TGT/PGT
PHYSICS
Revision Book

Importent Facts, Formulas & Oneliners
Chapter, Topic & Subtopic Wise

<u>Useful for</u> : TGT/PGT/LT-GRADE/NVS/KVS/DSSSB/GIC/GDC/Assistant Professor EMRS/AWES/DIET/AEES and Other Competitive Exam

Chief Editor
A. K. Mahajan

Written & Compiled By
YCT Expert Team
Computer Graphics by
Balkrishna, Charan Singh

Editorial Office
12, Church Lane Prayagraj-211002
9415650134
Email: yctap12@gmail.com
website : www.yctbooks.com/www.yctfastbook.com/www.yctbooksprime.com

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INDEX

Fundamental Physical Constants

S.No.	Physical Quantity	Symbol	Value	Units
1.	Speed of light	c	3×10^{8}	m/s
2.	Planck's constant	h	6.63×10^{-34}	J s
		$\hbar = h/2\pi$	1.055×10^{-34}	J s
		hc	1242	eV-nm
3.	Gravitation constant	G	6.67×10^{-11}	$m^3\ kg^{-1}\ s^{-2}$
4.	Acceleration due to gravity (at sea level)	g	9.81	ms^{-2}
5.	Boltzmann constant	k	1.38×10^{-23}	J/K
6.	Volume of Ideal gas (0°C Temperature and 1 atm Pressure)	V	22.4	$L\ mol^{-1}$
7.	Universal gas constant	R	8.314	J/(mol K)
8.	Mechanical Equivalent of Heat	J	4.184	$J\ cal^{-1}$
9.	Avogadro's number	NA	6.023×10^{23}	mol^{-1}
10.	Charge of electron	e	1.602×10^{-19}	C
11.	Permeability of vacuum	μ_0	$4\pi \times 10^{-7}$	N/A^2
12.	Permeability of vacuum	$\in_0$	8.85×10^{-12}	F/m
13.	Coulomb constant	$\frac{1}{4\pi \in_0}$	9×10^{9}	$N\ m^2/C^2$
14.	Faraday constant	F	6485	C/mol
15.	Mass of electron	m_e	9.1×10^{-31}	kg
16.	Charge of electron	e	1.6×10^{-19}	C
17.	Unified atomic mass unit	1 u	1.661×10^{-27}	kg
18.	Electron volt	eV	1.6×10^{-19}	J
19.	Energy Equivalent of 1 u	uc^2	931.5	MeV
20.	Mass of proton	m_p	1.6726×10^{-27}	kg
21.	Mass of neutron	m_n	1.6749×10^{-27}	kg
22.	Atomic mass unit	u	1.66×10^{-27}	kg
			931.49	MeV/c^2
23.	Stefan-Boltzmann constant	σ	5.67×10^{-8}	$W/(m^2\ K^4)$
24.	Rydberg constant	$R\infty$	1.097×10^{7}	m^{-1}
25.	Bohr magneton	μ_B	9.27×10^{-24}	J/T
26.	Bohr radius	a_0	0.529×10^{-10}	m
27.	Standard atmosphere	1 atm	1.01325×10^{5}	Pa
28.	Wien displacement constant	b	2.9×10^{-3}	mK

Basic Mathematics Used in Physics

▪ Quadratic Equation

- Roots of $ax^2 + bx + c = 0$ are $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
- Sum of roots : $x_1 + x_2 = -\frac{b}{a}$
- Produce of roots $x_1 x_2 = \frac{c}{a}$

▪ Binomial theorem

- $(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + ...$
- $(1-x)^n = 1 - nx + \frac{n(n-1)}{2!}x^2 - \frac{n(n-1)(n-2)}{3!}x^3 + ...$

If x << then $(1+x)^n \approx 1 + nx$ & $(1-x)^n \approx 1 - nx$

▪ Logarithm

- $\log mn = \log m + \log n$
- $\log \frac{m}{n} = \log m - \log n$
- $\log m^n = n \log m$
- $\ln m = \log_e m = 2.303 \log_{10} m$
- $\log 2 = 0.3010$ & ln2=0.693
- $\log 3 = 0.4771$ & ln3=1.098

▪ Componendo and dividendo theorem

- If $\frac{p}{q} = \frac{a}{b}$ then $\frac{p+q}{p-q} = \frac{a+b}{a-b}$

▪ Algebraic Expressions:

- $(a + b)^2 = a^2 + b^2 + 2ab$
- $(a - b)^2 = a^2 + b^2 - 2ab$
- $(a - b)^3 = a^3 - b^3 - 3ab(a-b)$
- $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$
- $a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$
- $a^3 - b^3 = (a - b)(a^2 + b^2 + ab)$
- $(a + b)(a - b) = a^2 - b^2$

▪ Arithmetic progression - AP

a, a + d, a + 2d, a + 3d, ..., a + (n-1)d   here d = common difference

Sum of n terms $= S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

Note: (i) $1+2+3+4+5.....+n=\dfrac{n(n+1)}{2}$

(ii) $1^2+2^2+3^2+.....+n^2=\dfrac{n(n+1)(2n+1)}{6}$

(iii) $1^3+2^3+3^3+.....+n^3=\left[\dfrac{n(n+1)}{2}\right]^2$

■ Geometrical progression - GP

a, ar, ar^2, ar^3, ... here, r = common ratio

Sum of n terms $S_n=\dfrac{a\left(1-r^n\right)}{1-r}$ Sum of ∞ terms $S_\infty=\dfrac{a}{1-r}$

■ Trigonometry

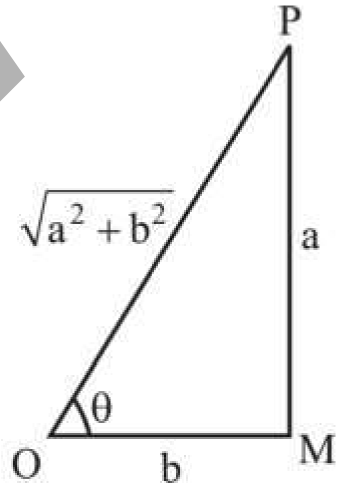
$\tan\theta=\dfrac{\text{Perpendicular}}{\text{Base}}=\dfrac{MP}{OM}$ $\cot\theta=\dfrac{\text{Base}}{\text{Perpendicular}}=\dfrac{OM}{MP}$

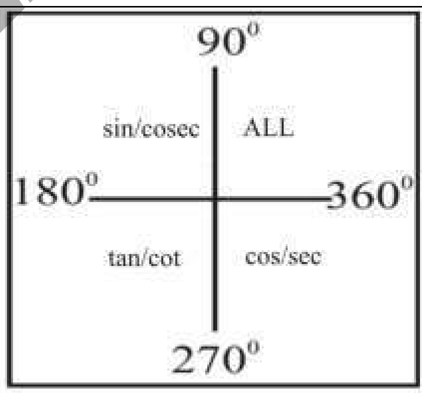
$\sec\theta=\dfrac{\text{Hypotenuse}}{\text{Base}}=\dfrac{OP}{OM}$ $\cos\theta=\dfrac{\text{Base}}{\text{Hypotenuse}}=\dfrac{OM}{OP}$

$\sin\theta=\dfrac{a}{\sqrt{a^2+b^2}}$ $\cos\theta=\dfrac{b}{\sqrt{a^2+b^2}}$

$\tan\theta=\dfrac{a}{b}$ $\operatorname{cosec}\theta=\dfrac{1}{\sin\theta}$

$\sec\theta=\dfrac{1}{\cos\theta}$ $\cot\theta=\dfrac{1}{\tan\theta}$





sin (90º + θ) = cos θ	sin (180º – θ) = sin θ	sin (–θ) = – sin θ	sin (90º – θ) = cos θ
cos (90º + θ) = – sin θ	cos (180º – θ) = – cos θ	cos (–θ) = cos θ	cos (90º – θ) = sin θ
tan (90º + θ) = – cot θ	tan (180º – θ) = – tan θ	tan (–θ) = –tan θ	tan (90º – θ) = cot θ
sin (180º + θ) = – sin θ	sin (270º – θ) = – cos θ	sin (270º + θ) = – cos θ	sin (360º – θ) = – sin θ

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$\cos(180^\circ + \theta) = -\cos\theta$ $\cos(270^\circ - \theta) = -\sin\theta$ $\cos(270^\circ + \theta) = \sin\theta$ $\cos(360^\circ - \theta) = \cos\theta$

$\tan(180^\circ + \theta) = \tan\theta$ $\tan(270^\circ - \theta) = \cot\theta$ $\tan(270^\circ + \theta) = -\cot\theta$ $\tan(360^\circ - \theta) = -\tan\theta$

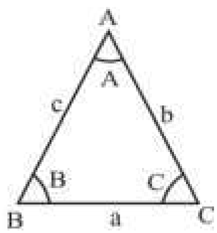
$\theta \rightarrow$	0° (0)	30° $\left(\frac{\pi}{6}\right)$	45° $\left(\frac{\pi}{4}\right)$	60° $\left(\frac{\pi}{3}\right)$	90° $\left(\frac{\pi}{2}\right)$	120° $\left(\frac{2\pi}{3}\right)$	135° $\left(\frac{3\pi}{4}\right)$	150° $\left(\frac{5\pi}{6}\right)$	180° (π)	270° $\left(\frac{3\pi}{2}\right)$	360° (2π)
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	-1	0
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	-1	0	1
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞	$-\sqrt{3}$	-1	$-\frac{1}{\sqrt{3}}$	0	∞	0

■ Trigonometric Identities-

- $\sin^2\theta + \cos^2\theta = 1$
- $1 + \tan^2\theta = \sec^2\theta$
- $1 + \cot^2\theta = \mathrm{cosec}^2\theta$
- $\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$
- $\sin 2A = 2\sin A\cos A$
- $\cos 2A = \cos^2 A - \sin^2 A = 1 - 2\sin^2 A = 2\cos^2 A - 1$
- $\tan 2A = \dfrac{2\tan A}{1 - \tan^2 A}$

■ Sine Rule

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$



■ Cosine Rule

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} \qquad \cos B = \frac{c^2 + a^2 - b^2}{2ca} \qquad \cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

- **For small θ**

$\sin\theta \approx \theta, \cos\theta \approx 1$ $\tan\theta \approx \theta, \sin\theta \approx \tan\theta$

- **Some Integration and Differentiation Formulae**

Differentiation	Integration
$\frac{d}{dx}[\text{Constant}(k)] = 0$	$\int dx = x + C$
$\frac{d}{dx}x^n = nx^{n-1}$	$\int x^n dx = \frac{x^{n+1}}{n+1} + C, n \neq -1$
$\frac{d}{dx}\ln\|x\| = \frac{1}{x}$	$\int \frac{1}{x} dx = \ln\|x\| + C$
$\frac{d}{dx}\sin x = \cos x$	$\int \cos x\, dx = \sin x + C$
$\frac{d}{dx}\cos x = -\sin x$	$\int \sin x\, dx = -\cos x + C$
$\frac{d}{dx}\tan x = \sec^2 x$	$\int \sec^2 x dx = \tan x + C$
$\frac{d}{dx}\cot x = -\operatorname{cosec}^2 x$	$\int \operatorname{cosec}^2 x dx = -\cot x + C$
$\frac{d}{dx}\sec x = \sec x \tan x$	$\int \sec x \tan x\, dx = \sec x + C$
$\frac{d}{dx}\operatorname{cosec} x = -\operatorname{cosec} x \cot x$	$\int \operatorname{cosec} x \cot x\, dx = -\operatorname{cosec} x + C$
$\frac{d}{dx}e^x = e^x$	$\int e^x\, dx = e^x + C$
$\frac{d}{dx}e^{\alpha x+\beta} = \alpha e^{\alpha x+\beta}$	$\int e^{\alpha x+\beta}\, dx = \frac{1}{\alpha}e^{\alpha x+\beta} + C$
$\frac{d}{dx}a^x = a^x \ln a$	$\int a^x\, dx = \frac{1}{\ln a}a^x + C$
$\frac{d}{dx}\sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$	$\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + C$
$\frac{d}{dx}\cos^{-1} x = \frac{-1}{\sqrt{1-x^2}}$	$\int \frac{dx}{\sqrt{1-x^2}} = -\cos^{-1} x + C$

$\frac{d}{dx}\tan^{-1}x = \frac{1}{1+x^2}$	$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$
$\frac{d}{dx}\cot^{-1}x = \frac{-1}{1+x^2}$	$\int \frac{dx}{1+x^2} = -\cot^{-1}x + C$
$\frac{d}{dx}\sec^{-1}x = \frac{1}{\lvert x\rvert\sqrt{x^2-1}}$	$\int \frac{dx}{\lvert x\rvert\sqrt{x^2-1}} = \sec^{-1}x + C$
$\frac{d}{dx}\operatorname{cosec}^{-1}x = \frac{-1}{\lvert x\rvert\sqrt{x^2-1}}$	$\int \frac{dx}{\lvert x\rvert\sqrt{x^2-1}} = -\operatorname{cosec}^{-1}x + C$
$\frac{d}{dx}[k\bullet f(x)] = k\bullet f'(x)$	$\int \ln x\,dx = x\ln x - x + C$
$\frac{d}{dx}[f(x)\pm g(x)] = f'(x)\pm g'(x)$	$\int \tan x\,dx = -\ln\lvert\cos x\rvert + C$
$\frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + g(x)f'(x)$	$\int \cot x\,dx = \ln\lvert\sin x\rvert + C$
$\frac{d}{dx}f(g(x)) = f'(g(x))\bullet g'(x)$	$\int \sec x\,dx = \ln\lvert\sec x + \tan x\rvert + C$
$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$	$\int \operatorname{cosec} x\,dx = -\ln\lvert\operatorname{cosec} x + \cot x\rvert + C$

■ Maxima & Minima of a function y = f(x)

- For maximum value $\frac{dy}{dx} = 0 \;\&\; \frac{d^2y}{dx^2} = -ve$
- For minimum value $\frac{dy}{dx} = 0 \;\&\; \frac{d^2y}{dx^2} = +ve$

■ Average of a varying quantity

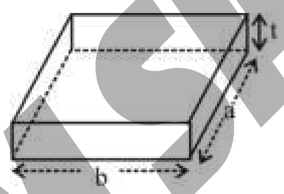
- If $y = f(x)$ than $< y >= \bar{y} = \frac{\int_{x_1}^{x_2} y\,dx}{\int_{x_1}^{x_2} dx} = \frac{\int_{x_1}^{x_2} y\,dx}{x_2 - x_1}$

■ Formulae for determination of area

- Area of a square = $(\text{side})^2$
- Area of rectangle = length × breadth
- Area of a triangle $= \frac{1}{2}\times \text{base}\times\text{height}$

- Area of a trapezoid $=\frac{1}{2}\times(\text{distance between parallel sides})\times(\text{sum of parallel sides})$
- Area enclosed by a circle = πr^2 (r = radius)
- Surface area of a sphere = $4\pi r^2$ (r = radius)
- Area of a parallelogram = base × height
- Area of curved surface of cylinder $= 2\pi r\ell$ $(r = \text{radius and } \ell = \text{length})$
- Area of whole surface of cylinder $= 2\pi r(r+\ell)$ $(\ell = \text{length})$
- Area of ellipse = πab (a & b are semi major and semi minor axis respectively)
- Surface area of a cube = 6 $(\text{side})^2$
- Total surface area of a cone $=\pi r^2 + \pi\ell$ where $=\pi r\left(r^2+h^2\right)$ = lateral area

■ Formulae for determination of volume:



- Volume of a rectangular slab = length × breadth × height = abt
- Volume of a cube = $(\text{side})^3$
- Volume of a sphere $=\frac{4}{3}\pi r^3$ (r = radius)
- Volume of a cylinder = $=\pi r^2\ell$ (r = radius and ℓ is length)
- Volume of a cone $=\frac{1}{3}\pi r^3 h$ (r = radius and h is height)

■ KEY POINTS:

- To convert an angle from degree to radian, we have to multiply if by $\frac{\pi}{180^{\circ}}$ and to convert an angle from radian to degree, we have to multiple it by $\frac{\pi}{180^{\circ}}$.
- By help of differentiation, if y is given, we can find $\frac{dy}{dx}$ and by help of integration, if $\frac{dy}{dx}$ is given, we can find y.
- The maximum and minimum values of function [A cosθ + B sinθ] are $\sqrt{A^2+B^2}$ and $\sqrt{A^2-B^2}$ respectively.

