts, and by partial functions. Integrations by substitution, by parts, and by partial functions. Integration using trigonometric identities.

Evaluation of simple integrals of the type

$\int \frac{dx}{x^2+a^2}$, $\int \frac{dx}{\sqrt{x^2 \pm a^2}}$, $\int \frac{dx}{a^2-x^2}$, $\int \frac{dx}{\sqrt{a^2-x^2}}$, $\int \frac{dx}{ax^2+bx+c}$, $\int \frac{dx}{\sqrt{ax^2+bx+c}}$, $\int \frac{(px+q)dx}{ax^2+bx+c}$, $\int \frac{(px+q)dx}{\sqrt{ax^2+bx+c}}$,

$\int \sqrt{a^2 \pm x^2}\, dx$, $\int \sqrt{x^2-a^2} dx$

The fundamental theorem of calculus, properties of definite integrals. Evaluation of definite integrals, determining areas of the regions bounded by simple curves in standard form.

UNIT 9 : DIFFERENTIAL EQUATION : Ordinary differential equations, their order, and degree, the solution of differential equation by the method of separation of variables, solution of a homegeneous and linear differential equation of the type

$$\frac{dy}{dx} + p(x)y = q(x)$$

UNIT 10 : CO-ORDINATE GEOMETRY : Cartesian system of rectangular coordinates in a plane, distance formula, sections formula, locus, and its equation, the slope of a line, parallel and perpendicular lines, intercepts of a line on the co-ordinate axis.

Straight line : Various forms of equations of a line, intersection of lines, angles between two lines, conditions for concurrence of three lines, the distance of a point form a line, co-ordinate of the centroid orthocentre, and circumcentre of a triangle.

Circle, conic sections : A standard form of equations of a circle, the general form of the equation of a circle, its radius and central, equation of a circle when the endpoints of a diameter are given, points of intersection of a line and a circle with the centre at the origin and sections of conics, equations of conic sections (parabola, ellipse, and hyperbola) in standard forms.

UNIT 11 : THREE DIMENSIONAL GEOMETRY : Coordinates of a point in space, the distance between two points, section formula, directions ratios, and direction consines, and the angle between two intersecting lines. Skew lines, the shortest distance between them, and its equation. Equations of a line

UNIT 12: VECTOR ALGEBRA: Vectors and scalars, the addition of vectors, components of a vector in two dimensions and three-dimensional space, scalar and vector products.

UNIT 13: STATISTICS AND PROBABILITY: Measures of discretion; calculation of mean, median, mode of grouped and ungrouped data calculation of standard deviation, variance, and mean deviation for grouped and ungrouped data.

Probability: Probability of an event, addition and multiplication theorems of probability, Baye's theorem, probability distribution of a random variate.

UNIT 14: TRIGONOMETRY : Trigonometrical identities and trigonometrical funtions, inverse trigonometrical functions, and their properties.

All India Engineering Entrance Examination & JEE-Main Previous Years Papers Analysis Chart

Sl No	Exam	Proposed Year		Total Question
Joint Entrance Examination (JEE) Main				
1.	NTA JEE Main	15.04.2023	Shift-I	30
2.	NTA JEE Main	13.04.2023	Shift-I	30
3.	NTA JEE Main	13.04.2023	Shift-II	30
4.	NTA JEE Main	12.04.2023	Shift-I	30
5.	NTA JEE Main	11.04.2023	Shift-I	30
6.	NTA JEE Main	11.04.2023	Shift-II	30
7.	NTA JEE Main	10.04.2023	Shift-I	30
8.	NTA JEE Main	10.04.2023	Shift-II	30
9.	NTA JEE Main	08.04.2023	Shift-I	30
10.	NTA JEE Main	08.04.2023	Shift-II	30
11.	NTA JEE Main	06.04.2023	Shift-I	30
12.	NTA JEE Main	06.04.2023	Shift-II	30
13.	NTA JEE Main	01.02.2023	Shift-I	30
14.	NTA JEE Main	01.02.2023	Shift-II	30
15.	NTA JEE Main	24.01.2023	Shift-I	30
16.	NTA JEE Main	24.01.2023	Shift-II	30
17.	NTA JEE Main	25.01.2023	Shift-I	30
18.	NTA JEE Main	25.01.2023	Shift-II	30
19.	NTA JEE Main	29.01.2023	Shift-I	30
20.	NTA JEE Main	29.01.2023	Shift-II	30
21.	NTA JEE Main	30.01.2023	Shift-I	30
22.	NTA JEE Main	30.01.2023	Shift-II	30
23.	NTA JEE Main	31.01.2023	Shift-I	30
24.	NTA JEE Main	31.01.2023	Shift-II	30
25.	NTA JEE Main	29.07.2022	Shift-I	30
26.	NTA JEE Main	29.07.2022	Shift-II	30
27.	NTA JEE Main	28.07.2022	Shift-I	30
28.	NTA JEE Main	28.07.2022	Shift-II	30
29.	NTA JEE Main	27.07.2022	Shift-I	30
30.	NTA JEE Main	27.07.2022	Shift-II	30
31.	NTA JEE Main	26.07.2022	Shift-I	30
32.	NTA JEE Main	26.07.2022	Shift-II	30
33.	NTA JEE Main	25.07.2022	Shift-I	30
34.	NTA JEE Main	25.07.2022	Shift-II	30
35.	NTA JEE Main	29.06.2022	Shift-I	30
36.	NTA JEE Main	29.06.2022	Shift-II	30
37.	NTA JEE Main	28.06.2022	Shift-I	30
38.	NTA JEE Main	28.06.2022	Shift-II	30
39.	NTA JEE Main	27.06.2022	Shift-I	30
40.	NTA JEE Main	27.06.2022	Shift-II	30
41.	NTA JEE Main	26.06.2022	Shift-I	30
42.	NTA JEE Main	26.06.2022	Shift-II	30
43.	NTA JEE Main	25.06.2022	Shift-I	30
44.	NTA JEE Main	25.06.2022	Shift-II	30
45.	NTA JEE Main	24.06.2022	Shift-I	30
46.	NTA JEE Main	24.06.2022	Shift-II	30
47.	NTA JEE Main	01.09.2021	Shift-I	30
48.	NTA JEE Main	01.09.2021	Shift-II	30

49.	NTA JEE Main	31.08.2021	Shift-I	30
50.	NTA JEE Main	31.08.2021	Shift-II	30
51.	NTA JEE Main	27.08.2021	Shift-I	30
52.	NTA JEE Main	27.08.2021	Shift-II	30
53.	NTA JEE Main	26.08.2021	Shift-I	30
54.	NTA JEE Main	26.08.2021	Shift-II	30
55.	NTA JEE Main	27.07.2021	Shift-I	30
56.	NTA JEE Main	27.07.2021	Shift-II	30
57.	NTA JEE Main	25.07.2021	Shift-I	30
58.	NTA JEE Main	25.07.2021	Shift-II	30
59.	NTA JEE Main	22.07.2021	Shift-I	30
60.	NTA JEE Main	22.07.2021	Shift-II	30
61.	NTA JEE Main	20.07.2021	Shift-I	30
62.	NTA JEE Main	20.07.2021	Shift-II	30
63.	NTA JEE Main	18.03.2021	Shift-I	30
64.	NTA JEE Main	18.03.2021	Shift-II	30
65.	NTA JEE Main	17.03.2021	Shift-I	30
66.	NTA JEE Main	17.03.2021	Shift-II	30
67.	NTA JEE Main	16.03.2021	Shift-I	30
68.	NTA JEE Main	16.03.2021	Shift-II	30
69.	NTA JEE Main	26.02.2021	Shift-I	30
70.	NTA JEE Main	26.02.2021	Shift-II	30
71.	NTA JEE Main	25.02.2021	Shift-I	30
72.	NTA JEE Main	25.02.2021	Shift-II	30
73.	NTA JEE Main	24.02.2021	Shift-I	30
74.	NTA JEE Main	24.02.2021	Shift-II	30
75.	NTA JEE Main	06.09.2020	Shift-I	30
76.	NTA JEE Main	06.09.2020	Shift-II	30
77.	NTA JEE Main	05.09.2020	Shift-I	30
78.	NTA JEE Main	05.09.2020	Shift-II	30
79.	NTA JEE Main	04.09.2020	Shift-I	25
80.	NTA JEE Main	04.09.2020	Shift-II	25
81.	NTA JEE Main	03.09.2020	Shift-I	30
82.	NTA JEE Main	03.09.2020	Shift-II	30
83.	NTA JEE Main	02.09.2020	Shift-I	25
84.	NTA JEE Main	02.09.2020	Shift-II	25
85.	NTA JEE Main	09.01.2020	Shift-I	30
86.	NTA JEE Main	09.01.2020	Shift-II	30
87.	NTA JEE Main	08.01.2020	Shift-I	30
88.	NTA JEE Main	08.01.2020	Shift-II	30
89.	NTA JEE Main	07.01.2020	Shift-I	30
90.	NTA JEE Main	07.01.2020	Shift-II	30
91.	NTA JEE Main	12.04.2019	Shift-I	30
92.	NTA JEE Main	12.04.2019	Shift-II	30
93.	NTA JEE Main	10.04.2019	Shift-I	30
94.	NTA JEE Main	10.04.2019	Shift-II	30
95.	NTA JEE Main	09.04.2019	Shift-I	30
96.	NTA JEE Main	09.04.2019	Shift-II	30
97.	NTA JEE Main	08.04.2019	Shift-I	30
98.	NTA JEE Main	08.04.2019	Shift-II	30
99.	NTA JEE Main	12.01.2019	Shift-I	30
100.	NTA JEE Main	12.01.2019	Shift-II	30
101.	NTA JEE Main	11.01.2019	Shift-I	30
102.	NTA JEE Main	11.01.2019	Shift-II	30
103.	NTA JEE Main	10.01.2019	Shift-I	30

104.	NTA JEE Main	10.01.2019	Shift-II	30
105.	NTA JEE Main	09.01.2019	Shift-I	30
106.	NTA JEE Main	09.01.2019	Shift-II	30
107.	JEE Main	16.04.2018		30
108.	JEE Main	15.04.2018	Shift-I	30
109.	JEE Main	15.04.2018	Shift-II	30
110.	JEE Main	08.04.2018		30
111.	JEE Main	09.04.2017		30
112.	JEE Main	08.04.2017		30
113.	JEE Main	02.04.2017		30
114.	JEE Main	2016		30
115.	JEE Main	2015		30
116.	JEE Main	2014		30
117.	JEE Main	2013		30
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119.	AIEEE	2011		30
120.	AIEEE	2010		30
121.	AIEEE	2009		30
122.	AIEEE	2008		30
	AIEEE	2007		30
123.	AIEEE	2006		30
124.	AIEEE	2005		30
125.	AIEEE	2004		30
126.	AIEEE	2003		30
127.	AIEEE	2002		30
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128.	ASSAM-CEE	2023		40
129.	ASSAM-CEE	2022		40
130.	ASSAM-CEE	2021		40
131.	ASSAM-CEE	2020		40
132.	ASSAM-CEE	2019		40
133.	ASSAM-CEE	2018		40
Andhra Pradesh EAMCET/EAPCET				
134.	A.P. EAPCET	15.05.2023	Shift-I	80
135.	A.P. EAPCET	15.05.2023	Shift-II	80
136.	A.P. EAPCET	16.05.2023	Shift-I	80
137.	A.P. EAPCET	16.05.2023	Shift-II	80
138.	A.P. EAPCET	17.05.2023	Shift-I	80
139.	A.P. EAPCET	17.05.2023	Shift-II	80
140.	A.P. EAPCET	18.05.2023	Shift-I	80
141.	A.P. EAPCET	18.05.2023	Shift-II	80
142.	A.P. EAPCET	19.05.2023	Shift-I	80
143.	A.P. EAMCET	04.07.2022	Shift-I	80
144.	A.P. EAMCET	04.07.2022	Shift-II	80
145.	A.P. EAMCET	05.07.2022	Shift-I	80
146.	A.P. EAMCET	05.07.2022	Shift-II	80
147.	A.P. EAMCET	06.07.2022	Shift-I	80
148.	A.P. EAMCET	06.07.2022	Shift-II	80
149.	A.P. EAMCET	07.07.2022	Shift-I	80
150.	A.P. EAMCET	07.07.2022	Shift-II	80
151.	A.P. EAMCET	08.07.2022	Shift-I	80
152.	A.P. EAMCET	08.07.2022	Shift-II	80
153.	A.P. EAMCET	07.09.2021	Shift-I	80
154.	A.P. EAMCET	23.08.2021	Shift-I	80
155.	A.P. EAMCET	23.08.2021	Shift-II	80

156.	A.P. EAMCET	19.08.2021	Shift-II	80
157.	A.P. EAMCET	20.08.2021	Shift-I	80
158.	A.P. EAMCET	20.08.2021	Shift-II	80
159.	A.P. EAMCET	19.08.2021	Shift-I	80
160.	A.P. EAMCET	19.08.2021	Shift-II	80
161.	A.P. EAMCET	05.10.2021	Shift-II	80
162.	A.P. EAMCET	25.08.2021	Shift-I	80
163.	A.P. EAMCET	25.08.2021	Shift-II	80
164.	A.P. EAMCET	24.08.2021	Shift-I	80
165.	A.P. EAMCET	24.08.2021	Shift-II	80
166.	A.P. EAMCET	22.09.2020	Shift-I	80
167.	A.P. EAMCET	22.09.2020	Shift-II	80
168.	A.P. EAMCET	23.09.2020	Shift-I	80
169.	A.P. EAMCET	21.09.2020	Shift-I	80
170.	A.P. EAMCET	21.09.2020	Shift-II	80
171.	A.P. EAMCET	18.09.2020	Shift-I	80
172.	A.P. EAMCET	18.09.2020	Shift-II	80
173.	A.P. EAMCET	17.09.2020	Shift-I	80
174.	A.P. EAMCET	17.09.2020	Shift-II	80
175.	A.P. EAMCET	07.10.2020	Shift-I	80
176.	A.P. EAMCET	20.04.2019	Shift-I	80
177.	A.P. EAMCET	20.04.2019	Shift-II	80
178.	A.P. EAMCET	21.04.2019	Shift-I	80
179.	A.P. EAMCET	21.04.2019	Shift-II	80
180.	A.P. EAMCET	22.04.2019	Shift-I	80
181.	A.P. EAMCET	22.04.2019	Shift-II	80
182.	A.P. EAMCET	23.04.2019	Shift-I	80
183.	A.P. EAMCET	22.04.2018	Shift-I	80
184.	A.P. EAMCET	22.04.2018	Shift-II	80
185.	A.P. EAMCET	23.04.2018	Shift-I	80
186.	A.P. EAMCET	23.04.2018	Shift-II	80
187.	A.P. EAMCET	24.04.2018	Shift-I	80
188.	A.P. EAMCET	24.04.2018	Shift-II	80
189.	A.P. EAMCET	2017		80
190.	A.P. EAMCET	2016		80
191.	A.P. EAMCET	2015		80
192.	A.P. EAMCET	2014		80
193.	A.P. EAMCET	2013		80
194.	A.P. EAMCET	2012		80
195.	A.P. EAMCET	2011		80
196.	A.P. EAMCET	2010		80
197.	A.P. EAMCET	2009		80
198.	A.P. EAMCET	2008		80
199.	A.P. EAMCET	2007		80
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206.	A.P. EAMCET	2000		80
207.	A.P. EAMCET	1999		80
208.	A.P. EAMCET	1998		80
209.	A.P. EAMCET	1997		80
210.	A.P. EAMCET	1996		80

211.	A.P. EAMCET	1995		80
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225.	AMU	2013		50
226.	AMU	2012		50
227.	AMU	2011		50
228.	AMU	2010		70
229.	AMU	2009		70
230.	AMU	2008		70
231.	AMU	2007		70
232.	AMU	2006		70
233.	AMU	2005		70
234.	AMU	2004		70
235.	AMU	2003		70
236.	AMU	2002		100
237.	AMU	2001		100
(Bihar) BCECE				
238.	BCECE	2018		50
239.	BCECE	2017		50
240.	BCECE	2016		50
241.	BCECE	2015		50
242.	BCECE	2014		50
243.	BCECE	2013		50
244.	BCECE	2012		50
245.	BCECE	2011		50
246.	BCECE	2010		50
247.	BCECE	2009		50
248.	BCECE	2008		50
249.	BCECE	2007		50
250.	BCECE	2006		50
251.	BCECE	2005		50
252.	BCECE	2004		50
253.	BCECE	2003		50
BITSAT				
254.	BITSAT	2023		40
255.	BITSAT	2022		40
256.	BITSAT	2021		40
257.	BITSAT	2019		40
258.	BITSAT	2018		40
259.	BITSAT	2017		40
260.	BITSAT	2016		40
261.	BITSAT	2015		40
262.	BITSAT	2014		40

263.	BITSAT	2013		40
264.	BITSAT	2012		40
265.	BITSAT	2011		40
266.	BITSAT	2010		40
267.	BITSAT	2009		40
268.	BITSAT	2008		40
269.	BITSAT	2007		40
270.	BITSAT	2006		40
271.	BITSAT	2005		40
Chhattisgarh-PET				
272.	Chhattisgarh-PET	2023		100
273.	Chhattisgarh-PET	2022		100
274.	Chhattisgarh-PET	2021		100
275.	Chhattisgarh-PET	2020		100
276.	Chhattisgarh-PET	2019		100
277.	Chhattisgarh-PET	2018		100
278.	Chhattisgarh-PET	2017		100
279.	Chhattisgarh-PET	2016		100
280.	Chhattisgarh-PET	2015		100
281.	Chhattisgarh-PET	2014		100
282.	Chhattisgarh-PET	2013		100
283.	Chhattisgarh-PET	2012		100
284.	Chhattisgarh-PET	2011		100
285.	Chhattisgarh-PET	2010		100
286.	Chhattisgarh-PET	2009		100
287.	Chhattisgarh-PET	2008		100
288.	Chhattisgarh-PET	2007		100
289.	Chhattisgarh-PET	2006		100
290.	Chhattisgarh-PET	2005		100
291.	Chhattisgarh-PET	2004		100
COMEDK				
292.	COMEDK-JEE	2023		60
293.	COMEDK-JEE	2022		60
294.	COMEDK-JEE	2021		60
295.	COMEDK-JEE	2020		60
296.	COMEDK-JEE	2019		60
297.	COMEDK-JEE	2018		60
298.	COMEDK-JEE	2017		60
299.	COMEDK-JEE	2016		60
300.	COMEDK-JEE	2015		60
301.	COMEDK-JEE	2014		60
302.	COMEDK-JEE	2013		60
303.	COMEDK-JEE	2012		60
304.	COMEDK-JEE	2011		60
Gujarat Common Entrance Test (GUJCET)				
305.	GUJCET	2023		40
306.	GUJCET	2022		40
307.	GUJCET	2021		40
308.	GUJCET	2020		40
309.	GUJCET	2019		40
310.	GUJCET	2018		40
311.	GUJCET	2017		40
312.	GUJCET	2016		40
313.	GUJCET	2015		40
314.	GUJCET	2014		40

315.	GUJCET	2011		40
316.	GUJCET	2010		40
317.	GUJCET	2009		40
318.	GUJCET	2008		40
319.	GUJCET	2007		40
HIMACHAL PRADESH-CET				
320.	HP-CET	2018		60
J & K-CET				
321.	J & K-CET	2020		75
322.	J & K-CET	2019		75
323.	J & K-CET	2018		75
324.	J & K-CET	2017		75
325.	J & K-CET	2016		75
326.	J & K-CET	2015		75
327.	J & K-CET	2014		75
328.	J & K-CET	2013		75
329.	J & K-CET	2012		75
330.	J & K-CET	2011		75
331.	J & K-CET	2010		75
332.	J & K-CET	2009		75
333.	J & K-CET	2008		75
334.	J & K-CET	2007		75
335.	J & K-CET	2006		75
336.	J & K-CET	2005		75
337.	J & K-CET	2004		75
338.	J & K-CET	2003		75
Jharkhand (JCECE)				
339.	JCECE	2019		50
340.	JCECE	2018		50
341.	JCECE	2017		50
342.	JCECE	2016		50
343.	JCECE	2015		50
344.	JCECE	2014		50
345.	JCECE	2013		50
346.	JCECE	2012		50
347.	JCECE	2011		50
348.	JCECE	2010		50
349.	JCECE	2009		50
350.	JCECE	2008		50
351.	JCECE	2007		50
352.	JCECE	2006		50
353.	JCECE	2005		50
354.	JCECE	2004		50
355.	JCECE	2003		50
356.	JCECE	2002		50
357.	JCECE	2001		50
Jamia Millia Islamia				
358.	Jamia Millia Islamia	2015		60
359.	Jamia Millia Islamia	2014		60
360.	Jamia Millia Islamia	2013		60
361.	Jamia Millia Islamia	2012		60
362.	Jamia Millia Islamia	2011		60
363.	Jamia Millia Islamia	2010		60
364.	Jamia Millia Islamia	2009		60
365.	Jamia Millia Islamia	2008		60

366.	Jamia Millia Islamia	2007		60
367.	Jamia Millia Islamia	2006		60
368.	Jamia Millia Islamia	2005		60
369.	Jamia Millia Islamia	2004		60
Kerala-KEAM				
370.	Kerala KEAM	2023		60
371.	Kerala KEAM	2022		60
372.	Kerala KEAM	2021		60
373.	Kerala KEAM	2020		60
374.	Kerala KEAM	2019		60
375.	Kerala KEAM	2018		60
376.	Kerala KEAM	2017		60
377.	Kerala KEAM	2016		60
378.	Kerala KEAM	2015		60
379.	Kerala KEAM	2014		60
380.	Kerala KEAM	2013		60
381.	Kerala KEAM	2012		60
382.	Kerala KEAM	2011		60
383.	Kerala KEAM	2010		60
384.	Kerala KEAM	2009		60
385.	Kerala KEAM	2008		60
386.	Kerala KEAM	2007		60
387.	Kerala KEAM	2006		60
388.	Kerala KEAM	2005		60
389.	Kerala KEAM	2004		60
Karnataka-CET (KCET)				
390.	Karnataka-CET	2023		60
391.	Karnataka-CET	2022		60
392.	Karnataka-CET	2021		60
393.	Karnataka-CET	2020		60
394.	Karnataka-CET	2019		60
395.	Karnataka-CET	2018		60
396.	Karnataka-CET	2017		60
397.	Karnataka-CET	2016		60
398.	Karnataka-CET	2015		60
399.	Karnataka-CET	2014		60
400.	Karnataka-CET	2013		60
401.	Karnataka-CET	2012		60
402.	Karnataka-CET	2011		60
403.	Karnataka-CET	2010		60
404.	Karnataka-CET	2009		60
405.	Karnataka-CET	2008		60
406.	Karnataka-CET	2007		60
407.	Karnataka-CET	2006		60
408.	Karnataka-CET	2005		60
409.	Karnataka-CET	2004		60
410.	Karnataka-CET	2003		60
411.	Karnataka-CET	2002		60
412.	Karnataka-CET	2001		60
413.	Karnataka-CET	2000		60
Kishore Vaigyanik Protsahan Yojana (KVPY)				
414.	KVPY-SB-SX	2023		15
415.	KVPY-SB-SX	2022		15
416.	KVPY-SB-SX	2021		15
417.	KVPY-SA	2021		15

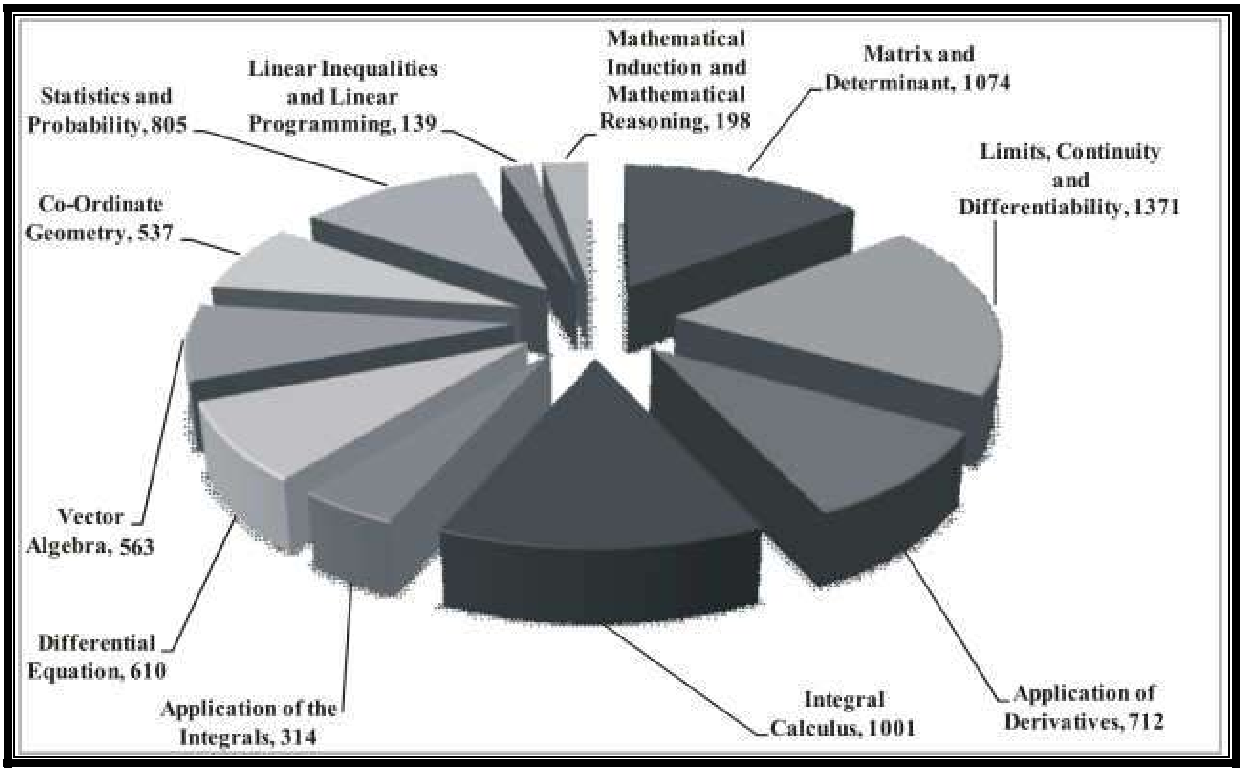
418.	KVPY-SA	2020		15
419.	KVPY-SB-SX	2018		15
420.	KVPY-SA	2017		15
421.	KVPY-SB-SX	2016		15
422.	KVPY-SB-SX	2015		15
423.	KVPY-SA	2014		15
424.	KVPY-SB-SX	2013		15
425.	KVPY-SA	2012		15
426.	KVPY-SA	2009		15
427.	KVPY-SB-SX	2009		15
Madhya Pradesh Pre Engineering Test (MPPET)				
428.	MPPET	2013		50
429.	MPPET	2012		50
430.	MPPET	2009		50
431.	MPPET	2008		50
Manipal-UGET				
432.	Manipal	2023		50
433.	Manipal	2022		50
434.	Manipal	2021		50
435.	Manipal	2020		50
436.	Manipal	2019		50
437.	Manipal	2018		50
438.	Manipal	2017		50
439.	Manipal	2016		50
440.	Manipal	2015		50
441.	Manipal	2014		50
442.	Manipal	2013		50
443.	Manipal	2012		50
444.	Manipal	2011		50
445.	Manipal	2010		50
446.	Manipal	2009		50
447.	Manipal	2008		50
(Maharashtra) MHT-CET				
448.	MHT-CET	2022	All Shifts	500
449.	MHT-CET	2021	All Shifts	500
450.	MHT-CET	13.10.2020	Shift-I	100
451.	MHT-CET	13.10.2020	Shift-II	100
452.	MHT-CET	14.10.2020	Shift-I	100
453.	MHT-CET	14.10.2020	Shift-II	100
454.	MHT-CET	15.10.2020	Shift-I	100
455.	MHT-CET	15.10.2020	Shift-II	100
456.	MHT-CET	16.10.2020	Shift-I	100
457.	MHT-CET	16.10.2020	Shift-II	100
458.	MHT-CET	19.10.2020	Shift-I	100
459.	MHT-CET	19.10.2020	Shift-II	100
460.	MHT-CET	20.10.2020	Shift-I	100
461.	MHT-CET	20.10.2020	Shift-II	100
462.	MHT-CET	02.05.2019	Shift-I	100
463.	MHT-CET	02.05.2019	Shift-II	100
464.	MHT-CET	03.05.2019		100

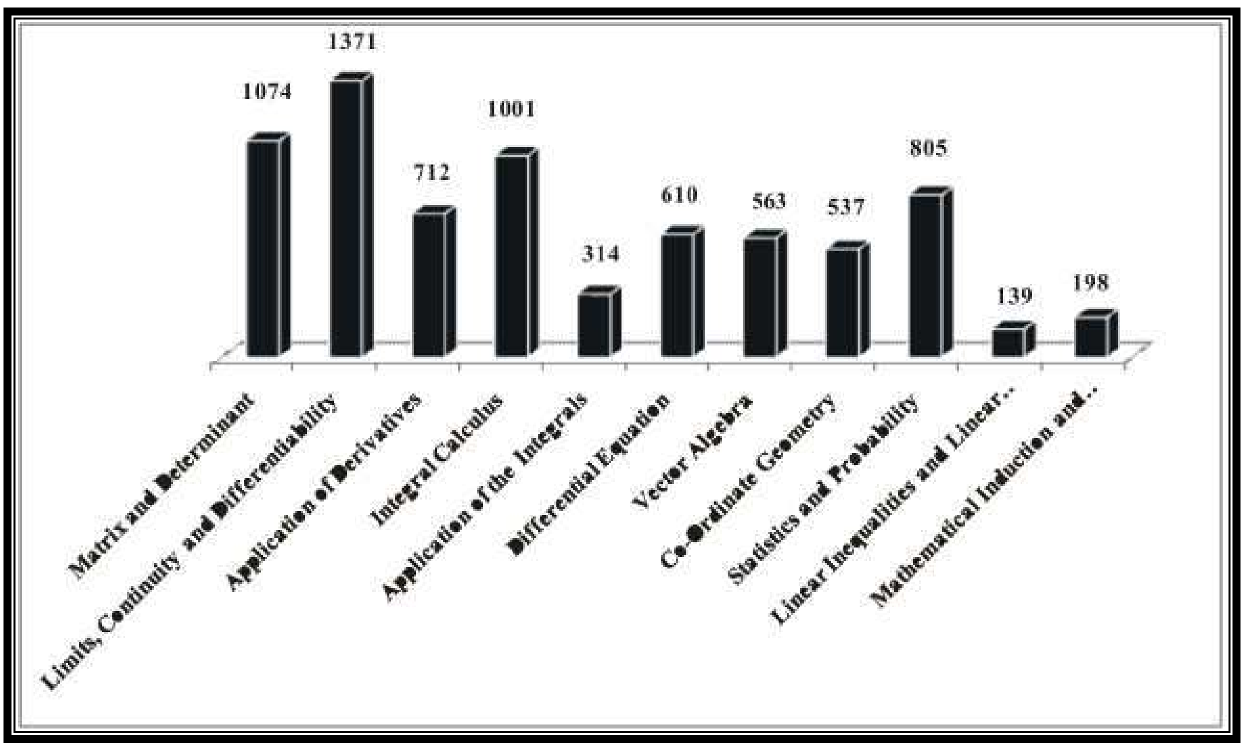
465.	MHT-CET	2018		100
466.	MHT-CET	2017		100
467.	MHT-CET	2016		100
468.	MHT-CET	2015		100
469.	MHT-CET	2014		100
470.	MHT-CET	2013		100
471.	MHT-CET	2012		100
472.	MHT-CET	2011		100
473.	MHT-CET	2010		100
474.	MHT-CET	2009		100
475.	MHT-CET	2008		100
476.	MHT-CET	2007		100
477.	MHT-CET	2006		100
478.	MHT-CET	2005		100
479.	MHT-CET	2004		100
Rajasthan PET				
480.	Rajasthan PET	2012		40
481.	Rajasthan PET	2011		40
482.	Rajasthan PET	2010		40
483.	Rajasthan PET	2009		40
484.	Rajasthan PET	2008		40
485.	Rajasthan PET	2007		40
486.	Rajasthan PET	2006		40
487.	Rajasthan PET	2005		40
488.	Rajasthan PET	2004		40
489.	Rajasthan PET	2003		40
490.	Rajasthan PET	2002		40
491.	Rajasthan PET	2001		40
SCRA				
492.	SCRA	2015		60
493.	SCRA	2014		60
494.	SCRA	2013		60
495.	SCRA	2012		60
496.	SCRA	2010		60
497.	SCRA	2009		60
SRM-JEEE				
498.	SRM-JEEE	2022		40
499.	SRM-JEEE	2021		40
500.	SRM-JEEE	2020		40
501.	SRM-JEEE	2019		40
502.	SRM-JEEE	2018		40
503.	SRM-JEEE	2016		40
504.	SRM-JEEE	2015		40
505.	SRM-JEEE	2014		40
506.	SRM-JEEE	2013		40
507.	SRM-JEEE	2012		40
508.	SRM-JEEE	2011		40
509.	SRM-JEEE	2010		40
510.	SRM-JEEE	2009		40
511.	SRM-JEEE	2008		40
512.	SRM-JEEE	2007		40

Telangana EAMCET				
513.	TS-EAMCET	12.05.2023	Shift-I	80
514.	TS-EAMCET	12.05.2023	Shift-II	80
515.	TS-EAMCET	13.05.2023	Shift-I	80
516.	TS-EAMCET	13.05.2023	Shift-II	80
517.	TS-EAMCET	14.05.2023	Shift-I	80
518.	TS-EAMCET	14.05.2023	Shift-II	80
519.	TS-EAMCET	18.07.2022	Shift-I	80
520.	TS-EAMCET	18.07.2022	Shift-II	80
521.	TS-EAMCET	19.07.2022	Shift-I	80
522.	TS-EAMCET	19.07.2022	Shift-II	80
523.	TS-EAMCET	20.07.2022	Shift-I	80
524.	TS-EAMCET	20.07.2022	Shift-II	80
525.	TS-EAMCET	06.08.2021	Shift-I	80
526.	TS-EAMCET	06.08.2021	Shift-II	80
527.	TS-EAMCET	05.08.2021	Shift-I	80
528.	TS-EAMCET	05.08.2021	Shift-II	80
529.	TS-EAMCET	04.08.2021	Shift-I	80
530.	TS-EAMCET	04.08.2021	Shift-II	80
531.	TS-EAMCET	09.09.2020	Shift-I	80
532.	TS-EAMCET	09.09.2020	Shift-II	80
533.	TS-EAMCET	10.09.2020	Shift-I	80
534.	TS-EAMCET	10.09.2020	Shift-II	80
535.	TS-EAMCET	11.09.2020	Shift-I	80
536.	TS-EAMCET	11.09.2020	Shift-II	80
537.	TS-EAMCET	14.09.2020	Shift-I	80
538.	TS-EAMCET	14.09.2020	Shift-II	80
539.	TS-EAMCET	03.05.2019	Shift-I	80
540.	TS-EAMCET	03.05.2019	Shift-II	80
541.	TS-EAMCET	04.05.2019	Shift-I	80
542.	TS-EAMCET	04.05.2019	Shift-II	80
543.	TS-EAMCET	06.05.2019	Shift-I	80
544.	TS-EAMCET	05.05.2018	Shift-I	80
545.	TS-EAMCET	05.05.2018	Shift-II	80
546.	TS-EAMCET	02.05.2018	Shift-I	80
547.	TS-EAMCET	04.05.2018	Shift-II	80
548.	TS-EAMCET	07.05.2018	Shift-I	80
549.	TS-EAMCET	24.04.2017	Shift-I	80
550.	TS-EAMCET	2016		80
551.	TS-EAMCET	2015		80
552.	TS-EAMCET	2014		80
Tripura JEE				
553.	Tripura JEE	2023		50
554.	Tripura JEE	2022		50
555.	Tripura JEE	2021		50
556.	Tripura JEE	2019		50
(Uttar Pradesh) UPTU/UPSEE				
557.	UPTU/UPSEE	2020		50
558.	UPTU/UPSEE	2019		50
559.	UPTU/UPSEE	2018		50
560.	UPTU/UPSEE	2017		50

561.	UPTU/UPSEE	2016		50
562.	UPTU/UPSEE	2015		50
563.	UPTU/UPSEE	2014		50
564.	UPTU/UPSEE	2013		50
565.	UPTU/UPSEE	2012		50
566.	UPTU/UPSEE	2011		50
567.	UPTU/UPSEE	2010		50
568.	UPTU/UPSEE	2009		50
569.	UPTU/UPSEE	2008		50
570.	UPTU/UPSEE	2007		50
571.	UPTU/UPSEE	2006		50
572.	UPTU/UPSEE	2005		50
573.	UPTU/UPSEE	2004		50
VITEEE				
574.	VITEEE	2023		40
575.	VITEEE	2022		40
576.	VITEEE	2021		40
577.	VITEEE	2020		40
578.	VITEEE	2019		40
579.	VITEEE	2018		40
580.	VITEEE	2017		40
581.	VITEEE	2016		40
582.	VITEEE	2015		40
583.	VITEEE	2014		40
584.	VITEEE	2013		40
585.	VITEEE	2012		40
586.	VITEEE	2011		40
587.	VITEEE	2010		40
588.	VITEEE	2009		40
589.	VITEEE	2008		40
590.	VITEEE	2007		40
591.	VITEEE	2006		40
WEST BENGAL				
592.	West Bengal	2023		30
593.	West Bengal	2022		30
594.	West Bengal	2021		30
595.	West Bengal	2020		30
596.	West Bengal	2019		30
597.	West Bengal	2018		30
598.	West Bengal	2017		30
599.	West Bengal	2016		30
600.	West Bengal	2015		30
601.	West Bengal	2014		30
602.	West Bengal	2013		30
603.	West Bengal	2012		30
604.	West Bengal	2011		30
605.	West Bengal	2010		30
606.	West Bengal	2009		30
607.	West Bengal	2008		30
	Total			**34700**

Trend Analysis of previous year paper of IIT JEE Mathematics through Bar Graph and Pie Chart





01. Matrix and Determinant

A. Matrices and Their Types Algebra of Matrices and Order of Matrices

1. The set of all 2×2 matrices over the real numbers is not a group under matrix multiplication because :
(a) identity element does not exist
(b) closure property is not satisfied
(c) association property is not satisfied
(d) Inverse axiom may not be satisfied

Karnataka CET-2000

Ans. (d) : If 2 × 2 matrix over the real number (R) is not a group under matrix multiplication because of inverse axiom may not be satisfied.

2. For how many values of x in the closed interval [–4, –1] the matrix $\begin{bmatrix} 3 & x-1 & 2 \\ 3 & -1 & x+2 \\ x+3 & -1 & 2 \end{bmatrix}$ is singular :
(a) zero (b) 2
(c) 1 (d) 3

Karnataka CET-2002
UPSEE-2008

Ans. (c) : Given that a singular matrix,

$$A = \begin{bmatrix} 3 & x-1 & 2 \\ 3 & -1 & x+2 \\ x+3 & -1 & 2 \end{bmatrix}$$

Then, $|A| = 0$

$$\Rightarrow \begin{vmatrix} 3 & -x+1 & 2 \\ 3 & -1 & x+2 \\ x+3 & -1 & 2 \end{vmatrix} = 0$$

On applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$ we get –

$$\begin{vmatrix} 3 & x-1 & 2 \\ 0 & -x & x \\ x & -x & 0 \end{vmatrix} = 0$$

On applying $C_1 \to C_1 + C_2 + C_3$ we get –

$$\begin{vmatrix} x+4 & -x+1 & 2 \\ 0 & -x & x \\ 0 & -x & 0 \end{vmatrix} = 0$$

$x + 4 (0 + x^2) = 0$
$4x^2 + x^3 = 0$
$x^2 (x + 4) = 0$
$x^2 = 0$ and $x + 4 = 0$
$x = 0$ and $x = -4$

Given interval $x \in [-4,-1]$
Hence, $x = -4$
So, x have only one value.

3. What must be the matrix X if $2X + \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 3 & 8 \\ 7 & 2 \end{bmatrix}$?

(a) $\begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & -3 \\ 2 & -1 \end{bmatrix}$

(c) $\begin{bmatrix} 2 & 6 \\ 4 & -2 \end{bmatrix}$ (d) $\begin{bmatrix} 2 & -6 \\ 4 & -2 \end{bmatrix}$

Karnataka CET-2004

Ans. (a) : Given that,

$$2X + \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 3 & 8 \\ 7 & 2 \end{bmatrix}$$

$$2X = \begin{bmatrix} 3 & 8 \\ 7 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$2X = \begin{bmatrix} 3-1 & 8-2 \\ 7-3 & 2-4 \end{bmatrix}$$

$$2X = \begin{bmatrix} 2 & 6 \\ 4 & -2 \end{bmatrix}$$

$$X = \frac{1}{2}\begin{bmatrix} 2 & 6 \\ 4 & -2 \end{bmatrix}$$

$$X = \begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix}$$

4. Let M be 2 × 2 symmetric matrix with integer entries, then M is invertible if
(a) the first column of M is the transpose of second row of M
(b) the second row of M is the transpose of first column of M
(c) M is diagonal matrix with non-zero entries in the principal diagonal
(d) the product of entries in the principal diagonal of M is the product of entries in the other diagonal

Karnataka CET-2021

Ans. (c, d) : If M is a diagonal matrix then,

$$M = \begin{bmatrix} a & 0 \\ 0 & d \end{bmatrix}$$

$|M| = ad - 0$
$|M| = ad$

Hence,
$|M| \neq 0$

Therefore M is invertible matrix.

And
$$M = \begin{bmatrix} a & b \\ b & d \end{bmatrix}$$

$|M| = ad - b^2$

Hence, $|M| \neq 0$
Therefore, M is an invertible matrix.
Hence, option (c) and (d) both are correct.

5. **If $\begin{pmatrix}2 & 1\\3 & 2\end{pmatrix}A=\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}$, then the matrix A is**

(a) $\begin{pmatrix}2 & -1\\-3 & 2\end{pmatrix}$ (b) $\begin{pmatrix}-2 & 1\\3 & -2\end{pmatrix}$

(c) $\begin{pmatrix}2 & -1\\3 & 2\end{pmatrix}$ (d) $\begin{pmatrix}2 & 1\\3 & 2\end{pmatrix}$

Karnataka CET-2020

Ans. (a) : Given that,

$$\begin{bmatrix}2 & 1\\3 & 2\end{bmatrix}A=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$$

Let,

$$A=\begin{bmatrix}a & b\\x & y\end{bmatrix}$$

$$\begin{bmatrix}2 & 1\\3 & 2\end{bmatrix}\begin{bmatrix}a & b\\x & y\end{bmatrix}=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$$

$$\begin{bmatrix}2a+x & 2b+y\\3a+2x & 3b+2y\end{bmatrix}=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$$

On comparing corresponding elements on both the side we get –

$2a + x = 1$

$2a = 1 - x$

$a=\frac{1-x}{2}$ …..(i)

And $2b + y = 0$

$2b = -y$

$b=-\frac{y}{2}$ …..(ii)

$3a + 2x = 0$

From equation (i),

$3\times\frac{1-x}{2}+2x=0$

$\frac{3-3x+4x}{2}=0$

$x = -3$

And $3b + 2y = 1$

from equation(ii),

$3\times-\frac{y}{2}+2y=1$

$\frac{-3y+4y}{2}=1$

$y = 2$

Putting the value of x and y in equation (i) and (ii) respectively–

$a = 2$ and $b = -1$

$\therefore\quad A=\begin{bmatrix}a & b\\x & y\end{bmatrix}=\begin{bmatrix}2 & -1\\-3 & 2\end{bmatrix}$

6. **If A and B are square matrices of same order and B is a skew symmetric matrix, then A'BA is**

(a) Null matrix
(b) Diagonal matrix
(c) Skew symmetric matrix
(d) Symmetric matrix

Karnataka CET-2020

Ans. (c) : According to question B is a skew symmetric matrix, it means –

$B' = -B$

Then, value of A' BA –

Consider, $[A'BA]' = (A')'B'A'$

$= A(-B)A'$ $[\because B' = -B]$

$= -(ABA')$

Hence, the transpose of given matrix is equal to its negative, so it is a skew – symmetric matrix.

7. **If $f(x)=\begin{bmatrix}x^3-x & a+x & b+x\\x-a & x^2-x & c+x\\x-b & x-c & 0\end{bmatrix}$, then**

(a) f(2) = 0 (b) f(0) = 0
(c) f(–1) = 0 (d) f(1) = 0

Karnataka CET-2020

Ans. (b) : Given that,

$$f(x)=\begin{bmatrix}x^3-x & a+x & b+x\\x-a & x^2-x & c+x\\x-b & x-c & 0\end{bmatrix}$$

$$f(0)=\begin{bmatrix}0 & a & b\\-a & 0 & c\\-b & -c & 0\end{bmatrix}$$

If all the diagonal of the matrix is zero then the matrix is zero matrix.

f(0) = 0

8. **If P and Q are symmetric matrices of the same order then PQ – QP is**

(a) identity matrix
(b) zero matrix
(c) symmetric matrix
(d) skew symmetric matrix

Karnataka CET-2019

Ans. (d) : We know that,

The product of symmetric matrix and skew – symmetric matrix is a skew – symmetric matrix.

Given that,

$P = P'$ and $Q = Q'$

$(PQ - QP)' = [(PQ)' - (QP)']$

$= [Q'P' - P'Q']$

$= QP - PQ$

$= -[PQ - QP]$

Hence, (PQ – QP) is Q Skew symmetric matrix.

9. **If $A=\begin{bmatrix}2 & -2\\-2 & 2\end{bmatrix}$, then $A^n = 2^k A$, where k =**

(a) 2^{n-1} (b) n + 1
(c) n – 1 (d) 2 (n – 1)

Karnataka CET-2018

Ans. (d) : Given that,

$$A=\begin{bmatrix}2 & -2\\-2 & 2\end{bmatrix}$$

$$A^2=\begin{bmatrix}2 & -2\\-2 & 2\end{bmatrix}\begin{bmatrix}2 & -2\\-2 & 2\end{bmatrix}$$

$$A^2 = \begin{bmatrix} 8 & -8 \\ -8 & 8 \end{bmatrix} = 4\begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}$$

$\therefore$ $A^2 = 4A = 2^2A$

$A^3 = A^2 A$

$A^3 = 4A \cdot A = 4A^2$

$A^3 = 4(4A)$

$A^3 = 2^4A$

$\therefore$ $A^4 = A^3 \cdot A = 16A \cdot A$

$A^4 = 16A^2$

$A^4 = 16(4A) = 64A$

$A^4 = 2^6 A$

Hence, $A^n = 2^k A$

$A^2 = 2^2 A$

$A^3 = 2^4 A$

$A^4 = 2^6 A$

Therefore,

$k = 2(n - 1)$

10. If a matrix A is both symmetric and skew symmetric, then

(a) A is diagonal matrix (b) A is a zero matrix
(c) A is scalar matrix (d) A is square matrix

Karnataka CET-2017

Ans. (b) : According to question, A is both symmetric and skew symmetric matrix it means –

$A^T = A$ (Symmetric)

And $A^T = -A$ (Skew-symmetric)

$\therefore$ $A = A^T = -A$

It is only possible in case of zero matrix,

$$A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = A^T = -A$$

11. If A and B are square matrices of order 'n' such that $A^2 - B^2 = (A - B)(A + B)$, then which of the following will be true?

(a) Either of A or B is zero matrix.
(b) A = B
(c) AB = BA
(d) Either of A or B is an identity matrix.

Karnataka CET-2013

Ans. (c) : Given that,

$A^2 - B^2 = (A - B)(A + B)$

Here, A, B are order $n \times n$

$\therefore$ $A^2 - B^2 = A^2 - BA + AB - B^2$

$A^2 - B^2 - A^2 + B^2 = AB - BA$

$0 = AB - BA$

Therefore, $AB = BA$

12. $\begin{bmatrix} \sin\alpha & \cos\alpha & \sin(\alpha+\delta) \\ \sin\beta & \cos\beta & \sin(\beta+\delta) \\ \sin\gamma & \cos\gamma & \sin(\gamma+\delta) \end{bmatrix} =$

(a) 0
(b) 1
(c) $1 + \sin\alpha \sin\beta \sin\gamma$
(d) $1 - (\sin\alpha - \sin\beta)(\sin\beta - \sin\gamma)(\sin\gamma - \sin\alpha)$

Karnataka CET-2011

Ans. (a) : Let,

$$A = \begin{bmatrix} \sin\alpha & \cos\alpha & \sin(\alpha+\delta) \\ \sin\beta & \cos\beta & \sin(\beta+\delta) \\ \sin\gamma & \cos\gamma & \sin(\gamma+\delta) \end{bmatrix}$$

On applying $C_1 \to C_1 \times \cos\delta$ and $C_2 \to C_2 \times \sin\delta$ we get–

$$A = \frac{1}{\sin\delta\cos\delta}\begin{vmatrix} \sin\alpha\cos\delta & \cos\alpha\sin\delta & \sin(\alpha+\delta) \\ \sin\beta\cos\delta & \cos\beta\sin\delta & \sin(\beta+\delta) \\ \sin\gamma\cos\delta & \cos\gamma\sin\delta & \sin(\gamma+\delta) \end{vmatrix}$$

On using sin (A + B) = sinA cosB + cosA sinB and $C_1 = C_1 + C_2$.

$$A = \frac{1}{\sin\delta\cos\delta}\begin{vmatrix} \sin(\alpha+\delta) & \cos\alpha\sin\delta & \sin(\alpha+\delta) \\ \sin(\beta+\delta) & \cos\beta\sin\delta & \sin(\beta+\delta) \\ \sin(\gamma+\delta) & \cos\gamma\sin\delta & \sin(\gamma+\delta) \end{vmatrix}$$

On applying $C_3 \to C_3 - C_1$ we get–

$$A = \frac{1}{\sin\delta\cos\delta}\begin{vmatrix} \sin(\alpha+\delta) & \cos\alpha\sin\delta & 0 \\ \sin(\beta+\delta) & \cos\beta\sin\delta & 0 \\ \sin(\gamma+\delta) & \cos\gamma\sin\delta & 0 \end{vmatrix}$$

$A = 0$

13. If $\begin{bmatrix} 1 & 2 & -1 \\ 1 & x-2 & 1 \\ x & 1 & 1 \end{bmatrix}$ is singular, then the value of x is

(a) 2 (b) 3
(c) 1 (d) 0

Karnataka CET-2011

Ans. (a) : Let,

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 1 & x-2 & 1 \\ x & 1 & 1 \end{bmatrix}$$

According to question A is singular matrix,

$|A| = 0$

$$\begin{vmatrix} 1 & 2 & -1 \\ 1 & x-2 & 1 \\ x & 1 & 1 \end{vmatrix} = 0$$

$1[1(x - 2) - 1] - 2[1 - x] - 1[1 - x^2 + 2x] = 0$

$x - 3 - 2 + 2x - 1 + x^2 - 2x = 0$

$x^2 + x - 6 = 0$

$x^2 + 3x - 2x - 6 = 0$

$x(x + 3) - 2(x + 3) = 0$

$(x + 3)(x - 2) = 0$

$x + 3 = 0$ and $x - 2 = 0$

$x = -3$ and $x = 2$

Hence, $x = -3$ is not possible.

Therefore, $x = 2$.

