YOUTH COMPETITION TIMES

VOLUME I
ALGEBRA
Chapterwise Solved Papers

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In the event of any dispute, the judicial area will be Prayagraj.

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Syllabus for JEE (Main) - 2024

Syllabus for JEE Main Paper-1 (B.E./B.Tech.)

MATHEMATICS

UNIT 1: SETS, RELATIONS, AND FUNCTIONS: Sets and their representation: Union, intersection, and complement of sets and their algebraic properties; Power set; Relation, Type of relations, equivalence relations, functions; one-one, into and onto functions, the composition of functions.

UNIT 2: COMPLEX NUMBERS AND QUADRATIC EQUATIONS: Complex numbers as ordered pairs of reals, Representation of complex numbers in the form a + ib and their representation in a plane, Argand diagram, algebra of complex number, modulus, and argument (or amplitude) of a complex number, Quadratic equations in real and complex number system and their solutions Relations between roots and co-efficient, nature of roots, the formation of quadratic equation with given roots.

UNIT 3: MATRICES AND DETERMINATS: Matrices, algebra of matrices, type of matrices, determinants, and matrices of order two and three, evaluation of determinants, area of triangles using determinants, Adjoint, and evaluation of inverse of a square matrix using determinants and, Test of consistency and solution of simultaneous linear equations in two or three variables using matrices.

UNIT 4: PERMUTATIONS AND COMBINATIONS: The fundamental principle of counting, permutation as an arrangement and combination as section, Meaning of P (n, r) and C(n, r), simple applications.

UNIT 5: BINOMIAL THEOREM AND ITS SIMPLE APPLICATIONS: Binomial theorem for a positive integral index, general term and middle term, and simple applications.

UNIT 6: SEQUENCE AND SERIES: Arithmetic and Geometric progressions, insertion of arithmetic, geometric means between two given numbers, Relation between A.M and G.M.

UNIT 7: LIMIT, CONTINUITY, AND DIFFERENTIABILITY: Real-valued functions, algebra of functions, polynomials, rational, trigonometric, logarithmic, and exponential functions, inverse function. Graphs of simple functions. Limits, continuity, and differentiability. Differentiation of the sum, difference, product, and quotient of two functions. Differentiation of trigonometric, inverse trigonometric, logarithmic, exponential, composite, and implicit functions; derivatives of order up to two, Applications of derivatives: Rate of change of quantities, monotonic-Increasing and decreasing functions, Maxima and minima of functions of one variable.

UNIT 8: INTEGRAL CALCULAS: Integral as an anti-derivative, Fundamental integral involving algebraic, trigonometric, exponential, and logarithmic functions. Integrations by substitution, by parts, and by partial functions. Integrations by substitution, by parts, and by partial functions. Integration using trigonometric identities.

Evaluation of simple integrals of the type

$$\int \frac{dx}{x^2+a^2},\ \int \frac{dx}{\sqrt{x^2 \pm a^2}},\ \int \frac{dx}{a^2-x^2},\ \int \frac{dx}{\sqrt{a^2-x^2}},\ \int \frac{dx}{ax^2+bx+c},\ \int \frac{dx}{\sqrt{ax^2+bx+c}},\ \int \frac{(px+q)dx}{ax^2+bx+c},\ \int \frac{(px+q)dx}{\sqrt{ax^2+bx+c}},$$

$$\int \sqrt{a^2 \pm x^2}\,dx,\ \int \sqrt{x^2-a^2}\,dx$$

The fundamental theorem of calculus, properties of definite integrals. Evaluation of definite integrals, determining areas of the regions bounded by simple curves in standard form.

UNIT 9 : DIFFERENTIAL EQUATION : Ordinary differential equations, their order, and degree, the solution of differential equation by the method of separation of variables, solution of a homegeneous and linear differential equation of the type

$$\frac{dy}{dx}+p(x)y=q(x)$$

UNIT 10 : CO-ORDINATE GEOMETRY : Cartesian system of rectangular coordinates in a plane, distance formula, sections formula, locus, and its equation, the slope of a line, parallel and perpendicular lines, intercepts of a line on the co-ordinate axis.

Straight line : Various forms of equations of a line, intersection of lines, angles between two lines, conditions for concurrence of three lines, the distance of a point form a line, co-ordinate of the centroid orthocentre, and circumcentre of a triangle.

Circle, conic sections : A standard form of equations of a circle, the general form of the equation of a circle, its radius and central, equation of a circle when the endpoints of a diameter are given, points of intersection of a line and a circle with the centre at the origin and sections of conics, equations of conic sections (parabola, ellipse, and hyperbola) in standard forms.

UNIT 11 : THREE DIMENSIONAL GEOMETRY : Coordinates of a point in space, the distance between two points, section formula, directions ratios, and direction consines, and the angle between two intersecting lines. Skew lines, the shortest distance between them, and its equation. Equations of a line

UNIT 12: VECTOR ALGEBRA: Vectors and scalars, the addition of vectors, components of a vector in two dimensions and three-dimensional space, scalar and vector products.

UNIT 13: STATISTICS AND PROBABILITY: Measures of discretion; calculation of mean, median, mode of grouped and ungrouped data calculation of standard deviation, variance, and mean deviation for grouped and ungrouped data.

Probability: Probability of an event, addition and multiplication theorems of probability, Baye's theorem, probability distribution of a random variate.

UNIT 14: TRIGONOMETRY : Trigonometrical identities and trigonometrical funtions, inverse trigonometrical functions, and their properties.

All India Engineering Entrance Examination & JEE-Main Previous Years Papers Analysis Chart

Sl No	Exam	Proposed Year		Total Question
Joint Entrance Examination (JEE) Main				
1.	NTA JEE Main (April Session)	April 2024	24 Paper	720
2.	NTA JEE Main (January Session)	January 2024	20 Paper	600
3.	NTA JEE Main	15.04.2023	Shift-I	30
4.	NTA JEE Main	13.04.2023	Shift-I	30
5.	NTA JEE Main	13.04.2023	Shift-II	30
6.	NTA JEE Main	12.04.2023	Shift-I	30
7.	NTA JEE Main	11.04.2023	Shift-I	30
8.	NTA JEE Main	11.04.2023	Shift-II	30
9.	NTA JEE Main	10.04.2023	Shift-I	30
10.	NTA JEE Main	10.04.2023	Shift-II	30
11.	NTA JEE Main	08.04.2023	Shift-I	30
12.	NTA JEE Main	08.04.2023	Shift-II	30
13.	NTA JEE Main	06.04.2023	Shift-I	30
14.	NTA JEE Main	06.04.2023	Shift-II	30
15.	NTA JEE Main	01.02.2023	Shift-I	30
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19.	NTA JEE Main	25.01.2023	Shift-I	30
20.	NTA JEE Main	25.01.2023	Shift-II	30
21.	NTA JEE Main	29.01.2023	Shift-I	30
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30.	NTA JEE Main	28.07.2022	Shift-II	30
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34.	NTA JEE Main	26.07.2022	Shift-II	30
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40.	NTA JEE Main	28.06.2022	Shift-II	30
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42.	NTA JEE Main	27.06.2022	Shift-II	30
43.	NTA JEE Main	26.06.2022	Shift-I	30
44.	NTA JEE Main	26.06.2022	Shift-II	30
45.	NTA JEE Main	25.06.2022	Shift-I	30
46.	NTA JEE Main	25.06.2022	Shift-II	30
47.	NTA JEE Main	24.06.2022	Shift-I	30
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74.	NTA JEE Main	25.02.2021	Shift-II	30
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76.	NTA JEE Main	24.02.2021	Shift-II	30
77.	NTA JEE Main	06.09.2020	Shift-I	30
78.	NTA JEE Main	06.09.2020	Shift-II	30
79.	NTA JEE Main	05.09.2020	Shift-I	30
80.	NTA JEE Main	05.09.2020	Shift-II	30
81.	NTA JEE Main	04.09.2020	Shift-I	25
82.	NTA JEE Main	04.09.2020	Shift-II	25
83.	NTA JEE Main	03.09.2020	Shift-I	30
84.	NTA JEE Main	03.09.2020	Shift-II	30
85.	NTA JEE Main	02.09.2020	Shift-I	25
86.	NTA JEE Main	02.09.2020	Shift-II	25
87.	NTA JEE Main	09.01.2020	Shift-I	30
88.	NTA JEE Main	09.01.2020	Shift-II	30
89.	NTA JEE Main	08.01.2020	Shift-I	30
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92.	NTA JEE Main	07.01.2020	Shift-II	30
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98.	NTA JEE Main	09.04.2019	Shift-II	30
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108.	NTA JEE Main	09.01.2019	Shift-II	30
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114.	JEE Main	08.04.2017		30
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117.	JEE Main	2015		30
118.	JEE Main	2014		30
119.	JEE Main	2013		30
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121.	AIEEE	2011		30
122.	AIEEE	2010		30
123.	AIEEE	2009		30
124.	AIEEE	2008		30
	AIEEE	2007		30
125.	AIEEE	2006		30
126.	AIEEE	2005		30
127.	AIEEE	2004		30
128.	AIEEE	2003		30
129.	AIEEE	2002		30
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131.	ASSAM-CEE	2022		40
132.	ASSAM-CEE	2021		40
133.	ASSAM-CEE	2020		40
134.	ASSAM-CEE	2019		40
135.	ASSAM-CEE	2018		40
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137.	A.P. EAPCET	15.05.2023	Shift-II	80
138.	A.P. EAPCET	16.05.2023	Shift-I	80
139.	A.P. EAPCET	16.05.2023	Shift-II	80
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141.	A.P. EAPCET	17.05.2023	Shift-II	80
142.	A.P. EAPCET	18.05.2023	Shift-I	80
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144.	A.P. EAPCET	19.05.2023	Shift-I	80
145.	A.P. EAMCET	04.07.2022	Shift-I	80
146.	A.P. EAMCET	04.07.2022	Shift-II	80
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150.	A.P. EAMCET	06.07.2022	Shift-II	80
151.	A.P. EAMCET	07.07.2022	Shift-I	80
152.	A.P. EAMCET	07.07.2022	Shift-II	80
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157.	A.P. EAMCET	23.08.2021	Shift-II	80
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206.	A.P. EAMCET	2002		80
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234.	AMU	2006		70
235.	AMU	2005		70
236.	AMU	2004		70
237.	AMU	2003		70
238.	AMU	2002		100
239.	AMU	2001		100
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240.	BCECE	2018		50
241.	BCECE	2017		50
242.	BCECE	2016		50
243.	BCECE	2015		50
244.	BCECE	2014		50
245.	BCECE	2013		50
246.	BCECE	2012		50
247.	BCECE	2011		50
248.	BCECE	2010		50
249.	BCECE	2009		50
250.	BCECE	2008		50
251.	BCECE	2007		50
252.	BCECE	2006		50
253.	BCECE	2005		50
254.	BCECE	2004		50
255.	BCECE	2003		50
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260.	BITSAT	2018		40
261.	BITSAT	2017		40
262.	BITSAT	2016		40

263.	BITSAT	2015		40
264.	BITSAT	2014		40
265.	BITSAT	2013		40
266.	BITSAT	2012		40
267.	BITSAT	2011		40
268.	BITSAT	2010		40
269.	BITSAT	2009		40
270.	BITSAT	2008		40
271.	BITSAT	2007		40
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280.	Chhattisgarh-PET	2017		100
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285.	Chhattisgarh-PET	2012		100
286.	Chhattisgarh-PET	2011		100
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288.	Chhattisgarh-PET	2009		100
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301.	COMEDK-JEE	2016		60
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331.	J & K-CET	2012		75
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335.	J & K-CET	2008		75
336.	J & K-CET	2007		75
337.	J & K-CET	2006		75
338.	J & K-CET	2005		75
339.	J & K-CET	2004		75
340.	J & K-CET	2003		75
Jharkhand (JCECE)				
341.	JCECE	2019		50
342.	JCECE	2018		50
343.	JCECE	2017		50
344.	JCECE	2016		50
345.	JCECE	2015		50
346.	JCECE	2014		50
347.	JCECE	2013		50
348.	JCECE	2012		50
349.	JCECE	2011		50
350.	JCECE	2010		50
351.	JCECE	2009		50
352.	JCECE	2008		50
353.	JCECE	2007		50
354.	JCECE	2006		50
355.	JCECE	2005		50
356.	JCECE	2004		50
357.	JCECE	2003		50
358.	JCECE	2002		50
359.	JCECE	2001		50
Jamia Millia Islamia				
360.	Jamia Millia Islamia	2015		60
361.	Jamia Millia Islamia	2014		60
362.	Jamia Millia Islamia	2013		60
363.	Jamia Millia Islamia	2012		60
364.	Jamia Millia Islamia	2011		60
365.	Jamia Millia Islamia	2010		60

366.	Jamia Millia Islamia	2009		60
367.	Jamia Millia Islamia	2008		60
368.	Jamia Millia Islamia	2007		60
369.	Jamia Millia Islamia	2006		60
370.	Jamia Millia Islamia	2005		60
371.	Jamia Millia Islamia	2004		60
Kerala-KEAM				
372.	Kerala KEAM	2023		60
373.	Kerala KEAM	2022		60
374.	Kerala KEAM	2021		60
375.	Kerala KEAM	2020		60
376.	Kerala KEAM	2019		60
377.	Kerala KEAM	2018		60
378.	Kerala KEAM	2017		60
379.	Kerala KEAM	2016		60
380.	Kerala KEAM	2015		60
381.	Kerala KEAM	2014		60
382.	Kerala KEAM	2013		60
383.	Kerala KEAM	2012		60
384.	Kerala KEAM	2011		60
385.	Kerala KEAM	2010		60
386.	Kerala KEAM	2009		60
387.	Kerala KEAM	2008		60
388.	Kerala KEAM	2007		60
389.	Kerala KEAM	2006		60
390.	Kerala KEAM	2005		60
391.	Kerala KEAM	2004		60
Karnataka-CET (KCET)				
392.	Karnataka-CET	2023		60
393.	Karnataka-CET	2022		60
394.	Karnataka-CET	2021		60
395.	Karnataka-CET	2020		60
396.	Karnataka-CET	2019		60
397.	Karnataka-CET	2018		60
398.	Karnataka-CET	2017		60
399.	Karnataka-CET	2016		60
400.	Karnataka-CET	2015		60
401.	Karnataka-CET	2014		60
402.	Karnataka-CET	2013		60
403.	Karnataka-CET	2012		60
404.	Karnataka-CET	2011		60
405.	Karnataka-CET	2010		60
406.	Karnataka-CET	2009		60
407.	Karnataka-CET	2008		60
408.	Karnataka-CET	2007		60
409.	Karnataka-CET	2006		60
410.	Karnataka-CET	2005		60
411.	Karnataka-CET	2004		60
412.	Karnataka-CET	2003		60
413.	Karnataka-CET	2002		60
414.	Karnataka-CET	2001		60
415.	Karnataka-CET	2000		60
Kishore Vaigyanik Protsahan Yojana (KVPY)				
416.	KVPY-SB-SX	2023		15
417.	KVPY-SB-SX	2022		15

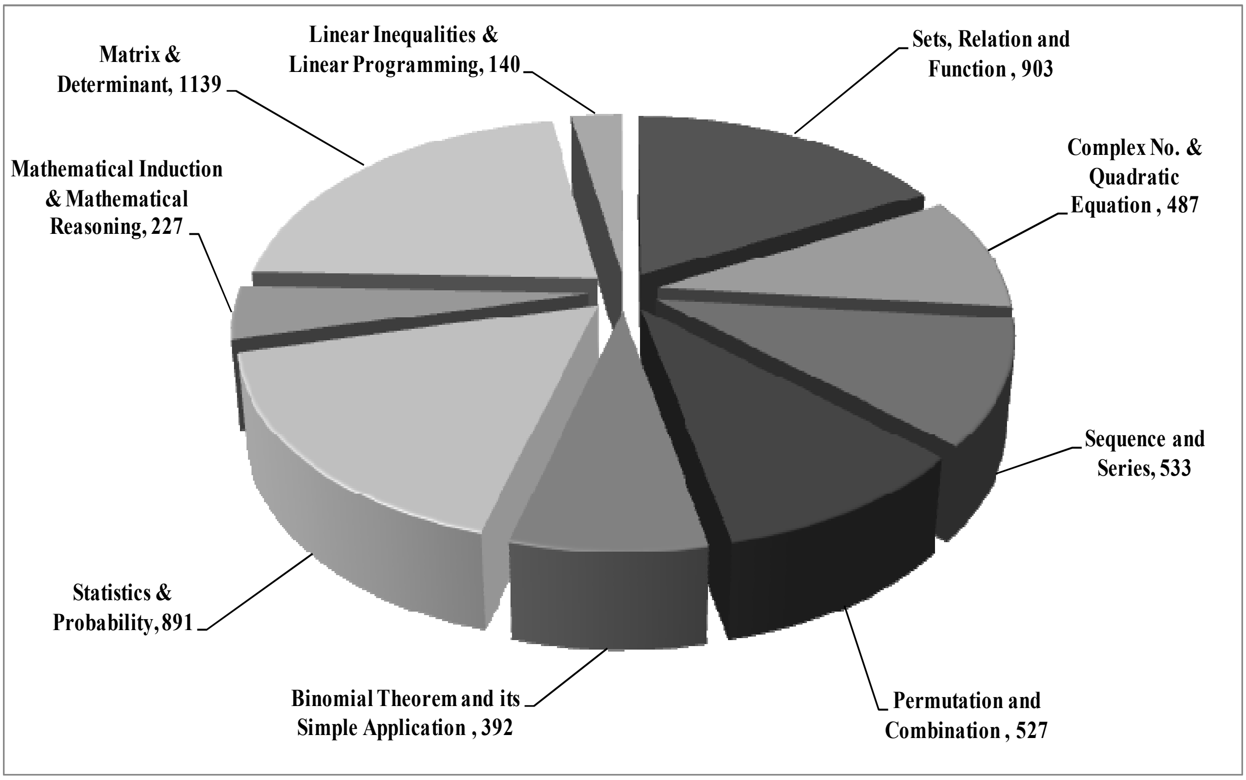
418.	KVPY-SB-SX	2021		15
419.	KVPY-SA	2021		15
420.	KVPY-SA	2020		15
421.	KVPY-SB-SX	2018		15
422.	KVPY-SA	2017		15
423.	KVPY-SB-SX	2016		15
424.	KVPY-SB-SX	2015		15
425.	KVPY-SA	2014		15
426.	KVPY-SB-SX	2013		15
427.	KVPY-SA	2012		15
428.	KVPY-SA	2009		15
429.	KVPY-SB-SX	2009		15
Madhya Pradesh Pre Engineering Test (MPPET)				
430.	MPPET	2013		50
431.	MPPET	2012		50
432.	MPPET	2009		50
433.	MPPET	2008		50
Manipal-UGET				
434.	Manipal	2023		50
435.	Manipal	2022		50
436.	Manipal	2021		50
437.	Manipal	2020		50
438.	Manipal	2019		50
439.	Manipal	2018		50
440.	Manipal	2017		50
441.	Manipal	2016		50
442.	Manipal	2015		50
443.	Manipal	2014		50
444.	Manipal	2013		50
445.	Manipal	2012		50
446.	Manipal	2011		50
447.	Manipal	2010		50
448.	Manipal	2009		50
449.	Manipal	2008		50
(Maharashtra) MHT-CET				
450.	MHT-CET	2022	All Shifts	500
451.	MHT-CET	2021	All Shifts	500
452.	MHT-CET	13.10.2020	Shift-I	100
453.	MHT-CET	13.10.2020	Shift-II	100
454.	MHT-CET	14.10.2020	Shift-I	100
455.	MHT-CET	14.10.2020	Shift-II	100
456.	MHT-CET	15.10.2020	Shift-I	100
457.	MHT-CET	15.10.2020	Shift-II	100
458.	MHT-CET	16.10.2020	Shift-I	100
459.	MHT-CET	16.10.2020	Shift-II	100
460.	MHT-CET	19.10.2020	Shift-I	100
461.	MHT-CET	19.10.2020	Shift-II	100
462.	MHT-CET	20.10.2020	Shift-I	100
463.	MHT-CET	20.10.2020	Shift-II	100
464.	MHT-CET	02.05.2019	Shift-I	100
465.	MHT-CET	02.05.2019	Shift-II	100

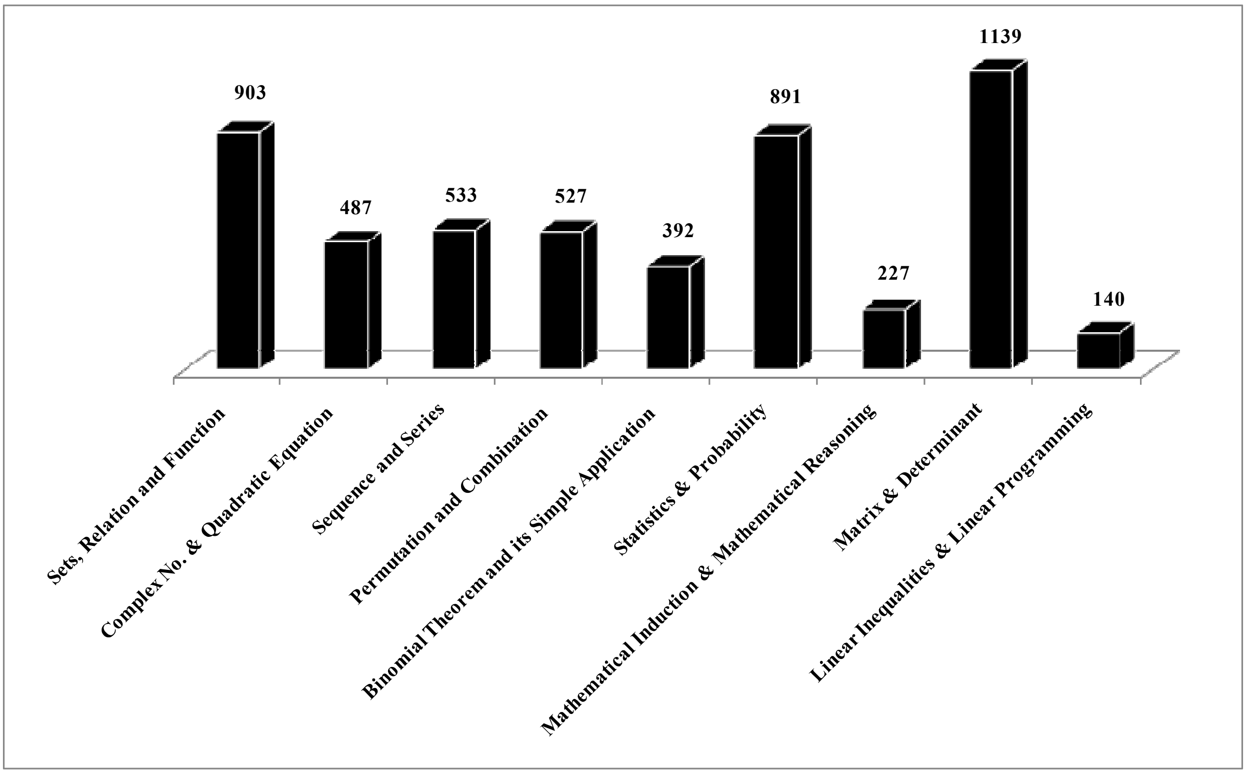
466.	MHT-CET	03.05.2019		100
467.	MHT-CET	2018		100
468.	MHT-CET	2017		100
469.	MHT-CET	2016		100
470.	MHT-CET	2015		100
471.	MHT-CET	2014		100
472.	MHT-CET	2013		100
473.	MHT-CET	2012		100
474.	MHT-CET	2011		100
475.	MHT-CET	2010		100
476.	MHT-CET	2009		100
477.	MHT-CET	2008		100
478.	MHT-CET	2007		100
479.	MHT-CET	2006		100
480.	MHT-CET	2005		100
481.	MHT-CET	2004		100
Rajasthan PET				
482.	Rajasthan PET	2012		40
483.	Rajasthan PET	2011		40
484.	Rajasthan PET	2010		40
485.	Rajasthan PET	2009		40
486.	Rajasthan PET	2008		40
487.	Rajasthan PET	2007		40
488.	Rajasthan PET	2006		40
489.	Rajasthan PET	2005		40
490.	Rajasthan PET	2004		40
491.	Rajasthan PET	2003		40
492.	Rajasthan PET	2002		40
493.	Rajasthan PET	2001		40
SCRA				
494.	SCRA	2015		60
495.	SCRA	2014		60
496.	SCRA	2013		60
497.	SCRA	2012		60
498.	SCRA	2010		60
499.	SCRA	2009		60
SRM-JEEE				
500.	SRM-JEEE	2022		40
501.	SRM-JEEE	2021		40
502.	SRM-JEEE	2020		40
503.	SRM-JEEE	2019		40
504.	SRM-JEEE	2018		40
505.	SRM-JEEE	2016		40
506.	SRM-JEEE	2015		40
507.	SRM-JEEE	2014		40
508.	SRM-JEEE	2013		40
509.	SRM-JEEE	2012		40
510.	SRM-JEEE	2011		40
511.	SRM-JEEE	2010		40
512.	SRM-JEEE	2009		40
513.	SRM-JEEE	2008		40

514.	SRM-JEEE	2007		40
Telangana EAMCET				
515.	TS-EAMCET	12.05.2023	Shift-I	80
516.	TS-EAMCET	12.05.2023	Shift-II	80
517.	TS-EAMCET	13.05.2023	Shift-I	80
518.	TS-EAMCET	13.05.2023	Shift-II	80
519.	TS-EAMCET	14.05.2023	Shift-I	80
520.	TS-EAMCET	14.05.2023	Shift-II	80
521.	TS-EAMCET	18.07.2022	Shift-I	80
522.	TS-EAMCET	18.07.2022	Shift-II	80
523.	TS-EAMCET	19.07.2022	Shift-I	80
524.	TS-EAMCET	19.07.2022	Shift-II	80
525.	TS-EAMCET	20.07.2022	Shift-I	80
526.	TS-EAMCET	20.07.2022	Shift-II	80
527.	TS-EAMCET	06.08.2021	Shift-I	80
528.	TS-EAMCET	06.08.2021	Shift-II	80
529.	TS-EAMCET	05.08.2021	Shift-I	80
530.	TS-EAMCET	05.08.2021	Shift-II	80
531.	TS-EAMCET	04.08.2021	Shift-I	80
532.	TS-EAMCET	04.08.2021	Shift-II	80
533.	TS-EAMCET	09.09.2020	Shift-I	80
534.	TS-EAMCET	09.09.2020	Shift-II	80
535.	TS-EAMCET	10.09.2020	Shift-I	80
536.	TS-EAMCET	10.09.2020	Shift-II	80
537.	TS-EAMCET	11.09.2020	Shift-I	80
538.	TS-EAMCET	11.09.2020	Shift-II	80
539.	TS-EAMCET	14.09.2020	Shift-I	80
540.	TS-EAMCET	14.09.2020	Shift-II	80
541.	TS-EAMCET	03.05.2019	Shift-I	80
542.	TS-EAMCET	03.05.2019	Shift-II	80
543.	TS-EAMCET	04.05.2019	Shift-I	80
544.	TS-EAMCET	04.05.2019	Shift-II	80
545.	TS-EAMCET	06.05.2019	Shift-I	80
546.	TS-EAMCET	05.05.2018	Shift-I	80
547.	TS-EAMCET	05.05.2018	Shift-II	80
548.	TS-EAMCET	02.05.2018	Shift-I	80
549.	TS-EAMCET	04.05.2018	Shift-II	80
550.	TS-EAMCET	07.05.2018	Shift-I	80
551.	TS-EAMCET	24.04.2017	Shift-I	80
552.	TS-EAMCET	2016		80
553.	TS-EAMCET	2015		80
554.	TS-EAMCET	2014		80
Tripura JEE				
555.	Tripura JEE	2023		50
556.	Tripura JEE	2022		50
557.	Tripura JEE	2021		50
558.	Tripura JEE	2019		50
(Uttar Pradesh) UPTU/UPSEE				
559.	UPTU/UPSEE	2020		50
560.	UPTU/UPSEE	2019		50
561.	UPTU/UPSEE	2018		50

562.	UPTU/UPSEE	2017		50
563.	UPTU/UPSEE	2016		50
564.	UPTU/UPSEE	2015		50
565.	UPTU/UPSEE	2014		50
566.	UPTU/UPSEE	2013		50
567.	UPTU/UPSEE	2012		50
568.	UPTU/UPSEE	2011		50
569.	UPTU/UPSEE	2010		50
570.	UPTU/UPSEE	2009		50
571.	UPTU/UPSEE	2008		50
572.	UPTU/UPSEE	2007		50
573.	UPTU/UPSEE	2006		50
574.	UPTU/UPSEE	2005		50
575.	UPTU/UPSEE	2004		50
VITEEE				
576.	VITEEE	2023		40
577.	VITEEE	2022		40
578.	VITEEE	2021		40
579.	VITEEE	2020		40
580.	VITEEE	2019		40
581.	VITEEE	2018		40
582.	VITEEE	2017		40
583.	VITEEE	2016		40
584.	VITEEE	2015		40
585.	VITEEE	2014		40
586.	VITEEE	2013		40
587.	VITEEE	2012		40
588.	VITEEE	2011		40
589.	VITEEE	2010		40
590.	VITEEE	2009		40
591.	VITEEE	2008		40
592.	VITEEE	2007		40
593.	VITEEE	2006		40
WEST BENGAL				
594.	West Bengal	2023		30
595.	West Bengal	2022		30
596.	West Bengal	2021		30
597.	West Bengal	2020		30
598.	West Bengal	2019		30
599.	West Bengal	2018		30
600.	West Bengal	2017		30
601.	West Bengal	2016		30
602.	West Bengal	2015		30
603.	West Bengal	2014		30
604.	West Bengal	2013		30
605.	West Bengal	2012		30
606.	West Bengal	2011		30
607.	West Bengal	2010		30
608.	West Bengal	2009		30
609.	West Bengal	2008		30
			Total	**36020**

Trend Analysis of previous year paper of IIT JEE Mathematics through Bar graph and Pie chart.