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01.

General Physics (Mechanics)

(1) Unit, Dimensions and Measurement

■ Physical Quantity-

- A quantity which can be measured directly or indirectly or can be explained and expressed in the form of laws of physics are called physical quantity.
- A physical quantity is completely represented by its magnitude and unit.
- The magnitude of physical quantity and unit are inversely proportional to each other. Larger the unit smaller will be the magnitude.

Types of physical Quantity -

- **Ratio (Numerical value only)-** When a physical quantity is a ratio of two similar quantities. It has no unit. For **example-**

$$\text{Relative density} = \frac{\text{Density of Object}}{\text{Density of Water}}$$

- **Scalar-** A physical quantity which has magnitude only and do not have any direction.
  Example- Work, Energy, Length, Time.
- **Vector-** A physical quantity which has magnitude and direction both.
  Example- Displacement, Velocity, Acceleration etc.
- **Units-** Measurement of any physical quantity involves comparison with a certain basic, arbitrarily chosen, internationally accepted reference standard called unit.

■ System of units-

- A system of unit is a complete set of unit. It is used to measure all kinds of fundamental and derived quantities.

Some system of units are as follows-

Physical quantity	CGS	MKS	FPS
Length	cm	meter	foot
Mass	gram	kg	pound
Time	second	second	second

Fundamental and Derived Unit-

- **Fundamental unit-** The units of those physical quantities which can neither be derived from one another, nor they can be further resolved into more simpler units. Example:- Units of Mass, Length etc.
- **Derived Unit-** Those units of physical quantities which are derived from units of fundamental quantities are Called Derived units. Example:- Units of Velocity, Acceleration, Force, Work etc.

S.I. Unit-

- The S.I. unit is the international system of units. This system contains seven fundamental units and two supplementary fundamental units.

Fundamental quantities in S.I. System and their units-

Sr. No.	Physical Quantity	Name of unit	Symbol of unit
1.	Mass	Kilogram	kg
2.	Length	Meter	m
3.	Time	Second	s
4.	Temperature	Kelvin	K
5.	Luminous intensity	Candela	Cd
6.	Electric Current	Ampere	A
7.	Amount of Substance	Mole	Mol

Supplementary S.I Unit- (Dimensionless Unit)

Sr. No.	Physical Quantity	Name of unit	Symbol of unit
1.	Plane angle	Radian	rad
2.	Solid angle	Steradian	Sr

■ Dimension of Physical Quantity-

- The powers to which fundamental quantities must be in order to express the given physical quantity is called its dimension.
- It is used to express derived quantity in terms of fundamental quantities.

For example- Force = Mass× Acceleration

$$= \frac{\text{Mass} \times \text{Velocity}}{\text{Time}}$$

$$= \text{Mass} \times \text{Length} \times \text{Time}^{-2}$$

$$= [MLT^{-2}]$$

S.I. Prefixes-

- The magnitudes of physical quantities vary over a wide range. The CGPM recommended standard prefixes for magnitude too large or too small to be expressed more compactly for certain powers of 10.

Power of 10	Prefix	Symbol	Power of 10	Prefix	Symbol
10^{18}	exa	E	10^{-1}	deci	d
10^{15}	peta	P	10^{-2}	centi	c
10^{12}	tera	T	10^{-3}	mili	m
10^{9}	giga	G	10^{-6}	mirco	μ
10^{6}	mega	M	10^{-9}	nano	n
10^{3}	kilo	K	10^{-12}	pico	p
10^{2}	hecto	h	10^{-15}	femto	f
10^{1}	deca	da	10^{-18}	atto	a

Units of Important Physical Quantities-

Physical Quantity	Unit	Physical Quantity	Unit
Angular Acceleration	Rad-s^{-2}	Frequency	Hertz
Moment of inertia	Kg-m^2	Resistance	Ohm
Self inductance	Henry	Surface tension	Newton/m
Magnetic Flux	Weber	Universal Gas Constant	Joule K^{-1} Mol^{-1}
Pole Strength	A-m	Dipole-moment	Coulomb-meter
Dynamic Viscosity	Pascal sec or kg/ms	Stefan Constant	Watt m^{-2} K^{-4}
Kinematic Viscosity	m^2/s	Permittivity of free space (ε_0)	$Coulomb^2$/N-m^2
Reactance	Ohm	Permeability of free space (μ_0)	Weber/A-m
Specific heat	J/Kg°C	Planck's constant	Joule-sec
Strength of magnetic field	Tesla	Entropy	J/K
Astronomical distance	Parsec	Angular Speed	Rad/sec

Dimensions of important Physical Quantities -

Physical Quantity	Dimensions	Physical Quantity	Dimensions
Momentum	$M^1L^1T^{-1}$	Capacitance	$M^{-1}L^{-2}T^4\,A^2$
Calorie	$M^1L^2T^{-2}$	Modulus of rigidity	$M^1L^{-1}T^{-2}$
Latent heat capacity	$M^0L^2T^{-2}$	Magnetic permeability	$M^1L^1T^{-2}\,A^{-2}$
Self inductance	$M^1L^2T^{-2}A^{-2}$	Solar constant	$M^1L^0T^{-3}$
Coefficient of thermal conductivity	$M^1L^1T^{-3}\,\theta^{-1}$	Magnetic flux	$M^1L^2T^{-2}\,A^{-1}$
Power	$M^1L^2T^{-3}$	Current density	$M^0L^{-2}T^0\,A^1$
Impulse	$M^1L^1T^{-1}$	Young's Modulus	$M^1L^{-1}T^{-2}$
Hole mobility of a semiconductor	$M^{-1}L^0A^1\,T^2$	Magnetic field intensity	$MT^{-2}\,A^{-1}$
Bulk modulus of elasticity	$M^1L^{-1}T^{-2}$	Magnetic induction	$M^1T^{-2}\,A^{-1}$
Light year	$M^0L^1T^0$	Permittivity	$M^{-1}L^{-3}T^4\,A^2$
Thermal resistance	$M^{-1}L^{-2}T^3\,\theta$	Electric field	$M^1L^1T^{-3}\,A^{-1}$
Coefficient of Viscosity	$M^1L^{-1}T^{-1}$	Resistance	$ML^2T^{-3}\,A^{-2}$

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Physical quantities which are dimensionless-

Sr. No.	Physical Quantity	Dimensional Formula
1.	Specific gravity	$[M^0L^0T^0]$
2.	Strain	
3.	Angle (θ)	
4.	Avogadro's number (N)	
5.	Reynold's number (N_R)	
6.	Refractive Index (μ)	
7.	Mechanical equivalent of heat (J)	
8.	Dielectric Constant (K) or relative permittivity	
9.	Relative density	
10.	Trigonometric-ratios	
11.	Distance gradient	
12.	Relative permeability	

Physical quantities which have same dimensional formula -

S.No.	Physical Quantity	Dimensional Formula
1. 2. 3. 4.	Speed or velocity Velocity of light in Vacuum (c) Distance travelled in n^{th} second (S_n^{th}) Relative Velocity	$M^0L^1T^{-1}$
5. 6. 7. 8.	Frequency (v) Angular frequency Angular velocity (ω) Velocity Gradient	$M^0L^0T^{-1}$
9. 10. 11. 12. 13. 14. 15. 16.	Work Moment of force Torque Internal energy Potential energy Kinetic energy Heat energy Light energy	$M^1L^2T^{-2}$
17. 18. 19. 20. 21. 22. 23.	Coefficient of elasticity Pressure Stress Young's Modulus Bulk Modulus Modulus of rigidity Energy density	$M^1L^{-1}T^{-2}$
24. 25. 26 27. 28.	Force Weight Thrust Energy gradient Tension	$M^1L^1T^{-2}$
29. 30. 31.	Acceleration Acceleration due to gravity Gravitational field intensity	$[M^0L^1T^{-2}]$
32. 33.	Plank's Constant (h) Angular momentum	$[M^1L^2T^{-1}]$
34.	Mass	$[M^1L^0T^0]$
35.	Momentum	$[M^1L^1T^{-1}]$

36.	Impulse	
37. 38. 39.	Length Radius of gyration (K) Wavelength (λ)	$[M^0L^1T^0]$
40. 41. 42.	Force constant Surface tension Surface energy	$M^1L^0T^{-2}$
43.	Area	$M^0L^2T^0$
44.	Volume	$M^0L^3T^0$
45.	Density	$M^1L^{-3}T^0$
46.	Universal gravitational constant (G)	$M^{-1}L^3T^{-2}$
47.	Moment of Inertia	$M^1L^2T^0$
48.	Angular acceleration	$M^0L^0T^{-2}$
49.	Rate of flow	$M^0L^3T^{-1}$
50.	Mass per unit length	$M^1L^{-1}T^0$
51.	Rydberg constant (R)	$M^0L^{-1}T^0$
52.	Coefficient of viscosity (η)	$M^1L^{-1}T^{-1}$
53.	kinematic viscosity	$M^0L^2T^{-1}$
54.	Surface potential	$M^0L^2T^{-2}$
55.	Specific Volume	$M^{-1}L^3T^0$
56	Power	$M^1L^2T^{-3}$

■ Dimensional Analysis and Its Applications:-

- Dimensional analysis helps up in deducing certain relations among different physical quantities checking the derivation accuracy and dimensional consistency or homogeneity of various mathematical expressions.
- **Checking dimensional consistency of equations:** According to principle of homogeneity, dimensions of each term on both side of an equation must be same.

Example -

(i) Work done = force × displacement ;

$[ML^2T^{-2}] = [MLT^{-2}] \times [M^0LT^0]$

$[ML^2T^{-2}] = [ML^2T^{-2}]$

(ii) $S = ut + \frac{1}{2}at^2$; Dimensionally,

$[S] = [ut] = [at^2]$

$[M^0LT^0] = [M^0LT^{-1}]\,[M^0L^0T] = [M^0LT^{-2}]\,[M^0L^0T^2]$

$[M^0LT^0] = [M^0LT^0] = [M^0LT^0]$

- **To convert a physical quantity from one to another system of units**: $Q_1n_1 = Q_2n_2$; Where Q_1 = unit in 1st system, Q_2 = units in 2nd system n_1 and n_2 be constant value in 1st and 2nd system.

$$\therefore\ n_2 = \frac{Q_1n_1}{Q_2} \Rightarrow n_2 = n_1\left[\frac{Q_1}{Q_2}\right]$$

Example -

Conversion of SI unit of force from Newton (MKS) into dyne (CGS),

Let $n_2 = x$, Q_2 = dyne (g cm s^{-2}), $n_1 = 1$, Q_1 = N(kg m/s^{-2})

Applying $n_2 = \left[\frac{Q_1}{Q_2}\right]n_1$

$$\therefore\quad x = 1\left[\frac{M_1}{M_2}\right]^a\left[\frac{L_1}{L_2}\right]^b\left[\frac{T_1}{T_2}\right]^c \text{ or } x = 1\times\left[\frac{kg}{g}\right]^1\left[\frac{m}{cm}\right]^1\left[\frac{s}{s}\right]^{-2}$$

$$\text{or } x = 1\times\left[\frac{1000\text{ g}}{1\text{ g}}\right]\times\left[\frac{100\text{ cm}}{\text{cm}}\right]\times\left[\frac{1\text{ s}}{\text{s}}\right] \Rightarrow \qquad x = 10^5$$

$\therefore$ 1 N = 10^5 dynes.

- **Deducing relation among the physical quantities:** If we know the dependency of a physical quantity on the other quantities then using dimensional analysis relation between them can be derived.

Example- Time period of simple pendulum depends on mass of bob (m), length (l) of string and acceleration due to gravity (g).

$\therefore\ T \propto m^a l^b g^c$ **;** $T = k\, m^a\, l^b\, g^c$

Here k is a dimensionless constant.

$[M^0L^0T] = [ML^0T^0]^a\ [M^0LT^0]^b\ [M^0LT^{-2}]^c$

$[M^0L^0T] = [M^aL^{b+c}T^{-2c}]$

Comparing the powers, we get a = 0, b + c = 0 and –2c = 1

$\therefore\ c = \dfrac{-1}{2}$ and $b = \dfrac{1}{2}$

Substituting values of a, b and c in equation (i), $T\ k\ m^0\ l^{1/2}\ g^{-1/2}$

$\therefore\ T = k\sqrt{\dfrac{l}{g}}$ (k = a constant cannot be determined using dimensions)

Limitations of dimensional analysis:

- Dimensional method can be used only if the dependency is of multiplication type. The formulae containing exponential, trigonometric and logarithmic function can't be derived using this method. Formulae containing more than one term which are added or subtracted like $S= ut + \dfrac{1}{2}at^2$ also can't be derived.
- We cannot determine the value of constants in a relation.
- It gives no information whether a physical quantity is a scalar or a vector.
- In mechanics, the physical quantities depends on more than three quantities cannot be derived by dimensional method as there will be less number (= 3) of equations than the unknowns (> 3). However still we can check the correctness of equation dimensionally.
- Physical quantities having identical dimensions may be of entirely different in nature.

Application of Dimensional Analysis -

1. To convert physical quantity from one system of units to another.
2. To check correctness of a given physical relation.
3. To derive a relationship between different physical quantities.

■ Accuracy and Precision -

- **Accuracy-** Accuracy is the term used to indicate the closeness of a measured value to its accurate value.

  $$\text{Accuracy} \propto \frac{1}{\text{Fractional or Relative error}}$$

- **Precision -** Precision is the closeness of a measurement of two or more measurement to each other.

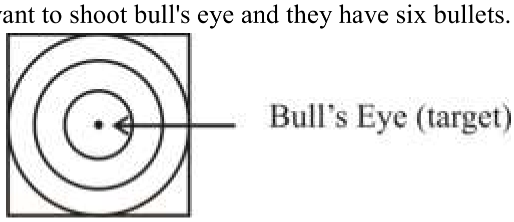
  limit of precision = ± ½(least count of instrument)

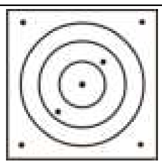
  $$\text{Precision} \propto \frac{1}{\text{least count}}$$

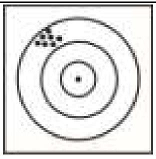
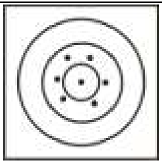
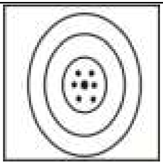
  Precision $\propto$ Fractional error/Relative error

Example-

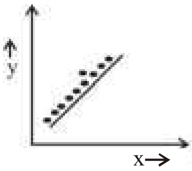
- Suppose, A cadate of soldier want to shoot bull's eye and they have six bullets.



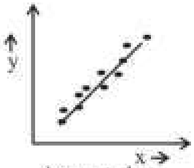
	Conditions	Conclusions
	(i) Maximum bullets are far from target. (ii) The separation between bullets are also large.	Low Accuracy and Low Precision

	(i) Bullets are for from target. (ii) Bullets are very close to each other.	Low Accuracy and High precision
	(i) Bullets are close to target. (ii) Separation between bullets are large.	High Accuracy and Low Precision
	(i) Bullets are close to target. (ii) Separation between bullets are low.	High Accuracy and High Precision

Thus,



Precise but inaccurate data



Accurate but imprecise data

■ Significant figure -

- The figure which express the required degree of accuracy, is called significant figure.
- In significant figure digits carry a meaningful representation.
- Accuracy $\propto$ No. of Significant figure.