14. If A and B are two square matrices of the same order such that AB = B and BA = A, then $A^2 + B^2$ is always equal to

(a) $A + B$ (b) I
(c) 2BA (d) 2AB

Karnataka CET-2010

Ans. (a) : Given that,

$AB = B$ ....(i)

$BA = A$ ....(ii)

$A^2 + B^2 = A \cdot A + B \cdot B$

From equation (i) and equation (ii), we get –

$A^2 + B^2 = A(BA) + B(AB)$

$= (AB) \cdot A + (BA) \cdot B$

$= B \cdot A + A \cdot B = A + B$

15. The value of $\begin{bmatrix} x & p & q \\ p & x & q \\ p & q & x \end{bmatrix}$ is

(a) $x(x-p)(x-q)$

(b) $(x-p)(x-q)(x+p+q)$

(c) $(p-q)(x-q)(x-p)$

(d) $pq(x-p)(x-q)$

Karnataka CET-2007

Ans. (b) : Let,

$$A = \begin{bmatrix} x & p & q \\ p & x & q \\ p & q & x \end{bmatrix}$$

$$|A| = \begin{vmatrix} x & p & q \\ p & x & q \\ p & q & x \end{vmatrix}$$

Expanding the determinant –

$|A| = x[x^2 - q^2] - p[px - pq] + q[pq - px]$

$= x^3 - xq^2 - p^2x + p^2q + pq^2 - pqx$

$= x(x-q)(x+q) - p^2(x-q) - pq(x-q)$

$= (x-q)(x^2 + xq - p^2 - pq)$

$= (x-q)(x-p)(x+p+q)$

16. If $A = \begin{bmatrix} 0 & x & 16 \\ x & 5 & 7 \\ 0 & 9 & x \end{bmatrix}$ is singular, then the possible values of x are

(a) 0, 12, –12 (b) 0, 1, –1

(c) 0, 4, –4 (d) 0, 5, –5

Karnataka CET-2007

Ans. (a) : Given that,

$$A = \begin{bmatrix} 0 & x & 16 \\ x & 5 & 7 \\ 0 & 9 & x \end{bmatrix}$$

According to question A is singular matrix it means –

$|A| = 0$

$$\begin{vmatrix} 0 & x & 16 \\ x & 5 & 7 \\ 0 & 9 & x \end{vmatrix} = 0$$

$0(5x - 63) - x(x^2 - 0) + 16(9x - 0) = 0$

$-x^3 + 144x = 0$

$x^3 - 144x = 0$

$x(x^2 - 144) = 0$

$x = 0$ and $x^2 - 144 = 0$

$x^2 = 144$

$x = \pm 12$

Hence, x = 0, –12, 12

17. If $(4\ 3\ 2)\begin{pmatrix} 1 \\ -2 \\ x \end{pmatrix} = (6)$ then x is

(a) 4 (b) 3 (c) 2 (d) 1

SRM JEEE-2018

Ans. (a) : Given that,

$$\begin{bmatrix} 4 & 3 & 2 \end{bmatrix}_{1\times3} \begin{bmatrix} 1 \\ -2 \\ x \end{bmatrix}_{3\times1} = 6$$

$4 - 6 + 2x = 6$

$2x = 8$

$x = 4$

18. The matrix $A = \begin{bmatrix} a & -1 & 4 \\ -3 & 0 & 1 \\ -1 & 1 & 2 \end{bmatrix}$ is not invertible only if a =

(a) –16 (b) 16 (c) 17 (d) –17

MHT CET-2020

Ans. (d) : Given that,

$$A = \begin{bmatrix} a & -1 & 4 \\ -3 & 0 & 1 \\ -1 & 1 & 2 \end{bmatrix}$$

According to question, A is not invertible it means –

$|A| = 0$

$$\begin{vmatrix} a & -1 & 4 \\ -3 & 0 & 1 \\ -1 & 1 & 2 \end{vmatrix} = 0$$

$a(0-1) + 1(-6+1) + 4(-3+0) = 0$

$-a - 5 - 12 = 0$

$a = -17$

19. The sum of the cofactors of the elements of second row of the matrix $\begin{bmatrix} 1 & 3 & 2 \\ -2 & 0 & 1 \\ 5 & 2 & 1 \end{bmatrix}$ is

(a) 23 (b) 5

(c) 3 (d) –23

MHT CET-2020

Ans. (b) : Let,

$$A = \begin{bmatrix} 1 & 3 & 2 \\ -2 & 0 & 1 \\ 5 & 2 & 1 \end{bmatrix}$$

Co-factors of second row of matrix are –

$$A_{21} = (-1)^{2+1}\begin{vmatrix} 3 & 2 \\ 2 & 1 \end{vmatrix} = (-1)^3(-1) = 1$$

$$A_{22} = (-1)^{2+2}\begin{vmatrix} 1 & 2 \\ 5 & 1 \end{vmatrix} = (-1)^4(-9) = -9$$

$$A_{23} = (-1)^{2+3}\begin{vmatrix} 1 & 3 \\ 5 & 2 \end{vmatrix} = (-1)^5(-13) = 13$$

Therefore, the sum of co-factor of second row,

$A_{21} + A_{22} + A_{23} = 1 - 9 + 13 = 5$

20. **The value of x such that the matrix $\begin{bmatrix} x & 2 & 3 \\ 4 & 5 & 6 \\ 2 & 3 & 5 \end{bmatrix}$ is not invertible is**

(a) $\frac{7}{10}$ (b) $\frac{10}{7}$

(c) $\frac{-7}{10}$ (d) $\frac{-10}{7}$

MHT CET-2020

Ans. (b) : Let,

$$A = \begin{bmatrix} x & 2 & 3 \\ 4 & 5 & 6 \\ 2 & 3 & 5 \end{bmatrix}$$

According to question, A is not in invertible it means–

$\therefore$ $|A| = 0$

$$\begin{vmatrix} x & 2 & 3 \\ 4 & 5 & 6 \\ 2 & 3 & 5 \end{vmatrix} = 0$$

$x(25-18) - 2(20-12) + 3(12-10) = 0$

$7x - 16 + 6 = 0$

$7x = 10$

$x = \frac{10}{7}$

21. **If $A = \begin{bmatrix} 1 & 0 & 2 \\ 2 & 1 & 3 \\ 0 & 3 & -5 \end{bmatrix}$, where A_{ij} is the cofactor of the element a_{ij} of matrix A, then $a_{21}A_{21} + a_{22}A_{22} + a_{23}A_{23} =$**

(a) 26 (b) –26

(c) 0 (d) –2

MHT CET-2020

Ans. (d) : Given that,

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 2 & 1 & 3 \\ 0 & 3 & -5 \end{bmatrix}$$

Here, $a_{21} = 2$, $a_{22} = 1$ and $a_{23} = 3$

To find co-factor –

$$A_{21} = (-1)^{2+1} \begin{vmatrix} 0 & 2 \\ 3 & -5 \end{vmatrix} = (-1)^3(-6) = 6$$

$$A_{22} = (-1)^{2+2} \begin{vmatrix} 1 & 2 \\ 0 & -5 \end{vmatrix} = (-1)^4(-5) = -5$$

$$A_{23} = (-1)^{2+3} \begin{vmatrix} 1 & 0 \\ 0 & 3 \end{vmatrix} = (-1)^5(3-0) = -3$$

So, $a_{21}A_{21} + a_{22}A_{22} + a_{23}A_{23}$

$= 2 \times 6 + 1 \times -5 + 3 \times -3$

$= 12 - 5 - 9$

$= -2$

22. **If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and X is a 2 × 2 matrix such that AX = I, then X =**

(a) $\begin{bmatrix} -2 & 1 \\ \frac{3}{2} & \frac{1}{2} \end{bmatrix}$ (b) $\begin{bmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{bmatrix}$

(c) $\begin{bmatrix} 2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{bmatrix}$ (d) $\begin{bmatrix} -2 & 1 \\ -\frac{3}{2} & -\frac{1}{2} \end{bmatrix}$

MHT CET-2020

Ans. (b) : Given that,

$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and AX = I

$\therefore$ $X = A^{-1}$

$\Rightarrow$ $X = \frac{1}{4-6}\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$

$=$ $X = \frac{1}{-2}\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ \frac{3}{2} & \frac{-1}{2} \end{bmatrix}$

23. **The cofactors of the elements of the first column of the matrix $A = \begin{bmatrix} 2 & 0 & -1 \\ 3 & 1 & 2 \\ -1 & 1 & 2 \end{bmatrix}$ are**

(a) 0, –7, 2 (b) 0, –1, 1

(c) 0, –8, 4 (d) –1, 3, –2

MHT CET-2020

Ans. (b) : Given that,

$$A = \begin{bmatrix} 2 & 0 & -1 \\ 3 & 1 & 2 \\ -1 & 1 & 2 \end{bmatrix}$$

We know that,

$$C_{11} = (-1)^{1+1} \times \begin{vmatrix} 1 & 2 \\ 1 & 2 \end{vmatrix} = (-1)^2 \times 0 = 0$$

$$C_{21} = (-1)^{2+1} \times \begin{vmatrix} 0 & -1 \\ 1 & 2 \end{vmatrix} = (-1)^3 \times (1) = -1$$

$$C_{31} = (-1)^{3+1} \times \begin{vmatrix} 0 & -1 \\ 1 & 2 \end{vmatrix} = (-1)^4 \times (1) = 1$$

24. **Which of the following matrix is invertible?**

$$A_1 = \begin{bmatrix} 4 & 2 \\ 2 & 1 \end{bmatrix}, A_2 = \begin{bmatrix} -1 & -2 & 3 \\ 4 & 5 & 7 \\ 2 & 4 & -6 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 1 & 0 & 0 \\ 5 & 2 & 1 \\ 7 & 2 & 1 \end{bmatrix}, A_4 = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix}$$

(a) A_3 (b) A_2

(c) A_1 (d) A_4

MHT CET-2020

Ans. (d) : Any matrix is said to be invertible only if $|A| \neq 0$

$$|A_4| = \begin{vmatrix} 1 & 0 & 1 \\ 0 & 2 & 3 \\ 1 & 2 & 1 \end{vmatrix} = (-4) - 0 + 1(-2) = -6$$

$|A_4| \neq 0$

Hence, A_4 is invertible matrix.

25. If Matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 5 \\ 2 & 4 & 7 \end{bmatrix}$, then the value of $a_{31}A_{31} + a_{32}A_{32} + a_{33}A_{33}$ is

(a) 1 (b) 13
(c) –1 (d) –13

MHT CET-2018

Ans. (c) : Given that,

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 5 \\ 2 & 4 & 7 \end{bmatrix}$$

Here, $a_{31} = 2$, $a_{32} = 4$ and $a_{33} = 7$

To find co-factor –

$$A_{31} = (-1)^{3+1}\begin{vmatrix} 2 & 3 \\ 1 & 5 \end{vmatrix} = (-1)^4 (10 - 3) = 7$$

$$A_{32} = (-1)^{3+2}\begin{vmatrix} 1 & 3 \\ 1 & 5 \end{vmatrix} = (-1)^5 (5 - 3) = -2$$

$$A_{33} = (-1)^{3+3}\begin{vmatrix} 1 & 2 \\ 1 & 1 \end{vmatrix} = (-1)^6 (1 - 2) = -1$$

So, $a_{31}A_{31} + a_{32}A_{32} + a_{33}A_{33}$
$= 2 \times 7 + 4 \times -2 + 7 \times -1$
$= 14 - 8 - 7$
$= -1$

26. If A is non-singular matrix such that $(A - 2I)(A - 4I) = 0$, then $A + 8A^{-1} =$

(a) 6I (b) 0
(c) 3I (d) I

MHT CET-2019

Ans. (a) :

A is non-singular matrix. i.e. $|A| \neq 0$. So A^{-1} exists.

$(A - 2I)(A - 4I) = 0$

$A \cdot A - 4AI - 2IA + 8I^2 = 0$

$A^2 - 6AI + 8I^2 = 0$

$A^2 - 6A + 8I = 0$

On multiplying by A^{-1} we get –

$A^2A^{-1} - 6AA^{-1} + 8IA^{-1} = 0$

$A - 6I + 8A^{-1} = 0$

$\therefore \quad A + 8A^{-1} = 6I$

27. If $A = \begin{bmatrix} 1 & 1 & 0 \\ 2 & 1 & 5 \\ 2 & 2 & 1 \end{bmatrix}$ then $a_{11}A_{21} + a_{12}A_{22} + a_{13}A_{23}$ = ________

(a) 1 (b) 0
(c) −1 (d) 2

MHT CET-2016

Ans. (b) : We know that,

a_{ij} stands for element in matrix in i^{th} row and j^{th} column, and A_{ij} stands for co-factor of element a_{ij} of matrix A.

Given that,

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 2 & 1 & 5 \\ 1 & 2 & 1 \end{bmatrix}$$

Here, $a_{11} = 1$, $a_{12} = 1$ and $a_{13} = 0$

$$A_{21} = (-1)^{2+1}\begin{vmatrix} 1 & 0 \\ 2 & 1 \end{vmatrix} = -1$$

$$A_{22} = (-1)^{2+2}\begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix} = 1$$

$$A_{23} = (-1)^{2+3}\begin{vmatrix} 1 & 1 \\ 2 & 2 \end{vmatrix} = 0$$

$\therefore \quad a_{11}A_{21} + a_{12}A_{22} + a_{13}A_{23}$
$= 1 \times (-1) + 1 \times (1) + 0 \times (0) = 0$

**28. If $A = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$, then the value of $a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13} =$
Where A_{11}, A_{12}, A_{13} are cofactors of a_{11}, a_{12}, a_{13} respectively.**

(a) –1 (b) 1
(c) 0 (d) $\frac{1}{2}$

MHT CET-2012

Ans. (b) : Given that,

$$A = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$a_{11} = \cos\theta$

And $A_{11} = (-1)^2\begin{vmatrix} \cos\theta & 0 \\ 0 & 1 \end{vmatrix} = (1)(\cos\theta) = \cos\theta$

$a_{12} = \sin\theta$

And $A_{12} = (-1)^3\begin{vmatrix} -\sin\theta & 0 \\ 0 & 1 \end{vmatrix} = (-1)(-\sin\theta) = \sin\theta$

$a_{13} = 0$

And $A_{13} = (-1)^4\begin{vmatrix} -\sin\theta & \cos\theta \\ 0 & 0 \end{vmatrix} = (1)(0) = 0$

Then,

$a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13}$
$= \cos\theta(\cos\theta) + \sin\theta(\sin\theta) + (0)(0)$
$= \cos^2\theta + \sin^2\theta = 1$

Note : $a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13} = |A|$

$\therefore |A| = \cos\theta(\cos\theta) - \sin\theta(-\sin\theta) = \cos^2\theta + \sin^2\theta = 1$

29. For the matrix $A = \begin{bmatrix} 3 & 2 & 4 \\ 1 & 4 & 1 \\ 2 & 6 & 3 \end{bmatrix}$ and where A_{ij} is co-factor of elements a_{ij}, then $a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13} =$

(a) 7 (b) 12
(c) –8 (d) 8

MHT CET-2009

Ans. (d) : Given that,

$$A = \begin{bmatrix} 3 & 2 & 4 \\ 1 & 4 & 1 \\ 2 & 6 & 3 \end{bmatrix}$$

$a_{11} = 3$ and $A_{11} = (-1)^2 \begin{vmatrix} 4 & 1 \\ 6 & 3 \end{vmatrix} = (1)(12-6) = 6$

$a_{12} = 2$ and $A_{12} = (-1)^3 \begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} = (-1)(3-2) = -1$

$a_{13} = 4$ and $A_{13} = (-1)^4 \begin{vmatrix} 1 & 1 \\ 2 & 6 \end{vmatrix} = (1)(6-8) = -2$

So,

$$a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13} = (3)(6) + (2)(-1) + (4)(-2)$$
$$= 18 - 2 - 8 = 8$$

Note : $a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13} = |A|$

$$\therefore \quad |A| = \begin{vmatrix} 3 & 2 & 4 \\ 1 & 4 & 1 \\ 2 & 6 & 3 \end{vmatrix}$$

$$= 3(12-6) - 2(3-2) + 4(6-8) = 18 - 2 - 8 = 8$$

30. Matrix A is of order m × n; matrix B is of order p ×q, such that AB exists, then

(a) m = n (b) p = n
(c) m = q (d) p = q

MHT CET-2007

Ans. (b) :
Matrix A is of order m × n and matrix B is of order p ×q then AB exist if number of columns of A = number of rows of B
$\therefore$ n = p

31. A is a square matrix $[a_{ij}]$ such that $A_{ij} = 0$ for $i \neq j = k$, for $i = j$, where k is a constant, then A is called as

(a) Null matrix (b) Unit matrix
(c) Diagonal matrix (d) Scalar matrix

MHT CET-2006

Ans. (d) :
Let A be 2 × 2 matrix. Then as per conditions given.

$$A = \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$$

Let A be a 3 × 3 matrix.
Then as per conditions given.

$$A = \begin{bmatrix} k & 0 & 0 \\ 0 & k & 0 \\ 0 & 0 & k \end{bmatrix}$$

A scalar matrix is a diagonal matrix with equal valued elements along the diagonal.
Thus, A is always a scalar matrix.

32. If $E(\theta) = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ then $|E^3(\theta)|$ =

(a) 1 (b) 0
(c) –2 (d) 2

MHT CET-2006

Ans. (a) : Given that,

$$E(\theta) = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$$|E(\theta)| = \begin{vmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{vmatrix} = \cos^2\theta + \sin^2\theta = 1$$

Now

$$|E^3(\theta)| = (1)^3 = 1$$

33. If $A + I = \begin{bmatrix} 3 & -2 \\ 4 & 1 \end{bmatrix}$, then (A + I) (A – I) =

(a) $\begin{bmatrix} 1 & 2 \\ 4 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 5 & -4 \\ 8 & -9 \end{bmatrix}$

(c) $\begin{bmatrix} -5 & -4 \\ 8 & -9 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & -2 \\ 4 & -1 \end{bmatrix}$

MHT CET-2005

Ans. (c) : Given that,

$$A + I = \begin{bmatrix} 3 & -2 \\ 4 & 1 \end{bmatrix}$$

$$A - I = (A + I) - 2I$$

$$A - I = \begin{bmatrix} 3 & -2 \\ 4 & 1 \end{bmatrix} - 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -2 \\ 4 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 4 & -1 \end{bmatrix}$$

$$(A+I)(A-I) = \begin{bmatrix} 3 & -2 \\ 4 & 1 \end{bmatrix}\begin{bmatrix} 1 & -2 \\ 4 & -1 \end{bmatrix} = \begin{bmatrix} 3-8 & -6+2 \\ 4+4 & -8-1 \end{bmatrix} = \begin{bmatrix} -5 & -4 \\ 8 & -9 \end{bmatrix}$$

34. If

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}, B = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 0 & 3 \\ 3 & 0 & 1 \end{bmatrix}, C = \begin{bmatrix} 4 & 5 & 6 \\ 1 & 2 & 3 \\ 7 & 6 & 5 \end{bmatrix},$$

then

(a) $A + B = B + A$ and $A + (B + C) = (A + B) + C$
(b) A + B = B + A and AB = BA
(c) BC = CB
(d) AC = CA

MHT CET-2004

Ans. (a) : For properties matrices A, B and C,
A + B = B + A and A + (B + C) = (A + B) + C

35. If $A = \begin{bmatrix} 2 & -4 \\ 1 & -2 \end{bmatrix}$, then A^2 is

(a) Null matrix (b) Unit matrix

(c) $\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

MHT CET-2004

Ans. (a) : Given that,

$$A^2 = A.A. = \begin{bmatrix} 2 & -4 \\ 1 & -2 \end{bmatrix}\begin{bmatrix} 2 & -4 \\ 1 & -2 \end{bmatrix} = \begin{bmatrix} 4-4 & -8+8 \\ 2-2 & -4+4 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

All elements of the matrix is zero then the matrix is null matrix.
Hence, A^2 is null matrix.

36. If A = $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ and $A^T + A = I_2$ then

(a) $\theta = n\pi, n \in Z$

(b) $\theta = (2n+1)\dfrac{\pi}{2}, n\in Z$

(c) $\theta = 2n\pi \pm \dfrac{\pi}{3}, n \in Z$

(d) None of these

COMEDK-2015

Ans. (c) : Given that,

$$A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$$A^T = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

We have, $A + A^T = I_2$

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} + \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \cos\theta+\cos\theta & -\sin\theta+\sin\theta \\ \sin\theta-\sin\theta & \cos\theta+\cos\theta \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2\cos\theta & 0 \\ 0 & 2\cos\theta \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$\Rightarrow$ $2\cos\theta = 1$

$\Rightarrow$ $\cos\theta = \dfrac{1}{2}$

$\Rightarrow$ $\cos\theta = \cos\dfrac{\pi}{3}$

$\Rightarrow$ $\theta = 2n\pi \pm \dfrac{\pi}{3}, \quad n \in Z$

37. If A = $\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$, then minor and cofactor of a_{32} are

(a) 1, –1 (b) 0, 1 (c) –1, 0 (d) 0, 0

COMEDK-2017

Ans. (d) : We have, $A = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Minor of $a_{32} = \begin{vmatrix} \cos\theta & 0 \\ \sin\theta & 0 \end{vmatrix} = 0 - 0 = 0$

And cofactor of a_{32} = – Minor of a_{32} = – 0 = 0

38. If A = $\begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$ is the sum of a symmetric matrix B and a skew-symmetric matrix C, then C is

(a) $\begin{bmatrix} 1 & -5/2 \\ 5/2 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & -5/2 \\ 5/2 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & -5/2 \\ 5/2 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & -3/2 \\ 5/2 & 1 \end{bmatrix}$

COMEDK-2019

Ans. (c) : Given that,

$$A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$$

According to theorem,

$$A = \left(\frac{A+A'}{2}\right) + \left(\frac{A-A'}{2}\right) = B + C$$

Where B and C are symmetric and skew - symmetric matrices respectively.

$$\text{Now, } C = \frac{A-A'}{2} = \frac{1}{2}\left\{\begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ -4 & -1 \end{bmatrix}\right\}$$

$$= \frac{1}{2}\begin{bmatrix} 0 & -5 \\ 5 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -5/2 \\ 5/2 & 0 \end{bmatrix}$$

39. The rank of the matrix $\begin{bmatrix} 3 & 4 & 5 & 6 & 7 \\ 4 & 5 & 6 & 7 & 8 \\ 5 & 6 & 7 & 8 & 9 \\ 10 & 11 & 12 & 13 & 14 \end{bmatrix}$ is equal to

(a) 3 (b) 2

(c) 4 (d) 5

SRM JEEE-2016

Ans. (b) : Let, $A = \begin{bmatrix} 3 & 4 & 5 & 6 & 7 \\ 4 & 5 & 6 & 7 & 8 \\ 5 & 6 & 7 & 8 & 9 \\ 10 & 11 & 12 & 13 & 14 \end{bmatrix}$

On applying $C_2 \to C_2 - C_1$, $C_4 \to C_4 - C_3$, we get –

$$A = \begin{bmatrix} 3 & 1 & 5 & 1 & 7 \\ 4 & 1 & 6 & 1 & 8 \\ 5 & 1 & 7 & 1 & 9 \\ 10 & 1 & 12 & 1 & 14 \end{bmatrix}$$

Applying $C_5 \to C_5 - C_1$, $C_2 \to C_2 - C_4$ we get –

$$A = \begin{bmatrix} 3 & 0 & 5 & 1 & 4 \\ 4 & 0 & 6 & 1 & 4 \\ 5 & 0 & 7 & 1 & 4 \\ 10 & 0 & 12 & 1 & 4 \end{bmatrix} = 4\begin{bmatrix} 3 & 0 & 5 & 1 & 1 \\ 4 & 0 & 6 & 1 & 1 \\ 5 & 0 & 7 & 1 & 1 \\ 10 & 0 & 12 & 1 & 1 \end{bmatrix}$$

Applying $C_4 \to C_4 - C_5$, we get –

$$A = 4\begin{bmatrix} 3 & 0 & 5 & 0 & 1 \\ 4 & 0 & 6 & 0 & 1 \\ 5 & 0 & 7 & 0 & 1 \\ 10 & 0 & 12 & 0 & 1 \end{bmatrix}$$

Applying $R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$, $R_4 \to R_4 - R_1$, we get –

$$A = 4\begin{bmatrix} 3 & 0 & 5 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 2 & 0 & 2 & 0 & 0 \\ 7 & 0 & 7 & 0 & 0 \end{bmatrix}$$

Applying $C_3 \to C_3 - C_1$, we get –

$$A = 4\begin{bmatrix} 3 & 0 & 2 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 \\ 7 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Applying $C_3 \to C_2 - 2C_5$, we get –

$$A = \begin{bmatrix} 3 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 \\ 7 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Hence, Rank of A = 2

40. If $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$, then $\lim\limits_{n\to\infty} \frac{1}{n} A^n$ is

(a) a null matrix (b) an identity matrix

(c) $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ (d) none of these

SRM JEEE-2007

Ans. (a) : We have,

$$A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$$A^2 = \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{bmatrix}$$

$$A^3 = \begin{bmatrix} \cos 3\theta & \sin 3\theta \\ -\sin 3\theta & \cos 3\theta \end{bmatrix}$$

$$A^n = \begin{bmatrix} \cos n\theta & \sin n\theta \\ -\sin n\theta & \cos n\theta \end{bmatrix}$$

$$\therefore \lim_{n\to\infty} \frac{1}{n} A^n = \begin{bmatrix} \lim\limits_{n\to\infty} \frac{\cos n\theta}{n} & \lim\limits_{n\to\infty} \frac{\sin n\theta}{n} \\ -\lim\limits_{n\to\infty} \frac{\sin n\theta}{n} & \lim\limits_{n\to\infty} \frac{\cos n\theta}{n} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \text{null matrix.}$$

41. If $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $J = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$, then B equals

(a) $I\cos\theta + J\sin\theta$ (b) $I\sin\theta + J\cos\theta$

(c) $I\cos\theta - J\sin\theta$ (d) $-I\cos\theta + J\sin\theta$

SRM JEEE-2011
VITEEE-2011

Ans. (a) : We have,

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, J = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \text{ and } B = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$$\therefore I\cos\theta + J\sin\theta = \begin{bmatrix} \cos\theta & 0 \\ 0 & \cos\theta \end{bmatrix} + \begin{bmatrix} 0 & \sin\theta \\ -\sin\theta & 0 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} = B$$

42. The value of θ in [0,2π] such that matrix

$$\begin{bmatrix} 2\sin\theta - 1 & \sin\theta & \cos\theta \\ \sin(\theta+\pi) & 2\cos\theta - \sqrt{3} & \tan\theta \\ \cos(\theta-\pi) & \tan(\pi-\theta) & 0 \end{bmatrix}$$

is skew-symmetric, is

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$

(c) $\frac{\pi}{4}$ (d) $\frac{\pi}{6}$

SRM JEEE-2013

Ans. (d) :

$$\text{Let, } A = \begin{bmatrix} 2\sin\theta - 1 & \sin\theta & \cos\theta \\ \sin(\theta+\pi) & 2\cos\theta - \sqrt{3} & \tan\theta \\ \cos(\theta-\pi) & \tan(\pi-\theta) & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 2\sin\theta - 1 & \sin\theta & \cos\theta \\ -\sin\theta & 2\cos\theta - \sqrt{3} & \tan\theta \\ -\cos\theta & -\tan\theta & 0 \end{bmatrix}$$

It is given that, A is skew symmetric, therefore diagonal elements of A must be all zero.

$2\sin\theta - 1 = 0$ and $2\cos\theta - \sqrt{3} = 0$

$\sin\theta = \frac{1}{2}$ $\cos\theta = \frac{\sqrt{3}}{2}$

$\theta = \frac{\pi}{6}$ $\theta = \frac{\pi}{6}$

Hence, $\theta = \frac{\pi}{6}$

43. If ω is a complex cube root of unity, then the matrix $A = \begin{bmatrix} 1 & \omega^2 & \omega \\ \omega^2 & \omega & 1 \\ \omega & 1 & \omega^2 \end{bmatrix}$ is a

(a) singular matrix
(b) non singular matrix
(c) skew symmetric matrix
(d) none of these

SRM JEEE-2014

Ans. (a) : Given that,

$$A = \begin{bmatrix} 1 & \omega^2 & \omega \\ \omega^2 & \omega & 1 \\ \omega & 1 & \omega^2 \end{bmatrix}$$

Applying $C_1 \to C_1 + C_2 + C_3$, we get –

$$|A| = \begin{vmatrix} 1+\omega+\omega^2 & \omega^2 & \omega \\ 1+\omega+\omega^2 & \omega & 1 \\ 1+\omega+\omega^2 & 1 & \omega^2 \end{vmatrix}$$

Now,

$$|A| = \begin{vmatrix} 1+\omega+\omega^2 & \omega^2 & \omega \\ 1+\omega+\omega^2 & \omega & 1 \\ 1+\omega+\omega^2 & 1 & \omega^2 \end{vmatrix}$$

$$|A| = \begin{vmatrix} 0 & \omega^2 & \omega \\ 0 & \omega & 1 \\ 0 & 1 & \omega^2 \end{vmatrix}$$

[$\because$ ω is a complex cube root of unity.
So, $1 + \omega + \omega^2 = 0$]

$\Rightarrow |A| = 0$

$\therefore$ A is singular matrix.

44. If $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$, $C = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ **then**

(a) $C = A\cos\theta - B\sin\theta$ (b) $C = A\sin\theta + B\cos\theta$
(c) $C = A\sin\theta - B\cos\theta$ (d) $C = A\cos\theta + B\sin\theta$

SRM JEEE-2015

Ans. (d) : Given that,

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Then, $A\cos\theta = \begin{bmatrix} \cos\theta & 0 \\ 0 & \cos\theta \end{bmatrix}$

Also, $B = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

So, $B\sin\theta = \begin{bmatrix} 0 & \sin\theta \\ -\sin\theta & 0 \end{bmatrix}$

Now,

$$A\cos\theta + B\sin\theta = \begin{bmatrix} \cos\theta & 0 \\ 0 & \cos\theta \end{bmatrix} + \begin{bmatrix} 0 & \sin\theta \\ -\sin\theta & 0 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$$= C$$

So, $A\cos\theta + B\sin\theta = C$

45. The value of $\begin{vmatrix} 1 & 2 & 3 \\ -4 & 3 & 6 \\ 2 & -7 & 9 \end{vmatrix}$ **is**

(a) 213 (b) –231
(c) 231 (d) 39

BITSAT-2010

Ans. (c) :

Let, $A = \begin{vmatrix} 1 & 2 & 3 \\ -4 & 3 & 6 \\ 2 & -7 & 9 \end{vmatrix}$

$$|A| = 1\begin{vmatrix} 3 & 6 \\ -7 & 9 \end{vmatrix} - 2\begin{vmatrix} -4 & 6 \\ 2 & 9 \end{vmatrix} + 3\begin{vmatrix} -4 & 3 \\ 2 & -7 \end{vmatrix}$$

$$|A| = 1(3\times9-6(-7)) - 2(-4\times9-2\times6) + 3(-4)(-7) - 3\times2$$

$$|A| = (27 + 42) - 2(-36 - 12) + 3(28 - 6) = 231$$

46. If $A = \begin{bmatrix} 1 & 3 \\ 3 & 2 \\ 2 & 5 \end{bmatrix}$ **and** $B = \begin{bmatrix} -1 & -2 \\ 0 & 5 \\ 3 & 1 \end{bmatrix}$ **and A + B – D = 0 (zero matrix), then D matrix will be-**

(a) $\begin{bmatrix} 0 & 2 \\ 3 & 7 \\ 6 & 5 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 2 \\ 3 & 7 \\ 5 & 6 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & 1 \\ 3 & 7 \\ 5 & 6 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & -2 \\ -3 & -7 \\ -5 & -6 \end{bmatrix}$

BITSAT-2010

Ans. (c) :

Let, $D = \begin{bmatrix} a & b \\ c & d \\ e & f \end{bmatrix}$

$$A + B - D = \begin{bmatrix} 1 & 3 \\ 3 & 2 \\ 2 & 5 \end{bmatrix} + \begin{bmatrix} -1 & -2 \\ 0 & 5 \\ 3 & 1 \end{bmatrix} - \begin{bmatrix} a & b \\ c & d \\ e & f \end{bmatrix}$$

$\therefore$

$A + B - D = 0$

$$\begin{bmatrix} 1-1-a & 3-2-b \\ 3+0-c & 2+5-d \\ 2+3-e & 5+1-f \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

On comparing corresponding elements on both side we get–

$-a = 0 \Rightarrow a = 0,$ $1 - b = 0 \Rightarrow b = 1$
$3 - c = 0 \Rightarrow c = 3,$ $7 - d = 0 \Rightarrow d = 7$
$5 - e = 0 \Rightarrow e = 5,$ $6 - f = 0 \Rightarrow f = 6$

$$\therefore D = \begin{bmatrix} 0 & 1 \\ 3 & 7 \\ 5 & 6 \end{bmatrix}$$

47. If $A = \frac{1}{3}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix}$ **is an orthogonal matrix, then**

(a) a = –2, b = –1 (b) a = 2, b = 1
(c) a = 2, b = –1 (d) a = –2, b = 1

BITSAT-2018

Ans. (a) : Given that,

$$A = \frac{1}{3}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix}$$

As A is an orthogonal matrix, $AA^T = I$

$$\Rightarrow \frac{1}{3}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix} \cdot \frac{1}{3}\begin{bmatrix} 1 & 2 & a \\ 2 & 1 & 2 \\ 2 & -2 & b \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \frac{1}{9}\begin{bmatrix}1 & 2 & 2\\ 2 & 1 & -2\\ a & 2 & b\end{bmatrix}\begin{bmatrix}1 & 2 & a\\ 2 & 1 & 2\\ 2 & -2 & b\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}9 & 0 & a+4+2b\\ 0 & 9 & 2a+2-2b\\ a+4+2b & 2a+2-2b & a^2+4+b^2\end{bmatrix}=\begin{bmatrix}9 & 0 & 0\\ 0 & 9 & 0\\ 0 & 0 & 9\end{bmatrix}$$

On comparing corresponding elements both side, we get –

$\Rightarrow a+4+2b=0,\ 2a+2-2b=0$ and $a^2+4+b^2=9$

$\Rightarrow a+2b+4=0,\ a-b+1=0$ and $a^2+b^2=5$

$\Rightarrow a=-2,\ b=-1$

48. If $R(t)=\begin{bmatrix}\cos t & \sin t\\ -\sin t & \cos t\end{bmatrix}$, then R(s) R(t) equals

(a) R(s + t) (b) R(s – t)
(c) R(s) + R(t) (d) None of these

BITSAT-2017

Ans. (a) : Given that,

$$R(t)=\begin{bmatrix}\cos t & \sin t\\ -\sin t & \cos t\end{bmatrix}$$

$$R(s)R(t)=\begin{bmatrix}\cos s & \sin s\\ -\sin s & \cos s\end{bmatrix}\times\begin{bmatrix}\cos t & \sin t\\ -\sin t & \cos t\end{bmatrix}$$

$$=\begin{bmatrix}\cos s\cos t-\sin s\sin t & \cos s\sin t+\sin s\cos t\\ -\sin s\cos t-\cos s\sin t & -\sin s\sin t+\cos s\cos t\end{bmatrix}$$

$$=\begin{bmatrix}\cos(s+t) & \sin(s+t)\\ -\sin(s+t) & \cos(s+t)\end{bmatrix}=R(s+t)$$

49. The value of x, for which the matrix

$$A=\begin{bmatrix}\frac{2}{x} & -1 & 2\\ 1 & x & 2x^2\\ 1 & \frac{1}{x} & 2\end{bmatrix}$$ **is singular, is**

(a) ± 1 (b) ± 2
(c) ± 3 (d) ± 4

VITEEE-2006

Ans. (a) : Given that,

$$A=\begin{bmatrix}\frac{2}{x} & -1 & 2\\ 1 & x & 2x^2\\ 1 & \frac{1}{x} & 2\end{bmatrix}$$

According to question, A is singular matrix then –

$|A|=0$

$$|A|=\begin{vmatrix}\frac{2}{x} & -1 & 2\\ 1 & x & 2x^2\\ 1 & \frac{1}{x} & 2\end{vmatrix}=0$$

$$\frac{2}{x}\left(2x-\left(2x^2\right)\left(\frac{1}{x}\right)\right)+1\left(2-2x^2\right)+2\left(\frac{1}{x}-x\right)=0$$

$$\frac{2}{x}(2x-2x)+2-2x^2+\frac{2}{x}-2x=0$$

$$\frac{2}{x}(0)+2-2x^2+\frac{2}{x}-2x=0$$

$$2x-2x^3+2-2x^2=0$$

$$x^3+x^2-x-1=0$$

$$x^2(x+1)-1(x+1)=0$$

$$(x+1)(x^2-1)=0$$

$x+1=0$ and $x^2-1=0$

$x=-1$ and $x^2=1$

Not possible $x=\pm 1$

Hence, $x=\pm 1$

50. If $A=\begin{bmatrix}1 & -5 & 7\\ 0 & 7 & 9\\ 11 & 8 & 9\end{bmatrix}$ then trace of matrix A is

(a) 17 (b) 25
(c) 3 (d) 12

VITEEE-2012

Ans. (a) : We know that, trace is the sum of principal diagonal of the square matrix.

$$\text{If } A=\begin{bmatrix}1 & -5 & 7\\ 0 & 7 & 9\\ 11 & 8 & 9\end{bmatrix}$$

Then, tr (A) = 1 + 7 + 9=17

51. If $A=\begin{pmatrix}2 & -1\\ -7 & 4\end{pmatrix}$ and $B=\begin{pmatrix}4 & 1\\ 7 & 2\end{pmatrix}$ then which statement is true ?

(a) $AA^T=I$ (b) $BB^T=I$
(c) $AB\neq BA$ (d) $(AB)^T=I$

VITEEE-2018

Ans. (d) : Given that,

$$A=\begin{pmatrix}2 & -1\\ -7 & 4\end{pmatrix}\text{ and } B=\begin{pmatrix}4 & 1\\ 7 & 2\end{pmatrix}$$

$(A B)^T=I$

Taking L.H.S –

$=(AB)^T$

$=B^T A^T$

$$=\begin{bmatrix}4 & 7\\ 1 & 2\end{bmatrix}\begin{bmatrix}2 & -7\\ -1 & 4\end{bmatrix}$$

$$=\begin{bmatrix}8-7 & -28+28\\ 2-2 & -7+8\end{bmatrix}$$

$$=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=I$$

Hence, option (d) is correct.

52. An orthogonal matrix is

(a) $\begin{bmatrix}\cos\alpha & 2\sin\alpha\\ -2\sin\alpha & \cos\alpha\end{bmatrix}$ (b) $\begin{bmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{bmatrix}$

(c) $\begin{bmatrix}\cos\alpha & \sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}$ (d) $\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}$

UPSEE-2009

Ans. (b) : We know that, condition of orthogonal matrix–

$$A A^T = I$$

Let,

$$A = \begin{bmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{bmatrix}$$

$$A^T = \begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}$$

$$AA^T = \begin{bmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{bmatrix}\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}$$

$$= \begin{bmatrix}\cos^2\alpha+\sin^2\alpha & -\sin\alpha\cos\alpha+\sin\alpha\cos\alpha\\ -\sin\alpha\cos\alpha+\cos\alpha\sin\alpha & \cos^2\alpha+\sin^2\alpha\end{bmatrix}$$

$$= \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

$$= I$$

Hence, option (b) is correct.

53. Which one of the following is not true?

(a) Matrix addition is commutative
(b) Matrix addition is associative
(c) Matrix multiplication is commutative
(d) Matrix multiplication is associative

UPSEE -2008

Ans. (c) : If three matrices are A, B & C of same order, then we can say that,

ABC = ABC

So, matrix multiplication is associative.

But, AB ≠ BA so matrix multiplication is not commutative.

54. If A and B are 2×2 matrices, then which of the following is true?

(a) $(A+B)^2 = A^2+B^2+2AB$
(b) $(A-B)^2 = A^2+B^2-2AB$
(c) $(A-B)(A+B) = A^2+AB-BA-B^2$
(d) $(A+B)(A-B) = A^2-B^2$

BITSAT-2009
UPSEE-2006

Ans. (c) : A and B are 2 × 2 matrices,

$(A-B)(A+B) = A\cdot A + A\cdot B - B\cdot A - B\cdot B$
$= A^2 + AB - BA - B^2$

Therefore, option (c) is correct.

55. If $A = \begin{bmatrix}6 & 8 & 5\\ 4 & 2 & 3\\ 9 & 7 & 1\end{bmatrix}$ is the sum of a symmetric matrix B and skew-symmetric matrix C, then B is:

(a) $\begin{bmatrix}6 & 6 & 7\\ 6 & 2 & 5\\ 7 & 5 & 1\end{bmatrix}$ (b) $\begin{bmatrix}0 & 2 & -2\\ -2 & 5 & -2\\ 2 & 2 & 0\end{bmatrix}$

(c) $\begin{bmatrix}6 & 6 & 7\\ -6 & 2 & -5\\ -7 & 5 & 1\end{bmatrix}$ (d) $\begin{bmatrix}0 & 6 & -2\\ 2 & 0 & -2\\ -2 & -2 & 0\end{bmatrix}$

UPSEE-2006

Ans. (a) : Given that,

$$A = \begin{bmatrix}6 & 8 & 5\\ 4 & 2 & 3\\ 9 & 7 & 1\end{bmatrix}$$

According to theorem,

$$A = \left(\frac{A+A'}{2}\right)+\left(\frac{A-A'}{2}\right) = B+C$$

Where B and C are symmetric and skew symmetric matrices respectively –

Now, $B = \frac{1}{2}(A+A')$

$$B = \frac{1}{2}\left\{\begin{bmatrix}6 & 8 & 5\\ 4 & 2 & 3\\ 9 & 7 & 1\end{bmatrix}+\begin{bmatrix}6 & 4 & 9\\ 8 & 2 & 7\\ 5 & 3 & 1\end{bmatrix}\right\}$$

$$B = \frac{1}{2}\begin{bmatrix}12 & 12 & 14\\ 12 & 4 & 10\\ 14 & 10 & 2\end{bmatrix}$$

$$B = \begin{bmatrix}6 & 6 & 7\\ 6 & 2 & 5\\ 7 & 5 & 1\end{bmatrix}$$

56. Let A, B and C be n × n matrices. Which one of the following is a correct statement?

(a) If AB = AC, then B = C
(b) If $A^3 + 2A^2 + 3A + 5I = 0$, the A is invertible
(c) If $A^2 = 0$, then A = 0
(d) None of the above

UPSEE-2005

Ans. (b) : We know that theorem of a square matrix A satisfies the equation –

$$a_0 + a_1x + a_2x^2 + \ldots\ldots + a_nx^n = 0$$

Here, $a_0 \neq 0$ then A is invertible matrix

According to question A, B and C are order n × n matrices.

And also satisfied the equation $x^3 + 2x^2 + 3x + 5 = 0$

As given equation $A^3 + 2A^2 + 3A + 5I = 0$

So, A is invertible matrix.

57. $\Delta_1 = \begin{vmatrix}x & b & b\\ a & x & b\\ a & a & x\end{vmatrix}$ and $\Delta_2 = \begin{vmatrix}x & b\\ a & x\end{vmatrix}$ are the given determinants, then

(a) $\Delta_1 = 3(\Delta_2)^2$ (b) $\frac{d}{dx}(\Delta_1) = 3\Delta_2$

(c) $\frac{d}{dx}(\Delta_1) = 3(\Delta_2)^2$ (d) $\Delta_1 = 3(\Delta_2)^{3/2}$

UPSEE-2013

Ans. (b) : We have,

$$\Delta_1 = \begin{vmatrix} x & b & b \\ a & x & b \\ a & a & x \end{vmatrix} \text{ and } \Delta_2 = \begin{vmatrix} x & b \\ a & x \end{vmatrix}$$

Δ_1 are expanding along R_1,

$$\Delta_1 = x\begin{vmatrix} x & b \\ a & x \end{vmatrix} - b\begin{vmatrix} a & b \\ a & x \end{vmatrix} + b\begin{vmatrix} a & x \\ a & a \end{vmatrix}$$

$\Delta_1 = x [x^2 - ab] - b [ax - ab] + b [a^2 - ax]$

$\Delta_1 = x^3 - abx - abx + ab^2 + ba^2 - abx$

$\Delta_1 = x^3 - 3abx + ab (a + b)$

On differential with respect to x, we get–

$$\frac{d}{dx}(\Delta_1) = \frac{d}{dx}\left[x^3 - 3abx + ab(a+b)\right]$$

$$\frac{d}{dx}(\Delta_1) = 3x^2 - 3ab + 0$$

$$\frac{d}{dx}(\Delta_1) = 3(x^2 - ab) \quad \text{.....(i)}$$

$$\Delta_2 = \begin{vmatrix} x & b \\ a & x \end{vmatrix} = x^2 - ab$$

Putting the Δ_2 in equation (i), we get –

$$\frac{d}{dx}(\Delta_1) = 3(\Delta_2)$$

58. If A and B are square matrices of the same order and A is non-singular, then for a positive integer n, $(A^{-1} BA)^n$ is equal to

(a) $A^{-n}B^n A^n$ (b) $A^nB^n A^{-n}$
(c) $A^{-1}B^n A$ (d) $n(A^{-1}BA)$

UPSEE-2013

Ans. (c) : According to question, A is non-singular matrix it means A^{-1} is possible.

$(A^{-1} BA)^2 = (A^{-1}BA) (A^{-1}BA)$
$= A^{-1}B (AA^{-1}) BA$
$= A^{-1}B^2 A$
$(A^{-1}BA)^3 = (A^{-1}B^2A) (A^{-1}BA)$
$= A^{-1}B^3A$

Similarly,
$(A^{-1}BA)^n = A^{-1}B^nA$

59. Matrix A is such that $A^2 = 2A - I$, where I is the identity matrix. Then for $n \geq 2$, A^n is equal to

(a) $nA - (n-1)I$ (b) $nA - I$
(c) $2^{n-1}A - (n-1)I$ (d) $2^{n-1}A - I$

JCECE-2014

Ans. (a) : Given that,

$A^2 = 2A - I$
$\therefore$ $A^3 = A (A^2)$
$A^3 = A (2A - I)$
$A^3 = 2A^2 - AI$
$A^3 = 2(2A - I) - A$
$A^3 = 3A - 2I$

Similarly, $A^4 = 4A - 3I$
Hence, $A^n = nA - (n - 1) I$

60. A skew-symmetric matrix M satisfies the relation M + I = 0 where I is the unit matrix. Then, MM' is equal to

(a) I (b) 2I
(c) –I (d) None of these

JCECE-2012

Ans. (a) : Given that,

$M + I = 0$
$M = - I$ $\because$ I is unit matrix.

$$M = -\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

Now,
$$MM' = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$MM' = \begin{bmatrix} 1+0 & 0+0 \\ 0+0 & 0+1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$M M' = I$

61. If B is an idempotent matrix and A = I – B, then

(a) $A^2 = A$ (b) $AB = 0$
(c) $BA = 0$ (d) All of the above

JCECE-2010

Ans. (d) : If B is an idempotent matrix then

$B^2 = B$ .....(i)
$A = I - B$
$A^2 = (I - B) (I - B)$
$= I - IB - BI + B^2$
$= I - 2B + B^2$
$= I - 2B + B$ $\because B^2 = B$
$= I - B$
$\therefore$ $A^2 = A$
And $AB = (I - B) B$
$AB = IB - B^2$
$AB = B - B$
$AB = 0$

Similarly,
$BA = B (I - B)$
$BA = BI - B^2$
$BA = B - B$
$BA = 0$

So, option all of these are correct.

62. If the matrix $\begin{bmatrix} \mathbf{0} & \mathbf{2\beta} & \boldsymbol{\gamma} \\ \boldsymbol{\alpha} & \boldsymbol{\beta} & \mathbf{-\gamma} \\ \boldsymbol{\alpha} & \mathbf{-\beta} & \boldsymbol{\gamma} \end{bmatrix}$ is orthogonal, then

(a) $\alpha = \pm\frac{1}{\sqrt{2}}$ (b) $\beta = \pm\frac{1}{\sqrt{6}}$
(c) $\gamma = \pm\frac{1}{\sqrt{3}}$ (d) All of these

JCECE-2010

Ans. (d) : Let,

$$M = \begin{bmatrix} 0 & 2\beta & \gamma \\ \alpha & \beta & -\gamma \\ \alpha & -\beta & \gamma \end{bmatrix}$$

According to question M is orthogonal, it means –

$M M' = I$

$$\begin{bmatrix} 0 & 2\beta & \gamma \\ \alpha & \beta & -\gamma \\ \alpha & -\beta & \gamma \end{bmatrix} \cdot \begin{bmatrix} 0 & \alpha & \alpha \\ 2\beta & \beta & -\beta \\ \gamma & -\gamma & \gamma \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0+4\beta^2+\gamma^2 & 0+2\beta^2-\gamma^2 & 0-2\beta^2+\gamma^2 \\ 0+2\beta^2-\gamma^2 & \alpha^2+\beta^2+\gamma^2 & \alpha^2-\beta^2-\gamma^2 \\ 0-2\beta^2+\gamma^2 & \alpha^2-\beta^2-\gamma^2 & \alpha^2+\beta^2+\gamma^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

On comparing corresponding elements on both the side we get–

$$4\beta^2+\gamma^2=1, \qquad 2\beta^2-\gamma^2=0$$

$$4\left(\frac{\gamma^2}{2}\right)+\gamma^2=1, \qquad 2\beta^2=\gamma^2$$

$$3\gamma^2=1, \qquad \beta^2=\frac{\gamma^2}{2}$$

$$\gamma^2=\frac{1}{3}$$

$$\gamma=\pm\frac{1}{\sqrt{3}}$$

$\Rightarrow$ $2\beta^2-\gamma^2=0,\ \alpha^2+\beta^2+\gamma^2=1,\ \alpha^2-\beta^2-\gamma^2=0$

$$\beta^2=\frac{\gamma^2}{2},\ \alpha^2+\frac{\gamma^2}{2}+\frac{\gamma^2}{1}=1$$

$$\beta=\frac{1}{6},\ \alpha^2+\frac{3}{2}\times\frac{1}{3}=1,\ \alpha^2+\frac{3\gamma^2}{2}=1$$

$$\beta=\pm\frac{1}{6} \qquad \alpha=\pm\frac{1}{\sqrt{2}}$$

63. A square matrix A is called an orthogonal matrix, if

(a) $A\bar{A}=I$ (b) $AA'=I$
(c) $AA^{\theta}=I$ (d) $A^2=I$

JCECE-2009

Ans. (b) : Let us matrix A of n × n (square matrix) order then, A is the transpose matrix of matrix A.
We know that,
$A\cdot A^T=I$
So, A is orthogonal matrix.

64. If $A=\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ I is the unit matrix of order 2 and a, b are arbitrary constants, then $(aI+bA)^2$ is equal to

(a) a^2I-abA (b) $a^2I+2abA$
(c) a^2I+b^2A (d) None of these

JCECE-2007

Ans. (b) : Given that,

$$A=\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \text{ and } I=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A^2=\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}=\begin{bmatrix} 0+0 & 0+0 \\ 0+0 & 0+0 \end{bmatrix}$$

$A^2=0$ (zero matrix)

Now,
$(aI+bA)^2$
$=(aI+bA)(aI+bA)$
$=a^2I+abIA+baAI+b^2A^2$
$=a^2I+2abIA+0$ $[\because A^2=0]$
$=a^2I+2abIA$

65. The matrix $\begin{bmatrix} 5 & 10 & 3 \\ -2 & -4 & 6 \\ -1 & -2 & b \end{bmatrix}$ is a singular matrix, if b is equal to:

(a) –3 (b) 3
(c) 0 (d) for any value of b

JCECE-2005

Ans. (d) : Let,

$$A=\begin{bmatrix} 5 & 10 & 3 \\ -2 & -4 & 6 \\ -1 & -2 & b \end{bmatrix}$$

According to question, A is singular matrix it means,
$|A|=0$

$$\begin{vmatrix} 5 & 10 & 3 \\ -2 & -4 & 6 \\ -1 & -2 & b \end{vmatrix}=0$$

$5(-4b+12)-10(-2b+6)+3(4-4)=0$
$-20b+60+20b-60+0=0$
$20b-60=20b-60$
$b-3=b-3$

For any value of b the above equation is equal on both side.

66. If A and B are matrices of same order, then (AB' – BA') is a

(a) Skew-symmetric (b) Symmetric
(c) Null (d) Unit

JCECE-2019

Ans. (a): Given that,
$(AB'-BA')$
$\because (AB'-BA')'=(AB')'-(BA')'$
$=(B')'A'-(A')'B'$
$=BA'-AB'=-(AB'-BA')$
Hence, (AB' – BA') is a skew symmetric matrix.

67. The matrix $\begin{bmatrix} 0 & 0 & 9 \\ 0 & 9 & 0 \\ 9 & 0 & 0 \end{bmatrix}$ is a

(a) scalar matrix (b) square matrix
(c) diagonal matrix (d) unit matrix

JCECE-2018

Ans. (b) : Let,

$$A=\begin{bmatrix} 0 & 0 & 9 \\ 0 & 9 & 0 \\ 9 & 0 & 0 \end{bmatrix}$$

If A matrix is square then diagonal elements are not same, so it is not a scalar matrix.
Clearly A is not a unit and diagonal matrix.
It is follow condition of square matrix of order 3 × 3.

68. If A is 3 × 4 matrix and B is a matrix such that A'B and B'A are both defined, then the order of B is

(a) 4 × 4 (b) 3 × 3
(c) 3 × 4 (d) 4 × 3

JCECE-2017
Karnataka CET-2014

Ans. (b,c) : Given that,
Order of A = 3×4
Order of A' = 4×3
Therefore, the number of columns in B should be equal to the number of rows in A' for BA' and also the number of columns in A' should be equal to the number of rows in A' for BA'.
Hence, the order of the matrix B = 3×4 .