01. Sets, Relation and Function

A. Set and type of Sets and its application

1. Let $f : (-\infty, \infty) - \{0\} \rightarrow R$ be a differentiable function such that $f'(1) = \lim\limits_{a\to\infty} a^2 f\left(\frac{1}{a}\right)$.

Then $\lim\limits_{a\to\infty} \frac{a(a+1)}{2}\tan^{-1}\frac{1}{a} + a^2 - 2\log_e a$ is equal to

(a) $\frac{3}{2}+\frac{\pi}{4}$ (b) $\frac{3}{8}+\frac{\pi}{4}$

(c) $\frac{5}{2}+\frac{\pi}{8}$ (d) $\frac{3}{4}+\frac{\pi}{8}$

JEE MAIN-06.04.2024, Shift-I

Ans. (c) : Given,

$f : (-\infty, \infty) - \{0\} \rightarrow R$

$f'(1) = \lim\limits_{a\to\infty} a^2 f\left(\frac{1}{a}\right)$

$\lim\limits_{a\to\infty} \frac{a(a+1)}{2}\tan^{-1}\left(\frac{1}{a}\right) + a^2 - 2\ell n(a)$

$\lim\limits_{a\to\infty} a^2\left(\frac{\left(1+\frac{1}{a}\right)}{2}\tan^{-1}\left(\frac{1}{a}\right)+1-\frac{2}{a^2}\ell n(a)\right)$

$f(x) = \frac{1}{2}(1+x)\tan^{-1}(x)+1-2x^2\ell n(x)$

differentiate w.r.t x

$f'(x) = \frac{1}{2}\left(\frac{1+x}{1+x^2}+\tan^{-1}(x)\right)+4x\ell n(x)+2x$

$f'(1) = \frac{1}{2}\left(1+\frac{\pi}{4}\right)+2$

$f'(1) = \frac{5}{2}+\frac{\pi}{8}$

2. Let $A = \{n \in [100, 700] \cap N$: n is neither a multiple of 3 nor a multiple of 4$\}$. Then the number of elements in A is

(a) 300 (b) 280

(c) 310 (d) 290

JEE MAIN-06.04.2024, Shift-I

Ans. (a) : n (3) $\Rightarrow$ multiple of 3

102, 105, 108, ..., 699 in A.P series

We know that, $l = a + (n-1)d$

$l = 699 = 102 + (n-1)(3)$

n = 200

n (3) = 200

$\because$ n (4) $\Rightarrow$ multiple of 4

100, 104, 108, ..., 700 in A.P series

Similarly, $l = 700 = 100 + (n-1)(4)$

n = 151

n (4) = 151

n $\left(\overline{3\cup 4}\right) \Rightarrow$ multiple of 3& 4 both

108, 120, 132, ..., 696 in A.P series

Similarly $l = 696 = 108 + (n-1)(12)$

n = 50

$n(3 \cap 4) = 50$

$n(3 \cup 4) = n(3) + n(4) - n(3 \cap 4)$

$= 200 + 151 - 50$

$= 301$

n $\left(\overline{3\cup 4}\right)$ = Total − n $(3 \cup 4)$ = neither a multiple of 3 nor a multiple of 4

= 601 − 301 = 300

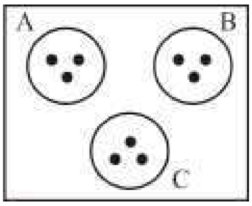
3. Let the set S = {2, 4, 8, 16, ...,512} be partitioned into 3 sets A, B, C with equal number of elements such that $A \cup B \cup C = S$ and $A \cap B = B \cap C = A \cap C = \phi$. The maximum number of such possible partitions of S is equal to:

(a) 1680 (b) 1520

(c) 1710 (d) 1640

JEE MAIN-05.04.2024, Shift-II

Ans. (a) : According to question,



$S = \{2, 2^2, 2^3 ....2^9\}$

n (S) = 9

Maximum number of possible partition of S

$= {}^9C_3 \times {}^6C_3 \times {}^3C_3$

$= \frac{9!}{3!6!}\times\frac{6!}{3!3!}\times 1$

$= \frac{9\times 8\times 7}{3\times 2}\times\frac{6\times 5\times 4}{3\times 2}\times 1$

$= 84\times 20$

$=1680$

4. The sum of all rational terms in the expansion of $\left(2^{\frac{1}{5}}+5^{\frac{1}{3}}\right)^{15}$ is equal to:

(a) 3133 (b) 633

(c) 931 (d) 6131

JEE MAIN-04.04.2024, Shift-I

Ans. (a) : $T_{r+1} = {}^{15}C_r\left(5^{\frac{1}{3}}\right)^r\left(2^{\frac{1}{5}}\right)^{15-r}$

$= {}^{15}C_r 5^{\frac{r}{3}}.2^{\frac{15-r}{5}}$

$r \in (0, 1, 2 ....15)$

$\frac{r}{3} \in$ integer and $\frac{r}{5} \in$ integer

So 3 and 5 divides r = 15 divides

= r = 0, 15

2 rational terms

$\Rightarrow {}^{15}C_0 2^3 + {}^{15}C_{15}(5)^5$

= 8 + 3125 = 3133

5. The number of elements in the set
$S \quad \{(x,y,z): x,y,z \quad Z, x \quad 2y \quad 3z \quad 42, x,y,z \quad 0\}$
equals________ .

JEE MAIN-01.02.2024, Shift-I

Ans.(169) : We have,

$x + 2y + 3z = 42, \quad x, y, z \geq 0$

$\Rightarrow \quad x + 2y = 42 - 3z$

There are following cases-

1) z = 0 $\quad x + 2y = 42 \rightarrow$ 22 case
2) z = 1 $\quad x + 2y = 39 \rightarrow$ 20 case
3) z = 2 $\quad x + 2y = 36 \rightarrow$ 19 case
4) z = 3 $\quad x + 2y = 33 \rightarrow$ 17 case
5) z = 4 $\quad x + 2y = 30 \rightarrow$ 16 case
6) z = 5 $\quad x + 2y = 27 \rightarrow$ 14 case
7) z = 6 $\quad x + 2y = 24 \rightarrow$ 13 case
8) z = 7 $\quad x + 2y = 21 \rightarrow$ 11 case
9) z = 8 $\quad x + 2y = 18 \rightarrow$ 10 case
10) z = 9x + 2y = 15 $\rightarrow$ 8 case
11) z = 10 $\quad x + 2y = 12 \rightarrow$ 7 case
12) z = 11 $\quad x + 2y = 9 \rightarrow$ 5 case
13) z = 12 $\quad x + 2y = 6 \rightarrow$ 4 case
14) z = 13 $\quad x + 2y = 3 \rightarrow$ 2 case
15) z = 14 $\quad x + 2y = 0 \rightarrow$ 1 case

Therefore the number of elements in the set = 169.

6. Let $S \quad \{x \quad R:(\sqrt{3} \quad \sqrt{2})^x \quad (\sqrt{3} \quad \sqrt{2})^x \quad 10\}$.
Then the number of elements in S is:

(a) 4 (b) 0
(c) 2 (d) 1

JEE MAIN-01.02.2024, Shift-I

Ans. (c) : We have,

$$\left(\sqrt{3}+\sqrt{2}\right)^x + (\sqrt{3}-\sqrt{2})^x = 10$$

Let, $\left(\sqrt{3}+\sqrt{2}\right)^x = t$

Now, multiplying by its conjugate-

$$\left(\frac{1}{\sqrt{3}-\sqrt{2}}\right)^x = t$$

$$\left(\sqrt{3}-\sqrt{2}\right)^x = \frac{1}{t}$$

$t \quad \frac{1}{t} \quad 10 \quad t^2 \quad 10t \quad 1 \quad 0$

$$\Rightarrow t = \frac{10 \pm 4\sqrt{6}}{2} = 5 \pm 2\sqrt{6}$$

$$\left(\sqrt{3}+\sqrt{2}\right)^x = \left(5 \pm 2\sqrt{6}\right)$$

Put, $\quad x = 2$

$$\left(\sqrt{3}+\sqrt{2}\right)^2 = 3+2+2\sqrt{6}$$

$$= 5 + 2\sqrt{6}$$

and $x = -2$

$$\left(\sqrt{3}+\sqrt{2}\right)^{-2} = 5-2\sqrt{6}$$

We have got 2 solution at x = 2, –2

So, number of elements = 2

7. Let the set $C = \left\{(x,y)\middle|x^2 - 2^y = 2023, x,y \in N\right\}$. Then $\sum_{(x,y)\in C}(x+y)$ is equal to _____.

JEE MAIN-29.01.2024, Shift-II

Ans. : (46) We have-

$x^2 \quad 2^y \quad 2023$

Let, x = 45 and y = 1,which is satisfying the given equation.

$45^2 = 2025$

$45^2 - 2^1 = 2023$

$\Rightarrow x = 45, y = 1$

So,

$(x \quad y) \quad 46.$
$(x,y) \quad C$

8. Let A and B be two finite sets with m and n elements respectively. The total number of subsets of the set A is 56 more than the total number of subsets of B. Then the distance of the point P (m,n) from the point Q (–2,–3) is.

(a) 10 (b) 6
(c) 4 (d) 8

JEE MAIN-27.01.2024, Shift-II

Ans. (a) : A and B be two finite sets with m and n elements.

Given, $2^m = 2^n + 56$

$2^m - 2^n = 56$

$2^n(2^{m-n} - 1) = 56 = 2^3 \times 7$

$n = 3 \quad 2^{m-n} = 8$

m = 6

$\therefore$ P(m, n) = P(6, 3) and Q(–2, –3)

Distance between P and Q are-

$PQ = \sqrt{64+36} = 10$

9. **Let S be the set of positive integral values of a for which $\frac{ax^2+2(a+1)x+9a+4}{x^2-8x+32}<0, \forall x \in \mathbb{R}$. Then, the number of elements in S is:**

(a) ∞ (b) 3
(c) 0 (d) 1

JEE MAIN-31.01.2024, Shift-I

Ans. (c) : We have,

$$\frac{ax^2+2(a+1)x+9a+4}{x^2-8x+32}<0$$

$D = 64 - 4 \times 32 < 0$
& $a = 1 > 0$
$\therefore x^2-8x+32>0 \forall x \in R$
$ax^2+2(a+1)x+9a+4<0 \forall x \in R$
Only possible when,
$a < 0$ & $D < 0$
But we need positive integral value of a.
So,
$|S| = 0$

10. **The number of elements in the set $\{n \in Z: |n^2 - 10n + 19| < 6\}$ is ______.**

JEE MAIN-10.04.2023, Shift-I

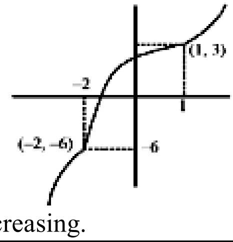
Ans. (6) : We have,
$-6 < n^2 - 10n + 19 < 6$
$\Rightarrow n^2 - 10n + 25 > 0$
$\Rightarrow (n-5)^2 > 0$
$\Rightarrow n \in Z - \{5\}$ ... (i)
and $n^2 - 10n + 13 < 0$
$\Rightarrow 5 - 2\sqrt{3} < n < 5 + 2\sqrt{3}$
$\Rightarrow 1.6 < n < 8.4$
$\Rightarrow n = \{2, 3, 4, 5, 6, 7, 8\}$ ... (ii)
From (i) $\cap$ (ii)
$N = \{2, 3, 4, 6, 7, 8\}$
So, Number of elements in the set = 6

11. **The set of all $a \in R$ for which the equation $x|x-1| + |x+2| + a = 0$ has exactly one real root, is**

(a) $(-\infty, -3)$ (b) $(-\infty, \infty)$
(c) $(-6, \infty)$ (d) $(-6, -3)$

JEE MAIN-13.04.2023, Shift-I

Ans. (b) :
Let, $f(x) = x|x-1| + |x+2|$
$x|x-1| + |x+2| + a = 0$
$x|x-1| + |x+2| = -a$



All values are increasing.

12. **Let $a \neq b$ be two-zero real numbers. Then the number of elements in the set $X = \{z \in \mathbb{C} : Re(az^2 + bz) = a$ and $Re(bz^2 + az) = b\}$ is equal to:**

(a) 0 (b) 2
(c) 1 (d) 3

JEE MAIN-06.04.2023, Shift-II

Ans. (a) : $z = x + iy$
$Z^2 = x^2 - y^2 + 2xyi$
$az^2 + bz = a(x^2 - y^2 + 2xyi) + b(x + iy)$
$az^2 + bz = a(x^2 - y^2 + 2xyi) + b(x + iy)$
$Re(az^2 + bz) = a(x^2 - y^2) + bx = a$ …(i)
$Re(bz^2 + az) = b(x^2 - y^2)\ ax = b$ ….(ii)
On subtracting equation (i) and (ii) we get–
$(a-b)(x^2-y^2)+(b-a)x = a-b$
$x^2 - y^2 - x = 1$ …(iii)
Adding equation (i) and (ii), we get –
$(a+b)(x^2-y^2) + (a+b) \times x = a+b$
$x^2 - y^2 + x = 1$ …(iv)
By equation (iii) and (iv)
$(x^2-y^2)-x=1$
$(x^2-y^2)+x=1$ $[\because x = 0]$
$x^2 - y^2 = 1$
$y^2 = -1$
Hence, No solution

13. **The number of elements in the set $\{n \in N : 10 \leq n \leq 100$ and $3^n - 3$ is a multiple of 7$\}$ is _____**

JEE MAIN-15.04.2023, Shift-I

Ans. (15) :
$n \in [10, 100]$
$3^n - 3$ is multiple of 7
$3^n = 7\lambda + 3$
$n = 1, 7, 13, 20, …..97$
Number of possible values of n = 15

14. **Let S= $\{1,2,3,5,7,10,11\}$. The number of non-empty subsets of S that have the sum of all elements a multiple of 3, is _____.**

JEE MAIN-25.01.2023, Shift-I

Ans. (43) : Elements of the type $3k = 3$
Elements of the type $3k + 1 = 1, 7, 9$
Element of the type $3k + 2 = 2, 5, 11$
Subsets containing one element $S_1 = 1$
Subsets containing two elements
$S_2 = {}^3C_1 \times {}^3C_1 = 9$
Subsets containing three elements
$S_3 = {}^3C_1 \times {}^3C_1 + 1 + 1 = 11$
Subsets containing four elements
$S_4 = {}^3C_3 + {}^3C_3 + {}^3C_2 \times {}^3C_2 = 11$

Subsets containing five elements
$S_5 = {}^3C_2 \times {}^3C_2 \times 1 = 9$
Subsets containing six elements $S_6 = 1$
Subsets containing seven elements $S_7 = 1$
$\Rightarrow$ sum = 43

15. Let S = {1, 2, 3, 4, 5, 6}. Then the number of one-one functions f : S → P(S), where P(S) denote the power set of S, such that f(n) ⊂ f(m) where n < m is _______.

JEE MAIN-30.01.2023, Shift-I

Ans. (3240) : Given,
S = {1, 2, 3, 4, 5, 6}
$\Rightarrow$ n(S) = 6
P(S) = {ϕ, {1}, … {6}, {1, 2}, … {5, 6}, … {1, 2, 3, 4, 5, 6}}
Case–1
f(6) = S i.e. 1 option.
f(5) = any 5 elements subset A of S i.e. 6 options.
f(4) = any 4 element subset B of A i.e. 5 options.
f(3) = any 3 element subset C of B i.e. 4 options.
f(2) = any 2 element subset D of C i.e. 3 options.
f(1) = any 1 element subset E of D or empty subset i.e 3 options.
Total function = 1080.
Case-2
f(6) = any 5 element subset A of S i.e. 6 options.
f(5) = any 4 elements subset B of A i.e. 5 options.
f(4) = any 3 element subset C of B i.e. 4 options.
f(3) = any 2 element subset D of C i.e. 3 options.
f(2) = any 1 element subset E of D i.e. 2 options.
f(1) = empty subset i.e. 1 option.
Total functions = 720.
Case-3
f(6) = S
f(5) = any 4 element subset A of S i.e. 15 options.
f(4) = any 3 elements subset B of A i.e. 4 options.
f(3) = any 2 element subset C of D i.e. 3 options.
f(2) = any 1 element subset D of C i.e. 2 options.
f(1) = empty subset i.e. 1 option.
Total functions = 360.
Case-4
f(6) = S
f(5) = any 5 element A of S i.e. 6 options.
f(4) = any 3 elements subset B of A i.e. 10 options.
f(3) = any 2 element subset C of B i.e. 3 options.
f(2) = any 1 element subset D of C i.e. 2 options.
f(1) = empty subset i.e. 1 option.
Total functions = 360.
Case-5
f(6) = S
f(5) = any 5 element A of S i.e. 6 options.
f(4) = any 4 elements subset B of A i.e. 5 options.
f(3) = any 2 element subset C of B i.e. 6 options.
f(2) = any 2 element subset D of C i.e. 2 options.
f(1) = empty subset i.e. 1 option.
Total functions = 360.
Case-6
f(6) = S
f(5) = any 5 element A of S i.e. 6 options.
f(4) = any 4 elements subset B of A i.e. 5 options.
f(3) = any 3 element subset C of B i.e. 4 options.
f(2) = any 2 element subset D of C i.e. 3 options.
f(1) = empty subset i.e. 1 option.
Total functions = 360.
∴ Number of such functions = 3240

16. Let $f^1(x) = \frac{3x+2}{2x+2}, x \in R - \left\{\frac{-3}{2}\right\}$
For n ≥ 2, define $f^n(x) = f^1 O f^{n-1}(x)$.
If $f^5(x) = \frac{ax+b}{bx+a}$, gcd (a, b) = 1, then a + b is equal to ________.

JEE MAIN-30.01.2023, Shift-I

Ans. (3125) : Given,
$$f'(x) = \frac{3x+2}{2x+3} = \frac{a_1x+b_1}{b_1x+a_1}, a_1 + b_1 = 3+2 = 5$$
$$f^2(x) = f^1 of^{2-1}(x) = \frac{3\left(\frac{3x+2}{2x+3}\right)+2}{2\left(\frac{3x+2}{2x+3}\right)+3}$$
$$= \frac{9x+6+4x+6}{6x+4+6x+9} = \frac{(3^2+2^2)x+2\times3\times2}{(3^2+2^2)+2.3.2x} = \frac{a_2x+b_2}{b_2x+a_2}$$
$$a_2 + b_2 = (3+2)^2 = 5^2$$
Similarly
$$f^3(x) = \frac{a_3x+b_3}{b_3x+a_3}$$
$$a_3 + b_3 = (3+2)^3 = 5^3$$
$$f^5(x) = a_5 + b_5 = (3+2)^5 = 5^5 = 3125$$

17. Consider a function f : IN → IR, satisfying f(1) + 2f(2) + 3f(3) + ... + xf(x) = x(x + 1)f(x); x ≥ 2 with f(1) = 1.
Then $\frac{1}{f(2022)} + \frac{1}{f(2028)}$ is equal to
(a) 8100
(b) 8200
(c) 8000
(d) 8400

JEE MAIN-29.01.2023, Shift-II

Ans. (a) : Given that,
f : N → R such that f(1) = 1
Now, f(1) + 2f(2) + 3f(3) + ... +xf(x) = x(x + 1) f(x), x ≥ 2

Here, $f(1) + 2f(2) = 2(2 + 1) f(2)$
$\Rightarrow f(1) + 2f(2) = 6 f(2)$
$\Rightarrow f(1) = 4f(2)$
$\Rightarrow f(2) = \frac{f(1)}{4}$
$\Rightarrow f(2) = \frac{1}{4}, \quad \{\because f(1) = 1\}$
And $f(1) + 2f(2) + 3f(3) = 3(3 + 1) f(3)$
$\Rightarrow 1 + 2\left(\frac{1}{4}\right) + 3f(3) = 12 f(3)$
$\Rightarrow 9f(3) = \frac{3}{2}$
$\Rightarrow f(3) = \frac{1}{6}$
Similarly, $f(1) + 2f(2) + 3f(3) + 4f(4) = 4(4 + 1)f(4)$
$\Rightarrow 16f(4) = 1 + 2\times\frac{1}{4} + 3\times\frac{1}{6} = = 1 + \frac{1}{2} + \frac{1}{2} = 2$
$\Rightarrow f(4) = \frac{1}{8}$
Now, In general, $f(x) = \frac{1}{2x}$, if x = x then
or $f(n) = \frac{1}{2n} \Rightarrow 2n = \frac{1}{f(n)}$
Here, $\frac{1}{f(2022)} = 2\times 2022$ and $\frac{1}{f(2028)} = 2\times 2028$
$\frac{1}{f(2022)} = 4044$ and $\frac{1}{f(2028)} = 4056$
now, $\frac{1}{f(2022)} + \frac{1}{f(2028)} = 4044 + 4056 = 8100$

18. Let $S = \{x \in R : 0 < x < 1$ and $2\tan^{-1}\left(\frac{1-x}{1+x}\right) = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)\}$.
If n (S) denotes the number of elements in S then:
(a) n(S) = 1 and the element in S is less than $\frac{1}{2}$
(b) n(S) = 0
(c) n(S) = 1 and the elements in S is more than $\frac{1}{2}$
(d) n(S) = 2 and only one element in S is less than $\frac{1}{2}$

JEE MAIN-01.02.2023, Shift-II

Ans. (a) :
$$S = \left\{x \in R : 0 < x < 1 \text{ and } 2\tan^{-1}\left(\frac{1-x}{1+x}\right) = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)\right\}$$
$$2\tan^{-1}\left(\frac{1-x}{1+x}\right) = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$$
Let $x = \tan\theta$
R.H.S
$= \cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)$
$= \cos^{-1}(\cos 2\theta)$
$= 2\theta$
L.H.S
$= 2\tan^{-1}\left(\frac{1-x}{1+x}\right)$
$= 2\tan^{-1}\left[\frac{1-\tan\theta}{1+\tan\theta}\right]$
$= 2\tan^{-1}\tan\left(\frac{\pi}{4}-\theta\right)$
$= 2\left(\frac{\pi}{4}-\theta\right)$
Given, L.H.S = R.H.S
$2\left(\frac{\pi}{4}-\theta\right) = 2\theta$
$\frac{\pi}{4} = 2\theta$
$\theta = \frac{\pi}{8}$
So, $x = \tan\frac{\pi}{8} = \sqrt{2}-1 = 1.414 - 1 = 0.414$
Thus there is only one element in S (n(S) = 1) less than 1/2.

19. Set A and B have 3 and 6 elements respectively. What can be the minimum number of elements in $A \cup B$?
(a) 3 (b) 6
(c) 9 (d) 1

SRMJEEE-2009

Ans. (b) : Given that, n(A) = 3, n(B) = 6
Then, $n(A \cap B) = 3$ (maximum)
We know that -
$n(A \cup B) = n(A) + n(B) - n(A \cap B)$
$n(A \cup B) = 3 + 6 - 3$
So, the minimum number of element $n(A \cup B) = 6$.

20. $X = \{8^n - 7n - 1 \mid n \in N\}$ and $Y = \{49(n-1) \mid n \in N\}$, then
(a) $X \subset Y$ (b) $Y \subset X$
(c) X = Y (d) none of these

JCECE-2016
SRMJEEE-2010

Ans. (a) : Given, $X = \{8^n - 7n - 1 \mid n \in N\}$
And, $Y = \{49(n-1) \mid n \in N\}$
X can be also written as –

$8^n - 7n - 1 = (7+1)^n - 7n - 1$

By Binomial expansion –

$(7+1)^n - 7n - 1 = ({}^nC_0 . 7^0 + {}^nC_1 . 7^1 + {}^nC_2 . 7^2 + {}^nC_3 . 7^3 + ...+ {}^nC_n . 7^n) - 7n - 1$

$= 1 + 7n + {}^nC_2 7^2 + ... + {}^nC_n 7^n - 7n - 1$

$= {}^nC_2 7^2 + {}^nC_3 7^3 + ... + {}^nC_n 7^n$

$= 49 ({}^nC_2 + 7\,{}^nC_3 + ... {}^nC_n 7^{n-2})$, for $n \ge 2$

We see that, $8^n - 7n - 1$ is multiple of 49 for $n \ge 2$ and 0 for n = 1.

Also written as –

$8^n - 7n - 1 = 49. K$

Where, $K = ({}^nC_2 + 7\,{}^nC_3 + ...+ {}^nC_n 7^{n-2})$

$\therefore$ X contains all positive integrals multiple of 49 and 0. and Y is also contains of all positive integral multiple of 49 together with zero.

So, $X \subset Y$.

21. If A = {x : x^2 = 1} and B = {x : x^4 = 1}, then A Δ B is equal to

(a) {i, –i}
(b) {–1, 1}
(c) {–1, 1, i, –i}
(d) {ϕ}

COMEDK-2019

Ans. (a) : Given that, A= { x : x^2=1}, B={x : x^4 = 1]

Then, A = square root of 1.

and, B = fourth root 1.

$\therefore$ A = { x : x^2 =1} = {–1, 1}

B = {x : x^4 = 1} = {–1, 1, i, – i}

We know that –

$A \Delta B = (A - B) \cup (B - A)$

Or, $A \Delta B = (A \cup B) - (A \cap B)$

Then, $A \cup B$ = {–1, +1, i, –i }

and, $A \cap B$ = {–1, 1}

So, $A \Delta B = (A \cup B) - (A \cap B)$

= {–1, +1, i, –i} – {–1, 1}

$A \Delta B$ = {i, –i}

22. A set contains 2n+1 elements. The number of subsets of this set containing more than n elements is equal to:

(a) 2^{n-1}
(b) 2^n
(c) 2^{n+1}
(d) 2^{2n}

UPSEE-2004

Ans. (d) : Given,

A set contains (2n+1) element consider the number of subset be N.

Then, number of subsets –

$= {}^{2n+1}C_0 + {}^{2n+1}C_1 + {}^{2n+1}C_2 + ...+ {}^{2n+1}C_n$

$N = 2^{2n+1} - N$

$2N = 2^{2n+1}$

$N = \dfrac{2^{2n+1}}{2}$

$N = \dfrac{2^{2n}.2^1}{2}$

$N = 2^{2n}$

23. Universal set,

$\mathbf{U = \{x \mid x^5 - 6x^4 + 11x^3 - 6x^2 = 0\}}$

$\mathbf{A = \{x \mid x^2 - 5x + 6 = 0\}}$

$\mathbf{B = \{x \mid x^2 - 3x + 2 = 0\}}$

What is $(A \cap B)'$ equal to ?

(a) {1, 3}
(b) {1, 2, 3}
(c) {0, 1, 3}
(d) {0, 1, 2, 3}

BITSAT-2015

Ans. (c) : Given, $U = \{x \mid x^5 - 6x^4 + 11x^3 - 6x^2 = 0\}$

$A = \{x \mid x^2 - 5x + 6 = 0\}$

$B = \{x \mid x^2 - 3x + 2 = 0\}$

Solve U,

U (0) = 0

U (1) = 0

U (2) = 0

U (3) = 0

Then, U = {0, 1, 2, 3}

Solve A,

A = {2, 3}

A(2) = 0

A (3) = 0

Solve B,

B (1) = 0

B (2) = 0

Then, B = {1, 2}

From solving U, A and B we get –

U = {0, 1, 2, 3}

A = {2, 3}

B = {1, 2}

Then, $A \cap B = \{2, 3\} \cap \{1, 2\}$

$A \cap B = \{2\}$

So, $(A \cap B)' = U - (A \cap B)$

= {0, 1, 2, 3} – {2}

$(A \cap B)'$ = {0, 1, 3}

24. Let $A = \{x : x \in R, |x| < 1\}$; $B = \{x : x \in R, |x - 1| \ge 1\}$ and $A \cup B = R - D$, then the set D is

(a) $\{x : 1 < x \le 2\}$
(b) $\{x : 1 \le x < 2\}$
(c) $\{x : 1 \le x \le 2\}$
(d) None of these

BITSAT-2010

Ans. (b) : Given,

$A = \{x : x \in R, |x| < 1\}$

$B = \{x : x \in R, |x - 1\} \ge 1\}$

And $A \cup B = R - D$

Then, A is also written as –

$A = \{x : x \in R, -1 < x < 1\}$

And, B is also written as –

$B = \{x : x \in R, x - 1 \ge 1 \text{ or } x - 1 \le -1\}$

i.e., $B = \{x : x \in R, x \ge 2 \text{ or } x \le 0\}$

$\therefore$ A = Range set = (–1, 1)

B = Range set = $x \ge 2$ or $x \le 0$

$= R - (0, 2) = (-\infty, 0] \cup [2, \infty)$

So, $A \cup B = (-1, 1) \cup (-\infty, 0] \cup [2, \infty)$

$= (-\infty, 1) \cup [2, \infty)$

Then, $A \cup B = R - \{(x : 1 \le x < 2)\}$

Since, $R = (-\infty, \infty)$

Hence, $A \cup B = R - D$

By comparing -

$A \cup B = R - \{x : 1 \le x < 2\}$

Hence, $D = \{x : 1 \le x < 2\}$

25. If A = {1, 2, 3, 4, 5} then the number of proper subsets of A is

(a) 31 (b) 38
(c) 48 (d) 54

BITSAT-2009

Ans. (a) : Given, $A = \{1, 2, 3, 4, 5\}$

Then, number of elements in A = 5

We know that,

Number of proper subsets of $A = 2^n - 1$

Where, n = number of elements in the given set.