Rules for counting the no. of Significant figure in a measured quantity-

(i) All non-zero digits are significant.
Ex- 13.75 S.f = 4

(ii) All zeros between two non- zero digits are significant.
Ex- 100.05 km S.f = 5

(iii) All zeros to the rights of a non-zero digits but to the left of an understood decimal point are not significant.
Ex- 86400 S.f = 3

But such zero are significant if they come from a measurement 86400 sec S.f = 5

(iv) All zero to the rights of a non-zero digits but to the left of the decimal point are significant.
Ex- 648700 S.f = 6

(v) All zeros to the rights of a decimal point are significant.

Ex-	S.f
161 cm	3
161.0 cm	4
161.00 cm	5

(vi) All zeros to the right of a decimal point but to the left of a non-zero digit are not significant.
0.161 cm $\rightarrow$ S.f = 3
0.0161 cm $\rightarrow$ S.f = 3

(vii) The no. of significant figure does not depend on the system of units.
16.4 cm $\rightarrow$ 3
0.164 m $\rightarrow$ 3
0.000164 km $\rightarrow$ 3

(viii) The power of 10 are not counted as significant digit.
$1.4\times10^{-7} \rightarrow 2$
$1.65\times10^{14} \rightarrow 3$

Rounding off a digit-

(i) If the no. lying to the right of cut off digit is less than 5, then the cut off digit is retained as such. However, if it is more than 5, then the cut off digit is increased by 1.

$x = 6.24$ rounding in 2 significant $\quad x = 6.2$

$x = 5.328$ rounding in 3 significant $\quad x = 5.33$

(ii) If the digit to be dropped is 5 followed by digits other than zero then the preceding digit is increased by 1.

$x = 14.252 \quad x = 14.3$ (upto 3)

(iii) If the digit to be dropped is 5 or 5 followed by zeros, then the preceding digit is unchanged if it is even.

$x = 6.250$ or $x = 6.25$

$x = 6.2$ (upto 2)

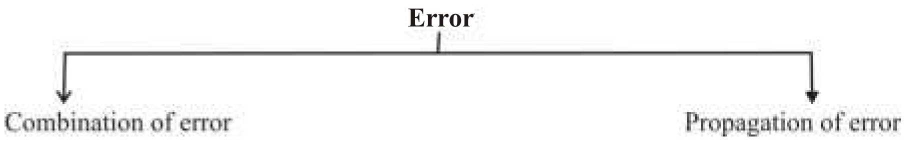
(iv) If the digit to be dropped is 5 or 5 followed by zeros then the preceding digits is raised by one if it is odd.

$x = 6.350$ or $x = 6.35$

$x = 6.4$ (rounding off two significant)

■ Error

- The difference between the measured value and the true value of a quantity is known as the error in the measurement.



- When we know actual error in observation.
- Systematic error
- Upto 10%

- When we do not know the actual error in observation.
- Random error

Combination of Error-

(i) Absolute error - The difference between the true value and the measured value of a quantity is called an absolute error.

$$\bar{x} = \frac{x_1 + x_2 + x_3 + ... + x_n}{n} = \text{measured/mean value}$$

Absolute error -

$$\Delta x_1 = \bar{x} - x_1$$
$$\Delta x_2 = \bar{x} - x_2$$
$$\Delta x_3 = \bar{x} - x_3$$
$$. \qquad .$$
$$. \qquad .$$
$$. \qquad .$$
$$\Delta x_n = \bar{x} - x_n$$

Absolute error may be + ve or – ve.

(ii) Mean absolute error-

$$\Delta x_{mean} = \frac{|\Delta x_1| + |\Delta x_2| + ... + |\Delta x_n|}{n}$$

Final result can be written as–

$x = \bar{x} \pm \Delta x_{mean}$

(iii) Relative or Fractional error-

$$= \frac{\Delta x_{mean}}{x_m} = \frac{\Delta x_{mean}}{\bar{x}}$$

$$\% = \frac{\Delta x_{mean}}{\bar{x}} \times 100$$

Operation	Formula (Z)	Absolute error (ΔZ)	Relative error $\left(\frac{\Delta Z}{Z}\right)$	Percentage error $\frac{\Delta Z}{Z}\times 100$
Sum	$A + B$	$\Delta A + \Delta B$	$\frac{\Delta A+\Delta B}{A+B}$	$\frac{\Delta A+\Delta B}{A+B}\times 100$
Difference	$A - B$	$\Delta A + \Delta B$	$\frac{\Delta A+\Delta B}{A-B}$	$\frac{\Delta A+\Delta B}{A-B}\times 100$
Multiplication	$A \times B$	$A\Delta B + B\Delta A$	$\frac{\Delta A}{A}+\frac{\Delta B}{B}$	$\left(\frac{\Delta A}{A}+\frac{\Delta B}{B}\right)\times 100$
Division	$\frac{A}{B}$	$\frac{B\Delta A+A\Delta B}{B^2}$	$\frac{\Delta A}{A}+\frac{\Delta B}{B}$	$\left(\frac{\Delta A}{A}+\frac{\Delta B}{B}\right)\times 100$
Power	A^n	$nA^{n-1}\Delta A$	$n\frac{\Delta A}{A}$	$n\frac{\Delta A}{A}\times 100$
Root	$A^{1/n}$	$\frac{1}{n}A^{1/n-1}\Delta A$	$\frac{1}{n}\frac{\Delta A}{A}$	$\frac{1}{n}\frac{\Delta A}{A}\times 100$

Propagation of Error-

Case	Propagation of Error
$Z = ax \pm b$	$\delta z = a\cdot\delta x$
$Z = x \pm y$	$\delta z = [(\delta x)^2 + (\delta y)^2]^{1/2}$
$Z = cxy$	$\frac{\delta z}{z} = \left[\left(\frac{\delta x}{x}\right)^2 + \left(\frac{\delta y}{y}\right)^2\right]^{1/2}$
$Z = c\frac{x}{y}$	$\frac{\delta z}{z} = \left[\left(\frac{\delta x}{x}\right)^2 + \left(\frac{\delta y}{y}\right)^2\right]^{1/2}$
$Z = cx^a$	$\frac{\delta z}{z} = a\frac{\delta x}{x}$
$Z = cx^a y^b$	$\frac{\delta z}{z} = \left[\left(a\frac{\delta x}{x}\right)^2 + \left(b\frac{\delta y}{y}\right)^2\right]^{1/2}$
$Z = \sin x$	$\frac{\delta z}{z} = \delta x \cot x$
$Z = \cos x$	$\frac{\delta z}{z} = \delta x \tan x$
$Z = \tan x$	$\frac{\delta z}{z} = \frac{\delta x}{\sin x \cos x}$

■ Measurement -

Pitch -

- The smallest value of length or any other units which can be read directly from a main scale accurately is called pitch.

$$\text{Pitch} = \frac{1\,\text{Unit}}{\text{No. of division in unit}}$$

Least Count-

- The minimum measurement which can be taken accurately by the measuring instrument.

$$\text{L.C} = \frac{\text{Value Measured}}{\text{No. of Division in that Measurement}}$$

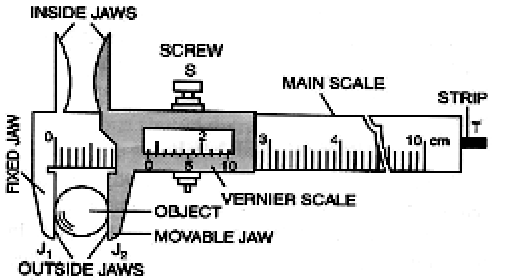
$$\text{or L.C} = \frac{\text{Pitch}}{\text{No. of V.S.D}} \quad \text{(for vernier)}$$

There are basically two types of precision instrument used for measurement:-

(i) Vernier Calipers
(ii) Screw Gauge

TEAM SPY

Vernier Calipers -

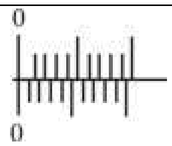
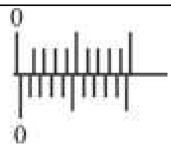
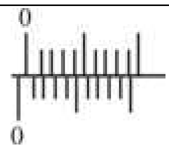


- A vernier scale is an auxiliary scale that slides along the main scale.
- The vernier scale is that in which a certain number, no. of division n on the vernier scale is equal in length to a different number (usually one less) of main-scale divisions.
  nV.S.D = (n – 1) M.S.D
  Where n = number of divisions on the vernier scale
  V.S.D = The length of one division on the vernier scale
  and M.S.D = Length of the smallest main-scale division
- Least count = M.S.D. – V.S.D = $\frac{1}{n}$M.S.D
- In the ordinary Vernier calipers, 1 M.S.D is 1 mm and 10 VSD coincide with 9 MSD

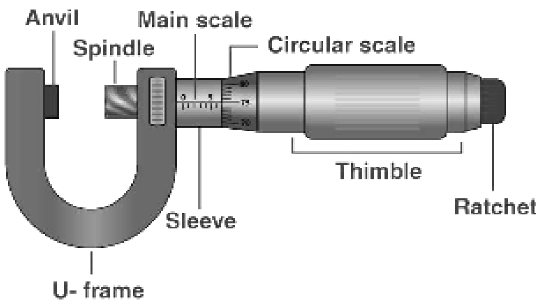
  1 VSD = $\frac{9}{10}$ MSD = 0.9 mm

  Least Count of Vernier 1 M.S.D – 1 V.S.D
  = 1 mm –0.9 mm
  = 0.1 mm

Types of zero error–

	No zero Error	**Positive Zero Error**	**Negative Zero Error**
Correction	None	Negative	Positive
Correction formula	None	– Coinciding division × L.C. of V.S.	+ Coinciding division × L.C. of V. S.
			
Corrected reading	Observed reading + correction		
Actual reading	Main scale reading + V.S. reading ± zero correction		

Screw Gauge-



- A Screw Gauge allows a measurement of the size of a body. It is one of the most accurate mechanical devices in common use.
- It consists of a main scale and a thimble
- Least Count = $\frac{\text{Pitch}}{\text{No. of Divisions on Circular Scale}}$
- L.C of Screw Gauge = 0.001 mm

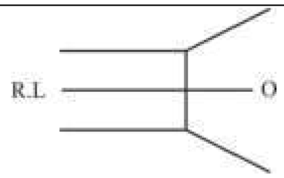
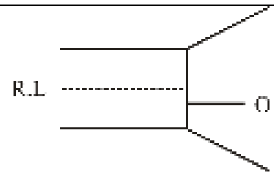
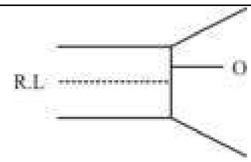
Method of Measurement

Step- I : Find the whole number of mm in the barrel

Step - II: Find the reading of barrel and multiply by 0.01

Step III: Add the value in Step- I and Step -II

Types of zero Error :–

	No zero Error	**Positive zero error**	**Negative zero error**
correction	None	Negative	Positive
Correction Formula	None	+zero error ×L.C	-zero error ×L.C
			
Actual reading	Observed reading -excess reading ×zero error		

Least Count of various measuring Instruments :-

Instrument	**Its least Count**
mm Scale	1 mm
Vernier Calipers	0.1 mm
Screw Gauge	0.001 mm

■ Key points -

- Trigonometric functions sin θ, cos θ, tan θ etc and their arrangements θ are dimensionless.
- Dimension of differential coefficients $\left[\frac{d^n y}{dx^n}\right] = \left[\frac{y}{x^n}\right]$.
- Dimension of integrals $[\int ydx] = [yx]$ we can not add or subtract two physical quantities of different dimensions.
- Independent quantity may be taken as fundamental quantities in a new system of units.
- Measure of a physical quantity = Numerical value of the physical quantity × Size of the unit

  i.e $Q = n \times u$

  Thus, the numerical value (n) is inversely proportional to the size (u) of the unit.

  $n \propto \frac{1}{u}$ or nu = constant.

(2) Scalar and Vectors

■ **Scalars:-** Those physical quantity which require only magnitude but no direction for their complete representation are called Scalars. Ex. Distance, Speed, Work, Mass, Energy, Power, Temperature etc.

■ **Vectors:-** A physical quantity which requires magnitude and direction both, when it is expressed. Ex. Force, Displacements, Momentum, Acceleration, Velocity, Impulse, Pressure, Gravity etc.

Types of Vectors:-

(i) Equal vectors:-

- Two vectors of equal magnitude and having same direction are called equal vector. fig (i)

(ii) Negative Vectors:-

- Two vectors of equal magnitude but having opposite direction are called negative vectors. Fig(ii)

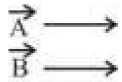


fig. (i)

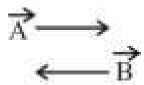


fig. (ii)

(iii) Zero Vector or null Vector:-

- A vector whose magnitude is zero known as a zero or null vector. Its direction is not defined. It is denoted by $\vec{0}$. Velocity of stationary object and resultant of two equal and opposite vectors are the example of null vector.

(iv) Unit vector:-

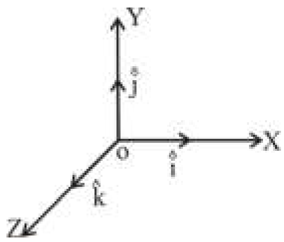
- A vector having unit magnitude is called a unit vector. A unit vector in the direction of vector A is given by

  $\hat{A} = \frac{\vec{A}}{|\vec{A}|}$

- A unit vector is unit less and dimensionless vector and represents direction only.

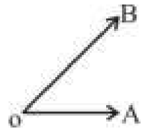
(v) Orthogonal Unit Vector:-

- The unit vectors along the direction of orthogonal axis i.e. x-axis, y-axis and z-axis are called orthogonal unit vectors. They are represented by $\hat{i}, \hat{j}$ and $\hat{k}$



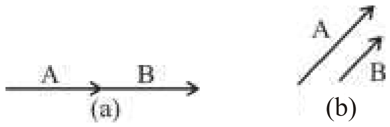
(vi) Co-initial vector :-

- Vectors having a common initial points are called co-initial vectors.



(vii) Colinear Vector :-

- Vector having equal or unequal magnitudes but acting along the same or parallel lines are called colinear vectors.



(viii) Co-planar vector:-

- Vectors acting in the same plane are called coplanar vectors.

(ix) Localised Vectors-

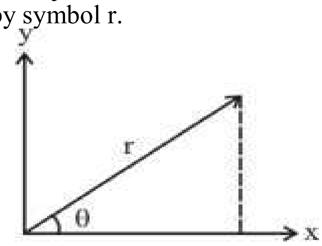
- A vector whose initial point is fixed is called localised vectors.

(x) Non-Localized or free vector-

- A vector whose initial point is not fixed is called a non-localized or free vectors.

(xi) Position Vectors-

- A vector which gives position of an object with reference to the origin of a co-ordinate system is called position vector. It is represented by symbol r.



$\vec{r} = x\hat{i} + y\hat{j}$

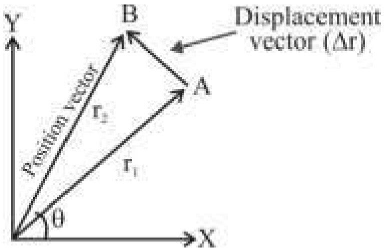
If $\vec{r}$ makes an angle θ with x-axis, then

x= rcos θ and y = rsin θ

$\vec{r} = r\left(\cos\theta\hat{i} + \sin\theta\hat{j}\right)$

(xii) Displacement Vector-

- The vector which tells how much and in which direction an object has changed its position in a given interval of time is called a displacement vector.



r_1 & r_2 are position vector.
The displacement vector for AB is-
$\Delta r = r_2 - r_1$

■ Tensors-

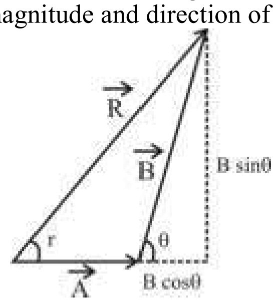
A quantity that has different values in different direction is called Tensor.
Tensors can be classified according to following order-

- **Zero-Order Tensors (Scalars):** Among some of the quantities that have magnitude but not direction are zero-order tensors e.g.: mass density, temperature, and pressure.
- **First-Order Tensors (Vectors):** Quantities that have both magnitude and direction e.g.: velocity, force. The first-order tensor is symbolized with a boldface letter and by and arrow at the top part of the vector, i.e.: $\vec{0}$.
- **Second-Order Tensors:** Quantities that have magnitude and two directions, e.g. stress and strain. The second-order and higher-order tensors are symbolized with a boldface letter.