69. If $A = \begin{bmatrix} 1 & x \\ x^2 & 4y \end{bmatrix}$ **and** $B = \begin{bmatrix} -3 & 1 \\ 1 & 0 \end{bmatrix}$ **adj.** $A+B=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$**, then values of x and y are**

(a) 1, 1 (b) ±1, 1
(c) 1, 0 (d) none of these

BCECE-2007

Ans. (a) : Given that,

$$A = \begin{bmatrix} 1 & x \\ x^2 & 4y \end{bmatrix} \text{ and } B = \begin{bmatrix} -3 & 1 \\ 1 & 0 \end{bmatrix}$$

Now, $\quad \text{adj } A= \begin{bmatrix} 4y & -x^2 \\ -x & 1 \end{bmatrix}^T$

$$\text{adj } A = \begin{bmatrix} 4y & -x \\ -x^2 & 1 \end{bmatrix}$$

According to question,

$$\text{adj } A + B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 4y & -x \\ -x^2 & 1 \end{bmatrix} + \begin{bmatrix} -3 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 4y-3 & -x+1 \\ -x^2+1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

On comparing corresponding elements on both the side, we get –

$4y - 3 = 1, \qquad -x + 1 = 0$
$y = 1 \qquad x = 1$

70. The matrix $\begin{vmatrix} 2 & 5 & -7 \\ 0 & 3 & 11 \\ 0 & 0 & 9 \end{vmatrix}$ **is known as :**

(a) symmetric matrix
(b) upper triangular matrix
(c) diagonal matrix
(d) skew-symmetric matrix

BCECE-2003

Ans. (b) : Let,

$$A = \begin{bmatrix} 2 & 5 & -7 \\ 0 & 3 & 11 \\ 0 & 0 & 9 \end{bmatrix}$$

According to theory of matrix we know that if all the elements below the diagonal in the matrix are zero then the matrix is upper triangular matrix,
Hence, A is upper triangular matrix.

71. If H is an orthogonal square matrix, then what is the determinant of H?

(a) 0 (b) 1 (c) 2 (d) 4

SCRA-2012

Ans. (b) : According to given summation,
If H is an orthogonal matrix, then –
$HH^T = 1$

$\therefore \ |HH^T| = 1 = |I|$

Or $|H||H^T| = 1$

Or $|H|^2 = 1$

$|H|^2 = \pm 1$

H = 1 or H = –1

72. If A is a skew-symmetric matrix of order 3, then matrix A^3 is a/an

(a) Orthogonal matrix
(b) Diagonal matrix
(c) Symmetric matrix
(d) Skew-symmetric matrix

SCRA-2012

Ans. (d) : According to given summation,
If A is a skew symmetric matrix of order 3 then,
$A^T = -A$
Now, $(A^3)^T = A^T \times A^T \times A^T$
$= (-A)(-A)(-A)$
$= -A^3$
Hence, A^3 is also skew – symmetric matrix.

73. Let $A = \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$**, the only correct statement about the matrix A, is**

(a) A is a zero matrix
(b) A = (–1)I, where I is a unit matrix
(c) A^{-1} does not exist
(d) $A^2 = I$

CG PET- 2005

Ans. (d) : We have,

$$A = \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$$

It is clear that A is not a zero matrix.

$$-I = -1\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \neq A$$

i.e, $(-1) \neq A$

$$|A| = A.A = \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

74. **If $A=\begin{bmatrix}2 & 3 & 1\\ 0 & -1 & 5\end{bmatrix}$ and $B=\begin{bmatrix}1 & 2 & -6\\ 0 & -1 & 3\end{bmatrix}$, then 3A – 4B is equal to**

(a) $\begin{bmatrix}1 & 2 & 3\\ 0 & 4 & 10\end{bmatrix}$ (b) $\begin{bmatrix}2 & 1 & 27\\ 0 & 1 & 3\end{bmatrix}$

(c) $\begin{bmatrix}3 & 5 & -5\\ 0 & -2 & 8\end{bmatrix}$ (d) None of these

CG PET- 2010

Ans. (b) : We have,

$$A=\begin{bmatrix}2 & 3 & 1\\ 0 & -1 & 5\end{bmatrix} \text{ and } B=\begin{bmatrix}1 & 2 & -6\\ 0 & -1 & 3\end{bmatrix}$$

Now, $3A-4B=\begin{bmatrix}6 & 9 & 3\\ 0 & -3 & 15\end{bmatrix}-\begin{bmatrix}4 & 8 & -24\\ 0 & -4 & 12\end{bmatrix}$

$$=\begin{bmatrix}2 & 1 & 27\\ 0 & 1 & 3\end{bmatrix}$$

Hence, option (b) is correct.

75. **For how many values of x in the interval [–4, –1] the matrix $\begin{bmatrix}3 & -1+x & 2\\ 3 & -1 & x+2\\ x+3 & -1 & 2\end{bmatrix}$ is singular?**

(a) 2 (b) 1

(c) 0 (d) 3

CG PET- 2012

Ans. (b) : Given, matrix is a singular matrix

$$\therefore \quad \begin{vmatrix}3 & -1+x & 2\\ 3 & -1 & x+2\\ x+3 & -1 & 2\end{vmatrix}=0$$

Operating $R_1 \to R_1 - R_2$

$$\begin{vmatrix}0 & x & -x\\ 3 & -1 & x+2\\ x+3 & -1 & 2\end{vmatrix}=0$$

Expanding along R_1, we get

$-x[6-(x^2+5x+6)]-x(-3+x+3)=0$

$-x(-x^2-5x)-x(x)=0$

$x(x^2+5x-x)=0$

$x^2(x+4)=0$

$x=0, -4$

But it is given that, $x\in[-4,-1]$.

$\therefore \quad O\in[-4,-1]$

Hence, only one solution exist.

76. **If $A=\begin{bmatrix}1 & 0 & 1\\ 0 & 1 & 1\\ 1 & 0 & 0\end{bmatrix}$, then A is a**

(a) singular (b) non- singular

(c) symmetric (d) Unit matrix

CG PET- 2013

Ans. (b) : We have,

$$A=\begin{bmatrix}1 & 0 & 1\\ 0 & 1 & 1\\ 1 & 0 & 0\end{bmatrix}$$

$$\therefore \quad |A|=\begin{vmatrix}1 & 0 & 1\\ 0 & 1 & 1\\ 1 & 0 & 0\end{vmatrix}$$

$=1(0-0)-0(0-1)+1(0-1)$

$=0-0-1$

$=|A|\neq 0$

Hence, A is non-singular matrix.

77. **If $2x-\begin{bmatrix}1 & 2\\ 7 & 4\end{bmatrix}=\begin{bmatrix}3 & 2\\ 0 & -2\end{bmatrix}$ the value of x will be**

(a) $\begin{bmatrix}2 & 2\\ 7 & 4\end{bmatrix}$ (b) $\begin{bmatrix}1 & 2\\ 7/2 & 2\end{bmatrix}$

(c) $\begin{bmatrix}2 & 2\\ 7/2 & 1\end{bmatrix}$ (d) None of the above

CG PET- 2015

Ans. (c) : Given,

$$2x-\begin{bmatrix}1 & 2\\ 7 & 4\end{bmatrix}=\begin{bmatrix}3 & 2\\ 0 & -2\end{bmatrix}$$

$$2x=\begin{bmatrix}1 & 2\\ 7 & 4\end{bmatrix}+\begin{bmatrix}3 & 2\\ 0 & -2\end{bmatrix}$$

$$2x=\begin{bmatrix}1+3 & 2+2\\ 7+0 & 4-2\end{bmatrix}$$

$$2x=\begin{bmatrix}4 & 4\\ 7 & 2\end{bmatrix}$$

$$x=\frac{1}{2}\begin{bmatrix}4 & 4\\ 7 & 2\end{bmatrix}$$

$$x=\begin{bmatrix}2 & 2\\ 7/2 & 1\end{bmatrix}$$

78. **If $\begin{bmatrix}0 & 2\beta & \gamma\\ \alpha & \beta & -\gamma\\ \alpha & -\beta & \gamma\end{bmatrix}$ is orthogonal, then the values of α, β and γ will be**

(a) $\alpha=\pm\frac{1}{\sqrt{5}}, \beta=\pm\frac{1}{\sqrt{3}}, \gamma=\pm\frac{1}{\sqrt{2}}$

(b) $\alpha=\pm\frac{1}{\sqrt{6}}, \beta=\pm\frac{1}{\sqrt{7}}, \gamma=\pm\frac{1}{\sqrt{3}}$

(c) $\alpha=\pm\frac{1}{\sqrt{3}}, \beta=\pm\frac{1}{\sqrt{3}}, \gamma=\pm\frac{1}{\sqrt{3}}$

(d) $\alpha=\pm\frac{1}{\sqrt{2}}, \beta=\pm\frac{1}{\sqrt{6}}, \gamma=\pm\frac{1}{\sqrt{3}}$

CG PET- 2017

Ans. (d) : Let $A=\begin{bmatrix}0 & 2\beta & \gamma\\ \alpha & \beta & -\gamma\\ \alpha & -\beta & \gamma\end{bmatrix}$

Since, A is an orthogonal matrix

$\therefore$ AA'=I

$$\Rightarrow\begin{bmatrix}0 & 2\beta & \gamma\\ \alpha & \beta & -\gamma\\ \alpha & -\beta & \gamma\end{bmatrix}\begin{bmatrix}0 & \alpha & \alpha\\ 2\beta & \beta & -\beta\\ \gamma & -\gamma & \gamma\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$\Rightarrow\begin{bmatrix}4\beta^2+\gamma^2 & 2\beta^2-\gamma^2 & -2\beta^2+\gamma^2\\ 2\beta^2-\gamma^2 & \alpha^2+\beta^2+\gamma^2 & \alpha^2-\beta^2-\gamma^2\\ -2\beta^2+\gamma^2 & \alpha^2-\beta^2-\gamma^2 & \alpha^2+\beta^2+\gamma^2\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

$\therefore$ $4\beta^2+\gamma^2=1, 2\beta^2-\gamma^2=0, \alpha^2+\beta^2+\gamma^2=1,$
$\alpha^2-\beta^2-\gamma^2=0$

(Comparing the corresponding entries)

On solving these equations, we get

$$\beta=\pm\frac{1}{\sqrt{6}}, \gamma=\pm\frac{1}{\sqrt{3}}, \alpha=\pm\frac{1}{\sqrt{2}}$$

79. Let $S=\left\{\begin{bmatrix}-1 & a\\ 0 & b\end{bmatrix}; a,b\in\{1,2,3.........100\}\right\}$ and let $T_n=\{A\in S : A^{n(n+1)}=I\}$. Then the number of elements in $\bigcap_{n=1}^{100} T_n$ is _____.

JEE Main-24.06.2022, Shift-II

Ans. (100) : According to given summation,

$$A=\begin{bmatrix}-1 & a\\ 0 & b\end{bmatrix}$$

And, $A^2=\begin{bmatrix}-1 & a\\ 0 & b\end{bmatrix}\begin{bmatrix}-1 & a\\ 0 & b\end{bmatrix}$

$$=\begin{bmatrix}1 & -a+ab\\ 0 & b^2\end{bmatrix}$$

$\therefore$ $T_n=\{A\in S; A^{n(n+1)}=I\}$

$\therefore$ b must be equal to I

$\therefore$ In this case A^2 will become identity matrix and can take any value from 1 to 100.

$\therefore$ Total number of common element will be 100.

80. For what value(s) of $n\ge 1$, where n is a natural number, $A^n-nA+nI=I$, where I is the identity matrix and $A=\begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix}$?

(a) $n=1$ only
(b) $n=2$ only
(c) For all values of n
(d) None of the values of n

SCRA-2014

Ans. (c): We have,

$A^n-nA+nI=I$

Where, $A=\begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix}$

For, $n=1$

$=A-A+I$

$$=\begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix}-\begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix}+\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=I$$

For, $n=2$

$=A^2-2A+2I$

$$=\begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix}-2\begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix}+2\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

$$=\begin{bmatrix}1 & 0\\ 2 & 1\end{bmatrix}-\begin{bmatrix}2 & 0\\ 2 & 2\end{bmatrix}+\begin{bmatrix}2 & 0\\ 0 & 2\end{bmatrix}$$

$$=\begin{bmatrix}3 & 0\\ 2 & 3\end{bmatrix}-\begin{bmatrix}2 & 0\\ 2 & 2\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=I$$

It is true for all value of n.

81. Let A, B C be 3 × 3 matrices such that A is symmetric and B and C are skew-symmetric. Consider the statements
(S_1) $A^{13}B^{26}-B^{26}A^{13}$ is symmetric
(S_2) $A^{26}C^{13}-C^{13}A^{26}$ is symmetric
Then,

(a) Only S_1 is true
(b) Both S_1 and S_2 are false
(c) Both S_1 and S_2 are true
(d) Only S_2 is true

JEE Main-25.01.2023, Shift-II

Ans. (d) : We have, $A^T=A, B^T=-B, C^T=-C$

Let $M=A^{13}B^{26}-B^{26}.A^{13}$

Then, $M^T=(A^{13}B^{26}-B^{26}A^{13})^T$
$=(A^{13}B^{26})^T-(B^{26}A^{13})^T$
$=(A^{13})^T(B^{26})^T-(B^{26}A^{13})^T$
$=(B^T)^{26}(A^T)^{13}-(A^T)^{13}(B^T)^{26}$
$=B^{26}A^{13}-A^{13}B^{26}=-M$

Therefore, M is skew symmetric

Let, $N=A^{26}C^{13}-C^{13}A^{26}$

Then, $N^T=(A^{26}C^{13})^T-(C^{13}.A^{26})^T$
$=-(C)^{13}(A)^{26}+A^{26}C^{13}=N$

Therefore N is symmetric.

$\therefore$ Only S_2 is true.

Hence, option (d) is correct.

82. If $A=\begin{bmatrix}1 & 2\\ -3 & 0\end{bmatrix}$ and $B=\begin{bmatrix}-1 & 0\\ 2 & 3\end{bmatrix}$ then

(a) AB = BA
(b) $B^2=B$
(c) AB ≠ BA
(d) $A^2=A$

AMU-2002

Ans. (c) : Given, $A=\begin{vmatrix}1 & 2\\ -3 & 0\end{vmatrix}$ and $B=\begin{vmatrix}-1 & 0\\ 2 & 3\end{vmatrix}$

Know,

$$AB=\begin{vmatrix}1 & 2\\ 3 & 0\end{vmatrix}\begin{vmatrix}-1 & 0\\ 2 & 3\end{vmatrix}$$

$$= \begin{vmatrix} -1+4 & 0+6 \\ -3+0 & 0+0 \end{vmatrix} = \begin{vmatrix} 3 & 6 \\ -3 & 0 \end{vmatrix}$$

$$BA = \begin{vmatrix} -1 & 0 \\ 2 & 3 \end{vmatrix} \begin{vmatrix} 1 & 2 \\ 3 & 0 \end{vmatrix} = \begin{vmatrix} -1+0 & -2+0 \\ 2+9 & 4+0 \end{vmatrix}$$

$$BA = \begin{vmatrix} -1 & -2 \\ 11 & 4 \end{vmatrix}$$

Here, $AB \neq BA$

83. The number of square matrices of order 5 with entries from the set {0, 1}, such that the sum of all the element in each row is 1 and the sum of all the elements in each column is also 1, is

(a) 225 (b) 120
(c) 125 (d) 150

JEE Main-24.01.2023, Shift-II

Ans. (b) :

$$\begin{vmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{vmatrix}$$

In each row and each column exactly one is to be placed

$\therefore$ No of such material = 5 × 4 × 3 × 2 × 1 = 120

84. Let $M = \begin{bmatrix} 0 & -\alpha \\ \alpha & 0 \end{bmatrix}$, where α is a non-zero real number an $N = \sum_{k=1}^{49} M^{2k}$. If $(I - M^2)N = -2I$, then the positive integral value of α is_____.

JEE Main-29.06.2022, Shift-II

Ans. (1) : According to given summation,

$$M = \begin{bmatrix} 0 & -\alpha \\ \alpha & 0 \end{bmatrix}$$

$$M^2 = \begin{bmatrix} 0 & -\alpha \\ \alpha & 0 \end{bmatrix}\begin{bmatrix} 0 & -\alpha \\ \alpha & 0 \end{bmatrix}$$

$$M^2 = \begin{bmatrix} -\alpha^2 & 0 \\ 0 & -\alpha^2 \end{bmatrix} = -\alpha^2 I$$

$N = M^2 + M^4 + \ldots\ldots + M^{98} = [-\alpha^2 + \alpha^4 - \alpha^6 + \ldots..]I$

$$= -\alpha^2 \frac{(1-(-\alpha^2)^{49})}{1+\alpha^2}.I$$

$I - M^2 = I + \alpha^2 I$

Now, $I - M^2 = (I + \alpha^2)\, I$

$(I - M^2)N = -\alpha^2(\alpha^{98} + 1) = -2$

$\alpha = 1$

Hence, the positive integral value of α is 1.

85. Let $A = \sum_{i=1}^{10}\sum_{j=1}^{10} \min\{i, j\}$ and $B = \sum_{i=1}^{10}\sum_{j=1}^{10} \max\{i, j\}$. Then A + B is equal to

JEE Main-26.06.2022, Shift-I

Ans. (1100) : According to given summation.

$$A = \sum_{i=1}^{10}\sum_{j=1}^{10} \min\ \{i, j\}$$

$$B = \sum_{i=1}^{10}\sum_{j=1}^{10} \min\ \{i, j\}$$

$$A = \sum_{j=1}^{10} \min(i,1) + \min(j,2) + \ldots..\min(i,10)$$

$$= \frac{1+1+1+\ldots+1}{19 \text{ times}} + \frac{2+2+2\ldots+2}{17 \text{ times}} + \frac{3+3+3\ldots+3}{15 \text{ times}} + \ldots(1)\ 1 \text{ times}$$

$$B = \sum_{j=1}^{10} \max(i,1) + \max(j,2) + \ldots.\max(i,10)$$

$$= \underbrace{10+10+\ldots.+10}_{19 \text{ times}} + \underbrace{9+9+\ldots.+9}_{19 \text{ times}} + \ldots..11 \text{ times.}$$

Now, $A + B = 20\ (1 + 2 + 3 + \ldots.. + 10)$

$$= 20 \times \frac{10\times(10+1)}{2}$$

$$= 20 \times \frac{10\times 11}{2} = 10\times 110 = 1100$$

Henc,e the value of A + B = 1100

86. Let A be a matrix of order 2 × 2, whose entries are from the set {0, 1, 2, 3, 4, 5}. If the sum of all the entries of A is a prime number p, 2 < p < 8, then the number of such matrices A is :

JEE Main-27.06.2022, Shift-II

Ans. (180) : According to given summation,

Let, $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Where, a, b, c, d ∈ {0, 1, 2, 3, 4, 5}

$a + b + c + d = P,\ P \in \{3, 5, 7\}$

Case-I,

$a + b + c + d = 3;\ a, b, c, d \in \{0, 1, 2, 3\}$

No of ways = ${}^{3+4-1}C_{4-1} = {}^6C_3 = 20$ …..(i)

Case-II,

$a + b + c + d = 5;\ a, b, c, d \in \{0, 1, 2, 3\}$

No of ways = ${}^{5+4-1}C_{4-1} = {}^8C_3 = 56$ ……(ii)

Case-III,

$a + b + c + d = 7$

No of ways = total ways when a, b, c, d ∈ {0, 1, 2, 3, 4, 5, 6, 7} – total ways when a, b, c, d ∉ {6, 7}

No of ways = ${}^{7+4-1}C_{4-1} - \left[\frac{\lfloor 4}{\lfloor 3} + \frac{\lfloor 4}{\lfloor 2}\right]$

$= {}^{10}C_3 - 16 = 104$ ……(iii)

Hence, total number of required matrices = 20 + 56 + 104 = 180.

87. Let $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 4 & -1 \\ 0 & 12 & -3 \end{pmatrix}$. Then the sum of the diagonal elements of the matrix $(A + I)^{11}$ is equal to

(a) 2050 (b) 4094
(c) 6144 (d) 4097

JEE Main-31.01.2023, Shift-I

Ans. (d) : According to given summation,

$$A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & -1 \\ 0 & 12 & -3 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & -1 \\ 0 & 12 & -3 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1+0+0 & 0+0+0 & 0+0+0 \\ 0+0+0 & 0+16-12 & 0-4+3 \\ 0+0+0 & 0+48-36 & 0-12+9 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & -1 \\ 0 & 12 & -3 \end{bmatrix} = A$$

So, $A^3 = A^4 = \ldots\ldots = A$

$(A + I)^{11} = {}^{11}C_0 A^{11} + {}^{11}C_1 A^{10} + \ldots.. {}^{11}C_{10}A + {}^{11}C_{11}I$

$= ({}^{11}C_0 + {}^{11}C_1 + \ldots.. {}^{11}C_{10}) A + I$

$= (2^{11} - 1) A + I = 2047 A + I$

$= (2^{11} - 1) A + I = 2047 A + I$

Now, Sum of diagonal elements = 2047 (1 + 4 – 3) + 3

= 4094 + 3 = 4097

Hence, option (d) is correct.

88. Let $A = \left|A_{ij}\right| \cdot a_{ij} \in Z \cap [0, 4], 1 \le I, j \le 2$. The number of matrices A such that the sum of all entries is a prime number $p \in (2, 13)$ is _____.

JEE Main-31.01.2023, Shift-II

Ans. (204) : Given,

prime number $p \in (2 , 13)$ is 3, 5, 7, & 11

If, a + b + c +d = 3 a, b, c, d ∈ (0, 1, 2, 3, 4)

Then, coefficient of x^3 in

$= (1+ x + x^2 + x^3 + x^4)^4$

$= (1 - x^5)^4 (1 - x)^{-4}$

$= (1 - {}^4C_1x^5 + {}^4C_2x^{10}........) (1 + {}^4C_1 + {}^5C_2x^2 + {}^6C_3x^3 + {}^7C_4x^4 + ...)$

Coefficient of $x^3 = {}^6C_3 = 20$

If, a + b +c + d = 5

Then, coefficient of x^5 in $(1 + x+ x^2 + x^3 + x^4)^4 = {}^8C_5 - 4$
$= 52$

If, a + b +c + d = 7

Then, coefficient of x^7 in $(1 + x+ x^2 + x^3 + x^4)^4 = {}^{10}C_7 - 4 \times {}^5C_2 = 120 - 40 = 80$

If, a + b +c + d = 11

Then, coefficient of x^{11} in $(1 + x+ x^2 + x^3 + x^4)^4 = {}^{14}C_{11} - 4 \cdot {}^9C_6 + {}^4C_2 \cdot {}^4C_1$

= 364– 336 + 24 = 52

Hence, sum of all entries = 20 + 52 + 80 + 52 = 204

89. Let $A = \begin{bmatrix} 2 & -1 & -1 \\ 1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}$ and B = A – I. If $\omega = \frac{\sqrt{3}i - 1}{2}$, then the number of elements in the set $\{n \in \{1, 2, \ldots\ldots. 100\} : A^n + (\omega B)^n = A + B\}$ is equal to ______.

JEE Main-25.07.2022, Shift-I

Ans. (17) : We have,

$$A = \begin{bmatrix} 2 & -1 & -1 \\ 1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix} \Rightarrow A^2 = A \Rightarrow A^n = A$$

$\forall\, n \in \{1, 2, \ldots\ldots. 100\}$

Now,

$$B = A - I = \begin{bmatrix} 2 & -1 & -1 \\ 1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & -1 \\ 1 & -1 & -1 \\ 1 & -1 & -1 \end{bmatrix}$$

$B^2 = -B$

$B^3 = -B^2 = B$

$B^5 = B$

$B^{99} = B$

Also, $\omega^{3k} = 1$

So, n = common of {1, 3, 5, …. 99} and {3, 6, 9….99}
= 17

90. Let A be a 3 × 3 matrix having entries from the set {–1, 0, 1}. The number of all such matrices A having sum of all the entries equal to 5, is _______

JEE Main-25.06.2022, Shift-I

Ans. (414) : According to the question,

Case-I: $1 \to 7$ times
and $-1 \to 2$ times

Number of possible matrix $= \frac{9!}{7!2!} = 36$

Case-II: $1 \to 6$ times
$-1 \to 1$ times
and $0 \to 2$ times

Number of possible matrix $= \frac{9!}{6!2!} = 252$

Case-III: $1 \to 5$ times,
and $0 \to 4$ times

Number of possible matrix $= = \frac{9!}{5!4!} = 126$

Hence total number of all such matrix A
= 36 + 252 + 126 = 414

91. Let $A = \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}$. If M and N are two matrices given by $M = \sum_{k=1}^{10} A^{2k}$ and $N = \sum_{k=1}^{10} A^{2k-1}$ then MN^2 is

(a) a non-identity symmetric matrix
(b) a skew-symmetric matrix
(c) neither symmetric nor skew-symmetric matrix
(d) an identify matrix

JEE Main-25.06.2022, Shift-I

Ans. (a) : We have,

$$A = \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} -4 & 0 \\ 0 & -4 \end{bmatrix} = -4I$$

$A^3 = -4A$
$A^4 = (-4I)(-4I) = (-4)^2 I$
$A^5 = (-4)^2 A$, $A_6 = (-4)^3 I$

$$M = \sum_{k=I}^{10} A^{2k} = A^2 + A^4 + ... + A^{20}$$

$= [-4 + (-4)^2 + (-4)^3 + ... + (-4)^{20}]I$
$= -4\lambda I$

M is symmetric matrix,

$$N = \sum_{k=I}^{10} A^{2k-1} = A + A^3 + ... + A^{19}$$

$= A[1 + (-4) + (-4)^2 + ... + (-4)^9]$
$= \lambda A \Rightarrow$ skew symmetric

N^2 is symmetric matrix

Hence, MN^2 is non identity symmetric matrix

92. Let A be a 3 × 3 real matrix such that

$$A\begin{bmatrix}1\\1\\0\end{bmatrix} = \begin{bmatrix}1\\1\\0\end{bmatrix}; A\begin{bmatrix}1\\0\\1\end{bmatrix} = \begin{bmatrix}-1\\0\\1\end{bmatrix} \text{ and } A\begin{bmatrix}0\\0\\1\end{bmatrix} = \begin{bmatrix}1\\1\\2\end{bmatrix}$$

If $X = (x_1, x_2, x_3)^T$ and I is an identity matrix of order 3, then the system $(A - 2I)X = \begin{bmatrix}4\\1\\1\end{bmatrix}$ has

(a) no solution
(b) infinitely many solutions
(c) unique solution
(d) exactly two solutions

JEE Main-25.06.2022, Shift-I

Ans. (b) : We have,

$$A = \begin{bmatrix}a_1 & b_1 & c_1\\a_2 & b_2 & c_2\\a_3 & b_3 & c_3\end{bmatrix}$$

$$A\begin{bmatrix}0\\0\\1\end{bmatrix} = \begin{bmatrix}c_1\\c_2\\c_3\end{bmatrix} = \begin{bmatrix}1\\1\\2\end{bmatrix}$$

$c_1 = 1, c_2 = 1, c_3 = 2$

$$A\begin{bmatrix}1\\0\\1\end{bmatrix} = \begin{bmatrix}c_1+a_1\\c_2+a_2\\c_3+a_3\end{bmatrix} = \begin{bmatrix}-1\\0\\1\end{bmatrix}$$

$\Rightarrow a_1 = -2, a_2 = -1, a_3 = -1$

$$A\begin{bmatrix}1\\1\\0\end{bmatrix} = \begin{bmatrix}a_1+b_1\\a_2+b_2\\a_3+b_3\end{bmatrix} = \begin{bmatrix}1\\1\\0\end{bmatrix}$$

$b_1 = 3, b_2 = 2, b_3 = 1$

$$A = \begin{bmatrix}-2 & 3 & 1\\-1 & 2 & 1\\-1 & 1 & 2\end{bmatrix}$$

$$A - 2I = \begin{bmatrix}-4 & 3 & 1\\-1 & 0 & 1\\-1 & 1 & 0\end{bmatrix}$$

$|A - 2I| = 0$

Now, $\begin{bmatrix}-4 & 3 & 1\\-1 & 0 & 1\\-1 & 1 & 0\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} = \begin{bmatrix}4\\1\\1\end{bmatrix}$

$-4x_1 + 3x_2 + x_3 = 4$ …(i)
$-x_1 + x_3 = 1$ ….(ii)
$-x_1 + x_2 = 1$ ….(iii)
$(1) - [(2) + 3(3)]$
$0 = 0 \Rightarrow$ infinite solutions

93. Let A and B be any two 3 × 3 symmetric and skew symmetric matrices respectively. Then which of the following is NOT true ?

(a) $A^4 - B^4$ is a symmetric matrix
(b) $AB - BA$ is a symmetric matrix
(c) $B^5 - A^5$ is a skew-symmetric matrix
(d) $AB + BA$ is a skew-symmetric matrix

JEE Main-28.07.2022, Shift-II

Ans. (c) : As per option

(a) $K = A^4 - B^4$
$K^T = (A^4 - B^4)^T = (A^T)^4 - (B^T)^4$
$= A^4 - (-B)^4 = A^4 - B^4 = K$

(b) $K = AB - BA$
$K^T = (AB - BA)^T = (AB)^T - (BA)^T$
$= B^T A^T - A^T B^T$
$= -BA - A(-B)$
$= AB - BA = K$

(c) $K = B^5 - A^5$
$K^T = (B^T)^5 - (A^T)^5 = (B^5 + A^5) \neq -K$

(d) $K = AB + BA$
$K^T = (AB)^T + (BA)^T$
$= B^T A^T + A^T B^T = -BA - AB = -K$

Hence, option (c) is correct.

94. Let $A = \begin{bmatrix}1 & a & a\\0 & 1 & b\\0 & 0 & 1\end{bmatrix}$ $a, b, \in R$. If for some $n \in N$.

$A^n = \begin{bmatrix}1 & 48 & 2160\\0 & 1 & 96\\0 & 0 & 1\end{bmatrix}$ then n + a + b is equal to _______

JEE Main-25.07.2022, Shift-II

Ans. (24) : Given,

$$A = \begin{bmatrix}1 & a & a\\0 & 1 & b\\0 & 0 & 1\end{bmatrix}, \quad a, b \in R$$

It can be also write as,

$$A = \begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix} + \begin{bmatrix}0 & a & a\\0 & 0 & b\\0 & 0 & 0\end{bmatrix}$$

$A = I + B$

Where, $B=\begin{bmatrix}0 & a & a\\0 & 0 & b\\0 & 0 & 0\end{bmatrix}$

$$B^2=\begin{bmatrix}0 & a & a\\0 & 0 & b\\0 & 0 & 0\end{bmatrix}\begin{bmatrix}0 & a & a\\0 & 0 & b\\0 & 0 & 0\end{bmatrix}$$

$$=\begin{bmatrix}0 & 0 & ab\\0 & 0 & 0\\0 & 0 & 0\end{bmatrix}$$

and $B^3=\begin{bmatrix}0 & 0 & ab\\0 & 0 & 0\\0 & 0 & 0\end{bmatrix}\begin{bmatrix}0 & a & a\\0 & 0 & b\\0 & 0 & 0\end{bmatrix}$

$$=\begin{bmatrix}0 & 0 & 0\\0 & 0 & 0\\0 & 0 & 0\end{bmatrix}$$

Now,
$A^n = (I + B)^n = {}^nC_0I + {}^nC_1B + {}^nC_2B^2 + {}^nC_3B^3 + ...+ {}^nC_nB^n$

$$=\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}+\begin{bmatrix}0 & na & na\\0 & 0 & nb\\0 & 0 & 0\end{bmatrix}+\begin{bmatrix}0 & 0 & \frac{n(n-1)}{2}ab\\0 & 0 & 0\\0 & 0 & 0\end{bmatrix}$$

$$A_n=\begin{bmatrix}1 & na & na+\frac{n(n-1)}{2}ab\\0 & 1 & nb\\0 & 0 & 1\end{bmatrix} \quad ....(i)$$

$$A_n=\begin{bmatrix}1 & 48 & 2160\\0 & 1 & 96\\0 & 0 & 1\end{bmatrix} \quad ....(ii) \text{ given}$$

On comparing equation (i) and (ii) we get -

na = 48 and nb = 96 and

$$=na+\frac{n(n-1)ab}{2}=2160$$

$$48+\frac{96(n-1)a}{2}=2160$$

48 + 48 (n – 1)a = 2160

= 1 + na – a = 45

a = 4 and n = 12

Then,

nb = 96

b = 8

∴ n + a + b = 12 + 4 + 8 = 24

95. If A, B are square matrices of order 3.A is non singular and AB = 0, then B is a

(a) null matrix
(b) non singular matrix
(c) singular matrix
(d) unit matrix

AP EAMCET-2002

Ans. (a) : It is given that A and B are square matrices of order 3.

∵ A is non-singular matrix

∴ A^{-1} exists

Here,

AB = 0

or $B = A^{-1}0$ [Pre-multiplying both sides of the equation by A^{-1}]

or B = 0

So, B is null matrix.

Hence, option (a) is correct.

96. If A and B are two square matrices of the same order and m is a positive integer, then $(A+B)^m$ $= {}^mC_0A^m + {}^mC_1A^{m-1}B + {}^mC_2A^{m-2}B^2 +....+ {}^mC_mB^m$, if

(a) AB = –BA
(b) $A^m = 0, B^m = 0$
(c) AB = 2BA
(d) AB = BA

AMU-2011

Ans. (d) : If A and B are two square matrix then $(A+B)^m = {}^mC_oA^m + {}^mC_1A^{m-1}B + {}^mC_2A^{m-2}B^2 +.....$

Putting m = 2

$(A + B)^2 = {}^2C_o A^2 + {}^2C_1 A^1B + {}^2C_2 A^o B^2$

$(A + B)^2 = A^2 + 2AB + B^2$ ....(i)

But $(A + B)^2 = (A + B). (A + B)$

A.(A + B) + B.(A + B) ....(ii)

$A^2 + B^2 + AB + BA$

From (i) and (ii)

$A^2 + 2AB + B^2 = A^2 + B^2 + AB + BA$

⇒ AB = BA

97. Value of $\begin{bmatrix}2 & 4 & 6\\2+3x & 4+3y & 6+3z\\2x & 2y & 2z\end{bmatrix}$ is

(a) 0
(b) 2
(c) 4
(d) 6

AMU-2010

Ans. (a) : We have,

$$\begin{vmatrix}2 & 4 & 6\\2+3x & 4+3y & 6+3z\\2x & 2y & 2z\end{vmatrix}$$

$$=\begin{vmatrix}2 & 4 & 6\\2 & 4 & 6\\2x & 2y & 2z\end{vmatrix}+\begin{vmatrix}0 & 0 & 0\\3x & 3y & 3z\\0 & 0 & 0\end{vmatrix}$$

= 0 + 0 (∴ $R_2 = R_3 = 0$)

98. What are the values of (x, y, z, t) where $3\begin{bmatrix}x & y\\z & t\end{bmatrix}=\begin{bmatrix}x & 6\\-1 & 2t\end{bmatrix}+\begin{bmatrix}4 & x+y\\z+t & 3\end{bmatrix}=?$

(a) (2, 4, 3, 1)
(b) (2, 4, 1, 3)
(c) (1, 3, 2, 4)
(d) (1, 3, 4, 2)

APEAPCET- 23.08.2021, Shift-2

Ans. (b): We have,

$$\begin{bmatrix} x & 6 \\ -1 & 2t \end{bmatrix} + \begin{bmatrix} 4 & x+y \\ z+t & 3 \end{bmatrix} = \begin{bmatrix} 3x & 3y \\ 3z & 3t \end{bmatrix}$$

$x + 4 = 3x$
$2x = 4 \quad x = 2$
$6 + x + y = 3y$
$2y = 8 \quad y = 4$
$2t + 3 = 3t$
$t = 3$
$-1 + z + t = 3z$
$-1 + z + 3 = 3z$
$2z = 2$
$z = 1$

therefore, the values are $(x, y, z, t) = (2, 4, 1, 3)$.

99. The trace of the matrix A = $\begin{bmatrix} 1 & -5 & 7 \\ 0 & 7 & 9 \\ 11 & 8 & 9 \end{bmatrix}$ is

(a) 17 (b) 25
(c) 3 (d) 12

AP EAPCET-25.08.2021, Shift-II

Ans. (a) : Trace of matrix A = 1 + 7 + 9 = 17
[Trace of matrix means sum of its diagonal elements].

100. Let A be a 2×2 matrix with real entries. Let I be the 2×2 identity matrix. Tr(A) denotes the sum of diagonal entries of A. Assume that A^2=I.
Statement I:If $A \neq I$ and $A \neq -I$ then $\det A = -I$
StatementII:If
$A\ A \neq I$ and $A \neq I$ and $A \neq -I$ then $\mathrm{Tr}\ A \neq 0$

(a) Statement I is true, statement II is true. Statement II is a correct explanation for statement I
(b) Statement I is true, statement II is true. Statement II is not a correct explanation for statement I
(c) Statement I is true, Statement II is false
(d) Statement I is false, statement II is true

AP EAPCET-24.08.2021, Shift-II

Ans. (c): Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \qquad I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow \quad$ Tr(A) = a + d
$A \neq I$
$a \neq 1, \quad b \neq 0, \quad c \neq 0, \quad d \neq 1$

$$A^2 = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a^2+bc & ab+bd \\ ac+cd & bc+d^2 \end{bmatrix}$$

Given A^2=I

$$\begin{bmatrix} a^2+bc & ab+bd \\ ac+cd & bc+d^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$a^2+bc=1=bc+d^2$ ...(i)
and ab + bd =0=ac + cd ...(ii)
$A \neq I$
From equation (ii) ab + bd = 0
b (a + d) = 0
a + d = 0 $\quad (\because b \neq 0)$
a = – d

From equation (i)
$a^2 = 1 - bc$
$a = \pm\sqrt{1-bc}$
$d^2 = 1 - bc$
$d = \pm\sqrt{1-bc}$
So,
$a = \sqrt{1-bc} \qquad [\because a = -d]$
$d = -\sqrt{1-bc}$

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} \sqrt{1-bc} & b \\ c & -\sqrt{1-bc} \end{bmatrix}$$

$|A| = -(1-bc) - bc = -1$
Statement I is true.
$\mathrm{Tr}(A) = a + d \Rightarrow \sqrt{1-bc} - \sqrt{1-bc}$
Statement II is false
So option **(c)** is true.

101. The sum of two lower triangular matrices is always ________

(a) an upper triangular matrix
(b) an lower triangular matrix
(c) a diagonal matrix
(d) a scalar matrix

AP EAPCET-24.08.2021, Shift-II

Ans. (b): $\begin{bmatrix} a & 0 & 0 \\ b & c & 0 \\ d & e & f \end{bmatrix} + \begin{bmatrix} g & 0 & 0 \\ h & i & 0 \\ j & k & 1 \end{bmatrix}$

$$= \begin{bmatrix} a+g & 0 & 0 \\ b+h & c+i & 0 \\ d+j & e+k & f+1 \end{bmatrix}$$

So, the sum of two lower triangular matrix is always lower triangular matrix.

102. Which of the following matrices is not a square-matrix?

(a) $[1]$ (b) $\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$

(c) $\begin{bmatrix} 3 & 3 & 3 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

AP EAPCET-24.08.2021, Shift-II

Ans. (c): A square matrix is a matrix in which Number of column is equal to row.
Hence, option (c) is correct.

103. If A is a non-singular matrix such that $A.A^T = A^T$. A and $B = A^{-1}.A^T$ then

(a) $A.B^T$=I (b) $B.B^T$=I
(c) $A^T.B^T$=I (d) $B^{-1}.B^T$=I

AP EAPCET-24.08.2021, Shift-II

Ans. (b): Given,
$B = A^{-1}A^T$
Multiplying both side by A, we get-
$BA = A^{-1}A^TA$
$BA = A^{-1}AA^T \qquad (\because A^TA = AA^T)$
$BA = A^T \qquad (\because A^{-1}A = I)$

Taking transpose both side, we get:-

$(BA)^T = A$

$\Rightarrow A^T B^T = A$

$\Rightarrow B^T = \dfrac{A}{A^T}$

$\Rightarrow B^T = \dfrac{A^{-1}A}{A^{-1}A^T}$

$\Rightarrow B^T = \dfrac{I}{B}$

$\Rightarrow \therefore \boxed{BB^T = I}$

104. If $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & \alpha & 1 \end{bmatrix}, A^{-1} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ -4 & 3 & \beta \\ \frac{5}{2} & -\frac{3}{2} & \frac{1}{2} \end{bmatrix}$, then

(a) $\alpha = 2, \beta = -\dfrac{1}{2}$ (b) $\alpha = 1, \beta = -1$

(c) $\alpha = -1, \beta = 1$ (d) $\alpha = \dfrac{1}{2}, \beta = \dfrac{1}{2}$

AMU-2018

Ans. (b): Given,

$$A = \begin{vmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & \alpha & 1 \end{vmatrix}$$

$$\text{adj}(A) = \begin{vmatrix} 2-3\alpha & 2\alpha-1 & -1 \\ 8 & -6 & 2 \\ \alpha-6 & 3 & -1 \end{vmatrix}$$

$$A^{-1} = \frac{\text{adj}A}{|A|} = \frac{1}{2\alpha-4}\begin{bmatrix} 2-3\alpha & 2\alpha-1 & -1 \\ 8 & -6 & 2 \\ \alpha-6 & 3 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ -4 & 3 & \beta \\ \frac{5}{3} & -\frac{3}{2} & \frac{1}{2} \end{bmatrix}$$

On comparing $\dfrac{8}{2\alpha-4} = -4 \Rightarrow -8\alpha + 16 = 8$

$\alpha = 1$

and $\dfrac{2}{2\alpha-4} = \beta \Rightarrow \beta = \dfrac{2}{2-4} = \dfrac{2}{-2} = -1$

$\alpha = 1$ and $\beta = -1$

105. From the matrix equation AB = AC, it can be concluded that B = C provided

(a) A is singular (b) A is non-singular

(c) A is symmetric (d) A is square

AMU-2008

Ans. (b) : According to given summation,

Let $|A| \neq 0$

$\therefore$ A^{-1} exists

Here, it is given that,

$AB = AC$

Pre-multiplying by A^{-1} on both sides, we get

$A^{-1}AB = A^{-1}AC$

$IB = IC$

$B = C$

So, A is non singular

106. If $A\begin{bmatrix} x & 1 & 2 \\ 2 & 4 & x \\ -3 & 3 & 2 \end{bmatrix}$ is a singular matrix and the distinct values of x are x_1 and x_2, then $x_1 + x_2 + x_1x_2 =$

(a) 9 (b) 11/3

(c) 15/3 (d) 7

AP EAMCET-06.07.2022, Shift-II

Ans. (*) : It is given that A is singular matrix.

$|A| = 0$

$$\begin{bmatrix} x & 1 & 2 \\ 2 & 4 & x \\ -3 & 3 & 2 \end{bmatrix} = 0$$

$x(8-3x) - (4+3x) + 2(6+12) = 0$

$8x - 3x^2 - 4 - 3x + 36 = 0$

$3x^2 - 5x - 32 = 0$

$$x = \frac{5 \pm \sqrt{25+384}}{6}$$

$$x = \frac{5 \pm \sqrt{409}}{6}$$

Hence,

$$x_1 = \frac{5+\sqrt{409}}{6}, x_2 = \frac{5-\sqrt{409}}{6}$$

Now,

$x_1 + x_2 + x_1x_2$

$$x = \frac{5+\sqrt{409}}{6} + \frac{5-\sqrt{409}}{6} + \left(\frac{5+\sqrt{409}}{6}\right)\left(\frac{5-\sqrt{409}}{6}\right)$$

$$= \frac{10}{6} + \frac{25-409}{36}$$

$$= \frac{60-384}{36} = \frac{-324}{36} = -9$$

107. The matrix $A = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \end{bmatrix}$ is...........

(a) Unitary (b) Orthogonal

(c) Nilpotent (d) Involutory

AP EAMCET-17.09.2020, Shift-I

Ans. (c) : Given,

$$A = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$$

$$A^2 = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$$

$$\Rightarrow \quad A^2 = \begin{bmatrix} \frac{1}{2}-\frac{1}{2} & \frac{1}{2}-\frac{1}{2} \\ -\frac{1}{2}+\frac{1}{2} & -\frac{1}{2}+\frac{1}{2} \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \text{null matrix}$$

So, A is nilpotent matrix

108. If $\begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$ is to be square root of the two rowed unit matrix, then α, β and γ should satisfy the relation

(a) $1+\alpha^2+\beta\gamma=0$ (b) $1-\alpha^2-\beta\gamma=0$
(c) $1-\alpha^2+\beta\gamma=0$ (d) $1+\alpha^2-\beta\gamma=0$

AMU-2016

Ans. (b) : Given,

$$\begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$$

is square root of $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Then,

$$\begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}\begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \alpha^2+\beta\gamma & 0 \\ 0 & \beta\gamma+\alpha^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

On comparing we get -

$\alpha^2+\beta\gamma=1$

$\alpha^2+\beta\gamma-1=0$

Or $\quad 1-\alpha^2-\beta\gamma=0$

109. If $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$, then A^3 =

(a) $\begin{bmatrix} -\cos 3\theta & -\sin 3\theta \\ \sin 3\theta & \cos 3\theta \end{bmatrix}$

(b) $\begin{bmatrix} \cos 3\theta & -\sin 3\theta \\ -\sin 3\theta & \cos 3\theta \end{bmatrix}$

(c) $\begin{bmatrix} \cos 3\theta & -\sin 3\theta \\ \sin 3\theta & \cos 3\theta \end{bmatrix}$

(d) $\begin{bmatrix} -\cos 3\theta & \sin 3\theta \\ -\sin 3\theta & -\cos 3\theta \end{bmatrix}$

AMU-2013

Ans. (c) : Given,

$$A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

then $A^3 = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

$$= \begin{bmatrix} \cos^2\theta-\sin^2\theta & -\sin\theta\cos\theta-\sin\theta\cos\theta \\ \cos\theta\sin\theta+\cos\theta\sin\theta & -\sin^2\theta+\cos^2\theta \end{bmatrix} \times \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{bmatrix}\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos 3\theta & -\sin 3\theta \\ \sin 3\theta & \cos 3\theta \end{bmatrix}$$

110. It for a matrix A, $A^2 + I = 0$, where I is the identity matrix of order 2, then A=

(a) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} i & 0 \\ 0 & i \end{bmatrix}$

(c) $\begin{bmatrix} i & 0 \\ 0 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

AMU-2012

Ans. (b) : For a matrices A, $A^2 + I = 0$

Since $\begin{vmatrix} i & 0 \\ 0 & i \end{vmatrix}\begin{vmatrix} i & 0 \\ 0 & i \end{vmatrix} = \begin{vmatrix} -1 & 0 \\ 0 & -1 \end{vmatrix}$

$A^2 = -I = \begin{vmatrix} -1 & 0 \\ 0 & -1 \end{vmatrix}$

$A^2 + I = -I + I = 0$

Hence $A = \begin{vmatrix} i & 0 \\ 0 & i \end{vmatrix}$

111. The inverse of a symmetric matrix is

(a) skew symmetric (b) symmetric
(c) diagonal matrix (d) none of these

AMU-2012

Ans. (b) : The inverse of a symmetric matrix is always symmetric

112. If $A = \begin{bmatrix} \alpha^2 & 5 \\ 5 & -\alpha \end{bmatrix}$ .and det $(A^{10}) = 1024$, then α =

(a) –2 (b) –1
(c) –3 (d) 0

AP EAMCET-04.07.2021, Shift-I

Ans. (c) : Given,

$$A = \begin{bmatrix} \alpha^2 & 5 \\ 5 & -\alpha \end{bmatrix}$$

and,

$$\det\left(A^{10}\right) = 1024$$

$$\begin{bmatrix} \alpha^2 & 5 \\ 5 & -\alpha \end{bmatrix} = (1024)^{1/10}$$

$= -\alpha^3 - 25 = 2$

$= \alpha = -3$

113. If A is a Slew-symmetric matrix then (given n $\in$ N)

1. A^{2n} is Skew-symmetric matrix.
2. A^{2n+1} is Skew-symmetric matrix.

(a) 1 is true, 2 is false
(b) Both 1 and 2 are true
(c) Both 1 and 2 are false
(d) 1 is false, 2 is true

AP EAMCET-18.09.2020, Shift-II

Ans. (d) : It is given that, A is a skew symmetric matrix
$\therefore\ A^T = -A \Rightarrow (A^{2n})^T = (-A)^{2n}$
$(A^{2n})^T = A^{2n}$
$\therefore\ A^{2n}$ is symmetric matrix (statement I is false)
$(A^{2n+1})^T = (A^T)^{2n+1} = (-A)^{2n+2}$
$(A^{2n+1})^T = -A^{2n+1}$
A^{2n+1} is skew symmetric (Statement II is true)

114. Let *A* and *B* be two symmetric matrices of same order. Then , the matrix *AB – BA* is
(a) a symmetric matrix
(b) a skew-symmetric matrix
(c) a null matrix
(d) the identity matrix

AP EAMCET-2009

Ans. (b) : Given,
A and B be two symmetric matrices of same order.
$\Rightarrow A' = A$ and $B' = B$
Now, $(AB - BA)' = (AB)' - (BA)'$
$= (B'A' - (A'B')$
$= (BA - AB)$
$= -(AB - BA)$
Thus,
$(AB - BA)' = -(AB - BA)$
Hence, $(AB - BA)'$ is a skew symmetric

115. Let A be a square matrix and I be the identity matrix of same order. If $A^2 = 2A - I$, then A^n equals
(a) $nA - I$
(b) $nA - (n - I)I$
(c) $2^{n-1} A - 2^{n-2} I$
(d) $2^{n-1} A - I$

AMU-2004

Ans. (b) : Given,
$A^2 = 2A - I$ …..(i)
A be on square matrix,
Multiplying by A both side,
$A^2.A = (2A - I)A$
$A^3 = 2A^2 - IA$
$= 2(2A - I) - IA$
$= 4A - 2I - IA$ $\{\therefore\ IA = A\}$
$A^3 = 3A - 2I$
Again differencing by A both side,
$A^3.A = (3A - 2I)\ A$
$= 3A^2 - 2IA$
$= 3(2A - I) - 2A$
$= 6A - 3I - 2A$
$A^4 = 4A - 3I$
Similarly, $A^5 = 5A - 4I$
$A^6 = 6A - 5I$
Hence, $A^n = nA - (n - I)I$

116. If $P = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, Q = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ then $(P\cos\theta + Q\sin\theta)^{-1}$ equals
(a) $P\sin\theta + Q\cos\theta$
(b) $P\sin\theta - Q\cos\theta$
(c) $P\cos\theta - Q\sin\theta$
(d) $-P\cos\theta - Q\sin\theta$

AMU-2004

Ans. (c) : Given,
$$P = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},\quad Q = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$
$$P\cos\theta + Q\sin\theta = \begin{bmatrix} \cos\theta & 0 \\ 0 & \cos\theta \end{bmatrix} + \begin{bmatrix} 0 & \sin\theta \\ -\sin\theta & 0 \end{bmatrix}$$
$$= \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$
$\because$ A is invertible,
$$\text{Then,}\quad A^{-1} = = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$
$$= \cos\theta \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \sin\theta \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$
$= P\cos\theta - Q\sin\theta$

117. If A is a square matrix then which of the following statements is true?
(a) if $A^2 = A$ and A is non-invertible then A = 0
(b) if $A^2 = 0$ then A = 0
(c) if A is invertible then A+A' is invertible
(d) if $A^2 = A$ and A is invertible then A = 1

AMU-2004

Ans. (d) : If A is a square matrix,
$$\text{Let,}\quad A = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$$
$$A^2 = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$$
$$A^2 = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = I$$
$A^2 = A = I$
Hence $A^2 = A$ and A is invertible then A = I

118. If $A = \begin{bmatrix} 0 & 2 \\ 3 & -4 \end{bmatrix}, k\,A = \begin{bmatrix} 0 & 3a \\ 2b & 24 \end{bmatrix}$, then the values of k, a, b are respectively
(a) –6, –12, –18
(b) –6, 4, 9
(c) –6, –4, –9
(d) –6, + 12, 18

AP EAMCET-2001

Ans. (c) : Given,
$$A = \begin{bmatrix} 0 & 2 \\ 3 & -4 \end{bmatrix} \text{ and } kA = \begin{bmatrix} 0 & 3a \\ 2b & 24 \end{bmatrix}$$
$$\begin{bmatrix} 0 & 2k \\ 3k & -4k \end{bmatrix} = \begin{bmatrix} 0 & 3a \\ 2b & 24 \end{bmatrix}$$
Comparing the above equations, we get –
$-4k = 24$
$k = -6$
And $3k = 2b$
$3(-6) = 2b$
$B = -9$
And $3a = 2k$
$3a = 2(-6)$
$a = -4$
Here,
$k = -6$ $a = -4$ and $b = -9$
Hence, option (c) is correct.

119. $\mathbf{A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}}$**, then** A^{-1} **is equal to**

(a) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

EAMCET-1994

Ans. (d) : Given,

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

Here, $|A| = 0 - 1 = -1$

Adj $A = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$

$\therefore$ $A^{-1} = \frac{1}{\text{Det.A}}.\text{Adj}(A)$

$$A^{-1} = \frac{-1}{1}\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

120. **If A =** $\mathbf{\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & \lambda \end{bmatrix}}$ **is a singular matrix, then** $\boldsymbol{\lambda}$ **is**

(a) 2 (b) 3

(c) 4 (d) 15

EAMCET-1995

Ans. (b) : Given,

$$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & \lambda \end{bmatrix}$$

For a singular matrix determinant of $|A| = 0$

Here,

$$|A| = 2(24 - 14) + 4(-32 + 12) + \lambda(56 - 36) = 0$$

$20 - 80 + 20\lambda = 0$

$\lambda = 3$

121. **The order of [x y z]** $\mathbf{\begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}}$ **is**

(a) 3×1 (b) 1×1

(c) 1×3 (d) 3×3

EAMCET-1994

Ans. (b) : Given,

$$[x\ y\ z]\begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Here, order of matrix [x y z] is 1 × 3

Order of matrix $\begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}$ is 3 × 3

And order of matrix $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ is 3 × 1

Now, we have to compute the order of

$$[x\ y\ z]\begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}.$$

So product the orders of the matrices to get the result. i.e.