So, the number of proper subsets of $A = 2^5 - 1$

$= 32 - 1 = 31$

26. Two finite sets have m and n elements. The number of subsets of the first set is 112 more than that of the second set. The values of m and n respectively are,

(a) 4, 7 (b) 7, 4
(c) 4, 4 (d) 7, 7

JCECE-2019
BITSAT-2016
CG – PET - 2018

Ans. (b) : Given, two finite sets have m and n elements.

Let, the finite set is A and B.

Then, $n(A) = m$

$n(B) = n$

$\therefore$ Number of subsets of finite set A and B is 2^m and 2^n.

According to given question –

$2^m = 112 + 2^n$

$2^m - 2^n = 112$

$2^n(2^{m-n} - 1) = 112$

$2^n(2^{m-n} - 1) = 16 \times 7$

$2^n(2^{m-n} - 1) = 2^4 \times (2^3 - 1)$

Comparing both sides, we get –

$n = 4$ and $m - n = 3$

Then, $m - 4 = 3 \Rightarrow m = 7$

Hence, $m = 7, n = 4$.

27. Let A and B be two sets such that $A \cap X = B \cap X = \phi$ and $A \cup X = B \cup X$ for same set X. Then

(a) A = B (b) A = X
(c) B = X (d) $A \cup B = X$

BITSAT-2015
AMU-2009

Ans. (a) : Given, A and B be two sets.

$A \cap X = B \cap X = \phi$ and $A \cup X = B \cup X$, for same set X.

Then, from, $A \cup X = B \cup X$.

Take intersection both sides by A –

$A \cap (A \cup X) = A \cap (B \cup X)$

By distributive law –

$(A \cap A) \cup (A \cap X) = (A \cap B) \cup (A \cap X)$

$A \cup \phi = (A \cap B) \cup \phi$

$A = A \cap B$ ....(i)

Again, take intersection both sides by B,

$B \cap (A \cup X) = B \cap (B \cup X)$

By distributive law –

$(B \cap A) \cup (B \cap X) = (B \cap B) \cup (B \cap X)$

$(B \cap A) \cup \phi = B \cup \phi$

$B \cap A = \phi \cup B = B$

$A \cap B = B$ ....(ii)

Since, $B \cap A = A \cap B$,

So, from equation (i) and equation (ii), we get–

$A = B$

28. If $A = \{(x, y) : x^2 + y^2 \le 1, x, y \in R\}$ and $B = \{(x, y) : x^2 + y^2 \le 4, x, y \in R\}$ then

(a) $A - B = A$ (b) $B - A = B$
(c) $A - B = \phi$ (d) $B - A = \phi$

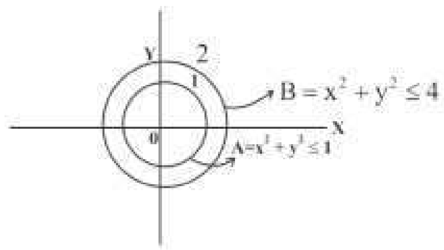
BCECE-2018

Ans. (c) : Given,

$A = \{(x, y) : x^2 + y^2 \le 1, x, y \in R\}$

And, $B = \{(x, y) : x^2 + y^2 \le 4, x\ y \in R\}$

We see that set A represents circle centered at origin and radius 1 and B represents circle centered at origin and radius 2.



Since both the circles are concentric

Hence, $A - B = \phi$

29. The remainder on dividing $1 + 3 + 3^2 + 3^3 + ... + 3^{2021}$ by 50 is ____.

JEE Main-24.06.2022, Shift-II

Ans. (4) : The given series are,

$1 + 3 + 3^2 + 3^3 + ... + 3^{2021}$

Which is in G.P series sum of the series $\frac{a(r^n - 1)}{r - 1}$

$= \frac{1.(3^{2022} - 1)}{3 - 1} = \frac{9^{1011} - 1}{2}$

$$= \frac{(10-1)^{1011}-1}{2}$$

$\because (10 - 1)^{1011} = (10)^{1011}.(1)^0 - {}^nC_1(10)^{1010}(1)^1 + {}^nC_2(10)^{1009}(1)^2 - {}^nC_3(10)^{1008}(1)^3 + \ldots\ldots + {}^{1011}C_{1011}(10)^0(1)^{1011}$

$$\therefore \quad \frac{100k+10110-2}{2}$$

$$= 50k + \frac{10108}{2}$$

Now, dividing by 50

$$\frac{50k}{50} + \frac{5054}{50} = 4$$

Remainder $\to 4$

30. The remainder when $(2021)^{2022} + (2022)^{2021}$ is divided by 7 is

(a) 0 (b) 1

(c) 2 (d) 6

JEE Main-27.07.2022, Shift-I

Ans. (a) : Given,

$(2021)^{2022} + (2022)^{2021}$

$$\frac{(x+y)^n}{x} = \frac{{}^nC_0 x^n}{x} + \frac{{}^nC_1 x^{n-1}y}{x} + \frac{{}^nC_2 x^{n-2}y^2}{x} + \ldots + \frac{{}^nC_n y^n x^0}{x}$$

$(2022)^{2021} = (2023 - 2)^{2022} + (2023 - 1)^{2021}$

$= (-2)^{2022} + (-1)^{2021}$

$= 2^{2022} - 1$

$= (2^3)^{674} - 1$

$= (8)^{674} - 1$

$= (7 + 1)^{674} - 1$

Divisible by 7,

So,

$(1)^{674} - 1$

$1 - 1 = 0$

31. Let A = {1, 2, 3, 4, 5, 6, 7} and B = {3, 6, 7, 9}. Then the number of elements in the set $\{C \subseteq A : C \cap B \neq \phi\}$ is _______.

JEE Main-26.07.2022, Shift-II

Ans. (112) : Given,

A = {1, 2, 3, 4, 5, 6, 7} and B = {3, 6, 7, 9}.

$\because$ The number of subset = 2^n

Then, number of subset A = 2^7

= 128

$C \cap B = \phi$ when set C contains the elements

C = {1, 2, 4, 5}

$S = \{C \subseteq A ; C \cap B \neq \phi\}$

$= \text{Total} - (C \cap B = \phi)$

$= 128 - 2^4 = 128 - 16 = 112$

32. The number of elements in the set $\{n \in Z : |n^2 - 10n + 19| < 6\}$ ' is ________.

JEE Main-10.04.2023, Shift-I

Ans. (6) : Given,

$n \in Z: |n^2 - 10n + 19| < 6$

$\Rightarrow \quad |(n-5)^2 - 6| < 6$

$\Rightarrow \quad -6 < (n-5)^2 - 6 < 6$

$0 < (n-5)^2 < 12$

$\Rightarrow \quad (n-5)^2 = 1, 4, 9$

$\Rightarrow \quad n - 5 = \pm 1, \pm 2, \pm 3$

So, the number of elements in the set is 6.

33. If A and B are disjoint sets, then $B \cap A'$, where A' is complement of A is equal to

(a) A (b) B

(c) A' (d) B'

AMU-2018

Ans. (b) : If A and B are disjoint set-

$\therefore \quad A \cap B = \phi$

$B \cap A' = B - A = B - (A \cap B) = B - \phi = B$

$B \cap A' = B$

34. Suppose A, B and C are three sets, each with three elements. The number of subsets of the set $A \times B \times C$ that have at least 2 elements is

(a) $(2^{27}) - 28$ (b) $(2^{27}) - 55$

(c) 27 (d) $(27^2) - 3$

J&K CET-2018

Ans. (a): No of element in $A \times B \times C = 3 \times 3 \times 3 = 27$

$\therefore$ No. of subsets of the set $A \times B \times C = 2^{27}$

No. of subsets having 1 element= 27

No. of subsets having 0 element=1

So, required no. of subsets = $2^{27} - (27+1)$

$= 2^{27} - 28$

35. If $P(A) = \frac{1}{4}$; $P(B) = \frac{1}{5}$ and $P(AB) = \frac{1}{8}$ then $P\left(\frac{A^C}{B^C}\right) =$

(a) $\frac{21}{32}$ (b) $\frac{25}{32}$

(c) $\frac{27}{32}$ (d) $\frac{29}{32}$

J&K CET-2017

Ans. (c) : Given,

$$P(A) = \frac{1}{4}$$

$$P(B) = \frac{1}{5}$$

$$P(AB) = \frac{1}{8}$$

$$P\left(\frac{A^C}{B^c}\right) = ?$$

$$P\left(\frac{A^C}{B^c}\right) = \frac{P(A^c \cap B^c)}{P(B^C)}$$

$$= \frac{P\left((A \cup B)^c\right)}{P(B^c)}$$

$$= \frac{1 - P(A \cup B)}{1 - P(B)}$$

$$= \frac{1 - \left[P(A) + P(B) - P(A \cap B)\right]}{1 - P(B)}$$

$$= \frac{1 - \frac{1}{4} - \frac{1}{5} + \frac{1}{8}}{1 - \frac{1}{5}}$$

$$= \frac{40 - 10 - 8 + 5}{40 \times \frac{4}{5}} = \frac{27}{32}$$

36. Suppose P, Q and R are three sets, each with three elements. The number of subsets of the set P × Q × R, that have at least 2 elements is

(a) 134217700 (b) 134217701
(c) 134217727 (d) 134217728

J&K CET-2017

Ans. (a) : Given,

x(p) = 3, x(Q) = 3, x(R) = 3

So, total number of set (x) = x(p) × x(Q) × x(R)

= 3 × 3 × 3

= 27

Total number of subset = $2^x = 2^{27}$ = 134217728

∴ Number of subsets of the set that have at least 2 element

= 134217728 – 1 – 27

= 134217700

37. If A = {1, 2, 3, 4, 5} and B = {2, 4, 6}, then A – B =

(a) {1, 3, 5, 6} (b) {0, 1, 3, 5, 6}
(c) {1, 3, 5} (d) {1, 2, 3, 4,5, 6}
(e) {2, 4}

Kerala CEE-2020

Ans. (c) : Given A ={ 1, 2, 3, 4, 5 }and B = { 2, 4, 6}

A – B means A contains the element which present in B.

Thus A –B = {1, 3, 5}

38. The set {x∈R:x –2 + x^2 = 0} is equal to

(a) {–1, 2} (b) {1, 2}
(c) {–1, –2} (d) {1, –2}

EAMCET-2000

Ans. (d) : Given,

set $\left\{x \in R : x - 2 + x^2 = 0\right\}$

Now,

$x^2 + x - 2 = 0$

$x^2 + 2x - x - 2 = 0$

x (x + 2) – 1(x + 2) = 0

(x – 1) (x + 2) = 0

x ∈ { 1, –2}

39. If A = {1, 2, 3, 4, 5, 6}, then the number of subsets of A which contains at least two elements is

(a) 63 (b) 57
(c) 58 (d) 64

Karnataka CET 2020

Ans. (b) : Given that, A = { 1, 2, 3, 4, 5, 6}

Then, the number of subsets of A = 2^6 = 64

Subsets are following –

{ϕ, {1}, {2}, {3},{4}, {5}, {6}, {1, 2}, {1, 3}.............
{1, 2, 3, 4, 5, 6}

So, the number of subsets of A which contains at least two elements is –

$= 64 - \left({}^6C_0 - {}^6C_1\right)$

= 64 – (1 + 6)

= 57

40. There is a set P of ordered pairs in which each pair has a vowel as first element and a consonant as second element. It is given that M = 4^{10}. How many element will be there in power set of P ?

(a) $32(M^5)$ (b) $16(M^5)$
(c) $32(M^4)$ (d) $16(M^4)$

J&K CET-2018

Ans. (a) : Total no. of vowels in English alphabets = 5

Total no. of consonants in English alphabets = 21

Since, set P has ordered pairs in which each pair has a vowel as first element and a consonants as second element.

Total no. of element in P = 21 + 21 +21 +21 + 21

= 105

Number of elements in power set of = 2^{105}

$= 2^{2 \times 50 + 5}$

$= 2^5 . 4^{50}$

$= 2^5 . (4)^{10 \times 5}$

$= 2^5 . M^5$

$= 32\ M^5$

41. If A = {2, 3, 4, 8, 10}, B = {3, 4, 5, 10, 12} and C = {4, 5, 6, 12, 14}, then (A ∪ B)∪(A ∪ C) is equal to

(a) {2, 3, 4, 5, 10, 12}
(b) {2, 3, 4, 5, 8, 10, 12}
(c) {2, 3, 4, 10, 12}
(d) None of these

COMEDK 2018

Ans. (b) : Given, A={2, 3, 4, 8, 10}, B = {3, 4, 5, 10, 12} and C ={4, 5, 6, 12, 14}

Then, A ∪ B = {2, 3, 4, 8, 10} ∪ {3, 4, 5, 10, 12}

= {2, 3, 4, 5, 8, 10, 12}

And A ∪ C = {2, 3, 4, 8, 10} ∪ {4, 6, 5, 12, 14}

= {2, 3, 4, 5, 6, 8, 10, 12, 14 }

So, (A ∪ B) ∩ (A ∪ C) = { 2, 3, 4, 5, 8, 10, 12}

42. If n(P) = 8, n (Q) = 10 and n(R) = 5 ('n' denotes cardinality) for three disjoint sets P, Q, R then n(P ∪ Q ∪ R) =
(a) 23 (b) 20
(c) 18 (d) 15

J&K CET-2017

Ans. (a) : Given,
n(P) = 8, n(Q) = 10, n(R) = 5
P, Q and R are disjoint set
∴ n(P ∪ Q ∪ R) = n(P) + n(Q) + n(R)
= 8 + 10 + 5
= 23

43. If A and B are two such events that P (A ∪ B) = P(A ∩ B), then which of the following is true?
(a) P(A)+P(B) = 0
(b) P(A)+P(B) =P(A) P(B/A)
(c) P(A)+P(B) =2P(A) P(B/A)
(d) None of the above

Manipal UGET-2020

Ans. (c) : Given that-
$$P(A\cup B)=P(A\cap B)$$
$$\Rightarrow \quad P(A)+P(B)-P(A\cap B)=P(A\cap B)$$
$$\Rightarrow \quad P(A)+P(B)=2P(A\cap B)$$
$$\Rightarrow \quad P(A)+P(B)=2\times P(A)P(B/A)$$
$$\left[\because P(B/A)=\frac{P(A\cap B)}{P(A)}\right]$$

44. If the set A contains 5 elements, then the number of elements in the power set P(A) is equal to
(a) 32 (b) 25
(c) 16 (d) 8
(e) 10

Kerala CEE-2011

Ans. (a) : Given n(A) = 5
$P(n) = 2^n$
Here n = 5
$n(P(A)) = 2^5 = 32$

45. If A = {1, 3, 5, 7, 9, 11, 13, 15, 16, 17}, B = {2, 4, 6, 8, 10, 12, 14, 16, 18} and N = {1, 2, 3, 4, 5, ……18} is the universal set, then A'∪ ((A ∪ B) ∩ B′) is
(a) A (b) N
(c) B (d) none of these

SRMJEEE-2013

Ans. (b) : Given A = { 1, 3, 5, 7, 9, 11, 13, 15, 16, 17}
B = {2, 4, 6, 8, 10, 12, 14, 16, 18}
N = {1, 2, 3, 4, 5, ……18}
Then, A∪B = {1, 2, 3, 4 ……18)
B' = {1, 3, 5, 7, 9, 11, 13, 15, 17, 19….} = B
A' = {2, 4, 6, 8, 10, 12, 14, 18, 19, 20….}= A
A'∪ (A ∪ B)∩B
A'∪(N∩B') [∴N∩B' = A]
A' ∪A
B ∪ A = N

46. If X and Y are two sets, then X∩ (Y∪X)^c equals
(a) X (b) Y
(c) ϕ (d) none of these

SRMJEEE-2014

Ans. (c) : Given, X and Y are two sets.
Then, $X \cap (Y \cup X)^c = X\cap(Y^c\cap X^c)$
Since $(A \cup B)^c = A^c \cap B^c$
$$\therefore \quad X\cap\left(Y^c\cap X^c\right)=X\cap\left(X^c\cap Y^c\right)$$
By Distributive law –
$$X\cap\left(X^c\cap Y^c\right)=\left(X\cap X^c\right)\cap Y^c$$
$$=\phi\cap Y^c=\phi$$
Since, $X \cap X^c = \phi$
So, $X \cap (Y \cup X)^c = \phi$

47. If a set A had 4 elements, then the total number of proper subsets of set A, is
(a) 16 (b) 14
(c) 15 (d) 17

COMEDK 2015

Ans. (c) : Given, A set had 4 elements. Then, total number of subsets of A = 2^4 = 16
So, the total number of proper subsets of
Set A, is $2^4 - 1 = 16 - 1 = 15$

48. Let A and B be two sets then $(A\cup B)'\cup(A'\cap B)$ is equal to
(a) A' (b) A
(c) B' (d) None of these

BITSAT-2012

Ans. (a) : Given, A and B be a two sets.
Find (A ∪ B)' ∪ (A' ∩ B) = ?
Then, by De Morgan's law –
(A ∪ B)' ∪ (A' ∩ B) = (A' ∩ B') ∪ (A' ∩ B)
= (A' ∪ A') ∩ (A' ∪ B) ∩ (B' ∪ A') ∩ (B' ∪ B)
= A' ∩ [{ A' ∪{ B ∩ B' }] ∩ U
= A' ∩ (A' ∪ ϕ) ∩ U
= A' ∩ A' ∩ U
= A' ∩ U
= A'

49. Let Z denotes the set of all integers and A = {(a, b) : $a^2+3b^2 = 28$, a, b ∈ Z} and B = {(a, b) : a < b, a, b ∈ Z}. Then, the number of elements in A ∩ B is
(a) 2 (b) 4
(c) 6 (d) 5

UPSEE-2015

Ans. (c) : Given, Z = Set of integers,
and A = {(a, b) : $a^2 + 3b^2 = 28$, a, b ∈ Z}
B = { (a, b) : a < b, a, b ∈ Z}
Then, in set A, $a^2 + 3b^2 = 28$ satisfies the following numbers are given below
$$=\begin{Bmatrix}(-1,-3),(-1,3),(1,-3),(1,3),(-4,-2),(-4,2)\\(4,-2),(4,2),(5,1),(-5,-1),(5,-1),(-5,1)\end{Bmatrix}$$
And in B, a < b,

Then,

$$\{(1,3),(-1,3),(-4,2),(-4,-2),(-5,-1),(-5,1)\}$$

So, $A \cap B = \{(1,3),(-1,3),(-4,2),(-4,-2),(-5,-1),(-5,1)\}$.

Hence, the number of element in $A \cap B$ is 6.

50. Let F_1 be the set of parallelograms, F_2 be the set of rectangles, F_3 be the set of rhombus, F_4 be the set of squares and F_5 be the set of trapeziums in a plane. Then, F_1 may be equal to

(a) $F_2 \cap F_3$ (b) $F_3 \cap F_4$
(c) $F_2 \cup F_5$ (d) $F_2 \cup F_3 \cup F_4 \cup F_1$

UPSEE-2014

Ans. (d) : Given,
F_1 be the set of parallelograms
F_2 be the set of rectangles
F_3 be the set of rhombus
F_4 be the set of squares
F_5 be the set of trapeziums.
We know that, in parallelograms opposite sides are equal and parallel and we also known in rectangles, rhombus and squares opposite sides are equal and parallel.
Then, $F_2 \subset F_1, F_3 \subset F_1, F_4 \subset F_1$
So, $F_1 = F_1 \cup F_2 \cup F_3 \cup F_4$.

51. If A = {x : x is a multiple of 4} and B = {x:x is a multiple of 6}, then A ∩ B consists of all multiples of

(a) 16 (b) 12
(c) 8 (d) 4

UPSEE-2014

Ans. (b) : Given, A = { x : x is a multiple of 4}
and B = { x : x is a multiple of 6}
Then, A = {4, 8, 12, 16, 20, 24, 28, 32, 36……..}
and B = {6, 12, 18, 24, 30, 36…….}
So, $A \cap B$ = {12, 24, 36 ....}
i.e. $A \cap B$ = {x : x is a multiple of 12}

52. The set $(A \cup B \cup C) \cap (A \cap B' \cap C')' \cap C'$ is equal to

(a) $B \cap C'$ (b) $A \cap C$
(c) $B' \cap C'$ (d) None of these

UPSEE-2013

Ans. (a) : Given, the set A, B and C
Find, $(A \cup B \cup C) \cap (A \cap B' \cap C')' \cap C' = ?$
Then, by Demorgan law,

$(A \cup B \cup C) \cap (A \cap B' \cap C')' \cap C' = (A \cup B \cup C) \cap (A' \cup B \cup C) \cap C'$
$= \{(A \cap A')\} \cup (B \cup C)\} \cap C'$
$= \{\phi \cup (B \cup C)\} \cap C'$

Since, $A \cap A' = \phi$
$= \{(B \cup C)\} \cap C'$
$= \{B \cap C'\} \cup \{C \cap C'\}$
$= (B \cap C') \cup \phi$
$= B \cap C'$

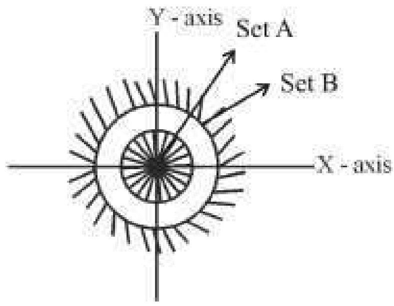
Hence, $(A \cup B \cup C) \cap (A \cap B' \cap C')' \cap C' = B \cap C'$

53. If $A = \{(x,y) : x^2 + y^2 \le 1; x, y \in R\}$ and $B = \{(x,y) : x^2 + y^2 \ge 4; x, y \in R\}$, then

(a) $A - B = \phi$ (b) $B - A = \phi$
(c) $A \cap B \ne \phi$ (d) $A \cap B = \phi$

UPSEE-2013

Ans. (d) : Given,
$A = \{(x, y) : x^2 + y^2 \le 1, x, y \in R\}$
and $B = \{(x, y) : x^2 + y^2 \ge 4, x, y \in R\}$
From equation, draw the graph of following above set A, B -



From figure we see that set A inside of the circle and set B outside of the circle.
So, $A \cap B = \phi$

54. The set A = {x : |2x + 3| < 7} is equal to the set

(a) D = {x : 0 < x + 5 < 7}
(b) B = {x : –3 < x < 7}
(c) E = {x : –7 < x < 7}
(d) C = {x : –13 < 2x < 4}

Karnataka CET 2014

Ans. (a) : Given,
Set A = {x : |2x + 3| < 7}
Then,
A = {x : –7 < 2x + 3 < 7}
A = {x : –7 –3 < 2x < 7 -3}
A = {x : –10 < 2x < 4}
A = {x : –5 < x < 2}
A = {x : –5 + 5 < x + 5 < 2+5 }
A = {x: 0 < x + 5 < 7}
So, the set A is equal to set D

55. The number of proper subsets of a set having n+1 elements is

(a) 2^{n+1} (b) $2^{n+1} - 1$
(c) $2^{n+1} - 2$ (d) 2^{n-2}

COMEDK 2014

Ans. (b) : We know that, if a set having n element then number of subsets = 2^n
Example – If a set A = {a, b, c} has 3 elements.
Then, subsets of A = $2^3 = 8$
Since, if a set having (n + 1) elements then its number of subsets = 2^{n+1}
So, the number of proper subsets of a set having n + 1 elements is $2^{n+1} - 1$

56. Set A and B have 3 and 6 elements respectively. What can be the minimum number of elements in A∪ B?

(a) 18 (b) 9
(c) 6 (d) 3

AMU-2014

Ans. (c) : From question,
No. of element in set A = 3
No. of element in set B = 6
Maximum no. of element can be in set $A \cap B = 3$
Minimum no. of element can be in set $A \cup B$ is,
$n(A \cup B) = n(A) + n(B) - n(A \cap B)$
$n(A \cup B) = 3 + 6 - 3$
$n(A \cup B) = 6$

57. If the sets A and B are as follows: A = {1, 2, 3, 4}, B = {3, 4, 5, 6}, then
(a) $A - B = \{1, 2\}$
(b) $B - A = \{5, 6\}$
(c) $[(A - B) - (B - A)] \cap A = \{1, 2\}$
(d) $[(A - B) - (B - A)] \cup A = \{3, 4)$

UPSEE-2011

Ans. (a,b,c) : Given, $A = \{1, 2, 3, 4\}$
$B = \{3, 4, 5, 6\}$
Then, by options –
Options a :- $A - B = \{1, 2, 3, 4\} - \{3, 4, 5, 6\}$
$A - B = \{1, 2\}$
Option b :- $B - A = \{3, 4, 5, 6\} - \{1, 2, 3, 4\}$
$= \{5, 6\}$
Options c :- $[(A - B) - (B - A)] \cap A$
$= [\{1, 2\} - \{5, 6\}] \cap \{1, 2, 3, 4\}$
$= \{1, 2\} \cap \{1, 2, 3, 4\}$
$= \{1, 2\}$
Option d :- $[(A - B) - (B - A)] \cup A$
$= [\{1, 2\} - \{5, 6\}] \cup \{1, 2, 3, 4\}$
$= \{1, 2,\} \cup \{1, 2, 3, 4\}$
$= \{1, 2, 3, 4\}$.
So, we see that option (a, b, c) are correct.

58. If $A = \{4^n - 3n - 1 : n \in N\}$ and $B = \{9(n - 1) : n \in N\}$, then

(a) $B \subset A$ (b) $A \cup B = N$
(c) $A \subset B$ (d) None of these

AMU-2012

Ans. (c) : If $A = \{4^n - 3n - 1 : n \in N\}$
And, $B = \{9(n - 1) : n \in N\}$
For n = 1,
$A = 4 - 3 - 1 = 0$
$B = 9(1 - 1) = 0$
For n = 2,
$A = 16 - 6 - 1 = 9$
$B = 9(2 - 1) = 9$
For n = 3,
$A = 4^3 - 3 \times 3 - 1$
$A = 64 - 10 = 54$
$B = 9(3 - 1) = 18$
Using roster method –
$A = \{0, 9, 54, 243, \ldots\ldots\}$
$B = \{0, 9, 18, 27, 36, 45, 54 \ldots\}$
So, $A \subset B$ but $A \neq B$

59. Let A and B be subsets of the universal set U. If n(A) = 24, $n(A \cap B) = 8$ and n(U) = 63, then $n(A' \cup B')$ is equal to

(a) 43 (b) 55
(c) 35 (d) 32
(e) 45

Kerala CEE-2021

Ans. (b) : Given n(A) = 24
$n(A \cap B) = 8$
$n(u) = 63$
$n(A' \cup B') = n(U) - n(A \cap B)$
$= 63 - 8 = 55$

60. The number of subsets containing exactly 4 elements of the set { 2, 4, 6, 8, 10, 12, 14, 16, 18 } is equal to

(a) 126 (b) 63
(c) 189 (d) 58
(e) 94

Kerala CEE-2022

Ans. (a) : Number of digits = 9
{ 2, 4, 6, 8, 10, 12, 14, 16, 18 }
Number of ways to choose 4 elements in given set are
$= {}^9C_4$
$= \frac{9!}{4! \times 5!}$
$= \frac{9 \times 8 \times 7 \times 6}{4 \times 3 \times 2}$
$= 9 \times 7 \times 2 = 126$

61. If $n(A \cup B) = 97$, $n(A \cap B) = 23$ and $n(A - B) = 39$, then n (B) is equal to

(a) 52 (b) 55
(c) 58 (d) 62
(e) 65

Kerala CEE-2022

Ans. (c) : Given, $n(A \cup B) = 97$
$n(A \cap B) = 23$
$n(A - B) = 39$
$n(A - B) = n(A \cup B) - n(B)$
$39 = 97 - n(B)$
$58 = n(B)$

62. If $N_a = \{an : n \in N\}$, then $N_5 \cap N_7$ is equal to :

(a) N_7 (b) N
(c) N_{35} (d) N_5
(e) N_{12}

Kerala CEE-2005

Ans. (c) : Given, $N_a = \{an : n \in N\}$
So,
$\because N_5 = \{5, 10, 15, 20, 25, 30, 35 \ldots\}$
$N_7 = \{7, 14, 21, 35 \ldots\}$
$\therefore N_5 \cup N_7 = \{35, 70 \ldots\ldots\} = N_{35}$

63. Given n(U) = 20, n(A) = 12, n(B) = 9, $n(A \cap B) = 4$, where U is the universal set, A and B are subsets of U, then $n[(A \cup B)^c]$ equals to:

(a) 17 (b) 9
(c) 11 (d) 3
(e) 16

Kerala CEE-2004

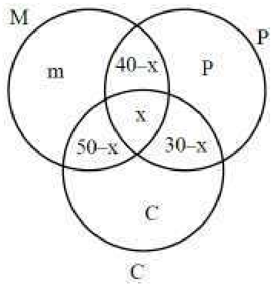
Ans. (d) : Given,
$n(U) = 20, n(A) = 12, n(B) = 9, n(A \cap B) = 4$
$\therefore n(A \cup B) = n(A) + n(B) - n(A \cap B)$
$= 12 + 9 - 4 = 17$
Hence,
$n[(A \cup B)^c] = n(U) - n(A \cup B)$
$= 20 - 17 = 3$

B. Operations on Set and Venn Diagram

64. In a survey of 220 students of a higher secondary school, it was found that at least 125 and at most 130 students studied Mathematics; at least 85 and at most 95 studied Physics; at least 75 and at most 90 studied Chemistry; 30 studied both Physics and Chemistry; 50 studied both Chemistry and Mathematics; 40 studied both Mathematics and Physics and 10 studied none of these subjects. Let m and n respectively be the least and the most number of students who studied all the three subjects. Then m + n is equal to _____

JEE MAIN-04.04.2024, Shift-I

Ans. 45



n(M∪P∪C) = 220 –10 = 210

n(M) ∈ [125, 130]

n(P) ∈ [85, 95]

n (C) ∈ [75, 90]

125 ≤ m + 90 – x ≤ 130 ...(i)

85 ≤ P + 70 – x ≤ 95 ...(ii)

75 ≤ C + 80 – x ≤ 90 ...(iii)

Also, m + P + C + 120 – 2x = 210

15 ≤ x ≤ 45 & 30 – x ≥ 0

15 ≤ x ≤ 30

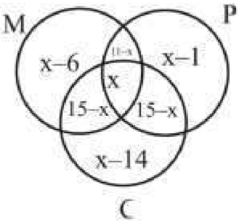
30 + 15 = 45

65. A group of 40 students appeared in an examination of 3 subjects – Mathematics, Physics and Chemistry. It was found that all students passed in atleast one of the subjects, 20 students passed in Mathematics, 25 students passed in Physics, 16 students passed in Chemistry, atmost 11 students passed in both Mathematics and Physics, atmost 15 students passed in both Physics and Chemistry, atmost 15 students passed in both Mathematics and Chemistry. The maximum number of students passed in all the three subjects is _______.