■ Vector addition-

(i) Triangle law of vector addition -

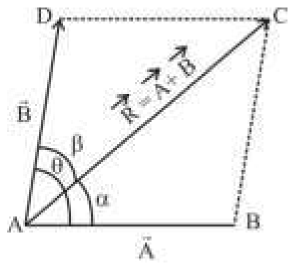
When two vectors are represented as two sides of the triangle with the order of magnitude and direction then the third side of the triangle represents the magnitude and direction of the resultant vector.



- $\vec{R} = \vec{A} + \vec{B}$

(ii) Parallelogram law of vector addition-

If two vectors are represented by two adjacent sides of a parallelogram which are directed away from their common point then their sum (i.e. resultant vector) is given by the diagonal of the parallelogram passing away through that common point.



$\overrightarrow{AB} + \overrightarrow{AD} = \overrightarrow{AC} = \vec{R}$ or $\vec{A} + \vec{B} = \vec{R}$

(a) Resultant of vectors A and B is given by -

$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

(b) If the resultant vector R subtend an angle β with vector B and α with vector A, then

$$\tan\alpha = \frac{\beta\sin\theta}{A + B\cos\theta} \;\&\; \tan\beta = \frac{A\sin\theta}{B + A\cos\theta}$$

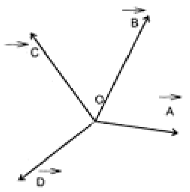
Case1- If A=B then R= $2A\cos\frac{\theta}{2}$ & $\alpha = \frac{\theta}{2}$

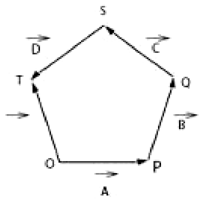
Case2- If θ = 0° then
$R_{max} = A+B$

Case3- If θ = 180° then
$R_{min} = A-B$

(iii) Law of polygon (Addition of more than two vectors)-

If some vectors are represented by sides of a polygon in same order, then their resultant vector is represented by the closing side of polygon in the opposite order.





$$\vec{R} = \vec{A} + \vec{B} + \vec{C} + \vec{D}$$

OT = OP + PQ + QS + ST

Properties of vector Addition-

1. $\vec{A} + \vec{B} = \vec{B} + \vec{A}$

2. $\left(\vec{A} + \vec{B}\right) + \vec{C} = \vec{A} + \left(\vec{B} + \vec{C}\right)$ (associative property)

3. $\vec{A} + \vec{0} = \vec{A}$ (additive identity)

4. $\vec{A} + (-\vec{A}) = \vec{O}$ (additive inverse)

5. $\left|\vec{A} + \vec{B}\right| \le \left|\vec{A}\right| + \left|\vec{B}\right|$

6. $m\left(\vec{A} + \vec{B}\right) = m\vec{A} + m\vec{B}$ (distributive property)

■ Vector subtraction-

Subtraction of vector B from a vector A is defined as the addition of vector B (negative of vector B) to vector A.

Thus, $\vec{A} - \vec{B} = \vec{A} + \left(-\vec{B}\right)$

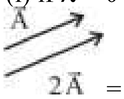
Multiplication of a vector-

1. By a real no.-

When a vector A is multiplied by a + ve real no. λ gives a vector whose magnitude is changed by the factor λ but the direction is the same as that of $\vec{A}$

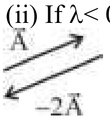
$$\left|\lambda\vec{A}\right| = \lambda\left|\vec{A}\right|$$

Example (i) if $\lambda > 0$



= direction is same but magnitude is different.

Example (ii) If $\lambda < 0$ then



= Magnitude and direction both are different.

2. By a scalar-

Let $\vec{A}$ be a vector and λ be a scalar then the product of it is called a multiplication of a vector by the scalar λ i.e.

$$\left|\lambda\vec{a}\right| = |\lambda|\left|\vec{a}\right|$$

When a vector is multiplied by a scalar quantity then the magnitude of the vector changes in accordance with the magnitude of the scalar but the direction of the vector remains unchanged.

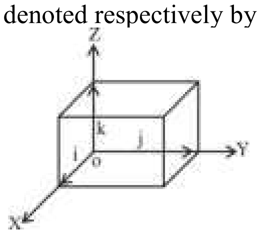
■ Rotation of a vector -

(i) If a vector is rotated through an angle θ, which is not an integral multiple of 2π the vector changes.

(ii) If the frame of reference is rotated or translated the given vector does not change, the components of a vector may change.

■ The rectangular unit vector-

It is an important set of unit vectors and are those vectors having the direction of the positive x,y and z axis of a three dimensional co-ordinate system and denoted respectively by i,j and k



Rectangular component-

When a vector is resolved along two mutually perpendicular directions the components so obtained are called rectangular components of a given vector.

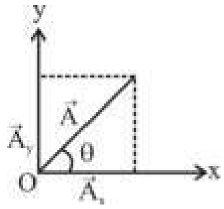
Rectangular components of a vector in a plane-

$\vec{A} = \vec{A}_x + \vec{A}_y$

$\vec{A} = A_x\hat{i} + A_y\hat{j}$

If $\vec{A}$ makes an angle θ with x-axis then-

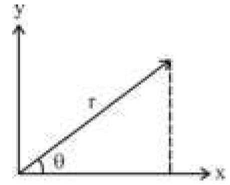
$A_x = A\cos\theta$ $A_y = A\sin\theta$



Magnitude of vector -

A= $\sqrt{A_x^2 + A_y^2}$ $\tan\theta = \dfrac{A_y}{A_x} \Rightarrow \theta = \tan^{-1}\left(\dfrac{A_y}{A_x}\right)$

General vector in x-y plane-



$\vec{r} = x\hat{i} + y\hat{j}$

If $\vec{r}$ makes an angle θ with x-axis, then

x= rcos θ and y = rsin θ

$\vec{r} = r\left(\cos\theta\hat{i} + \sin\theta\hat{j}\right)$

Example-

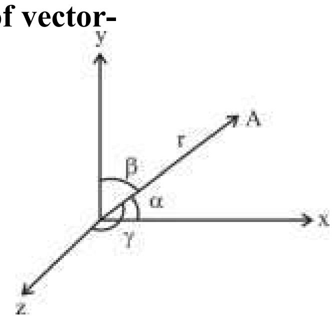
1. construct a vector of magnitude 6 units making an angle of 60° with x-axis-

solution- $\vec{r} = 6\left(\cos 60^\circ\,\hat{i} + \sin 60^\circ\,\hat{j}\right) = 3\hat{i} + 3\sqrt{3}\hat{j}$

2. construct an unit vector making an angle of 135° with x-axis-

solution- $\vec{r} = 1\left(\cos 135^\circ\hat{i} + \sin 135^\circ\hat{j}\right) = \dfrac{1}{\sqrt{2}}\left(-\hat{i} + \hat{j}\right)$

■ Direction cosine of vector-



- Angle made with x-axis $\cos\alpha = \dfrac{A_x}{A} = \dfrac{A_x}{\sqrt{A_x^2 + A_y^2 + A_z^2}} = \ell$
- Angle made with y-axis $\cos\beta = \dfrac{A_y}{A} = \dfrac{A_y}{\sqrt{A_x^2 + A_y^2 + A_z^2}} = m$
- Angle made with z-axis $\cos\gamma = \dfrac{A_z}{A} = \dfrac{A_z}{\sqrt{A_x^2 + A_y^2 + A_z^2}} = n$

Where, ℓ, m & n are called direction cosines.

- $\ell^2 + m^2 + n^2 = \cos^2\alpha + \cos^2\beta + \cos^2\gamma = \dfrac{A_x^2 + A_y^2 + A_z^2}{\sqrt{A_x^2 + A_y^2 + A_z^2}} = 1$.
- $\sin^2\alpha + \sin^2\beta + \sin^2\gamma\ \theta = 2$

■ The Dot or Scalar product -

The dot or scalar product of two vector is defined as the product of the magnitudes of A and B and the cosine of the angle θ between them.

$\vec{A}\cdot\vec{B} = |A||B|\cos\theta \qquad (0 \le \theta \le \pi)$

Note:-

- $\vec{A}\cdot\vec{B}$ is a scalar not a vector.
- $\vec{A}\cdot\vec{B}$ is +ve if θ is acute.
- $\vec{A}\cdot\vec{B}$ is –ve if θ is obtuse.
- $\vec{A}\cdot\vec{B}$ is zero if θ is right angle.

Properties of Dot product-

1. $\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$ (Commutative law)
2. $\vec{A}.\left(\vec{B}+\vec{C}\right) = \vec{A}.\vec{B} + \vec{A}.\vec{C}$ (Distributive law)
3. $m\left(\vec{A}\cdot\vec{B}\right) = \left(m\vec{A}\right)\vec{B} = \vec{A}.\left(m\vec{B}\right) = \left(\vec{A}.\vec{B}\right)m$, where m is scalar
4. $\hat{i}.\hat{i} = \hat{j}.\hat{j} = \hat{k}.\hat{k} = 1$
5. $\hat{i}.\hat{j} = \hat{j}.\hat{k} = \hat{k}.\hat{i} = 0$
6. If A = $A_1\hat{i} + A_2\hat{j} + A_3\hat{k}$ and B = $B_1\hat{i} + B_2\hat{j} + B_3\hat{k}$, then

   $\vec{A}.\vec{B} = A_1B_1 + A_2B_2 + A_3B_3$

   $\vec{A}.\vec{A} = A^2 = A_1^2 + A_2^2 + A_3^2$

   $\vec{B}\cdot\vec{B} = B^2 = B_1^2 + B_2^2 + B_3^2$
7. $\cos\theta = \dfrac{\vec{A}\cdot\vec{B}}{|\vec{A}||\vec{B}|}$

Example of dot product-

- Work W = $\vec{F}.\vec{d} = F\,d\cos\theta$ where, F → force, d → displacement
- Power P = $\vec{F}.\vec{v} = F.v\cos\theta$ Where, F → force, v → velocity
- Electric Flux ϕ_E = $\vec{E}.\vec{A} = E\,A\cos\theta$ Where, E → Electric field A → Area
- Magnetic flux ϕ_B = $\vec{B}.\vec{A} = B\,A\cos\theta$ Where, B → magnetic field A → area
- Potential energy of dipole in uniform field $\vec{U} = -\vec{p}.\vec{E}$ Where, $\vec{p}$ → dipole moment, E → electric field.

■ Cross Product (or vector product)-

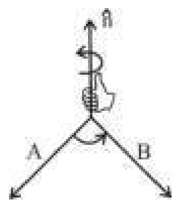
The magnitude of cross or vector product of A and B i.e. A×B is defined as the product of the magnitude of A and B and the sine of the angle θ between then,

$\vec{A}\times\vec{B} = |A||B|\sin\theta\hat{n}$

where, $\hat{n}$ is a vector perpendicular to $\vec{A}$ & $\vec{B}$ or their plane and its direction given by right hand thumb rule.

Right hand thumb rule-

Curl the fingers of your right hand from A to B through the smaller angle between them. Then, the direction of thumb represents A×B or $\hat{n}$.



Properties of cross product-

1. $\vec{A}\times\vec{B} = -\vec{B}\times\vec{A}$ (Anti Commutative law)
2. $\vec{A}\times\left(\vec{B}\times\vec{C}\right) = \vec{A}\times\vec{B}+\vec{A}\times\vec{C}$ (Distributive law)
3. $m\left(\vec{A}\times\vec{B}\right) = \left(m\vec{A}\right)\times\vec{B} = \left(\vec{A}\times\vec{B}\right)m$, where m is scalar
4. $\hat{i}\times\hat{i} = \hat{j}\times\hat{j} = \hat{k}\times\hat{k}; =0$
5. $\hat{i}\times\hat{j} = \hat{k}, \hat{j}\times\hat{k} = \hat{i}, \hat{k}\times\hat{i} = \hat{j}$
6. If $\vec{A} = A_x\hat{i}+A_y\hat{j}+A_z\hat{k}$ and $\vec{B} = B_x\hat{i}+B_y\hat{j}+B_z\hat{k}$, then

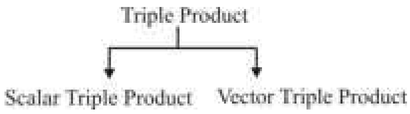
$$\vec{A}\times\vec{B} = \begin{vmatrix} i & j & k \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

7. The magnitude of $\vec{A}\times\vec{B}$ is the same as the area of a parallelogram with sides A and B.
8. If $\vec{A}\times\vec{B} = 0$ Then $\vec{A}$ and $\vec{B}$ are parallel and $\theta = 0°$

Examples of Cross product-

- Torque $\vec{\tau} = \vec{r}\times\vec{F}$ Where, $\vec{r} \rightarrow$ position vector, $F \rightarrow$ force
- Angular momentum $\vec{J} = \vec{r}\times\vec{p}$ Where, $\vec{r} \rightarrow$ position vector, $p \rightarrow$ linear momentum
- Linear velocity $\vec{V} = \vec{\omega}\times\vec{r}$ Where, $\vec{r} \rightarrow$ position vector, $\omega \rightarrow$ angular velocity
- Torque on dipole placed in electric field $\vec{\tau} = \vec{p}\times\vec{E}$ Where, $p \rightarrow$ dipole moment, $E \rightarrow$ Electric field

■ Triple product-



Scalar Triple Product - It is the dot product of a vector with the cross product of two other vectors. If a, b, c are three vectors, then, their scalar product is $\vec{a}.\left(\vec{b}\times\vec{c}\right)$.

Symbolically it is also written as [a b c] = $\vec{a}.\left(\vec{b}\times\vec{c}\right)$

Properties of Scalar Triple Product-

- The scalar triple product of three vectors is zero if any two of them are parallel, i.e., [a a b] = 0
- [(a + b) c d] = [a c d] + [b c d]
  LHS = [(a + b) c d]
  = (a + b) · (c × d)
  = a· (c × d) + b· (c × d)
  = [a c d] + [b c d]
  = RHS
- [λa b c] = λ [a b c], where λ is a real number.
- The scalar triple product of three non-zero vectors is zero if and only if they are coplanar,
- Since the scalar product is commutative, therefore we have
  - a · (b × c) = (b × c) · a
  - b· (c × a) = (c ×a) · b
  - c· (a × b) = (a × b) · c

Vector Triple Product - The vector triple product is the cross product of a vector with the cross-product of the other two vectors.

Mathematically, it can be represented by $\left(\vec{a}\times\vec{b}\times\vec{c}\right)$

Properties of Vector Triple Product–

- Vector triple product is a vector quantity.
- Unit vector coplanar with $\vec{a}$ and $\vec{b}$ and perpendicular to $\vec{c}$ is $\pm\dfrac{\left(\vec{a}\times\vec{b}\right)\times\vec{c}}{\left|\left(\vec{a}\times\vec{b}\right)\times\vec{c}\right|}$
- $\vec{a}\times\left(\vec{b}\times\vec{c}\right) \neq \left(\vec{a}\times\vec{b}\right)\times\vec{c}$
- $\left[\vec{a}\times\vec{b} \quad \vec{b}\times\vec{c} \quad \vec{c}\times\vec{a}\right] = \left[\vec{a} \ \vec{b} \ \vec{c}\right]^2$

■ Reciprocal vectors-

The set of vectors a*, b*, c* are the reciprocal vectors of a, b, c respectively.