(1 × 3) (3 × 3) (3 × 1)
= (1 × 3) (3 × 1)
= 1 × 1

Note : If k is a matrix with order a × b and L is a matrix with order b × c, then matrix KL has the order a × c.

122. The order of matrix A is 3×5 and that of B is 2×3, then the order of matrix BA is

(a) 2×3 (b) 3×2

(c) 2×5 (d) 5×2

EAMCET-1995

Ans. (c) : Note : Two matrices are said to be conformable for multiplication, if the number of columns of the first matrix is equal to the number of route of the second matrix which is equal to 3 in the above problem.

$\therefore$ BA exists.

i.e, if $A = [a_{ij}]_{n \times m}$ and $B = [b_{jk}]_{p \times n}$

then, multiplication of the matrices $[B]_{p \times n} \times [A]_{n \times m}$ routs in matrix $[BA]_{p \times n}$

Given,

$[A]_{3 \times 5}$ and $[B]_{2 \times 3}$

Hence, order of the matrix [BA] is 2 × 5

123. **The matrix** $\mathbf{\begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 0 \\ 3 & 1 & 1 \end{bmatrix}}$ **is**

(a) non-singular (b) singular

(c) skew-symmetric (d) symmetric

EAMCET-1998

Ans. (b) : Given,

$$\begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 0 \\ 3 & 1 & 1 \end{bmatrix}$$

Here,

$$\begin{vmatrix} 1 & 0 & 1 \\ 2 & 1 & 0 \\ 3 & 1 & 1 \end{vmatrix} = 1\begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix} - 0\begin{vmatrix} 2 & 0 \\ 3 & 1 \end{vmatrix} + 1\begin{vmatrix} 2 & 1 \\ 3 & 1 \end{vmatrix}$$

= 1(1 – 0) – 0 + 1 (2 – 3)
= 1 – 1
= 0

$\therefore$ It is a singular matrix

124. **The real part of $\begin{vmatrix} \cos\alpha + i\sin\alpha & \cos\beta + i\sin\beta \\ \sin\beta + i\cos\beta & \sin\alpha + i\cos\alpha \end{vmatrix}$ is equal to**
(a) 0 (b) 1
(c) 2 cos α (d) 2 sin β

EAMCET-1999

Ans. (a) : Given,,
$$\begin{vmatrix} \cos\alpha + i\sin\alpha & \cos\beta + i\sin\beta \\ \sin\beta + i\cos\beta & \sin\alpha + i\cos\alpha \end{vmatrix}$$
Here [(cosα + i sinα) (sinα + i cosα) – (sinβ + i cosβ) (cosβ + i sinβ)]
= [(cosα sinα + i $\cos^2\alpha$ + i $\sin^2\alpha$ sinα cosα) – (sinβ cosβ + i $\sin^2\beta$ + i^2 sinβ cosβ)
We know that, $i^2 = 1$
= [cosα sinα + i($\cos^2\alpha + \sin^2\alpha$) – sinα cosα] – [sinβ cosβ + i ($\sin^2\beta + \cos^2\beta$ – cosβ sinβ)
After solving, we get
[i(1) – i(1)] = 0
Real part = 0 + i0 = 0

125. **If the matrix $\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is singular, then θ is equal to**
(a) π (b) $\frac{\pi}{2}$
(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{4}$

EAMCET-1999

Ans. (d) : Given,,
$$\begin{bmatrix} \cos\theta & \sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
For a singular matrix, determinant is zero
$$\Rightarrow \begin{vmatrix} \cos\theta & \sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{vmatrix} = 0$$
⇒ cosθ cosθ – sinθ sinθ = 0
⇒ $\cos^2\theta = \sin^2\theta$
$\therefore\ \theta = \frac{\pi}{4}$

126. **If A is a square matrix then,**
(a) A + A^T is symmetric
(b) AA^T s symmetric
(c) A^T + A is skew-symmetric
(d) A^TA is skew symmetric

WB JEE-2009

Ans. (a) : We know that,
$(A + A^T)^T = A^T + (A^T)^T$
$= A^T + A$
∴ A + A^T is symmetric matrix

127. **If A and B are two matrices such that A + B and AB are both defined, then**
(a) A and B can be any matrices
(b) A, B are square matrices not necessarily of the same order
(c) A, B are square matrices of the same order
(d) number of columns of A = number of rows of B

WB JEE-2011

Ans. (c) : Note : "Addition of matrices is possible only when they are of same order."
Let M be a matrix of order m × n and N be a matrix of order p × q
∴ M + N is defined so, order of M and N must be same.
⇒ m = p and n = q
Moreover, product of two matrices M, N is possible if number of column in M is same as Number of rows in B.
⇒ n = p ⇒ m = n
∴ M and N are square matrices of same order.
Hence, option (c) is correct.

128. **If $A = \begin{bmatrix} 3 & x-1 \\ 2x+3 & x+2 \end{bmatrix}$ is a symmetric matrix, then the value of x is**
(a) 4 (b) 3
(c) –4 (d) –3

WB JEE-2011

Ans. (c) : It is given that, A is a symmetric matrix i.e., $A = A^T$
$$\Rightarrow \begin{bmatrix} 3 & x-1 \\ 2x+3 & x+2 \end{bmatrix} = \begin{bmatrix} 3 & 2x+3 \\ x-1 & x+2 \end{bmatrix}$$
⇒ x – 1 = 2x + 3
⇒ x = –4

129. **The rank of the matrix $\begin{bmatrix} 4 & 2 & (1-x) \\ 5 & k & 1 \\ 6 & 3 & (1+x) \end{bmatrix}$ is 1, then**
(a) $k = \frac{5}{2}, x = \frac{1}{5}$ (b) $k = \frac{5}{2}, x \ne \frac{1}{5}$
(c) $k = \frac{1}{5}, x = \frac{5}{2}$ (d) $k \ne \frac{5}{2}, x = \frac{1}{5}$

AP EAMCET-19.08.2021, Shift-I

Ans. (a): Note : Rank of a matrix is defined as the Number of non-zero rows of a matrix in its echelon form.
Here, We have
$$\begin{bmatrix} 4 & 2 & 1-x \\ 5 & k & 1 \\ 6 & 3 & 1+x \end{bmatrix}_{3\times3}$$
$R_3 \to R_1 + R_3$
$$\begin{bmatrix} 4 & 2 & 1-x \\ 5 & k & 1 \\ 10 & 5 & 2 \end{bmatrix}$$
$R_3 \to R_3 - 2R_2$
$$\begin{bmatrix} 4 & 2 & 1-x \\ 5 & k & 1 \\ 0 & 5-2k & 0 \end{bmatrix}$$

$R_2 \to R_2 - \frac{5}{4}R_1$

$$\begin{bmatrix} 4 & 2 & 1-x \\ 0 & k-\frac{5}{2} & \frac{5x-1}{4} \\ 0 & 5-2k & 0 \end{bmatrix}$$

For rank 1 one raw must be non-zero

So, $5 - 2k = 0 \Rightarrow k = \frac{5}{2}$

$$\frac{5x-1}{4} = 0 \Rightarrow x = \frac{1}{5}$$

To get rank 1 of matrix then $k = \frac{5}{2}$, $x = \frac{1}{5}$

130. Let I denote the 3 × 3 identity matrix and P be a matrix obtained by rearranging the columns of I. then

(a) there are six distinct choices for P and det (P) = 1

(b) there are six distinct choices for P and det (P) = ± 1

(c) there are more than one choices for P and some of them are not invertible

(d) there are more than one choices for P and P^{-1} = 1 in each choice

WB JEE-2014

Ans. (b) : Given,

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Det (I) = 1

If we take I as

$$M = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Then, det(M) = 1

In the same way, these are four other possibilities,

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

Which will give a determinant either –1 or 1.

Hence, these are six distinct choices for P and det (P) = ±1.

131. Let $n \geq 2$ be an integer,

$$A = \begin{bmatrix} \cos(2\pi/n) & \sin(2\pi/n) & 0 \\ -\sin(2\pi/n) & \cos(2\pi/n) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ **and I is the identity matrix of order 3. Then,**

(a) $A^n = I$ and $A^{n-1} \neq I$

(b) $A^m \neq I$ for any positive integer m

(c) A is not invertible

(d) $A^m = O$ for a positive integer m

WB JEE-2014

Ans. (a) : Given,

$$A = \begin{bmatrix} \cos\left[\frac{2\pi}{n}\right] & \sin\left(\frac{2\pi}{n}\right) & 0 \\ -\sin\left(\frac{2\pi}{n}\right) & \cos\left(\frac{2\pi}{n}\right) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Here, A × A

$$= \begin{bmatrix} \cos\left(\frac{2\pi}{n}\right) & \sin\left(\frac{2\pi}{n}\right) & 0 \\ -\sin\left(\frac{2\pi}{n}\right) & \cos\left(\frac{2\pi}{n}\right) & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} \cos\left(\frac{2\pi}{n}\right) & \sin\left(\frac{2\pi}{n}\right) & 0 \\ -\sin\left(\frac{2\pi}{n}\right) & \cos\left(\frac{2\pi}{n}\right) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2\left(\frac{2\pi}{n}\right) - \sin^2\left(\frac{2\pi}{n}\right) & \cos\left(\frac{2\pi}{n}\right)\sin\left(\frac{2\pi}{n}\right) + \sin\left(\frac{2\pi}{n}\right)\cos\left(\frac{2\pi}{n}\right) & 0 \\ -\sin\left(\frac{2\pi}{n}\right)\cos\left(\frac{2\pi}{n}\right) - \sin\left(\frac{2\pi}{n}\right)\cos\left(\frac{2\pi}{n}\right) & -\sin^2\left(\frac{2\pi}{n}\right) + \cos^2\left(\frac{2\pi}{n}\right) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\left(2\times\frac{2\pi}{n}\right) & 2\sin\left(\frac{2\pi}{n}\right)\cos\left(\frac{2\pi}{n}\right) & 0 \\ -2\sin\left(\frac{2\pi}{n}\right)\cos\left(\frac{2\pi}{n}\right) & \cos\left(2\times\frac{2\pi}{n}\right) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\left(2\times\frac{2\pi}{n}\right) & \sin\left(2\times\frac{2\pi}{n}\right) & 0 \\ -\sin\left(2\times\frac{2\pi}{n}\right) & \cos\left(2\times\frac{2\pi}{n}\right) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Similarly,

$$A^n = \begin{bmatrix} \cos\left(2^{n-1}\times\frac{2\pi}{n}\right) & \sin\left(2^{n-1}\times\frac{2\pi}{n}\right) & 0 \\ -\sin\left(2^{n-1}\times\frac{2\pi}{n}\right) & \cos\left(2^{n-1}\times\frac{2\pi}{n}\right) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

And

$$A^{n-1}\begin{bmatrix} \cos\left(2^{n-2}\times\frac{2\pi}{n}\right) & \sin\left(2^{n-2}\times\frac{2\pi}{n}\right) & 0 \\ -\sin\left(2^{n-2}\times\frac{2\pi}{n}\right) & \cos\left(2^{n-2}\times\frac{2\pi}{n}\right) & 0 \\ 0 & 0 & 1 \end{bmatrix} \neq I$$

$\therefore A^{n-1} \neq I$ for any positive integer n.

132. If A and B are two matrices such that AB = B and BA = A, then $A^2 + B^2$ equals

(a) 2AB (b) 2BA
(c) A + B (d) AB

WB JEE-2015

Ans. (c) : Given,

$AB = A$ …(i)

$BA = B$ …(ii)

From eq^n (ii)

$BA = B$

$B \times (AB) = B$

$B^2A = B$

From eq^n (ii)

$B^2A = BA$

$B^2 = B$

From eq^n (i)

$A \times (BA) = A$

$A^2B = A$

From eq^n (i)

$A^2B = AB$

$A^2 = A$

Hence, $A^2 + B^2 = A + B$

133. If the matrix $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 2 & 0 & 2 \end{bmatrix}$ then

$$A^n = \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ b & 0 & a \end{bmatrix}, n \in N, \text{ where}$$

(a) $a = 2^n, b = 2^n$ (b) $a = 2^n, b = 2n$
(c) $a = 2^n, b = n\,2^{n-1}$ (d) $a = 2^n, b = n\,2^n$

WB JEE-2016

Ans. (d) : Given,

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 2 & 0 & 2 \end{bmatrix}$$

$$2\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow A^2 = 2^2\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} = 2^2\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow A^3 = 2^3\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 3 & 0 & 1 \end{bmatrix} \text{ and so on}$$

We can observe that, $A^n = 2^n \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ n & 0 & 1 \end{bmatrix}$

By comparing we get $a = 2^n$, $b = n2^n$

134. Let P be the set of all non-singular matrices of order 3 over R and Q be the set of all orthogonal matrices of order 3 over R. Then,

(a) P is proper subset of Q
(b) Q is proper subset of P
(c) Neither P is proper subset of Q nor Q is proper subset of P
(d) $P \cap Q = \phi$ the void set

WB JEE-2017

Ans. (b) : Since, All the orthogonal matrix are non – singular matrix.

$\therefore$ Q is proper subset of P.

135. In a third order matrix A, a_{ij}, denotes the element in the i th row and it j th column.
If $a_{ij} = 0$ for I = j
= i for i > j
= –1 for i < j
Then the matrix is

(a) skew symmetric (b) symmetric
(c) not invertible (d) non- singular

WB JEE-2018

Ans. (a,c) : Given,

$$A = \begin{bmatrix} 0 & -1 & -1 \\ 1 & 0 & -1 \\ 1 & 1 & 0 \end{bmatrix} \text{ skew symmetric}$$

$$|A| = \begin{vmatrix} 0 & -1 & -1 \\ 1 & 0 & -1 \\ 1 & 1 & 0 \end{vmatrix}$$

$$|A| = 0 + 1(0+1) - 1(1-0)$$

$$|A| = 0$$

$|A| = 0 \Rightarrow$ non-invertible

136. The least positive integer n such that $\begin{bmatrix} \cos\frac{\pi}{4} & \sin\frac{\pi}{4} \\ -\sin\frac{\pi}{4} & \cos\frac{\pi}{4} \end{bmatrix}^n$ is an identity matrix of order 2 is

(a) 4 (b) 8
(c) 12 (d) 16

WB JEE-2018

Ans. (b) : Given,

$$\begin{bmatrix} \cos\frac{\pi}{4} & \sin\frac{\pi}{4} \\ -\sin\frac{\pi}{4} & \cos\frac{\pi}{4} \end{bmatrix}^n$$

It can be written as

$$\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

Let $A = \begin{bmatrix} \lambda & \lambda \\ -\lambda & \lambda \end{bmatrix}$ where, $\lambda = \frac{1}{\sqrt{2}}$

$$A^2 = \begin{bmatrix} \lambda & \lambda \\ -\lambda & \lambda \end{bmatrix}\begin{bmatrix} \lambda & \lambda \\ -\lambda & \lambda \end{bmatrix} = \begin{bmatrix} 0 & 2\lambda^2 \\ -2\lambda^2 & 0 \end{bmatrix}$$

$$A^4 = \begin{bmatrix} 0 & 2\lambda^2 \\ -2\lambda^2 & 0 \end{bmatrix}\begin{bmatrix} 0 & 2\lambda^2 \\ -2\lambda^2 & 0 \end{bmatrix} = \begin{bmatrix} -4\lambda^4 & 0 \\ 0 & -4\lambda^4 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$A^8 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

137. If $\begin{vmatrix} -1 & 7 & 0 \\ 2 & 1 & -3 \\ 3 & 4 & 1 \end{vmatrix} = A$, Then $\begin{vmatrix} 13 & -11 & 5 \\ -7 & -1 & 25 \\ -21 & -3 & -15 \end{vmatrix}$ is

(a) A^2 (b) $A^2 - A + l_3$

(c) $A^2 - 3A + l_3$ (d) $3A^2 + 5A - 4l_3$

(I_3 denote the det of the identity matrix of order 3)

WB JEE-2018

Ans. (a) : Given,

$$A = \begin{vmatrix} -1 & 7 & 0 \\ 2 & 1 & -3 \\ 3 & 4 & 1 \end{vmatrix}$$

$$\text{Adj } A = \begin{vmatrix} 13 & -7 & -21 \\ -11 & -1 & -3 \\ 5 & 25 & -15 \end{vmatrix}$$

Det (adj (A)) = $|A|^{n-1} = |A|^{3-1} = |A|^2 = A^2$

138. Let A be a square matrix of order 3. Choose the correct option regarding the following statements

I. There exists a matrix B of order 3 such that AB = I_3

II. There exists a matrix C of order 3 such that CA = I_3

III. A is invertible

(a) Only III implies I and II

(b) I, II and III are equivalent statements

(c) In I and II, B can be different from C

(d) None of the above

AP EAMCET-22.09.2020, Shift-II

Ans. (b) : From statement (I), AB = I_3

$B = I_3A^{-1}$

$= B = A^{-1}$ ....(i)

From statement (II) CA = I_3

$C = I_3 A^{-1}$

$C = A^{-1}$ ....(ii)

From equation (i) & (ii) A is a invertible matrix

So, statement (I), (II) & (III) are equivalent

139. If A $\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$, then AA^T is a

(a) Symmetric matrix

(b) Skew- Symmetric matrix

(c) Singular matrix

(d) Inverse of A

AP EAMCET-05.07.2022, Shift-I

Ans. (a) : Given,

$$A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 3 & 2 & 0 \\ -3 & -3 & -1 \\ 4 & 4 & 1 \end{bmatrix}$$

$$\text{Now, } AA^T = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} 3 & 2 & 0 \\ -3 & -3 & -1 \\ 4 & 4 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 9+9+16 & 6+9+16 & 0+3+4 \\ 6+9+16 & 4+9+16 & 0+3+4 \\ 0+3+4 & 0+3+4 & 0+1+1 \end{bmatrix}$$

$$\begin{bmatrix} 34 & 31 & 7 \\ 31 & 29 & 7 \\ 7 & 7 & 2 \end{bmatrix}$$

Here, $AA^T = (AA^T)^T$

Hence, the given matrix is symmetric matrix.

140. Determinant of skew-symmetric matrix of order "three" is always

(a) 0 (b) 1

(c) Depends on elements (d) –1

AP EAMCET-21.09.2020, Shift-II

Ans. (a) : Determinant of skew – symmetric matrix of order 'three' is always zero.

141. Let A and B be two square matrices of order 3 and AB = O_3 where O_3 denotes the null matrix of order 3. Then,

(a) must be $A = O_3$, $B = O_3$

(b) if $A \neq O_3$, must be $B \neq O_3$

(c) if $A = O_3$, must be $B \neq O_3$

(d) may be $A \neq O_3$, $B \neq O_3$

WB JEE-2019

Ans. (d) : $\because$ the product of two non-null matrix can be a null matrix.

$\therefore$ It may be

$A \neq O_3$, $B \neq O_3$

142. Let A be a square matrix of order 3 whose all entries are 1 and let I_3 be the identity matrix of order 3. Then the matrix A – $3I_3$ is

(a) invertible

(b) orthogonal

(c) non-invertible

(d) real Skew Symmetric matrix

WB JEE-2019

Ans. (c) : We know that,

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

Here, det A = 1

ad – cb = 1

$$\text{Now, } A - \lambda I_2 = \begin{bmatrix} a-\lambda & b \\ c & d-\lambda \end{bmatrix}$$

$\Rightarrow \det(A - \lambda I_2) = 0$

$\lambda^2 - \lambda(a + d) + ad - cb = 0$

$\because$ roots are imaginary

$\therefore$ $D < 0$

$(a + d)^2 - 4(ad - cb) < 0$

$(a + d)^2 < 4$

143. Let $A = \begin{bmatrix} 12 & 24 & 5 \\ x & 6 & 2 \\ -1 & -2 & 3 \end{bmatrix}$. The value of x for which the matrix A is not in variable is

(a) 6 (b) 12

(c) 3 (d) 2

WB JEE-2020

Ans. (c) : Given,

$$A = \begin{bmatrix} 12 & 24 & 5 \\ x & 6 & 2 \\ -1 & -2 & 3 \end{bmatrix}$$

For matrix to be non-invertible, det A = 0

$$|A| = 2\begin{vmatrix} 12 & 12 & 5 \\ x & 3 & 2 \\ -1 & -1 & 3 \end{vmatrix} = 0$$

$\Rightarrow x = 3 (\because C_1$ and C_2 are identical)

144. Let A and B be two non-singular skew symmetric matrices such that AB = BA, then $A^2B^2 (A^TB)^{-1} (AB^{-1})^T$ is equal to

(a) A^2 (b) $-B^2$

(c) $-A^2$ (d) AB

WB JEE-2021

Ans. (c):

$A^T = -A$, $B^T = -B$

$= A^2B^2 (A^T B)^{-1} (A B^{-1})^T$

$= A^2B^2 B^{-1}(A^T)^{-1} (B^{-1})^T A^T$

$= A^2B. BB^{-1} (-A)^{-1} (B^T)^{-1} (-A)$

$= A^2B. I A^{-1} (-B)^{-1} A$

$= -A^2 B. A^{-1} B^{-1} A$

$= -A. AB (BA)^{-1}A$

$= -A (BA) (BA)^{-1} A$ ($\because AB = BA$)

$= -A I, A = -A, A = -A^2$

Hence option (c) is correct.

$|A| \neq 0, |B| \neq 0$

B^{-1}, A^{-1} exist

$(AB)^{-1} = B^{-1} A^{-1}$

$(AB)^T = B^TA^T$

$(A^T)^{-1} = (A^{-1})^T$

$BB^{-1} = I$

145. If M is a 3 × 3 matrix such that [0 1 2] M = [1 0 0], [3 4 5] M = [0 1 0], then [6 7 8] M is equal to

(a) [2 1 –2] (b) [0 0 1]

(c) [–1 2 0] (d) [9 10 8]

WB JEE-2021

Ans. (c): Given M = 3×3 Matrix

$$M = \begin{vmatrix} x_1 & x_2 & x_3 \\ x_4 & x_5 & x_6 \\ x_7 & x_8 & x_9 \end{vmatrix}$$

$$[0 \quad 1 \quad 2]\begin{vmatrix} x_1 & x_2 & x_3 \\ x_4 & x_5 & x_6 \\ x_7 & x_8 & x_9 \end{vmatrix} = [1 \quad 0 \quad 0]$$

$|x_4 + 2x_7, x_5 + 2x_8, x_6 + 2x_9|$ [1 0 0] compare

$x_4 + 2x_7 = 1$ $x_5 + 2x_8 = 0$, $x_6 + 2x_9 = 0$

$$[3 \quad 4 \quad 5]\begin{vmatrix} x_1 & x_2 & x_3 \\ x_4 & x_5 & x_6 \\ x_7 & x_8 & x_9 \end{vmatrix} = [0 \quad 1 \quad 0]$$

$\Rightarrow [3x_1 + 4x_4 + 5x_7 \quad 3x_2 + 4x_5 + 5x_8 \quad 3x_3 + 4x_6 + 5x_9] = [0 \quad 1 \quad 0]$

$\Rightarrow 3x_1 + 4x_4 + 5x_7 = 0 \quad 3x_2 + 4x_5 + 5x_8 = 1$

$\Rightarrow 3x_3 + 4x_6 + 5x_9 = 0$

$$[6 \quad 7 \quad 8]\begin{vmatrix} x_1 & x_2 & x_3 \\ x_4 & x_5 & x_6 \\ x_7 & x_8 & x_9 \end{vmatrix}$$

$= [6x_1 + 7x_4 + 8x_7 \quad 6x_2 + 7x_5 + 8x_8 \quad 6x_3 + 7x_6 + 8x_9]$

$6x_1 + 7x_4 + 8x_7 = 2(3x_1 + 4x_5 + 5x_7) - x_4 - 2x_7$

$= 2 \times 0 - (x_4 + 2x_7) = 0 - 1 = -1$

– 1 is lies in option (c)

So, the answer is [–1 2 0]

Hence option (c) is correct.

146. Let M and N be two matrices over R of order 2. Then, MN = NM if.....

(a) One of M and N is a diagonal matrix

(b) Both M and N are diagonal matrices

(c) Both M and N are invertible matrices

(d) None of these options are true in general

AP EAMCET-17.09.2020, Shift-II

Ans. (b) : Let,

$$M = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, \quad N = \begin{bmatrix} p & q \\ r & s \end{bmatrix}$$

Then, $MN = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} p & q \\ r & s \end{bmatrix}$

$$= \begin{bmatrix} ap + br & aq + bs \\ pc + dr & cq + ds \end{bmatrix}$$

and $NM = \begin{bmatrix} p & q \\ r & s \end{bmatrix}\begin{bmatrix} a & b \\ c & c \end{bmatrix}$

$$= \begin{bmatrix} ap + qc & pb + qd \\ ar + cs & br + ds \end{bmatrix}$$

Clearly, MN = NM

When,

$br = 0 \quad qc = 0$

$pc = 0 \quad dr = 0$

or if $b = c = q = r = 0$

Both M & N must be diagonal matrices.

147. If P and Q are two non - zero square matrices of the same order such that the product PQ = 0, then....

(a) Exactly one of them must be singular

(b) Both P and Q must be singular

(c) Both P and Q must be non-singular

(d) None of the options are correct

AP EAMCET-17.09.2020, Shift-II

Ans. (d) : According to given summation,

PQ = 0 (given)

Taking determinant on both sides

$|PQ| = |P||Q| = 0$

$\Rightarrow$ Either P or Q should be a singular matrix.

Hence, option (d) is correct.

148. **Match the items of List-I with the items of List-II and choose the correct option.**

List-I	List-II
A. If A is a non singular matrix of order 3 and $\|A\| = a$, then $\|(\text{adj } A^{-1})^{-1}\| =$	**I. null matrix**
B. A is non singular matrix of order 3 and B is any matrix of order 3 such that AB = O, then B is	**II. a^2**
C. $\begin{vmatrix} 1 & x & x^2 \\ \cos(a-b)y & \cos ay & \cos(a+b)y \\ \sin(a-b)y & \sin ay & \sin(a+b)y \end{vmatrix}$ does not depend on	**III. b**
D. A is a square matrix of order 3 and $B = A - A^T$, then $\|B\|$ is	**IV. a**
	V. 0

The correct answer is

	A	B	C	D
(a)	II	IV	III	I
(b)	III	I	IV	V
(c)	II	V	III	I
(d)	II	I	IV	V

AP EAMCET-21.04.2019, Shift-II

Ans. (d) : According to given problem,

(A) We have a non-singular matrix of order 3, $|A| = a$

$$\because \left|(\text{adj A})^{-1}\right| = \frac{1}{\left|\text{adj } A^{-1}\right|} \qquad \left\{\because \left|A^{-1}\right| = \frac{1}{|A|}\right\}$$

$$= \frac{1}{\left|A^{-1}\right|^2} \qquad \left[\because |\text{adj A}| = |A|^2, \text{ if order of A is 3.}\right]$$

$$= \frac{1}{\left(\frac{1}{|A|^2}\right)} = |A|^2 = a^2$$

(B) Here, It is given that for a non-singular matrix A of order B, AB = 0

$\Rightarrow |AB| = 0$

$\Rightarrow |A||B| = 0$

$\Rightarrow |B| = 0 \qquad (\because |A| \neq 0)$

And matrix B should be null matrix.

(c) Here, $\Delta = \begin{vmatrix} 1 & x & x^2 \\ \cos(a-b)y & \cos ay & \cos(a+b)y \\ \sin(a-b)y & \sin ay & \sin(a+b)y \end{vmatrix}$

= 1[cos ay sin (a + b) y – cos (a + b)y sin ay] –x [cos (a – b) y sin (a + b) y – cos (a + b) y sin (a – b)y]

+ x^2 [cos (a – b) y sin ay – sin (a – b) y cos ay]

= sin (by) – x sin (2by) + x^2 sin (b)

$\therefore$ Determinant Δ does not depend on a.

(d) $\because B = A - A^T$

$\Rightarrow B^T = (A - A^T)^T = A^T - A = -(A - A^T)$

$\Rightarrow B^T = B$

$\therefore$ Matrix B is a skew symmetric matrix

$\therefore \quad |B| = 0$

149. **Let A be a square-matrix. The matrix AA^T is always a ________**

(a) Skew – symmetric matrix
(b) symmetric matrix
(c) symmetric & symmetric matrix
(d) Zero matrix

AP EAMCET-05.10.2021, Shift-II

Ans. (b) : Here, A is a square matrix then,

$(AA^T)^T = [(A^T)^T.A^T]$ (By reversal law)

$= AA^T$ $(\because (A^T)^T = A$

$\therefore$ AA^T is symmetric

Hence, option b is correct.

150. **In the set of all 3×3 real matrices a relation is defined as follow. A matrix A is related to a matrix B, if and only if there is a non-singular 3×3 matrix P, such that $B = P^{-1}AP$.**

This relation is

(a) reflexive, symmetric but not transitive
(b) reflexive, transitive but not symmetric
(c) symmetric, transitive but not reflexive
(d) an equivalence relation

WB JEE-2013

Ans. (d) : According to given summation, let the relation defined as

R = (A, B)| B P^{-1}AP

For reflexive A = I^{-1} AI

(A,A)∈ R

R is reflexive

Now, for symmetric:

let (A, B) ∈ R

∵ B = P^{-1} AP

PB = P^{-1} AP

PB = AP

PBP^{-1} = A

(B,A) ∈ R ⇒ is symmetric

For transitive:

let (A, B) ∈ R, (B, C) ∈ R

∵ A = P^{-1} BP and B = Q^{-1}CQ

A = $P^{-1}Q^{-1}$ CAP = $(QP)^{-1}$ C(QP)

(A, C) ∈ R

R is transitive

∵ R is reflexive, symmetric and transitive

So, R is an equivalence relation.

151. **If $I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{bmatrix}$. Then, the matrix p^3+2P^2 is equal to**

(a) P
(b) I– P
(c) 2I + P
(d) 2I – P

WB JEE-2013

Ans. (c) : We have,

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ and } \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{bmatrix}$$

Here,

The characteristic equation of P is

$|P - \pi I| = 0$

$$\begin{bmatrix} 1-\pi & 0 & 0 \\ 0 & -1-\pi & 0 \\ 0 & 0 & -2-\pi \end{bmatrix} = 0$$

$(1 - \pi)\ \{(1 + \pi)(2 + \pi)\} = 0$

$(1 - \pi^2)\ (2 + \pi) = 0$

$2 - 2\pi^2 + \pi - \pi^3 = 0$

$\pi^3 + 2\pi^2 - \pi - 2 = 0$

We know that, caylay Hamilton theorem states that 'Every square matrix satisfy its characteristic equation

$\therefore\quad P^3 + 2P^2 - P - 2I = 0$

$P^3 + 2P^2 = P + 2I$

152. If $P = \begin{bmatrix} \cos\frac{\pi}{4} & -\sin\frac{\pi}{4} \\ \sin\frac{\pi}{4} & \cos\frac{\pi}{4} \end{bmatrix}$ and $X = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}$ Then, P^3X is equation ....

(a) $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$

(b) $\begin{bmatrix} -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}$

(c) $\begin{bmatrix} -1 \\ 0 \end{bmatrix}$

(d) $\begin{bmatrix} -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{bmatrix}$

WB JEE-2013

Ans. (c) : We have,

$$P = \begin{bmatrix} \cos\frac{\pi}{4} & -\sin\frac{\pi}{4} \\ \sin\frac{\pi}{4} & \cos\frac{\pi}{4} \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

$$P = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$

Now, $P^2 = P.P = \frac{1}{2}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$

$$= \frac{1}{2}\begin{bmatrix} 1-1 & -1-1 \\ 1+1 & -1+1 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$$P^3 = P.P^2 = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \times \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$$= \frac{1}{\sqrt{2}}\begin{bmatrix} 0-1 & -1-0 \\ 0+1 & -1+0 \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} -1 & -1 \\ 1 & -1 \end{bmatrix}$$

Also, given $x = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$\therefore\quad P^3 x = \frac{1}{\sqrt{2}}\begin{bmatrix} -1 & -1 \\ 1 & -1 \end{bmatrix} . \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$$= \frac{1}{2}\begin{bmatrix} -1 & -1 \\ 1 & -1 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} -2 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$$

153. If $2A + 3B = \begin{bmatrix} 2 & -1 & 4 \\ 3 & 2 & 5 \end{bmatrix}$ and $A + 2B = \begin{bmatrix} 5 & 0 & 3 \\ 1 & 6 & 2 \end{bmatrix}$, then B is

(a) $\begin{bmatrix} 8 & -1 & 2 \\ -1 & 10 & -1 \end{bmatrix}$

(b) $\begin{bmatrix} 8 & 1 & 2 \\ -1 & 10 & -1 \end{bmatrix}$

(c) $\begin{bmatrix} 8 & 1 & -2 \\ -1 & 10 & -1 \end{bmatrix}$

(d) $\begin{bmatrix} 8 & 1 & 2 \\ 1 & 10 & 1 \end{bmatrix}$

Jamia Millia Islamia-2008

Ans. (b) : Given,

$2A + 3B = \begin{bmatrix} 2 & -1 & 4 \\ 3 & 2 & 5 \end{bmatrix}$ ...(i)

$A + 2B = \begin{bmatrix} 5 & 0 & 3 \\ 1 & 6 & 2 \end{bmatrix}$ …(ii)

Multiply equation (ii) by 2 and subtracting equation (i) from (ii)

We get,

$B = 2\begin{bmatrix} 5 & 0 & 3 \\ 1 & 6 & 2 \end{bmatrix} - \begin{bmatrix} 2 & -1 & 4 \\ 3 & 2 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 8 & 1 & 2 \\ -1 & 10 & -1 \end{bmatrix}$

154. If $\begin{vmatrix} x-4 & 2x & 2x \\ 2x & x-4 & 2x \\ 2x & 2x & x-4 \end{vmatrix} = (A + Bx)\ (x - A)^2$, then the ordered pair (A, B) is equal to

(a) (–4, –5)
(b) (–4, 3)
(c) (–4, 5)
(d) (4, 5)

JEE Main-2018

Ans. (c) :

Let,

$$A = \begin{vmatrix} x-4 & 2x & 2x \\ 2x & x-4 & 2x \\ 2x & 2x & x-4 \end{vmatrix}$$

$C_1 \to C_1 + C_2 + C_3$

$$A = \begin{vmatrix} 5x-4 & 2x & 2x \\ 5x-4 & x-4 & 2x \\ 5x-4 & 2x & x-4 \end{vmatrix}$$

$$= (5x-4)\begin{vmatrix} 1 & 2x & 2x \\ 1 & x-4 & 2x \\ 1 & 2x & x-4 \end{vmatrix}$$

$R_1 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$$= (5x-4)\begin{vmatrix} 1 & 2x & 2x \\ 0 & -x-4 & 0 \\ 0 & 0 & -x-4 \end{vmatrix}$$

Expanding along C_1, we get -

$= (5x-4)[(x+4)(x+4)-0]$

$= (5x-4)(x+4)^2$

$(5x-4)(x+4)^2 = (A+Bx)(x-A)^2$ (Given)

$A = -4$ and $B = 5$

$(A, B) = (-4, 5)$

155. If $\begin{bmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{bmatrix} = (a+b+c)(x+a+b+c)^2$, $x \neq 0$ and $a+b+c \neq 0$, then x is equal to

(a) $-(a+b+c)$ (b) $-2(a+b+c)$
(c) $2(a+b+c)$ (d) abc

JEE Main 11.01.2019, Shift - II

Ans. (b) : $\Delta = \begin{bmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{bmatrix}$

$R_1 \to R_1 + R_2 + R_3$

$$\Delta = \begin{bmatrix} a+b+c & a+b+c & a+b+c \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{bmatrix}$$

$$\Delta = (a+b+c)\begin{bmatrix} 1 & 1 & 1 \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{bmatrix}$$

$C_2 \to C_2 - C_1$

$C_3 \to C_3 - C_1$

$$\Delta = \begin{bmatrix} 1 & 0 & 0 \\ 2b & -(a+b+c) & 0 \\ 2c & 0 & -(a+b+c) \end{bmatrix}(a+b+c)$$

$= (a+b+c)\{(a+b+c)^2 - 0\}$

$= (a+b+c)^3$

$(a+b+c)(x+a+b+c)^2 = (a+b+c)^3$

$(x+a+b+c)^2 = (a+b+c)^2$

$x+a+b+c = \pm(a+b+c)$

$x = -2(a+b+c)$

156. If $\Delta_1 = \begin{bmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{bmatrix}$ and $\Delta_2 = \begin{bmatrix} x & \sin2\theta & \cos2\theta \\ -\sin2\theta & -x & 1 \\ \cos2\theta & 1 & x \end{bmatrix}$ $x \neq 0$, then for all $\theta \in \left(0, \frac{\pi}{2}\right)$

(a) $\Delta_1 + \Delta_2 = -2(x^3 + x - 1)$
(b) $\Delta_1 - \Delta_2 = -2x^3$
(c) $\Delta_1 + \Delta_2 = -2x^3$
(d) $\Delta_1 + \Delta_2 = x(\cos2\theta - \cos4\theta)$

JEE Main 10.04.2019, Shift - I

Ans. (c) : Given,

$$\Delta_1 = \begin{bmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{bmatrix}$$

$= x(-x^2-1) - \sin\theta(-x\sin\theta - \cos\theta)$
$\quad + \cos\theta(-\sin\theta + x\cos\theta)$

$= -x^3 - x + x(\sin^2\theta + \cos^2\theta)$

$= -x^3 - x + x$

$= -x^3$

$$\Delta_2 = \begin{bmatrix} x & \sin2\theta & \cos2\theta \\ -\sin2\theta & -x & 1 \\ \cos2\theta & 1 & x \end{bmatrix}$$

$= x[(-x^2-1)] - \sin2\theta(-x\sin2\theta - \cos2\theta)$
$\quad + \cos2\theta\{-\sin2\theta + x\cos2\theta\}$

$= -x^3 - x + x\sin^2 2\theta + \sin2\theta\cos2\theta$
$\quad - \sin2\theta\cos2\theta + x\cos^2 2\theta$

$= -x^3 = \Delta_1 + \Delta_2 = -x^3 - x^3 = -2x^3$

157. Let $A = \begin{bmatrix} x & y & z \\ y & z & x \\ z & x & y \end{bmatrix}$, where x, y and z are real numbers, such that $x+y+z > 0$ and $xyz = 2$. If $A^2 = I_3$, then the value of $x^3 + y^3 + z^3$ is ....

JEE Main 25.02.2021, Shift - I

Ans. (7) : Given,

$$A = \begin{bmatrix} x & y & z \\ y & z & x \\ z & x & y \end{bmatrix}$$

$$|A| = \begin{bmatrix} x & y & z \\ y & z & x \\ z & x & y \end{bmatrix}$$

$= x(yz - x^2) - y(y^2 - zx) + z(xy - z^2)$

$= -(x^3 + y^3 + z^3 - 3xyz)$

$A^2 = I_3$ (Given)

$|A^2| = |I_3| = 1$

$|A^2| = 1$

$\{-(x^3 + y^3 + z^3 - 3xyz)\}^2 = 1$

$x^3 + y^3 + z^3 - 3xyz = 1$

$x^3 + y^3 + z^3 = 1 + 3xyz$

$= 1 + 3(2)$ $(\because xyz = 2)$

$= 7$

158. Let A be a 2 × 2 real matrix with entries from {0, 1} and $|A| \neq 0$. Consider the following two statements
(P) If $A \neq I_2$, then $|A| = -1$
(Q) If $|A| = 1$, then tr(A) = 2
where I_2 denotes 2 × 2 identity matrix and tr(A) denotes the sum of the diagonal entries of (a) Then,

(a) (P) is false and (Q) is true
(b) Both (P) and (Q) are false
(c) (P) is true and (Q) is false
(d) Both (P) and (Q) are true

JEE Main 02.09.2020, Shift - I

Ans. (a) : Given, A be a 2×2 real matrix with entries from {0, 1} and $|A| \neq 0$, then following matrices are possible cases.

(i) $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, here $|A| = 1$

(ii) $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ or $\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$, here $|A| = 1$

and tr (A) = 2

(iii) $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, here $A \neq I_2$ and $|A| = -1$

(iv) $A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$ or $\begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$, here $A \neq I_2$

and $|A| = -1$

So, the given statements,

(P) It $A \neq I_2$, then $|A| = -1$ is prove as cases (iii) and (iv), but in case (ii) $A \neq I_2$ and $|A|k = 1$

(Q) it $|A| = 1$, then tr (A) = 2, is true as cases (i)f and (ii) .

159. Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $B = \begin{bmatrix} a \\ b \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ such that AB = B and a + d = 2021, then the value of ad – bc is equal to ....

JEE Main 17.03.2021, Shift - II

Ans. (2020) : Given,

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \text{ and } B = \begin{bmatrix} a \\ b \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

i.e. $B \neq 0$ and $AB = B$

$|AB - B| = 0$

$|B(A - I)| = 0$

$\because$ $|B \neq 0$

$\therefore$ $|A - I| = 0$

$$\begin{vmatrix} a-1 & b \\ c & d-1 \end{vmatrix} = 0$$

$(a - 1)(d - 1) - bc = 0$

$ad - a - d + 1 - bc = 0$

$ad - (a + d) + 1 - bc = 0$

$ad - 2021 + 1 - bc = 0$

$ad - bc = 2020$

160. Let $A = \begin{bmatrix} 2 & 3 \\ a & 0 \end{bmatrix}$, $a \in R$ be written as P + Q, where P is a symmetric matrix and Q is skew symmetric matrix . If det (Q) = 9, then the modulus of the sum of all possible values of determinant of P is equal to

(a) 36
(b) 24
(c) 45
(d) 18

JEE Main 20.07.2021, Shift - I

Ans. (a) : Given,

$$A = \begin{bmatrix} 2 & 3 \\ a & 0 \end{bmatrix}, A^T = \begin{bmatrix} 2 & a \\ 3 & 0 \end{bmatrix}$$

$$A = \frac{A + A^T}{2} + \frac{A - A^T}{2}$$

Let, $P = \dfrac{A + A^T}{2}$ and $Q = \dfrac{A - A^T}{2}$

$$Q = \begin{bmatrix} \frac{2-2}{2} & \frac{3-a}{2} \\ \frac{a-3}{2} & \frac{0-0}{2} \end{bmatrix} = \begin{bmatrix} 0 & \frac{3-a}{2} \\ \frac{a-3}{2} & 0 \end{bmatrix}$$

Det (Q) = 9 (Given)

$$0 - \left(\frac{3-a}{2}\right)\left(\frac{a-3}{2}\right) = 9$$

$(a - 3)^2 = 9 \times 4$

$a - 3 = \pm 6$

$a = 9, -3$

$$P = \begin{bmatrix} 2 & \frac{a+3}{2} \\ \frac{a+3}{2} & 0 \end{bmatrix} \text{ or } \begin{bmatrix} 2 & \frac{-3+3}{2} \\ \frac{-3+3}{2} & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & \frac{9+3}{2} \\ \frac{9+3}{2} & 0 \end{bmatrix} \text{ or } \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix} \quad [\because a = -3, 9]$$

$$= \begin{bmatrix} 2 & 6 \\ 6 & 0 \end{bmatrix} \text{ or } \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}$$

$|P| = -36$ or 0

$|-36 + 0| = 36$

161. If $A = \begin{bmatrix} a & b \\ b & a \end{bmatrix}$ and $A^2 = \begin{bmatrix} \alpha & \beta \\ \beta & \alpha \end{bmatrix}$, then

(a) $\alpha = a^2 + b^2$ and $\beta = ab$
(b) $\alpha = a^2 + b^2$ and $\beta = 2ab$
(c) $\alpha = a^2 + b^2$ and $\beta = a^2 - b^2$
(d) $\alpha = 2ab$ and $\beta = a^2 + b^2$

AIEEE-2003

Ans. (b) : Given that,

$$A = \begin{bmatrix} a & b \\ b & a \end{bmatrix} \text{ and } A^2 = \begin{bmatrix} \alpha & \beta \\ \beta & \alpha \end{bmatrix}$$

$$A \cdot A = A^2 = \begin{bmatrix} a & b \\ b & a \end{bmatrix}\begin{bmatrix} a & b \\ b & a \end{bmatrix}$$

$$= \begin{bmatrix} a^2 + b^2 & ab + ba \\ ab + ba & b^2 + a^2 \end{bmatrix} = \begin{bmatrix} \alpha & \beta \\ \beta & \alpha \end{bmatrix}$$

So, $\alpha = a^2 + b^2$ and $\beta = 2ab$

162. Let $A = \begin{pmatrix} 0 & 2q & r \\ p & q & -r \\ p & -q & r \end{pmatrix}$. If $AA^T = I_3$, then |p| is

(a) $\dfrac{1}{\sqrt{5}}$
(b) $\dfrac{1}{\sqrt{2}}$
(c) $\dfrac{1}{\sqrt{3}}$
(d) $\dfrac{1}{\sqrt{6}}$

JEE Main 11.01.2019, Shift - I

Ans. (b) : Given that,

$AA^T = I_3$

$$\begin{bmatrix} 0 & 2q & r \\ p & q & -r \\ p & -q & r \end{bmatrix}\begin{bmatrix} 0 & p & p \\ 2q & q & -q \\ r & -r & r \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0+4q^2+r^2 & 0+2q^2-r^2 & 0-2q^2+r^2 \\ 0+2q^2-r^2 & p^2+q^2+r^2 & p^2-q^2-r^2 \\ 0-2q^2+r^2 & p^2-q^2-r^2 & p^2+q^2+r^2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

We know that, if two matrices are equal then corresponding elements are also equal, so

$4q^2 + r^2 = 1 = p^2 + q^2 + r^2$ ...(i)

$2q^2 - r^2 = 0,\ r^2 = 2q^2$ ...(ii)

And, $p^2 - q^2 - r^2 = 0$ ...(iii)

Using equation (i) + (iii), we get –

$$p^2 = \frac{1}{2}$$

$$|p| = \frac{1}{\sqrt{2}}$$

163. Let P = $\begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 9 & 3 & 1 \end{bmatrix}$ and Q = $[q_{ij}]$ be two 3 × 3 matrices such that $Q - P^5 = I_3$. Then $\dfrac{q_{21}+q_{31}}{q_{32}}$ is equal to

(a) 10 (b) 135

(c) 9 (d) 15

JEE Main 12.01.2019, Shift- I

Ans. (a) : Given that,

$$P = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 9 & 3 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 3 & 0 & 0 \\ 9 & 3 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Let, $P = X + I$

$P^5 = (I + X)^5$

$= I + {}^5C_1(X) + {}^5C_2(X^2) + {}^5C_3(X^3) + \ldots.. {}^5C_5(X^5)$

$$\left[\because I^n = I, I\cdot A = A \text{ and } (a+X)^n = {}^nC_0\, a^n + {}^nC_1\ a^{n-1}X + \ldots\ldots + {}^nC_n\ r^n\right]$$

Here, $$X^2 = \begin{bmatrix} 0 & 0 & 0 \\ 3 & 0 & 0 \\ 9 & 3 & 0 \end{bmatrix}\begin{bmatrix} 0 & 0 & 0 \\ 3 & 0 & 0 \\ 9 & 3 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 9 & 0 & 0 \end{bmatrix}$$

And, $X^3 = X^2 \cdot X =$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 9 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 0 & 0 \\ 3 & 0 & 0 \\ 9 & 3 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$X^4 = X^5 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

So, $$P^5 = I + 5\begin{bmatrix} 0 & 0 & 0 \\ 3 & 0 & 0 \\ 9 & 3 & 0 \end{bmatrix} + 10\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 9 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 15 & 1 & 0 \\ 135 & 15 & 1 \end{bmatrix}$$

And, $$Q = I + P^5 = \begin{bmatrix} 2 & 0 & 0 \\ 15 & 2 & 0 \\ 135 & 15 & 2 \end{bmatrix} = \left[q_{ij}\right]$$

$Q_{21} = 15$, $Q_{31} = 135$ and $Q_{32} = 15$

So, $$\frac{Q_{21}+Q_{31}}{Q_{32}} = \frac{15+135}{15} = \frac{150}{15} = 10$$

164. If A an 3×3 non-singular matrix such that AA' = A'A and B = A^{-1}A'. then BB' is equal to

(a) I + B (b) I

(c) B^{-1} (d) (B)'

JEE Main-2014

Ans. (b) : Given,

If A an 3 × 3 non-singular matrix such that AA' = A'A and B = $A^{-1}A'$

If A is non-singular matrix, then $|A| \neq 0$

Given, $AA^T = A^TA$ and $B = A^{-1}A^T$

$BB' = (A^{-1}A^T)(A^{-1}A^T)$

$= A^{-1}A^T A(A^{-1})^T$ $[\because (A^T) = A]$

$= A^{-1}AA^T (A^{-1})^T$ $[\because AA^T = A^TA]$

$= A^T(A^{-1})^T = A^T(A^{-1})^T$

$= (A^{-1}A)^T = I^T = I$ $[\because [AB]^T = B^TA^T]$

165. If $\omega \neq 1$ is the complex cube root of unity and matrix H = $\begin{bmatrix} \omega & 0 \\ 0 & \omega \end{bmatrix}$, then H^{70} is equal to

(a) H (b) 0

(c) –H (d) H^2

AIEEE-2011

Ans. (a) : Given that, $\omega \neq 1$ is complex cube root of unity and matrix H = $\begin{bmatrix} \omega & 0 \\ 0 & \omega \end{bmatrix}$

$$H = \begin{bmatrix} \omega & 0 \\ 0 & \omega \end{bmatrix}$$

$H = \omega^2$

$H^{70} = (\omega^2)^{70}$

$H^{70} = \omega^{140}$

As we know that, if ω is a cube root of unity, then

$\therefore$ $H_{70} = \omega^{140} = \omega^{(3\times46)+2}$

$H_{70} = \omega^{3\times46} \cdot \omega^2$

$H_{70} = 1 \cdot \omega^2$ $[H = \omega^2]$

$\therefore$ $H_{70} = H$

166. **Let $A = \begin{pmatrix} \cos a & -\sin a \\ \sin a & \cos a \end{pmatrix}$, $(\alpha \in R)$ such that $A^{32} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$. Then the value of α is**

(a) $\frac{\pi}{32}$ (b) 0

(c) $\frac{\pi}{64}$ (d) $\frac{\pi}{16}$

JEE Main 08.04.2019, Shift - I

Ans. (c) : Given,

$$A = \begin{pmatrix} \cos a & -\sin a \\ \sin a & \cos a \end{pmatrix}$$

$$A^2 = \begin{bmatrix} \cos a & -\sin a \\ \sin a & \cos a \end{bmatrix}\begin{bmatrix} \cos a & -\sin a \\ \sin a & \cos a \end{bmatrix}$$

$$= \begin{bmatrix} \cos 2a & -\sin 2a \\ \sin 2a & \cos 2a \end{bmatrix}$$

$$A^3 = \begin{bmatrix} \cos 2a & -\sin 2a \\ \sin 2a & \cos 2a \end{bmatrix}\begin{bmatrix} \cos a & -\sin a \\ \sin a & \cos a \end{bmatrix}$$

$$= \begin{bmatrix} \cos 3a & -\sin 3a \\ \sin 3a & \cos 3a \end{bmatrix}$$

Similarly,

$$A^{32} = \begin{bmatrix} \cos 32a & -\sin 32a \\ \sin 32a & \cos 32a \end{bmatrix}$$

According to question,

$$A^{32} = \begin{vmatrix} 0 & -1 \\ 1 & 0 \end{vmatrix}$$

$$\begin{bmatrix} \cos 32a & -\sin 32a \\ \sin 32a & \cos 32a \end{bmatrix} = \begin{vmatrix} 0 & -1 \\ 1 & 0 \end{vmatrix}$$

$\cos 32a = 0$ and $-\sin 32a = -1$

$$32a = \frac{\pi}{2}\ , n \in I$$

$$a = \frac{\pi}{64}$$

167. **The total number of matrices $A = \begin{bmatrix} 0 & 2y & 1 \\ 2x & y & -1 \\ 2x & -y & 1 \end{bmatrix}, (x, y \in R, x \neq y)$ for which $A^T A = 3I_3$ is**

(a) 2 (b) 4

(c) 3 (d) 6

JEE Main 09.04.2019, Shift-II

Ans. (b) : We have,

$$A = \begin{bmatrix} 0 & 2y & 1 \\ 2x & y & -1 \\ 2x & -y & 1 \end{bmatrix}$$

Now, $A^T = \begin{bmatrix} 0 & 2x & 2x \\ 2x & y & -y \\ 1 & -1 & 1 \end{bmatrix}$

Now given condition, $A^T A = 3I_3$

$$\begin{bmatrix} 0 & 2x & 2x \\ 2y & y & -y \\ 1 & -1 & 1 \end{bmatrix}\begin{bmatrix} 0 & 2y & 1 \\ 2x & y & -1 \\ 2x & -y & 1 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 8x^2 & 0 & 0 \\ 0 & 6y^2 & 0 \\ 0 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$8x^2 = 3,\ \ x = \pm\sqrt{3/8}$

$6y^2 = 3\ ,\ \ y = \pm\sqrt{1/2}$

168. **If $A = \begin{bmatrix} \cos\theta & i\sin\theta \\ i\sin\theta & \cos\theta \end{bmatrix}$, $\left(\theta = \frac{\pi}{24}\right)$ and $A^5 = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, where $i = \sqrt{-1}$, them which one of the following is not true?**

(a) $a^2 - d^2 = 0$ (b) $a^2 - c^2 = 1$

(c) $a^2 - b^2 = \frac{1}{2}$ (d) $0 \le a^2 + b^2 \le 1$

JEE Main 04.09.2020 Shift - I

Ans. (c) : $A = \begin{bmatrix} \cos\theta & i\sin\theta \\ i\sin\theta & \cos\theta \end{bmatrix}$

Now, $A^2 = \begin{bmatrix} \cos\theta & i\sin\theta \\ i\sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} \cos\theta & i\sin\theta \\ i\sin\theta & \cos\theta \end{bmatrix}$

$$= \begin{bmatrix} \cos^2\theta - \sin^2\theta & 2i\sin\theta\cos\theta \\ 2i\sin\theta\cos\theta & \cos^2\theta - \sin^2\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos 2\theta & i\sin 2\theta \\ i\sin 2\theta & \cos 2\theta \end{bmatrix}$$

Similarly,

$$A^5 = \begin{bmatrix} \cos 5\theta & i\sin 5\theta \\ i\sin 5\theta & \cos 5\theta \end{bmatrix}$$

$$A^5 = \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} \cos 5\theta & i\sin 5\theta \\ i\sin 5\theta & \cos 5\theta \end{bmatrix}$$

(i) $a^2 - d^2 = \cos^2 5\theta - \cos^2 5\theta = 0$

(ii) $a^2 - c^2 = \cos^2 5\theta + \sin^2 5\theta = 1$

(iii) $a^2 - b^2 = \cos^2 5\theta + \sin^2 5\theta = 1 \neq 1/2$

Which is not true.