JEE MAIN-30.01.2024, Shift-I

Ans.(10) : According to question,

n(M) = 20, n(P) =25, n(C) = 16



11 x 0   15–x ≥ 0

x 11   x ≤15

x = number of student pass in all 3 subjects.

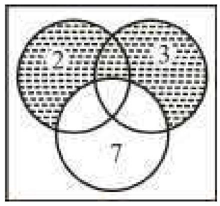
Max(x) = 11 it is not possible

Max (x) = 10

66. The number of 3-digit numbers that are divisible by either 2 or 3 but not divisible by 7, is __________.

JEE MAIN- 01.02.2023, Shift-I

Ans. (514) :



= n(2∪3∪7) – n(7)

= n(2)+ n(3)+n(7) – n(7) – n(2∩3) – n(3∩7) – n(2∩7) + n(2∩3∩7)

$= \left(\frac{900}{2}\right)+\left(\frac{900}{3}\right)-\left(\frac{900}{6}\right)-\left(\frac{900}{14}\right)+\left(\frac{900}{42}\right)$

= Divisible by 2 = 450

= Divisible by 3 = 300

= Divisible by 7 = 128

= Divisible by 2 or 7 = 64

= Divisible by 3 or 7 = 43

= Divisible by 2 or 3 = 150

= Divisible by 2, 3 or 7 = 21

Total number = 450 + 300 – 150 – 64 – 43 + 21

= 514

67. A survey shows that 63% of the Indians like tea whereas 76% like coffee. If x% of the Indians like both tea and coffee, then

(a) x = 39 (b) x = 63

(c) 39 ≤ x ≤ 63 (d) none of these

SRMJEEE-2011

JEE Main 04.09.2020 Shift-I

Ans. (c) : Given, number of the Indians like tea –

n(T) = 63

Number of the Indians like coffee

n(C) = 76

And number of the Indians like both tea and coffee

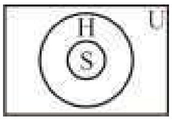
n(T ∩ C) = x

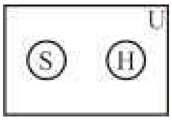
Then, n(T ∪ C) = n(T) + n (C) – n (T ∩ C)

100 = 63 + 76 –x

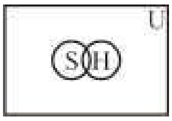
x = 139 – 100
x = 39
Also, n (T ∩ C) ≤ n(T)
x ≤ 63
So, 39 ≤ x ≤ 63

68. If U : Set of all days, S : Set of Sundays, H : Set of holidays, then, Venn diagram for "Sunday implies holiday" is

(a) 

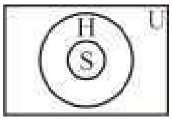
(b) 

(c) 

(d) 

MHT-CET 2004

Ans. (a): Given, U = Set of all days
S = Set of Sundays
H = Set of Holidays
Then Venn diagram for "Sunday implies holiday" is –



69. There are 20 students in a chemistry class and 30 students in a physics class. If ten students are to be enrolled in both the courses, then the number of students which are either in physics class or chemistry class is
(a) 50, if two classes meet at the same hour.
(b) 40, if two classes meet at the different hour.
(c) both (a) and (b) correct
(d) (a) correct but (b) incorrect

BITSAT-2007

Ans. (c) : Given,
n(C) = 20, n(P) = 30 and n(C ∩ P) = 10
Where C and P be the number of students in chemistry and physics class.
Find, n (C ∪ P) = ?
Here are two conditions –
Condition no I : – When both classes meet at the same time, then n(C ∩ P) = ϕ
Then, n(C ∪ P) = n (C) + n (P)
n (C ∪ P) = 20 + 30 = 50
Condition no II :- When both classes meet at different hours.
Then n(C ∩ P) = 10 (Given)
So, n(C ∪ P) = n(C) + n (P) – n (C ∩ P)
n(C ∪ P) = 20 + 30 – 10
n(C ∪ P) = 50 – 10
n(C ∪ P) = 40
Hence, both (a) and (b) correct

70. Which of the following is an empty set?
(a) $\{x \mid x \text{ is a real number and } x^2 - 1 = 0\}$
(b) $\{x \mid x \text{ is a real number and } x^2 + 3 = 0\}$
(c) $\{x \mid x \text{ is a real number and } x^2 - 9 = 0\}$
(d) $\{x \mid x \text{ is a real number and } x^2 = x + 2\}$

COMEDK 2014

Ans. (b) : We check the following is an empty set by options –
By option a: $x^2 - 1 = 0$
$x^2 = 1$
$x = \pm 1 \in R$
This is not empty set.
By option b: $x^2 + 3 = 0$
$x^2 = -3$
$x = \sqrt{-3} \in C$
This is a empty set.
By option c: $x^2 - 9 = 0$
$x^2 = 9$
$x = \pm 3 \in R$
This is not a empty set.
By option d: $x^2 = x + 2$
$x^2 - x - 2 = 0$
$x^2 - 2x + x - 2 = 0$
$x(x - 2) + 1(x - 2) = 0$
$(x - 2)(x + 1) = 0$
$x = -1, 2$
This is not empty set.

71. In a class of students, 25 students play cricket, 20 student play tennis and 10 students play both the games. Then the number of students who play tennis only is
(a) 25 (b) 10
(c) 15 (d) None of these

JCECE-2019

Ans. (b): From question, Let 'C' class of students play cricket and 'T' class of student play tennis respectively. Then, given –
n(C) = 25, n (T) = 20
n(C ∩ T) = 10
∴ n(C ∪ T) = n (C) + n (T) – n (C ∩ T)
n (C ∪ T) = 25 + 20 – 10 = 35
So, number of student who play tennis only –
= n(C ∪ T) – n(C)
= 35 – 25
= 10.

72. In a certain town, 25% families own a cell phone, 15% families own a scooter and 65% families own neither a cell phone nor a scooter. If 1500 families own both a cell phone and a scooter, then the total number of families in the town is

(a) 10000 (b) 20000
(c) 30000 (d) None of these

BCECE-2014

Ans. (c): Given, 25 % families own a cell phone
15% families own a scooter
65% families own neither cell phone nor a scooter.
And, 1500 families own both a cell phone and a scooter.
Let, the total number of families in the town is x.
Then,

$$\frac{25x}{100}+\frac{15x}{100}-1500+\frac{65x}{100}=x$$

$$\frac{105x}{100}-x=1500$$

$$\frac{5x}{100}=1500$$

So, $x=\frac{1500\times100}{5}$

x = 30000

73. If A and B are two events associated to some experiment E such that P(A) = 0.5 P(B) = 0.4, $P(A\cap B)=0.3$ then $P(A^C/B^C)$ is equal to

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$
(c) $\frac{2}{3}$ (d) $\frac{3}{4}$

AMU-2017

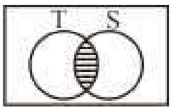
Ans. (c) : Given,
P(A) = 0.5, P(B) = 0.4 and $P(A\cap B)=0.3$
$\because$ $P(A\cup B)=P(A)+P(B)-P(A\cap B)$
$P(A\cup B)=0.5+0.4-0.3\Rightarrow 0.6$
$P(A\cup B)^c=1-P(A\cup B)$
$P(A\cup B)^c=1-0.6=0.4$

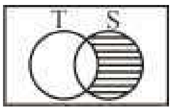
Now,
$$P\left(\frac{A^c}{B^c}\right)=\frac{P(A^c\cap B^c)}{P(B^c)}$$

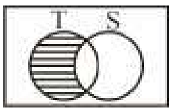
$$\frac{P(A\cap B)^c}{P(B^c)}=\frac{0.4}{1-0.4}=\frac{2}{3}$$

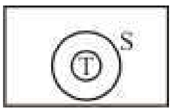
$$P\left(\frac{A^C}{B^C}\right)=\frac{2}{3}$$

74. All teachers are not sincere is represented by

(a) 

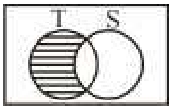
(b) 

(c) 

(d) 

MHT CET-2006

Ans. (c) :

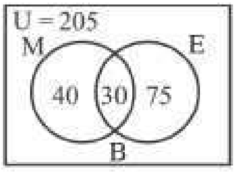


75. 205 students take an examination of whom 105 pass in English, 70 students pass in mathematics and 30 students pass in both. How many students in both subjects?

(a) 60 (b) 145
(c) 175 (d) 30

AP EAMCET-05.07.2022, Shift-I

Ans. (a) : Given U = 205
Pass in English = 105
Pass in Math = 70
Pass in Both = 30



The number of student in both subjects-
= 205 –105–70 + 30
= 30 + 30
= 60

76. If the total number of m-element subsets of the set $A=\{a_1,a_2,...,a_n\}$ is k times the number of m element subsets containing a_4, then n is

(a) $(m-1)k$ (b) mk
(c) $(m+1)k$ (d) $(m+2)k$

WB JEE-2020

Ans. (b) : From set A
n element selecting a subset of m element = ^nC_m
From given condition-
$^nC_m = k.\ ^{n-1}C_{m-1}$ { a_4 is already contains)

$$\frac{n!}{m!(n-m)!}=k\frac{(n-1)!}{(m-1)!(n-m)!}$$

$$\frac{n(n-1)!}{m(m-1)!(n-m)!}=K\frac{(n-1)!}{(m-1)!(n-m)!}$$

$$\frac{n(n-1)!}{m(n-m)!}=k\frac{(n-1)!}{(n-m)!}$$

$$\frac{n}{m}=k$$

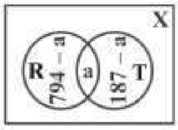
$$n=mk$$

77. In a statistical investigation of 1003 families of Calcutta, it was found that 63 families has neither a radio nor a T.V, 794 families has a radio and 187 has T.V. The number of families in that group having both a radio and a T.V is

(a) 36 (b) 41
(c) 32 (d) None of these

BITSAT-2020

Ans. (b) : Given, in a statistical investigation of 1003 families of Calcutta.
Let T be the set of families having a T.V. and R be the set of families having a radio.
Then, n(T) = 187
n (R) = 794



From, Venn diagram –
Where,
X = Total families who have T.V. and radio both.
X = 1003 – 63 = 940
187 – a = number of families who have only T.V.
794 – a = Number of families who have only radio.
Where, a = Number of families having both a radio and a T.V.
So, by Venn diagram –
$187 - a + a + 794 - a = 940$
$981 - a = 940$
$a = 981 - 940$
$a = 41$
Hence, the required number of families having both a radio and a T.V. is 41.

78. Let A, B, C be finite sets, Suppose that n(A) = 10, n(B) = 15, n(C) = 20, $n(A \cap B) = 8$ and $n(B \cap C) = 9$. Then the possible value of $n(A \cup B \cup C)$ is
(a) 26 (b) 27
(c) 28 (d) Can be 26 or 27 or 28

BITSAT-2017

Ans. (d) : Given,
n(A) = 10, n(B) = 15, n(C) = 20,
$n(A \cap B) = 8$ and $n(B \cap C) = 9$.
We know that,
$n(A\cup B\cup C) = n(A) + n(B) + n(C) - n(A\cap B) - n(B \cap C) - n(C \cap A) + n(A\cap B\cap C)$
Then, $n(A \cup B \cup C) = 10 + 15 + 20 - 8 - 9 - n(C \cap A) + n(A \cap B \cap C)$
$= 28 - n(C \cap A) - n(A \cap B \cap C)$
$n(A\cup B\cup C) = 28 - \{n(C\cap A) - n(A\cap B\cap C)\}$ ....(i)
$\because$ We know, $n(C \cap A) \geq n(A \cap B \cap C)$
Then $n(C \cap A) - n(A \cap B \cap C) \geq 0$ ....(ii)
$\therefore$ From equation (i) and equation (ii), we get –
$n(A \cup B \cup C) \leq 28$ ....(iii)
And also, $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
$n(A \cup B) = 10 + 15 - 8 = 17$
And, $n(B \cup C) = n(B) + n(C) - n(B \cap C)$
$= 15 + 20 - 9$
$= 26$
$\because$ $n(A \cup B \cup C) \geq n(A \cup C)$ and
$n(A \cup B \cup C) \geq n(B \cup C)$.
Then, $n(A \cup B \cup C) \geq 17$ and $n(A \cup B \cup C) \geq 26$
Thus, $n(A \cup B \cup C) \geq 26$ ....(iv)
So, from equation (iii) and equation (iv), we get–
$26 \leq n(A \cup B \cup C) \leq 28$
Hence, $n(A \cup B \cup C)$ is can be 26 or 27 or 28.

79. In a group of 100 persons, 80 people can speak Malayalam and 60 can speak English. The number of people who speak English only is
(a) 40 (b) 30
(c) 20 (d) 25
(e) 35

Kerala CEE-2020

Ans. (c) : Total number of person = 100
• Let A be the set of person who speak malayalum
• Let B be the set of person who speak English
n(A) = 80 n(B) = 60
$n(A \cup B) = 100$
$n(A \cup B) = n(A) + n(B) - n(A \cap B)$
$100 = 80 + 60 - n(A\cap B)$
$140 - 100 = n(A\cap B)$
$40 = n(A \cap B)$
$\therefore$ The person who speak English only $n(B) - n(A\cap B)$
= 60 – 40
= 20

80. Let A and B be finite sets such that n (A – B) = 18, $n(A \cap B) = 25$ and $n(A \cup B) = 70$. Then n(B) is equal to
(a) 52 (b) 25
(c) 27 (d) 43
(e) 45

Kerala CEE-2020

Ans. (a) : Given, n(A–B) = 18, $n(A \cup B) = 70$
$n(A\cup B) = n(A - B) + n(A\cap B) + n(B-A)$
70 = 18 + 25 + n(B–A)
70 – 43 = n(B–A)
27 = n (B – A)
Now,
$n(B) = n(A \cap B) + n(B - A)$
= 25 + 27 = 52
n(B) = 52

81. In a class of 100 student, there are 70 boys whose average marks in a subject are 75. If the average marks of the complete class is 72, then what is the average of the girls?

(a) 73 (b) 85
(c) 68 (d) 74
(e) 65

Kerala CEE-2019

Ans. (e) : Total no. of student in class = 100
Number of boys = 70
Average marks of boys = 75
So,
Total marks of boys = 70 × 75 = 5250
And, Total marks of the class = 72 × 100
= 7200
Total marks of girls = 7200 – 5250
= 1950
Average of the girls = $\frac{1950}{30}$
= 65

82. In a flight 50 people speak Hindi, 20 speak English and 10 speak both English and Hindi. The number of people who speak at least one of the languages is
(a) 40 (b) 50
(c) 20 (d) 80
(e) 60

Kerala CEE-2017

Ans. (e) : Let H = people who speak Hindi
E = People who speak English
Given
N(H) = 50 , n (E) = 20 , n(H ∩E) = 10
∴ Number of people who speak at least two language
$n(H \cup E) = n(H) + n(E) - n(H \cap E)$
= 50 + 20 – 10
= 60

83. If a class of 175 students the following data shows the number of students opting one or more subjects. Mathematics 100, Physics 70, Chemistry 40, Mathematics and Physics 30, Mathematics and Chemistry 28, Physics and Chemistry 23, Mathematics, Physics and Chemistry is 18. The number of students who have opted Mathematics alone is
(a) 35 (b) 48
(c) 60 (d) 22

COMEDK 2015
BITSAT-2013

Ans. (c) : Given, n (M) = 100, n(P) = 70, n(C) = 40
$n(M \cap P) = 30, n(M \cap C) = 28,$
$n(P \cap C) = 23, n(M \cap P \cap C) = 18$
Where M, P and C be the set of students who opted mathematics, physics and chemistry respectively
Then, the number of students who opted mathematics alone is –
$n(M \cap P' \cap C') = n\{M \cap (P \cup C)'\}$
$= n(M) - n\{M \cap (P \cup C)\}$
$= n(M) - n\{(M \cap P) \cup (M \cap C)\}$
$= n(M) - \{n(M \cap P) + n(M \cap C) - n(M \cap P \cap C)\}$
$= n(M) - n(M \cap P) - n(M \cap C) + n(M \cap P \cap C)$
= 100 – 30 – 28 + 18
= 118 – 58
= 60

84. Out of 800 boys in a school, 224 played cricket, 240 played hockey and 336 played basketball. Of the total, 64 played both basketball and hockey; 80 played cricket and basketball and 40 played cricket and hockey; 24 played all the three games. The number of boys who did not play any game is
(a) 128 (b) 216
(c) 240 (d) 160

UPSEE-2012

Ans. (d) : Given,
n (C) = 224
n (H) = 240, n (B) = 336
$n(B \cap H) = 64$
$n(C \cap B) = 80$
$n(C \cap H) = 40$
$n(C \cap H \cap B) = 24$
Where C, H and B are show that cricket, Hockey and Basketball.
We know –
$n(C \cup H \cup B) = n(C) + n(H) + n(B) - n(C \cap H) - n(C \cap B) - n(B \cap H) + n(C \cap H \cap B)$
$n(C \cup H \cup B)$ = 224 + 240 + 336 – 40 – 80 – 64 + 24
$n(C \cup H \cup B)$ = 640
So, the number of boys who did not play any game is —
= 800 – 640
= 160

85. In a survey of 200 students of a school it was found that 120 study Mathematics, 90 study Physics and 70 study chemistry, 40 study Mathematics and Physics, 30 study Physics and Chemistry, 50 study Chemistry and Mathematics and 20 none of these subjects. The number of student who study all the three subject is
(a) 30 (b) 20
(c) 22 (d) 25

BCECE-2016

Ans. (b): Given,
n (M) = 120
n (P) = 90
n (C) = 70
$n(M \cap P) = 40$
$n(P \cap C) = 30,$

$n(C \cap M) = 50,$
Find $n(M \cap P \cap C) = ?$
Where, M = Mathematics,
P = Physics
C = Chemistry
And, $n(M' \cup P' \cup C') = 20$
Then, $n(M \cup P \cup C) = 200 - n(M' \cup P' \cup C')$
$n(M \cup P \cup C) = 200 - 20 = 180$
We know -
$n(M \cup P \cup C) = n(M) + n(P) + n(C)$
$-n(M \cap P) - n(P \cap C)$
$-n(C \cap M) + n(M \cap P \cap C)$
$n(M \cup P \cup C) = 120 + 90 + 70 - 40 - 30 - 50 + n(M \cap P \cap C)$
$180 = 160 + n(M \cap P \cap C)$
$n(M \cap P \cap C) = 180 - 160 = 20$
So, the number of student who study all the three subject is 20.

86. From 50 students taking examinations in Mathematics, Physics and Chemistry, 37 passed in Mathematics, 24 in Physics and 43 in Chemistry. Atmost 19 passed in Mathematics and Physics, atmost 29 passed in Mathematics and Chemistry and atmost 20 passed in Physics and Chemistry. The largest possible number that could have passed all three exminations, is
(a) 11 (b) 12
(c) 13 (d) 14

BCECE-2015

Ans. (d): Let m be the set of students passing in mathematics, P be the set of students passing in physics and C be the set of students passing in chemistry.
Given,
$n(M) = 37, n(P) = 24$
$n(C) = 43, n(M \cap P) = 19$
$n(M \cap C) = 29, n(P \cap C) = 20$
$n(M \cup P \cup C) = 50$
Where, M = Mathematics
P = Physics
C = Chemistry
We know,
$n(M \cup P \cup C) = n(M) + n(P) + n(C) - n(M \cap P) - n(M \cap C) - n(P \cap C) + n(M \cap P \cap C)$
$50 = 37 + 24 + 43 - 19 - 29 - 20 + n(M \cap P \cap C)$
$50 = 36 + n(M \cap P \cap C)$
$n(M \cap P \cap C) = 50 - 36 = 14$
So, the largest possible number that could have passed all three examination is 14.

87. If n(A) = 43, n(B) = 51 and n(A∪B) = 75, then n[(A – B) ∪ (B – A)] is equal to
(a) 53 (b) 45
(c) 56 (d) 66
(e) 46

Kerala CEE-2013

Ans. (c) : Given $n(A) = 43$
$n(B) = 51$
$n(A \cup B) = 75$
Now by addition theorem of probability,
$n(A \cap B) = n(A) + n(B) - n(A \cup B)$
$= 43 + 51 - 75$
$= 19$
$\because n[(A - B) \cup (B - A)]$
$= n(A \cup B) - n(A \cap B)$
$= 75 - 19 = 56$

88. if n(A) = 1000, n(B) = 500 and if n(A ∩ B) ≥ 1 and n(A ∪ B) = p, then
(a) $500 \le p \le 1000$ (b) $1001 \le p \le 1498$
(c) $1000 \le p \le 1498$ (d) $999 \le p \le 1499$
(e) $1000 \le p \le 1499$

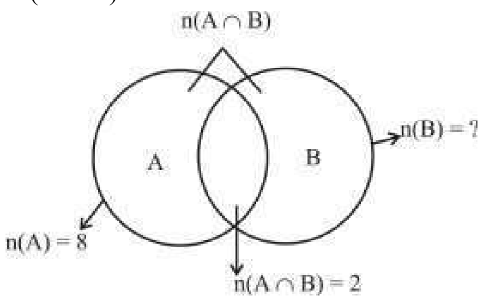
Kerala CEE-2012

Ans. (e) : Given, $n(A) = 1000$, $n(B) = 500$
and $n(A \cap B) \ge 1$ and $n(A \cup B) = p$
$\because n(A \cap B) = n(A) + n(B) - n(A \cup B)$
$= 1000 + 500 - p$
$= 1500 - p$
$\because n(A \cap B) \ge 1$
$1500 - p \ge 1$
$p \le 1499$ ... (i)
Also, $n(A \cup B) \ge n(A)$
$p \ge 1000$
$\because$ From equation (i) and (ii) we get
$1000 \le P \le 1499$

89. If n(A) = 8 and n(A ∩ B) = 2, then n((A ∩ B) ∩ A) is equal to
(a) 2 (b) 4
(c) 6 (d) 8
(e) 10

Kerala CEE-2011

Ans. (c) : Given $n(A) = 8$
$n(A \cap B) = 2$



As we know that $(A \cap B) \cap A$ can be written as

$= A - (A \cap B)$

$\therefore$ $n[(A \cap B) \cap A] = n(A) - n(A \cap B)$

$= 8 - 2 = 6$

90. There are 100 students in a class. In an examination, 50 of them failed in Mathematics, 45 failed in Physics, 40 failed in Biology and 32 failed in exactly two of the three subjects. Only one student passed in all the subjects. Then, the number of students failing in all the three subjects.

(a) is 12
(b) is 4
(c) is 2
(d) cannot be determined from the given information

WB JEE-2012

Ans. (c) : Given that,

$n(M) = 50$ $n(P) = 45$

$n(B) = 40$

Exactly 32 failed in two of the three subjects $n(M \cap P) + n(M \cap B) + n(P \cap B) - 3n(M \cap P \cap B) = 32$.

Number of student passed in all the three subject = 1 therefore, the number of student who failed

$n(M \cup P \cup B) = 99$

$n(M \cup P \cup B) = n(M) + n(P) + n(B) - n(M \cap P)$
$-n(M \cap B) - n(P \cap B) + n(M \cap P \cap B)$

$99 = 50+45+40-32-3n(M \cap P \cap B) + n(M \cap P \cap B)$

$99 = 103 - 2n(M \cap P \cap B)$

$2n(M \cap P \cap B) = 4$

$n(M \cap P \cap B) = 2$

91. In a class of 60 students, 25 students play cricket and 20 students play tennis and 10 students play both the games, then the number of students who play neither is

(a) 45 (b) 0
(c) 25 (d) 35

Karnataka CET 2014

Ans. (c) : Given, $n(C) = 25$

$n(T) = 20$

$n(C \cap T) = 10$

Where, C = Number of students play cricket

T = Number of students play tennis

Then, $P(C \cup T) = P(C) + P(T) - P(C \cap T)$

$P(C \cup T) = 25 + 20 - 10$

$P(C \cup T) = 45 - 10$

$P(C \cup T) = 35$

So, the number of student who play neither is –

$= 60 - 35$

$= 25.$

92. If U is the universal set with 100 elements; A and B are two sets such that n(A) = 50, n(B) = 60, n $(A \cap B) = 20$ then n $(A' \cap B')$ =

(a) 40 (b) 90
(c) 20 (d) 10

Karnataka CET 2019

Ans. (d) : Given,

$n(U) = 100$

$n(A) = 50, n(B) = 60$

$n(A \cap B) = 20$, then $n(A' \cap B') = ?$

We know that –

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$n(A \cup B) = 50 + 60 - 20$

$n(A \cup B) = 50 + 40$

$n(A \cup B) = 90$

Since, $n(A' \cap B') = n(A \cap B)' = n(U) - n(A \cup B)$

Hence, $n(A' \cap B') = n(U) - n(A \cup B)$

$n(A' \cap B') = 100 - 90$

$n(A' \cap B') = 10$

93. Suppose the number of elements in set A is p, the number of elements in set B is q and the number of elements in A × B is 7. Then $p^2 + q^2$ is equal to :

(a) 42
(b) 49
(c) 50
(d) 51
(e) 55

Kerala CEE-2006

Ans. (c) : It is given that,

$n(A) = p$, $n(B) = q$ and $n(A \times B) = 7$

$\Rightarrow$ $pq = 7$

$\therefore$ Only Possible valves are $p = 7$, $q = 1$

Or $p = 1$, $q = 7$

$\therefore$ $p^2 + q^2 = 49 + 1 = 50$

Hence, option (c) is correct.