Properties of reciprocal vectors-

(i) $\vec{a}*\vec{a}=1$ (ii) $\vec{a}*\vec{b}=0$ (iii) $\vec{a}*\vec{c}=0$

$\vec{b}*\vec{b}=1$ $\vec{b}*\vec{a}=0$ $\vec{b}*\vec{c}=0$

$\vec{c}*\vec{c}=1$ $\vec{c}*\vec{a}=0$ $\vec{c}*\vec{b}=0$

Where, $$\vec{a}^* = 2\pi\frac{\vec{b}\times\vec{c}}{\left[\vec{a}\vec{b}\vec{c}\right]}$$

$$\vec{b}^* = 2\pi\frac{\vec{c}\times\vec{a}}{\left[\vec{a}\vec{b}\vec{c}\right]}$$

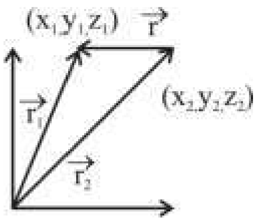
$$\vec{c}^* = 2\pi\frac{\vec{a}\times\vec{b}}{\left[\vec{a}\,\vec{b}\,\vec{c}\right]}$$

■ Linearly Independent and dependent vector- Let $\vec{A},\vec{B}$ and $\vec{C}$ are set of vectors,

- If $\left[\vec{A}\vec{B}\vec{C}\right]=0$ then these set of vectors are linearly dependent and coplanar
- If $\left[\vec{A}\vec{B}\vec{C}\right]\neq 0$ then these set of vectors are linearly independent and non-coplanar
- When a particle moved from (x_1, y_1, z_1) to (x_2, y_2, z_2) then its displacement vector-

$$\vec{r}=\vec{r}_2-\vec{r}_1=\left(x_2\hat{i}+y_2\hat{j}+z_2\hat{k}\right)-\left(x_1\hat{i}+y_1\hat{j}+z_1\hat{k}\right)$$

$$=\left(x_2-x_1\right)\hat{i}+\left(y_2-y_1\right)\hat{j}+\left(z_2-z_1\right)\hat{k}$$



Magnitude -

$$r=\left|\vec{r}\right|=\sqrt{\left(x_2-x_1\right)^2+\left(y_2-y_1\right)^2+\left(z_2-z_1\right)^2}$$

■ Key points-

- A Scalar is a zero rank tensor
- A vector is a first rank tensor.
- Electric current is not a vector as it does not obey the law of vector addition.
- A unit vector has no unit.
- A scalar or a vector can never be divided by a vector
- To a vector only a vector of same type can be added and resultant is a vector of same type.

(3) Curl Divergence, Gauss, Stokes Theorem and their application

■ The Vector differential Operator Del-

$$\nabla \equiv \frac{\partial}{\partial x}\hat{i}+\frac{\partial}{\partial y}\hat{j}+\frac{\partial}{\partial z}\hat{k}\equiv \hat{i}\frac{\partial}{\partial x}+\hat{j}\frac{\partial}{\partial y}+\hat{k}\frac{\partial}{\partial z}$$

The operator ∇ is also known as nabla.

■ The Gradient-

- Let (x, y, z) be defined as differential at each point in a certain region of space (i.e ϕ defines a differential scalar field). Then the $\nabla\phi$ can be written as-

$$\nabla\phi=\left(\frac{\partial}{\partial x}\hat{i}+\frac{\partial}{\partial y}\hat{j}+\frac{\partial}{\partial z}\hat{k}\right)\phi=\left(\frac{\partial\phi}{\partial x}\hat{i}+\frac{\partial\phi}{\partial y}\hat{j}+\frac{\partial\phi}{\partial z}\hat{k}\right)$$

- $\nabla\phi$ defines a vector field.

Note :- The component of $\nabla\phi$ in the direction of a unit vector a is given by $\nabla\phi$.a and is called the directional derivative of ϕ in the direction a. Physically, it is the rate of change of ϕ (x, y, z) in the direction a.

■ The Divergence –

- Let V(x, y, z) = $V_1\hat{i}+V_2\hat{j}+V_3\hat{k}$ can be defined as differential at each point (x, y, z) in a certain region of space (i.e V defines a differential vector field). Then the divergence of V i.e ∇.V can be written as -

$$\nabla\cdot V=\left(\frac{\partial}{\partial x}\hat{i}+\frac{\partial}{\partial x}\hat{j}+\frac{\partial}{\partial x}\hat{k}\right)\cdot\left(V_1\hat{i}+V_2\hat{j}+V_3\hat{k}\right)$$

$$\nabla\cdot V=\left(\frac{\partial V_1}{\partial x}+\frac{\partial V_2}{\partial y}+\frac{\partial V_3}{\partial z}\right)$$

Note- $\nabla\cdot V \neq V\cdot\nabla$

■ The Curl -

- If V(x, y, z) is a differential vector field then curl of V can be written as -

$$\nabla\times V=\left(\frac{\partial}{\partial x}\hat{i}+\frac{\partial}{\partial y}\hat{j}+\frac{\partial}{\partial z}\hat{k}\right)\times\left(V_1\hat{i}+V_2\hat{j}+V_3\hat{k}\right)$$

$$=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ V_1 & V_2 & V_3\end{vmatrix}$$

Note :-

- $\nabla\phi$ is vector so we can take divergence and curl of it.

Formula involving ∇ –

- If A and B are differentiable vector functions and ϕ and ψ are differentiable scalar function of positions (x, y, z) then-

1. $\nabla(\phi+\psi)=\nabla\phi+\nabla\psi$
2. $\nabla\cdot(A+B)=\nabla\cdot A+\nabla\cdot B$
3. $\nabla\times(A+B)=\nabla\times A+\nabla\times B$
4. $\nabla\cdot(\phi A)=(\nabla\phi)\cdot A+\phi(\nabla\cdot A)$
5. $\nabla\times(\phi A)=(\nabla\phi)\times A+\phi(\nabla\times A)$
6. $\nabla\cdot(A\times B)=B\cdot(\nabla\times A)-A\cdot(\nabla\times B)$
7. $\nabla\times(A\times B)=(B\cdot\nabla)A-B(\nabla\cdot A)-(A\cdot\nabla)B+A(\nabla\cdot B)$
8. $\nabla(A\cdot B)=(B\cdot\nabla)A+(A\cdot\nabla)B+B\times(\nabla\times A)+A\times(\nabla\times B)$
9. $\nabla\cdot(\nabla\phi)=\nabla^2\phi=\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}+\frac{\partial^2\phi}{\partial z^2}$

Where, $\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}$ is called the Laplacian operator

10. $\nabla\times(\nabla\phi)=0$ The curl of the gradient of ϕ is zero.
11. $\nabla\cdot(\nabla\times A)=0$ The divergence of curl of A is.
12. $\nabla\times(\nabla\times A)=\nabla(\nabla\cdot A)-\nabla^2 A$

■ Integral Calculus-

(i) Line integral-

- The line integral expression is -

$$I=\int_a^b \vec{A}\cdot\vec{d\ell}\ =0$$

where, $\vec{A}$ = vector field

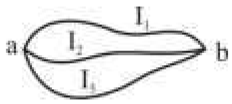
$\vec{d\ell}$ = infinitesimal displacement vectors

$\vec{d\ell}$ = $dx\hat{i}+dy\hat{j}+dz\hat{k}$

- The close line integral -

$$\oint_{Path}\vec{A}.\vec{d\ell}$$

- In general the line integral is path dependent.



- There are some special vector function for which the line integral is independent of path

$I = I_1 = I_2 = I_3$

For example -

(a) $I = \int_a^b \vec{A} \cdot \vec{d\ell}$ = path independent (will depends on end points a and b,)

where, $\vec{A}$ = special vector function

(b) $\int \nabla T \cdot \vec{d\ell} = T(\vec{b}) - T(\vec{a})$

= independent of path

$\oint \nabla T \cdot \vec{d\ell} = 0$

Where, T is a function and ∇T is grad T

(ii) Surface (Double) integral -

The expression is of the form

$$I = \int_{surface} \vec{A} \cdot \vec{da}$$

where, $\vec{A}$ = vector function

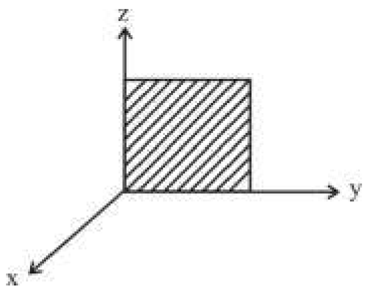
$\vec{da}$ = elementary area is called surface integral.

Case-1 When surface is parallel to fundamental plane (xy, yz, zx)

$\vec{da} = dydz\hat{i}$

$\vec{da} = dzdx\hat{j}$

$\vec{da} = dxdy\hat{k}$



Case-2 When surface is not parallel to fundamental plane

$\vec{da} = da\,\hat{n}$

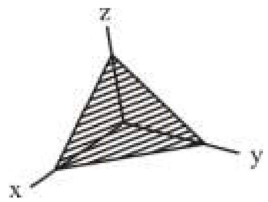
$\phi = c$ (equation of surface)

$\hat{n} = \dfrac{\nabla\phi}{|\nabla\phi|}$

$\vec{da} = \dfrac{dxdy}{|\hat{k}\cdot\hat{n}|}\hat{n} \rightarrow$ In xy -plane

$\vec{da} = \dfrac{dydz}{|\hat{i}\cdot\hat{n}|}\hat{n} \rightarrow$ In yz -plane

$\vec{da} = \dfrac{dzdx}{|\hat{j}\cdot\hat{n}|}\hat{n} \rightarrow$ In xz -plane



(iii) Volume Integral -

- The expression of the volume integral is -

$$I = \int T dV$$

Where, T = Scalar function
dV = infinitesimal volume element
dV = dxdydz $\rightarrow$ In Cartesian co-ordinate system

■ **Fundamental theorem of Calculus-**

- The integral of a derivative is equal to the value of the function at end point or boundary

$$\frac{\mathbf{d}}{\mathbf{dx}}\int_a^x f(t)dt = f(x) \Rightarrow \int_a^b f(x)dx = F(b) - F(a)$$

Fundamental theorem of Gradient -

- The line integral of gradient is given by the value of the function at the boundaries (a and b)

$$\int_a^b \nabla T \cdot \vec{d\ell} = T(b) - T(a)$$

Where, $\vec{dl}$ = infinitesimal displacement vector in Cartesian co-ordinate.

The Fundamental theorem for Divergence (Green's Theorem, Gauss Divergence Theorem)

This theorem is applicable only for closed surfaces and this theorem is used to convert surface integral into volume integral and vice verse.

$$\oiint_s \vec{A} \cdot \vec{ds} = \oiint_s \vec{A} \cdot \hat{n} ds = \iiint_v (\nabla \cdot \vec{A}) dv$$

Where $\hat{n}$ is the outward normal to s indicating the +ve direction of s.

Fundamental theorem of curl-

- This theorem is applicable only for open surfaces and this theorem is used to convert surface integral into line integral and vice versa.
- If S is an open, two sided surface bounded by a closed, non-intersecting curve C and $\vec{A}$ is vector function of position with continuous derivatives then-

$$\oint_c \vec{A} \cdot \vec{dr} = \iint_s (\nabla \times \vec{A}) \hat{n}\, ds = \iint_s (\nabla \times \vec{A}) \cdot \vec{ds}$$

Where C is traversed in the position (counter clockwise direction)

■ Co-ordinate System–

Co-ordinate system	$\vec{dl}$	h_1 h_2 h_3	u_1 u_2 u_3
Cartesian	$dx\hat{i} + dy\hat{j} + dz\hat{k}$	1 1 1	x y z
Spherical	$dr\hat{r} + r\, d\theta\hat{\theta} + r\sin\theta\, d\phi\hat{\phi}$	1 r r sin θ	r θ ϕ
Cylindrical	$ds\, \hat{s} + s\, d\phi\hat{\phi} + dz\hat{z}$	1 S 1	s ϕ z

■ General Expression for Gradient, Divergence and Curl-

(i) Gradient - $\nabla\phi = \left(\frac{1}{h_1}\frac{\partial}{\partial u_1}\hat{u}_1 + \frac{1}{h_2}\frac{\partial}{\partial u_2}\hat{u}_2 + \frac{1}{h_3}\frac{\partial}{\partial u_3}\hat{u}_3\right)\phi(u_1u_2u_3)$

(ii) Divergence - $\nabla \cdot \vec{A} = \frac{1}{h_1h_2h_3}\left[\frac{\partial}{\partial u_1}(h_2h_3A_1) + \frac{\partial}{\partial u_2}(h_1h_3A_2) + \frac{\partial}{\partial u_3}(h_1h_2A_3)\right]$

(iii) Curl - $\nabla \times \vec{A} = \frac{1}{h_1h_2h_3}\begin{bmatrix} h_1\hat{u}_1 & h_2\hat{u}_2 & h_3\hat{u}_3 \\ \frac{\partial}{\partial u_1} & \frac{\partial}{\partial u_2} & \frac{\partial}{\partial u_3} \\ h_1A_1 & h_2A_2 & h_3A_3 \end{bmatrix}$

Formula for Gradient in co-ordinate system -

Cartesian - $\nabla\phi = \left(\frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y} + \frac{\partial}{\partial z}\hat{z}\right)\phi(x, y, z)$

Spherical - $\nabla\phi = \left(\frac{\partial}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial}{\partial\theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}\hat{\phi}\right)\phi(r, \theta, \phi)$

Cylindrical - $\nabla\phi = \left(\frac{\partial}{\partial s}\hat{s} + \frac{1}{s}\frac{\partial}{\partial\phi}\hat{\phi} + \frac{\partial}{\partial z}\hat{z}\right)\phi(s, \phi, z)$

Polar - $\nabla\phi = \left(\frac{\partial}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial}{\partial\theta}\hat{\theta}\right)\phi(r, \theta)$

Formula for divergence in co-ordinate system-

Cartesian - $\nabla\cdot\vec{A} = \left[\frac{\partial}{\partial x}A_1 + \frac{\partial}{\partial y}A_2 + \frac{\partial}{\partial z}A_3\right]$

Spherical - $\nabla\cdot\vec{A} = \frac{1}{r^2\sin\theta}\left[\frac{\partial}{\partial r}\left(r^2\sin\theta\, A_1\right) + \frac{\partial}{\partial\theta}\left(r\sin\theta\, A_2\right) + \frac{\partial}{\partial\phi}\left(r\, A_3\right)\right]$

Cylindrical - $\nabla\cdot\vec{A} = \frac{1}{s}\left[\frac{\partial}{\partial s}\left(s\, A_1\right) + \frac{\partial}{\partial\phi}A_2 + \frac{\partial}{\partial z}\left(s\, A_3\right)\right]$

Formula for curl in co-ordinate system -

Cartesian - $\nabla\times\vec{A} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_1 & A_2 & A_3 \end{vmatrix}$

Spherical - $\nabla\times\vec{A} = \frac{1}{r^2\sin\theta}\begin{vmatrix} \hat{r} & r\hat{\theta} & (r\sin\theta)\hat{\phi} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\phi} \\ A_1 & rA_2 & r\sin\theta A_3 \end{vmatrix}$

Cylindrical - $\nabla\times\vec{A} = \frac{1}{s}\begin{vmatrix} \hat{s} & s\hat{\phi} & \hat{z} \\ \frac{\partial}{\partial s} & \frac{\partial}{\partial\phi} & \frac{\partial}{\partial z} \\ A_1 & sA_2 & A_3 \end{vmatrix}$

■ Key Points-

(i) ∇ is not a vector, it mimics the behaviour of an ordinary vector.

(ii) ∇ is a vector operator that acts upon:

- A scalar function T : ∇T (the gradient)
- A vector function V : $\nabla\cdot V$ (the divergence)
- A vector function V : $\nabla\times V$ (the curl)

(iii) ∇ is not a vector that multiplies T.