169. **Let α be a root of the equation $x^2 + x + 1 = 0$ and the matrix $A = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha^4 \end{bmatrix}$, then the matrix A^{31} is equal to**

(a) A^3 (b) I_3

(c) A^2 (d) A

JEE Main 07.01.2020, Shift- I

Ans. (a) : We have equation $x^2 + x + 1 = 0$

$$\alpha = \frac{-1 \pm \sqrt{3}}{2}$$

$$\omega = \frac{-1+\sqrt{3}}{2} \text{ and } \omega^2 = \frac{-1-\sqrt{3}}{2}$$

Matrix, $A = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \omega^4 \end{bmatrix}$

Putting, $\alpha = \omega$

$$A = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega^4 \end{bmatrix}$$

We know that,

$\omega^3 = 1$

$$A = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{bmatrix}$$

Now, $A^2 = \left(\frac{1}{\sqrt{3}}\right)^2 \begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{bmatrix}$

$$A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$A^4 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$A^4 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Now, $= A^{31} = A^{28}.\ A^3$

$= (A^4)^7 A^3 = I\ A^3$

$A^{31} = A^3$

170. The number of all 3 × 3 matrices A, with entries from the set {–1, 0, 1} such that the sum of the diagonal elements of AA^T is 3, is ....

JEE Main 08.01.2020, Shift - I

Ans. (672) : Let A =

$$AA^T = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}, A^T = \begin{bmatrix} a & b & g \\ d & e & h \\ c & f & i \end{bmatrix}$$

$$= \begin{bmatrix} a^2+b^2+c^2 & - & - & - \\ - & a^2+e^2+f^2 & - \\ - & -g^2+h^2+i^2 \end{bmatrix}$$

$= a^2 + b^2 + c^2 + d^2 + e^2 + f^2 + g^2 + h^2 + i^2 = 3$

$9c_3 \times 1 = 9c_3$

$9c_3 \times 1 = 9c_3$

$9c_3 \times \frac{3!}{2!} = 9c_3(3)$

$9c_3 \times \frac{3!}{2!} = 9c_3(3)$

Sum of diagonal element

$9c_3 + 9c_3 + 3\ 9c_3 + 3\ 9c_3$

$\Rightarrow \quad 8\ .\ 9c_3$

$= \quad \frac{9!}{3!6!} \times 8$

$= \quad \frac{9\times 8\times 7}{3\times 2} \times 8$

$= \quad 672$

171. Let A be a symmetric matrix of order 2 with integer entries. If the sum of the diagonal elements of A^2 is 1, then the possible number of such matrices is

(a) 4 (b) 1

(c) 6 (d) 12

JEE Main 26.02.2021, Shift - I

Ans. (a) : Let $A = \begin{bmatrix} a & b \\ b & c \end{bmatrix}$

$$A^2 = \begin{bmatrix} a & b \\ b & c \end{bmatrix}\begin{bmatrix} a & b \\ b & c \end{bmatrix}$$

$$= \begin{bmatrix} a^2+b^2 & b(a+c) \\ b(a+c) & b^2+c^2 \end{bmatrix}$$

Now sum of diagonal element

$A^2 = a^2 + 2b^2 + c^2 = 1$

a = 1 then b = 0 and c = 0

a = 0 then b = 0, c = 1

a = –1 b = 0 and c = 0

a = 0 , b = 0 c = –1

Hence the possible number of matrix is 4

172. If for the matrix, $A = \begin{bmatrix} 1 & -\alpha \\ \alpha & \beta \end{bmatrix}$, $AA^T = I_2$, then the value of $\alpha^4 + \beta^4$ is

(a) 4 (b) 1

(c) 2 (d) 3

JEE Main 25.02.2021, Shift - II

Ans. (b) : $A = \begin{bmatrix} 1 & -\alpha \\ \alpha & \beta \end{bmatrix}$

$$A^T = \begin{bmatrix} 1 & +\alpha \\ -\alpha & \beta \end{bmatrix}$$

$$AA^T = \begin{bmatrix} 1 & -\alpha \\ \alpha & \beta \end{bmatrix}\begin{bmatrix} 1 & \alpha \\ -\alpha & \beta \end{bmatrix}$$

$$= \begin{bmatrix} 1+\alpha^2 & \alpha-\alpha\beta \\ \alpha-\alpha\beta & \alpha^2+\beta^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

By comparing $1 + \alpha^2 = 1$

$\alpha^2 = 0 \qquad \alpha = 0$

And $\alpha^2 + \beta^2 = 1$

$\beta^2 = 1$

$\beta = \pm 1$

Then $\alpha^4 + \beta^4 = 0 + (1)^4$

$= 1$

173. Let M be any 3 × 3 matrix with entries from the set {0, 1, 2}. The maximum number of such matrices, for which the sum of diagonal elements of M^T M is seven, is ....

JEE Main 24.02.2021, Shift - I

Ans. (540) : Given,
M is 3 × 3 matrix,

$$M = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

$$M^T = \begin{bmatrix} a & d & g \\ b & e & h \\ c & f & i \end{bmatrix}$$

Now, $M^TM = \begin{bmatrix} a & d & g \\ b & e & h \\ c & f & i \end{bmatrix}\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$

It is given that,
Sum of diagonal element of $M^TM = 7$
$\therefore\ a^2 + b^2 + c^2 + d^2 + e^2 + f^2 + g^2 + h^2 + i^2 = 7$
Case-I: Sum (1's) and two (0's)
$^9C_2 = 36$
Similarly,
Case-II: One (2's) and three (1's) and five (0's)

$$\frac{9!}{5!\,3!} = 504$$

Hence, the total number of such matrices = 540

174. Let a, b, c ∈ R be all non - zero and satisfy $a^3 + b^3 + c^3 = 2$. If the matrix A = $\begin{pmatrix} a & b & c \\ b & c & a \\ c & a & b \end{pmatrix}$ satisfies $A^TA = I$, then a value of abc can be

(a) $-\frac{1}{3}$ (b) $\frac{1}{3}$
(c) 3 (d) $\frac{2}{3}$

JEE Main 02.09.2020, Shift - II

Ans. (b) : Given,
$A^TA = I$
$a^2 + b^2 + c^2 = 1$
And, $ab + bc + ca = 0$
Now,
$(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$
$(a + b + c)^2 = 1 + 0$
$a + b + c = \pm 1$
So,
$a^3 + b^3 + c^3 - 3abc = (a+b+c)\ (a^2 + b^2 + c^2 - ab - bc - ca)$
$= 1(1 - 0)$
$a^3 + b^3 + c^3 - 3abc = 1$
$2 - 3abc = 1$
$abc = \frac{1}{3}$

175. The total number of 3 × 3 matrices A having entries from the set (0, 1, 2, 3), such that the sum of all the diagonal entries of AA^T is 9, is equal to ....

JEE Main 16.03.2021, Shift - I

Ans. (766) : Given,
$A.A^T = 9$
Set (0, 1, 2, 3)

Let, $A = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$

$$A^T = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$$

Now,

$$A.A^T = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$$

Sum of diagonal element = 9
$a_1^2 + b_1^2 + c_1^2 + a_2^2 + b_2^2 + c_2^2 + a_3^2 + b_3^2 + c_3^2 = 9$
$9 = (1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1)$
Or, $9 = (1 + 4 + 4 + 0 + 0 + 0 + 0 + 0 + 0)$
Or, $9 = (9 + 0 + 0 + 0 + 0 + 0 + 0 + 0 + 0)$
Or, $9 = (4 + 1 + 1 + 1 + 1 + 1 + 0 + 0 + 0)$
Total permutation in case-I = 1
Total permutation s in case-II = $\frac{9!}{6!2!} = 252$
In case III = $\frac{9!}{8!} = 9$
In case IV = $\frac{9!}{5!3!} = 504$
Now, Total permutations = 1 + 252 + 9 + 504 = 766

176. The number of elements in the sets

$$\left\{ \mathbf{A} = \begin{pmatrix} \mathbf{a} & \mathbf{b} \\ \mathbf{0} & \mathbf{d} \end{pmatrix} : \mathbf{a, b\ and\ d} \in \{-1, 0, 1\} \ \mathbf{and}\ (\mathbf{I} - \mathbf{A})^3 = \mathbf{I} - \mathbf{A}^3 \right\}$$

where I is 2 × 2 identity matrix, is

JEE Main 31.08.2021, Shift - II

Ans. (8) : We have,
$(I - A)^3 = I - A^3$
$I^3 - A^3 - 3A + 3A^2 = I - A^3$
$3A^2 - 3A = 0$
$3A(A - 1) = 0$
$A^2 = A$
For, $A^2 = A$

$$\begin{bmatrix} a & b \\ 0 & d \end{bmatrix}\begin{bmatrix} a & b \\ 0 & d \end{bmatrix} = \begin{bmatrix} a & b \\ 0 & d \end{bmatrix}$$

$$\begin{bmatrix} a^2 & ab + bd \\ 0 & d^2 \end{bmatrix} = \begin{bmatrix} a & b \\ 0 & d \end{bmatrix}$$

Comparing both the equality,
$a^2 = a$
$a = 0, 1$

$d^2 = d$
$d = 0, 1$
$b(a + d) = b$
$a + d = 1$
If $b = 0 \Rightarrow (a, d) = (1, 0), (0, 0), (1, 1), (0, 1)$
If $a + d = 1 \Rightarrow (1, 0), (0, 1)$ and $b = \pm 1$
Total number of elements = 4 + 4 = 8

177. If the matrix $A = \begin{pmatrix} 0 & 2 \\ K & -1 \end{pmatrix}$ satisfies $A(A^3 + 3I) = 2I$, then the value of K is

(a) $\frac{1}{2}$ (b) $-\frac{1}{2}$
(c) –1 (d) 1

JEE Main 27.08.2021, Shift - I

Ans. (a) : Given,

$$\text{matrix } A = \begin{bmatrix} 0 & 2 \\ K & -1 \end{bmatrix}$$

Characteristic equation of A is

$|A - \lambda I| = 0$

$$\begin{vmatrix} -\lambda & 2 \\ K & -1-\lambda \end{vmatrix} = 0$$

$\lambda(1 + \lambda) - 2K = 0$
$\lambda^2 + \lambda - 2K = 0$

$\because$ Every square matrix satisfied its own characteristic equation,

$\therefore$ $A^2 + A - 2KI = 0$
$A^2 = 2KI - A$
$A^4 = 4K^2 I + A^2 - 2(2KI)(A)$
$A^4 = 4K^2 I + 2KI - A - 4KA$
$A^4 = (4K^2 + 2K)I - (1 + 4K)A$ .....(i)

Now, $A(A^3 + 3I) = 2I$
$A^4 = 2I - 3A$ ....(ii)

Comparing equation (i) and (ii), we get –
$1 + 4K = 3$
$K = \frac{1}{2}$

178. Let $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix}$. Then $A^{2025} - A^{2020}$ is equal to

(a) $A^6 - A$ (b) A^5
(c) $A^5 - A$ (d) A^6

JEE Main 26.08.2021, Shift - II

Ans. (a) : Given,

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

Now,

$$A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$A^4 = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$A^n = \begin{bmatrix} 1 & 0 & 0 \\ n-2 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ n-1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$A^{2025} - A^{2020} = \begin{bmatrix} 1 & 0 & 0 \\ 2024 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 2019 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 5 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Now,

$$A^6 - A = \begin{bmatrix} 1 & 0 & 0 \\ 5 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 5 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$\therefore$ $A^{2025} - A^{2020} = A^6 - A$

179 If $P = \begin{bmatrix} 1 & 0 \\ 1/2 & 1 \end{bmatrix}$, then P^{50} is

(a) $\begin{bmatrix} 1 & 0 \\ 25 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 50 \\ 0 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 25 \\ 0 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 \\ 50 & 1 \end{bmatrix}$

JEE Main 25.07.2021, Shift - II

Ans. (a) : Given,

$$P = \begin{bmatrix} 1 & 0 \\ \frac{1}{2} & 1 \end{bmatrix}$$

$$P^2 = \begin{bmatrix} 1 & 0 \\ \frac{1}{2} & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ \frac{1}{2} & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$

$$P^3 = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ \frac{1}{2} & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{3}{2} & 1 \end{bmatrix}$$

$$P^n = \begin{bmatrix} 1 & 0 \\ \frac{n}{2} & 1 \end{bmatrix}$$

Here, $P^{50} = \begin{bmatrix} 1 & 0 \\ \frac{50}{2} & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 25 & 1 \end{bmatrix}$

180. Let A + 2B = $\begin{bmatrix} 1 & 2 & 0 \\ 6 & -3 & 3 \\ -5 & 3 & 1 \end{bmatrix}$ and 2A – B = $\begin{bmatrix} 2 & -1 & 5 \\ 2 & -1 & 6 \\ 0 & 1 & 2 \end{bmatrix}$. If Tr(A) denotes the sum of all diagonal elements of the matrix A, then Tr(A) – Tr(B) has value equal to

JEE Main 18.03.2021, Shift - I

Ans. (2) : Given,

$$A + 2B = \begin{bmatrix} 1 & 2 & 0 \\ 6 & -3 & 3 \\ -5 & 3 & 1 \end{bmatrix} \quad .....(i)$$

And,

$$2A - B = \begin{bmatrix} 2 & -1 & 5 \\ 2 & -1 & 6 \\ 0 & 1 & 2 \end{bmatrix} \quad ....(ii)$$

Equation (ii) × 2, we get –

$$4A - 2B = \begin{bmatrix} 4 & -2 & 10 \\ 4 & -2 & 12 \\ 0 & 2 & 4 \end{bmatrix} \quad .....(iii)$$

Adding equation (i) and (iii),

$$5A = \begin{bmatrix} 5 & 0 & 10 \\ 10 & -5 & 15 \\ -5 & 5 & 5 \end{bmatrix}$$

$$\therefore \quad A = \begin{bmatrix} 1 & 0 & 2 \\ 2 & -1 & 3 \\ -1 & 1 & 1 \end{bmatrix}$$

$\therefore$ Tr(A) = 1 – 1 + 1 = 1

From equation (i)

$$B = \frac{1}{2}\left\{\begin{bmatrix} 1 & 2 & 0 \\ 6 & -3 & 3 \\ -5 & 3 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 2 \\ 2 & -1 & 3 \\ -1 & 1 & 1 \end{bmatrix}\right\}$$

$$= \begin{bmatrix} 0 & 1 & -1 \\ 2 & -1 & 0 \\ -2 & 1 & 0 \end{bmatrix}$$

$\therefore$ Tr(B) = 0 – 1 + 0 = –1

Hence, Tr (A) – Tr(B) = 1 – (– 1) = 2.

181. Let $A = \begin{bmatrix} b^2+c^2 & a^2 & a^2 \\ b^2 & c^2+a^2 & b^2 \\ c^2 & c^2 & a^2+b^2 \end{bmatrix}$, If a = sin π/6, b = cos π/4 and c = cot π/2 then A is

(a) Symmetric matrix
(b) Skew-Symmetric matrix
(c) Singular matrix
(d) Non-singular matrix

AP EAMCET-07.07.2022, Shift-II

Ans. (c) : Let Matrix,

$$\begin{bmatrix} b^2+c^2 & a^2 & a^2 \\ b^2 & c^2+a^2 & b^2 \\ c^2 & c^2 & a^2+b^2 \end{bmatrix}$$

Given,

a = sin π/6 = 1/2

b = cos π/4 = $\frac{1}{\sqrt{2}}$

c = $\cot \pi/2 = 0$

Then,

$$A = \begin{bmatrix} 0+\frac{1}{2} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{2} & 1/4 & 1/2 \\ 0 & 0 & 1/4+1/2 \end{bmatrix}$$

$$= \begin{vmatrix} \frac{1}{2} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{2} & \frac{1}{4} & \frac{1}{2} \\ 0 & 0 & \frac{3}{4} \end{vmatrix}$$

$$|A| = \frac{3}{4}\left[\frac{1}{2}\times\frac{1}{4} - \frac{1}{2}\times\frac{1}{4}\right] = 0$$

|A| = 0

It is a singular matrix.

182. Suppose A and B are two square matrices of same order. If A and B are symmetric matrix, then AB – BA is

(a) a symmetric matrix (b) a skew-symmetric
(c) a scalar matrix (d) a triangular matrix

TS EAMCET-2016

Ans. (b) : Given,

∵ A & B symmetric matrix, then]
A' = A and B' = B

Now, Let p = AB – BA
p' = (AB – BA)'
= (AB)' – (BA)'
= B'A' – A'B' [∵(AB)' = B' A']
= BA – AB
= –(AB – BA) [∵A' = A, B' = B]
= –P

Hence, P is a skew - symmetric material as p' = –p,

183. Let $\begin{bmatrix} x^2+x+1 & x+1 & 2x-3 \\ 3x^2-1 & x+2 & x-1 \\ x^2+5x+1 & 2x+3 & x+4 \end{bmatrix} = ax^4 + bx^3 + cx^2 + dx + e$ be an identity in x. If a, b, c, d are known, then the value of e is

(a) 29 (b) 24
(c) 16 (d) 9

TS EAMCET-2015

Ans. (a) : Given,

$$\begin{vmatrix} x^2+x+1 & x+1 & 2x-3 \\ 3x^2-1 & x+2 & x-1 \\ x^2+5x+1 & 2x+3 & x+4 \end{vmatrix} = ax^4+bx^3+cx^2+dx+e$$

Hence, on expanding the determinate we get an identify, it will satisfy along for x = 0

Put x = 0 on both sides, we get-

$$\begin{vmatrix} 1 & 1 & -3 \\ -1 & 2 & -1 \\ 1 & 3 & 4 \end{vmatrix} = e$$

$e = 1(8+3) - 1(-4+1) - 3(-3-2)$

$e = 11 + 3 + 15 = 29$

184. Let A, B be two 3 × 3 matrices and C be a 3 × 3 unit matrix such that AB – C is a non-singular matrix. Let $D = (AB - C)^{-1}$. Then, consider the following statements.

Statement I : det (BA) = det (BA – C) det (BDA)

Statement II : ABD = DAB

Which of the above statement is (are) true ?

(a) Statement I is true, but statement II is false

(b) Statement II is true, but statement I is false

(c) Both statement I and statement II are true

(d) Both statement I and Statement II are false

TS EAMCET-19.07.2022, Shift-II

Ans. (c) : A and B are square matrices of order.

C is a unit matrix of orders.

$D = (AB - C)^{-1}$

O and AB – C are inverse of each other,

$D \times (AB - C) = I$

$|D| \times |AB - C| = 1$

$|AB - C| = \dfrac{1}{|D|} \Rightarrow |D| = \dfrac{1}{|AB - C|}$

Now, $|BA - C| \times |BDA| = \dfrac{1}{|D|} \times |BDA|$

$= \dfrac{|B| \times |D| \times |A|}{|D|}$

$= |B| \times |A| = |BA|$

If D and AB – C are inverse of each other

$(AB - C) \times D = D \times (AB - C) = I$

$ABD - CD = DAB - CD = I$

$ABD = DAB = CD + I$

$ABD = DAB$

Hence, both statement are true.

185. If $\begin{bmatrix} x^2+3x & x+1 & x-3 \\ x-1 & 2-x & x+4 \\ x-3 & x-3 & 3x \end{bmatrix} = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4$, then $(a_1 + a_3) + 2(a_0 + a_2 + a_4) =$

(a) –1
(b) 0
(c) 1
(d) –29

TS EAMCET-03.05.2019, Shift-I

Ans. (a) : Given,

$$\begin{vmatrix} x^2+3x & x+1 & x-3 \\ x-1 & 2-x & x+4 \\ x-3 & x-3 & 3x \end{vmatrix} = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4$$

Put x = 1 both sides,

$$\begin{vmatrix} 4 & 2 & -2 \\ 0 & 1 & 5 \\ -2 & -2 & 3 \end{vmatrix} = a_0 + a_1 + a_2 + a_3 + a_4$$

$4(3+10) - 2(0+10) - 2(0+2) = a_0 + a_1 + a_2 + a_3 + a_4$

$52 - 20 - 4 = a_0 + a_1 + a_2 + a_3 + a_4$

$a_0 + a_1 + a_2 + a_3 + a_4 = 28$ ....(i)

Put x = –1, both sides, we get-

$$\begin{vmatrix} -2 & 0 & -4 \\ -2 & 3 & 3 \\ -4 & -4 & -3 \end{vmatrix} = a_0 - a_1 + a_2 - a_3 + a_4$$

$-2(-9+12) - 0 + (-4)(8+12) = a_0 - a_1 + a_2 - a_3 + a_4$

$-6 - 80 = a_0 - a_1 + a_2 - a_3 + a_4$

$a_0 - a_1 + a_2 - a_3 + a_4 = -86$ ....(ii)

On adding equation (i) and (ii), we get-

$2(a_0 + a_2 + a_4) = -58$

On subtracting equation (ii) and (i), we get-

$2(a_1 + a_3) = 114$

$a_1 + a_3 = 57$

Then, $(a_1 + a_3) + 2(a_0 + a_2 + a_4) = 57 + (-58) = -1$

186. A is a singular matrix of order five. B is another matrix having the rank ρ(B) equal to the rank ρ(A) and B has a non-zero minor of order 3. Then which one of the following is true?

(a) B is a 4 × 4 matrix

(b) ρ(A) = ρ(B) = 4, irrespective of the order of B

(c) ρ(A) = ρ(B) = 3, when all the fourth order minors of A are zero

(d) |B| = 0

TS EAMCET-11.09.2020, Shift-II

Ans. (c) : Given that, Rank of matrix A = rank of matrix B

f(A) = f(B) and f(B) = 3

∴ Order of matrix B ≥ Rank of matrix B.

187. Let $[A]_{3\times3}$ be a non-singular matrix such that $A^{-1} = \dfrac{1}{3}\left(A^2 - 5A + 7I\right)$.

Then $17A^8 - 85A^7 + 119A^6 - 51A^5 - 19A^4 + 95A^3 - 133A^2 + 58A + I =$

(a) 0
(b) A
(c) A+I
(d) A^2+A+I

TS EAMCET-11.09.2020, Shift-I

Ans. (c) : Given,

$$\frac{1}{3}\left(A^2-5A+7I\right).$$

Let, $y = 17A^8 - 85A^7 + 119A^6 - 51A^5 - 19A^4 + 95A^3 - 133A^2 + 58A + I$

$y = 17A^5[A^3-5A^2+7A-3I]-19A^2[A^2-5A+7I]+58A+I$

$y = 17A^5[A(A^2-5A+7I)-3I]-19A^2[A^2-5A+7I]+58A+I$

Given, $\because A^2 - 5A + 7I = 3A^{-1}$

$y = 17A^5[A \times 3A^{-1} - 3I] - 19A^2 \times 3A^{-1} + 58A + I$

$y = 17A^5 \times 0 - 57A + 58A + I$

$y = A + I$

$17A^8 - 85A^7 + 119A^6 - 51A^5 - 19A^4 + 95A^3 - 133A^2 + 58A + I = A + I$

188. If $\begin{bmatrix} 0 & 2 & a \\ b & 0 & 4 \\ -3 & c & 0 \end{bmatrix}$ is a skew-symmetric matrix, then $\begin{bmatrix} a & b \\ b & a \end{bmatrix}\begin{bmatrix} b & c \\ c & b \end{bmatrix} =$

(a) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 2 & -8 \\ -8 & 2 \end{bmatrix}$ (d) $\begin{bmatrix} 2 & 8 \\ 8 & 2 \end{bmatrix}$

TS EAMCET-19.07.2022, Shift-I

Ans. (c) : The given, matrix $\begin{bmatrix} 0 & 2 & a \\ b & 0 & 4 \\ -3 & c & 0 \end{bmatrix}$ will be skew symmetric if-

$A = -A^T$

$$\begin{bmatrix} 0 & 2 & a \\ b & 0 & 4 \\ -3 & c & 0 \end{bmatrix} = -\begin{bmatrix} 0 & b & -3 \\ 2 & 0 & c \\ a & 4 & 0 \end{bmatrix}$$

By compare,

$a_{13} = a = -a_{31} = 3$

$a_{21} = b = -a_{12} = -2$

$a_{32} = c = -a_{23} = -4$

$$\begin{bmatrix} a & b \\ b & a \end{bmatrix}\begin{bmatrix} b & c \\ c & b \end{bmatrix} = \begin{bmatrix} 3 & -2 \\ -2 & 3 \end{bmatrix}\begin{bmatrix} -2 & -4 \\ -4 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} -6+8 & -12+4 \\ 4-12 & 8-6 \end{bmatrix} = \begin{bmatrix} 2 & -8 \\ -8 & 2 \end{bmatrix}$$

189. If $\begin{bmatrix} -1 & 2 & b \\ a & 5 & 6 \\ 3 & c & 7 \end{bmatrix}$ is a symmetric matrix, then $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} =$

(a) 0 (b) –121

(c) 143 (d) –143

TS EAMCET-19.07.2022, Shift-I

Ans. (d) : The given matrix $A = \begin{bmatrix} -1 & 2 & b \\ a & 5 & 6 \\ 3 & c & 7 \end{bmatrix}$ will be symmetric if $A = A^T$

$$\begin{bmatrix} -1 & 2 & b \\ a & 5 & 6 \\ 3 & c & 7 \end{bmatrix} = \begin{bmatrix} -1 & a & 3 \\ 2 & 5 & c \\ b & 6 & 7 \end{bmatrix}$$

By compare,

$a = 2$

$b = 3$

$c = 6$

$$\therefore \quad \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = \begin{vmatrix} 2 & 3 & 6 \\ 3 & 6 & 2 \\ 6 & 2 & 3 \end{vmatrix}$$

$= 2(18-4) - 3(9-12) + 6(6-36)$

$= 28 + 9 - 180 = -143$

190. A is a m × n matrix of rank 4. If A contains an m^{th} order non singular sub matrix and $A^T A$ is a 7 × 7 matrix, then the number of rows of A is

(a) 5 (b) 6

(c) 7 (d) 4

TS EAMCET-10.09.2020, Shift-II

Ans. (d) : According to question,

A is m × n matrix of rank 4

A contains m^{th} order of non-singular sub matrix.

A is non-singular matrix

$\therefore$ A is a square matrix of order M

$\therefore$ Rank of A is M [rank of A = 4]

no. of rows of A = 4.

191. If C and D are two n × n non-singular matrices over the set of real number R such that CD = –DC, then n is

(a) a natural number of the form 3k + 5, $k \in N$

(b) an odd integer

(c) n even integer

(d) equal to one

TS EAMCET-10.09.2020, Shift-II

Ans. (c) : Given, C and D are non-singular matrix of order n

$\because \quad |C| \neq 0, \ |D| \neq 0$

$CD = -DC$

$|CD| = |-DC|$

$|C||D| = (-1)^n |D||C|$

$1 = (-1)^n$

So, n is an even integer.

192. If $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then for all $n \in N$

(a) $A^n = nA$ (b) $A^n = nA + (n-1)I$

(c) $A^n = (n-1)A - nI$ (d) $A^n = nA - (n-1)I$

TS EAMCET-07.05.2018, Shift-I

Ans. (d) : Given,

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \text{and } I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Now, $A^2 = A.A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$

$= 2\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 2A - (2-1)I$

Again, $A^3 = A^2A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$

$= 3\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} - 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= 3A - (3-1)I$

$\therefore \quad a^n = nA - (n-1)I$

193. If $A = \begin{bmatrix} 0 & 5 \\ 0 & 0 \end{bmatrix}$ and $f(x) = x + x^2 + ... + x^{2018}$, then f(A) + I =

(a) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 5 \\ 0 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & 5 \\ 1 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 5 \\ 0 & 1 \end{bmatrix}$

TS EAMCET-04.05.2019, Shift-I

Ans. (d) : Given,

$A = \begin{bmatrix} 0 & 5 \\ 0 & 0 \end{bmatrix}$

$A^2 = \begin{bmatrix} 0 & 5 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 5 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$A^3 = A^2 \cdot A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 5 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Similarly, $A^4 = A^5 = ... A^n = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

As, $f(x) = x + x + x^2 + ... + x^{2018}$

$\therefore f(A) = A + A^2 + A^3 + ... + A^{2018}$

$f(A) = A + 0 + ... + 0 = A$

$f(A) + I = \begin{bmatrix} 0 & 5 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 5 \\ 0 & 1 \end{bmatrix}$

194. If $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & -1 & 4 \end{bmatrix}$, A = B + C, B = B^T and C = $-C^T$, then C =

(a) $\begin{bmatrix} 0 & 0.5 & 0 \\ -0.5 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0.5 \\ 0 & -0.5 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & -0.5 & 0.5 \\ 0.5 & 0 & 0 \\ -0.5 & 0 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & 0.5 & 0 \\ -0.5 & 0 & 0.5 \\ 0 & -0.5 & 0 \end{bmatrix}$

TS EAMCET-2017

Ans. (b) : Given,

$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & -1 & 4 \end{bmatrix}$

$\because$ A = B + C, B = B^T and C = $-C^T$

A can be represented as,

$A = \frac{1}{2}(A + A^T) + \frac{1}{2}(A - A^T)$

$\therefore \quad B = \frac{1}{2}(A + A^T)$ and $C = \frac{1}{2}(A - A^T)$

$C = \frac{1}{2}\begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & -1 & 4 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & -1 \\ 1 & 0 & 4 \end{bmatrix}$

$C = \frac{1}{2}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}$

$C = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0.5 \\ 0 & -0.5 & 0 \end{bmatrix}$

195. Let P be a square matrix such that $P^2 = I - P$. For $\alpha, \beta, \gamma\ \delta \in N$. If $P^\alpha + P^\beta = \gamma I - 29P$ and $P^\alpha - P^\beta = \delta I - 13P$, then $\alpha + \beta + \gamma - \delta$ is equal to

(a) 18 (b) 40

(c) 24 (d) 22

JEE Main-06.04.2023, Shift-II

Ans. (c) : Given,

$P^2 = I - P$

$P^\alpha + P^\beta = \gamma I - 29P$ ...(i)

$P^\alpha - P^\beta = \delta I - 13P$ ...(ii)

$P^4 = (I - P)^2 = I - 2P + P^2 = 2I - 3P$

$P^6 = (2I - 3P)(I - P) = 5I - 8P$

$P^8 = (2I - 3P)^2 = 4I - 12P + 9(I - P)$

$= 13I - 21P$

$P^8 + P^6 = 18I - 29P$ ...(iii)

$P^8 - P^6 = 8I - 13P$ ...(iv)

comparing the equation (iii) with (i)

and (iv) with (ii)

$\alpha = 8,\ \beta = 6,\ \gamma = 18,\ \delta = 8$

$\alpha + \beta + \gamma - \delta = 8 + 6 + 18 - 8 = 24$

196. If A and B are non-zero n × n matrics such that $A^2 + B = A^2B$, then

(a) AB = I (b) $A^2B = I$

(c) $A^2 = I$ or B = I (d) $A^2B = BA^2$

JEE Main-24.01.2023, Shift-I

Ans. (d) : $A^2 + B = A^2B$

$(A^2 - I)(B - I) = I$ .....(1)

$A^2 + B = A^2B$

$A^2(B - I) = B$

$A^2 = B(B - I)^{-1}$

$A^2 = B(A^2 - I)$

$A^2 = BA^2 - B$

$A^2 + B = BA^2$

$A^2B = BA^2$

197. Given $A = \begin{bmatrix} x & 3 & 2 \\ 1 & y & 4 \\ 2 & 2 & z \end{bmatrix}$, if xyz = 60 and 8x + 4y + 3z = 20, then A

(a) $\begin{bmatrix} 68 & 0 & 0 \\ 0 & 68 & 0 \\ 0 & 0 & 68 \end{bmatrix}$ (b) $\begin{bmatrix} 108 & 0 & 0 \\ 0 & 108 & 0 \\ 0 & 0 & 108 \end{bmatrix}$

(c) $\begin{bmatrix} 60 & 0 & 0 \\ 0 & 60 & 0 \\ 0 & 0 & 60 \end{bmatrix}$ (d) $\begin{bmatrix} 20 & 0 & 0 \\ 0 & 20 & 0 \\ 0 & 0 & 20 \end{bmatrix}$

MHT CET-2022

Ans. (a) : Given, $A = \begin{bmatrix} x & 3 & 2 \\ 1 & y & 4 \\ 2 & 2 & z \end{bmatrix}$

$xyz = 60$

$8x + 4y + 3z = 20$

Now, $|A| = \begin{vmatrix} x & 3 & 2 \\ 1 & y & 4 \\ 2 & 2 & z \end{vmatrix}$

$= x(yz - 8) - 3(z - 8) + 2(2 - 2y)$

$= xyz - 8x - 3z + 24 + 4 - 4y$

$= xyz - (8x + 4y + 3z) + 24 + 4$

$= 60 - 20 + 28$

$|A| = 68$

A. adj (A) $= \begin{bmatrix} 68 & 0 & 0 \\ 0 & 68 & 0 \\ 0 & 0 & 68 \end{bmatrix}$

198. If A = $[a_{ij}]$ 3 × 3 = $\begin{bmatrix} 1 & 3 & 3 \\ 1 & 2 & 2 \\ 1 & 1 & 4 \end{bmatrix}$ and A_{ij} is a cofactor of a_{ij} then

$a_{31}A_{31} + a_{32}A_{32} + a_{33}A_{33}$ is equal to

(a) 0 (b) 5

(c) 20 (d) 15

MHT CET-2022

Ans. (d) : Given matrix A= $\begin{bmatrix} 1 & 3 & 3 \\ 1 & 2 & 2 \\ 1 & 1 & 4 \end{bmatrix}$

$a_{31} = 2, a_{32} = 4, a_{33} = 7$

$A_{31} = 10 - 3 = 7, A_{32} = 5 - 3 = 2, A_{33} = 1 - 2 = -1$

$a_{31}A_{31} + a_{32}A_{32} + a_{33}A_{33}$

$= 7 \times 2 + 4 \times 2 + 7 \times -1$

$= 14 + 8 - 7$

$= 15$

199. If $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & -3 \\ -1 & 2 & 3 \end{bmatrix}$, then $A_{31} + A_{32} + A_{33}$ =

Where A_{ij} is cofactor of a_{ij}, where A = $[a_{ij}]_{3\times3}$

(a) 10 (b) 1

(c) 0 (d) 11

MHT CET-2022

Ans. (c) : Given, $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & -3 \\ -1 & 2 & 3 \end{bmatrix}$

$M_{31} = \begin{vmatrix} 1 & 1 \\ 1 & -3 \end{vmatrix} = -3 - 1 = -4$

$M_{32} = \begin{vmatrix} 1 & 1 \\ 2 & -3 \end{vmatrix} = -3 - 2 = -5$

$M_{33} = \begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} = 1 - 2 = -1$

$A_{31} = (-1)^{3+1}(-4) = -4$

$A_{32} = (-1)^{3+2}(-5) = 5$

$A_{33} = (-1)^{3+3}(-1) = -1$

$A_{31} + A_{32} + A_{33} = -4 + 5 - 1 = 0$

200. If $A = \begin{bmatrix} 5 & 6 & 3 \\ -4 & 3 & 2 \\ -4 & -7 & 3 \end{bmatrix}$, then cofactors of all elements of second row are respectively.

(a) –39, 27, 11 (b) –39, 3, 11

(c) 39, –3, 11 (d) –39, –27, 11

MHT CET-2021

Ans. (a) : Given,

$A = \begin{bmatrix} 5 & 6 & 3 \\ -4 & 3 & 2 \\ -4 & -7 & 3 \end{bmatrix}$

$A_{21} = (-1)^{2+1}\begin{vmatrix} 6 & 3 \\ -7 & 3 \end{vmatrix} = -(18 + 21) = -39$

$A_{22} = (-1)^{2+2}\begin{vmatrix} 5 & 3 \\ -4 & 3 \end{vmatrix} = 15 + 12 = 27$

$$A_{23} = (-1)^{2+3}\begin{vmatrix} 5 & 6 \\ -4 & -7 \end{vmatrix} = -(-35+24) = 11$$

Hence, $A_{21}, A_{22}, A_{23} = -39, 27, 11$

201. The sum of three numbers is 6. Thrice the third number when added to the first number gives 7. On adding three times first number to the sum of second and third number we get 12. The product of these numbers is

(a) 3 (b) $\frac{5}{3}$

(c) 20 (d) $\frac{20}{3}$

MHT CET-2021

Ans. (d) : Let, a, b, c be the three number

$a + b + c = 6$ …..(i)

$a + 3c = 7$ …..(ii)

$3a + b + c = 12$ …..(iii)

Using matrix property for finding value of a, b, c we get.

$Ax = B$

$X = A^{-1}B$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 3 \\ 3 & 1 & 1 \end{bmatrix}\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 6 \\ 7 \\ 12 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 3 \\ 3 & 1 & 1 \end{bmatrix}$$

As we know that,

$$A^{-1} = \frac{\text{adj}A}{|A|}$$

$|A| = 1\,(0-3) - 1(1-9) + 1\,(1-0)$

$|A| = -3 + 8 + 1 = 6$

$$\text{Adj}A = \begin{bmatrix} -3 & 0 & 3 \\ 8 & -2 & -2 \\ 1 & 2 & -1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{6}\begin{bmatrix} -3 & 0 & 3 \\ 8 & -2 & -2 \\ 1 & 2 & -1 \end{bmatrix}$$

Now, $$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \frac{1}{6}\begin{bmatrix} -3 & 0 & 3 \\ 8 & -2 & -2 \\ 1 & 2 & -1 \end{bmatrix}\begin{bmatrix} 6 \\ 7 \\ 12 \end{bmatrix}$$

$$= \begin{bmatrix} -18 & 0 & 36 \\ 48 & -14 & -24 \\ 6 & 14 & -12 \end{bmatrix}$$

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 18 \\ 10 \\ 8 \end{bmatrix}$$

On comparing both sides, we get –

$$a = \frac{18}{6}$$

$$b = \frac{10}{6}$$

$$c = \frac{8}{6}$$

$$a = 3,\ b = \frac{5}{3},\ c = \frac{4}{3}$$

$$abc = 3 \times \frac{5}{3} \times \frac{4}{3}$$

$$abc = \frac{20}{3}$$

202. The co-factors of the elements of second column of $\begin{bmatrix} 1 & -1 & 2 \\ 3 & 2 & 1 \\ -1 & 3 & 4 \end{bmatrix}$ are

(a) –13, 6, 5 (b) –13, –6, 5

(c) 13, 5, 6 (d) 13, –6, –5

MHT CET-2021

Ans. (a) : Given, $A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 2 & 1 \\ -1 & 3 & 4 \end{bmatrix}$

$$C_{21} = (-1)^{1+2}\begin{vmatrix} 3 & 1 \\ -1 & 4 \end{vmatrix} = -(12+1) = -13$$

$$C_{22} = (-1)^{2+2}\begin{vmatrix} 1 & 2 \\ -1 & 4 \end{vmatrix} = (4+2) = 6$$

$$C_{23} = (-1)^{3+2}\begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix} = -(1-6) = 5$$

$C_{21}, C_{22}, C_{23} = -13, 6, 5$

203. If the matrix $\begin{bmatrix} 1 & 2 & -1 \\ -3 & 4 & k \\ -4 & 2 & 6 \end{bmatrix}$ is singular, then the value of k is equal to

(a) 3 (b) 4

(c) 5 (d) 6

(e) 7

Kerala CEE-2020

Ans. (c) : Given,

$$A = \begin{vmatrix} 1 & 2 & -1 \\ -3 & 4 & k \\ -4 & 2 & 6 \end{vmatrix}$$

Since, A is a singular matrix

Then, $|A| = 0$

Now, $|A| = \begin{vmatrix} 1 & 2 & -1 \\ -3 & 4 & k \\ -4 & 2 & 6 \end{vmatrix} = 0$

$1(24 - 2k) - 2(-18 + 4k) - 1(-6 + 16) = 0$
$24 - 2k + 36 - 8k + 6 - 16 = 0$
$10k = 50$
$k = 5$

204. If $A = \begin{pmatrix} 6 & 2 \\ 7 & -5 \end{pmatrix}$ and $A - B = \begin{pmatrix} -2 & 1 \\ 4 & -9 \end{pmatrix}$, then B =

(a) $\begin{pmatrix} -8 & -1 \\ 3 & 4 \end{pmatrix}$ (b) $\begin{pmatrix} 8 & 1 \\ -3 & -4 \end{pmatrix}$

(c) $\begin{pmatrix} 4 & 3 \\ 11 & -14 \end{pmatrix}$ (d) $\begin{pmatrix} 8 & 1 \\ 3 & 4 \end{pmatrix}$

(e) $\begin{pmatrix} 4 & 1 \\ 3 & 2 \end{pmatrix}$

Kerala CEE-2020

Ans. (d) : Given,

$A = \begin{pmatrix} 6 & 2 \\ 7 & -5 \end{pmatrix}$

$A - B = \begin{pmatrix} -2 & 1 \\ 4 & -9 \end{pmatrix}$

$A - (A - B) = \begin{vmatrix} 6 & 2 \\ 7 & -5 \end{vmatrix} - \begin{vmatrix} -2 & 1 \\ 4 & -9 \end{vmatrix}$

$B = \begin{pmatrix} 8 & 1 \\ 3 & 4 \end{pmatrix}$

205. If $\begin{pmatrix} 3x - y & x + 3y \\ 2x - z & 2y + z \end{pmatrix} = \begin{pmatrix} 7 & 9 \\ 5 & 5 \end{pmatrix}$, then x + y + z equals

(a) 3 (b) 6
(c) 9 (d) 12
(e) 11

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Ans. (b) : $\begin{pmatrix} 3x - y & x + 3y \\ 2x - z & 2y + z \end{pmatrix} = \begin{pmatrix} 7 & 9 \\ 5 & 5 \end{pmatrix}$

Comparing both sides,

$3x - y = 7$ …..(i)
$x + 3y = 9$ …..(ii)
$2x - z = 5$ …..(iii)
$2y + z = 5$ ……(iv)

From equation (i) × 3 and equation (ii),

$9x - 3y = 21$
$\underline{x + 3y = 9}$
$10x = 30$
$x = 3$
$y = 2$

By equation (iii) put the value of x we get
$z = 1$
Now, $x + y + z = 3 + 2 + 1 = 6$

206. The number of 3 × 3 matrices with entries – 1 or + 1 is

(a) 2^{-4} (b) 2^5
(c) 2^6 (d) 2^7
(e) 2^9

Kerala CEE-2017

Ans. (e) : In 3 × 3 matrix, total numbers of elements = 3 × 3 = 9

∴ Total number of 3 × 3 matrices with entries either –1 or 1 = 2^9

207. If $\begin{pmatrix} 2x + y & x + y \\ p - q & p + q \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}$, then (x, y, p, q) is equal to

(a) (0, 1, 0, 0) (b) (0, –1, 0, 0)
(c) (1, 0, 0, 0) (d) (0, 1, 0, 1)
(e) (1, 0, 1, 0)

Kerala CEE-2017

Ans. (a) : $\begin{pmatrix} 2x + y & x + y \\ p - q & p + q \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}$

Comparing both sides, we get –

$2x + y = 1$ …..(i)
$x + y = 1$ …..(ii)
$p - q = 0$ …..(iii)
$p + q = 0$ …..(iv)

By equation (iii) and (iv), we get –
$2p = 0$
$p = 0, q = 0$

By equation (i) and (ii), we get –
$x = 0$
$y = 1$
$(x, y, p, q) = (0, 1, 0, 0)$

208. If $\begin{pmatrix} x + y & x - y \\ 2x + z & x + z \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix}$, then the values of x, y and z are respectively

(a) 0, 0, 1 (b) 1, 1, 0
(c) –1, 0, 0 (d) 0, 0, 0
(e) 1, 1, 1

Kerala CEE-2017

Ans. (a) : $\begin{pmatrix} x + y & x - y \\ 2x + z & x + z \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix}$

$x + y = 0$ ……(i)
$2x + z = 1$ …..(ii)
$x - y = 0$ …..(iii)
$x + z = 1$ …..(iv)

By equation (i) and (iii), we get –

$2x = 0$

$x = 0,$

$y = 0$

By equation (iv),

$0 + z = 1$

$z = 1$

$x, y, z = 0, 0, 1$

209. If A and B are square matrices of the same order and if $A = A^T$, $B = B^T$, then $(ABA)^T$ is equal to

(a) BAB (b) ABA
(c) ABAB (d) AB^T
(e) $(AB)^T$

Kerala CEE-2015

Ans. (b) : Given,
$A = A^T$, $B = B^T$ then $(ABA)^T$
$(ABA)^T = A^T B^T A^T = ABA$

210. If $\begin{vmatrix} 0 & 3 & 2b \\ 2 & 0 & 1 \\ 4 & -1 & 6 \end{vmatrix}$ is singular, then the value of b is equal to

(a) –3 (b) 3
(c) –6 (d) 6
(e) –2

Kerala CEE-2015

Ans. (c) : Given, $A = \begin{vmatrix} 0 & 3 & 2b \\ 2 & 0 & 1 \\ 4 & -1 & 6 \end{vmatrix}$

Since given matrix is a singular matrix then

$|A| = 0$

So, $\begin{vmatrix} 0 & 3 & 2b \\ 2 & 0 & 1 \\ 4 & -1 & 6 \end{vmatrix} = 0$

$0(0 + 1) - 3(12 - 4) + 2b(-2 - 0) = 0$

$0 - 24 - 4b = 0$

$-4b = 24$

$b = -6$

211. If A is a square matrix of order 3 such that $A^2 + A + 4I = 0$, where 0 is the zero matrix and I is the unit matrix of order 3, then

(a) A is singular and A + 1 is non-singular
(b) A is non-singular and A + 1 is non-singular
(c) A is non-singular and A + 1 is singular
(d) A is singular and A + 1 is singular
(e) A is non-singular and A – 1 is singular

Kerala CEE-2013

Ans. (b) : Given, $A^2 + A + 4I = 0$

$A^2 + A = -4I$

$A(A + 1) = -4I$

$|A(A + I)| = |-4I|$

$|A|\,|A + I)| = |-4|\,|I|$ (by property)

$|A|\,(A + 1) = 4.1$ ($\because |I| = 1$)

$|A|\,|A + I| = 4 \neq 0$

So, both A and (A + 1) are nonsingular.

212. If $e\begin{bmatrix} e^x & e^y \\ e^y & e^x \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$, then the value of x and y are respectively

(a) –1, –1 (b) 1, 1
(c) 0, 1 (d) 1, 0
(e) 0, 0

Kerala CEE-2012

Ans. (a) : Given,

$e\begin{vmatrix} e^x & e^y \\ e^y & e^x \end{vmatrix} = \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix}$

$\begin{vmatrix} ee^x & ee^y \\ ee^y & ee^x \end{vmatrix} = \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix}$

$\begin{vmatrix} e^{(1+x)} & e^{(1+y)} \\ e^{(1+y)} & e^{(1+x)} \end{vmatrix} = \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = \begin{vmatrix} e^0 & e^0 \\ e^0 & e^0 \end{vmatrix}$ $\{\because e^\circ = 1\}$

$e^{1+x} = e^\circ$ ……(i)

$e^{1+y} = e^\circ$ ……(ii)

By comparing power we get –

$1 + x = 0$

$x = -1$

and $1 + y = 0$

$y = -1$

So, value of x and y is –1, –1 respectively.

213. If $\begin{bmatrix} x - y - z \\ -y + z \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 5 \\ 3 \end{bmatrix}$, then the values of x, y and z are respectively

(a) 5, 2, 2 (b) 1, –2, 3
(c) 0, –3, 3 (d) 11, 8, 3
(e) 4, 1, 3

Kerala CEE-2010

Ans. (b) : Given, $\begin{vmatrix} x - y - z \\ -y + z \\ z \end{vmatrix} = \begin{vmatrix} 0 \\ 5 \\ 3 \end{vmatrix}$

$x - y - z = 0$ …(i)

$-y + z = 5$ …(ii)

$z = 3$ …(iii)

By eqn. (ii) and (iii) we get

$-y + 3 = 5$

$y = -2$

and put the value of y and z in equation (i), we get –

$x - (-2) - 3 = 0$

$x = 1$

Now, $x, y, z = 1, -2, 3$ respectively.

B. Product of Matrices and its Properties

214. If A and B are two matrices such that AB = B and BA = A then $A^2 + B^2 =$

(a) 2AB (b) 2BA
(c) A + B (d) AB

Karnataka CET-2000

Ans. (c) : Given that,
A and B are two matrices and AB = B and BA = A
Now, A B = B
On multiply by A on both the side we get –

(AB) A = BA
A (BA) = BA
A (A) = A [∵ BA = A]
$A^2 = A$

And AB = B
On multiply by B on both the side we get –

B (AB) = BB
(BA) B = B^2 [∵BA = A]
AB = B^2 [∵AB = B]
B = B^2

Hence, $A^2 + B^2 = A + B$

215. $G = \left\{\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} : \theta \in R\right\}$ is a group under matrix multiplication. Then which one of the following statements in respect of G is true.

(a) $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$ is the inverse of itself

(b) G is a finite group

(c) $\begin{pmatrix} 1/2 & -\sqrt{3}/2 \\ \sqrt{3}/2 & 1/2 \end{pmatrix}$ is not an element of G

(d) $\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$ is an element of G.

Karnataka CET-2002

Ans. (a) : Given that,

$$G = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \quad \because \theta \in R$$

For option (a),
If the matrix is the inverse of itself when multiplied by itself it should yield identity matrix.

$$= \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+0 & 0+0 \\ 0+0 & 0+1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Hence, option (a) true.