C. Cartesian Product of Sets

94. Consider the matrix $f(x) \begin{matrix} \cos x & \sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{matrix}$

Given below are two statements:
Statement I: f(–x) is the inverse of the matrix f(x).
Statement II: f(x)f(y) = f(x+y)
In the light of the above statements, choose the correct answer from the options given below.
(a) Statement I is true but Statement II is false
(b) Both Statement I and Statement II are false
(c) Both Statement I and Statement II are true
(d) Statement I is false but Statement II is true

JEE MAIN-27.01.2024, Shift-I

Ans. (c) :

$$f(x) \begin{matrix} \cos x & \sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{matrix}$$

Statement I: $f(\ \ x) \begin{matrix} \cos x & \sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{matrix}$

$$f^{\ 1}(x) \begin{matrix} \cos x & \sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{matrix}^{T}$$

$$f^{\ 1}(x) \begin{matrix} \cos x & \sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{matrix} \quad f(\ \ x)$$

Statement-I is True

Statement II :

$$f(x)f(y) \begin{matrix} \cos x & \sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{matrix} \begin{matrix} \cos y & \sin y & 0 \\ \sin y & \cos y & 0 \\ 0 & 0 & 1 \end{matrix}$$

$$\begin{matrix} \cos(x\ \ y) & \sin(x\ \ y) & 0 \\ \sin(x\ \ y) & \cos(x\ \ y) & 0 \\ 0 & 0 & 1 \end{matrix} \quad f(x\ \ y)$$

Statement II is True

95. Let the number of elements in sets A and B be five and two respectively. Then the number of subsets of A × B each having at least 3 and at most 6 elements is:
(a) 752
(b) 772
(c) 782
(d) 792

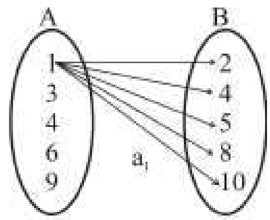
JEE MAIN-08.04.2023, Shift-I

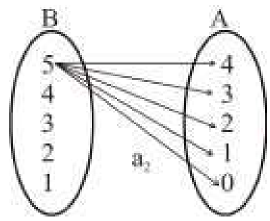
Ans. (d) : Given,
Number of elements in set A, n (A)= 5
For set n(B) = 2
n(A × B) = 10
No of ways of selection of r things out of n things $= {}^{n}C_{r}$
$= {}^{10}C_3 + {}^{10}C_4 + {}^{10}C_5 + {}^{10}C_6 = 792$

96. Let A = {1, 3, 4, 6, 9} and B = {2, 4, 5, 8, 10}. Let R be a relations defined on A×B such that R = {((a_1, b_1), (a_2,b_2)): $a_1 \le b_2$ and $b_1 \le a_2$}. Then the number of elements in the set R is
(a) 52
(b) 160
(c) 26
(d) 180

JEE MAIN-11.04.2023, Shift-II

Ans. (b) :



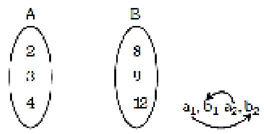


Total element = 5 ×5 = 25
Total Subset = 2^{25}
= 5(4+3+2+1+0) = 5 ×10 = 50
= 4× 10 = 40
= 4 ×10 = 40
= 2× 10 = 20
= 1×10 = 10
Total = 160

97. Let A = {2, 3, 4} and B = {8, 9, 12}. Then the number of elements in the relation R = {((a_1, b_1), (a_2, b_2))∈ (A × B, A × B) : a_1 divides b_2 and a_2 divides b_1} is
(a) 18
(b) 24
(c) 12
(d) 36

JEE MAIN-10.04.2023, Shift-II

Ans. (d) : Given,



For a_1 divides b_2, each elements has 2 choices
⇒ 3 × 2 = 6
Also, for a_2 divides b_1, each elements has 2 choices
⇒ 3 × 2 = 6
∴ Number of elements in R = 6 × 6 = 36

98. If two sets A and B have 99 elements in common, then the number of elements common to the sets A × B and B × A is

(a) 2^{99} (b) 99^2
(c) 100 (d) 18

COMEDK 2015 / UPSEE-2015
Kerala CEE-2004

Ans. (b) : Given,
$n(A \cap B) = 99$
Find, $n\{(A \times B) \cap (B \times A)\} = ?$
Then,
$n\{(A \times B) \cap (B \times A)\} = n\{(A \cap B) \times (B \cap A)\}$
$= n(A \cap B) \times n(B \cap A) = 99 \times 99 = 99^2$

99. If A and B be two sets such that A × B consists of 6 elements. If three elements A × B are (1, 4) (2, 6) and (3, 6), find B×A.

(a) $\{(1,4),(1,6),(2,4),(2,6),(3,4),(3,6)\}$
(b) $\{(4,1),(4,2),(4,3),(6,1),(6,2),(6,3)\}$
(c) $\{(4,4),(6,6)\}$
(d) $\{(4,1),(6,2).(6,3)\}$

VITEEE-2011

Ans. (b) : Given, A and B be two sets.
And (1,4) , (2,6) and (3,6) are the elements of A ×B
Then by ordered pair 1, 2, 3 are the elements of A and 4, 6 are the elements of B.
$\therefore$ A = {1, 2, 3}, B = {4, 6}
So, B×A = {(4, 1) , (4, 2) , (4, 3) ,(6, 1), (6, 2) , (6, 3)}

100. If A and B have n elements in common, then the number of elements common to A × B and B × A is

(a) 0 (b) n (c) 2n (d) n^2

Karnataka CET 2012

Ans. (d) : Given, A and B have n elements in common.
So, the number of elements common to A × B and B × A is
$= n \times n = n^2$

101. Let the number of elements in sets A and B five and two respectively. Then the number of subsets of A × B each having at least 3 and at most 6 element is:

(a) 792 (b) 752 (c) 782 (d) 772

JEE Main-08.04.2023, Shift-I

Ans. (a) : Number of element in set A = 5
And no. of element in set B = 2
The no. of element in ordered pair A × B = 2 × 5 = 10
n (A + B) = 10
Then, The number of subsets of A × B each having at least 3 and at most 6 elements is-
$= {}^{10}C_3 + {}^{10}C_4 + {}^{10}C_5 + {}^{10}C_6$
We know that,
$${}^{n}c_r = \frac{n!}{r!(n-r)!}$$
$$= \frac{10!}{3!\times7!} + \frac{10!}{4!\times6!} + \frac{10!}{5!\times5!} + \frac{10!}{6!\times4!}$$
$$= \frac{10\times9\times8\times7!}{3\times2\times1\times7!} + \frac{10\times9\times8\times7\times6!}{4\times3\times2\times1\times6!}$$
$$+ \frac{10\times9\times8\times7\times6\times5!}{5\times4\times3\times2\times1\times5!} + \frac{10\times9\times8\times7\times6!}{4\times3\times2\times1\times6!}$$
$= 120 + 210 + 252 + 210 = 792$

102. **If n(A) = 4, n(B) = 3, n(A × B × C) = 24, then n(C) is equal to :**

(a) 288 (b) 1
(c) 12 (d) 17
(e) 2

Kerala CEE-2005

Ans. (e) : Given, n(A) = 4 n(B) = 3
n(A × B × C) = 24
$\therefore$ n(A × B × C) = n(A) × n(B) × n(C)
24 = 4 × 3 × n(c)
$$n(c) = \frac{24}{4\times3} = 2$$

103. If A = {a, b, c}, B = {b, c, d} and C = {a, d, c}, then (A – B) × (B ∩ C) =

(a) {(a, c), (a, d)} (b) {(a, b), (c, d)}
(c) {(c, a), (d, a)} (d) {(a, c), (a, d), (b, d)}

Karnataka CET 2006

Ans. (a) : Given, A = { a, b, c} B = { b, c, d} and c = {a, d, c }
Then, A – B = {a, b, c} – {b, c, d}
A – B = {a}
and, B ∩ C = { b, c, d,} ∩ { a, d, c }
B ∩ C = { d, c}
So, (A – B) × (B ∩ C) = { a}× { d, c} = {a} × { c, d}.
(A – B) × (B ∩ C) = { (a, c), (a, d)}.

104. If n(A) = 5 and n(B) = 7, then the number of relations on A × B is

(a) 2^{35} (b) 2^{49}
(c) 2^{25} (d) 2^{70}
(e) $2^{35 \times 35}$

Kerala CEE-2012

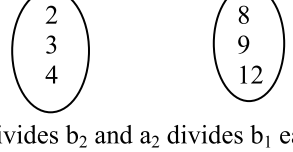
Ans. (e) : Given, n(A) = 5
n (B) = 7
$\therefore$ Number of relation on
$A \times B = 2^{[n(A)\times n(B)]}$
$A \times B = 2^{[n(A)\times n(B)]^2}$
$2^{(5\times7)^2} = 2^{35\times35}$

105. **Let A = {2, 3, 4} and B = {8, 9, 12}. Then the number of elements in the relation R = {((a_1, b_1), (a_2, b_2)) ∈ (A × B, A × B) : a_1 divides b_2 and a_2 divides b_1} is :**

(a) 36 (b) 12
(c) 18 (d) 24

JEE Main-10.04.2023, Shift-II

Ans. (a) : Let A = { 2, 3, 4}
And, B = {8, 9, 12}



a_1 divides b_2 and a_2 divides b_1 each element has 2 choice
3 × 2 = 6 and 3 × 2 = 6
Now total number of elements = 6 × 6 = 36 .

D. Relations and Type of Relation

106. Let A = {1, 2, 3, 4, 5}. Let R be a relation on A defined by xRy if and only if $4x \le 5y$. Let m be the number of element in R and n be the minimum number of elements from A × A that are required to be added to R to make it a symmetric relation. Then m + n is equal to:

(a) 24 (b) 23
(c) 25 (d) 26

JEE MAIN-06.04.2024, Shift-II

Ans. (c) : Given: $4x \le 5y$
if x = 1
So, 4 < 5y i.e (1, 1), (1, 2), (1, 3), (1, 4) ,(1, 5)
x = 2, 8 < 5y i.e (2, 2), (2, 3),(2,4), (2, 5)
x = 3, 12 < 5y i.e. (3, 3), (3, 4), (3, 5)
x = 4, 16 < 5y i.e (4, 4), (4, 5)
x = 5, 20 < 5y i.e (5, 4), (5, 5)
Then
R = {(1,1) , (1, 2), (1, 3), (1, 4) ,(1, 5), (2, 2), (2, 3),(2,4) (2, 5), (3, 3), (3, 4), (3, 5), (4, 4), (4, 5), (5, 4), (5, 5)}
i.e. 16 elements.
i.e. m = 16
Now to make R a symmetric relation add
{(2, 1) (3, 2)(4, 3)(3, 1)(4, 2)(5, 3)(4, 1)(5,2)(5,1)}
i.e. n = 9
So m + n = 25

107. Let the relations R_1 and R_2 on the set X = {1, 2, 3, ..., 20} be given by $R_1 = \{(x, y) : 2x - 3y = 2\}$ and $R_2 = \{(x, y) : -5x + 4y = 0\}$. If M and N be the minimum number of elements required to be added in R_1 and R_2, respectively, in order to make the relations symmetric, then M + N equals

(a) 8 (b) 16
(c) 12 (d) 10

JEE MAIN-06.04.2024, Shift-I

Ans. (d) : x = {1, 2, 3, ...20}
$R_1 = \{(x, y) : 2x - 3y = 2\}$
$R_2 = \{(x, y) : -5x + 4y = 0\}$
R_1 ={(4, 2), (7, 4), (10, 6), (13, 8), (16, 10), (19, 12)}
For symmetry
={(2, 4), (4, 7), (6,10), (8, 13), (10, 16), (12, 19)}
R_2 ={(4, 5), (8, 10), (12, 15), (16, 20)}
For symmetry
R_2 ={(5,4), (10, 8), (15,12), (20, 16)}
in R_1 6 element needed i. e. M = 6
in R_2 4 element needed i. e. N = 4
So, the value of M + N = 6 + 4 = 10 element

108. Let a relation R on N×N be defined as:
(x_1, y_1) $R(x_2, y_2)$ if and only if $x_1 \le x_2$ or $y_1 \le y_2$
Consider the two statements:
(I) R is reflexive but not symmetric.
(II) R is transitive
Then which one of the following is true?

(a) Only (II) is correct
(b) Only (I) is correct
(c) Both (I) and (II) are correct
(d) Neither (I) nor (II) is correct

JEE MAIN-04.04.2024, Shift-II

Ans. (b) : All $((x_1, y_1), (x_1, y_1))$ are is R where
$x_1, y_1 \in N$ $\therefore$ R is reflexive
$((1, 1), (2, 3)) \in R$ but $((2, 3), (1, 1)) \notin R$
$\therefore$ R is not symmetric
$((2,4), (3,3))\in R$ and $((3,3), (1,3))\in R$ but $((2,4), (1,3))\notin R$
$\therefore$ R is not transitive

109. Let A = {2, 3, 6, 7} and B = {4, 5, 6, 8}. Let R be a relation defined on A × B by (a_1, b_1) R (a_2, b_2) such that $a_1 + a_2 = b_1 + b_2$. Then the number of element in R is _________.

JEE MAIN-09.04.2024, Shift-I

Ans. (25) : A = {2, 3, 6, 7}
B = {4, 5, 6, 8}
(a_1, b_1) R (a_2, b_2)
$a_1 + a_2 = b_1 + b_2$

1.	(2, 4) R (6, 4)	2. (2, 4) R (7, 5)	
3.	(2, 5) R (7, 4)	4. (3, 4) R (6, 5)	
5.	(3, 5) R (6, 4)	6. (3, 5) R (7, 5)	
7.	(3, 6) R (7, 4)	8. (3, 4) R (7, 6)	× 2
9.	(6, 5) R (7, 8)	10. (6, 8) R (7, 5)	
11.	(7, 8) R (7, 6)	12. (6, 8) R (6, 4)	
13.	(6, 6) R (6, 6)		

Total 24 + 1 = 25

110. Let A = {2, 3, 6, 8, 9, 11} and B = {1, 4, 5, 10, 15} Let R be a relation on A × B define by (a, b) R (c, d) if and only if 3ad – 7bc is an even integer. Then the relation R is

(a) reflexive but not symmetric.
(b) transitive but not symmetric.
(c) reflexive and symmetric but not transitive.
(d) an equivalence relation.

JEE MAIN-08.04.2024, Shift-II

Ans. (c) : A = {2, 3, 6, 8, 9, 11)
B = {1, 4, 5, 10, 15}
R is defined as (a, b) R (c, d) such that 3ad – 7bc is an even integer.
Reflexive : (a, b) R (a, b)
$\Rightarrow$ 3ab – 7ba = –4ab always even so it is reflexive.
Symmetric : If 3ad – 7bc = Even

Case- I : odd no. odd no.
Case-II : even no. even no.
(c, d) R (a, b) $\Rightarrow$ 3bc – 3ab
Case-I : odd no. odd no.
Case-II : even no. even no.
so it has symmetric relation on R
Transitive :
(3, 1) R (6, 4)
$\Rightarrow$ 12 – 6 = 6, which is an even integer, satisfying the above relation
(6, 4) R (3, 1)
$\Rightarrow$ 6 – 12 = –6, which is an even integer, satisfying the above relation
but (3, 4) R (3, 1) does not satisfy relation
so it is not transitive.

111. The number of 3-digit numbers, formed using the digits 2, 3, 4, 5 and 7, when the repetition of digits is not allowed, and which are not divisible by 3, is equal to _______.

JEE MAIN-08.04.2024, Shift-I

Ans. (36) : 2, 3, 4, 5, 7
total number of three digit numbers not divisible by 3 will be formed by using the digits
(4, 5, 7)
(3, 4, 7)
(2, 5, 7)
(2, 4, 7)
(2, 4, 5)
(2, 3, 5)
number of ways = 6 × 3! = 36

112. Consider the relations R_1 and R_2 defined as
$aR_1b \quad a^2 \quad b^2 \quad 1$ for all a, b $\in$ R and
$(a,b)R_2(c,d) \quad a \quad d \quad b \quad c$ for all
$(a,b),(c,d) \quad N \quad N$. Then
(a) R_1 and R_2 both are equivalence relations
(b) Only R_1 is an equivalence relation
(c) Only R_2 is an equivalence relation
(d) Neither R_1 nor R_2 is an equivalence relation

JEE MAIN-01.02.2024, Shift-II

Ans. (c) :
$a R_1 b \Leftrightarrow a^2 + b^2 = 1 \quad a, b \in R$
For Reflexive-
$a R_1 b \Leftrightarrow a^2 + a^2 = 1$
Which is not true $\forall$ a $\in$ R.
Hence R_1 is not reflexive.
Therefore, R_1 is not equivalence relation.
(a, b) R_2 (c, d) $\Rightarrow$ a + d = b + c
For reflexive:-
(a, b) R_2 (a, b) = a + b = b + a
It's true $\forall$ (a , b) $\in$ N × N
Hence, R_2 is reflexive.
For symmetric-
(a, b), (c, d) $\in$ N × N
(a, b) R_2 (c, d) = a + d = b + c
(c, d) R_2 (a, b) = c + d = d + a
$\therefore$ a + b = b + c
(a, b) R_2 (c, d) $\Rightarrow$ (c, d) R_2 (a, b) $\forall$ (a, b), (c, d) $\in$N × N
Hence R_2 is symmetric.
For transitive:-
(a, b), (c, d), (e, f) $\in$ N × N
(a, b) R_2 (c, d) $\Rightarrow$ a + d = b + c
(c, d) R_2 (e, f) $\Rightarrow$ c + f = d + e
$\therefore$ a + b + c + f = b + d + c + e
a + f = b + c
(a, b) R_2 (e, f)
Hence, R_2 is transitive.
Therefore, R_2 is equivalence relation.

113. Let A = {1,2,3,….,20}. Let R_1 and R_2 be the two relation on A such that
$R_1 \quad \{(a,b) : \text{b is divisible by a}\}$
$R_2 \quad \{(a,b) : \text{a is an integral multiple of b}\}$.
Then, number of elements in $R_1 - R_2$ is equal to_______.

JEE MAIN-01.02.2024, Shift-I

Ans. (46) : We have,
A = {1, 2, 3,….20}
R_1 = {(a, b) : b is divisible by a}
R_1 = {(1,1), (1,2)…(1, 20), (2,2), (2,4)…(2,20)
(3,3), (3,6) ….(3, 18), (4,4), (4,8)….(4,20)
(5,5), (5,10) (5,15), (5,20), (6,6), (6,12), (6,18)
(7,7), (7,14), (8,8), (8,16), (9,9), (9,18), (10,10)
(10, 20), (11, 11), (12, 12)……. (20, 20)}
$n(R_1) = 20+10+6+5+4+3+2+2+2+2+1+....+1$
$n(R_1) = 66$
$\because n(R_1 - R_2) = n(R_1) - n(R_1 \cap R_2)$
And n ($R_1 \cap R_2$) = {(1,1), (2,2) (3,3) …. (20,20)}=20
n($R_1 - R_2$) = 66 – 20 = 46

114. If R is the smallest equivalence relation on the set {1, 2, 3, 4} such that $\{(1, 2),(1, 3)\} \subset$ R, then the number of elements in R is _____
(a) 15 (b) 8
(c) 12 (d) 10

JEE MAIN-29.01.2024, Shift-II

Ans. (d) : Given,
set {1, 2, 3, 4}
Smallest equivalence relation ={(1, 1), (2, 2), (3, 3), (4, 4), (3, 1),(2, 1), (2, 3),(3,2), (1, 2), (1, 3)}
Thus, no. of elements = 10

115. Let R be a relation on Z × Z defined by (a, b) R (c, d) if and only if ad–bc is divisible by 5. Then R is :

(a) Reflexive and transitive but not symmetric
(b) Reflexive, symmetric and transitive
(c) Reflexive and symmetric but not transitive
(d) Reflexive but neither symmetric not transitive

JEE MAIN-29.01.2024, Shift-I

Ans. (c) : For reflexive:

(a, b) R (a, b)

$\Rightarrow$ ab – ab = 0 is divisible by 5

So, (a, b) R (a,b) $\forall$ ab $\in$ Z

$\therefore$ R is reflexive relation.

For symmetric:

(a, b) R (b, c)

If $ac - b^2$ is divisible by 5

Then, $-(b^2-ac)$ is also divisible by 5.

$\Rightarrow$ (b, c) R (a, b) $\forall$ a, b, c, d $\in$ Z

$\therefore$ R is symmetric relation on R.

For transitive:

If (a, b) R (c, d)

$\Rightarrow$ ad – bc divisible by 5 and (c, d) R (e, f)

$\Rightarrow$ cf – de divisible by 5

$ad - bc = 5k_1 \quad \because k_1, k_2 \in Z$

$cf - de = 5k_2$

$\therefore \quad afd - bcf = 5k_1 f$

$bcf - bde = 5 k_2 b$

$afd - bde = 5(k_1 f + k_2 b)$

$d(af - be) = 5(k_1 f + k_2 b)$

af – be is not divisible by 5 for every a. b, c, d, e, f $\in$ Z.

$\therefore$ R is not transitive.

Thus R is reflexive and symmetric but not transitive.

Hence, option (c) is correct.

116. Let S = {1,2,3,…,10}. Suppose M is the set of all the subsets of S, then the relation R {(A,B) : A B ;A,B M}is:

(a) reflexive only
(b) symmetric and reflexive only
(c) symmetric and transitive only
(d) symmetric only

JEE MAIN-27.01.2024, Shift-I

Ans. (d) : Let S = {1, 2, 3, ... 10}

R = {(A, B) : A $\cap$B $\neq \phi$; A, B $\in$M}

For reflexive-

m is subset of 'S'

So, $\phi \in$ m

for $\phi \cap \phi = \phi$

but relation is A $\cap$ B $\neq \phi$

So it is not reflexive.

For symmetric,

ARB = A $\cap$ B $\neq \phi$

= BRA = A $\cap$ B $\neq \phi$

So it is symmetric

For transitive

if A = {(1, 2) (2, 3)}

B = {(2, 3) (3, 4)}

C = {(3, 4) (5, 6)}

ARB and BRC but A does not relate to C so it not transitive.

117. The number of symmetric relations defined on the set {1, 2, 3, 4} which are not reflexive is ______.

JEE MAIN-30.01.2024, Shift-II

Ans. : (960) We know that,

Total number of relation which reflexive and symmetric both = $2^{\frac{n^2-n}{2}}$

Total number of relation which symmetric = $2^{\frac{n^2+n}{2}}$

Number of relation which are not reflexive

$= 2^{\frac{n^2+n}{2}} - 2^{\frac{n^2-n}{2}}$

$\therefore \quad n = 4$

$= 2^{\frac{16+4}{2}} - 2^{\frac{16-4}{2}}$

$= 2^{10} - 2^6$

$= 2^6(16-1)$

$= 64 \times 15 = 960$

118. Let A = {1, 2, 3, 4} and R = {(1, 2)(2, 3)(1, 4)} be a relation on A. Let S be the equivalence relation on A such the R $\subset$ S and the number of elements in S is n. Then, the minimum value of n is ________.

JEE MAIN-31.01.2024, Shift-I

Ans. (16) : Given,

A = {1, 2, 3, 4}

R = {(1, 2)(2, 3)(1, 4)}

S is equivalence relation, relation must be reflexive, symmetric & transitive.

For Reflexive,

{(1, 1),(2, 2),(3, 3),(4 ,4)}

For Symmetric,

{(2, 1),(4, 1),(3, 2)}

For transitive,

{(1, 3),(3, 1),(4, 2),(2, 4)}

S = {(1, 1)(2, 2)(3, 3)(4, 4)(1, 2)(2, 1)(2, 3)(3, 2)(1, 4)(4, 1)(1, 3)(3, 1)(2, 4)(4, 2)(4, 3)(3, 4)}

All elements are included,

$\therefore$ The number of elements are 16

119. Let A = {1,2,3…….100}. Let R be a relation on A defined by (x,y)$\in$R if and only if 2x = 3y. Let R_1 be a symmetric relation on A such that R R_1 and the number of elements in R_1 is n. Then, the minimum value of n is_______.

JEE MAIN-31.01.2024, Shift-II

Ans. :(66) Given,

A $\{1,2,...100\}$

And R= $\{(x, y): 2x= 3y\}$

$\Rightarrow R \quad \{(3,2),(6,4),(9,6)....(99,96)\}$

$\Rightarrow n(R) \quad 33$

$\because$ R $\quad R_1$ and R_1 be a symmetric relation on A i.e

R_1 contains (y,x) such that 2y = 3x

i.e, $\quad R_1 \quad \{(3,2),(6,4),(9,6).....(99,66),$

$(2,3),(4,6),(6,9)......(66,99)\}$

$\Rightarrow$minimum number of elements in R_1 = 66

120. Let A = {0, 3, 4, 6, 7, 8, 9, 10} and R be the relation defined on A such that R = {x, y) ∈ A × A : x – y is odd positive integer or x – y = 2}. The minimum number of elements that must be added to the relation R, so that it is a symmetric relation, is equal to ____.

JEE MAIN-08.04.2023, Shift-I

Ans. (19) : Given,

A = {0, 3, 4, 6, 7, 8, 9, 10}

R = {x–y = odd positive integer x –y = 2}

Here, 3,7,9, are odd number i.e. 3_{C_1}

0,4,6,8,10 are even number so 5_{C_1}

minimum order pair to be added must be = 15 + 4 = 19

R ={(6, 4),(8, 6),(9, 7),(10, 8),(3, 0), (7, 0), (9, 0), (4, 3), (6, 3), (8, 3), (10, 3), (7, 4), (9, 4), (7, 6), (9, 6), (8, 7), (10, 7), (9, 8), (10, 9)}

Hence 19 element should be add in R for making its.

121. Let A = $\{-4,-3,-2,0,1,3,4\}$ and R = $\{(a,b) \in A\times A : b = |a| \text{or } b^2 = a+1\}$ be a relation on A. Then the minimum number of elements, that must be added to relation R so that it becomes reflexive and symmetric, is ______.

JEE MAIN-13.04.2023, Shift-II

Ans. (7) : A = $\{-4,-3,-2,0,1,3,4\}$ and R = $\{(a,b) \in A\times A : b = |a| \text{or } b^2 = a+1\}$

$R = [(-4,4),(-3,3),(3,-2),(0,1),(0,0),(1,1),(4,4),(3,3)]$

For reflexive, add $\Rightarrow (-2,-2),(-4,-4),(-3,-3)$

For symmetric, add $\Rightarrow (4,-4),(3,-3),(-2,3),(1,0)$

Total = 3 + 4 = 7

122. Let A = {1, 2, 3, 4, 5, 6, 7}. Then the relation R = {(x, y) ∈A × A : x + y = 7} is

(a) Symmetric but neither reflexive nor transitive
(b) Transitive but neither symmetric nor reflexive
(c) An equivalence relation
(d) Reflexive but neither symmetric nor transitive

JEE MAIN-08.04.2023, Shift-II

Ans. (a) : Sol.

A = {1, 2, 3, 4, 5, 6, 7}

R = {(x, y) ∈ A×A : x + y = 7}

y = 7 – x

R = {(1, 6) (2, 5) (3, 4) (4, 3) (5, 2) (6, 1)}

Symmetric but neither reflexive nor transitive

R = {(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)}

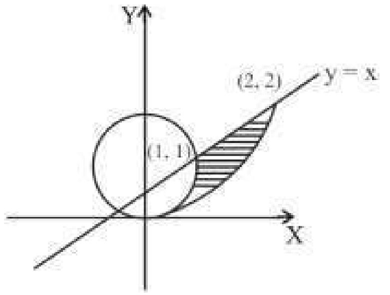
123. Let A = {1, 2, 3, 4, ….., 10} and B = {0, 1, 2, 3, 4}. The number of elements in the relation R = {(a, b) ∈ A × A : 2 (a – b)^2 + 3 (a – b) ∈ B} is ___.

JEE MAIN-06.04.2023, Shift-I

Ans. (18) : Given,

A = {1, 2, 3, 4,.............10}

and B = {0, 1, 2, 3, 4}



R = {(a, b) ∈ A × A : 2 (a – b)^2 + 3(a – b) ∈ B}

Now $\quad 2 (a – b)^2 + 3 (a – b) = (a – b) (2 (a – b) + 3)$

$\Rightarrow$ a = b or a – b = –2∈ B

When a = b ⇒ 10 order pairs

Number of order pair, $\quad$ a – b = – 2 ⇒ 8 order pairs

Number of total elements = 18

124. Let A = {1, 2, 3, 4} and R be a relation on the set A × A defined by R = {((a, b, (c, d):2a + 3b = 4c + 5d}. Then the number of elements in R is ______

JEE MAIN-15.04.2023, Shift-I

Ans. (6) : Given,

A = {1, 2, 3, 4}

R = {(a, b), (c, d)}

2a + 3b = 4c + 5d = α (let)

2a = {2, 4, 6, 8} $\qquad$ 4c = {4, 8, 12, 16}

3b = {3, 6, 9, 12} $\qquad$ 5d = {5, 10, 15, 20}

$$2a+3b=\begin{Bmatrix}5,8,11,14\\7,10,13,16\\9,12,15,18\\11,14,17,20\end{Bmatrix}=4c+5d=\begin{Bmatrix}9,14,19,24\\13,18....\\17,22....\\21,26....\end{Bmatrix}$$

Possible value of α = 9, 13, 14, 14, 17, 18

Pairs of {(a, b), (c, d)} = 6

125. The number of the relations, on the set {1, 2, 3} containing (1, 2) and (2, 3) which are reflexive and transitive but not symmetric, is______.

JEE MAIN-12.04.2023, Shift-I

Ans. (3) :

A = {1, 2, 3}

For Reflexive (1, 1) (2, 2) (3, 3) $\in$R

For transitive : (1, 2) and (2, 3) $\in$R $\Rightarrow$ (1, 3) $\in$R

Not symmetric : (2, 1) and (3, 2) $\notin$ R

R_1 = {(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3)}

R_2 = {(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3), (2, 1)}

R_3 = {(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3), (2, 1)}

126. The relation R = {(a, b): gcd (a, b) = 1, 2a $\neq$ b, a, b $\in$ Z } is :

(a) reflexive but not symmetric
(b) neither symmetric nor transitive
(c) symmetric but not transitive
(d) transitive but not reflexive

JEE MAIN-24.01.2023, Shift-I

Ans. (b) : Given that

R = {(a,b) : gcd (a,b) = 1, 2a $\neq$ b, a b $\in$ z }

• For reflexive relation :

(a,a) $\Rightarrow$ gcd (a,a) = 1

Which is true for every a $\in$ z.