(iv) Operator is meaningless without a function (scalar field, vector field). For eg. without a computer mobile pen drive (operator) is useless.

(v)

Function	Outcome
$\nabla\phi$	Vector
$\nabla\cdot\vec{A}$	Scalar
$\nabla\times\vec{A}$	Vector

(vi) $\nabla\cdot(\nabla\phi)$ = Divergence of gradient
$\nabla\times(\nabla\phi)$ = Curl of gradient
$\nabla(\nabla\cdot\vec{A})$ = Gradient of divergence
$\nabla\cdot(\nabla\times\vec{A}) = 0$ Divergence of curl
$\nabla\times(\nabla\times\vec{A})$ = Curl of curl

(vii) $\hat{i}$, $\hat{j}$ and $\hat{k}$ Constant unit vector and it is taken out from differential and integral sign.

(viii) Surface integral represent flow of vector through the surface or flux.

(ix) In flux through a closed surface become zero then it is Solenoid vector.

$$\oint \hat{A}\cdot\vec{ds} = 0 \text{ then } \vec{\nabla}\cdot\hat{A} = 0$$

(x) Volume integral represent amount of quantity in volume.

(xi) If line integral of a field along closed loop is zero then it is irrotational vector field.

$$L \cdot I = \oint \vec{F} \cdot \vec{dr} = 0$$

So, $\nabla \times \vec{F} = 0$

(xii) For conservative field

$$\nabla \times \vec{F} = 0$$

and $F = \nabla\phi$, then

$$\nabla \times (\nabla\, \phi) = 0 \text{ (always)}$$

(xiii) Physically gradient represent normal vector to the surface.

$$\vec{n} = \nabla \mathbf{T}$$

$$\hat{n} = \frac{\nabla T}{|\nabla T|}$$

(xiv) The divergence measure how much the vector v spreads out (diverges) from point.

(xv) The curl is $\nabla \times V$ is measure of how much the vector V curl around the point .

(xvi) General form of Laplacian -

$$\nabla^2\varphi = \frac{1}{h_1 h_2 h_3}\left[\frac{\partial}{\partial u_1}\left(\frac{h_2 h_3}{h_1}\frac{\partial\phi}{\partial u_1}\right) + \frac{\partial}{\partial u_2}\left(\frac{h_1 h_3}{h_2}\frac{\partial\phi}{\partial u_2}\right) + \frac{\partial}{\partial u}\left(\frac{h_1 h_2}{h_3}\frac{\partial\phi}{\partial u_3}\right)\right]$$

(4) Motion in a Straight Line

■ **Mechanics-:** Mechanics is the branch of physics that deals with the condition of rest or motion of the material object around us.

■ **Rest:** When a body does not change its position with respect to time, the body is said to be in rest.

Example: A bed lying in a room is in the state of rest, because it does not change its position with respect to time.

■ **Motion :** When a body changes its position with respect to its surrounding, it is said to be in motion.

Example: A train moving on rails

Rest and motion as relative terms - Rest and motion are relative states. It means an object which is at rest in one frame of reference can be in motion in another frame of reference.

■ Types of Motion –

On the basis of direction:-

1. One dimensional Motion- if only one out of three co-ordinates specifying the position of the object with respect to time. Then it is called one dimensional motion or rectilinear motion.

For Example- (i) Motion of car on straight road.
(ii) Motion of a body under gravity.

2.Two dimensional Motion -

If only two out of three co-ordinates specifying the position of the object with respect to time, then the motion is called two dimensional motion.

For Example- (i) A gymnast on a balance beam.
(ii) Motion of planets around the sun.
(iii) A car moving along zig-zag path on a level road.

3. Three dimensional motion -

If all three coordinates specify the position of object with respect to time then it is called 3-D motion

For example- (i) Movement of gyroscope.
(ii) A like flying on a windy day.
(iii) Motion of an aeroplane in space.

On the basis of moving object in space:-

1. Uniform Motion: When moving objects cover equal distances in equal time intervals.

2. Non Uniform Motion: When moving objects cover different distances in unequal time intervals.

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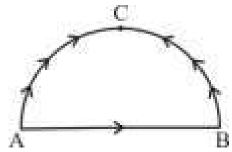
■ Distance and Displacement -

Distance (x)

- Total path ACB traveled by the body between initial and final position in definite interval is called Distance
- It is a scalar quantity.
- It have no direction
- Distance will be always positive.
- Distance have infinite function.

Displacement ($\vec{x}$)

- Displacement is the minimum possible path (AB) between initial and final position.
- It is a vector quantity.
- Its direction will be always from initial to final position.
- It may be +ve, –ve or zero.
- it have only one unique function.



■ Speed and Velocity-

(i) Speed-: The rate of change of position of an object with respect to time in any direction is called its speed.

$$\text{Speed}(V) = \frac{\text{distance travelled}(s)}{\text{time taken}(t)}$$

- It is a scalar quantity
- It is always +ve
- It's S.I unit is m/sec.

Uniform Speed- If a body covers equal distance in equal intervals of time it is said to be moving with uniform speed.

Example- (i) A rotating fan

(ii) A rocket moving in a space.

Variable speed or Non-Uniform speed:-

If a body covers unequal distances in equal intervals of time. It is said to be moving with a variable speed.

Example- (i) A train starting from a station.

(ii) a dog chasing a cat.

Average speed : The ratio of total distance travelled by the object to the total time taken is called average speed.

$$\text{Average speed} = \frac{\text{Distance travelled}}{\text{Total time taken}}$$

Instantaneous speed: If the speed of a body is continuously changing with time. Then the speed at some particular instant during the motion is called instantaneous speed.

For example- Speedometer of a moving automobile measures instantaneous speed.

(ii) Velocity : The rate of change of displacement with respect to time of body in specified direction is called velocity.

$$\text{Velocity} = \frac{\text{Displacement}}{\text{Time taken}}$$

- It is a vector quantity.
- It may be +ve, -ve or zero.
- It's S.I. unit is m/sec.

Uniform velocity -

When a body covers equal distances in equal intervals of time in a particular direction the body is said to be moving with uniform velocity.

Non-uniform velocity- when a body covers unequal distances in equal intervals of time in a particular direction the body is said to be non-uniform velocity .

Average Velocity- The ratio of the total displacement to the total time taken by the body is called average velocity.

$$\text{Average velocity} = \frac{\text{Total displacement}}{\text{Total time taken}}$$

Instantaneous Velocity - The velocity of a particle at any instant of time is known as instantaneous velocity

$$\text{Instantaneous velocity} = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

■ Acceleration -

The rate of change of velocity with respect to time is known as acceleration.

$$\text{Acceleration} = \frac{\text{Change in velocity } (\Delta V)}{\text{Time interval } (\Delta t)}$$

- Its S.I unit is m/sec^2
- It is a vector quantity
- It may be +ve, -ve or zero
- If velocity increases then acceleration is +ve
- If velocity decreases then it is known as retardation and 'it' is -ve.
- If velocity is constant then a = 0 (i.e uniform motion)

Uniform Acceleration - When a body describes equal changes in velocity in equal intervals of time , it is said to be moving with uniform acceleration.

Non- Uniform Acceleration-

If an object is moving with non-uniform acceleration , it means that change in velocity is unequal for equal interval of time.

Average Acceleration-

It is defined as the ratio of total change in velocity in given interval to the total time taken. Unlike acceleration the average acceleration is calculated for a given interval.

Instantaneous Acceleration-

It is defined as the acceleration of body at any instant of time.

$$\text{Instantaneous Acceleration} = \lim_{\Delta t \to 0} \frac{\Delta V}{\Delta t} = \frac{dV}{dt}$$

Formula and concept for uniformly accelerated motion in a straight line

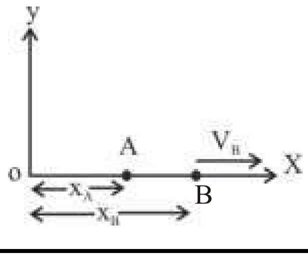
Scalar form	**Vector form**
• $v = u + at$	$\vec{v} = \vec{u} + \vec{a}t$
• $s = ut + \frac{1}{2}at^2$	$\vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2$
• $v^2 = u^2 + 2as$	$\vec{v}.\vec{v} - \vec{u}.\vec{u} = 2\vec{a}\vec{s}$
• $s = \left(\frac{u+v}{2}\right)t$	$\vec{s} = \frac{1}{2}(\vec{u} + \vec{v})t$
• $s_n = u + \frac{a}{2}(2n-1)$	$\vec{s}_n = \vec{u} + \frac{\vec{a}}{2}(2n-1)$

- Displacement of a particle in n^{th} second of its motion in uniformly accelerated motion-

  $D_n = u + \frac{a}{2}(2n-1)$

■ Relative motion in one Dimension :-

Let A and B are two objects and if $\vec{x}_A$ and $\vec{x}_B$ are their respective displacements with respect to the fixed origin. Then



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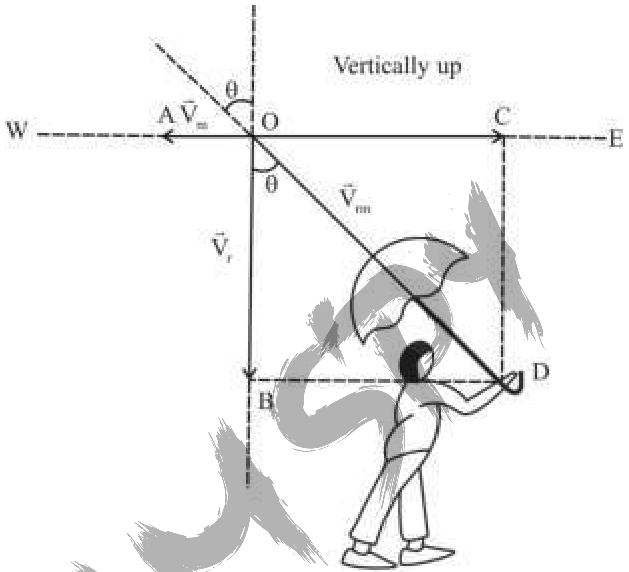
- The relative displacement of B with respect to A is defined as
  $\vec{x}_{BA} = \vec{x}_B - \vec{x}_A$
- The relative velocity of B with respect to A is defined as-
  $\vec{V}_{BA} = \vec{V}_B - \vec{V}_A$
- The relative acceleration of B with respect to A is defined as -
  $\vec{a}_{BA} = \vec{a}_B - \vec{a}_A$

Relative velocity of Rain with respect to the Moving man -

A man walking west with velocity $\vec{v}_m$, represented by $\overrightarrow{OA}$. Let the rain be falling vertically downwards with velocity $\vec{v}_r$ represented by $\overrightarrow{OB}$ as shown in



The relative velocity of rain with respect to man $\vec{V}_{rm} = \vec{V}_r - \vec{V}_m$ will be represented by diagonal $\overrightarrow{OD}$ of rectangle OBDC.

$\therefore\ V_{rm} = \sqrt{V_r^2 + V_m^2 + 2V_rV_m \cos 90^\circ} = \sqrt{V_r^2 + V_m^2}$

If θ is the angle which $\vec{V}_{rm}$ makes with the vertical direction then

$\tan\theta = \dfrac{BD}{OB} = \dfrac{V_m}{V_r} \Rightarrow \theta = \tan^{-1}\dfrac{V_m}{V_r}$

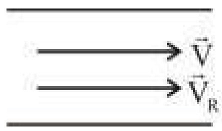
- **Swimming into the River-**

  A man can swim with velocity $\vec{V}$ i.e it is the velocity of man with respect to still water. If water is also flowing with velocity $\vec{V}_R$, then velocity of man relative to ground.

  $\vec{V}_m = \vec{V} + \vec{V}_R$

Case I -

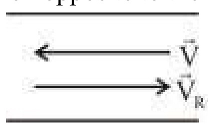
- If the swimming is in the direction of flow of water or downstream then-



$\vec{V}_m = \vec{V} + \vec{V}_R$

Case II -

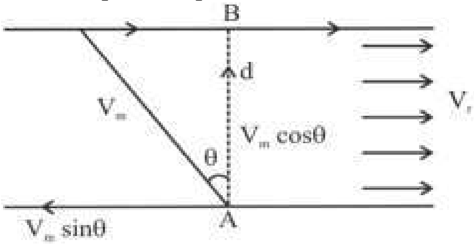
- If the swimming is in the direction opposite to the flow of water or then-



$\vec{V}_m = \vec{V} - \vec{V}_R$

Case-III To cross the river from one bank to another bank.

(i) To cross the river in minimum possible path.



d = width of river to reach from A to B,

$$V_m \sin\theta = V_r$$

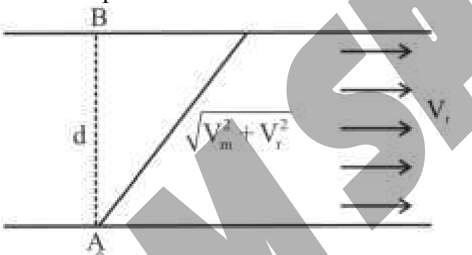
$$\sin\theta = \frac{V_r}{V_m}$$

$$\theta = \sin^{-1}\frac{V_r}{V_m}$$

(ii) Time taken to cross the river -

$$t = \frac{d}{V_m \cos\theta} = \frac{d}{\sqrt{V_m^2 - V_r^2}}$$

(iii) To cross the river in minimum possible time-



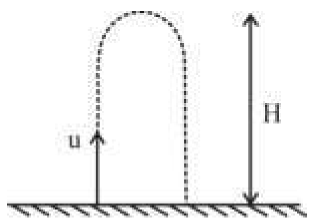
$$t = \frac{d}{V_m \cos\theta}$$

For minimum, $\theta = 0^o$

$$\left[t_{min} = \frac{d}{V_m}\right]$$

■ Motion Under Gravity –

- If a body is thrown vertically up with a velocity u in the uniform gravitational field (neglecting air resistance), then-



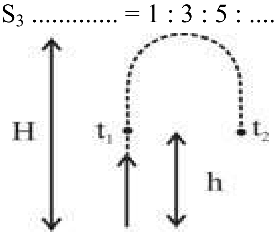
(i) Maximum height attained $H = \frac{u^2}{2g}$

(ii) Time of ascent = time of descent = $\frac{u}{g}$

(iii) Total time of flight = $\frac{2u}{g}$

(iv) Velocity of body at the point of projection = u (downwards)

(v) **Galileo's law of odd numbers :** For a freely falling body ratio of successive distance covered in equal time interval 't' $S_1 : S_2 : S_3$ .... = 1 : 3 : 5 : .... : 2n – 1.



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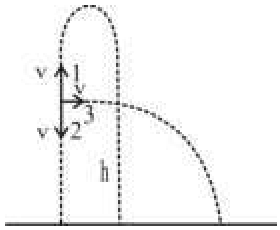
(vi) At any point on its path the body will have same speed for upward journey and downward journey.

(vii) If a body throws upward crosses point in time t_1 & t_2 respectively, then height of point h = $\frac{1}{2}gt_1t_2$ and maximum height H = $\frac{1}{2}g\left(t_1+t_2\right)^2$.

(viii) A body is thrown upward, downward & horizontally with same speed takes time t_1, t_2 & t_3 respectively to reach the ground then $t_3=\sqrt{t_1t_2}$ & height from where the particle was thrown is- $H=\frac{1}{2}g\,t_1t_2$.