216. If $A = \begin{bmatrix} 1 & -2 & 1 \\ 2 & 1 & 3 \end{bmatrix}, B = \begin{bmatrix} 2 & 1 \\ 3 & 2 \\ 1 & 1 \end{bmatrix}$ then $(AB)^T$ is equal to

(a) $\begin{bmatrix} -3 & -2 \\ 10 & 7 \end{bmatrix}$ (b) $\begin{bmatrix} -3 & 10 \\ -2 & 7 \end{bmatrix}$

(c) $\begin{bmatrix} -3 & 7 \\ 10 & 2 \end{bmatrix}$ (d) $\begin{bmatrix} -3 & 7 \\ 10 & -2 \end{bmatrix}$

Karnataka CET-2021

Ans. (b) : Given that,

$$A = \begin{bmatrix} 1 & -2 & 1 \\ 2 & 1 & 3 \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 & 1 \\ 3 & 2 \\ 1 & 1 \end{bmatrix}$$

Then,

$$AB = \begin{bmatrix} 1 & -2 & 1 \\ 2 & 1 & 3 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 3 & 2 \\ 1 & 1 \end{bmatrix}$$

$$AB = \begin{bmatrix} 2-6+1 & 1-4+1 \\ 4+3+3 & 2+2+3 \end{bmatrix}$$

$$AB = \begin{bmatrix} -3 & -2 \\ 10 & 7 \end{bmatrix}$$

So, $(AB)^T = \begin{bmatrix} -3 & 10 \\ -2 & 7 \end{bmatrix}$

217. Let A be a square matrix of order 3 × 3, then |5A| =

(a) 5|A| (b) 125|A|
(c) 25|A| (d) 15|A|

Karnataka CET-2018

Ans. (b) : We have A be a square matrix of order 3 × 3
Then,

|5A|
$= 5^3$ |A| [∵ $|KA| = k^n|A|$]
= 125 |A| (where n is the order of the matrix)

218. If (x_1, y_1), (y_2, y_2) and (x_3, y_3) are the vertices of a triangle whose area is k square units, then $\begin{vmatrix} x_1 & y_1 & 4 \\ x_2 & y_2 & 4 \\ x_3 & y_3 & 4 \end{vmatrix}^2$ is

(a) $32k^2$ (b) $16k^2$
(c) $64k^2$ (d) $48k^2$

Karnataka CET-2018

Ans. (c) : We know that,

$$\text{Area of triangle } = \frac{1}{2}\begin{bmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{bmatrix} = k$$

From the given question,

$$\therefore \begin{bmatrix} x_1 & y_1 & 4 \\ x_2 & y_2 & 4 \\ x_3 & y_3 & 4 \end{bmatrix}^2$$

$$= 4^2\begin{bmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{bmatrix}^2$$

$= 16\,(2k)^2 = 64\,k^2$

219. If $2\begin{bmatrix}1 & 3\\0 & x\end{bmatrix}+\begin{bmatrix}y & 0\\1 & 2\end{bmatrix}=\begin{bmatrix}5 & 6\\1 & 8\end{bmatrix}$ then the value of x and y are

(a) $x = 3, y = 3$ (b) $x = -3, y = 3$
(c) $x = 3, y = -3$ (d) $x = -3, y = -3$

Karnataka CET-2017

Ans. (a) : Given that,

$$2\begin{bmatrix}1 & 3\\0 & x\end{bmatrix}+\begin{bmatrix}y & 0\\1 & 2\end{bmatrix}=\begin{bmatrix}5 & 6\\1 & 8\end{bmatrix}$$

$$\begin{bmatrix}2 & 6\\0 & 2x\end{bmatrix}+\begin{bmatrix}y & 0\\1 & 2\end{bmatrix}=\begin{bmatrix}5 & 6\\1 & 8\end{bmatrix}$$

$$\begin{bmatrix}2+y & 6+0\\0+1 & 2x+2\end{bmatrix}=\begin{bmatrix}5 & 6\\1 & 8\end{bmatrix}$$

On comparing corresponding elements on both the side we get –

$2 + y = 5,$ $2x + 2 = 8$
$y = 5 - 2,$ $2x = 6$
$y = 3$ $x = 3$

220. If A is a square matrix of order 3 × 3, then |KA| is equal to

(a) $K|A|$ (b) $K^2|A|$
(c) $3K|A|$ (d) $K^3|A|$

Karnataka CET-2017

Ans. (d) : A is a square matrix of order 3 × 3

Then, $|KA|$

$= K^3|A|$ $[\because |KA| = K^n |A|]$

221. If $A\begin{bmatrix}3 & 1\\-1 & 2\end{bmatrix}$ then $A^2 - 5A$ is equal to

(a) I (b) –I
(c) 7I (d) –7I

Karnataka CET-2016

Ans. (d) : Given that,

$$A=\begin{bmatrix}3 & 1\\-1 & 2\end{bmatrix}$$

$$A^2 = A\cdot A=\begin{bmatrix}3 & 1\\-1 & 2\end{bmatrix}\begin{bmatrix}3 & 1\\-1 & 2\end{bmatrix}$$

$$A^2=\begin{bmatrix}9-1 & 3+2\\-3-2 & -1+4\end{bmatrix}$$

$$A^2=\begin{bmatrix}8 & 5\\-5 & 3\end{bmatrix}$$

Now, $A^2 - 5A$

$$=\begin{bmatrix}8 & 5\\-5 & 3\end{bmatrix}-5\begin{bmatrix}3 & 1\\-1 & 2\end{bmatrix}$$

$$=\begin{bmatrix}8 & 5\\-5 & 3\end{bmatrix}-\begin{bmatrix}15 & 5\\-5 & 10\end{bmatrix}$$

$$=\begin{bmatrix}-7 & 0\\0 & -7\end{bmatrix}$$

$$B=\begin{bmatrix}6 & 6 & 7\\6 & 2 & 5\\7 & 5 & 1\end{bmatrix}=-17$$

222. If A is a matrix of order m × n and B is a matrix such that AB' and B'A are both defined, the order of the matrix B is

(a) m × m (b) n × n
(c) n × m (d) m × n

Karnataka CET-2016

Ans. (d) : We have,

Order of A matrix = m × n

Let order of B matrix = x × p

Order of B' matrix = p × x

If AB' is defined then the order of AB' is m × x if n = p

If B'A is defined then order of B'A is p × n when x = m

Now, Order of B' = p × x

∴ Order of B = x × p

= m × n ∵ (n = p, x = m)

223. If A is any square matrix of order 3 × 3 then |3A| is equal to

(a) $3|A|$ (b) $\frac{1}{3}|A|$
(c) $27|A|$ (d) $9|A|$

Karnataka CET-2016

Ans. (c) : A is square matrix of order 3 × 3.

Then, $|3A|$

$= 3^3|A|$ $\because |KA| = K^n|A|$

$= 27|A|$

224. If $A=\begin{bmatrix}\alpha & 2\\2 & \alpha\end{bmatrix}$ and $|A|^3 = 27$ then α = _______

(a) + 2 (b) $\pm\sqrt{5}$
(c) ± 1 (d) $\pm\sqrt{7}$

Karnataka CET-2015

Ans. (d) : Given that,

$$A=\begin{bmatrix}\alpha & 2\\2 & \alpha\end{bmatrix}$$

$|A| = \alpha^2 - 4$ …..(i)

$\because$ $|A^3| = 27$

$|A|^3 = 27$ $\because |M^n| = |M|^n$

$|A| = 3$ …..(ii)

From equation (i) and (ii), we get –

$\alpha^2 - 4 = 3$

$\alpha^2 = 7$

$\alpha = \pm\sqrt{7}$

225. If $A=\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}$, then A^2 is equal to ________

(a) $\begin{bmatrix}1 & 0\\1 & 0\end{bmatrix}$ (b) $\begin{bmatrix}0 & 1\\0 & 1\end{bmatrix}$

(c) $\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}$ (d) $\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$

Karnataka CET-2015

Ans. (d) : Given that,

$$A=\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}$$

$A^2 = A \cdot A$

$$A^2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$
$$A^2 = \begin{bmatrix} 0+1 & 0+0 \\ 0+0 & 1+0 \end{bmatrix}$$
$$A^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

226. If $A = \begin{bmatrix} \alpha & 2 \\ 2 & \alpha \end{bmatrix}$ and $|A^3| = 125$ then α =

(a) ± 1 (b) ± 2
(c) ± 3 (d) ± 5

Karnataka CET-2013

Ans. (c) : Given that,
$$A = \begin{bmatrix} \alpha & 2 \\ 2 & \alpha \end{bmatrix}$$
$|A| = \alpha^2 - 4$ .....(i)
$\because$ $|A^3| = 125$
$|A|^3 = 5^3$ $\because |M^n| = |M|^n$
$|A| = 5$ .....(ii)
From equation (i) and (ii), we get –
$\alpha^2 - 4 = 5$
$\alpha^2 = 9$
$\alpha = \pm 3$

227. If ω is an imaginary cube root of unity, then the value of $\begin{vmatrix} 1 & \omega^2 & 1-\omega^4 \\ \omega & 1 & 1+\omega^5 \\ 1 & \omega & \omega^2 \end{vmatrix}$ is

(a) -4 (b) $\omega^2 - 4$
(c) ω^2 (d) 4

Karnataka CET-2006

Ans. (b) : Given,
$$\begin{vmatrix} 1 & \omega^2 & 1-\omega^4 \\ \omega & 1 & 1+\omega^5 \\ 1 & \omega & \omega^2 \end{vmatrix}$$
$$= \begin{vmatrix} 1 & \omega^2 & 1-\omega \\ \omega & 1 & -\omega \\ 1 & \omega & \omega^2 \end{vmatrix} \quad [\because \omega^3 = 1]$$
On applying $C_1 \to C_1 + C_2 + C_3$ we get–
$$\begin{vmatrix} 1+\omega^2+1-\omega & \omega^2 & 1-\omega \\ \omega+1-\omega & 1 & -\omega \\ 1+\omega+\omega^2 & \omega & \omega^2 \end{vmatrix}$$
$$= \begin{vmatrix} 1-2\omega & \omega^2 & 1-\omega \\ 1 & 1 & -\omega \\ 0 & \omega & \omega^2 \end{vmatrix}$$
$[\because 1 + \omega + \omega^2 = 0]$

Expending on C_1
$= (1 - 2\omega)(\omega^2 + \omega^2) - 1[\omega^4 - (\omega - \omega^2)]$
$= (1 - 2\omega)(2\omega^2) - (\omega^4 - \omega + \omega^2)$
$= 2\omega^2 - 4\omega^3 - (\omega - \omega + \omega^2)$
$= 2\omega^2 - 4 - \omega^2$
$= \omega^2 - 4$

228. If A and B are symmetric matrices of the same order, then which one of the following is NOT true?

(a) A + B is symmetric
(b) A – B is symmetric
(c) AB + BA is symmetric
(d) AB – BA is symmetric

Karnataka CET-2011

Ans. (d) : Given that,
A and B are symmetric matrix of same order.
$A = A'$ and $B = B'$
AB – BA is not a symmetric matrix because,
$(AB - BA)' = (AB)' - (BA)'$
$= B'A' - A'B'$
$= BA - AB$ $[\because A = A', B = B']$
$= -(AB - BA)$
$M^T = -M$
So, it is a skew symmetric matrix.
Hence, option (d) is not true.

229. If $A = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$, then $A^2 + xA + yI = 0$ for (x, y) =

(a) (–1, 3) (b) (–4, 1)
(c) (1, 3) (d) (4, 1)

Karnataka CET-2010

Ans. (b) : Given that,
$$A = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix} \text{ and } I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$
$$A^2 = A \cdot A = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} 9+2 & 6+2 \\ 3+1 & 2+1 \end{bmatrix}$$
$$= \begin{bmatrix} 11 & 8 \\ 4 & 3 \end{bmatrix}$$
Now, $A^2 + xA + yI = 0$
$$\begin{bmatrix} 11 & 8 \\ 4 & 3 \end{bmatrix} + \begin{bmatrix} 3x & 2x \\ x & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 0 & y \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$
$$\begin{bmatrix} 11+3x+y & 8+2x+0 \\ 4+x+0 & 3+x+y \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$
On comparing corresponding elements on both side we get –
$11 + 3x + y = 0,$ $8 + 2x = 0$
$11 + 3 \times (-4) + y = 0,$ $2x = -3$
$11 - 12 + y = 0,$ $x = -4$
$y = 1$
Hence, $(x, y) \equiv (-4, 1)$

230. The characteristic roots of the matrix $\begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 0 \\ 4 & 5 & 6 \end{bmatrix}$ are

(a) 4, 5, 6 (b) 2, 4, 6

(c) 1, 3, 6 (d) 1, 2, 4

Karnataka CET-2008

Ans. (c) : Let,

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 0 \\ 4 & 5 & 6 \end{bmatrix}$$

A is upper triangular matrix. Then the characteristic roots are the diagonal element of matrix A is 1, 3, 6.

231. If O(A) = 2 × 3, O(B) = 3 × 2, and O(C) = 3 × 3, which one of the following is not defined?

(a) CB + A' (b) BAC

(c) C(A + B')' (d) C(A + B')

Karnataka CET-2006

Ans. (d) :

For option (a)

CB + A'

$= [C]_{3\times3} \cdot [B]_{3\times2} + [A']_{3\times2}$

$= [CB]_{3\times2} + [A']_{3\times2}$

It is defined.

For option (b)

B A C

$= [B]_{3\times2}\,[A]_{2\times3}\,[C]_{3\times3}$

$= [BA]_{3\times3}\,[C]_{3\times3}$

It is also defined.

For option (c)

C (A + B')'

$= [C]_{3\times3}\,[A + B']'_{3\times2}$

$= 0\,[C(A + B')'] = 3\times2$

It is also defined.

For option (d)

C (A + B')

$= [C]_{3\times3}\,[A + B']_{2\times3}$

It is not defined.

232. If $2A+3B=\begin{bmatrix} 2 & -1 & 4 \\ 3 & 2 & 5 \end{bmatrix}$ and $A+2B=\begin{bmatrix} 5 & 0 & 3 \\ 1 & 6 & 2 \end{bmatrix}$ then B =

(a) $\begin{bmatrix} 8 & -1 & 2 \\ -1 & 10 & -1 \end{bmatrix}$ (b) $\begin{bmatrix} 8 & 1 & 2 \\ -1 & 10 & -1 \end{bmatrix}$

(c) $\begin{bmatrix} 8 & 1 & -2 \\ -1 & 10 & -1 \end{bmatrix}$ (d) $\begin{bmatrix} 8 & 1 & 2 \\ 1 & 10 & 1 \end{bmatrix}$

Karnataka CET-2006

Ans. (b) : Given that,

$$2A+3B=\begin{bmatrix} 2 & -1 & 4 \\ 3 & 2 & 5 \end{bmatrix} \quad \text{.....(i)}$$

$$A+2B=\begin{bmatrix} 5 & 0 & 3 \\ 1 & 6 & 2 \end{bmatrix}$$

On multiply by 2 on both the side we get –

$$2A+4B=\begin{bmatrix} 10 & 0 & 6 \\ 2 & 12 & 4 \end{bmatrix} \quad \text{.....(ii)}$$

On subtracting equation (ii) by (i), we get –

$$2A+4B-2A-3B=\begin{bmatrix} 10-2 & 0+1 & 6-4 \\ 2-3 & 12-2 & 4-5 \end{bmatrix}$$

$$B=\begin{bmatrix} 8 & 1 & 2 \\ -1 & 10 & -1 \end{bmatrix}$$

233. If $A=\begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}, I=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $A^2 = 8A + kI$, then the values of k is

(a) 7 (b) −7

(c) $\frac{1}{7}$ (d) $\frac{-1}{7}$

MHT CET-2020

Ans. (b) : We have,

$$A=\begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix} \text{ and } I=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A^2 = A\times A = \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -8 & 49 \end{bmatrix}$$

Given,

$A^2 = 8A + kI$

$$A^2 = 8\begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix} + k\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 8 & 0 \\ -8 & 56 \end{bmatrix} + \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ -8 & 49 \end{bmatrix} = \begin{bmatrix} 8+k & 0 \\ -8 & 56+k \end{bmatrix}$$

On comparing corresponding elements on both the side we get–

$8 + k = 1$

$k = -7$

234. If AX = B, where $A=\begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 4 \\ 1 & 3 & 4 \end{bmatrix}$, $X=\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B=\begin{bmatrix} 12 \\ 15 \\ 13 \end{bmatrix}$, then $x^2 + y^2 + z^2 = 0$

(a) 14 (b) 19

(c) 21 (d) 6

MHT CET-2020

Ans. (a) : Given,

AX = B

$$\begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 4 \\ 1 & 3 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ 15 \\ 13 \end{bmatrix}$$

On applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$, we get

$$\begin{bmatrix} 1 & 3 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ 3 \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} x+3y+3z \\ y+z \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ 3 \\ 1 \end{bmatrix}$$

On comparing corresponding elements on both the side we get–

$\therefore x+3y+3z=12$

$y+z=3$

$z = 1$
Thus, $z = 1, y = 2, x = 3$
$\therefore x^2 + y^2 + z^2 = 9 + 4 + 1 = 14$

235. The matrix A satisfying $A\begin{bmatrix}1 & 5\\0 & 1\end{bmatrix} = \begin{bmatrix}3 & -1\\6 & 0\end{bmatrix}$ is

(a) $\begin{bmatrix}3 & 2\\6 & -3\end{bmatrix}$ (b) $\begin{bmatrix}3 & -16\\6 & -30\end{bmatrix}$

(c) $\begin{bmatrix}3 & -16\\6 & 30\end{bmatrix}$ (d) $\begin{bmatrix}3 & -3\\6 & 2\end{bmatrix}$

MHT CET-2007

Ans. (b) : We have,

$A\begin{bmatrix}1 & 5\\0 & 1\end{bmatrix} = \begin{bmatrix}3 & -1\\6 & 0\end{bmatrix}$

On applying $C_2 \to C_2 - 5C_1$ we get –

$A\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} = \begin{bmatrix}3 & -16\\6 & -30\end{bmatrix}$

$\begin{bmatrix}3 & -16\\6 & -30\end{bmatrix}\{\because AI = A\}$

$\Rightarrow A = \begin{bmatrix}3 & -16\\6 & -30\end{bmatrix}$

236. Let $A = \begin{bmatrix}3 & 1\\-1 & 2\end{bmatrix}$ and I is a unit matrix of order 2. Also if $A^2 - kA + 7I = 0$, then k = ?

(a) 2 (b) 5
(c) –5 (d) 4

MHT CET-2008

Ans. (b) : Given that,

$A = \begin{bmatrix}3 & 1\\-1 & 2\end{bmatrix}$

$A^2 = A.A = \begin{bmatrix}3 & 1\\-1 & 2\end{bmatrix}\begin{bmatrix}3 & 1\\-1 & 2\end{bmatrix} = \begin{bmatrix}9-1 & 3+2\\-3-2 & -1+4\end{bmatrix} = \begin{bmatrix}8 & 5\\-5 & 3\end{bmatrix}$

Now,

$A^2 - kA + 7I = 0$

$\begin{bmatrix}8 & 5\\-5 & 3\end{bmatrix} - k\begin{bmatrix}3 & 1\\-1 & 2\end{bmatrix} + 7\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} = 0$

$\begin{bmatrix}8 & 5\\-5 & 3\end{bmatrix} + \begin{bmatrix}7 & 0\\0 & 7\end{bmatrix} = \begin{bmatrix}3k & k\\-k & 2k\end{bmatrix}$

$\begin{bmatrix}15 & 5\\-5 & 10\end{bmatrix} = \begin{bmatrix}3k & k\\-k & 2k\end{bmatrix}$

By equating of matrices, k = 5

237. If $A = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}, B = \begin{bmatrix}1 & 0\\1 & 0\end{bmatrix}$, then AB = ?

(a) Null matrix (b) Scalar matrix
(c) B (d) A

MHT CET-2006

Ans. (c) : Given that,

$A = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$ and $B = \begin{bmatrix}1 & 0\\1 & 0\end{bmatrix}$

$AB = \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\1 & 0\end{bmatrix}$

$AB = \begin{bmatrix}1+0 & 0+0\\0+1 & 0+0\end{bmatrix}$

$AB = \begin{bmatrix}1 & 0\\1 & 0\end{bmatrix}$

AB = B

238. If $A = \begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}$, then $A^2 - 5A =$

(a) 2I (b) 3I
(c) –2I (d) Null Matrix

MHT CET-2004

Ans. (a) : Given that,

$A = \begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}$

$A^2 - 5A = A.A. - 5A = \begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix} - 5\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}$

$= \begin{bmatrix}7 & 10\\15 & 22\end{bmatrix} - \begin{bmatrix}5 & 10\\15 & 20\end{bmatrix} = \begin{bmatrix}2 & 0\\0 & 2\end{bmatrix} = 2\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} = 2I$

239. If $A = \begin{bmatrix}1 & -1\\2 & -1\end{bmatrix}, B = \begin{bmatrix}1 & a\\4 & b\end{bmatrix}$ and $(A + B)^2 = A^2 + B^2$, then

(a) $a = 1, b = 1$ (b) $a = 1, b = -1$
(c) $a = -1, b = -1$ (d) $a = -1, b = 1$

MHT CET-2005

Ans. (b) : Given that,

$A = \begin{bmatrix}1 & -1\\2 & -1\end{bmatrix}$ and $B = \begin{bmatrix}1 & a\\4 & b\end{bmatrix}$

$\because (A + B)^2 = A^2 + B^2$

$(A + B)(A + B) = A^2 + B^2$

$A^2 + AB + BA + B^2 = A^2 + B^2 \Rightarrow AB + BA = 0$

$AB = \begin{bmatrix}1 & -1\\2 & -1\end{bmatrix}\begin{bmatrix}1 & a\\4 & b\end{bmatrix} = \begin{bmatrix}-3 & a-b\\-2 & 2a-b\end{bmatrix}$

$BA = \begin{bmatrix}1 & a\\4 & b\end{bmatrix}\begin{bmatrix}1 & -1\\2 & -1\end{bmatrix} = \begin{bmatrix}1+2a & -1-a\\4+2b & -4-b\end{bmatrix}$

$AB + BA = \begin{bmatrix}-3 & a-b\\-2 & 2a-b\end{bmatrix} + \begin{bmatrix}1+2a & -1-a\\4+2b & -4-b\end{bmatrix} = 0$

$AB + BA = \begin{bmatrix}-2+2a & -b-1\\2+2b & 2a-2b-4\end{bmatrix} = \begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$

On comparing corresponding elements on both side, we get –

$\therefore \quad -2 + 2a = 0 \quad$ and $\quad -b - 1 = 0$

$2a = 2 \quad$ and $\quad -b = 1$

$a = 1 \quad$ and $\quad b = -1$

240. If $A = \begin{bmatrix}2 & -2 & -4\\-1 & 3 & 4\\1 & -2 & -3\end{bmatrix}$, then $A^2 =$

(a) Null matrix (b) Unit matrix
(c) A (d) None of these

MHT CET-2005

Ans. (c) : Given that,

$$A=\begin{bmatrix}2&-2&-4\\-1&3&4\\1&-2&-3\end{bmatrix}$$

$$A^2=A.A=\begin{bmatrix}2&-2&-4\\-1&3&4\\1&-2&-3\end{bmatrix}\begin{bmatrix}2&-2&-4\\-1&3&4\\1&-2&-3\end{bmatrix}$$

$$A^2=\begin{bmatrix}4+2-4&-4-6+8&-8-8+12\\-2-3+4&2+9-8&4+12-12\\2+2-3&-2-6+6&-4-8+9\end{bmatrix}=\begin{bmatrix}2&-2&-4\\-1&3&4\\1&-2&-3\end{bmatrix}=A$$

241. If A and B are 2 matrices such that AB=A and BA=B, then B^2 is equal to

(a) B (b) A
(c) zero matrix (d) 1

COMEDK-2015

Ans. (a) : We have,

BA=B

On multiplying B on both side, we get –

$$(BA)B=BB$$

$$B(AB)=B^2$$

$$BA=B^2$$

$$B=B^2 \qquad \because BA=B$$

242. The matrix $\begin{bmatrix}\mathbf{a-b-c}&\mathbf{2a}&\mathbf{2a}\\\mathbf{2b}&\mathbf{b-c-a}&\mathbf{2b}\\\mathbf{2c}&\mathbf{2c}&\mathbf{c-a-b}\end{bmatrix}$ is singular, if

(a) $a=0$ (b) $a+b=0$
(c) $a-b=0$ (d) $a+b+c=0$

COMEDK-2016

Ans. (d) :

$$\begin{bmatrix}a-b-c&2a&2a\\2b&b-c-a&2b\\2c&2c&c-a-b\end{bmatrix}$$

Applying $R_1\to R_1+R_2+R_3$, we get –

$$\Delta=(a+b+c)\begin{vmatrix}1&1&1\\2b&b-c-a&2b\\2c&2c&c-a-b\end{vmatrix}$$

Applying $C_2\to C_2-C_1$, $C_3\to C_3-C_1$, we get –

$$\Delta=(a+b+c)^3\begin{vmatrix}1&0&0\\2b&-1&0\\2c&0&-1\end{vmatrix}=(a+b+c)^3.$$

Since, the given matrix is singular,

$$\therefore (a+b+c)^3=0\Rightarrow a+b+c=0$$

243. If $A=\begin{bmatrix}\mathbf{1}&\mathbf{3}\\\mathbf{3}&\mathbf{4}\end{bmatrix}$ and $A^2-KA-5I_2=O$, then the value of K is

(a) 3 (b) 5 (c) 7 (d) –7

SRM JEEE-2010

Ans. (b) : Given that,

$$A=\begin{bmatrix}1&3\\3&4\end{bmatrix}$$

And, $A^2-KA-5I_2=O$

Now,

$$A^2=\begin{bmatrix}1&3\\3&4\end{bmatrix}\begin{bmatrix}1&3\\3&4\end{bmatrix}=\begin{bmatrix}10&15\\15&25\end{bmatrix}$$

So,

$$\Rightarrow \begin{bmatrix}10&15\\15&25\end{bmatrix}-K\begin{bmatrix}1&3\\3&4\end{bmatrix}-5\begin{bmatrix}1&0\\0&1\end{bmatrix}=\begin{bmatrix}0&0\\0&0\end{bmatrix}$$

$$\begin{bmatrix}10-K-5&15-3K\\15-3K&25-4K-5\end{bmatrix}=\begin{bmatrix}0&0\\0&0\end{bmatrix}$$

On comparing corresponding elements on both side, we get –

$$10-K-5=0$$
$$5-K=0$$
$$\therefore \quad K=5$$

244. If $\begin{bmatrix}\boldsymbol{\alpha}&\boldsymbol{\beta}\\\boldsymbol{\gamma}&\boldsymbol{-\alpha}\end{bmatrix}$ is square root of identity matrix of order 2 then-

(a) $1+\alpha^2+\beta\gamma=0$ (b) $1+\alpha^2-\beta\gamma=0$
(c) $1-\alpha^2+\beta\gamma=0$ (d) $\alpha^2+\beta\gamma=1$

BITSAT-2014

Ans. (d) : According to question,

$$\begin{bmatrix}\alpha&\beta\\\gamma&-\alpha\end{bmatrix}=\sqrt{I_2}$$

On squaring both side, we get –

$$\begin{bmatrix}\alpha&\beta\\\gamma&-\alpha\end{bmatrix}\begin{bmatrix}\alpha&\beta\\\gamma&-\alpha\end{bmatrix}=\begin{bmatrix}1&0\\0&1\end{bmatrix}$$

$$\begin{bmatrix}\alpha^2+\beta\gamma&\alpha\beta-\alpha\beta\\\alpha\gamma-\alpha\gamma&\beta\gamma+\alpha^2\end{bmatrix}=\begin{bmatrix}1&0\\0&1\end{bmatrix}$$

$$\begin{bmatrix}\alpha^2+\beta\gamma&0\\0&\alpha^2+\beta\gamma\end{bmatrix}=\begin{bmatrix}1&0\\0&1\end{bmatrix}$$

On equating corresponding both the side, we get –

$$\Rightarrow \quad \alpha^2+\beta\gamma=1$$

245. The matrix $A^2+4A-5I$, where I is identity matrix and $A=\begin{bmatrix}\mathbf{1}&\mathbf{2}\\\mathbf{4}&\mathbf{-3}\end{bmatrix}$, equals :

(a) $4\begin{bmatrix}2&1\\2&0\end{bmatrix}$ (b) $4\begin{bmatrix}0&-1\\2&2\end{bmatrix}$

(c) $32\begin{bmatrix}2&1\\2&0\end{bmatrix}$ (d) $32\begin{bmatrix}1&1\\1&0\end{bmatrix}$

BITSAT-2011

Ans. (a) : According to question,

$$A^2+4A-5I$$
$$=\begin{bmatrix}1&2\\4&-3\end{bmatrix}\times\begin{bmatrix}1&2\\4&-3\end{bmatrix}+4\begin{bmatrix}1&2\\4&-3\end{bmatrix}-5\begin{bmatrix}1&0\\0&1\end{bmatrix}$$
$$=\begin{bmatrix}9&-4\\-8&17\end{bmatrix}+\begin{bmatrix}4&8\\16&-12\end{bmatrix}-\begin{bmatrix}5&0\\0&5\end{bmatrix}$$
$$=\begin{bmatrix}9+4-5&-4+8-0\\-8+16-0&17-12-5\end{bmatrix}=\begin{bmatrix}8&4\\8&0\end{bmatrix}=4\begin{bmatrix}2&1\\2&0\end{bmatrix}$$

246. $\begin{bmatrix}\mathbf{0}&\mathbf{a}\\\mathbf{b}&\mathbf{0}\end{bmatrix}^4=\mathbf{I}$ **, then**

(a) a = 1 = 2b
(b) a = b
(c) $a=b^2$
(d) ab = 1

VITEEE-2008

Ans. (d) : Here,

$$\begin{bmatrix}0&a\\b&0\end{bmatrix}^2=\begin{bmatrix}0&a\\b&0\end{bmatrix}\begin{bmatrix}0&a\\b&0\end{bmatrix}$$
$$\begin{bmatrix}0+ab&0+0\\0+0&ab+0\end{bmatrix}=\begin{bmatrix}ab&0\\0&ab\end{bmatrix}$$

Similarly, $\begin{bmatrix}0&a\\b&0\end{bmatrix}^4=\begin{bmatrix}ab&0\\0&ab\end{bmatrix}\begin{bmatrix}ab&0\\0&ab\end{bmatrix}$

$$=\begin{bmatrix}a^2b^2+0&0+0\\0+0&0+a^2b^2\end{bmatrix}=\begin{bmatrix}a^2b^2&0\\0&a^2b^2\end{bmatrix}$$

Now,

$$\begin{bmatrix}0&a\\b&0\end{bmatrix}^4=I$$
$$\begin{bmatrix}a^2b^2&0\\0&a^2b^2\end{bmatrix}=\begin{bmatrix}1&0\\0&1\end{bmatrix}$$

On comparing corresponding elements on both side, we get –

$\Rightarrow$ $a^2b^2=1$

$(ab)^2=1$

ab = 1

247. If A= $\begin{bmatrix}\mathbf{0}&\mathbf{c}&\mathbf{-b}\\\mathbf{-c}&\mathbf{0}&\mathbf{a}\\\mathbf{b}&\mathbf{-a}&\mathbf{0}\end{bmatrix}$ **and B =** $\begin{bmatrix}\mathbf{a^2}&\mathbf{ab}&\mathbf{ac}\\\mathbf{ab}&\mathbf{b^2}&\mathbf{bc}\\\mathbf{ac}&\mathbf{bc}&\mathbf{c^2}\end{bmatrix}$**.**

The AB is equal to

(a) B
(b) A
(c) O
(d) I

VITEEE-2019

Ans. (c) : Given that,

$$A=\begin{bmatrix}0&c&-b\\-c&0&a\\b&-a&0\end{bmatrix}\text{ and } B=\begin{bmatrix}a^2&ab&ac\\ab&b^2&bc\\ac&bc&c^2\end{bmatrix}$$
$$AB=\begin{bmatrix}0&c&-b\\-c&0&a\\b&-a&0\end{bmatrix}\begin{bmatrix}a^2&ab&ac\\ab&b^2&bc\\ac&bc&c^2\end{bmatrix}$$
$$AB=\begin{bmatrix}abc-abc&b^2c-b^2c&bc^2-bc^2\\-a^2c+a^2c&-abc+abc&-ac^2+ac^2\\a^2b-a^2b&ab^2-ab^2&abc-abc\end{bmatrix}$$
$$AB=\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}=O$$

AB = O

248. If [1 x 1] $\begin{bmatrix}\mathbf{1}&\mathbf{3}&\mathbf{2}\\\mathbf{0}&\mathbf{5}&\mathbf{1}\\\mathbf{0}&\mathbf{3}&\mathbf{2}\end{bmatrix}\begin{bmatrix}\mathbf{x}\\\mathbf{1}\\\mathbf{-2}\end{bmatrix}$ **= 0, then x is**

(a) $-\frac{1}{2}$
(b) $\frac{1}{2}$
(c) 1
(d) –1

VITEEE-2018

Ans. (b) : We have,

$$[1\ \ x\ \ 1]\begin{bmatrix}1&3&2\\0&5&1\\0&3&2\end{bmatrix}\begin{bmatrix}x\\1\\-2\end{bmatrix}=0$$
$$[1+0+0\ \ 3+5x+3\ \ 2+x+2]\begin{bmatrix}x\\1\\-2\end{bmatrix}=0$$
$$[1\ \ 5x+6\ \ x+4]\begin{bmatrix}x\\1\\-2\end{bmatrix}=0$$

$\Rightarrow$ $x+5x+6-2(x+4)=0$

$\Rightarrow$ $x+5x+6-2x-8=0$

$\Rightarrow$ $4x-2=0$

$\Rightarrow$ $4x=2$

$x=\frac{1}{2}$

249. If X is any matrix of order n × p (n and p are integers) and I is an identity matrix of order n × n, then the matrix M = I – X $(X'X)^{-1}$X' is
(i) idempotent matrix (ii) MX = O
Choose the correct answer

(a) (i) is correct
(b) (ii) is correct
(c) (i) is incorrect
(d) (ii) is incorrect

UPSEE-2011

Ans. (a,b) : We have,

$M=I-X(X'X)^{-1}X'$

$M=I-X[X^{-1}X']X'$

$M=I-(XX^{-1})[(X')^{-1}X']$

$M=I-I\cdot I$

$M=O$

Therefore, $M^2=M$ and $MX=O$

250. The value of $\begin{bmatrix} 1 & -\tan\frac{\theta}{4} \\ \tan\frac{\theta}{4} & 1 \end{bmatrix}\begin{bmatrix} 1 & \tan\frac{\theta}{4} \\ -\tan\frac{\theta}{4} & 1 \end{bmatrix}^{-1}$ is

(a) $\begin{bmatrix} \cos\frac{\theta}{2} & \sin\frac{\theta}{2} \\ -\sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{bmatrix}$ (b) $\begin{bmatrix} \cos\frac{\theta}{2} & -\sin\frac{\theta}{2} \\ +\sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{bmatrix}$

(c) $\begin{bmatrix} \sin\frac{\theta}{2} & \cos\frac{\theta}{2} \\ -\cos\frac{\theta}{2} & \sin\frac{\theta}{2} \end{bmatrix}$ (d) $\begin{bmatrix} \sin\frac{\theta}{2} & -\cos\frac{\theta}{2} \\ \cos\frac{\theta}{2} & \sin\frac{\theta}{2} \end{bmatrix}$

UPSEE-2011

Ans. (b) : Let,

$$A = \begin{bmatrix} 1 & -\tan\frac{\theta}{4} \\ \tan\frac{\theta}{4} & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & \tan\frac{\theta}{4} \\ -\tan\frac{\theta}{4} & 1 \end{bmatrix}^{-1}$$

We know that,

$$B^{-1} = \frac{\text{adj } B}{|B|}$$

$$\text{adj}(B) = \begin{bmatrix} 1 & \tan\frac{\theta}{4} \\ -\tan\frac{\theta}{4} & 1 \end{bmatrix}^T = \begin{bmatrix} 1 & -\tan\frac{\theta}{4} \\ \tan\frac{\theta}{4} & 1 \end{bmatrix}$$

$$|B| = \begin{vmatrix} 1 & \tan\frac{\theta}{4} \\ -\tan\frac{\theta}{4} & 1 \end{vmatrix} = 1-\left(-\tan\frac{\theta}{4}\right)\tan\frac{\theta}{4}$$

$$= 1+\tan^2\frac{\theta}{4}$$

$$= \sec^2\frac{\theta}{4}$$

Now, $$B^{-1} = \frac{1}{\sec^2\frac{\theta}{4}}\begin{bmatrix} 1 & -\tan\frac{\theta}{4} \\ \tan\frac{\theta}{4} & 1 \end{bmatrix}$$

$$B^{-1} = \begin{bmatrix} \frac{1}{\sec^2\frac{\theta}{4}} & -\tan\frac{\theta}{4}\cdot\frac{1}{\sec^2\frac{\theta}{4}} \\ \tan\frac{\theta}{4}\cdot\frac{1}{\sec^2\frac{\theta}{4}} & \frac{1}{\sec^2\frac{\theta}{4}} \end{bmatrix}$$

$$B^{-1} = \begin{bmatrix} \cos^2\frac{\theta}{4} & -\sin\frac{\theta}{4}\cdot\cos\frac{\theta}{4} \\ \sin\frac{\theta}{4}\cos\frac{\theta}{4} & \cos^2\frac{\theta}{4} \end{bmatrix}$$

According to question,

$$A\,B^{-1} = \begin{bmatrix} 1 & -\tan\frac{\theta}{4} \\ \tan\frac{\theta}{4} & 1 \end{bmatrix}\begin{bmatrix} \cos^2\frac{\theta}{4} & -\sin\frac{\theta}{4}\cos\frac{\theta}{4} \\ \sin\frac{\theta}{4}\cos\frac{\theta}{4} & \cos^2\frac{\theta}{4} \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2\frac{\theta}{4}-\tan\frac{\theta}{4}\cdot\sin\frac{\theta}{4}\cos\frac{\theta}{4} & -\sin\frac{\theta}{4}\cos\frac{\theta}{4}-\tan\frac{\theta}{4}\cdot\cos^2\frac{\theta}{4} \\ \tan\frac{\theta}{4}\cdot\cos^2\frac{\theta}{4}+\sin\frac{\theta}{4}\cdot\cos\frac{\theta}{4} & -\tan\frac{\theta}{4}\cdot\sin\frac{\theta}{4}\cdot\cos\frac{\theta}{4}+\cos^2\frac{\theta}{4} \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2\frac{\theta}{4}-\sin^2\frac{\theta}{4} & -\sin\frac{\theta}{4}\cos\frac{\theta}{4}-\sin\frac{\theta}{4}\cdot\cos\frac{\theta}{4} \\ \sin\frac{\theta}{4}\cdot\cos\frac{\theta}{4}+\sin\frac{\theta}{4}\cdot\cos\frac{\theta}{4} & -\sin^2\frac{\theta}{4}+\cos^2\frac{\theta}{4} \end{bmatrix}$$

$$= \begin{bmatrix} \cos 2\cdot\frac{\theta}{4} & -\sin\left(2\cdot\frac{\theta}{4}\right) \\ \sin 2\cdot\frac{\theta}{4} & \cos 2\cdot\frac{\theta}{4} \end{bmatrix} \quad \because 2\sin\theta\cos\theta = \sin 2\theta$$

$$= \begin{bmatrix} \cos\frac{\theta}{2} & -\sin\frac{\theta}{2} \\ \sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{bmatrix}$$

251. If $A = \begin{bmatrix} 2x & 0 \\ x & x \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}$, then x equals

(a) 2 (b) $-\frac{1}{2}$

(c) 1 (d) $\frac{1}{2}$

UPSEE -2008

Ans. (d) : Given that,

$$A = \begin{bmatrix} 2x & 0 \\ x & x \end{bmatrix} \text{ and } A^{-1} = \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}$$

We know that,

$$AA^{-1} = I$$

$$\begin{bmatrix} 2x & 0 \\ x & x \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2x-0 & 0+0\times 2 \\ x-x & 0+2x \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2x & 0 \\ 0 & 2x \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

On equating corresponding elements on both the side, we get –

$$2x = 1$$

$$x = \frac{1}{2}$$

252. $\begin{bmatrix} \mathbf{1} \\ \mathbf{-1} \\ \mathbf{2} \end{bmatrix} [\mathbf{2} \quad \mathbf{1} \quad \mathbf{-1}]$ **is equal to**

(a) $\begin{bmatrix} 2 \\ -1 \\ -2 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & 1 & -1 \\ -2 & -1 & 1 \\ 4 & 2 & -2 \end{bmatrix}$

(c) $[-1]$ (d) not defined

UPSEE-2007

Ans. (b) : Given that,

$$\begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}_{3\times1} [2 \quad 1 \quad -1]_{1\times3}$$

$$= \begin{bmatrix} 2 & 1 & -1 \\ -2 & -1 & 1 \\ 4 & 2 & -2 \end{bmatrix}_{3\times3}$$

253. **If** $\begin{bmatrix} \mathbf{1} & \mathbf{1} & \mathbf{1} \\ \mathbf{1} & \mathbf{-2} & \mathbf{-2} \\ \mathbf{1} & \mathbf{3} & \mathbf{1} \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ \mathbf{y} \\ \mathbf{z} \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{3} \\ \mathbf{4} \end{bmatrix}$**, then** $\begin{bmatrix} \mathbf{x} \\ \mathbf{y} \\ \mathbf{z} \end{bmatrix}$ **is equal to:**

(a) $\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}$

(c) $\begin{bmatrix} 5 \\ -2 \\ 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}$

UPSEE-2006

Ans. (b) : According to question,

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & -2 & -2 \\ 1 & 3 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} x+y+z \\ x-2y-2z \\ x+3y+z \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \\ 4 \end{bmatrix}$$

On equating corresponding elements on both the side we get–

$x + y + z = 0$
$x - 2y - 2z = 3$
$x + 3y + z = 4$

On solving these equation, we get –

$x = 1$, $y = 2$ and $z = -3$

$\therefore$ $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}$

254. **If** $\mathbf{A} = \begin{bmatrix} \mathbf{1} & \mathbf{1} \\ \mathbf{1} & \mathbf{1} \end{bmatrix}$**, then** $\mathbf{A^{100}}$ **is equal to:**

(a) $2^{100}A$ (b) 2^{99} A

(c) 100 A (d) 299 A

UPSEE-2006, 2009

Ans. (b) : Given that,

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$A^2 = A \cdot A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1+1 & 1+1 \\ 1+1 & 1+1 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$$

$$A^2 = 2\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$A^2 = 2A$

Now, $A^4 = A^2 \cdot A^2$

$A^4 = 2A \cdot 2A$

$A^4 = 4A^2$

$A^4 = 4 \times 2A$

$A^4 = 8A = 2^3$ A

Similarly, $A^8 = 2^7$ A

Hence, $A^{100} = 2^{99}$ A

255. **If** $\mathbf{A} = \begin{bmatrix} \mathbf{1} & \mathbf{2} \\ \mathbf{2} & \mathbf{1} \end{bmatrix}$ **and** $\boldsymbol{f}(\mathbf{x}) = \dfrac{\mathbf{1+x}}{\mathbf{1-x}}$ **find the value of** $\boldsymbol{f}$**(A).**

(a) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix}$

(c) $\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$ (d) None the these

JCECE-2012

Ans. (b) : Given that,

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$$

Now, $f(A) = \dfrac{I+A}{I-A}$

$f(A) = (I+A)(I-A)^{-1}$

Therefore, $I + A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 1+1 & 0+2 \\ 0+2 & 1+1 \end{bmatrix}$

$$I + A = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$$

And $I - A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 1-1 & 0-2 \\ 0-2 & 1-1 \end{bmatrix}$

$$I - A = \begin{bmatrix} 0 & -2 \\ -2 & 0 \end{bmatrix}$$

For the inverse of (I – A)

$$(I-A)^{-1} = \frac{\text{adj}(I-A)}{|I-A|}$$

$|I - A| = 0 - 4 = -4$

And adj $(I - A) = \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}$

$$(I-A)^{-1} = -\frac{1}{4}\begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1/2 \\ -1/2 & 0 \end{bmatrix}$$

Hence,

$$f(A) = (I + A)(I - A)^{-1}$$

$$f(A) = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} \cdot \begin{bmatrix} 0 & -1/2 \\ -1/2 & 0 \end{bmatrix}$$

$$f(A) = \begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix}$$

256. If A = diag(2, –1, 3), B = diag(–1, 3, 2) then A^2B is equal to

(a) diag (–4, 3, 18) (b) diag (5, 4, 11)
(c) diag (3, 1, 8) (d) None of these

JCECE-2008

Ans. (a) : Given that,

A = diag (2, –1, 3)

And B = diag(–1, 3, 2)

So, $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

Now,

$$A^2 = A.A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 3 \end{bmatrix} \cdot \begin{bmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$$= = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 9 \end{bmatrix}$$

Hence,

$$A^2 . B = = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 9 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$A^2.B = \begin{bmatrix} -4 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 18 \end{bmatrix}$$

Hence, the diagonal of is diag(–4, 3, 18).

257. If A and B are square matrices of the same order such that (A + B) (A – B) = $A^2 - B^2$ then $(ABA^{-1})^2$ us equal to

(a) B^2 (b) I
(c) A^2B^2 (d) A^2

BCECE-2016

Ans. (a) : Given that,

$(A + B)(A - B) = A^2 - B^2$

$A^2 - BA + AB - B^2 = A^2 - B^2$

$A^2 - B^2 - A^2 + B^2 + AB = BA$

$AB = BA$

Now, $(A BA^{-1})^2$

$= (B AA^{-1})^2$ $\quad \therefore AA^{-1} = I$

$= (BI)^2$

$= B^2$

258. If A is a square matrix such that $A^2 = A$ and B = I–A, then AB + BA + I – $(I - A)^2$ is equal to

(a) A (b) 2A
(c) – A (d) I – A

BCECE-2015

Ans. (a) : Given that,

$A^2 = A$ and $B = I - A$

Now,

$AB + BA + I - (I - A)^2$

Putting the value B we get –

$= A(I - A) + (I - A)A + I - (I - A)(I - A)$

$= AI - A^2 + AI - A^2 + I - (I - 2A + A^2)$

$= A - A^2 + A - A^2 + I - (I - 2A - A^2)$

$= A - A + A - A + I - (I - 2A + A) \because (A^2 = A)$

$= A$

259. If $A = \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}$ and $A^2 = 8A + KI$, then K is equal to

(a) –1 (b) 1
(c) –7 (d) 7

BCECE-2013

Ans. (c) : Given that,

$$A = \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix}$$

Now, $A^2 = \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1 & 7 \end{bmatrix} = \begin{bmatrix} 1+0 & 0+0 \\ -1-7 & 0+49 \end{bmatrix}$

$$A^2 = \begin{bmatrix} 1 & 0 \\ -8 & 49 \end{bmatrix}$$

Therefore,

$$A^2 = 8A + K I$$

$$\begin{bmatrix} 1 & 0 \\ -8 & 49 \end{bmatrix} = \begin{bmatrix} 8 & 0 \\ -8 & 56 \end{bmatrix} + \begin{bmatrix} K & 0 \\ 0 & K \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ -8 & 49 \end{bmatrix} = \begin{bmatrix} 8+K & 0+0 \\ -8+0 & 56+K \end{bmatrix}$$

On equating corresponding elements on both the side we get –

$8 + K = 1$

$K = -7$

260. If A = $[a_{ij}]_{2\times2}$, where $a_{ij} = i + j$, then A is equal to

(a) $\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ (d) $\begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix}$

BCECE-2012

Ans. (d) : Given that,

$A = [a_{ij}]_{2\times2}$

$a_{ij} = i + j$

So, $a_{11} = 1 + 1 = 2$

$a_{12} = 1 + 2 = 3$

$a_{21} = 2 + 1 = 3$

$a_{22} = 2 + 2 = 4$

Hence, $A = \begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix}$

261. If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is such that |A| = 0 and $A^2 - (a + d)A + kI = 0$, then k is equal to

(a) b + c (b) a + d
(c) ab + cd (d) zero

BCECE-2009

Ans. (d) : Given that,

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$A^2 = A \,.\, A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$= \begin{bmatrix} a^2+bc & ab+bd \\ ac+cd & bc+d^2 \end{bmatrix}$$

$$(a+d)A = \begin{bmatrix} a^2+ad & ab+bd \\ ac+cd & ad+d^2 \end{bmatrix}$$

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$kI = \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$$

Now,

$$A^2 - (a+d)A + kI = 0$$

$$\begin{bmatrix} a^2+bc & ab+bd \\ ac+cd & bc+d^2 \end{bmatrix} - \begin{bmatrix} a^2+ad & ab+bd \\ ac+cd & ad+d^2 \end{bmatrix} + \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} bc-ad & 0 \\ 0 & bc-ad \end{bmatrix} + \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} bc-ad & 0 \\ 0 & bc-ad \end{bmatrix} + \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

On equation corresponding elements on both the side we get –

$$bc - ad + k = 0$$
$$k = ad - cd$$

262. If a square matrix A is such that $AA^T = I = A^T A$, then $|A|$ is equal to :

(a) 0 (b) ±1
(c) ±2 (d) none of these

BCECE-2006

Ans. (b) : Given that,

$$AA^T = I = A^TA$$

Taking modulus on the side –

$$|AA^T| = |I| = |A^TA|$$
$$|A|\,|A^T| = 1 = |A^T|\,|A|$$
$$|A|^2 = 1 \qquad [\because |A^T| = A]$$
$$|A| = \pm 1$$

263. Matrix A is such that $A^2 = 2A - I$, where I is the identity matrix, then for n ≥ 2, A^n is equal to :

(a) nA – (n – 1) I (b) nA – I
(c) 2^{n-1} A – (n – 1)I (d) 2^{n-1} A – I

BCECE-2003

Ans. (a) : Given that,

$$A^2 = 2A - I$$

On multiply by A on both the side, we get –

$$A^2 \cdot A = (2A - I)A$$
$$A^3 = 2A^2 - AI \qquad \because A^2 = 2A - I$$
$$A^3 = 2(2A - I) - A$$
$$A^3 = 4A - 2I - A$$
$$A^3 = 3A - 2I$$

And $A^4 = A^3 \cdot A$

$$= (3A - 2I)A$$
$$= 3A^2 - 2AI$$
$$= 3(2A - I) - 2AI \qquad \because A^2 = 2A - I$$
$$= 6A - 3I - 2AI$$
$$= 4A - 3I$$

Hence,

$$A^n = nA - (n-1)I$$

264. Let

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix}$$

If U_1, U_2 and U_3 are column matrices satisfying

$$AU_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, AU_2 = \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}, AU_3 = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$$ **If U_1, U_2 and U_3 are column matrices satisfying**

$$AU_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, AU_2 = \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}, AU_3 = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$$ **and U is 3×3 matrix whose columns are U_1, U_2 and U_3, then what is $|U|$ equal to?**

(a) –3 (b) $\frac{3}{2}$
(c) 2 (d) 3

SCRA-2009

Ans. (d): $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix}$

Let, $U_1 = \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix}$ $U_2 = \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix}$ $U_3 = \begin{bmatrix} x_3 \\ y_3 \\ z_3 \end{bmatrix}$

$$AU_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{matrix} x_1 = 1 & \Rightarrow x_1 = 1 \\ 2x_1 + y_1 = 0 & \Rightarrow y_1 = -2 \\ 3x_1 + 2y_1 + z_1 = 0 & \Rightarrow z_1 = 1 \end{matrix}$$

$$AU_2 = \begin{pmatrix} 2 \\ 3 \\ 0 \end{pmatrix}$$

$$\begin{bmatrix}1&0&0\\2&1&0\\3&2&1\end{bmatrix}\begin{bmatrix}x_2\\y_2\\z_2\end{bmatrix}=\begin{bmatrix}2\\3\\0\end{bmatrix}\Rightarrow \begin{matrix}x_2=2 & \Rightarrow x_2=2\\ 2x_2+y_2=3 & \Rightarrow y_2=-1\\ 3x_2+2y_2+z_2=0 & \Rightarrow z_2=-4\end{matrix}$$

$$AU_3=\begin{bmatrix}2\\3\\1\end{bmatrix}$$

$$\begin{bmatrix}1&0&0\\2&1&0\\3&2&1\end{bmatrix}\begin{bmatrix}x_3\\y_3\\z_3\end{bmatrix}=\begin{bmatrix}2\\3\\1\end{bmatrix}\Rightarrow \begin{matrix}x_3=2 & \Rightarrow x_3=2\\ 2x_3+y_3=3 & \Rightarrow y_3=-1\\ 3x_3+2y_3+z_3=1 & \Rightarrow z_3=-3\end{matrix}$$

So, $|U|=$

$$\begin{vmatrix}1&2&2\\-2&-1&-1\\1&-4&-3\end{vmatrix}\Rightarrow 1(3-4)-2(6+1)+2(8+1)$$

$\Rightarrow -1-14+18=-15+18=3$

265. If $A=\begin{bmatrix}2&0&0\\0&2&0\\0&0&2\end{bmatrix}$, then A^5 is equal to

(a) 5A (b) 10A
(c) 16A (d) 32A

CG PET- 2005

Ans. (c) : We have,

$$A=\begin{bmatrix}2&0&0\\0&2&0\\0&0&2\end{bmatrix}$$

$$I=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

$A = 2I$
$A^5 = (2I)^5$
$A^5 = 16 \times 2I$

$$A^5=16\begin{bmatrix}2&0&0\\0&2&0\\0&0&2\end{bmatrix}$$

$A^5 = 16A$

266. If A and B are square matrices of order 3 such that $|A|=-1$, $|B|=3$, then $|3AB|$ is equal to

(a) –9 (b) –81
(c) –27 (d) 81

CG PET- 2005

Ans. (b) : Here, A and B are square matrices of order 3 such that $|A| = -1$, $|B| = 3$

Find; $|3AB|$

As we know that

$|AB| = |A|\ |B|$

Also for a square matrix of order 3

$|kA| = k^3 |A|$ because each element of the matrix A is multiplied by k and hence in this case we will have k^3 common.