$\Rightarrow$ For symmetric relation:

Taking a = 2 , b = 1 $\Rightarrow$ gcd (2,1) = 1

Also, 2a = 4 $\neq$b

Now, when a = 1, b = 2$\Rightarrow$ gcd (1,2) = 1

Also, now 2a = 2 = b

Hence, a = 2b

$\Rightarrow$ R is not symmetric.

• For transitive relation:

Let a = 14, b = 19 , c = 21

gcd (a,b) = 1

gcd (b,c) = 1

gcd (a,c) = 7

Hence, R is not transitive.

Therefore, R is neither Symmetric nor transitive.

Thus, option (b) is correct answer.

127. The minimum number of elements that must be added to the relation R = {(a, b), (b, c), (b, d)} on the set {a, b, c, d} so that it is an equivalence relation, is _______.

JEE MAIN-24.01.2023, Shift-II

Ans. (13) : Set = {a, b, c, d}

R = {(a, b), (b, c), (b, d)}

To make the given relation R as an equivalence relation-

Reflexive $\rightarrow$ (a, a), (b, b), (c, c), (d, d)

Symmetric $\rightarrow$ (a, b) $\in$R

$\Rightarrow$ (b, a) $\in$R

$(a,b)\ (b,c)\ (b,d)$

$(b,a)\ (c,b)\ (d,b)$

Transitive$\rightarrow$ (a, b) and (b, c) $\in$ R

(a, c), (a, d), (c, d), (d, c), (d, a), (c, a)

n = 4

set (A) = n^2

set (A) = 4^2

set A = 16

So, 13 elements more to be added to make an equivalence relation.

128. Let R be a relation on N×N defined by (a, b) R (c, d) if and only if ad (b–c) = bc (a–d). Then R is

(a) transitive but neither reflexive nor symmetric
(b) symmetric but neither reflexive nor transitive
(c) symmetric and transitive but not reflexive
(d) reflexive and symmetric but not transitive

JEE MAIN-31.01.2023, Shift-I

Ans. (b) : (a, b) R (c, d) $\Rightarrow$ ad (b – c) = bc (a–d)

Symmetric :

(c, d) R (a, b) $\Rightarrow$ cb (d–a) = da (c– b)

$\Rightarrow \quad bc = (a-b) = ad(b-c)$

Reflexive :

(a, b) R (a, b) $\Rightarrow$ ab (b – a) $\neq$ ba (a – b) $\Rightarrow$

Not reflexive

Transitive : (2, 3) R (3, 2) and (3, 2) R (5, 30) but

((2, 3), (5, 30) $\notin$ R $\Rightarrow$ Not transitive

129. For $\alpha, \beta \in \mathbb{R}$, suppose the system of linear equations

$x - y + z = 5$

$2x + 2y + \alpha z = 8$

$3x - y + 4z = \beta$

has infinitely many solutions. Then α and β are the roots of

(a) $x^2 + 18x + 56 = 0$
(b) $x^2 - 10x + 16 = 0$
(c) $x^2 - 18x + 56 = 0$
(d) $x^2 + 14x + 24 = 0$

JEE MAIN-30.01.2023, Shift-II

Ans. (c) : Given linear equation-

$x - y + z = 5$

$2x + 2y + \alpha z = 8$

$3x - y + 4z = \beta$

for infinite solution-

$$\begin{vmatrix} 1 & -1 & 1 & 5 \\ 2 & 2 & \alpha & 8 \\ 3 & -1 & 4 & \beta \end{vmatrix}$$

$R_2 \rightarrow R_2 - 2R_1, R_3 \rightarrow R_3 - 3R_1$

$$\begin{vmatrix} 1 & -1 & 1 & 5 \\ 0 & 4 & \alpha-2 & -2 \\ 0 & 2 & 1 & \beta-15 \end{vmatrix}$$

$R_3 \to 2R_3 - R_2$

$$\begin{vmatrix} 1 & -1 & 1 & 5 \\ 0 & 4 & \alpha-2 & -2 \\ 0 & 0 & 2-\alpha+2 & 2\beta-30+2 \end{vmatrix}$$

Now, for infinite solution

$2 - \alpha + 2 = 0$ & $2\beta - 30 + 2 = 0$

$\alpha = 4$ & $\beta = 14$

Equation having roots α & β is $x^2 - (\alpha + \beta) x + \alpha\beta$

$x^2 - (4 + 14)x + 4 \times 14 = 0$

$x^2 - 18x + 56 = 0$

130. The minimum number of elements that must be added to the relation R = {(a, b), (b, c)} on the set {a, b, c} so that it becomes symmetric and transitive is:

(a) 7
(b) 3
(c) 5
(d) 4

JEE MAIN-30.01.2023, Shift-I

Ans. (a) : Given relation R = {(a, b), (b, c)}

For symmetric (a, b), (b, c) $\in$R

$\Rightarrow$ (b, a), (c, b) $\in$R

For transitive, (a, b), (b, c) $\in$R

(a, c) $\in$R

Now, For symmetric– $\because$ (a, c) $\in$R $\Rightarrow$ (c, a) $\in$R

And, For transitive– $\because$ (a, b), (b, a) $\in$R

$\Rightarrow$ (a, a) $\in$R

And, (b, c), (c, b) $\in$R

$\Rightarrow$ (b, b) & (c, c) $\in$R

Therefore, elements to be added

{(b, a), (c, b) (a, c) (c, a), (a, a) (b, b), (c, c)}

$\therefore$ Number of elements to be added = 7

131. Let R be a relation defined on $\mathbb{N}$ as a R b if 2s + 3b is a multiple of 5, a, b $\in$ N. Then R is

(a) transitive but not symmetric
(b) an equivalence relation
(c) not reflexive
(d) symmetric but not transitive

JEE MAIN-29.01.2023, Shift-II

Ans. (b) : For (a,a) $\Rightarrow$ 2a+3b

= 2a+3a = 5a which is divisible by 5

So, (a,a) $\in$ R , a$\in$ N reflexive

Let (a,b) $\in$ R $\Rightarrow$ 2a + 3b = $5k_1$

and 5a + 5b = $5k_2$

then,

5a + 5b –2a– 3b = 5 (k_2–k_1)

2b + 3a : 5k

(b,a) $\in$ R is symmetric

Let (a,b) and (b, c) both $\in$ R

2a +3b = $5k_1$

2b + 3c = $5k_2$

then , 2a + 3b + 3c = 5(k_1+ k_2)

2a + 3c = 5k – 5b

(a,c) $\in$ R for transitive

So, it is equivalence relation-

132. Among the relations

$$\mathbf{S = \{(a,b) : a, b \in R - \{0\}, 2 + \frac{a}{b} > 0\}}$$

$$\mathbf{\text{and } T = \{(a,b) : a, b \in R,\ a^2 - b^2 \in Z\},}$$

(a) Both S and T are symmetric
(b) S is transitive but T is not
(c) Neither S nor T is transitive
(d) T is symmetric but S is not

JEE MAIN-31.01.2023, Shift-II

Ans. (d) : From 2nd relation T = $a^2 - b^2 = -$ I a, b $\in$R

Then, (b, a) on relation T

$\Rightarrow b^2 - a^2 = -$ I b, a $\in$R

$\therefore$ T is symmetric

Now, from equation first

$$S = \left\{(a,b) : a, b \in R - \{0\}, 2 + \frac{a}{b} > 0\right\}$$

$$2 + \frac{a}{b} > 0$$

$$\Rightarrow \frac{a}{b} > -2,$$

$$\Rightarrow \frac{b}{a} < \frac{-1}{2}$$

If (b, a) one relation S then

$2 + \frac{b}{a}$ not necessarily positive

$\therefore$ S is not symmetric

133. Let f : R – {0, 1} $\to$ R be a function such that $\mathbf{f(x) + f\left(\frac{1}{1-x}\right) = 1 + x}$. Then f (2) is equal to

(a) $\frac{7}{3}$

(b) $\frac{7}{4}$

(c) $\frac{9}{2}$

(d) $\frac{9}{4}$

JEE MAIN-01.02.2023, Shift-II

Ans. (d) : f : R – {0, 1} $\to$ R

$$f(x) + f\left(\frac{1}{1-x}\right) = 1 + x$$

Put x = 2,

f(2) + f(–1) = 3 ...(i)

Put x = –1,

$f(-1) + f\left(\frac{1}{2}\right) = 0$ ...(ii)

Put x = 1/2

$f\left(\frac{1}{2}\right) + f(2) = \frac{3}{2}$ ...(iii)

Subtracting equation (i) and (ii), we get -

$f(2) + f(-1) - f(-1) - f(-1) - f\left(\frac{1}{2}\right) = 3$

$f(2) - f\left(\frac{1}{2}\right) = 3$ ...(iv)

On adding equation (iii) and (iv), we get-

$f(2) - f\left(\frac{1}{2}\right) + f(2) + f\left(\frac{1}{2}\right) = 3 + \frac{3}{2}$

$2f(2) = \frac{9}{2}$

$f(2) = \frac{9}{4}$

134. Let P(S) denote the power set S = {1, 2, 3, ..., 10} . Define the relations R_1 and R_2 on P(S) as AR_1B if $(A \cap B^c) \cup (B \cap A^c) = \phi$ and AR_2B if $A \cup B^c = A^c \cup B, \forall A, B \in P(S)$. Then:

(a) Only R_1 is an equivalence relation
(b) Both R_1 and R_2 are not equivalence relations
(c) both R_1 and R_2 are equivalence relations
(d) only R_2 is an equivalence relation

JEE MAIN-01.02.2023, Shift-II

Ans. (c) : Given,

$S = \{1, 2, 3.........10\}, n = 10$

Total number of element in P(S) = 2^{10}

$AR_1B \text{ is defined as} : (A \cap B^c) \cup (B \cap A^c) = \phi$

$\Rightarrow A \cap B^c = \phi \text{ and } B \cap A^c = \phi$

$\Rightarrow A = B.$

Thus AR_1B is an equivalence relation.

and AR_2B is defined as $A \cup B^c = B \cup A^c \forall A, B \in P(S)$

$\Rightarrow A = B.$

Thus AR_2B is an equivalence relation.

So, both of them have an equivalence relation on S.

135. Let R be a relation of ℝ, given by $R = \{(a,b) : 3a - 3b + \sqrt{7} \text{ is an irrational number}\}$. Then R is

(a) reflexive and transitive but not symmetric.
(b) an equivalence relation
(c) reflexive but neither symmetric nor transitive
(d) reflexive and symmetric but not transitive

JEE MAIN- 01.02.2023, Shift-I

Ans. (c) :

$R = \{(a,b) : 3a - 3b + \sqrt{7} \text{ is an irrational number}\}$

Reflexive - let (a, a)∈R

$\Rightarrow \quad 3a - 3a + \sqrt{7} = \sqrt{7}$

(a, a) : $\sqrt{7}$ ∈R is an irrational number and it is Reflexive over R.

for symmetric-

Let $\left(\frac{\sqrt{7}}{3}, 0\right) \in R$

$\Rightarrow \quad 3 \times \frac{\sqrt{7}}{3} - 3 \times 0 + \sqrt{7} = 2\sqrt{7} \in Q^c$,i.e. $2\sqrt{7}$ is an irrational no.

but for $\left(0, \frac{\sqrt{7}}{3}\right)$

$3(0) - 3 \times \frac{\sqrt{7}}{3} + \sqrt{7} = 0 \notin Q^C$,i.e. not an irrational no.

$\Rightarrow \left(0, \frac{\sqrt{7}}{3}\right) \notin R$

∴ R is not symmetric.

For transitive -

Let (0, 3) ∈ R and $\left(3, \frac{\sqrt{7}}{3}\right) \in R$

but $\left(0, \frac{\sqrt{7}}{3}\right) \notin R$

So, R is not transitive .

136. The negation of the expression $q \vee ((\sim q) \wedge p)$ is equivalent to

(a) $(\sim p) \vee q$ (b) $(\sim p) \wedge (\sim q)$
(c) $(\sim p) \vee (\sim q)$ (d) $p \wedge (\sim q)$

JEE MAIN- 01.02.2023, Shift-I

Ans. (b) : Given,

$q \vee (\sim q \wedge p)$

$\sim q \wedge (q \wedge \sim p)$

$(\sim q \wedge q) \vee (\sim q \wedge \sim p)$

$= F \vee (\sim q \wedge \sim p) = (\sim p) \wedge (\sim q)$

137. A set A contains 10 elements, then the number of relations on A into A is

(a) 2^{10} (b) 10^2
(c) 2^{100} (d) 2^{1000}

SRM JEEE 2018

Ans. (c) : Given,

Set A contain 10 elements.

We know that, A set contains n elements then the number of relations on set into set is 2^{n^2} .

So, then the number of relations A into A is-

$2^{10^2} = 2^{100}$

138. Let A = {1, 2, 3, 4,} and let R = {(2, 2) (3, 3), (4, 4), (1, 2)} be a relation on A, then R is
(a) reflexive (b) symmetric
(c) transitive (d) equivalence relation

SRMJEEE-2013

Ans. (c) : Given, A = {1, 2, 3, 4}
R = { (2, 2), (3, 3), (4, 4), (1, 2)}
From question, we see that in relation R, $(1, 1) \notin R$,
Then, R is not reflexive.
And, $(1, 2) \in R$ but $(2, 1) \notin R$
Then, R is not symmetric.
But it is transitive because –
$(1, 2) \in R, (2, 2) \in R \Rightarrow (1, 2) \in R$
So, the R is only transitive relation.

139. If R is a relation on the set A = {1, 2, 3, 4, 5, 6, 7, 8, 9} given by xRy $\Leftrightarrow$ y = 3x, then R =
(a) {(3, 1,), (6, 2), (8, 2), (9, 3)}
(b) {(3, 1), (6, 2), (9, 3)}
(c) {(3,1), (2, 6), (3, 9)}
(d) {(1, 3), (2, 6), (3, 9)}

SRMJEEE-2011

Ans. (d) : Given,
R is a relation on the set A.
A = {1, 2, 3, 4, 5, 6, 7, 8, 9}
and xRy $\Leftrightarrow$ y = 3x
Since, R = { (x, y)}
For x = 1, y = 3
For x = 2, y = 6
For x = 3, y = 9
So, R = { (x, 3x)} = { (1, 3), (2, 6), (3, 9)}

140. If A = {a, b, c, d} then a relation R = {(a, b), (b, a), (a, a)} on A is
(a) symmetric and transitive
(b) reflexive and transitive only
(c) symmetric only
(d) transitive

SRMJEEE-2010

Ans. (a) : Given, A = {a, b, c, d}
and Relation R = { (a, b), (b, a), (a, a)}
Then, check relation –
(1) Reflexive :– Here, R is not reflexive.
$\because$ $(b, b) \notin R$.
(2) Symmetric :– Here R is symmetric.
$\because$ $(a, b) \in R \Rightarrow (b, a) \in R$.
(3) Transitive : – Here, R is transitive.
$\because$ $(a, b) \in R, (b, a) \in R = (a, a) \in R$
So, A is on relation symmetric and transitive.

141. If $A = \{x, y, z\}, B = \{1, 2\}$, then the total number of relations from set A to set B are
(a) 16 (b) 32 (c) 8 (d) 64

MHT-CET 2020

Ans. (d) : Given, A = { x, y, z} , B = { 1, 2}
Then, A × B = { (x, 1), (y, 1), (z, 1) (x, 2) (y, 2), (z, 2)}
Then number of element – n(A × B) = 6
So, the total number of relations from set A to set B are $= 2^n = 2^6 = 64$.

142. The relation R defined on set $A = \{x : |x| < 3, x \in I\}$ by $R = \{(x, y) : y = |x|\}$ is
(a) $\{-2,2),(-1,1),(0,0),(1,1),(2,2)\}$
(b) $\{-2,2),(-2,2),(-1,1),(0,0),(1,-2),(1,2),$
$(2,-1),(2,-2)\}$
(c) $\{(0,0),(1,1),(2,2)\}$
(d) None of the above

VITEEE-2013

Ans. (a): Given,
A = { x : | x| < 3, x∈ I} by R = { (x, y) : y = | x| }
Then, A = { x : | x| < 3, x∈I }
A = { x : –3 < x < 3, x∈ I}
$\because$ A = { –2, –1, 0, 1, 2 }
Now, R = { (x, y) : y = |x|}
So, R = { (–2, 2), (–1, 1) (0, 0) (1, 1) (2, 2) }

143. The relation R defined on the set of natural numbers as {(a, b) : a differs from b by 3} is given
(a) {(1, 4), (2, 5), (3, 6),....}
(b) {(4, 1), (5, 2), (6, 3),....}
(c) {(1, 3), (2, 6), (3, 9),....}
(d) None of the above

VITEEE-2012

Ans. (b) : Given,
The relation R defined on the set of natural number as {(a, b): a differs from b by 3} can be also written as.
R = { (a, b) : a, b ∈ N, a – b = 3}
R = { (a, b) : a, b ∈ N, a = b + 3}
R = [{ b + 3, b }, b ∈ N]
Or R = [{ n + 3, n} n ∈ N }]
If n = 1, 2, 3, 4….. so, the relation becomes
R = { (4, 1), (5, 2), (6, 3)…….}

144. If R be a relation from $A = \{1,2,3,4\}$ to $B = \{1,3,5\}$ such that $(a,b) \in R \Leftrightarrow a < b$, then ROR^{-1} is
(a) $\{(1,3),(1,5),(2,3),(2,5),(3,5),(4,5)\}$
(b) $\{(3,1),(5,1),(3,2),(5,2),(5,3),(5,4)\}$
(c) $\{(3,3),(3,5),(5,3),(5,5)\}$
(d) $\{(3,3),(3,4),(4,5)\}$

VITEEE-2011

Ans. (c) : Given,

$A = \{1, 2, 3, 4\}$

and $B = \{1, 3, 5\}$

Such that, $(a, b) \in R \Leftrightarrow a < b$

Then,

So, $R = \{(1, 3)(1, 5)(2, 3)(2, 5)(3, 5)(4, 5)\}$

and $R^{-1} = \{(3, 1)(5, 1)(3, 2)(5, 2)(5, 3)(5, 4)\}$

For composition ROR^{-1}, we will pickup an element of R^{-1} first then of R.

Eg. $(3, 1) \in R^{-1}, (1, 3) \in R \Rightarrow (3, 3) \in ROR^{-1}$

Hence, $ROR^{-1} = \{(3, 3), (3, 5)(5, 3), (5, 5)\}$

145. If R be a relation defined as a R b if f |a – b| > 0, then the relation is

(a) reflexive (b) symmetric
(c) transitive (d) symmetric and transitive

VITEEE-2008

Ans. (d) : Given, R be a relation defined as $_aR_b$ if f $|a - b| > 0$

Then, checking for the relation-

(a) Reflexive : – Consider a be an arbitrary element

$\therefore |a - a| = 0$ which shows $a \notin R$

Then, it is not reflexive relation on R.

(b) Symmetric : –

$|a - b| > 0 \Rightarrow |b - a| > 0$

$\Rightarrow$ aRb = bRa

Since, $|a - b| = |b - a|$ Then, R is symmetric.

(c) Transitive : –

$|a - b| > 0, |b - c| > 0 \Rightarrow |a - c| > 0$

Therefore, $(a, c) \in R$ Then, R is transitive.

So, the relation is symmetric and transitive.

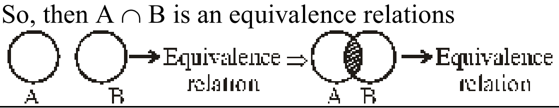
146. If A and B are two equivalence relations defined on set C, then

(a) $A \cap B$ is an equivalence relations
(b) $A \cap B$ is not an equivalence relation
(c) $A \cup B$ is an equivalence relation
(d) $A \cup B$ is not an equivalence relation

UPSEE-2011

Ans. (a) : Given, A and B are two equivalence relations defined on set C.

So, then $A \cap B$ is an equivalence relations



147. The relation R defined on the set A = {1, 2, 3} as R = {(1, 1), (2, 2), (3, 3), (1, 3)} is

(a) equivalence (b) not symmetric
(c) not reflexive (d) not transitive

JCECE-2018

Ans. (b) : Given,

$A = \{1, 2, 3\}$, $R = \{(1, 1), (2, 2), (3, 3), (1, 3)\}$

Then, check relations -

(a) Reflexive : –

$(1, 1) \in R \Rightarrow {_1R_1}$

Then, R is reflexive.

(b) Symmetric : –

$(1, 3) \in R \Rightarrow (3, 1) \notin R$

${_1R_3} \not\Rightarrow {_3R_1}$

Then, R is not symmetric.

(c) Transitive :-

$(1, 1) \in R, (3, 3) \in R \Rightarrow (1, 3) \in R$

${_1R_1}, {_3R_3} \Rightarrow {_1R_3}$

Then, R is transitive.

So, the relation R is not symmetric but reflexive and transitive.

148. Let R be a relation from the set {1, 2, 3.........,60} to itself such that R = {(a, b) : b = pq, where p, q ≥ 3 are prime numbers}. Then the number of elements in R is :

(a) 600 (b) 660
(c) 540 (d) 720

JEE Main-29.07.2022, Shift-I

Ans. (b) : Given set,

$A = \{1, 2, 3, 4 \ldots.60\}$

And, function $R = \{(a, b) : b = pq\}$

$1 \le pq \ge 60$

Number of possible values of a = 60 for b = pq

If p = 3, q = 3, 5, 7, 11, 13, 17, 19

If p = 5, q = 5, 7, 11

If p = 7, q = 7

a = 60 b = 11

$a.b = 60 \times 11$

So, the number of elements in R is = 660.

149. Let R_1 and R_2 be relations on the set {1, 2, …, 50} such that

$R_1 = \{(p, p^n) : p$ is a prime and $n \ge 0$ is an integer$\}$ and $R_2 = \{(p, p^n) : p$ is a prime and $n = 0$ or $1\}$. Then, the number of elements in $R_1 - R_2$ is ______.

JEE Main-28.06.2022, Shift-I

Ans. (8) : Here, $\{p, p^n\} \in \{1, 2, \ldots.50\}$

Possible choice of P are –

2, 3, 5, 7, 11, 13, 17, 23, 29, 31, 37, 41, 43 and 47. we can calculate no. of elements in R_1 as $(2, 2^0), (2, 2^1)$ .. $(2, 2^5)$

$(3, 3^0), \ldots,(3, 3^3)$

$(5, 5^0), \ldots,(5, 5^2)$

$(7, 7^0), \ldots,(7, 7^2)$

$(11, 11^0), \ldots,(11, 11^1)$

Every number of P^n should lie in the given set $\{1, 2, 3\ldots\ldots50\}$

And rest for all other two elements each

$n(R_1) = 6 + 4 + 3 + 3 + (2 \times 10) = 36$

Similarly for R_2

$(2, 2^0), (2, 2^1)$

$(47, 47^0), (47, 47^1)$

$\therefore$ $n(R_2) = 2 \times 14 = 28$

$\therefore$ $n(R_1) - n(R_2) = 36 - 28 = 8$

150. Let P(S) denote the power set of S = {1, 2, 3…..,10}. Define the relation R_1 and R_2 on P(S) as $AR_1 B$ if $(A \cap B^C) \cup (B \cap A^C) = \phi$ and AR_2B if $A \cup B^C = B \cup A^C, \forall$ A,B ∈ P(S). Then

(a) both R_1 and R_2 are not equivalence relations
(b) only R_2 is an equivalence relation
(c) only R_1 is an equivalence relation
(d) both R_1 and R_2 are equivalence relations

JEE Main-01.02.2023, Shift-II

Ans. (d) : P (S) = power set S

S = {1, 2, 3, ….10}

Given, $AR_1B \Rightarrow (A \cap B^c) \cup (B \cap A^c) = \phi$

$\Rightarrow A \cap B^c = \phi$ and $(B \cap A^c) = \phi$

$\Rightarrow A = B$

$\therefore$ AR_1B is an equivalence relation.

$AR_2B \Rightarrow A \cup B^c = B \cup A^c$

$\Rightarrow AB$

$\therefore$ AR_2B is an equivalence relation.

Hence, R_1 and R_2 are equivalence relation.

151. Let a relation R in the set N of natural numbers be defined as (x, y) ⇔ $x^2 - 4xy + 3y^2 = 0 \ \forall$ x, y∈ N. The relation R is

(a) reflexive (b) symmetric
(c) transitive (d) an equivalence relation

AMU-2009

Ans. (a) : We have –

$R = \{(x, y) \Leftrightarrow x^2 - 4xy + 3y^2 = 0 \ \forall x, y \in N\}$

For reflexive –

Let, x ∈ N

$x^2 - 4xx + 3x^2 = 4x^2 - 4x^2 = 0$

(x, x) ∈ R

So, R is reflective

For symmetric –

Let $(x , y) = (3, 1) \Rightarrow (3)^2 - 4(3)(1) + 3(1)^2$

$\Rightarrow 9 - 12 + 3 = 0$

$\therefore$ (3, 1) ∈ R

But $(1, 3) \Rightarrow (1)^2 - 4(3)(1) + 3(3)^2$

$= 1 - 12 + 27 = 16$

$(1, 3) \notin R$

Hence, R is not symmetric so given relation R is reflexive.

152. Let r be a relation from R (set of real numbers) to R defined by r = {(a, b) | a, b ∈ R and a – b + $\sqrt{3}$ is an irrational number}. The relation r is

(a) an equivalence relation (b) reflexive only
(c) symmetric only (d) transitive only

AMU-2009

JEE Main – 01.02.2023 Shift-1

Ans. (a) : Given,

r = {a, b | a, b ∈ R}

And , $r \Rightarrow a - b + \sqrt{3}$ is an irrational number.

For reflexive relation –

Then, $aRa = a - a + \sqrt{3}$

$\Rightarrow$ $aRa = \sqrt{3}$

And, $bRb = b - b + \sqrt{3} \Rightarrow$ $bRb = \sqrt{3}$

Therefore r is reflexive.

For symmetric relation –

Let, a, b ∈ R

$a - b + \sqrt{3} = b - a + \sqrt{3}$ is an irrational number

b,a ∈ R

Therefore r is symmetric.

For transitive relation –

Let (a, b) ∈ R and (b, c) ∈ R

$a - b + \sqrt{3} = b - c + \sqrt{3}$ is an irrational number

Now, $a - c + 2\sqrt{3}$ is an also irrational number

$\therefore$ (a, c) ∈ R

Thus r is transitive relation

Hence, r is an equivalence relation.

153. The minimum number of elements that must be added to the relation R = {(a, b), (b, c)} on the set {a, b, c} so that it becomes symmetric and transitive is :

(a) 3 (b) 7 (c) 4 (d) 5

JEE Main-30.01.2023, Shift-I

Ans. (b) : Given relation,

R = { (a, b) , (b, c)} on the set {a, b, c}

Now, required elements in sets for symmetrices and transitive are –

R = { (a, a), (b, b), (c, c), (b, a), (c, b), (a, c), (c, a) }

R = { (a, b), (b, c)}

Then, total number is 9.

So, minimum 7 elements must be added to becomes symmetric and transitive.

154. The minimum number of elements that must be added to the relation R = {(a, b), (b, c), (b, d)} on the set {a, b, c, d} so that it is an equivalence relation, is.

JEE Main-24.01.2023, Shift-II

Ans. (13) : Given that, R = { (a, b), (b, c), (b, d)}

On the set {a, b, c, d} to become equivalence.

For symmetric

(b, a) (c, a) (c, d), (d, c) (a, d) (d, a) (a, c)

For reflexive

(a, a) (b, b) (c, c) , (d, d)

For transitive

(c, b) (d, b)

Total number of element to be added = 7 + 4 + 2 = 13

155. Let R be a relation defined an N as a R b is 2a + 3b is a multiple of 5, a, b ∈ N. Then R is

(a) transitive but not symmetric
(b) an equivalence relation
(c) symmetric but no transitive
(d) not reflexive

JEE Main-29.01.2023, Shift-II

Ans. (b) : Given Relation, R = {(2a + 3b)} multiple of 5, a, b $\in$ N

Let $(a, b) \in R$

$f(a, b) = 2a + 3b$

For reflexive –

$f(a, a) = 2a + 3a = 5a$

i.e. it is divisible by 5.

$\Rightarrow (a, a) \in R$

For symmetric –

$f(a, b) = 2a + 3b = 5\alpha$

$f(b, a) = 2b + 3a$

$= 2b + \left(\frac{5\alpha - 3b}{2}\right) \times 3$

$= \frac{15\alpha}{2} - \frac{5}{2}b = \frac{5}{2}(3\alpha - b)$

$= \frac{5}{2}(2a + 2b - 2\alpha) = 5(a + b - \alpha)$

f (b, a) is divisible by 5 $\Rightarrow$ (b, a) $\Rightarrow$ R

For transitive –

$f(a, b) = 2a + 3b$ is divisible by 5

$\Rightarrow 2a + 3b = 5\alpha$

$f(b, c) = 2b + 3c$, is divisible by 5

$2b + 3c = 5\beta$

$2a + 5b + 3c = 5(\alpha + \beta)$

$2a + 3c = 5(\alpha + \beta - b)$

$\Rightarrow$ aRc

So, 2a + 3c is divisible by 5

$\Rightarrow (a, c) \in R$

Which is transitive.