Important points about graphical analysis of motion -

- Instantaneous velocity is the slope of position time curve $\left(V=\frac{dx}{dt}\right)$.
- Slope of velocity time curve = instantaneous acceleration $\left(a=\frac{dv}{dt}\right)$.
- V-t curve area gives displacement, $\left[\Delta x=\int vdt\right]$.
- a-t curve area gives change in velocity $\left[\Delta v=\int adt\right]$

■ Key points -

- $\boxed{\text{Displacement}} \underset{\text{Integration}}{\overset{\text{Differentiation}}{\rightleftarrows}} \boxed{\text{Velocity}} \underset{\text{Integration}}{\overset{\text{Differentiation}}{\rightleftarrows}} \boxed{\text{Acceleration}}$
- Displacement ≤ Distance.
- $\frac{\text{Velocity}}{\text{Speed}}\le 1$
- $\frac{\text{Average velocity}}{\text{Average speed}}\le 1$
- $\frac{\text{Instantaneous velocity}}{\text{Instantaneous speed}}=1$
- If distance > |displacement| this implies -
  (a) Atleast at one point in path, velocity is zero.
  (b) The body must have retarded during the motion.
- If particle travels distances S_1, S_2, S_3, ... with speeds V_1, V_2, V_3, ... then,

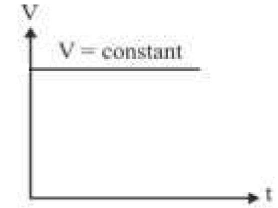
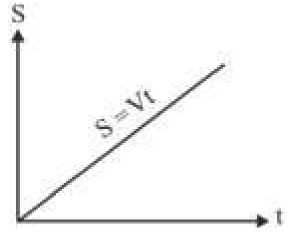
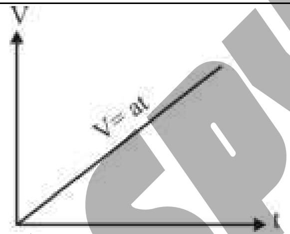
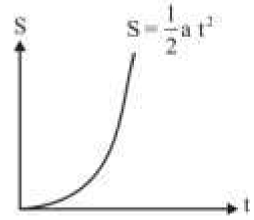
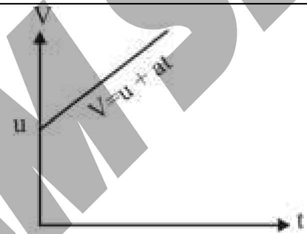
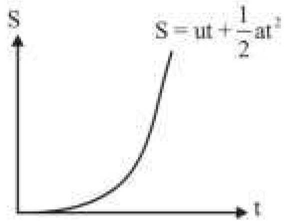
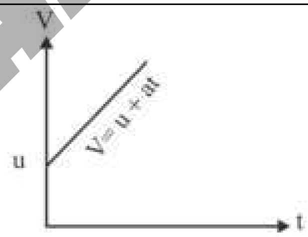
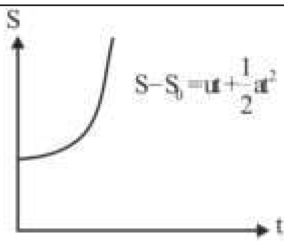
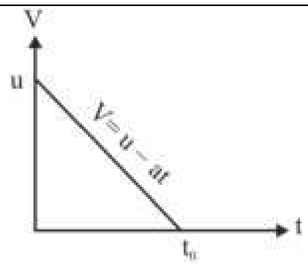
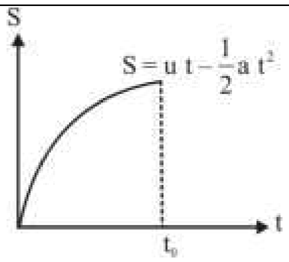
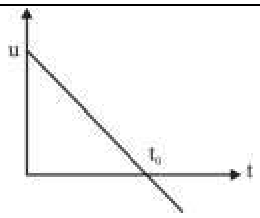
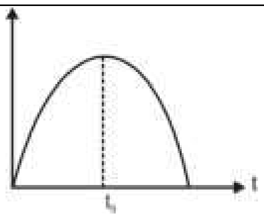
$$\text{Average speed}=\frac{S_1+S_2+S_3}{\left(\frac{S_1}{V_1}+\frac{S_2}{V_2}+\frac{S_3}{V_3}......\right)}$$

- If particle travels equal distances ($S_1 = S_2 = S$) with velocities V_1, V_2, V_3, ... during time intervals t_1, t_2, t_3 then, $\text{Average speed}=\frac{V_1t_1+V_2t_2+V_3t_3}{t_1+t_2+t_3+........}$

- If particle travels with speed V_1 and V_2 for equal time intervals i.e $t_1 = t_2 = t$, then
  Average speed = $\dfrac{V_1 + V_2}{2}$.
- When a body travels equal distances with speed V_1 and V_2, the average speed (V) is the harmonic mean of two speeds i.e

$$\frac{2}{V} = \frac{1}{V_1} + \frac{1}{V_2}$$

Different Motions and their Graphs :

Different Cases	**V-t Graph**	**S-t Graph**
1. Uniform motion		
2. Uniform accelerated motion with u = 0 at t = 0		
3. Uniformly accelerated with u ≠ 0 at t = 0		
4. Uniformly accelerated motion with u ≠ 0 and S= S_0 at t = 0		
5. Uniformly retarded motion till velocity becomes zero		
6. Uniformly retarded then accelerated in opposite direction		

TEAM SPY

(5) Motion in a Plane

■ Motion in a Plane

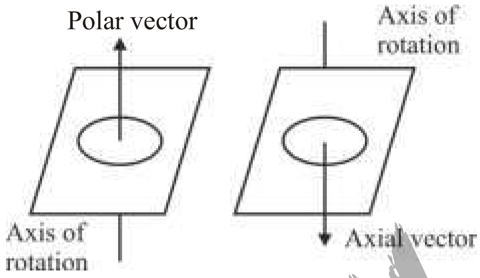
- Motion in a plane is also called as a motion in two dimension.
  For example- circular motion, projectile motion etc.

Polar Vectors - The polar vectors which have a starting point or point of application are called polar vectors.

Example- Displacement, velocity, force etc are polar vectors.

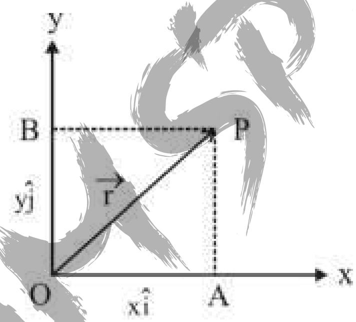
Axial Vectors- The vector which represent rotational effect and act along the axis of rotation in accordance with right hand screw rule are called axial vector.

Example:- Angular velocity, Torque, Angular momentum etc.



■ Terms related to motion in a plane -

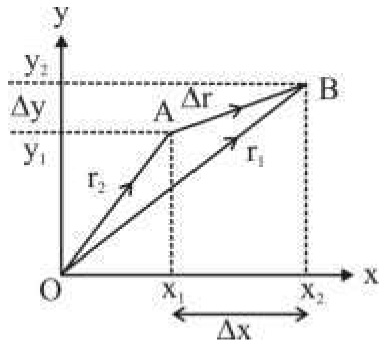
Position vector



$$\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{OB}$$
$$\vec{r} = x\hat{i} + y\hat{j}$$

- This equation express position vector $\vec{r}$ in terms of its rectangular component x and y.

Displacement Vector -



- In plane, displacement can be represented as -

$$\Delta r = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j}$$

- Magnitude of displacement vector

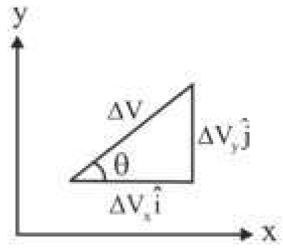
$$|\Delta r| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

- Direction of the displacement vector Δr is given by -

$$\tan\theta = \frac{\Delta y}{\Delta x}$$

Velocity Vector-

(i) Average Velocity -



$$V_{av} = \frac{\Delta r}{\Delta t} = \frac{r_2 - r_1}{t_2 - t_1}$$

Average velocity in component form-

$$V_{av} = \frac{\Delta x}{\Delta t}\hat{i} + \frac{\Delta y}{\Delta t}\hat{j}$$

$$= \Delta V_x\hat{i} + \Delta V_y\hat{j}$$

Direction of the velocity ΔV is given by-

$$\tan\theta = \frac{\Delta V_y}{\Delta V_x}$$

(ii) Instantaneous Velocity-

$$V = \lim_{x\to 0}\frac{\Delta r}{\Delta t} = \frac{dr}{dt}$$

$$V = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j}$$

$$V = V_x\hat{i} + V_y\hat{j}$$

Magnitude of Instantaneous Velocity-

$$|V| = \sqrt{V_x^2 + V_y^2}$$

Direction of V is given by-

$$\tan\theta = \frac{V_y}{V_x}$$

Acceleration Vector -

(i) Average Acceleration-

- The average acceleration vector is defined as the rate at which the velocity changes. It is in the direction of the change in velocity $\overrightarrow{\Delta V}$

$$\vec{a}_{av} = \frac{\overrightarrow{\Delta V}}{\Delta t}$$

$$\vec{a}_{av} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$$

(ii) Instantaneous Acceleration -

- It is defined as the limit of the average acceleration as Δt approaches zero.

$$\vec{a} = \lim_{\Delta t\to 0}\frac{\Delta\vec{V}}{\Delta t} = \frac{d\vec{V}}{dt}$$

$$\vec{a} = \lim_{\Delta t\to 0}\left(\frac{\Delta V_x}{\Delta t}\hat{x} + \frac{\Delta V_y}{\Delta t}\hat{y} + \frac{\Delta V_z}{\Delta t}\hat{z}\right)$$

■ Motion in a plane with uniform acceleration-

$V_x = V_{ox} + a_x t$

$V_y = V_{oy} + a_y t$

TEAM SPY

Path of particle under constant Acceleration-

$$x = x_0 + V_{ox}t + \frac{1}{2}a_x t^2 \text{ .... along x-axis}$$

$$y = y_0 + V_{oy}t + \frac{1}{2}a_y t^2 \text{ .... along y-axis}$$

■ Circular Motion -

- When object is moving on a circular path on the circumference of the circle, then the motion is called circular motion.

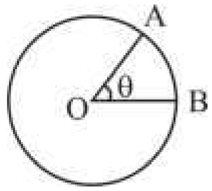
Uniform circular Motion-

- When object is moving on a circular path on the circumference of the circle, covers equal distances in equal intervals of time then the motion is called uniform circular motion.

Angular displacement (θ)-

- It is the angle traced out by the radius vector at the circular path.

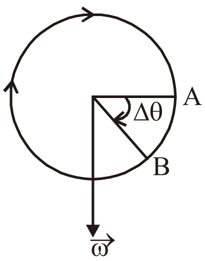
Angle (θ) = $\frac{\text{arc}}{\text{radius}}$



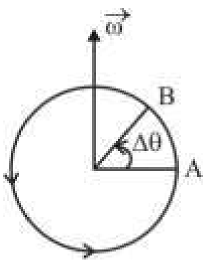
It is a vector quantity.

Angular Velocity $(\vec{\omega})$ -

- It is the time rate of change of angular displacement.
  SI unit is rad/sec.
  $\vec{\omega} = \frac{\text{Angular displacement}}{\text{Time taken}}$
- Instantaneous angular velocity $\omega = \frac{d\theta}{dt}$
- Average angular velocity $\vec{\omega}_{av} = \frac{\text{Total angular displacement}}{\text{Total time taken}} = \frac{\Delta\theta}{\Delta t}$
- For clockwise rotation $\vec{\omega}$ is directed downwards



- For anti-clockwise rotation $\vec{\omega}$ is directed upwards.



■ **Time Period (T):-** The taken by object to complete one revolution on its circular path.
■ **Frequency (ν):-**The number of revolution per unit time on the circular path.
■ **Angular acceleration (α) -**
- It is the time rate of change of angular velocity

$$\alpha = \frac{d\vec{\omega}}{dt}$$

- SI unit radian/second^2
- When a body moves with constant angular velocity, its angular acceleration is zero.

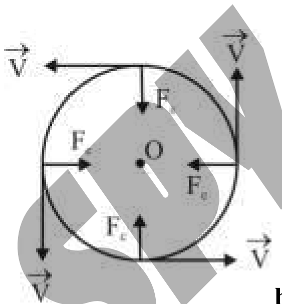
Centripetal Acceleration (a_c) -
- Acceleration of an object moving uniformly on the circular path and is along the radius towards the centre of the circular path.

$$a_c = \omega^2 r = \frac{V^2}{r} = \omega V \qquad \left\{ \because \quad \omega = \frac{V}{r} \right\}$$

$$\vec{a}_c = \vec{\omega} \times \vec{V}$$

Centripetal Force (F_c) -

$$F_c = \frac{mv^2}{r}$$
$$F_c = ma_c$$
$$F_c = m\omega^2 r$$
$$F_c = m\omega v$$
$$\vec{F}_c = m\left(\vec{v} \times \vec{\omega}\right)$$



b

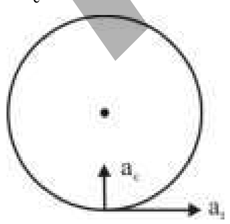
- The work done by centripetal force is zero.
- Centripetal force is essential for circular motion, without it the body cannot move in circular path.
- The K.E. and angular momentum cannot be increased by centripetal force.

Tangential Acceleration (a_t) -
- The acceleration which acts along the tangent to the circular path.

$$a_t = \alpha r$$
$$\vec{a}_t = \vec{\alpha} \times \vec{r}$$

Total acceleration ($\vec{a}$) -

$$\vec{a} = \alpha \times \vec{r} + \vec{\omega} \times \vec{v}$$
$$a = \sqrt{a_t^2 + a_c^2}$$



Where, a_t = Tangential acceleration -
a_c = Centripetal acceleration

Some relations -

(i) Relation between time period and frequency $\nu = \frac{1}{T}$

(ii) Relation between frequency angular velocity and time. $\omega = \frac{\theta}{t} = \frac{2\pi}{T} = 2\pi\nu$

(iii) Relation between linear acceleration and angular acceleration.

$$a = \alpha r$$
$$\vec{a} = \alpha \times \vec{r}$$

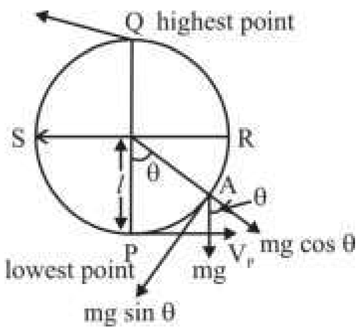
TEAM SPY

■ Motion in Vertical circle -

- Motion in a vertical circle is non-uniform circular motion.

- Tension at the lowest point P

$$T_p = \frac{mV_p^2}{l} + mg$$



- Tension at the highest point Q.

$$T_Q = \frac{mV_Q^2}{l} - mg$$

$$T_Q = \frac{mV_p^2}{l} - 5\text{ mg}$$

- Tension at point R-

$$T_R = \frac{mV_R^2}{l}$$

$$T_R = \frac{mV_p^2}{l} - 2\text{ mg}$$

- $T_p > T_R > T_Q$
- $T_p - T_Q = 6$ mg
- $T_p - T_R = 3$ mg
- Tension at any point A -

$$T = \frac{mV^2}{r} + mg\cos\theta$$

- **Minimum velocity for vertical circular motion -**

(a) V_P at P $\geq \sqrt{5gl}$

(b) V_Q at Q $\geq \sqrt{gl}$

(c) V_R at R $\geq \sqrt{3gl}$

- **In case of minimum velocity-**

(a) $T_p \geq 6$ mg

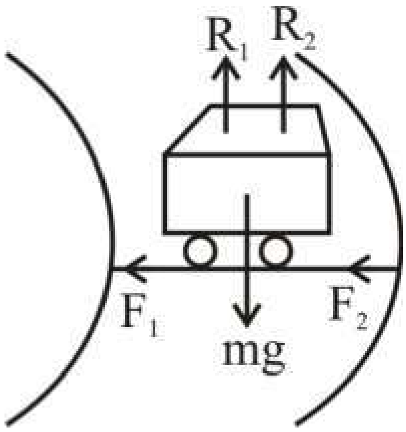
(b) $T_Q = 0$

(c) $T_R \geq 3$ mg

- If $T_{min} < 0$, the string will slack and the body will fall down from the highest point. Hence, for "looping the loop" or completing the vertical circle $T_{min} \geq 0$.
- If $V_P = \sqrt{2gl}$, velocity and tension becomes zero at R and S and particle will oscillate along semi-circular path.
- If $V_P < \sqrt{2gl}$, velocity becomes zero between P and R and particle oscillate about with the lower point P.
- If $V_P > \sqrt{5gl}$ tension never becomes zero and particle will just complete the circle.
- For leaving the vertical circle somewhere between $90° < \theta < 180°$. Tension becomes zero (T = 0) at the point of leaving but the velocity will not be zero.