$\therefore$ $|3AB| = 3^3 |A|\ |B|$
$= 27 (-1) (3)$
$= -81$

267. If $U=[2\ -3\ 4]$ $X=[0\ 2\ 3]$, $V=\begin{bmatrix}3\\2\\1\end{bmatrix}$ and $Y=\begin{bmatrix}2\\2\\4\end{bmatrix}$, then UV + XY is equal to

(a) 20 (b) [–20]
(c) –20 (d) [20]

CG PET- 2006

Ans. (d) : We have,

$U = [2\ \ -3\ \ 4]$, $X = [0\ \ 2\ \ 3]$

$$V=\begin{bmatrix}3\\2\\1\end{bmatrix} Y=\begin{bmatrix}2\\2\\4\end{bmatrix}$$

Now,

$$UV=[2\ \ -3\ \ 4]\begin{bmatrix}3\\2\\1\end{bmatrix}=[6-6+4]=[4]$$

And $$XY=[0\ \ 2\ \ 3]\begin{bmatrix}2\\2\\4\end{bmatrix}=[0+4+12]=[16]$$

Then, $UV + XY = [4] + [16] = [20]$

268. If matrix $A=\begin{bmatrix}1&-1\\1&1\end{bmatrix}$, then

(a) $A'=\begin{bmatrix}1&1\\1&-1\end{bmatrix}$ (b) $A^{-1}=\begin{bmatrix}1&1\\-1&1\end{bmatrix}$

(c) $A\cdot\begin{bmatrix}1&1\\-1&1\end{bmatrix}=2I$ (d) $\lambda A=\begin{bmatrix}\lambda&-\lambda\\-1&1\end{bmatrix}$,

where λ is non-zero scalar

CG PET- 2006

Ans. (c) : We have,

$$A=\begin{bmatrix}1&-1\\1&1\end{bmatrix}$$

As per given option,

Option (a) : $A^T \text{or } A'=\begin{bmatrix}1&1\\-1&1\end{bmatrix}$

Option (b) : $A^{-1}=\dfrac{1}{|A|}.\text{adj}A$

$|A| = 1 + 1 = 2$

$|A| \neq 0 \Rightarrow A^{-1}$ exist

$$\text{Adj }(A)=\begin{bmatrix}1&1\\-1&1\end{bmatrix}$$

$$A^{-1}=\frac{1}{2}\begin{bmatrix}1&1\\-1&1\end{bmatrix}$$

Option (c): L.H.S.

$$A\begin{bmatrix}1&1\\-1&1\end{bmatrix}$$

$$\begin{bmatrix}1 & -1\\1 & 1\end{bmatrix}\begin{bmatrix}1 & 1\\-1 & 1\end{bmatrix}=\begin{bmatrix}2 & 0\\0 & 2\end{bmatrix}$$

$$2\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}$$

2I = R.H.S

Option (d): $\lambda A = \lambda\begin{bmatrix}1 & -1\\1 & 1\end{bmatrix}$

$$\begin{bmatrix}\lambda & -\lambda\\1 & 1\end{bmatrix}$$

As per the above calculation option c is correct.

269. If $A=\begin{bmatrix}\alpha & 0\\1 & 1\end{bmatrix}$, $B=\begin{bmatrix}1 & 0\\5 & 1\end{bmatrix}$, then the value of α, if $A^2 = B$, will be

(a) 4 (b) 3 (c) 5 (d) None of these

AMU-2017, 2013 / CG PET- 2009

Ans. (d) : We have,

$$A=\begin{bmatrix}\alpha & 0\\1 & 1\end{bmatrix} \text{ and } B=\begin{bmatrix}1 & 0\\5 & 1\end{bmatrix}$$

Here,

$$A^2=\begin{bmatrix}\alpha & 0\\1 & 1\end{bmatrix}\begin{bmatrix}\alpha & 0\\1 & 1\end{bmatrix}$$

$$=\begin{bmatrix}\alpha^2+0 & 0+0\\\alpha+1 & 0+1\end{bmatrix}=\begin{bmatrix}\alpha^2 & 0\\\alpha+1 & 1\end{bmatrix}$$

It is given that, $A^2 = B$

$$=\begin{bmatrix}\alpha^2 & 0\\\alpha+1 & 1\end{bmatrix}=\begin{bmatrix}1 & 0\\5 & 1\end{bmatrix}$$

$\Rightarrow$ $\alpha^2 = 1$ and $\alpha + 1 = 5$

Which is not possible at the same time.

$\therefore$ No real values of α exists.

270. If $A=\begin{bmatrix}2 & +1\\-1 & 2\end{bmatrix}$ and I is the unit matrix of order 2, then A^2 is equal to

(a) 2A – 3I (b) 4A + 5I
(c) 4A – 5I (d) None of these

CG PET- 2018

Ans. (c) : Given, $A=\begin{bmatrix}2 & 1\\-1 & 2\end{bmatrix}$

$\therefore$ $A^2 = A \times A$

$$=\begin{bmatrix}2 & 1\\-1 & 2\end{bmatrix}\begin{bmatrix}2 & 1\\-1 & 2\end{bmatrix}$$

$$=\begin{bmatrix}4-1 & 2+2\\-2-2 & -1+4\end{bmatrix}=\begin{bmatrix}3 & 4\\-4 & 3\end{bmatrix} \quad \text{...(i)}$$

Now, $4A-5I=\begin{bmatrix}8 & 4\\-4 & 8\end{bmatrix}-\begin{bmatrix}5 & 0\\0 & 5\end{bmatrix}$

$$=\begin{bmatrix}3 & 4\\-4 & 3\end{bmatrix} \quad \text{...(ii)}$$

$\therefore$ From Eqs. (i) and (ii), we have

$A^2=4A-5I$

271. Let $A=\begin{bmatrix}1 & \frac{1}{5I}\\0 & 1\end{bmatrix}$. If $B=\begin{bmatrix}1 & 2\\-1 & -1\end{bmatrix}A\begin{bmatrix}-1 & -2\\1 & 1\end{bmatrix}$, then the sum of all the elements of the matrix $\sum_{n=1}^{50} B^n$ is equal to

(a) 100 (b) 50
(c) 75 (d) 125

JEE Main-12.04.2023, Shift-I

Ans. (a) : Let $C=\begin{bmatrix}1 & 2\\-1 & -1\end{bmatrix}$, $D=\begin{bmatrix}-1 & -2\\1 & 1\end{bmatrix}$

$$CD=\begin{bmatrix}1 & 2\\-1 & -1\end{bmatrix}\begin{bmatrix}-1 & -2\\1 & 1\end{bmatrix}=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}=I$$

B = CAD

$$B^n = \underbrace{(CAD)(CAD)(CAD)....(CAD)}_{n-\text{times}}$$

$\Rightarrow$ $B^n = CA^nD$

$$A^2=\begin{bmatrix}1 & \frac{1}{51}\\0 & 1\end{bmatrix}\begin{bmatrix}1 & \frac{1}{51}\\0 & 1\end{bmatrix}=\begin{bmatrix}1 & \frac{2}{51}\\0 & 1\end{bmatrix}$$

$$A^3=\begin{bmatrix}1 & \frac{3}{51}\\0 & 1\end{bmatrix}$$

Similarly $A^n=\begin{bmatrix}1 & \frac{n}{51}\\0 & 1\end{bmatrix}$

$$B^n=\begin{bmatrix}1 & 2\\-1 & -1\end{bmatrix}\begin{bmatrix}1 & \frac{n}{51}\\0 & 1\end{bmatrix}\begin{bmatrix}-1 & -2\\1 & 1\end{bmatrix}$$

$$=\begin{bmatrix}1 & \frac{n}{51}+2\\-1 & -\frac{n}{51}-1\end{bmatrix}\begin{bmatrix}-1 & -2\\1 & 1\end{bmatrix}$$

$$=\begin{bmatrix}\frac{n}{51}+1 & \frac{n}{51}\\-\frac{n}{51} & 1-\frac{n}{51}\end{bmatrix}$$

$$\sum_{n-1}^{50} B^n=\begin{bmatrix}25+50 & 25\\-25 & 50-25\end{bmatrix}=\begin{bmatrix}75 & 25\\-25 & 25\end{bmatrix}$$

Sum of the elements = 100

272. Let $A = [a_{ij}]$ be a square matrix of order 3 such that $a_{ij} = 2^j$, for all i, j = 1, 2, 3. Then, the matrix $A^2 + A^3 + ...+A^{10}$ is equal to :

(a) $\left(\frac{3^{10}-3}{2}\right)A$ (b) $\left(\frac{3^{10}-1}{2}\right)A$

(c) $\left(\frac{3^{10}+1}{2}\right)A$ (d) $\left(\frac{3^{10}+3}{2}\right)A$

JEE Main-29.06.2022, Shift-I

Ans. (a) : According to given summation,

$$A=\begin{bmatrix}a_{11}&a_{12}&a_{13}\\a_{21}&a_{22}&a_{23}\\a_{31}&a_{32}&a_{33}\end{bmatrix}=\begin{bmatrix}2^0&2^1&2^2\\2^{-1}&2^0&2^1\\2^{-2}&2^{-1}&2^0\end{bmatrix}$$

$$A^2=\begin{bmatrix}1&2&4\\\frac{1}{2}&1&2\\\frac{1}{4}&\frac{1}{2}&1\end{bmatrix}\begin{bmatrix}1&2&4\\\frac{1}{2}&1&2\\\frac{1}{4}&\frac{1}{2}&1\end{bmatrix}=\begin{bmatrix}3&6&12\\\frac{3}{2}&3&6\\\frac{3}{4}&\frac{3}{2}&3\end{bmatrix}=3A$$

Here,

$A^2 = 3A$

$A^3 = A.A^2 = A(3A) = 3A^2 = 3^2A$

$A^4 = 3^3A$

Now,

$A^2 + A^3 + \text{----} + A^{10}$

$A[3^1 + 3^2 + 3^3 + \text{----} + 3^9]$

$$=\frac{3\left[3^9-1\right]}{3-1}A=\frac{\left(3^{10}-3\right)}{2}A$$

273. Let $A=\begin{bmatrix}1&2\\-2&-5\end{bmatrix}$. Let α, $\beta \in$ R be such that $\alpha A^2 + \beta A = 2I$. Then $\alpha + \beta$ is equal to

(a) –10 (b) –6

(c) 6 (d) 10

JEE Main-27.07.2022, Shift-I

Ans. (d) : We have,

$$A=\begin{bmatrix}1&2\\-2&-5\end{bmatrix}$$

Characteristic equation of matrix A is

$|A-\pi I| = 0$

$$\begin{vmatrix}1-\pi&2\\-2&-5-\pi\end{vmatrix}=0$$

$\pi^2 + 4\pi = 1$

$A^2 + 4A = I$

$2A^2 + 8A = 2I$ ….(i)

Given that,

$\alpha A^2 + \beta A = 2I$ ….(ii)

Comparing equation (i) and (ii) we get

$\alpha = 2, \quad \beta = 8$

$\therefore \quad \alpha + \beta = 10$

274. Let $A=\begin{bmatrix}0&1&2\\a&0&3\\1&c&0\end{bmatrix}$, where a, c $\in$R. If $A^3 = A$ and the positive value of a belongs to the interval (n – 1 , n], where n $\in$ N, then n is equal to _______.

JEE Main-11.04.2023, Shift-I

Ans. (2) : $A=\begin{bmatrix}0&1&2\\a&0&3\\1&c&0\end{bmatrix}$

$A^3 = A$

$$A^2=\begin{bmatrix}0&1&2\\a&0&3\\1&c&0\end{bmatrix}\begin{bmatrix}0&1&2\\a&0&3\\1&c&0\end{bmatrix}$$

$$A^2=\begin{bmatrix}a+2&2c&3\\3&a+3c&2a\\ac&1&2+3c\end{bmatrix}$$

$$A^3=\begin{bmatrix}a+2&2c&3\\3&a+3c&2a\\ac&1&2+3c\end{bmatrix}\begin{bmatrix}0&1&2\\a&0&3\\1&c&0\end{bmatrix}$$

$$A^3=\begin{bmatrix}2ac+3&a+2+3c&2a+4+6c\\a(a+3c)+2a&3+2ac&6+3a+9c\\a+2+3c&ac+c(2+3c)&2ac+3\end{bmatrix}$$

Given $A^3 = A$

$2ac + 3 = 0$ and $a + 2 + 3c = 1$

$$c=\frac{-3}{2a}$$

$a + 1 + 3c = 0$

$$a+1-\frac{9}{2a}=0$$

$2a^2 + 2a - 9 = 0$

$f(1) < 0, f(2) > 0$

$a \in (1, 2]$

$n = 2$

275. If $A=\frac{1}{2}\begin{bmatrix}1&\sqrt{3}\\-\sqrt{3}&1\end{bmatrix}$, then

(a) $A^{30} - A^{25} = 2 + I$ (b) $A^{30} = A^{25}$

(c) $A^{30} + A^{25} - A = I$ (d) $A^{30} + A^{25} + A = I$

JEE Main-01.02.2023, Shift-II

Ans. (c) : We have,

$$A=\frac{1}{2}\begin{bmatrix}1&\sqrt{3}\\-\sqrt{3}&1\end{bmatrix}$$

$$A=\begin{bmatrix}\cos 60^\circ&\sin 60^\circ\\-\sin 60^\circ&\cos 60^\circ\end{bmatrix}$$

If, $$A=\begin{bmatrix}\cos\alpha&\sin\alpha\\-\sin\alpha&\cos\alpha\end{bmatrix}$$

$$A^2=\begin{bmatrix}\cos\alpha&\sin\alpha\\-\sin\alpha&\cos\alpha\end{bmatrix}\begin{bmatrix}\cos\alpha&\sin\alpha\\-\sin\alpha&\cos\alpha\end{bmatrix}$$

$$=\begin{bmatrix}\cos 2\alpha&\sin 2\alpha\\-\sin 2\alpha&\cos 2\alpha\end{bmatrix}$$

$$A^{30}=\begin{bmatrix}\cos 30\alpha&\sin 30\alpha\\-\sin 30\alpha&\cos 30\alpha\end{bmatrix}$$

$$A^{30}=\begin{bmatrix}1&0\\0&1\end{bmatrix}=I$$

$$A^{25} = \begin{bmatrix} \cos 25\alpha & \sin 25\alpha \\ -\sin 25\alpha & \cos 25\alpha \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{-\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$$

$A^{25} = A$
$A^{25} - A = 0$
$A^{30} + A^{25} - A = I$

276. If $A = \begin{bmatrix} i & -i \\ -i & i \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$, then A^8 equals
(a) 4 B (b) 128 B
(c) – 128 B (d) – 64 B

AMU-2007

Ans. (b) : $A = \begin{bmatrix} i & -i \\ -i & i \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

$iB = \begin{bmatrix} i & -i \\ -i & i \end{bmatrix}$

Therefore, $A = iB$
$A^2 = i^2B^2 = -B^2$

$B^2 = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}$

$-B^2 = -2B \; \{\because A^2 = -B^2\}$
$A^4 = (-2B)^2 = 4B^2 \Rightarrow 4(2B) = 8B$
$(A^4)^2 = (8\,B)^2 = 64\,B^2 = 64 \times 2B$
$A^8 = 128\,B$

277. The number of non-zero diagonal matrices of order 4 satisfying $A^2 = A$ is
(a) 2 (b) 4
(c) 16 (d) 15

AMU-2015

Ans. (d) : A non-zero diagonal matrix of order 4 satisfying $A^2 = A$

Let $A = \begin{vmatrix} d_1 & 0 & 0 & 0 \\ 0 & d_2 & 0 & 0 \\ 0 & 0 & d_3 & 0 \\ 0 & 0 & 0 & d_4 \end{vmatrix}$

$A^2 = A.A = \begin{bmatrix} d_1^2 & 0 & 0 & 0 \\ 0 & d_2^2 & 0 & 0 \\ 0 & 0 & d_3^2 & 0 \\ 0 & 0 & 0 & d_4^2 \end{bmatrix}$

Given, $A^2 = A$

$d_i^2 = d_i \left(i = 1,2,3,4\right)$

$d_i\left(d_i - 1\right) = 0$

$d_i = 0$ or 1

Each diagonal element can be chosen in 2 ways either (0 or 1) As there are 4 diagonal elements.
No of ways $\Rightarrow 2 \times 2 \times 2 \times 2 \Rightarrow 16$
No of Non zero diagonal matrix
$\Rightarrow 16 - 1 = 15$

278. If each element, of a determinant of third order with value A, is multiplied by 3, then the value of newly formed determinant is ______
(a) 3A (b) 9A
(c) 27A (d) –27 A

APEAPCET-20.08.2021, Shift-I

Ans. (c) : By property:- if we have a determinant of form

$A = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$

Then,

$kA = \begin{vmatrix} ka_{11} & ka_{12} & ka_{13} \\ a_{21} & a_{22} & 3a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$

Multiplication by some scalar k gives a new determinant with any of 3 columns or rows multiplied with k.
So if the transformed determinant is-

$\begin{vmatrix} 3a_{11} & 3a_{12} & 3a_{13} \\ 3a_{21} & 3a_{22} & 3a_{23} \\ 3a_{31} & 3a_{32} & 3a_{33} \end{vmatrix}$ then

A would have been transformed to $(3\times3\times3)\,A = 27\,A$

279. If A is a matrix $\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$, then $A^n = \begin{bmatrix} 1 & 0 \\ n & 1 \end{bmatrix} . \forall n \in N$
(a) not true for n = 3 (b) not true for n = 2
(c) true for n = 3 (d) not true for n = 1

APEAPCET-20.08.2021, Shift-I

Ans. (d): $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$

$A^2 = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$

$A^3 = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$

So option (c) is correct.

280. If $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$, then A^n is

(a) A (b) $\begin{bmatrix} 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \end{bmatrix}$

(c) $\begin{bmatrix} 3^n & 3^n & 3^n \\ 3^n & 3^n & 3^n \\ 3^n & 3^n & 3^n \end{bmatrix}$ (d) none of these

AMU-2010

Ans. (c) : $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

$$A^2 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 3 & 3 \\ 3 & 3 & 3 \\ 3 & 3 & 3 \end{bmatrix}$$

Similarly, $A^3 = \begin{bmatrix} 3^2 & 3^2 & 3^2 \\ 3^2 & 3^2 & 3^2 \\ 3^2 & 3^2 & 3^2 \end{bmatrix}$

$$\therefore A^n = \begin{bmatrix} 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \end{bmatrix}$$

281. Let X = $\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$ Let Y be a 2 × 2 real matrix satisfying the condition XY = YX. Then the smallest possible value of det (Y) is ______

(a) 0 (b) –2
(c) –1 (d) 1/2

APEAPCET- 23.08.2021, Shift-2

Ans. (a):

$X = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}, y = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$XY = \begin{bmatrix} a-c & b-d \\ a+c & b+d \end{bmatrix}$

$Y = \begin{bmatrix} a & b \\ c & d \end{bmatrix} X = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$

$YX = \begin{bmatrix} a+b & -a+b \\ c+d & -c+d \end{bmatrix}$

Given, XY = YX

$a-c = a+b$	$b-d = -a+b$	$a+c = c+d$	$b+d = -c+d$
$b = -c$	$a = d$	$a = d$	$b = -c$

=0

$|y| = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$

$|y| = \begin{vmatrix} a & -c \\ c & a \end{vmatrix}$

$|y| = a^2 + c^2$ which is always non-negative for a, c $\in$ R

$\therefore$ Smallest value of $|y| = 0$

282. The sum of the values of x so that the matrix $\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is singular, is

(a) 3 (b) 5
(c) 7 (d) 9

AP EAMCET-20.04.2019, Shift-II

Ans. (c) : According to given summation,

Let $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix} - \begin{bmatrix} x & 0 & 0 \\ 0 & x & 0 \\ 0 & 0 & x \end{bmatrix}$

$A = \begin{bmatrix} 2-x & 2 & 1 \\ 1 & 3-x & 1 \\ 1 & 2 & 2-x \end{bmatrix}$

Matrix is singular, $\therefore$ |A| = 0

$A = \begin{bmatrix} 2-x & 2 & 1 \\ 1 & 3-x & 1 \\ 1 & 2 & 2-x \end{bmatrix} = 0$

Operating $C_1 \to C_1 + C_2 + C_3$

$|A| = \begin{bmatrix} 2-x+2+1 & 2 & 1 \\ 1+3-x+1 & 3-x & 1 \\ 1+2+2-x & 2 & 2-x \end{bmatrix} = 0$

$|A| = \begin{bmatrix} 5-x & 2 & 1 \\ 5-x & 3-x & 1 \\ 5-x & 2 & 2-x \end{bmatrix} = 0$

$|A| = (5-x)\begin{bmatrix} 1 & 2 & 1 \\ 1 & 3-x & 1 \\ 1 & 2 & 2-x \end{bmatrix} = 0$

$R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$|A| = (5-x)\begin{bmatrix} 1 & 2 & 1 \\ 0 & 1-x & 0 \\ 0 & 0 & 1-x \end{bmatrix} = 0$

$|A| = (5-x)(1-x)^2 = 0$

(x = 5) (x = 1, 1)

Sum of values = 5 + 1 + 1 = 7

283. If A and B are square matrix of the same order such that AB = A and BA = B, then A and B are both

(a) singular (b) non-singular
(c) idempotent (d) involutory

AMU-2001

Ans. (c) : We have,

AB = A and BA = B

and A, B are square matrix of same order

Now, AB = A

A(BA) = A ($\because$ BA = B)

(AB)A = A ($\because$ AB = A)

A(A) = A

$A^2 = A$

In the same way, $B^2 = B$

So, A and B are idempotent matrix.

284. If A and B are two square matrices such that AB =A and BA =B, then

(a) A and B are idempotent
(b) only A is idempotent
(c) only B is idempotent
(d) none of these

AMU-2016

Ans. (a) : A and B are two square matrix given AB = A and BA = B

Consider $A^2 = (AB)^2 = (AB)(AB)$

$\Rightarrow A(BA)B \Rightarrow ABB = (AB)B = AB = A$

and $B^2 = (BA)^2 = (BA)(BA) = B(AB)A)$

$\Rightarrow BAA \Rightarrow (BA)A = BA = B$

We get $A^2 = A$ and $B^2 = B$

Both A and B are idempotent matrix

285. If $A = \begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix}$ then $4A^2 - A^3$ is equal to

(a) A (b) Identity
(c) 5A (d) zero

AMU-2004

Ans. (c) : It $A = \begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix}$

$$A^2 = \begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 4-1 & -2-2 \\ 2+2 & -1+4 \end{bmatrix} = \begin{bmatrix} 3 & -4 \\ 4 & 3 \end{bmatrix}$$

$$A^3 = A^2 A = \begin{bmatrix} 3 & -4 \\ 4 & 3 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 6-4 & -3-8 \\ 8+3 & -4+6 \end{bmatrix} = \begin{bmatrix} 2 & -11 \\ 11 & 2 \end{bmatrix}$$

$$4A^2 - A^3 = 4\begin{bmatrix} 3 & -4 \\ 4 & 3 \end{bmatrix} - \begin{bmatrix} 2 & -11 \\ 11 & 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 12 & -16 \\ 16 & 12 \end{bmatrix} - \begin{bmatrix} 2 & -11 \\ 11 & 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 10 & -5 \\ 5 & 10 \end{bmatrix}$$

$$\Rightarrow 5\begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix}$$

$$\Rightarrow 5A$$

286. If $A = \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix}$, then $A^3 - A^2$ is equal to

(a) 2A (b) 2I
(c) A (d) I

AP EAMCET-2005

Ans. (a) : We have,

$$A = \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix}$$

Here,

$$A^2 = A \times A = \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix}$$

$$A^3 = A^2 \times A = \begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix}\begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 8 \end{bmatrix}$$

$$A^3 - A^2 = \begin{bmatrix} -1 & 0 \\ 0 & 8 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ 0 & 4 \end{bmatrix} = 2\begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix} = 2A$$

287. If $A = \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$, then (A – 2I) (A – 3I) is equal to

(a) 0 (b) I
(c) –I (d) 4I

EAMCET-1995,1992

Ans. (a) : We have,

$$A = \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix}$$

Here,

$$A - 2I = \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix} - 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 4-2 & 2-0 \\ -1-0 & 1-2 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ -1 & -1 \end{bmatrix}$$

And, $$A - 3I = \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix} - 3\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 2 \\ -1 & 1 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 4-3 & 2-0 \\ -1-0 & 1-3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ -1 & -2 \end{bmatrix} \quad \text{....(i)}$$

$$\therefore \quad (A-2I)(A-3I) = \begin{bmatrix} 2 & 2 \\ -1 & -1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ -1 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 2\times1+2\times(-1) & 2\times2+2\times(-2) \\ -1\times1+(-1)\times(-1) & -1\times2+(-1)\times(-2) \end{bmatrix}$$

$$= \begin{bmatrix} 2-2 & 4-4 \\ -1+1 & 2+2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$$

288. If $A(\alpha) = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$, then $A(\alpha) . A(\beta)$ is equal to

(a) $A(\alpha) - A(\beta)$ (b) $A(\alpha) + A(\beta)$
(c) $A(\alpha + \beta)$ (d) $A(\alpha - \beta)$

EAMCET-1999

Ans. (c) : We have,

$$A(\alpha) = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$$

Here,

$$A(\alpha).A(\beta) = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}\begin{bmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{bmatrix}$$

$$= \begin{bmatrix} \cos\alpha.\cos\beta - \sin\alpha.\sin\beta & \cos\alpha.\sin\beta + \sin\alpha.\cos\beta \\ -\sin\alpha.\cos\beta - \cos\alpha.\sin\beta & \cos\alpha.\cos\beta - \sin\alpha.\sin\beta \end{bmatrix}$$

$$= \begin{bmatrix} \cos(\alpha+\beta) & \sin(\alpha+\beta) \\ -\sin(\alpha+\beta) & \cos(\alpha+\beta) \end{bmatrix}$$

$= A(\alpha + \beta)$

289. If the matrices $A=\begin{bmatrix}2 & 1 & 3\\ 4 & 1 & 0\end{bmatrix}$ and $B=\begin{bmatrix}1 & -1\\ 0 & 2\\ 5 & 0\end{bmatrix}$ then AB will be

(a) $\begin{bmatrix}17 & 0\\ 4 & -2\end{bmatrix}$ (b) $\begin{bmatrix}4 & 0\\ 0 & 4\end{bmatrix}$

(c) $\begin{bmatrix}17 & 4\\ 0 & -2\end{bmatrix}$ (d) $\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$

WB JEE-2010

Ans. (a) : We have,

$$A=\begin{bmatrix}2 & 1 & 3\\ 4 & 1 & 0\end{bmatrix} \text{ and } B=\begin{bmatrix}1 & -1\\ 0 & 2\\ 5 & 0\end{bmatrix}$$

Now,

$$AB=\begin{bmatrix}2 & 1 & 3\\ 4 & 1 & 0\end{bmatrix}.\begin{bmatrix}1 & -1\\ 0 & 2\\ 5 & 0\end{bmatrix}$$

$$AB=\begin{bmatrix}2+0+15 & -2+2+0\\ 4+0+0 & -4+2+0\end{bmatrix}=\begin{bmatrix}17 & 0\\ 4 & -2\end{bmatrix}$$

290. If $P=\begin{bmatrix}1 & 2 & 1\\ 1 & 3 & 1\end{bmatrix}$, $Q=PP^T$, then the value of the determinant of Q is

(a) 2 (b) –2
(c) 1 (d) 0

WB JEE-2012

Ans. (a) : We have,

$$P=\begin{bmatrix}1 & 2 & 1\\ 1 & 3 & 1\end{bmatrix}$$

To find: $Q = PP^T$

Here, $P^T=\begin{bmatrix}1 & 1\\ 2 & 3\\ 1 & 1\end{bmatrix}$

$$Q=PP^T=\begin{bmatrix}1 & 2 & 1\\ 1 & 3 & 1\end{bmatrix}\begin{bmatrix}1 & 1\\ 2 & 3\\ 1 & 1\end{bmatrix}$$

$$Q=\begin{bmatrix}1+4+1 & 1+6+1\\ 1+6+1 & 1+9+1\end{bmatrix}=\begin{bmatrix}6 & 8\\ 8 & 11\end{bmatrix}$$

$Q = 66 - 64 = 2$

291. Let a, b be non zero real numbers such that ab = 5/2 and given $A=\begin{bmatrix}a & -b\\ b, & a\end{bmatrix}$ and $AA^T = 20\,I$ (I is a unit matrix). Then the equation whose roots are a and b is

(a) $x^2 \mp 10x+5=0$ (b) $2x^2 \pm 10x+5=0$

(c) $x^2-5x+\frac{5}{2}=0$ (d) $x^2-25x+\frac{5}{2}=0$

AP EAMCET-19.08.2021, Shift-I

Ans. (b):

$$A=\begin{bmatrix}a & -b\\ b & a\end{bmatrix}$$

Then, $A^T=\begin{bmatrix}a & b\\ -b & a\end{bmatrix}$

$\because AA^T = 20I$

$$\Rightarrow \begin{bmatrix}a & -b\\ b & a\end{bmatrix}\begin{bmatrix}a & b\\ -b & a\end{bmatrix}=20\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}a^2+b^2 & 0\\ 0 & a^2+b^2\end{bmatrix}=\begin{bmatrix}20 & 0\\ 0 & 20\end{bmatrix}$$

$\therefore\ a^2+b^2=20$

$(a+b)^2-2ab=20$

$\Rightarrow (a+b)^2 = 20 + 2\times\frac{5}{2}$ $\left[\because ab=\frac{5}{2}\right]$

or $(a+b)^2=25$

$(a+b)=\pm 5$

Equation whose roots are a and b is given by:-
$x^2 + (a + b)\,x + ab = 0$

$x^2 \pm 5\,x + \frac{5}{2}=0$

or $2x^2 \pm 10x + 5 = 0$

292. Let $A=\begin{bmatrix}7 & 5\\ 4 & 8\end{bmatrix}$, $B=\begin{bmatrix}4 & 3\\ 7 & 5\end{bmatrix}$ and $C=\begin{bmatrix}-5 & 3\\ 7 & -4\end{bmatrix}$

If Tr(S) denotes the trace of a square matrix S then

$$\sum_{k=0}^{\infty}\frac{1}{3^k}\mathrm{Tr}\{A(BC)^k\}=$$

(a) $\frac{45}{2}$ (b) 36

(c) $\frac{81}{2}$ (d) 9

AP EAMCET-22.04.2019, Shift-II

Ans. (a) : We have,

$$A=\begin{bmatrix}7 & 5\\ 4 & 8\end{bmatrix}, B=\begin{bmatrix}4 & 3\\ 7 & 5\end{bmatrix}, C=\begin{bmatrix}-5 & 3\\ 7 & -4\end{bmatrix}$$

Here,

$$BC=\begin{bmatrix}4 & 3\\ 7 & 5\end{bmatrix}\begin{bmatrix}-5 & 3\\ 7 & -4\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=I$$

$(BC)k = I$

$$\sum_{k=0}^{\infty}\frac{1}{3^k}\mathrm{Tr}\{A(BC)^k\}=\mathrm{Tr}(A)+\frac{1}{3}\mathrm{Tr}(A)+\frac{1}{3^2}\mathrm{Tr}(A)+----\infty$$

$$\sum_{k=0}^{\infty}\frac{1}{3^k}\mathrm{Tr}\{A(BC)^k\}=\mathrm{Tr}(A)+\left(1+\frac{1}{3}+\frac{1}{3^2}+---\right)=[7+8]\left[\frac{1}{1-\frac{1}{3}}\right]$$

$$=\frac{45}{2}$$

293. **Let** $G(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$**, If x + y = 0, then G(x) G(y) =**

(a) Null Matrix
(b) Skew Symmetric Matrix
(c) Identity Matrix
(d) Symmetric Matrix

AP EAMCET-05.07.2022, Shift-I

Ans. (c) : Given,

$G(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$x + y = 0$
$y = (-x)$

$G(y) = \begin{bmatrix} \cos y & -\sin y & 0 \\ \sin y & \cos y & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos x & \sin x & 0 \\ -\sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Now,

$G(x) \cdot G(y) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos x & \sin x & 0 \\ -\sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$

Hence, G(x)· G(y) = Identity Matrix

294. **If** $A = \begin{bmatrix} 1 & 1 \\ 0 & i \end{bmatrix}$ **and** $A^{2018} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ **then (a + d) equals**

(a) 1 + i
(b) 0
(c) 2
(d) 2018

WB JEE-2022

Ans. (b) : We have,

$A = \begin{bmatrix} 1 & 1 \\ 0 & i \end{bmatrix}$

$A^2 = \begin{bmatrix} 1 & 1 \\ 0 & i \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & i \end{bmatrix} = \begin{bmatrix} 1 & 1+i \\ 0 & -1 \end{bmatrix}$

$A^3 = \begin{bmatrix} 1 & 1+i \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & i \end{bmatrix} = \begin{bmatrix} 1 & 1+i+i^2 \\ 0 & -i \end{bmatrix} = \begin{bmatrix} 1 & i \\ 0 & -i \end{bmatrix}$

$A^4 = \begin{bmatrix} 1 & i \\ 0 & -i \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & i \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Repeat after 4^{th} cycle
Here,

$A^{2018} = A^{2016} . A^2$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1+i \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 1+i \\ 0 & -1 \end{bmatrix}$

$\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 1+i \\ 0 & -1 \end{bmatrix}$

$a = 1, d = -1$
Now,

$a + d = 1 - 1 = 0$

295. **If** $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ **then the incorrect option among the following is**

(a) $A^3 - I = A(A - I)$
(b) $(A^3 + I) = A(A^3 - I)$
(c) $A^4 - I = A^2 + I$
(d) $A^2 + I = A(A^2 - I)$

AP EAMCET-23.04.2018, Shift-II

Ans. (d) : Given,

$A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

The characteristics equation is-

$A^2 + I = 0$

So, $A^3 = -A$ and $A^4 = I$

Now, if we analyse that,

$A^2 + I = A(A^2 - I)$ is incorrect.

296. **If** $A = \begin{bmatrix} p & q & r \\ r & p & q \\ q & r & p \end{bmatrix}$ **and** $AA^T = I$ **then,** $P^3 + q^3 + r^3 =$

(a) ±1
(b) pqr
(c) 3pqr
(d) 3pqr ± I

AP EAMCET-21.04.2019, Shift-II

Ans. (d) : We have,

$A = \begin{bmatrix} p & q & r \\ r & p & q \\ q & r & p \end{bmatrix}$ and $AA^T = I$

Here, A represents orthogonal matrix

∴ Det A = I

$|A| = \begin{vmatrix} p & q & r \\ r & p & q \\ q & r & p \end{vmatrix} = \pm I$

$p(p^2 - qr) - q(pr - q^2) + r(r^2 - pq) = \pm I$

$p^3 - pqr - pqr + q^3 + r^3 - pqr = \pm I$

$p^3 + q^3 + r^3 = 3pqr \pm I$

297. **If** $P = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$ **then** P^5 **is equal to**

(a) P
(b) 2P
(c) –P
(d) –2P

WB JEE-2013

Ans. (a) : We have,

$$P = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$$

$$P^2 = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}\begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$$

$$= \begin{vmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{vmatrix} = P$$

Here,

$P^2 = P, \quad P^5 = P^2 \times P^3 \Rightarrow P^5 = P^4$

(since, $P^2 = P$) $\Rightarrow P^5 = P^2 \times P^2$

$P^5 = P \times P$

$\Rightarrow \quad P^5 = P^2 = P$

$\therefore \quad P^5 = P$

298. Let P and Q are matrices such that PQ = Q and QP = P, then $P^2 + Q^2$=

(a) P (b) Q

(c) P + Q (d) P – Q

J&K CET-2017

Ans. (c) : Given,

PQ = Q and QP = P

Now, PQ = Q

Q(P Q) = Q . Q

(Q P) Q = Q^2 [$\because$ Q P = P]

P Q = Q^2 [$\because$ PQ = Q]

Q = Q^2 …… (i)

QP = P

P(QP) = P.P

(PQ)P = P^2 [$\because$ PQ = Q]

QP = P^2 [$\because$ QP = P]

P = P^2 ….. (ii)

On adding equation (i) & (ii), we get

$P^2 + Q^2 = P + Q$

299. A is a 3 × 3 matrix where its first row is (1 0 0), second row is (2 1 0) third row is (3 2 1). P, Q and R are column matrices such that AP = $(100)^T$, AQ = $(2\ 3\ 0)^T$ and AR = $(001)^T$. If P, Q and R are three columns of matrix U, then $|U|$

(a) 0 (b) 1

(c) 3 (d) 9

J&K CET-2017

Ans. (c) : $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix}$

P, Q and R are column matrix

Let, $P = \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix}, Q = \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix}$, and $R = \begin{bmatrix} x_3 \\ y_3 \\ z_3 \end{bmatrix}$

AP = $[1\ 0\ 0]^T$

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ 2x_1 + y_1 \\ 3x_1 + 2y_1 + z_1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$x_1 = 1$, $y_1 = -2$ and $Z_1 = 1$

AQ = [Z_1= 1]

AQ = $[2\ 3\ 0]^T$ (Given)

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix}\begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x_2 \\ 2x_2 + y_2 \\ 3x_2 + 2y_2 + z_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}$$

$x_2 = 2$, $y_2 = -1$, $z_2 = -4$

AR = $[0\ 0\ 1]^T$

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix}\begin{bmatrix} x_3 \\ y_3 \\ z_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x_3 \\ 2x_3 + y_3 \\ 3x_3 + 2y_3 + z_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$x_3 = 0$, $y_3 = 0$ and z_3=1

So, $U = \begin{bmatrix} 1 & 2 & 0 \\ -2 & -1 & 0 \\ 1 & -4 & 1 \end{bmatrix}$

Expanding on C_3

$|U| = 1(-1 + 4)$

$= 3$

300. Let $X = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$, then $|X^{100}|$=

(a) 1024 (b) 100

(c) 1 (d) –1

J&K CET-2016

Ans. (c) : $x = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$

$|x| = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$

$= \cos^2\alpha + \sin^2\alpha = 1$

Now,

$|x^{100}| = (1 \times 1)^{100} = (1)^{100} = 1$

301. Let $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$, then $A^3 - 6A^2 + 12A - 8I$

(a) $\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}$ (b) $\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$

(c) $\begin{bmatrix}3&0&0\\0&3&0\\0&0&3\end{bmatrix}$ (d) $\begin{bmatrix}2&0&0\\0&2&0\\0&0&0\end{bmatrix}$

J&K CET-2016

Ans. (a) : Given,

$$A=\begin{bmatrix}2&0&0\\0&2&0\\0&0&2\end{bmatrix}=2\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}=2I$$

$A^2 = 2I \cdot 2I = 2^2 I$

Similarly, $A^3 = 2^3I$

Now,

$A^3 - 6A^2 + 12A - 8I$

$= 8I - 6(4I) + 12(2I) - 8I = 0$

$$=\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}$$

302. Let E (θ) = $\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$ then E(α) E(β) is equal to

(a) $E\left(\frac{\alpha+\beta}{2}\right)$ (b) $E(\alpha\beta)$

(c) $E(\alpha + \beta)$ (d) $E(\alpha - \beta)$

J&K CET-2019

Ans. (c) : Given,

$$E(\theta)=\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$$

Now,

$$E(\alpha)E(\beta)=\begin{bmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{bmatrix}\begin{bmatrix}\cos\beta & \sin\beta\\ -\sin\beta & \cos\beta\end{bmatrix}$$

$$=\begin{bmatrix}\cos\alpha\cdot\cos\beta-\sin\alpha\cdot\sin\beta & \cos\alpha\cdot\sin\beta+\sin\alpha\cdot\cos\beta\\ -(\sin\alpha\cdot\cos\beta+\cos\alpha\cdot\sin\beta) & -\sin\alpha\cdot\sin\beta+\cos\alpha\cdot\cos\beta\end{bmatrix}$$

$$=\begin{bmatrix}\cos(\alpha+\beta) & \sin(\alpha+\beta)\\ -\sin(\alpha+\beta) & \cos(\alpha+\beta)\end{bmatrix}$$

$= E(\alpha + \beta)$

303. If A = $\begin{bmatrix}3&4\\5&6\end{bmatrix}$ and B = $\begin{bmatrix}x&0\\0&y\end{bmatrix}$, $x,y\in N$, then

(a) There is exactly one such matrix B such that AB = 1
(b) There is no matrix B such that AB = BA
(c) There exist only a finite number of matrices B such that AB = BA
(d) There exist infinite number of matrices B such that AB = BA

AP EAMCET-06.07.2022, Shift-I

Ans. (d) : Given,

$$A=\begin{bmatrix}3&4\\5&6\end{bmatrix}\text{and } B=\begin{bmatrix}x&0\\0&y\end{bmatrix}$$

$$AB=\begin{bmatrix}3&4\\5&6\end{bmatrix}\begin{bmatrix}x&0\\0&y\end{bmatrix}=\begin{bmatrix}3x&4y\\5x&6y\end{bmatrix}$$

$$BA=\begin{bmatrix}x&0\\0&y\end{bmatrix}\begin{bmatrix}3&4\\5&6\end{bmatrix}=\begin{bmatrix}3x&4x\\5y&6y\end{bmatrix}$$

If, AB = BA, then

$x = y$

Hence, there is infinite number of matrices B such that AB = BA

304. Let A = $\begin{bmatrix}2&-5\\3&1\end{bmatrix}$ what is f (A) =? Where f (x) = $x^3 - 2x^2 - 5$.

(a) $\begin{bmatrix}-50&70\\42&36\end{bmatrix}$ (b) $\begin{bmatrix}-50&70\\42&-36\end{bmatrix}$

(c) $\begin{bmatrix}-50&70\\-42&-36\end{bmatrix}$ (d) $\begin{bmatrix}-50&70\\-42&36\end{bmatrix}$

AP EAMCET-23.08.2021, Shift-I

Ans. (c):

Given, $\mathbf{A}=\begin{bmatrix}2&-5\\3&1\end{bmatrix}$

Here,

$f(x) = x^3 - 2x^2 - 5$

$\therefore f(A) = A^3 - 2A^2 - 5I$

$\therefore$ Let's first find out the value of A^3

$$A^3 = A\times A\times A=\begin{bmatrix}2&-5\\3&1\end{bmatrix}\begin{bmatrix}2&-5\\3&1\end{bmatrix}\begin{bmatrix}2&-5\\3&1\end{bmatrix}$$

$$=\begin{bmatrix}4-15&-10-5\\6+3&-15+1\end{bmatrix}\begin{bmatrix}2&-5\\3&1\end{bmatrix}$$

$$=\begin{bmatrix}-11&-15\\9&-14\end{bmatrix}\begin{bmatrix}2&-5\\3&1\end{bmatrix}$$

$$=\begin{bmatrix}-22-45&55-15\\18-42&-45-14\end{bmatrix}$$

$$A^3=\begin{bmatrix}-67&40\\-24&-59\end{bmatrix}$$

$$\therefore A^2=\begin{bmatrix}-11&-15\\9&-14\end{bmatrix}$$

$$2A^2=2\begin{bmatrix}-11&-15\\9&-14\end{bmatrix}$$

$$=\begin{bmatrix}-22&-30\\18&-28\end{bmatrix}$$

$$5I=5\begin{bmatrix}1&0\\0&1\end{bmatrix}=\begin{bmatrix}5&0\\0&5\end{bmatrix}$$

$f(A) = A^3 - 2A^2 - 5I$

$$= \begin{bmatrix} -67 & 40 \\ -24 & -59 \end{bmatrix} - \begin{bmatrix} -22 & -30 \\ 18 & -28 \end{bmatrix} - \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} -67+22-5 & 40+30-0 \\ -24-18-0 & -59+28-5 \end{bmatrix}$$

$$= \begin{bmatrix} -50 & 70 \\ -42 & -36 \end{bmatrix}$$

305. $\begin{bmatrix} 1 & 2 & 3 \\ -1 & 1 & 2 \\ 3 & 0 & 2 \end{bmatrix}^{\begin{vmatrix} 2022 & 2024 \\ 2021 & 2023 \end{vmatrix}} = ?$

(a) $\begin{bmatrix} 8 & 4 & 11 \\ 4 & -1 & 3 \\ 9 & 6 & 13 \end{bmatrix}$ (b) $\begin{bmatrix} 8 & 4 & 13 \\ 4 & -1 & 3 \\ 9 & 6 & 12 \end{bmatrix}$

(c) $\begin{bmatrix} 8 & 4 & 13 \\ 4 & -1 & 3 \\ 9 & 6 & 13 \end{bmatrix}$ (d) $\begin{bmatrix} 8 & 4 & 11 \\ 4 & 1 & 13 \\ 9 & 6 & 13 \end{bmatrix}$

AP EAMCET-23.08.2021, Shift-I

Ans. (c): Given,

$$\begin{bmatrix} 1 & 2 & 3 \\ -1 & 1 & 2 \\ 3 & 0 & 2 \end{bmatrix}^{\begin{bmatrix} 2022 & 2024 \\ 2021 & 2023 \end{bmatrix}}$$

$$= \begin{bmatrix} 1 & 2 & 3 \\ -1 & 1 & 2 \\ 3 & 0 & 2 \end{bmatrix}^{((2022)\times(2023)-(2024)\times(2021))}$$

$$= \begin{bmatrix} 1 & 2 & 3 \\ -1 & 1 & 2 \\ 3 & 0 & 2 \end{bmatrix}^{(4090506-4090504)}$$

$$= \begin{bmatrix} 1 & 2 & 3 \\ -1 & 1 & 2 \\ 3 & 0 & 2 \end{bmatrix}^2 = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 1 & 2 \\ 3 & 0 & 2 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ -1 & 1 & 2 \\ 3 & 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1-2+9 & 2+2 & 3+4+6 \\ -1-1+6 & -2+1 & -3+2+4 \\ 3+6 & 6 & 9+4 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 4 & 13 \\ 4 & -1 & 3 \\ 9 & 6 & 13 \end{bmatrix}$$

306. If A is a symmetric matrix and B is a skew - symmetric matrix such that A + B = $\begin{bmatrix} 2 & 3 \\ 5 & -1 \end{bmatrix}$, then AB is equal to

(a) $\begin{bmatrix} -4 & -2 \\ -1 & 4 \end{bmatrix}$ (b) $\begin{bmatrix} 4 & -2 \\ -1 & -4 \end{bmatrix}$

(c) $\begin{bmatrix} 4 & -2 \\ 1 & -4 \end{bmatrix}$ (d) $\begin{bmatrix} -4 & 2 \\ 1 & 4 \end{bmatrix}$

JEE Main 12.04.2019, Shift - I

Ans. (b) : Given,

$$A+B = \begin{bmatrix} 2 & 3 \\ 5 & -1 \end{bmatrix}$$

Where, A is a symmetric matrix

B is a skew - symmetric matrix

Let, $A = \begin{bmatrix} a & c \\ c & b \end{bmatrix}$ and $B = \begin{bmatrix} 0 & d \\ -d & 0 \end{bmatrix}$

$$A+B = \begin{bmatrix} a & c+d \\ c-d & b \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 5 & -1 \end{bmatrix}$$

$a = 2$, $b = -1$, $c - d = 5$, $c + d = 3$

$a = 2$, $b = -1$, $c = 4$, $d = -1$

$$AB = \begin{bmatrix} 2 & 4 \\ 4 & -1 \end{bmatrix}\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 4 & -2 \\ -1 & -4 \end{bmatrix}$$

307. Let A = $\begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}$, x ∈ R and $A^4 = [a_{ij}]$. If a_{11} = 109, then a_{22} is equal to ....

JEE Main 03.09.2020, Shift - I

Ans. (10) : Given,

$$A = \begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} x^2+1 & x \\ x & 1 \end{bmatrix}$$

$$A^4 = \begin{bmatrix} x^2+1 & x \\ x & 1 \end{bmatrix}\begin{bmatrix} x^2+1 & x \\ x & 1 \end{bmatrix}$$

$$= \begin{bmatrix} (x^2+1)^2+x^2 & x(x^2+2) \\ x(x^2+2) & x^2+1 \end{bmatrix}$$

∵ $A^4 = [a_{ij}]$ and $a_{11} = 109$

$(x^2 + 1)^2 + x^2 = 109$

$(x^2 + 1)^2 + x^2 = 100 + 9$

$x^2 = 9$

∴ $a_{22} = x^2 + 1 = 9 + 1$

$a_{22} = 10$

308. Compute the product $\begin{bmatrix} -1 & 3 & 0 \\ 2 & 1 & 4 \end{bmatrix} \times \begin{bmatrix} 1 & 3 \\ 0 & 2 \\ -2 & -1 \end{bmatrix}$

(a) $\begin{bmatrix} 1 & 3 \\ 6 & -4 \end{bmatrix}$ (b) $\begin{bmatrix} -1 & -3 \\ 6 & 4 \end{bmatrix}$

(c) $\begin{bmatrix} -1 & 3 \\ -6 & 4 \end{bmatrix}$ (d) $\begin{bmatrix} -1 & 3 \\ -6 & -4 \end{bmatrix}$

J&K CET-2014

Ans. (c) : Given,

$$\begin{bmatrix} -1 & 3 & 0 \\ 2 & 1 & 4 \end{bmatrix} \times \begin{bmatrix} 1 & 3 \\ 0 & 2 \\ -2 & -1 \end{bmatrix}$$

$$\begin{bmatrix} -1(1)+3(0)+0(-2) & -1(3)+3(2)+0(-1) \\ 2(1)+1(0)+4(-2) & 2(3)+1(2)+4(-1) \end{bmatrix}$$

$\therefore$ Product of above matrices

$$= \begin{bmatrix} -1 & 3 \\ -6 & 4 \end{bmatrix}$$

309. If A $= \begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix}$, then $A^2 + 2A - 3I =$

(a) $\begin{bmatrix} 4 & -6 \\ 6 & 4 \end{bmatrix}$ (b) 0

(c) $\begin{bmatrix} -6 & 2 \\ -2 & 6 \end{bmatrix}$ (d) 5I

J&K CET-2014

Ans. (a) : Given that,

$$A = \begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 3 & -4 \\ 4 & 3 \end{bmatrix}$$

Now, $A^2 + 2A - 3I$

$$= \begin{bmatrix} 3 & -4 \\ 4 & 3 \end{bmatrix} + 2\begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix} - 3\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -4 \\ 4 & 3 \end{bmatrix} + \begin{bmatrix} 4 & -2 \\ 2 & 4 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -4 \\ 4 & 3 \end{bmatrix} + \begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & -6 \\ 6 & 4 \end{bmatrix}$$

310. If $\begin{bmatrix} 2+x & 3 & 4 \\ 1 & -1 & 2 \\ x & 1 & -5 \end{bmatrix}$ is a singular matrix, then x is

(a) $\frac{13}{25}$ (b) $-\frac{25}{13}$

(c) $\frac{5}{13}$ (d) $\frac{25}{13}$

Manipal UGET-2017

Ans. (b) : We have martrix,

$$\begin{bmatrix} 2+x & 3 & 4 \\ 1 & -1 & 2 \\ x & 1 & -5 \end{bmatrix} \text{ is a singular matrix.}$$

$$\therefore \begin{vmatrix} 2+x & 3 & 4 \\ 1 & -1 & 2 \\ x & 1 & -5 \end{vmatrix} = 0$$

$$(2+x)(5-2)-3(-5-2x)+4(1+x)=0$$

$$6+3x+15+6x+4+4x=0$$

$$13x+25=0$$

$$\therefore \quad x = -\frac{25}{13}$$

311. If $P = \begin{bmatrix} \sqrt{3}/2 & 1/2 \\ -1/2 & \sqrt{3}/2 \end{bmatrix}, A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and Q = PAP', then $P'Q^{2005}P$ is

(a) $\begin{bmatrix} 1 & 1 \\ 2005 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 2005 \\ 0 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 2005 \\ 2005 & 1 \end{bmatrix}$

Manipal UGET-2016

Ans. (b) : Given,

$$P = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} \text{ and } Q = PAP'$$

$$\therefore \quad P' = \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$$

Now, $P'P = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

$P'Q^{2005}P = P'(PAP')^{2005}P$

$= P'[(PAP')(PAP')(PAP')\ldots\ldots 2005 \text{ times}]P = A^{2005}$

$= (P'P)A(P'P)A\ldots\ldots\ldots 2005$ times

$$\because \quad A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

$$A^2 = A \cdot A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

$$A^2 = A^2 \cdot A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$$

. . . .

. . . .