Hence, R is equivalence relation.

156. Let R be a relation on N×N defined by (a, b) R (c, d) if and only if ad (b–c) = bc (a–d). Then R is

(a) transitive but neither reflexive nor symmetric
(b) symmetric but neither reflexive nor transitive
(c) symmetric and transitive but not reflexive
(d) reflexive and symmetric but not transitive

JEE Main-31.01.2023, Shift-I

Ans. (b) : Let R be relation defined by (a, b) R (c, d) $\Leftrightarrow$ ad (b – c) = bc (a – d)

For reflexive –

(a, b) R (a, b) $\Rightarrow$ ab (b – a) = ba (a – b)

$\therefore$ It is not reflexive.

For symmetric $\Rightarrow$ (a, b) R (c, d) = ad (b – c) = bc (a – d) and

(c, d) R (a, b) = cb (d – a) = da (c – b)

It is true

Which is symmetric.

For transitive –

(a, b) R (c, d) = ad (b – c) = bc (a – d)

(c, d) R (e, f) = cf (d – e) = de (c – f)

So,

adcf (b – c) (d – e) = bcde (c – d) (c – f)

af (b – c) (d – e) = be (a – d) (c – f)

It is not transitive .

157. Among the relations

S = {(a, b) : a, b $\in$ R – {0}, 2 + $\frac{a}{b}$ > 0} and T = {(a, b) : a, b $\in$ R, $a^2 - b^2 \in Z$}.

(a) S is transitive but T is not transitive
(b) both S and T are symmetric
(c) neither S nor T is transitive
(d) T is symmetric but S is not symmetric

JEE Main-31.01.2023, Shift-II

Ans. (d) : Given relations

$S = \{ (a, b) : a, b \in R - \{0\} , 2 + \frac{a}{b} > 0 \}$

And, $T = \{ (a, b) : a, b \in R, a^2 - b^2 \in Z \}$.

Now, $T = a^2 - b^2 \in Z$

Then (b, a) on Relation R

$b^2 - a^2 \in Z$

Hence T is symmetric.

For,

$$S = \left\{(a,b) : a, b \in R - \{0\}, 2 + \frac{a}{b} > 0\right\}$$

$$2 + \frac{a}{b} > 0 \Rightarrow \frac{a}{b} > -2 \Rightarrow \frac{b}{a} < \frac{-1}{2}$$

If $(b, a) \in S$ then,

$2 + \frac{b}{a}$ not necessarily positive.

So, S is not symmetric.

158. Let R be the relation on the set R of all real Numbers defined by setting aRb iff $|a - b| \le \frac{1}{2}$

Then R is

(a) Reflexive and symmetric but not transitive
(b) Symmetric and transitive but not reflexive
(c) Reflexive and transitive but not symmetric
(d) Transitive but neither reflexive nor symmetric

AMU-2021

Ans. (a) : Given relation –

${}_aR_b \Rightarrow |a - b| \le \frac{1}{2}$

For reflexive, aRa

$|a - a| = 0 \le \frac{1}{2}$

Hence, R is reflexive.

For symmetric $\Rightarrow |a - b| \le \frac{1}{2}$ and $|b - a| \le \frac{1}{2}$

$\Rightarrow$ aRb = bRa

Hence, it is symmetric.

For transitive –

aRb and bRc then aRc

$|a - b| \le \frac{1}{2}$ And, $|b - c| \le \frac{1}{2}$

Then, $|a - c| \not\le \frac{1}{2}$ Hence, R is not transitive.

159. Given the relation R = {(1, 2), (2, 3)} on the set A = {1, 2, 3}, the number of ordered pairs which when added to R make it an equivalence relation is

(a) 5 (b) 6
(c) 7 (d) none of these.

AMU-2008

Ans. (c) : Given relation R = {(1, 2), (2, 3)}
on the set A = {1, 2, 3}
R is symmetric if contains
{(2, 1), (3, 2)} $\in$ R
R is reflexive if contains
{(1, 1) (2, 2), (3, 3)}
R is transitive if it contains
{(3, 1) , (1, 3)}
number of ordered pair to be added
{(1, 1) , (2, 2) , (3, 3) , (2, 1), (3, 2) (1, 3), (1, 2)}
Hence, total number = 7

160. Let R be a reflexive relation on a finite set A having n elements and let there be m ordered pairs in R then

(a) $m \geq n$ (b) $m \leq n$
(c) m=n (d) none of these

AMU-2016

Ans. (a) : The set consists of n elements and for relation to be reflexive it must have at least n ordered pairs. It has m ordered pair
Therefore, $m \geq n$

161. Let A = {(x, y) : y = e^{-x}} and B = {(x, y) : y = –x} Then the correct statement is :

(a) $A \cap B = \phi$ (b) $A \subset B$
(c) $B \subset A$ (d) $A \cap B = \{(0, 1), (0, 0)\}$

AMU-2013

Ans. (a) : We have ,

$$A = \{(x, y) : y = e^{-x}\},\ B = \{(x, y) : y = -x\}$$

Now, A = (x, y) = (x, e^{-x}), B = (x, y) = (x,– x)
Since image of x in A cannot be equal to image of x in B i.e.

$e^{-x} \neq -x$
$A \cap B = \phi$

162. For any two real numbers θ and φ, we define θRφ, if and only if $\sec^2\theta - \tan^2\phi = 1$. The relation R is

(a) reflexive but not transitive
(b) symmetric but not reflexive
(c) both reflexive and symmetric but not transitive
(d) an equivalence relation

WB JEE-2014

Ans. (d) : Given,
The relation is $\theta R\phi \Rightarrow \sec^2\theta - \tan^2\phi = 1$
For reflexive:- $\theta R\theta \Rightarrow \text{Sec}^2\theta - \tan^2\theta = 1$
1 = 1, Which is true
$\therefore$ It is reflexive.

For symmetric:
$\theta R\phi \Rightarrow \sec^2\theta - \tan^2\phi = 1$
$(1 + \tan^2\theta) - (\sec^2\phi - 1) = 1$
$1 + \tan^2\theta - \sec^2\phi + 1 = 1$
$2 + \tan^2\theta - \sec^2\phi = 1$
$\tan^2\theta - \sec^2\phi = -1$
$\sec^2\phi - \tan^2\theta = 1$
$\phi R\theta$
$\therefore$ It is symmetric.

For transitive:-
Let θRφ and φRψ, then-
$\sec^2\theta - \tan^2\phi = 1$
and, $\sec^2\phi - \tan^2\psi = 1$
$\therefore$ $\theta R\psi \Rightarrow \sec^2\theta - \tan^2\psi = 1$
$\Rightarrow$ $\sec^2\theta - \tan^2\psi + 1 = 1 + 1$
$\Rightarrow$ $\sec^2\theta - \tan^2\psi + \sec^2\phi - \tan^2\phi = 1 + 1$
$\Rightarrow$ θRφ and φRψ
Then, it is transitive.
So, it is an equivalence relation.

163. Let the number of elements of the sets A and B be p and q, respectively. Then, the number of relations from the set A to the set B is

(a) 2^{p+q} (b) 2^{pq} (c) $p+q$ (d) pq

WB JEE-2014

Ans. (b) : Given, the sets A and B .
And, number of elements of the set A = p
number of elements of the set B = q
Then, the cartesian product of A and B is –
$A \times B = \{ (a ,b) : (a \in A) \text{ and } (b \in B)\}$
$\therefore$ Number of elements in $|A \times B| = |A| \cdot |B| = pq$
Then, any relation from A to B is a subset of A × B.
So, the number of relations from A to B is the number of subsets of A × B is-
$= 2^{|A \times B|} = 2^{pq}$

164. A relation P on the set of real number R is defined as {xPy : xy > 0}. Then, which of the following is/are true?

(a) P is reflexive and symmetric
(b) P is symmetric but not reflexive
(c) P is symmetric and transitive
(d) P is an equivalence relation

WB JEE-2015

Ans. (a) : Given, a relation P on the set of real number R is defined as –

$$\{xPy : xy > 0\}$$

For reflexive:– $x \cdot x = x^2 \geq 0\ \forall\ x \in R$
$\therefore$ P is reflexive.
For symmetric :–
Let, x, y $\in$R such that $xy \geq 0$
$\Rightarrow$ $yx \geq 0,\ \forall\ x, y \in R$
$\Rightarrow$ yPx
$\Rightarrow$ P is symmetric

For transitive:- $(1, 0), (0, -2) \in P$
but, $(1, -2) \notin P$
$\therefore$ P is not transitive.
So, P is reflexive and symmetric but not transitive.

165. For any two real numbers a and b, we define a R b if and only if $\sin^2 a + \cos^2 b = 1$. The relation R is
(a) reflexive but not symmetric
(b) symmetric but not transitive
(c) transitive but not reflexive
(d) an equivalence relation

WB JEE-2013

Ans. (d) : Given, for any two real number a and b.
We define $aRb \Leftrightarrow \sin^2 a + \cos^2 b = 1$
For reflexive : – $aRa \Rightarrow \sin^2 a + \cos^2 a = 1 \ \forall \ a \in R$.
$\therefore$ It is reflexive relation.
For symmetric:-
$aRb \Rightarrow \sin^2 a + \cos^2 b = 1$
$\Rightarrow 1 - \cos^2 a + 1 - \sin^2 b = 1$
$\Rightarrow 2 - \cos^2 a - \sin^2 b = 1$
$\Rightarrow \sin^2 b + \cos^2 a = 1$
$\Rightarrow bRa \ \forall \ a, b \in R$
$\therefore$ It is symmetric relation.
For transitive:-
aRb and bRc $\Rightarrow$ aRc
$\Rightarrow \sin^2 a + \cos^2 b = 1$ and $\sin^2 b + \cos^2 c = 1$
$\therefore$ Adding these two equation we get-
$\sin^2 a + \cos^2 b + \sin^2 b + \cos^2 c = 2$
$\Rightarrow \sin^2 a + \cos^2 c = 1$
$\Rightarrow$ aRc
$\therefore$ It is transitive relation.
So, R is an equivalence relation.

166. The number of equivalence relations on the set {1, 2, 3} containing (1, 2) and (2, 1) is
(a) 3 (b) 1 (c) 2 (d) None of these

AMU-2015

Ans. (c) : Equivalence relation of the set {(1, 2, 3)} containing (1, 2) and (2, 1)
$A_1 = \{(1, 1) (2, 2) (3, 3) (1, 2) (2, 1)\}$
$A_2 = \{(1, 1), (2, 2), (3, 3) (1, 2) (2, 1), (2, 3) (3, 1) (3, 2), (1, 3)\}$
So, There are only two equivalence relation are possible.

167. Let R and S be two equivalence relations on a non-void set A. Then
(a) $R \cup S$ is a equivalence relation
(b) $R \cap S$ is equivalence relation
(c) $R \cap S$ is not equivalence relegation
(d) $R \cup S$ is not a equivalence relation

WB JEE-2022

Ans. (b) : Given, R and S be two equivalence relations on a non-void set A.
For reflexive : -
R and S are reflexive this means for any $a \in A$.
$\therefore (a, a) \in R$ and $(a, a) \in S$
$\Rightarrow (a, a) \in R \cap S$
$\therefore R \cap S$ is reflexive.
For symmetric:-
$(a, b) \in R \cap S$
Then, $(a, b) \in R, (a, b) \in S$
Since, R and S are symmetric.
$\therefore (b, a) \in R$ and $(b, a) \in S$
For transitive:-
Let, $(a, b), (b, c) \in R \cap S$
$\Rightarrow (a, b), (b, c) \in R$
$\therefore (a, c) \in R$, since, R is transitive.
And, $(a, b), (b, c) \in S$
This means $(a, c) \in S$ since, S is transitive.
$\therefore (a, c) \in R \cap S$.
So, $R \cap S$ is transitive.
Hence, $R \cap S$ is an equivalence relation.

168. If there are 2 elements in a set A, then what would be the number of possible relations from the set A to set A?
(a) 2 (b) 4
(c) 16 (d) 32

J&K CET-2019

Ans. (c) : Given,
$n(A) = 2$
Hence, number of possible relation from set A to set A
$\Rightarrow 2^{n^2} = 2^{2^2} = 2^4 = 2 \times 2 \times 2 \times 2 = 16$

169. Let X = {a, b, c, d, e} and R = {(a, a), (b, b), (c, c), (a, b), (b, a)}. Then the relation R on X is
(a) reflexive and symmetric
(b) not reflexive but symmetric
(c) symmetric and transitive, but not reflexive
(d) reflexive but not transitive

J&K CET-2015

Ans. (c) : Given,
X = {a, b, c, d, e} and
R = {(a, a), (b, b), (c, c), (a, b), (b, a)}
Since, $(a, b) \in R$, $(b, a) \in R$ and $(a, a) \in R$
So, Relation is transitive for all $a, b \in X$
$(a, b) \in R$ and $(b, a) \in R$ so relation R is symmetric
The relation r is not reflexive because $(d, d) \notin R$ and $(e, e) \notin R$

170. Let R be the set of real numbers and let $G \subseteq R^2$ be a relation defined by $G = \{[(a, b), (c, d)] [b - a = d - c]\}$ then G is
(a) reflexive only
(b) symmetric only
(c) transitive only
(d) an equivalence relation

J&K CET-2015

Ans. (d) : Given the relation –

$G = \{ [(a, b), (c , d)] \mid b - a = d - c \}$

For reflexive –

Let $(x, y) \in R^2$

$\Rightarrow \; y - x = y - x$

$\therefore [(x, y), (x, y)] \in G \; \forall \; (x, y) \in R^2$

$\therefore$ G is reflexive

For symmetric –

Let $[(a, b) (c, d)] \in G$

$\Rightarrow \; b - a = d - c \Rightarrow d - c = b - a$

$\Rightarrow \; [(c, d), (a, b)] \in G$

$\therefore$ G is symmetric –

For transitive –

Let $[(a, b), (c, d)] \in G$ .....(i)

And $[(c, d), (x, y)] \in G$ .....(ii)

$\Rightarrow b - a = d - c$ (form equation (i))

$\Rightarrow d - c = y - x$ (form equation (ii))

$\Rightarrow b - a = y - x$

$\Rightarrow [(a, b), (x, y)] \in G$

$\therefore$ G is transitive.

Hence, G is an equivalence relation.

171. Let N denote the set of all natural numbers and R be the relation on N × N defined by (a, b) R (c, d) if ad(b + c) = bc (a + d). Then R is

(a) symmetric only
(b) symmetric and reflexive
(c) transitive only
(d) an equivalence relation

Assam CEE-2021
VITEEE – 2013

Ans. (d) : Given, N is a set of natural numbers and R is a relation on N × N defined by

(a, b) R (c, d) if ad (b + c) = bc (a + d)

For reflexive –

$(a, a) \in R \; \forall a \in A$

Let, (a, b) R (a, b)

Therefore,

$ab (b + a) = ba (a + b)$

$ab (b + a) = ab (b + a)$

This implies that R is reflexive.

For symmetric -

Let (a, b) R (c, d)

Therefore,

$ad (b + c) = bc (a + d)$

$\Rightarrow \; bc (a + d) = ad (b + c)$

$\Rightarrow \; cb (d + a) = da (c + b)$

$\Rightarrow$ (c, d) R (a, b)

This implies that R is symmetric

For transitive –

$(a, b) \in R$ and $(b, c) \in R$ then –

$(a, c) \in R \; \forall \; a, b, c \in A$

Let, (a, b) R (c, d)

Therefore,

$ad(b + c) = bc (a + d)$

$adb + adc = abc + bcd$

$abd - abc = bcd - acd$

$ab (d - c) = cd (b - a)$

$\dfrac{ab}{b-a} = \dfrac{cd}{d-c}$ .....(i)

And let (c, d) R (e, f)

Therefore,

$cf (d + e) = de (c + f)$

$cfd + cef = ced + edf$

$cfd - ced = edf - cef$

$cd (f - e) = ef (d - c)$

$\dfrac{cd}{d-c} = \dfrac{ef}{f-e}$ ....(ii)

From (i) and (ii), we get –

$\dfrac{ab}{b-a} = \dfrac{ef}{f-e}$

$\Rightarrow \; abf - abe = efb - efa$

$\Rightarrow \; abf + efa = efb + abe$

$\Rightarrow \; af (b + e) = be(a + f)$

$\Rightarrow$ (a, b) R (e, f)

This implies that R is transitive.

So, it is an equivalence relation.

172. Let A = {1, 3, 4, 6, 9) and B = {2, 4, 5, 8, 10}. Let R be a relation defined on A × B such that R = $\{((a_1, b_1), (a_2, b_2)) : a_1 \le b_2$ and $b_1 \le a_2\}$. Then the number of elements in the set R is

(a) 26 (b) 160 (c) 180 (d) 52

JEE Main-11.04.2023, Shift-II

Ans. (b) : Given set,

A = {1, 3, 4, 6, 9)

and B = {2, 4, 5, 8, 10}

$R = A \times B \Rightarrow \{(a_1\, b_1)\, (a_2\, b_2) : a_1 \le b_2\; b_1 \le a_2\}$

Let,

$a_1 = 1$	then	b_2 has	5 choices
$a_1 = 4$	then	b_2 has	4 choices
$a_1 = 6$	then	b_2 has	2 choices
$a_1 = 9$	then	b_2 has	1 choices

Now,

$b_1 = 2$	then	a_2 has	4 choices
$b_1 = 4$	then	a_2 has	3 choices
$b_1 = 5$	then	a_2 has	2
$b_1 = 8$		a_2 has	1 choices

So, total number of element

R = 160

173. Let R be the relation in the set $\{x : x \in N, x \le 4\}$ given by R = {(1, 1), (2, 2), (3, 3)} then, R is ________ .

(a) Reflexive and symmetric but not transitive
(b) Symmetric and transitive but not reflexive
(c) Reflexive and transitive but not symmetric
(d) An equivalence relation.

GUJCET-2021

Ans. (b) : $\{x : x \in N, \; x \le 4\}$

Let A = {1, 2, 3, 4}

R = {(1, 1), (2, 2), (3, 3)}

For symmetry –

$(a, b) \in R \Rightarrow (b, a) \in R$

$(3, 3) \in R$

$(3, 3) \in R$

So, R is symmetry

For transitive –

$(a, b), (b, a) \in R$

Then, $(a, a) \in R$

$(2, 2), (2, 2) \in R$

Also $(2, 2) \in R$

So, R is transitive.

For reflexive –

For all $x \in A$

$(x, x) \in R$

But here

$(4, 4) \in A$

and $(4, 4) \notin R$

So, R is not reflexive.

Hence, this relation is symmetric and transitive but not reflexive.

174. Relation S = {(1, 2), (2, 1), (2, 3)} is defined on the set {1, 2, 3} is______.

(a) not transitive (b) symmetric
(c) reflexive (d) equivalence

GUJCET-2017

Ans. (a) : Given, Relation S = {(1,2), (2,1), (2,3)} is defiened on the set {1, 2, 3}. Then by definition of Relation –

For reflexive : – (1,1) is not belongs to the set S.

So, set S is not Reflexive.

For symmetric: – From set S, $(2,3) \in S$ but $(3,2) \not\in S$

So this is not symmetric.

For transitive : – From set S, $(1,2) \in S$ and $(2,3) \in S$ but $(1, 3) \not\in S$

So, this is not transitive.

175. Relation R in the set (π,π^2, π^3) defined by R = $\{(\pi, \pi), (\pi^2, \pi^2), (\pi^3, \pi^3), (\pi, \pi^2), (\pi^2, \pi^3)\}$ is :

(a) Reflexive but neither symmetric nor transitive
(b) Symmetric but neither reflexive nor transitive
(c) Transitive but neither reflexive nor symmetric
(d) Only symmetric and transitive

GUJCET-2023

Ans. (a) : Given,

Set (π,π^2, π^3) defined by

$R = \{(\pi, \pi), (\pi^2, \pi^2), (\pi^3, \pi^3), (\pi, \pi^2), (\pi^2, \pi^3)\}$

For symmetric –

Since, $(\pi, \pi^2) \in R$ but $(\pi^2, \pi) \notin R$ so R is not symmetric.

For Reflexive –

Since, $(\pi, \pi) \in R$, $(\pi^2, \pi^2) \in R$ and $(\pi^3, \pi^3) \in R$ so R is Reflexive.

For transitive –

Since, $(\pi, \pi^2) \in R$, and $(\pi^2, \pi^3) \in R$ but $(\pi, \pi^3) \notin R$ so R is not transitive.

176. When R is the set of all real numbers,

$$\left\{x \in R : \frac{\sqrt{12-x-x^2}}{x+10} \le \frac{\sqrt{12-x-x^2}}{2x+9}\right\} =$$

(a) $(-4, 1] \cup \{3\}$ (b) $[-4, 1]$
(c) $[-4, 1] \cup \{3\}$ (d) ϕ, the empty set

TS EAMCET 14.09.2020, Shift-II

Ans. (c) : We have, $\frac{\sqrt{12-x-x^2}}{x+10} \le \frac{\sqrt{12-x-x^2}}{2x+9}$

$\Rightarrow \sqrt{12-x-x^2}\,(2x+9-x-10) \le 0$

$\Rightarrow \sqrt{12-x-x^2}\,(x-1) \le 0$

$\therefore 12-x-x^2 \ge 0$ and $x \le 1$

$\Rightarrow x^2+x-12 \le 0$ and $x \le 1$

$\Rightarrow (x+4)(x-3) \le 0$ and $x \le 1$

$\Rightarrow x \in [-4, 3]$ and $x \in (-\infty, 1]$

$\therefore x \in [-4, 1] \cup \{3\}$

177. Let A = {1, 2, 3, 4, 5, 6, 7}. Then the relation R = $\{(x, y) \in A \times A : x + y = 7\}$ is

(a) transitive but neither symmetric nor reflexive
(b) reflexive but neither symmetric nor transitive
(c) an equivalence relation
(d) symmetric but neither reflexive nor transitive

JEE Main-08.04.2023, Shift-II

Ans. (d) : A = {1, 2, 3, 4, 5, 6, 7}. defined on the set

$R = \{(x, y) \in A \times A : x + y = 7\}$

$R = \{(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)\}$

For symmetric:- xRy = yRx

$(1, 6) \in R, (6, 1) \in R$ and $(5, 2) \in R, (2, 5) \in R$

So R is symmetric

For Reflexive:- xRx

$(1, 1) \notin R$ $(2, 2) \notin R$ $(3, 3) \notin R$ and $(5, 5) \notin R$

So, R is not reflexive

For transitive

$(1, 6) \in R$ and $(6, 1) \in R$ but $(1, 1) \notin R$ and $(2, 5) \in R$ $(5, 2) \in R$ but $(2, 2) \notin R$ so R is not transitive.

178. Let A = {0, 3, 4, 6, 7, 8, 9, 10} and R be the relation defined on A such that R = $\{(x, y) \in A \times A$: x –y is odd positive integer or x – y = 2}. The minimum number of elements that must be added to the relation R, so that it is a symmetric relation, equal to_____

JEE Main-08.04.2023, Shift-I

Ans. (19) : Given,

Set A = {0, 3, 4, 6, 7, 8, 9, 10}

Relation R defined in A.

R = [{x, y}$\in$ A × A : x – y is odd positive integer or x – y = 2]

R = {(6, 4), (8, 6), (9, 7), (10, 8), (3, 0), (7, 0), (9, 0), (4, 3), (6, 3), (8, 3), (10, 3), (7, 9), (9, 4), (7, 6), (9, 6), (8, 7), (10, 7), (9, 8), (10, 9)}

Hence, 19 element should be add in R for making it symmetric.

179. The relations R defined in the set {1, 2, 3, 4, 5, 6} as R = {(a, b) : b = a +1} is

(a) reflexive (b) symmetric
(c) transitive (d) None of these

BCECE-2013

Ans. (d) : Consider, A = {1, 2, 3, 4, 5, 6}
Given, as R = {(a, b)} : b = a + 1}.
Then, check relation is -
(a) Reflexive relation :-
Then, ${}_aR_a \neq {}_aR_{a+1} \neq {}_aR_a, {}_1R_2, a \in A$
So, it is not reflexive relation.
(b) Symmetric relation
$${}_aR_b = {}_bR_a$$
Then ${}_aR_{a+1} \neq {}_{a+1}R_a$ is not defined.
So, it is not a symmetric relation.
(c) Transitive Relation : -
If $a, b, c \in R$
Then, $aRb, bRc \Rightarrow aRc$
It is not transitive because -
${}_5R_6, {}_6R_7 \not\Rightarrow {}_5R_7$
is not defined because $7 \notin A$.
So, it is not a transitive relation.
Hence, the relation R is not a reflexive not a symmetric and not a transitive relation.

180. Let R be the relation on the set R, of all real numbers defined by aRb if $f(x) = |a-b| \leq 1$. Then, R is

(a) reflexive and symmetric
(b) symmetric only
(c) transitive only
(d) anti-symmetric only

BCECE-2012

Ans. (a): Given, R be the relation on the set R defined by aRb if $f(x) = |a - b| \leq 1$.
Then, R is reflexive and symmetric relation but transitive relation.
Check relations –
(a) Reflexive relation : –
$aRa, \quad a \in R$
Then, $|a - a| \leq 1$
$|0| \leq 1$
$0 \leq 1$
It is reflexive relation.
(b) Symmetric Relation : -
${}_aR_b = {}_bR_a, \ a, b \in R$
Then, $|a - b| \leq 1 = |b - a| \leq 1$
It is true because modulus gives the value.
So, it is symmetric relation.
(c) Transitive relation –
${}_aR_b, {}_bR_c \Rightarrow {}_aR_c, \ a, b, c \in R$
It is not true.
Let a = 1, b = 2 and c = 3
Then, $|a - b| \leq 1, \ |b - c| \leq 1 \Rightarrow |a - c| \leq 1$
$|1 - 2| \leq 1, |2 - 3| \leq 1 \Rightarrow |1 - 3| \leq 1$
$1 \leq 1, \ 1 \leq 1 \not\Rightarrow 2 \nleq 1$
It is not transitive relations .
So, R is reflexive and symmetric relation but not transitive relation.

181. On set A = {1, 2, 3}, relations R and S are given by R = {(1, 1), (2, 2), (3, 3), (1, 2), (2, 1)}, S = {(1, 1), (2, 2), (3, 3), (1, 3), (3, 1)}. Then,

(a) $R \cup S$ is an equivalence relation
(b) $R \cup S$ is reflexive and transitive but not symmetric
(c) $R \cup S$ is reflexive and symmetric but not transitive
(d) $R \cup S$ is symmetric and transitive but no reflexive

WB JEE-2017

Ans. (c) : We have,
R = {(1, 1), (2, 2), (3, 3), (1, 2), (2, 1)}
S = {(1, 1), (2, 2), (3, 3), (1, 3), (3, 1)}
$\therefore$ $R \cup S$ = {(1, 1),(2, 2),(3, 3),(1, 2),(2, 1), (1, 3), (3, 1)}
Since, $(2, 1) \in R \cup S, (2, 3) \in R \cup S$ but $(2, 3) \notin R \cup S$
$\therefore$ $R \cup S$ is reflexive and symmetric but not transitive.

182. If A = {1, 2, 3, 4}, then which one of the following is reflexive?

(a) {(1, 1),(2, 3),(3, 3)}
(b) {(1, 1),(2, 2),(3, 3),(4, 4)}
(c) {(1, 2),(2, 1),(3, 2),(2, 3)}
(d) {1, 2),(1, 3),(1, 4)}

COMEDK 2014

Ans. (b) : Given, A={1, 2, 3, 4}
Let R be a reflexive relation on a then for each $a \in A, (a, a) \in R$
For reflexive (1, 1) (2, 2) (3, 3) (4, 4)
$\therefore$ Option (b) is true.

183. $x^2 = xy$ is a relation which is

(a) Symmetric (b) Reflexive
(c) Transitive (d) None of these

BITSAT-2008

Ans. (b) : Given, $x^2 = xy$
The relation is only reflexive relation because -
xRx, is only define in this relation.
So, $x^2 = xy$ is a relation which is reflexive.

184. Let a relation R be defined on set of all real numbers by a R b if and only if 1 + ab > 0. Then, R is

(a) reflexive, transitive but not symmetric
(b) reflexive, symmetric but not transitive
(c) Symmetric, transitive but not reflexive
(d) an equivalence relation

UPSEE-2009

Ans. (b) : Given,
A relation R be defined on set of all real numbers.
and, aRb is 1 + ab > 0.
Then, check reaction R is -
(a) Reflexive relation : –
aRa = 1 + a^2, here a^2 is always a positive real number.
Then, $1 + a^2 > 0$
So, R is reflexive relation.
(b) Symmetric relation :-
aRb = bRa
1 + ab > 0 = 1+ ba > 0
Since, ab = ba So, R is a symmetric relation.
(c) Transitive relation : -
aRb, bRc $\not\Rightarrow$ aRc
$1 + ab > 0, 1 + bc > 0 \not\Rightarrow 1 + ac > 0$
So, R is not transitive relation.
Hence, R is reflexive symmetric but not transitive relation.