  $\sqrt{2gl} < V_P < \sqrt{5gl}$

■ Rounding a level curved Road -



- $\frac{mv^2}{r} \le (F_1 + F_2)$

  Where, $F_1 = \mu R_1$ and $F_2 = \mu R_2$ μ = Coefficient of friction between tyres and road.

- $V \le \sqrt{\mu rg}$, $V_{max} = \sqrt{\mu rg}$

  This is the maximum speed without skidding.

- If centripetal force is obtained only by the banking of roads, then the speed (v) of the vehicle for a safe turn.

  $v = \sqrt{r\, g \tan\theta}$

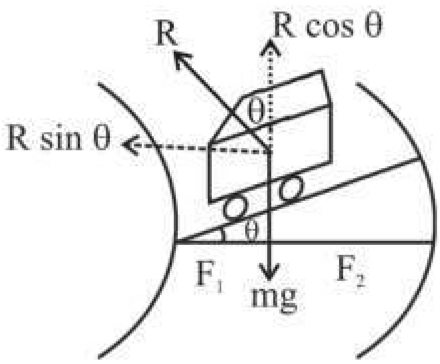
- If speed of vehicle is less than $\sqrt{r\, g \tan\theta}$. Then it will move inward (down) and r will decrease and if speed is more than $\sqrt{r\, g \tan\theta}$ then it will move toward (up) and r will increases.

- In normal life, the centripetal force is obtained by the friction force between the road and tyres as well as by the banking of the roads.

- Therefore the maximum permissible speed for the vehicle is much greater than the optimum value of the speed on a banked road.

- When centripetal force is obtained from friction force as well as banking of roads then maximum safe value of speed of vehicle.

  $V_{max} = \sqrt{\frac{rg(\tan\theta + \mu_s)}{(1 - \mu_s \tan\theta)}}$ Where μ_s = coefficient of static friction



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Bending of cyclist-

- When a cyclist takes turn at road, he inclines himself from the vertical slows down his speed and moves on a circular path of larger radius.

  If a cyclist is inclined at an angle θ, then

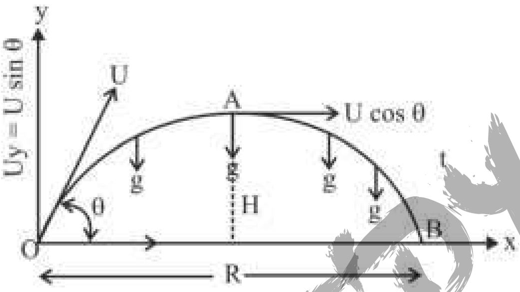
  $$\tan\theta = \frac{V^2}{rg}$$

  Where, V = Speed of the cyclist
  r = Radius of path
  g = Acceleration due to gravity

■ Projectile Motion –

- When any object is thrown from horizontal at an angle θ except 90º then it moves on a parabolic known as trajectory. The object is called projectile and its motion is called projectile motion.

Projectile motion in two dimensional motion :-



$U_x = U\cos\theta$
= Horizontal motion
= Responsible for range produced
= Constant acceleration ($\because a = 0$)

But,
$U_y = U\sin\theta$
= Vertical component
= Variable ($a_y = -g$)
= Responsible for height produced.

Concept -

x-axis	y-axis
$U_x = U\cos\theta$	$U_y = U\sin\theta$
$a_x = 0$	$a_y = -g$

- **Time of assending (t) -**
  along y axis-
  $V_y = U_y + a_y t$
  $$t = \frac{U\sin\theta}{g}$$
- **Time of Flight (T) -**
  $T = 2t$
  $$T = \frac{2U\sin\theta}{g}$$
- **Height attained by the body in projectile motion -**
  $$H = \frac{U^2\sin^2\theta}{2g}$$
- **Condition for maximum height attained (H_{max}) –**
  For maximum height,
  $\theta = 90º$, $\sin\theta = 1$
  $$H_{max} = \frac{U^2}{2g}$$

- **Range produced by the body in projectile motion (R) -**

  $R = \frac{U^2 \sin 2\theta}{g}$

- **Condition for maximum Range : (R_{max}) –**

  Sin 2θ = 1 = max = sin90º

  θ = 45º

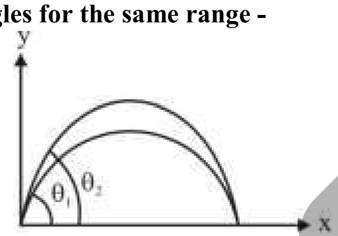
  $R_{max} = \frac{U^2}{g}$

- **Ratio -**

  $\frac{R_{max}}{H_{max}} = \frac{\frac{U^2}{g}}{\frac{U^2}{2g}} = 2$

  $R_{max} = 2\times H_{max}$

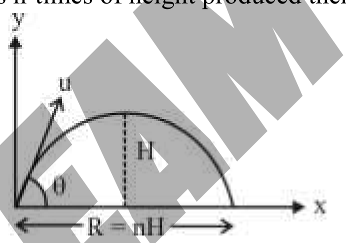
- **Two projective angles for the same range -**



  $\theta_1 + \theta_2 = 90^o = \frac{\pi}{2}$

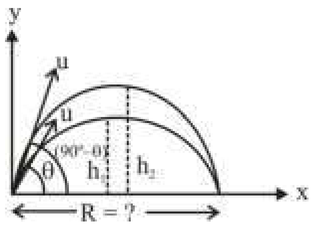
Special Cases –

- If Horizontal range is n-times of height produced then to determine projection angle.



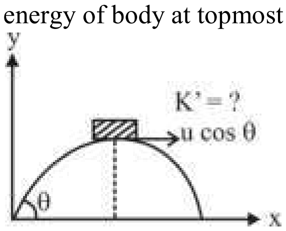
  $\theta = \tan^{-1}\left(\frac{u}{H}\right)$

- If two bodies are projected with equal speed u such that their range produced are same but height produced are different.



  $R = 4\sqrt{h_1 h_2}$

- To determine kinetic energy of body at topmost point in projectile motion ⇒ $K' = K\cos^2\theta$ where K=initial K.E



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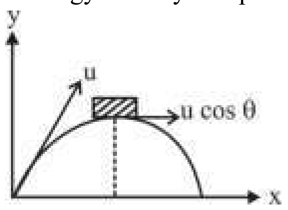
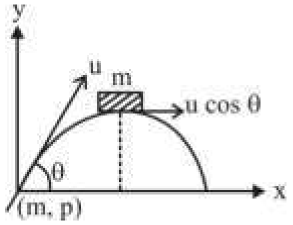
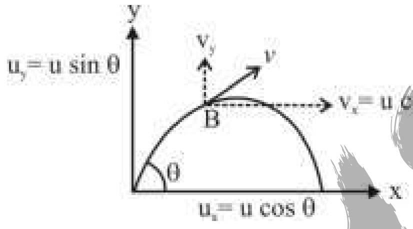
- To determine potential energy of body at topmost point in projectile $\Rightarrow U = K \sin^2\theta$



- Ratio of potential energy and kinetic energy at topmost point in projectile motion $\Rightarrow \frac{U}{K'} = \tan^2\theta$
- To determine linear momentum of body at topmost point in projectile of initial linear momentum p is given by- $p' = p \cos\theta$



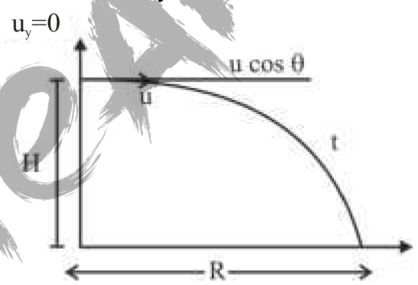
- To determine the change in linear momentum of body after time t in projectile motion (ΔP = ?)



$|\Delta p| = |\Delta p_y|$

$|\Delta p| = mg \times t$

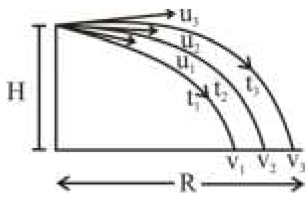
- When projectile projected horizontally-



Time of flight = $T = \sqrt{\frac{2H}{g}}$

Range = $u_x \times \sqrt{\frac{2H}{g}}$

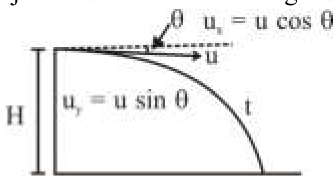
Concept -



$$T = T_1 = T_2 = T_3 = \sqrt{\frac{2H}{g}}$$

$$V_3 > V_2 > V_1 > V$$

- When projectile is projected downward at an angle θ with Horizontal -

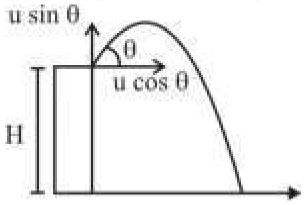


$R = u\cos\theta \times t$

$H = (u\sin\theta)t + \frac{1}{2}gt^2$

Time of flight, $T = \frac{-2u\sin\theta}{2g} \pm \frac{\sqrt{4u^2\sin^2\theta + 8gh}}{2g}$

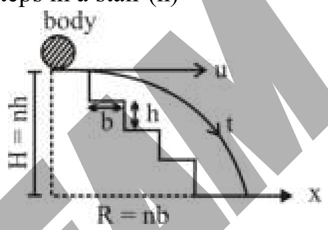
- When projectile is projected upward at an angle θ with Horizontal.



$H = -(u\sin\theta)t + \frac{1}{2}gt^2$

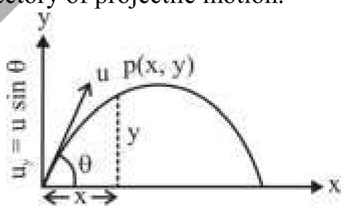
$T = \frac{u\sin\theta}{g} \pm \sqrt{\frac{u^2\sin^2\theta}{g^2} + \frac{2h}{g}}$

- To determine no of steps in a stair (n)



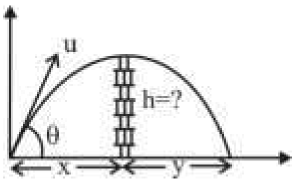
$n = \frac{2hu^2}{gb^2}$

- To determine equation of trajectory of projectile motion.



$t = \frac{x}{u\cos\theta}$

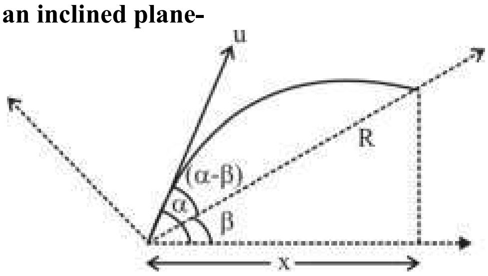
$y = x\tan\theta - \frac{gx}{2u^2\cos^2\theta}$

- 

$h = \left(\frac{xy}{x+y}\right)\tan\theta$

TEAM SPY

- **Projectile motion on an inclined plane-**



Initial velocity along the inclined plane = u cos (α – β).
Initial velocity perpendicular to the inclined plane = u sin (α – β).
Acceleration along the inclined plane = g sin β.
Acceleration perpendicular to the inclined plane = g cos β.

Time of flight, $T = \dfrac{2u\sin(\alpha-\beta)}{g\cos\beta}$

Maximum height, $H = \dfrac{u^2\sin^2(\alpha-\beta)}{2g\cos\beta}$

Horizontal range, $x = \dfrac{2u^2\sin^2(\alpha-\beta)\cos\alpha}{g\cos\beta}$

Range on inclined plane, $R = \dfrac{x}{\cos\beta} = \dfrac{2u^2\sin(\alpha-\beta)\cos\alpha}{g\cos^2\beta}$

Range on inclined plane will be maximum,

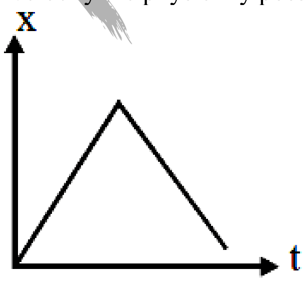
when, $\alpha = 45^\circ + \dfrac{\beta}{2}$

$R_{max} = \dfrac{u^2}{g(1+\sin\beta)}$

For angle of projectile α and (90° – α + β), The range an inclined plane are same.
If the projectile is thrown downwards, then maximum range is -

$R_{max} = \dfrac{u^2}{g(1-\sin\beta)}$

- A positive acceleration can be associated with a "slowing down" of the body because the origin and the positive direction of motion are a matter of choice.
- The x-t graph for a particle undergoing rectilinear motion cannot be as shown in figure because infinitesimal changes in velocity are physically possible only in infinitesimal time.



- In oblique projection of a projectile the speed gradually decreases up to the highest point and then increases because the tangential acceleration opposes the motion till the particle reaches the highest point, and then it favours the motion of the particle.
- In free fall the initial velocity of a body may not be zero.
- A body can have acceleration even if its velocity is zero at an instant.
- Average velocity of a body may be equal to its instantaneous velocity.
- The trajectory of an object moving under constant acceleration can be straight line or parabola.
- The path of one projectile as seen from another projectile is a straight line as relative acceleration of one projectile with respect to another projectile is zero.

(6) Newtons's Law of Motion

■ Force -

- A push or pull that one object exerts on another.

Force in Nature -

- There are four fundamental forces in nature-

1. Gravitational force   2. Electromagnetic force
3. Strong nuclear force   4. Weak force

Interaction	Particle affected	Range	Relative Strength	Characterstics time	Particle exchange	Role in universe
Strong Nuclear force	Quarks Hadrons	$\sim 10^{-15}$ m	1	10^{-23} Sec	Gluons Mesons	Holds quark together to form nucleon. Hold nucleons together to form atomic nuclei.
Electromagnetic	Charged particles	∞	10^{-2}	10^{-20} sec	Photons	Determine structure of atoms, solids and liquid is important factor in astronomical universe.
Weak nuclear force	Quark & Leptons	$\sim 10^{-16}$ m	10^{-13}	10^{-10} sec	Intermediate boson	Mediate transformations of quarks & leptons helps determine composition of atomic nuclei
Gravitational	All	∞	10^{-39}	10^{-16}	Gravitons Not experimentally detected	Assemble matter into planet, stars and galaxies

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■ Types of forces on macroscopic objects –

(a) Field Force or Range Forces -

- These are the forces in which contact between two objects is not necessary.
  Ex. (i) Gravitational force between two bodies.
  (ii) Electrostatic force between two charges.

(b) Contact force -

- Contact forces exist only as long as the objects are touching each other.
  (i) Normal force   (ii) Frictional force.

(c) Attachment to another body -

- Tension (T) in a string and spring force (F = kx) comes in this group.

■ Newton's Law of Motion –

(i) First law (Galileo's law of inertia)
(ii) Second law (Law of force)
(iii) Third law (Law of action and reaction)

(i) Newton's First law (Galileo's law of inertia)

- If $\vec{F}_{external} = 0$ and $\vec{V} = 0$ i.e body is in rest then it will always remain in rest.

Ex. A person who is standing freely in bus is thrown backward when the bus starts suddenly.

(ii) Newton's Second law (Law of force) -

- The rate of change of linear momentum w.r.t. time is equal to applied force and change in momentum takes place in the direction of applied force.

$$F = \frac{\Delta P}{\Delta t}$$

If $P = f(t)$ then,

$$F = \frac{dP}{dt}$$