. . . .

$$\therefore \quad A^{2005} = \begin{bmatrix} 1 & 2005 \\ 0 & 1 \end{bmatrix}$$

$$\therefore \quad P'Q^{2005}P = \begin{bmatrix} 1 & 2005 \\ 0 & 1 \end{bmatrix}$$

$$\therefore \quad P'Q^{2005}P = \begin{bmatrix} 1 & 2005 \\ 0 & 1 \end{bmatrix}$$

312. If $\begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$ is to be the square root of two rowed unit matrix, then α, β and γ should satisfy the relation

(a) $1+\alpha^2+\beta\gamma=0$ (b) $1-\alpha^2-\beta\gamma=0$
(c) $1-\alpha^2+\beta\gamma=0$ (d) $\alpha^2+\beta\gamma-1=0$

Manipal UGET-2016

Ans. (b) : Given,

$$\begin{bmatrix}\alpha & \beta\\ \gamma & -\alpha\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

$\because$ 2 rowed unit matrix $=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

$$\begin{bmatrix}\alpha & \beta\\ \gamma & -\alpha\end{bmatrix}\begin{bmatrix}\alpha & \beta\\ \gamma & -\alpha\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

$$\begin{bmatrix}\alpha^2+\beta\gamma & 0\\ 0 & \alpha^2+\beta\gamma\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

$\alpha^2+\beta\gamma=1$

$1-\alpha^2-\beta\gamma=0$

313. If $\begin{bmatrix}a & 2 & 3\\ b & 5 & -1\end{bmatrix}\times\begin{bmatrix}1 & 2\\ 3 & 4\\ -1 & 1\end{bmatrix}=\begin{bmatrix}4 & 13\\ 12 & 11\end{bmatrix}$, (a,b) is

(a) (1,–2) (b) (–1,–4)
(c) (1, 3) (d) (1,–4)

Manipal UGET-2018

Ans. (d) : Given,

$$\begin{bmatrix}a & 2 & 3\\ b & 5 & -1\end{bmatrix}\begin{bmatrix}1 & 2\\ 3 & 4\\ -1 & 1\end{bmatrix}=\begin{bmatrix}4 & 13\\ 12 & 11\end{bmatrix}$$

$$\begin{bmatrix}a+6-3 & 2a+8+3\\ b+15+1 & 2b+20-1\end{bmatrix}=\begin{bmatrix}4 & 13\\ 12 & 11\end{bmatrix}$$

$$\begin{bmatrix}a+3 & 2a+11\\ b+16 & 2b+19\end{bmatrix}=\begin{bmatrix}4 & 13\\ 12 & 11\end{bmatrix}$$

$a+3=4 \quad\Rightarrow\quad a=1$

$b+16=12 \quad\Rightarrow\quad b=12-16=-4$

$\therefore\quad (a, b) = (1, -4)$

314. If A is a 3 × 3 matrix satisfying $A\begin{bmatrix}0\\1\\0\end{bmatrix}=\begin{bmatrix}-1\\2\\3\end{bmatrix}$, $A\begin{bmatrix}1\\-1\\0\end{bmatrix}=\begin{bmatrix}1\\1\\-1\end{bmatrix}$ and $A\begin{bmatrix}1\\1\\1\end{bmatrix}=\begin{bmatrix}0\\0\\12\end{bmatrix}$, then sum of the diagonal elements in A is

(a) 8 (b) 9
(c) 10 (d) 11

Assam CEE-2019

Ans. (b) : Let matrix,

$$A=\begin{bmatrix}a & b & c\\ d & e & f\\ g & h & i\end{bmatrix}$$

According to question- $A\begin{bmatrix}0\\1\\0\end{bmatrix}=\begin{bmatrix}-1\\2\\3\end{bmatrix}$

$$\therefore\quad \begin{bmatrix}a & b & c\\ d & e & f\\ g & h & i\end{bmatrix}\begin{bmatrix}0\\1\\0\end{bmatrix}=\begin{bmatrix}-1\\2\\3\end{bmatrix}$$

$\therefore\quad b=-1, e=2, h=3$

So, $A=\begin{bmatrix}a & -1 & c\\ d & 2 & f\\ g & 3 & i\end{bmatrix}$

Now,

$$\begin{bmatrix}a & -1 & c\\ d & 2 & f\\ g & 3 & i\end{bmatrix}\begin{bmatrix}1\\-1\\0\end{bmatrix}=\begin{bmatrix}1\\1\\-1\end{bmatrix}$$

$$\begin{bmatrix}a+1+0\\ d-2+0\\ g-3+0\end{bmatrix}=\begin{bmatrix}1\\1\\-1\end{bmatrix}$$

$\therefore\quad a+1=1\Rightarrow a=0$

$d-2=1\Rightarrow d=3$

$g-3=-1\Rightarrow g=2$

Now,

$$A=\begin{bmatrix}0 & -1 & c\\ 3 & 2 & f\\ 2 & 3 & i\end{bmatrix}$$

$$\begin{bmatrix}0 & -1 & c\\ 3 & 2 & f\\ 2 & 3 & i\end{bmatrix}\begin{bmatrix}1\\1\\1\end{bmatrix}=\begin{bmatrix}0\\0\\12\end{bmatrix}$$

$$\begin{bmatrix}0 & -1 & +c\\ 3 & +2 & +f\\ 2 & +3 & +i\end{bmatrix}=\begin{bmatrix}0\\0\\12\end{bmatrix}$$

$c-1=0,\ 3+2+f=0$

$c=1,\ f=-5$

$\therefore\quad 2+3+i=12$

$i=7$

Therefore sum of diagonal of matrix A

$a+e+i=0+2+7=9$

315. Suppose $A=\begin{bmatrix}a & b\\ c & d\end{bmatrix}$ is a real matrix with nonzero entries, ad – bc = 0, and A^2 = A. Then a + d equals

(a) 1 (b) 2
(c) 3 (d) 4

KVPY SB/SX-2018

Ans. (a) : We have,

$$A=\begin{bmatrix}a & b\\ c & d\end{bmatrix}$$

and $A^2=A$

$$\begin{bmatrix}a & b\\ c & d\end{bmatrix}\begin{bmatrix}a & b\\ c & d\end{bmatrix}=\begin{bmatrix}a & b\\ c & d\end{bmatrix}$$

$$\begin{bmatrix}a^2+bc & ab+bd\\ ac+cd & bc+d^2\end{bmatrix}=\begin{bmatrix}a & b\\ c & d\end{bmatrix}$$

$a^2 + bc = a$
$[\because ad - bc = 0]$
$a^2 + ad = a$
$a(a + d) = a$
$a + d = 1$

316. $\mathbf{A = \begin{bmatrix} i & -i \\ -i & i \end{bmatrix}, B = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \Rightarrow A^8}$

(a) 4B (b) 8B
(c) 64B (d) 128B

AP EAMCET-2012

Ans. (d) : We have,

$$A = \begin{bmatrix} i & -i \\ -i & i \end{bmatrix} \text{and } B = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$\therefore \quad A = i\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$A = iB$

$$A^2 = i^2B^2 = -\begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}$$

$A^2 = -2B$
$A^4 = 4B^2 = 4(2B)$
$A^4 = 8B$
$A^8 = 64B^2$
$A^8 = 64 \times 2B$
$A^8 = 128B$

317. **If** $\mathbf{A = \begin{bmatrix} x & 0 \\ 1 & 1 \end{bmatrix}}$ **and** $\mathbf{B = \begin{bmatrix} 8 & 0 \\ 7 & 1 \end{bmatrix}}$ **and** $\mathbf{A^3 = B}$, **then x =**

(a) –2 or 3 (b) –2
(c) 2 or –3 (d) 2

AP EAMCET-05.07.2022, Shift-II

Ans. (d) : Given,

$$A = \begin{bmatrix} x & 0 \\ 1 & 1 \end{bmatrix} \text{and} B = \begin{bmatrix} 8 & 0 \\ 7 & 1 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} x & 0 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} x^2 & 0 \\ x+1 & 1 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} x^2 & 0 \\ x+1 & 1 \end{bmatrix}\begin{bmatrix} x & 0 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} x^3 & 0 \\ x^2+x+1 & 1 \end{bmatrix}$$

$A^3 = B$ (given)

$$\begin{bmatrix} x^3 & 0 \\ x^2+x+1 & 1 \end{bmatrix} = \begin{bmatrix} 8 & 0 \\ 7 & 1 \end{bmatrix}$$

$x^3 = 8$
$x = 2$

318. **If** $\mathbf{A = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix}}$ **and** $\mathbf{B = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}}$, **then which one of the following is incorrect.**

(a) $(AB)' = A'B'$ (b) $A \,.\, \text{adj } A = |A|\, I$
(c) $(A + B)' = B' + A'$ (d) $(AB)^{-1} = B^{-1} . A^{-1}$

GUJCET-2021

Ans. (a) : $A = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$

By option (A)

$$AB = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1+5 & 0+5 \\ 6+7 & 0+7 \end{bmatrix}$$

$$AB = \begin{bmatrix} 6 & 5 \\ 13 & 7 \end{bmatrix}$$

$$(AB)' = \begin{bmatrix} 6 & 13 \\ 5 & 7 \end{bmatrix}$$

Now,

$$A' = \begin{bmatrix} 1 & 6 \\ 5 & 7 \end{bmatrix} \quad B' = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

$$A'B' = \begin{bmatrix} 1 & 6 \\ 5 & 7 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+0 & 1+6 \\ 5+0 & 5+7 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 7 \\ 5 & 12 \end{bmatrix} \neq (AB)'$$

319. **If** $\mathbf{AB = \begin{bmatrix} -6 & 26 \\ -1 & 19 \end{bmatrix}}$ **and** $\mathbf{11B^{-1} = \begin{bmatrix} 5 & -3 \\ 2 & 1 \end{bmatrix}}$, **then A = ________ .**

(a) $\begin{bmatrix} -2 & 4 \\ 3 & -2 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}$

(c) $\begin{bmatrix} 2 & -4 \\ -3 & 2 \end{bmatrix}$ (d) $\begin{bmatrix} -2 & 4 \\ 3 & 2 \end{bmatrix}$

GUJCET-2021

Ans. (b): Given,

$$AB = \begin{bmatrix} -6 & 26 \\ -1 & 19 \end{bmatrix}$$

$$11B^{-1} = \begin{bmatrix} 5 & -3 \\ 2 & 1 \end{bmatrix}$$

$$B^{-1} = \frac{1}{11}\begin{bmatrix} 5 & -3 \\ 2 & 1 \end{bmatrix}$$

$$ABB^{-1} = \frac{1}{11}\begin{bmatrix} -6 & 26 \\ -1 & 19 \end{bmatrix}\begin{bmatrix} 5 & -3 \\ 2 & 1 \end{bmatrix}$$

$$AI = \frac{1}{11}\begin{bmatrix} -30+52 & 18+26 \\ -5+38 & 3+19 \end{bmatrix}$$

$$A = \frac{1}{11}\begin{bmatrix} 22 & 44 \\ 33 & 22 \end{bmatrix}$$

$$\text{Hence, } A = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}$$

320. **If** $\mathbf{[2\ 3\ 4]\begin{bmatrix} 1 & x & 3 \\ 2 & 4 & 5 \\ 3 & 2 & x \end{bmatrix}\begin{bmatrix} x \\ 2 \\ 0 \end{bmatrix} = 0}$, **then x______.**

(a) $\frac{7}{3}$ (b) $\frac{5}{3}$

(c) $-\frac{5}{3}$ (d) $-\frac{7}{3}$

GUJCET-2017

Ans. (c) : Given,

$$[2\ 3\ 4]\begin{bmatrix}1 & x & 3\\ 2 & 4 & 5\\ 3 & 2 & x\end{bmatrix}\begin{bmatrix}x\\ 2\\ 0\end{bmatrix}=0$$

$$[2+6+12 \quad 2x+12+8 \quad 6+15+4x]\begin{bmatrix}x\\ 2\\ 0\end{bmatrix}=0$$

$$[20 \quad 2x+20 \quad 21+4x]\begin{bmatrix}x\\ 2\\ 0\end{bmatrix}=0$$

$$[20x+4x+40+0]=0$$

$$24x+40=0$$

$$x=\frac{-40}{24}$$

$$x=\frac{-5}{3}.$$

321. If A = [1 2] and B = $\begin{bmatrix}\mathbf{3}\\ \mathbf{4}\end{bmatrix}$, then (BA)' = ______.

(a) $\begin{bmatrix}3 & 6\\ 4 & 8\end{bmatrix}$ (b) $\begin{bmatrix}3 & 4\\ 6 & 8\end{bmatrix}$

(c) $\begin{bmatrix}4 & 8\\ 6 & 3\end{bmatrix}$ (d) [11]

GUJCET-2023

Ans. (b) : Given,

$$A=[1,2] \quad B=\begin{bmatrix}3\\ 4\end{bmatrix}$$

$$\therefore \quad BA=\begin{bmatrix}3 & 6\\ 4 & 8\end{bmatrix}$$

$$(BA)'=\begin{bmatrix}3 & 4\\ 6 & 8\end{bmatrix}$$

322. If $A=\begin{bmatrix}\mathbf{0} & \mathbf{0} & \mathbf{-5}\\ \mathbf{0} & \mathbf{-5} & \mathbf{0}\\ \mathbf{-5} & \mathbf{0} & \mathbf{0}\end{bmatrix}$, then A^2 = ________

(a) –5 I (b) 5 A

(c) 25 A (d) 25 I

GUJCET-2023

Ans. (d) : Given,

$$A=\begin{bmatrix}0 & 0 & -5\\ 0 & -5 & 0\\ -5 & 0 & 0\end{bmatrix}$$

$$A^2=\begin{bmatrix}0 & 0 & -5\\ 0 & -5 & 0\\ -5 & 0 & 0\end{bmatrix}\begin{bmatrix}0 & 0 & -5\\ 0 & -5 & 0\\ -5 & 0 & 0\end{bmatrix}$$

$$=\begin{bmatrix}0+0+25 & 0+0+0 & 0+0+0\\ 0+0+0 & 0+25+0 & 0+0+0\\ -0+0+0 & 0+0+0 & 25+0+0\end{bmatrix}$$

$$=\begin{bmatrix}25 & 0 & 0\\ 0 & 25 & 0\\ 0 & 0 & 25\end{bmatrix}$$

$$=25\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$=25I$$

323. Let A = $\begin{pmatrix}\mathbf{0} & \mathbf{1}\\ \mathbf{1} & \mathbf{k}\end{pmatrix}$, k $\in$ R and $A^3=\begin{pmatrix}\mathbf{a} & \mathbf{b}\\ \mathbf{c} & \mathbf{d}\end{pmatrix}$. If d = 228, then b + c =

(a) 52 (b) 74

(c) 2 (d) 100

TS EAMCET-19.07.2022, Shift-II

Ans. (b) : Given that,

$$A=\begin{bmatrix}0 & 1\\ 1 & k\end{bmatrix}$$

$$A^2=\begin{bmatrix}0 & 1\\ 1 & k\end{bmatrix}\begin{bmatrix}0 & 1\\ 1 & k\end{bmatrix}=\begin{bmatrix}1 & k\\ k & 1+k^2\end{bmatrix}$$

$$A^3=A^2\times A=\begin{bmatrix}1 & k\\ k & 1+k^2\end{bmatrix}\begin{bmatrix}0 & 1\\ 1 & k\end{bmatrix}$$

$$=\begin{bmatrix}k & 1+k^2\\ 1+k^2 & 2k+k^3\end{bmatrix}$$

After comparing with given value

$$A^3=\begin{bmatrix}a & b\\ c & d\end{bmatrix} \text{ we get ,}$$

$a = k$, $b = 1 + k^2$, $c = 1 + k^2$ and $d = 2k + k^3$

Given d = 228

$2k + k^3 = 228$

$k^3 + 2k - 228 = 0$

By hit and trial method , k = 6

$b + c = 1 + k^2 + 1 + k^2$

$= 2 + 2k^2$

$= 2(1 + k^2)$

$= 2(1 + 6^2)$

$= 2(1 + 36) = 2(37) = 74$

324. If A, P, B are 3 × 3 matrices. If |–B | = 5, $|BA^T|$ = 15, $|P^TAP|$ = –27, then one of the values of |P| is

(a) 3 (b) –5

(c) 9 (d) 6

TS EAMCET-18.07.2022, Shift-II

Ans. (a) : Given, A, P, B are 3 ×3 matrices

$|-B|=|5|, \ |BA^T|=15, |P^TAP|=-27$

$$(-1)^3|B| = 5$$
$$|B| = -5$$
$$|BA^T| = 15$$
$$|B||A^T| = 15 \qquad [\because |A^T| = |A|]$$
$$(-5)|A| = 15$$
$$|A| = -3$$

Again $|P^TAP| = -27$

$$|P^T||A||P| = -27$$
$$|P||A||P| = -27 \qquad [\because |P^T| = |P|]$$
$$|P|^2(-3) = -27$$
$$|P|^2 = 9$$
$$\therefore |P| = \pm 3$$

325. If $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ then $(AA')' =$

(a) $\begin{bmatrix} 14 & 32 & 50 \\ 32 & 122 & 194 \\ 50 & 194 & 256 \end{bmatrix}$ (b) $\begin{bmatrix} 14 & 50 & 32 \\ 32 & 122 & 194 \\ 50 & 194 & 122 \end{bmatrix}$

(c) $\begin{bmatrix} 14 & 32 & 50 \\ 32 & 194 & 122 \\ 32 & 122 & 77 \end{bmatrix}$ (d) $\begin{bmatrix} 14 & 32 & 50 \\ 32 & 77 & 122 \\ 50 & 122 & 194 \end{bmatrix}$

TS EAMCET-05.05.2018, Shift-I

Ans. (d) : Given that -

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

$$\therefore A' = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$$

$$\text{Now, } AA' = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}\begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$$

$$= \begin{bmatrix} 1+4+9 & 4+10+18 & 7+16+27 \\ 4+10+18 & 16+25+36 & 28+40+54 \\ 7+16+27 & 28+40+54 & 49+64+81 \end{bmatrix}$$

$$AA' = \begin{bmatrix} 14 & 32 & 50 \\ 32 & 77 & 122 \\ 50 & 122 & 194 \end{bmatrix}$$

$$\therefore (AA')' = \begin{bmatrix} 14 & 32 & 50 \\ 32 & 77 & 122 \\ 50 & 122 & 194 \end{bmatrix}$$

326. Let A, B, C be 3 × 3 non-singular matrices and I be the identity matrix of order three. If $ABA = BA^2B$ and $A^3 = I$, then $AB^4 - B^4A =$

(a) $O_{3\times3}$ (b) $\frac{I}{2}$

(c) I (d) 2I

TS EAMCET-07.05.2018, Shift-I

Ans. (a) : Given that,

$ABA = BA^2B$

And, $A^3 = I$

$ABA = BA^2B$

$ABAA^2 = BA^2BA^2$

$ABA^3 = BA^2BA^2$

$AB = BA^2BA^2$

$AB^2 = BA^2BA^2B$

$AB^2 = BA^2ABA$

$AB^2 = BA^3BA$

$AB^2 = BIBA$

$AB^4 = B^2B^2A$

$AB^4 = B^4A$

$AB^4 - B^4A = 0$

327. Let $x = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ and $A = \begin{bmatrix} -1 & 2 & 3 \\ 0 & 1 & 6 \\ 0 & 0 & -1 \end{bmatrix}$. For $k \in N$, if $X'A^kX = 33$, then k is equal to:

JEE Main-29.07.2022, Shift-II

Ans. (10) : Given,

$$x = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}; A = \begin{bmatrix} -1 & 2 & 3 \\ 0 & 1 & 6 \\ 0 & 0 & -1 \end{bmatrix}$$

$X'A^kX = 33$

$$\begin{bmatrix} 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} -1 & 2 & 3 \\ 0 & 1 & 6 \\ 0 & 0 & -1 \end{bmatrix}^k\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = 33$$

As

$$A^2 = \begin{bmatrix} -1 & 0 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, A^3 = \begin{bmatrix} -1 & 2 & 3 \\ 0 & 1 & 6 \\ 0 & 0 & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 6 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^4 = \begin{bmatrix} 1 & 0 & 6 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 6 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 12 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^8 = \begin{bmatrix} 1 & 0 & 24 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^{10} = \begin{bmatrix} 1 & 0 & 6 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 24 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 30 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\text{For } K \to \text{Even } A^k = \begin{bmatrix} 1 & 0 & 3K \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$X' A^k X = 33$ (This is not correct)

$$= \begin{bmatrix}1 & 1 & 1\end{bmatrix}\begin{bmatrix}1 & 0 & 3K\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}\begin{bmatrix}1\\1\\1\end{bmatrix}$$

$$\begin{bmatrix}1 & 1 & 1\end{bmatrix}\begin{bmatrix}1+3k\\1\\1\end{bmatrix} = 33$$

$\therefore$ $3K + 3 = 33 \therefore K = 10$

But it should be dropped as 33 is not matrix

If K is odd

$X' A^k X = 33$

$X' A A^{k-1} X = 33$

$$\begin{bmatrix}1 & 1 & 1\end{bmatrix}\begin{bmatrix}-1 & 2 & 3\\0 & 1 & 6\\0 & 0 & -1\end{bmatrix}\begin{bmatrix}1 & 0 & 3k-3\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}\begin{bmatrix}1\\1\\1\end{bmatrix} = 33$$

$$\begin{bmatrix}-1 & 3 & 8\end{bmatrix}\begin{bmatrix}3k-2\\1\\1\end{bmatrix} = [33]$$

$[-3k + 13] = [33]$

$K = 20/3$ (not possible)

328. Let $P = \begin{bmatrix}\frac{\sqrt{3}}{2} & \frac{1}{2}\\ -\frac{1}{2} & \frac{\sqrt{3}}{2}\end{bmatrix}, A = \begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}$ and $Q = PAP^T$. If $P^TQ^{2007}P = \begin{bmatrix}a & b\\c & d\end{bmatrix}$, then 2a + b – 3c – 4d equal to

(a) 2007 (b) 2005

(c) 2006 (d) 2004

JEE Main-08.04.2023, Shift-I

Ans. (b) : It is given,

$$A = \begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}, p = \begin{bmatrix}\frac{\sqrt{3}}{2} & \frac{1}{2}\\ -\frac{1}{2} & \frac{\sqrt{3}}{2}\end{bmatrix}$$

$$A^2 = \begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}, p^T = \begin{bmatrix}\frac{\sqrt{3}}{2} & -\frac{1}{2}\\ \frac{1}{2} & \frac{\sqrt{3}}{2}\end{bmatrix}$$

$$A^3 = \begin{bmatrix}1 & 2\\0 & 1\end{bmatrix}\begin{bmatrix}1 & 1\\0 & 1\end{bmatrix} = \begin{bmatrix}1 & 3\\0 & 1\end{bmatrix}$$

Therefore,

$$A^n = \begin{bmatrix}1 & n\\0 & 1\end{bmatrix}$$

$$pp^T = \begin{bmatrix}\frac{\sqrt{3}}{2} & \frac{1}{2}\\ -\frac{1}{2} & \frac{\sqrt{3}}{2}\end{bmatrix}\begin{bmatrix}\frac{\sqrt{3}}{2} & -\frac{1}{2}\\ \frac{1}{2} & \frac{\sqrt{3}}{2}\end{bmatrix}$$

$$= \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} = I$$

Now, $\begin{bmatrix}a & b\\c & d\end{bmatrix} = p^TQ^{2007}p$

$= P^T(PAP)^T\ (PAP^T)\ (PAP^T)\ \ldots\ldots..(PAP^T)P$

$= IAIA\ \ldots\ldots\ldots.IAI = A^{2007}$

$$\begin{bmatrix}a & b\\c & d\end{bmatrix} = \begin{bmatrix}1 & 2007\\0 & 1\end{bmatrix}$$

$\because$ $A^n = \begin{bmatrix}1 & n\\0 & 1\end{bmatrix}$

On comparing both sides,

$a = 1, b = 2007, c = 0, d = 1$

$2a + b - 3c - 4d = 2005$

329. The matrix product satisfies [5 6 2] . A^T = [4 8 1 7 8] where A^T denotes the transpose of the matrix A. then order of the matrix A equals to

(a) 1 ×2 (b) 5 × 1

(c) 3× 5 (d) 5×3

J&K CET-2011

Ans. (d) : Given,

$\begin{bmatrix}5 & 6 & 2\end{bmatrix}$ is of order 1 × 3

$\begin{bmatrix}4 & 8 & 1 & 7 & 8\end{bmatrix}$ is of order 1 × 5

(1 × 3) × (p × q) gives 1 × 5

So,

p = 3 and q = 5

Thus A^T is of order 3 × 5

$\therefore$ A is of order 5 × 3

330. If A and B are square matrices such that AB = A and BA = B, then $A^2 + B^2$ =

(a) AB (b) A + B

(c) BA + B (d) AB + A

J&K CET-2010

Ans. (b) : Given,

AB = A and BA = B

Consider $A^2 + B^2 = AA + BB$

$(AB)A + (BA)B$ [$\because$ AB = A and BA = B]

$= A (BA) + B (AB) = AB + BA = A + B$

331. If $A = \begin{bmatrix}2 & -3\\-4 & 1\end{bmatrix}$, then adj $(3A^2 + 12A)$ is equal to

(a) $\begin{bmatrix}72 & -84\\-63 & 51\end{bmatrix}$ (b) $\begin{bmatrix}51 & 63\\84 & 72\end{bmatrix}$

(c) $\begin{bmatrix}51 & 84\\63 & 72\end{bmatrix}$ (d) $\begin{bmatrix}72 & -63\\-84 & 51\end{bmatrix}$

MHT CET-2022

Ans. (c) : Given, $A = \begin{bmatrix}2 & -3\\-4 & 1\end{bmatrix}$, $(3A^2 + 12A) = ?$

$$A^2 = A.A = \begin{vmatrix}2 & -3\\-4 & 1\end{vmatrix}.\begin{vmatrix}2 & -3\\-4 & 1\end{vmatrix} = \begin{vmatrix}4+12 & -6-3\\-8-4 & 12+1\end{vmatrix} = \begin{vmatrix}16 & -9\\-12 & 13\end{vmatrix}$$

$$(3A^2 + 12A) = 3\begin{vmatrix}16 & -9\\-12 & 13\end{vmatrix} + 12\begin{vmatrix}2 & -3\\-4 & 1\end{vmatrix}$$

$$= \begin{vmatrix} 48 & -27 \\ -36 & 39 \end{vmatrix} + \begin{vmatrix} 24 & -36 \\ -48 & 12 \end{vmatrix}$$

$$3A^2 + 12A = \begin{vmatrix} 72 & -63 \\ -84 & 51 \end{vmatrix}$$

$$\text{Adj}\left(3A^2 + 12A\right) = \begin{bmatrix} 51 & 84 \\ 63 & 72 \end{bmatrix}^T$$

$$\therefore \text{Adj}\left(3A^2 + 12A\right) = \begin{bmatrix} 51 & 84 \\ 63 & 72 \end{bmatrix}$$

332. If $A = \begin{bmatrix} 1 & a & 3 \\ 1 & 1 & 5 \\ 2 & 4 & 7 \end{bmatrix}$ and $A^{-1} \begin{bmatrix} 13 & 2 & -7 \\ -3 & b & 2 \\ -2 & 0 & 1 \end{bmatrix}$, then the values of a and b are respectively

(a) 2, –1 (b) –1, 2

(c) 2, 1 (d) 1, 2

MHT CET-2022

Ans. (a) : Given,

$$A = \begin{bmatrix} 1 & a & 3 \\ 1 & 1 & 5 \\ 2 & 4 & 7 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 13 & 2 & -7 \\ -3 & b & 2 \\ -2 & 0 & 1 \end{bmatrix}$$

As we know that,

$$A \cdot A^{-1} = I$$

$$\begin{vmatrix} 1 & a & 3 \\ 1 & 1 & 5 \\ 2 & 4 & 7 \end{vmatrix} \begin{vmatrix} 13 & 2 & -7 \\ -3 & b & 2 \\ -2 & 0 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$$\begin{bmatrix} -3a+7 & ab+2 & 2a-4 \\ 0 & b+2 & 0 \\ 0 & 4b+4 & 1 \end{bmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

Comparing between both sides,

$-3a + 7 = 1$

$3a = 6$

$a = 2$

and $ab + 2 = 0$

$2b = -2$

$b = -1$

$a, b = 2, -1$

333. If $\begin{bmatrix} 1 & 1 & 1 \\ 1 & -2 & -2 \\ 1 & 3 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \\ 4 \end{bmatrix}$, then $2x - y + z =$

(a) 2 (b) 3

(c) –3 (d) 1

MHT CET-2022

Ans. (c) : Given, $\begin{bmatrix} 1 & 1 & 1 \\ 1 & -2 & -2 \\ 1 & 3 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \\ 4 \end{bmatrix}$

$x + y + z = 0$ .....(i)

$x - 2y - 2z = 3$ .....(ii)

$x + 3y + z = 4$ ......(iii)

Subtracting equation (i) from equation (iii)

$x + 3y + z = 4$

$x + y + z = 0$

$2y = 4$

$y = 2$

Adding equation (i) , (ii) and (iii)

$3x + 2y = 7$

$3x + 2 \times 2 = 7$

$3x = 3$

$x = 1, z = -3$

Now, $2x - y + z = 2 \times 1 - 2 + (-3) = -3$

$2x - y + z = -3$

334. If $A = \left[a_{ij}\right]_{3\times3} = \begin{bmatrix} 3 & 2 & 4 \\ 1 & 4 & 1 \\ 2 & 6 & 3 \end{bmatrix}$ and a_{ij}, is a cofactor of a_{ij} then the value of $a_{21}A_{21} + a_{22}A_{22} + a_{23}A_{23}$ is equal to

(a) 18 (b) 8

(c) –8 (d) 0

MHT CET-2022

Ans. (b) : Given, $A = [a, j]_{3\times3} = \begin{bmatrix} 3 & 2 & 4 \\ 1 & 4 & 1 \\ 2 & 6 & 3 \end{bmatrix}$

$a_{21} = 1, a_{22} = 4, a_{23} = 1$

$$A_{21} = (-1)^{2+1} \begin{vmatrix} 2 & 4 \\ 6 & 3 \end{vmatrix} = -(6-24) = 18$$

$$A_{22} = (-1)^{2+2} \begin{bmatrix} 3 & 4 \\ 2 & 3 \end{bmatrix} = 1$$

$$A_{23} = (-1)^{2+3} \begin{vmatrix} 3 & 2 \\ 2 & 6 \end{vmatrix} = -(18-4) = -14$$

Now, $a_{21}A_{21} + a_{22}A_{22} + a_{33}A_{33}$

$= 1 \times 18 + 4 \times 1 + 1 \times -14$

$= 18 + 4 - 14 = 8$

335. If $A = [a_{ij}]_{3\times3} = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 5 \\ 2 & 4 & 7 \end{bmatrix}$ and A_{ij} is a cofactor of a_{ij}, then $a_{11}A_{21} + a_{12}A_{22} + a_{13}A_{23}$ is equal to

(a) 2 (b) 0

(c) –1 (d) 1

MHT CET-2022

Ans. (b) : Given, $A\left[a_{ij}\right]_{3\times3} = \begin{vmatrix} 1 & 2 & 3 \\ 1 & 1 & 5 \\ 2 & 4 & 7 \end{vmatrix}$

$a_{11} = 1, a_{12} = 2, a_{13} = 3$

$$A_{21} = (-1)^{2+1} \begin{vmatrix} 2 & 3 \\ 4 & 7 \end{vmatrix} = -2$$

$$A_{22} = (-1)^{2+2} \begin{vmatrix} 1 & 3 \\ 2 & 7 \end{vmatrix} = 1$$

$A_{23} = (-1)^{2+3}\begin{vmatrix}1 & 2\\ 2 & 4\end{vmatrix} = 0$

Now, $a_{11}A_{21} + a_{12}A_{22} + a_{13}A_{23}$
$= 1 \times -2 + 2 \times 1 + 3 \times 0$
$= 0$

336. If A = [2 0 6] and B = $\begin{bmatrix}3 & 5\\ 7 & -2\\ 6 & 6\end{bmatrix}$, then AB =

(a) $\begin{bmatrix}42 & 46\end{bmatrix}$ (b) $\begin{bmatrix}42\\ 46\end{bmatrix}$

(c) $\begin{bmatrix}6 & 10\\ 0 & 0\\ 36 & 36\end{bmatrix}$ (d) [17 19]

(e) $\begin{bmatrix}2 & 12\\ 14 & -4\end{bmatrix}$

Kerala CEE-2022

Ans. (a) : Given, $A = \begin{bmatrix}2 & 0 & 6\end{bmatrix}$, $B = \begin{bmatrix}3 & 5\\ 7 & -2\\ 6 & 6\end{bmatrix}$

$$AB = \begin{bmatrix}2 & 0 & 6\end{bmatrix}\begin{bmatrix}3 & 5\\ 7 & -2\\ 6 & 6\end{bmatrix}$$

$$AB = \begin{bmatrix}6+0+36 & 10+0+36\end{bmatrix}$$

$$AB = \begin{bmatrix}42 & 46\end{bmatrix}$$

337. If $\begin{bmatrix}-1 & 3\\ 4 & -5\\ 0 & 2\end{bmatrix}\begin{bmatrix}1 & 2\\ 0 & 7\end{bmatrix} = \begin{bmatrix}-1 & 19\\ \alpha & -27\\ 0 & 14\end{bmatrix}$, then the value of α is

(a) 5 (b) 4
(c) 7 (d) – 14
(e) – 5

Kerala CEE-2020

Ans. (b) : Given that,

$$\begin{bmatrix}-1 & 3\\ 4 & -5\\ 0 & 2\end{bmatrix}\begin{bmatrix}1 & 2\\ 0 & 7\end{bmatrix} = \begin{bmatrix}-1 & 19\\ \alpha & -27\\ 0 & 14\end{bmatrix}$$

$$\begin{bmatrix}-1+0 & -2+21\\ 4+0 & 8-35\\ 0+0 & 0+14\end{bmatrix} = \begin{bmatrix}-1 & 19\\ \alpha & -27\\ 0 & 14\end{bmatrix}$$

$$\begin{bmatrix}-1 & 19\\ 4 & -27\\ 0 & 14\end{bmatrix} = \begin{bmatrix}-1 & 19\\ \alpha & -27\\ 0 & 14\end{bmatrix}$$

By comparing
$\alpha = 4$

338. Let $A = \begin{bmatrix}\alpha & 0\\ 1 & 1\end{bmatrix}$ and $B = \begin{bmatrix}1 & 0\\ 5 & 1\end{bmatrix}$ be two matrices where α is a real number. Then

(a) $A^2 = B$ for some α (b) $A^2 \neq B$ for any α
(c) $A^2 = -B$ for some α (d) $|A^2| \neq |B|$ for any α
(e) $A = -B$ for some α

Kerala CEE-2019

Ans. (b) : Given that,

$$A = \begin{bmatrix}\alpha & 0\\ 1 & 1\end{bmatrix},\ B = \begin{bmatrix}1 & 0\\ 5 & 1\end{bmatrix}$$

$$A.A = A^2 = \begin{bmatrix}\alpha & 0\\ 1 & 1\end{bmatrix}\begin{bmatrix}\alpha & 0\\ 1 & 1\end{bmatrix} = \begin{bmatrix}\alpha^2+0 & 0+0\\ \alpha+1 & 0+1\end{bmatrix} = \begin{bmatrix}\alpha^2 & 0\\ \alpha+1 & 1\end{bmatrix}$$

So, for any value of α
$A^2 \neq B$

339. If $A = \begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix}$, then A^{2017} is equal to

(a) 2^{1015} A (b) 2^{2016} A
(c) 2^{2014} A (d) 2^{2017} A
(e) 2^{2020} A

Kerala CEE-2017

Ans. (b) : Given that,

$$A = \begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}$$

$$\therefore\quad A^2 = A.A = \begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix} = \begin{bmatrix}2 & 2\\ 2 & 2\end{bmatrix} = 2\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix} = 2A$$

Again $A^3 = A^2.\ A$

$$= \begin{bmatrix}2 & 2\\ 2 & 2\end{bmatrix}\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix} = \begin{bmatrix}4 & 4\\ 4 & 4\end{bmatrix} = 4\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix} = 2^2.A$$

$\therefore\quad A^n = 2^{n-1} A$
$\therefore\quad A^{2017} = 2^{2016}A$

340. If $A = \begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix}$, then $A^n + nI$ is equal to

(a) I (b) nA
(c) I + nA (d) I – nA
(e) nA – I

Kerala CEE-2017

Ans. (c) : Given that,

$$A = \begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix}$$

$$\therefore\quad A^2 = A.A = \begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix} = \begin{bmatrix}1 & 0\\ 2 & 1\end{bmatrix}$$

$$A^3 = A^2.A = \begin{bmatrix}1 & 0\\ 2 & 1\end{bmatrix}\begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix} = \begin{bmatrix}1 & 0\\ 3 & 1\end{bmatrix}$$

$$\therefore\quad A^n = \begin{bmatrix}1 & 0\\ n & 1\end{bmatrix}$$

Now, $A^n + nI = \begin{pmatrix}1 & 0\\ n & 1\end{pmatrix} + n\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$

$$= \begin{pmatrix} 1 & 0 \\ n & 1 \end{pmatrix} + \begin{pmatrix} n & 0 \\ 0 & n \end{pmatrix}$$

$$= \begin{pmatrix} 1+n & 0 \\ n & 1+n \end{pmatrix}$$

Again $\quad I + nA = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + n\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} n & 0 \\ n & n \end{pmatrix}$$

$$= \begin{pmatrix} 1+n & 0 \\ n & 1+n \end{pmatrix}$$

$\therefore \quad A^n + nI = I + nA$

341. If $A = \begin{pmatrix} 1 & 5 \\ 0 & 2 \end{pmatrix}$, then

(a) $A^2 - 2A + 2I = 0$ (b) $A^2 - 3A + 2I = 0$
(c) $A^2 - 5A + 2I = 0$ (d) $2A^2 - A + I = 0$
(e) $A^2 + 3A + 2I = 0$

Kerala CEE-2017

Ans. (b) : Given that,

$$A = \begin{pmatrix} 1 & 5 \\ 0 & 2 \end{pmatrix}$$

$$A^2 = A.A = \begin{pmatrix} 1 & 5 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} 1 & 5 \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} 1+0 & 5+10 \\ 0+0 & 0+4 \end{pmatrix} = \begin{pmatrix} 1 & 15 \\ 0 & 4 \end{pmatrix}$$

$$A^2 - 3A + 2I = \begin{pmatrix} 1 & 15 \\ 0 & 4 \end{pmatrix} - 3\begin{pmatrix} 1 & 5 \\ 0 & 2 \end{pmatrix} + 2\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 15 \\ 0 & 4 \end{pmatrix} - \begin{vmatrix} 3 & 15 \\ 0 & 6 \end{vmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 15 \\ 0 & 6 \end{bmatrix} - \begin{bmatrix} 3 & 15 \\ 0 & 6 \end{bmatrix}$$

$$A^2 - 3A + 2I = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$$

342. If $\begin{pmatrix} 1 & 2 & -3 \\ 0 & 4 & 5 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$, then (x, y, z) is equal to

(a) (1, 6, 6) (b) (1, –6, 1)
(c) (1, 1, 6) (d) (6, –1, 1)
(e) (–1, 6, 1)

Kerala CEE-2017

Ans. (d) : Given that,

$$\begin{pmatrix} 1 & 2 & -3 \\ 0 & 4 & 5 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$x + 2y - 3z = 1$ ….(i)
$4y + 5z = 1$ …..(ii)
$z = 1$ …..(iii)

By equation (i), (ii) and (iii)
$y = -1, z = 1, x = 6$
Now, $(x, y, z) = (6, -1, 1)$

343. $\begin{pmatrix} 7 & 1 & 5 \\ 8 & 0 & 0 \end{pmatrix}\begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} + 5\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ is equal to

(a) $\begin{pmatrix} 16 \\ 27 \end{pmatrix}$ (b) $\begin{pmatrix} 27 \\ 16 \end{pmatrix}$
(c) $\begin{pmatrix} 15 \\ 16 \end{pmatrix}$ (d) $\begin{pmatrix} 16 \\ 15 \end{pmatrix}$
(e) $\begin{pmatrix} 16 \\ 16 \end{pmatrix}$

Kerala CEE-2017

Ans. (b) : Given,

$$\begin{pmatrix} 7 & 1 & 5 \\ 8 & 0 & 0 \end{pmatrix}\begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} + 5\begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} 14+3+5 \\ 16+0+0 \end{pmatrix} + \begin{pmatrix} 5 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} 22 \\ 16 \end{pmatrix} + \begin{pmatrix} 5 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} 27 \\ 16 \end{pmatrix}$$

344. Let $A = \begin{bmatrix} 1 & \frac{-1-i\sqrt{3}}{2} \\ \frac{-1+i\sqrt{3}}{2} & 1 \end{bmatrix}$. Then, A^{100} is equal to

(a) 2^{100} A (b) 2^{99} A
(c) 2^{98} A (d) A
(e) A^2

Kerala CEE-2016

Ans. (b) : Given,

$$A = \begin{pmatrix} 1 & \frac{-1-i\sqrt{3}}{2} \\ \frac{-1+i\sqrt{3}}{2} & 1 \end{pmatrix}$$

Let, $\quad \frac{-1+i\sqrt{3}}{2} = \omega$

$$\omega^2 = \frac{\left(-1+i\sqrt{3}\right)^2}{(2)^2} = \frac{-2-2i\sqrt{3}}{4} = \frac{-1-i\sqrt{3}}{2}$$

$\therefore \quad A = \begin{pmatrix} 1 & \omega^2 \\ \omega & 1 \end{pmatrix}$

$$A^2 = \begin{pmatrix} 1 & \omega^2 \\ \omega & 1 \end{pmatrix}\begin{pmatrix} 1 & \omega^2 \\ \omega & 1 \end{pmatrix} = \begin{pmatrix} 2 & 2\omega^2 \\ 2\omega & 2 \end{pmatrix}$$

$(\because \omega^3 = 1)$

$$= 2\begin{pmatrix} 1 & \omega^2 \\ \omega & 1 \end{pmatrix} = 2A$$

$\therefore$ $A^n = 2^{n-1}A$

$A^{100} = 2^{100-1}A$

$A^{100} = 2^{99}A$

345. If $A = \begin{vmatrix} 8 & 27 & 125 \\ 2 & 3 & 5 \\ 1 & 1 & 1 \end{vmatrix}$, then the value of A^2 is equal to

(a) 0 (b) 36
(c) 64 (d) 2400
(e) 3600

Kerala CEE-2016

Ans. (e) : Given that,

$$A = \begin{vmatrix} 8 & 27 & 125 \\ 2 & 3 & 5 \\ 1 & 1 & 1 \end{vmatrix}$$

$= 8(3-5) - 27(2-5) + 125(2-3)$

$= 8(-2) - 27(-3) + 125(-1)$

$= -16 + 81 - 125 = -60$

$\therefore$ $A^2 = (-60)^2 = 3600$

346. If $[1\ x\ 1]\begin{bmatrix} 1 & 3 & 2 \\ 0 & 5 & 1 \\ 0 & 2 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ x \end{bmatrix} = 0$, the values of x are

(a) 1, 5 (b) –1, –5
(c) 1, 6 (d) –1, –6
(e) 3, 3

Kerala CEE-2016

Ans. (d) : Given that,

$$[1\ \ x\ \ 1]\begin{bmatrix} 1 & 3 & 2 \\ 0 & 5 & 1 \\ 0 & 2 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ x \end{bmatrix} = 0$$

$$[1\ \ x\ \ 1]\begin{bmatrix} 1+3+2x \\ 0+5+x \\ 0+2+0 \end{bmatrix} = 0$$

$$[1\ \ x\ \ 1]\begin{bmatrix} 4+2x \\ 5+x \\ 2 \end{bmatrix} = 0$$

$[4 + 2x + 5x + x^2 + 2] = 0$

$x^2 + 7x + 6 = 0$

$x^2 + x + 6x + 6 = 0$

$x(x + 1) + 6(x + 1) = 0$

$(x + 1)(x + 6) = 0$

$x = -1, -6$

347. If the square of the matrix $\begin{bmatrix} a & b \\ a & -a \end{bmatrix}$ is the unit matrix, then b is equal to

(a) $\frac{a}{1+a^2}$ (b) $\frac{1-a^2}{a}$
(c) $\frac{1+a^2}{a}$ (d) $\frac{a}{1-a^2}$
(e) $1 + a^2$

Kerala CEE-2016

Ans. (b) : Given that,

Let, $A = \begin{bmatrix} a & b \\ a & -a \end{bmatrix}$

$A = -(ab + a^2)$

$\therefore$ Square of the matrix A is unit matrix

$$A^2 = (ab + a^2)^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 1 - 0$$

$(ab + a^2)^2 = (1)^2$

$ab + a^2 = 1$

$$b = \frac{1-a^2}{a}$$

348. If $A = (a_{ij})_{m\times n}$, $B = (b_{ij})_{n\times p}$, $C = (c_{ij})_{p\times q}$, then the product (BC) A is possible only when

(a) m = q (b) n = q
(c) p = q (d) m = p
(e) m = n

Kerala CEE-2012

Ans. (a) : Since, given

$A = (a_{ij})_{m\times n}$,

$B = (b_{ij})_{n\times p}$

$C = (c_{ij})_{p\times q}$

Product of BC is of n × q but order of A is m × n. Therefore, the product of BC and A is only possible if q = m

349. If $A = \begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}$ and A^2 is the unit matrix, then the value of $x^3 + x - 2$ is equal to

(a) – 8 (b) – 2
(c) 0 (d) 1
(e) 8

Kerala CEE-2011

Ans. (b) : Given, $A = \begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}$

$$A^2 = A.A = \begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} x & 1 \\ 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} x^2+1 & x \\ x & 1 \end{bmatrix}$$

$$\therefore \quad A^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 1 = \begin{bmatrix} x^2+1 & x \\ x & 1 \end{bmatrix}$$

By comparing we get –

$x = 0$

Now, $x^3 + x - 2 = 0 + 0 - 2 = -2$

350. If $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$, then A^{10} is equal to

(a) $\begin{pmatrix} \cos^{10}\alpha & \sin^{10}\alpha \\ -\sin^{10}\alpha & \cos^{10}\alpha \end{pmatrix}$

(b) $\begin{pmatrix} \cos^{10}\alpha & -\sin^{10}\alpha \\ \sin^{10}\alpha & \cos^{10}\alpha \end{pmatrix}$

(c) $\begin{pmatrix} \cos^{10}\alpha & \sin^{10}\alpha \\ -\sin^{10}\alpha & -\cos^{10}\alpha \end{pmatrix}$

(d) $\begin{pmatrix} \cos 10\alpha & \sin 10\alpha \\ -\sin 10\alpha & \cos 10\alpha \end{pmatrix}$

(e) $\begin{pmatrix} \cos 10\alpha & -\sin 10\alpha \\ -\sin 10\alpha & -\cos 10\alpha \end{pmatrix}$

Kerala CEE-2011

Ans. (d) : Given,

$$A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$$

$$A^2 = A.A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}\begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2\alpha - \sin^2\alpha & \cos\alpha\sin\alpha + \sin\alpha\cos\alpha \\ -\sin\alpha\cos\alpha - \cos\alpha\sin\alpha & -\sin^2\alpha + \cos^2\alpha \end{bmatrix}$$

$$A^2 = \begin{bmatrix} \cos 2\alpha & \sin 2\alpha \\ -\sin 2\alpha & \cos 2\alpha \end{bmatrix} \quad \ldots\ldots(i)$$

$$A^3 = A^2.A = \begin{bmatrix} \cos(2\alpha+\alpha) & \sin(2\alpha+\alpha) \\ -\sin(2\alpha+\alpha) & \cos(2\alpha+\alpha) \end{bmatrix} = \begin{bmatrix} \cos 3\alpha & \sin 3\alpha \\ -\sin 3\alpha & \cos 3\alpha \end{bmatrix}$$

On observing equation (i) and (ii), we get –

$$A^n = \begin{bmatrix} \cos n\alpha & \sin n\alpha \\ -\sin n\alpha & \cos n\alpha \end{bmatrix}$$

$$A^{10} = \begin{bmatrix} \cos 10\alpha & \sin 10\alpha \\ -\sin 10\alpha & \cos 10\alpha \end{bmatrix}$$

351. If $A = \begin{bmatrix} x & -2 \\ 3 & 7 \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} \frac{7}{34} & \frac{1}{17} \\ \frac{-3}{34} & \frac{2}{17} \end{bmatrix}$, then the value of x is

(a) 2 (b) 3
(c) –4 (d) 4
(e) –2

Kerala CEE-2010

Ans. (d) : Given,

$$A = \begin{bmatrix} x & -2 \\ 3 & 7 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} \frac{7}{34} & \frac{1}{17} \\ \frac{-3}{34} & \frac{2}{17} \end{bmatrix}$$

$|A| = 7x + 6$,

$$\text{Adj}(A) = \begin{bmatrix} 7 & 2 \\ -3 & x \end{bmatrix}$$

$$A^{-1} = \frac{\text{adj}(A)}{|A|}$$

$$\therefore \quad A^{-1} = \begin{bmatrix} \frac{7}{7x+6} & \frac{2}{7x+6} \\ \frac{-3}{7x+6} & \frac{x}{7x+6} \end{bmatrix}$$

On comparing it with given inverse matrix

$$A^{-1} = \begin{bmatrix} \frac{7}{34} & \frac{1}{17} \\ \frac{-3}{34} & \frac{2}{17} \end{bmatrix}$$

$$\frac{7}{7x+6} = \frac{7}{34}$$

$7x + 6 = 34$

$$x = \frac{34-6}{7}$$

$x = 4$

352. If $A = \begin{vmatrix} 3 & 3 & 3 \\ 3 & 3 & 3 \\ 3 & 3 & 3 \end{vmatrix}$, then A^4 is equal to

(a) 27A (b) 81A
(c) 243A (d) 729A
(e) 3A

Kerala CEE-2008

Ans. (d) : Given, $A = \begin{bmatrix} 3 & 3 & 3 \\ 3 & 3 & 3 \\ 3 & 3 & 3 \end{bmatrix} = 3\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

$$A^2 = A.A = 3\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \cdot 3\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= 9\begin{bmatrix} 3 & 3 & 3 \\ 3 & 3 & 3 \\ 3 & 3 & 3 \end{bmatrix} = 9A$$

$A^4 = A^2 \cdot A^2$
$= 9A \cdot 9A$
$= 81A^2$
$= 81 \cdot 9A$
$= 729A$

353. If $f(x) = x^2 + 4x - 5$ and $A = \begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix}$, then $f(A)$ is equal to

(a) $\begin{bmatrix} 0 & -4 \\ 8 & 8 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & 1 \\ 2 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 8 & 4 \\ 8 & 0 \end{bmatrix}$

(e) $\begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix}$

Kerala CEE-2007

Ans. (d) : Given,

$$A = \begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix}$$

$f(x) = x^2 + 4x - 5$

$f(A) = A^2 + 4A - 5$

Now, $A^2 = A.A = \begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix}$

$$= \begin{bmatrix} 9 & -4 \\ -8 & 17 \end{bmatrix}$$

$$f(A) = \begin{bmatrix} 9 & -4 \\ -8 & 17 \end{bmatrix} + 4\begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix} - 5\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & -4 \\ -8 & 17 \end{bmatrix} + \begin{bmatrix} 4 & 8 \\ 16 & -12 \end{bmatrix} - \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 13 & 4 \\ 8 & 5 \end{bmatrix} - \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

$$f(A) = \begin{bmatrix} 8 & 4 \\ 8 & 0 \end{bmatrix}$$

354. If $X = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$, then X^n, for $n \in N$, is equal to:

(a) $2^{n-1} X$ (b) $n^2 X$

(c) $n X$ (d) $2^{n+1} X$

(e) $2^n X$

Kerala CEE-2004

Ans. (a) : If

$$X = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

Then, $X^2 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = 2\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

$$X^3 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$$

$$= 2\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$= 2\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$$

$$= 2^2\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

Similarly,

$$X^n = 2^{n-1}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$= 2^{n-1} X$

355. Suppose A is a matrix of order 3 and $B = |A|A^{-1}$, if $|A| = -5$, then $|B|$ is equal to :

(a) 1 (b) –5

(c) –1 (d) 25

(e) –125

Kerala CEE-2006

Ans. (d) : It is given that, A is a matrix of order 3 and

$B = |A|A^{-1}$

$|A| = -5$

Now, $B = |A| \dfrac{(\text{adj } A)}{|A|}$

$B = (\text{adj } A)$

$|B| = |\text{adj } A| = |A|^{3-1}$

$= |A|^2$

$= (-5)^2$

$= 25$

356. If $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} A \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ then A is equal to:

(a) $-\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$

(e) $\begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}$

Kerala CEE-2006

Ans. (a) : Given matrix to find A are.

$$\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} A \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

So, $A = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}^{-1}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix}^{-1}$

$$= \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} -3 & -2 \\ -5 & -3 \end{bmatrix}$$

$$= -\begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} 3 & 2 \\ 5 & 3 \end{bmatrix}$$

$$A = -\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$$

Hence, option (A) is correct.

357. If A is a square matrix such that $A^2 = A$ and $(1+A)^n = I + \lambda A$, then λ is equal to

(a) $2n - 1$ (b) $2^n - 1$

(c) $2n + 1$ (d) None of these

Manipal UGET-2015

Ans. (b) : We have, $A^2 = A$

$\therefore (I + A)^2 = (I + A)(I + A)$

$= I + 2A + A^2 = I + 3A$

and $(I + A)^3 = (I + A)^2 (I + A)$

$= I + 4A + 3A^2 = I + 7A$

Thus, we get $(I+A)^2 = I + 3A$

$(I+A)^3 = I + 7A$

$(I + A)^3 = I + (2^3 - 1)A$

Hence,

$(I + A)^n = I + (2^n - 1)A$

$\therefore$ $(I + A)^n = I + \lambda A$

$\Rightarrow \lambda = 2^n - 1$