185. Let R = {(3, 3), (6, 6), (9, 9), (12, 12), (6, 12), (3, 9), (3, 12), (3, 6)} be a relation on the set A = {3, 6, 9, 12}. Then, the relations is
(a) an equivalence relation
(b) reflexive and symmetric
(c) reflexive and transitive
(d) only reflexive

BITSAT-2014

Ans. (c) : (3, 3), (6, 6), (9, 9), (12, 12), $\in$ R
R is not symmetric as (6, 12) $\notin$ R but (12, 6) $\in$ R
R is transitive as the only pair which needs verification is (3, 6) and (6, 12) $\in$ R $\Rightarrow$ (3, 12) $\in$ R

186. An integer m is said to be related to another integer n, if m is a multiple of n. Then, the relation is
(a) reflexive and symmetric
(b) reflexive and transitive
(c) symmetric and transitive
(d) an equivalence relation

UPSEE-2012

Ans. (b) : Given, an integer m is said to be related to another integer n, if m is a multiple of n.
Then, check relation are –
(a) Reflexive relation : -
mRm $\Rightarrow$ nRn
Means (m, m) $\in$ R, (n, n) $\in$ R
Then, R is reflexive relation.
(b) Symmetric relation :
(m, n) $\in$ R but (n, m) $\notin$ R.
Example : –
(3, 9) $\in$ R but (9, 3) $\notin$ R
Then, R is not symmetric relation.
(c) Transitive relation : –
(m, n) $\in$ R, (n,p) $\in$ R $\Rightarrow$ (m, p) $\Rightarrow$ $\in$ R

Example : –
(2, 6) $\in$ R, (6, 12) $\in$ R $\Rightarrow$ (2, 12) $\Rightarrow\in$ R
Then, it is transitive relation.
So, the relation R is reflexive and transitive but not symmetric.

187. The relation R in R defined by R = {(a, b) : a ≤ b^3), is
(a) reflexive
(b) symmetric
(c) transitive
(d) None of these

UPSEE-2014

Ans. (d) : Given, the relation R in R.
R = { (a, b) : a ≤ b^3}.
Then, check relation R are -
(a) Reflexive relation : -
R = { (a, a) ; a ≤ a^3}
It is not true because –
Let a = 2, then R = { (2, 2); 2 ≤ 2^3 } is not true.
So, it is not reflexive relation.
(b) Symmetric Relation : –
Since, R = { (a, b); a ≤ b^3 }
R = { (b, a); b ≤ a^3}
It is not true because a is less then b but a not equal to b.
So. it is not reflexive relation.
(c) Transitive relation :
Let a, b, c in R.
Then, R = { (a, b) ; a ≤ b^3}, R = { (b, c); b ≤ c^3}
R = { (a, b) ; a ≤ c^3}
It is not true because, a is less than c but a not equal to c.
So, it is not transitive relation.
Hence, the relation R in R defined by
R = { (a, b) : a ≤ b^3} is not reflexive not symmetric and not transitive relation .

188. Let R be the relation on the set R of all real number defined by aRb if | a – b| ≤ 1, then R is
(a) Reflexive and symmetric
(b) Symmetric only
(c) Transitive only
(d) Anti symmetric only

J&K CET-2004
AMU - 2014

Ans. (a) : Let R be the relation defined by if |a – b| ≤ 1
For reflexive :- aRa
|a – a| ≤ 1
0 ≤ 1, which is true.
So, R is reflexive.
For symmetric:–
aRb $\Rightarrow$ |a – b| ≤ 1 $\Rightarrow$ |b – a| ≤ 1
bRa $\Rightarrow$ R is symmetric.
For transitive:-
Now, aRb and bRc
$\Rightarrow$ |a – b| ≤ 1 and |b – c| ≤ 1
|a – c| = |a – b + b – c|
≤ |a – b| + | b – c|

$\leq 1 + 1$

≤ 2 not true.

Hence, a is not related to c.

$\Rightarrow$ R is reflexive and symmetric but not transitive.

189. Let R = {(1, 3) (4, 2), (2, 4), (2, 3), (3, 1)} be a relation on the set A = {1, 2, 3, 4}. The relation R is

(a) a function (b) transitive
(c) not symmetric (d) reflexive

AIEEE-2004
Rajsthan PET-2007

Ans. (c) : Given,

set A = {1, 2, 3, 4}

and R = {(1, 3) (4, 2), (2, 4), (2, 3), (3, 1)}

For symmetric :- Since, $(2, 3) \in R$ But $(3, 2) \notin R$ so R is not symmetric.

For transitive :- $(1, 3) \in R$ and $(3, 1) \in R$ But $(1, 1) \notin R$

So, R is not transitive.

Hence, R is not symmetric.

190. On the set R of real numbers we define xPy if and only if $xy \geq 0$. Then, the relation P is

(a) reflexive but not symmetric
(b) symmetric but not reflexive
(c) transitive but not reflexive
(d) reflexive and symmetric but not transitive

WB JEE-2017

Ans. (d) : For every real number x, $x^2 \geq 0$

$\therefore$ $(x, x) \in P$

Hence, P is reflexive.

Now, let $(x, y) \in P$

$= xy \geq 0$

$= yx \geq 0 = (y, x) \in P$

191. Let R_1 and R_2 be two relations defined as follows

$R_1 = \{(a, b) \in R^2 : a^2 + b^2 \in Q\}$

$R_2 = \{(a, b) \in R^2 : a^2 + b^2 \notin Q\}$

where Q is the set of all rational numbers. Then

(a) R_1 and R_2 are both transitive
(b) Neither R_1 nor R_2 is transitive
(c) R_1 is transitive but R_2 is not transitive
(d) R_2 is transitive but R_1 is not transitive

JEE Main 03.09. 2020 Shift-II

Ans. (b) : Let R_1 and R_2 be two relations

$R_1 = \{(a, b) \in R^2 : a^2 + b^2 \in Q\}$

$R_2 = \{(a, b) \in R^2 : a^2 + b^2 \notin Q\}$

For R_1 –

Consider,

$a = 1 + \sqrt{2}$, $b = 1-\sqrt{2}$ and $c = 8^{1/4}$

$(a, b) \in R_1$ because,

$a^2 + b^2 = \left(1+\sqrt{2}\right)^2 + \left(1-\sqrt{2}\right)^2$

$= 1 + 2 + 2\sqrt{2} + 1 + 2 - 2\sqrt{2} = 6 \in Q$

And $(b, c) \in R_1$ because,

$b^2 + c^2 = \left(1-\sqrt{2}\right)^2 + \left(8^{\frac{1}{4}}\right)^2 = 1+2-2\sqrt{2}+2\sqrt{2} = 3 \in Q$

But $(a, c) \notin R_1$ because,

$a^2 + c^2 = \left(1+\sqrt{2}\right)^2 + \left(8^{1/4}\right)^2 = 1+2+2\sqrt{2}+2\sqrt{2}$

$= 3+4\sqrt{2} \notin Q$

Hence, R_1 is not transitive.

Now, For R_2 -

Consider, $a = 1 + \sqrt{3}$, $b = \sqrt{3}$, $c = 1-\sqrt{3}$

$(a, b) \in R_2$ because,

$a^2 + b^2 = \left(1+\sqrt{3}\right)^2 + \left(\sqrt{3}\right)^2$

$= 1 + 3 + 2\sqrt{3} + 3 = 7 + 2\sqrt{3} \notin Q$

$(b, c) \in R_2$ because,

$b^2 + c^2 = \left(\sqrt{3}\right)^2 + \left(1-\sqrt{3}\right)^2$

$= 3 + 1 + 3 = 2\sqrt{3} = 7 - 2\sqrt{3} \notin Q$

But $(a, c) \notin R_2$ because,

$a^2 + c^2 = \left(1+\sqrt{3}\right)^2 + \left(1-\sqrt{3}\right)^2$

$= 1 + 3 + 2\sqrt{3} + 1 + 3 - 2\sqrt{3} = 8 \in Q$

So, R_2 is not transitive.

Hence, neither R_1 nor R_2 is transitive.

192. Let W denotes the words in the English dictionary define the relation R by R = {(x, y)∈ W×W: the words x and y have at least one letter in common}. Then, R is

(a) reflexive, symmetric and not transitive
(b) reflexive, symmetric and transitive
(c) reflexive, not symmetric and transitive
(d) not reflexive, symmetric and transitive

AIEEE-2006

Ans. (a) : Let x be a word , x have every letter common,

Therefore,

$(x, x) \in R$.

So, r is reflexive.

Let, us Consider $(x, y) \in R$ thus x, y have at least one letter is common. y, x have atleast one letter is common. thus, R is symmetric.

Assume, x = AND, y = NEXT, Z = HER

Then, $(x, y) \in R$ and $(y, z) \in R$

But, $(y, z) \notin R$.

Thus, R is not transitive.

193. Let the relation ρ be defined on R as aρb if 1 + ab > 0. Then

(a) ρ is reflexive only.
(b) ρ is equivalence relation.
(c) ρ is reflexive and transitive but not symmetric
(d) ρ is reflexive and symmetric but not transitive.

WB JEE-2019

Ans. (d) : We observe the following properties:
Reflexivity : Let a be an arbitrary element of R,
Then, $a \in R$

$$1 + a.a = 1 + a^2 > 0 \quad [\because a^2 > 0 \text{for all } a \in R]$$

$$(a, a) \in R_1 \qquad \text{[By def. of } R_1]$$

Thus, $(b, a) \in R$, for all $a, b \in R$
So, R_1 is symmetric on R,

Transitivity, we observe that $\left(1, \frac{1}{2}\right) \in R_1$ and

$$\left(\frac{1}{2}, -1\right) \in R_1 \text{ but } (1, -1) \neq R_1 \text{because}$$

$$1 + 1 \times (-1) = 0 \not> 0$$

So, R_1 is not transitive on R.

194. Let A = {2, 3, 4, 5, ... 30} and '≃' be an equivalence relation on A×A, defined by (a, b) ≃ (c, d), if and only if ad = bc. Then, the number of ordered pairs, which satisfy this equivalence relation with ordered pair (4, 3) is equal to

(a) 5 (b) 6
(c) 8 (d) 7

JEE Main 16.03.2021 Shift-II

Ans. (d) : Given,
Set A = {2, 3, 4, 5, ... 30} where A×A is defined by
(a, b) ≃ (c, d). Hence, (a, b) ≃ (c, d) implies that it reflexive, symmetric and transitive conditions.
Given, (a, b) ≃ (c, d)

ad = bc

Now ordered pair (4, 3)

(4, 3) ≃ (c, d)

4d = 3c

$$\frac{4}{3} = \frac{c}{d}$$

$(c, d) \in \{2, 3, 4, 5,\ldots.. 30\}$

$$\frac{c}{d} = \frac{4}{3}$$

(c, d) = (4, 3) (8, 6) (12, 9) (16, 12) (20, 15) (24, 18) (28, 21)

Hence, n. of order pair = 7.

195. Let R = {(P, Q)|, P and Q are at the same distance from the origin} be a relation, then the equivalence class of (1, –1) is the set

(a) $S = \{(x, y)|x^2 + y^2 = 4\}$
(b) $S = \{(x, y)|x^2 + y^2 = 1\}$
(c) $S = \{(x, y)|x^2 + y^2 = \sqrt{2}\}$
(d) $S = \{(x, y)|x^2 + y^2 = 2\}$

JEE Main 26.02.2021 Shift-I

Ans. (d) : Equivalence class of (1, –1) is a circle with centre.
Radius of circle at (1, –1) from origin

$$r = \sqrt{(1-0)^2 + (-1+0)^2} = \sqrt{2}$$

Equation of circle

$x^2 + y^2 = r^2$

$x^2 + y^2 = \left(\sqrt{2}\right)^2$

$x^2 + y^2 = 2$

Which is symmetric, reflexive and transitive.
So relation

$S = \{(x, y)|\ x^2 + y^2 = 2\}$

is equivalence relation.

196. Which of the following is not correct for relation R on the set of real numbers?

(a) $(x, y) \in R \Leftrightarrow 0 < |x| - |y| \le 1$ is neither transitive nor symmetric.
(b) $(x, y) \in R \Leftrightarrow 0 < |x-y| \le 1$ is symmetric and transitive.
(c) $(x, y) \in R \Leftrightarrow |x| - |y| \le 1$ is reflexive but not symmetric
(d) $(x, y) \in R \Leftrightarrow |x-y| \le 1$ is reflexive and symmetric.

JEE Main 31.08.2021 Shift-I

Ans. (b) : $(x, y) \in R \Rightarrow 0 < |x-y| \le 1$

$(1, 2) \in R \Rightarrow 0 < |1-2| \le 1$

$\Rightarrow 0 < |-1| \le 1$

$(2, 3) \in R \Rightarrow 0 < |2-3| \le 1$

$\Rightarrow 0 < |1-1| \le 1$

But $(1, 3) \in R \Rightarrow 0 < |1-3| \le 1$

$\Rightarrow 0 < |-2| \le 1$

Hence, it is not transitive.

197. Define a relation R over a class of n × n real matrices A and B as "ARB, if there exists a non -singular matrix P such that $PAP^{-1} = B$". Then which of the following is true?

(a) R is symmetric, transitive but not reflexive.
(b) R is reflexive, symmetric but not transitive.
(c) R is an equivalence relation.
(d) R is reflexive, transitive but not symmetric.

JEE Main 18.03.2021, Shift-II

Ans. (c) : A and B are matrices of n × n order and ARB if there exists a non-singular matrix P(det (P) ≠ 0)
Such that $PAP^{-1} = B$

For reflexive –

$ARA \Rightarrow PAP^{-1} = A$ ….(i) must be true

For $P = I$, Equation (i) is true so 'R' is reflexive

For symmetric –

$ARB \Leftrightarrow PAP^{-1} = B$ …..(i) is true

For BRA if $PBP^{-1} = A$ ……(ii) must be true

$\because PAP^{-1} = B$

$P^{-1} PAP^{-1} = P^{-1}B$

$IAP^{-1}P = P^{-1}BP$

$A = P^{-1}BP$ …..(iii)

From equation (ii) and (iii) $PBP^{-1} = P^{-1}BP$ can be true some $P = P^{-1}$

$\Rightarrow \quad P^2 = I \quad (\because \det(P) \neq 0)$

So, R is symmetric.

For transitive –

$ARB \Leftrightarrow PAP^{-1} = B$ ….is true

$BRC \Leftrightarrow PBP^{-1} = C$ is true

Now, $P\,PAP^{-1}P^{-1} = C$

$P^2A\,(P^2)^{-1} = C$

$\Rightarrow \quad ARC$

So, 'R' is transitive relation

$\Rightarrow$ Hence, R is equivalence.

198. If A = {2, 3, 4, 5}, B = {36, 45, 49, 60, 77, 90} and let R be the relation 'is factor of ' from A to B. Then the range of R is the set

(a) {60}
(b) {36, 45, 60, 90}
(c) {49, 77}
(d) {49, 60, 77}
(e) {36, 45, 49, 60 77, 90}

Kerala CEE-2020

Ans. (b) : We have,

A { 2, 3, 4, 5}

B = {36, 45, 49, 60, 77, 90}

R : number form B with factor from A

$$\begin{array}{c|c} 2 & 36 \\ \hline 2 & 18 \\ \hline 2 & 9 \\ \hline 3 & 3 \\ \hline & 1 \end{array} \qquad \begin{array}{c|c} 3 & 45 \\ \hline 3 & 15 \\ \hline & 5 \end{array} \qquad \begin{array}{c|c} 7 & 49 \\ \hline 7 & 7 \\ \hline & 1 \end{array}$$

Factor of 49 = 7

$\therefore$ factor of 36 = 2, 3

factor of 45 = 3, 5

$$\begin{array}{c|c} 2 & 60 \\ \hline 2 & 30 \\ \hline 3 & 15 \\ \hline 5 & 5 \\ \hline & 1 \end{array} \qquad \begin{array}{c|c} 2 & 77 \\ \hline 11 & 11 \\ \hline & 1 \end{array}$$

$\therefore$ factor of 77 = 7 and 11

Fctor of 60 = 2, 3, 5

$$\begin{array}{c|c} 2 & 90 \\ \hline 3 & 45 \\ \hline 3 & 15 \\ \hline 5 & 5 \\ \hline & 1 \end{array}$$

factor of 90 = 2, 3, 5

$R = \{36, 45, 60, 90\}$

199. On the set N of all natural numbers define the relation R by a Rb if and only of the GCD of a and b is 2, then R is

(a) reflexive but not symmetric
(b) symmetric only
(c) reflexive and transitive
(d) reflexive symmetric and transitive
(e) not reflexive not symmetric and not transitive

Kerala CEE-2007

Ans. (b): The relation R is defined by aRb, if and only if the GCD of a and b is 2.

aRb = GCD of a and b is 2

(i) aRb , then GCD of a and a is a.

$\therefore$ R is not reflexive.

(ii) aRb $\Rightarrow$ bRa

if GCD of a and b is 2, then GCD of b and a is 2.

$\therefore$ R is symmetiric .

(iii) aRb, bRc $\Rightarrow$ cRa

if GCD of a and b is 2 and GCD of b and c is 2. Then it is need net to be GCD of c and is 2.

$\therefore$ R is transitive.

200. If n(A) = 2 and total number of possible relations from set A to set B is 1024, then n(B) is

(a) 20 (b) 10 (c) 5 (d) 512

Karnataka CET 2020

Ans. (c) : Given, n(A) = 2

And, total number of possible relations from set A to set B is 1024.

Then, find n(B) = ?

$2^{\{n(A).\,n(B)\}}$ = Total number of possible relations from set A to set B

$2^{\{n(A).\,n(B)\}} = 1024$

$2^{\{n(A).\,n(B)\}} = 2^{10}$

$n(A).n(B) = 10$

$2n(B) = 10$

$n(B) = 5$

201. If a relation R on the set {1, 2, 3} be defined by R = {(1, 1)}, then R is

(a) Reflective and transitive
(b) Symmetric and transitive
(c) Only symmetric
(d) Reflexive and symmetric

Karnataka CET 2020

Ans. (b) : Given, A set {1, 2, 3} be defined by relation R = { (1, 1)}

Then, check relation are -

(a) Reflexive :-

In this relation,

(1, 1) $\in$R but (2, 2) (3, 3) $\in$R

So, it is not reflexive relation.

(b) Symmetric : –

${}_1R_2 \Rightarrow {}_2R_1$

Means – (a, b) = (1, 1) $\Rightarrow \in$R

Then, (b, a) = (1, 1) $\in$R

So, it is symmetric relation
(c) Transitive -
If $(a, b) = (1, 1) \in R$
$(b, c) = (1, 1) \in R$
Then, $(a, c) = (1,1) \in R$
So, it is transitive relation.
Hence, R is symmetric and transitive but not reflexive relation.

202. Let S be the set of all real numbers. A relation R has been defined on S by aRb $\Leftrightarrow |a - b| \leq 1$, then R is
(a) symmetric and transitive but not reflexive
(b) reflexive and transitive but not symmetric
(c) reflexive and symmetric but not transitive
(d) an equivalence relation

Karnataka CET 2014
BITSAT-2013

Ans. (c) : Given,
S = set of all real numbers
and ${}_aR_b \Leftrightarrow |a–b| \leq 1$
Then, cheek relation are –
(a) Reflexive :–
Given, $aRb = |a - b| \leq 1$
For aRa then,
$|a - a| \leq 1$
$0 \leq 1$
Then, it is reflexive relation
(b) Symmetric : –
Let, $aRb = | a - b| \leq 1$, then
$bRa \Rightarrow | b - a| \leq 1$
$|-(a - b)| \leq 1$
$| a - b| \leq 1$
Then, it is symmetric relation.
(c) Transitive :-
Let, $aRb = | a - b| \leq 1$and $bRc = | b - c| \leq 1$
Then, $| a - c| \leq 1$ is not always true.
Then, it is not transitive relation .
So, R is reflexive and symmetric but not transitive.

203. Let R be an equivalence relation defined on a set containing 6 elements. The minimum number of ordered pairs that R should contain is
(a) 6 (b) 12 (c) 36 (d) 64

Karnataka CET 2010

Ans. (a) : Given, R be an equivalence relation defined on a set containing 6 element.
Let A = { 1, 2, 3, 4, 5, 6}
Here, R is an equivalence relation on set A.
Then, it must be satisfies reflexive property
${}_1R_1, \forall\ 1 \in A$.
It is true for set A. Then it is reflexive relation.
So, the minimum number of ordered pairs that R should contain is
R = {(1, 1), (2, 2), (3, 3), (4, 4), (5, 5) (6, 6)}

204. Define a relation R on A = {1, 2, 3, 4} as xRy if x divides y. R is
(a) reflexive and transitive
(b) reflexive and symmetric
(c) symmetric and transitive
(d) equivalence

Karnataka CET 2011

Ans. (a) : Given, A relation R on A = {1, 2, 3, 4} as xRy if x divides y.
Then, check relation R are -
(a) Reflexive :-
x divides x, $x \in A$
It is true because 1 divides 1
2 divides 2
3 divides 3
4 divides 4
Then, satisfies the condition -
$xRx,\ x \in A$.
It is a reflexive relation
(b) Symmetric relation :-
Since, x divides y $\not\Rightarrow$ y divides x, $x,y \in A$
It is not true because -
1 divides 2 $\not\Rightarrow$ 2 not divides 1
2 divides 4 $\not\Rightarrow$ 4 not divides 2
Then, does not satisfies the condition -
$xRy \not\Rightarrow yRx,\ x \in A, y \in A$.
It is not a symmetric relation -
(c) Transitive relation : -
Let $x, y, z \in A$
Since, x divides y, y divides $\Rightarrow$ x divides z
It is true, because -
1 Divides 2, 2 divides 4 $\Rightarrow$ 1 divides 4
Then satisfies the condition -
$xRy, yRz \Rightarrow xRz$
It is a transitive relation –
So, R is reflexive and transitive relation but not symmetric relation.

205. R is a relation on N given by R = {(x, y) |4x + 3y = 20}. Which of the following belongs to R?
(a) (3, 4) (b) (2, 4)
(c) (–4, 12) (d) (5, 0)

Karnataka CET 2008

Ans. (b) : Given, R is a relation on N given by –
R = { (x, y)| 4x + 3y = 20}.
Check from options –
(a) $4 \times 3 + 3 \times 4 = 12 + 12 = 24 \neq 20$
(b) $4 \times 2 + 3 \times 4 = 8 + 12 = 20 = 20$
(c) $4 \times -4 + 3 \times 12 = -16 + 36 = 20$
(d) $4 \times 5 + 3 \times 0 = 20 + 0 = 20 = 20$
Here, option (a) does not satisfies the condition. and option (c) and option (d) is not natural number.
So, (2, 4) is belongs to R.

E. Properties of Functions and its Graphs

206. The number of real solution of the equation $x|x+5| + 2|x+7| - 2 = 0$ _____.

JEE MAIN-05.04.2024, Shift-II

Ans. (3) : Given equation,

$x|x+5| + 2|x+7| - 2 = 0$

Case I: $x \le -7$

$-x^2 - 5x - 2x - 14 - 2 = 0$

$x^2 + 7x + 16 = 0$

$D = 49 - 64 < 0$ (i.e there is no real roots)

Case II : $-7 \le x < -5$

$-x^2 - 5x + 2x + 14 - 2 = 0$

$-x^2 - 3x + 12 = 0$

$x = \dfrac{-3 \pm \sqrt{9+48}}{2}$

$= \dfrac{-3 \pm \sqrt{57}}{2}$

$x = \dfrac{-3-\sqrt{57}}{2}, \dfrac{-3+\sqrt{57}}{2}$ (Rejected because $-7 \le x < -5$)

Then we have only $x = \dfrac{-3-\sqrt{57}}{2}$

Case III : $-5 \le x \le \infty$

$x^2 + 5x + 2x + 12 = 0$

$x^2 + 7x + 12 = 0$

$x = -3, -4$

Hence, total number of real solution = 3

207. If $S = \{a \in R : |2a - 1| = 3[a] + 2\{a\}\}$, where [t] denotes the greatest integer less than or equal to t and {t} represent the fractional part of t, then $72\sum_{a \in S} a$ is equal to ________.

JEE MAIN-05.04.2024, Shift-I

Ans. (18) : Given,

$S = \{a \in R : |2a - 1| = 3[a] + 2\{a\}\}$

$|2a - 1| = 3[a] + 2\{a\}$

$|2a - 1| = [a] + 2a$

Case-1: $a > \dfrac{1}{2}$

$2a - 1 = [a] + 2a$

$[a] = -1$ $\quad\therefore a \in [-1, 0)$ Reject

Case -2 $a < \dfrac{1}{2}$

$-2a + 1 = [a] + 2a$

$-2a + 1 = 0 + 2a$

$4a = 1$

Hence, $a = \dfrac{1}{4}$

Therefore, $72\sum_{a \in S} a = 72 \times \dfrac{1}{4} = 18$

208. If a function f satisfies $f(m + n) = f(m) + f(n)$ for all $m, n \in N$ and $f(1) = 1$, then the largest natural number λ such that $\sum_{k=1}^{2022} f(\lambda + k) \le (2022)^2$ is equal to ______.

JEE MAIN-09.04.2024, Shift-I

Ans. (1010) : $f(m + n) = f(m) + f(n)$

$\Rightarrow$ $f(x) = kx$

$\Rightarrow$ $f(1) = 1$

$\Rightarrow$ $k = 1$

$f(x) = x$

Now $\sum_{k=1}^{2022} f(\lambda + k) \le (2022)^2$

$\Rightarrow$ $\sum_{k=1}^{2022} (\lambda + k) \le (2022)^2$

$\Rightarrow$ $2022\lambda + \dfrac{2022 \times 2023}{2} \le (2022)^2$

$\Rightarrow$ $\lambda \le 2022 - \dfrac{2023}{2}$

$\Rightarrow$ $\lambda \le 1010.5$

$\therefore$ Largest natural number λ is 1010.

209. Let the set of all values of p, for which $f(x) = (p^2 - 6p + 8)(\sin^2 2x - \cos^2 2x) + 2(2-p)x + 7$ does not have any critical point, be the interval (a, b). Then 16ab is equal to ______

JEE MAIN-09.04.2024, Shift-II

Ans. (252) : Given,

$f(x) = -(p^2 - 6p + 8)\cos 4x + 2(2-p)x + 7$

$f'(x) = +4(p^2 - 6p + 8)\sin 4x + (4 - 2p) \ne 0$

$\sin 4x \ne \dfrac{2p-4}{4(p-4)(p-2)}$

$p \ne 2, 4$

$\Rightarrow$ $\sin 4x \ne \dfrac{1}{2(p-4)}$

$\Rightarrow$

$\left|\dfrac{1}{2(p-4)}\right| > 1$

[For not having any critical point in the interval (a, b)]

on solving we get

$\therefore p \in \left(\dfrac{7}{2}, \dfrac{9}{2}\right)$

Hence $a = \dfrac{7}{2}, b = \dfrac{9}{2}$

$\therefore$ 16ab = 252

210. Let $f(x) = 2^x - x^2$, $x \in R$. If m and n are respectively, the number of points at which the curves $y = f(x)$ and $y = f'(x)$ intersect the x-axis, then the value of m + n is _____.

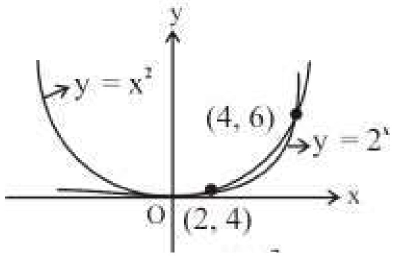
JEE MAIN-29.01.2024, Shift-I

Sol. (5) : $y = 2^x - x^2$ meet the x – axis

$y = 0$

$\Rightarrow \quad 2^x - x^2 = 0$

$2^x = x^2$



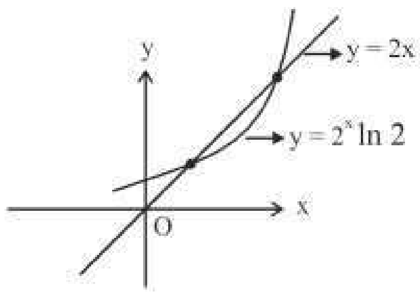
Number of point of intersection = 3

$m = 3$

$y = f'(x)$

$y = 2^x \ln 2 - 2x$ meet the x-axis at $y = 0$

$2^x \ln 2 = 2x$



Number of point of intersection = 2

$n = 2$

so, $m + n = 5$

211. Let A = {1, 2, 3, 4, 5} and B = { 1, 2, 3, 4, 5, 6}. Then the number of functions f:A→B satisfying f(1) + f(2) = f(4)–1 is equal to ....

JEE MAIN-11.04.2023, Shift-II

Ans. (360) : $f(1) + f(2) + 1 = f(4) \leq 6$

$f(1) + f(2) \leq 5$

Case (i) $f(1) = 1 \Rightarrow f(2) = 1, 2, 3, 4 \Rightarrow$ 4 mappings

Case (ii) $f(1) = 2 \Rightarrow f(2) = 1, 2, 3 \Rightarrow$ 3 mappings

Case (iii) $f(1) = 3 \Rightarrow f(2) = 1, 2 \Rightarrow$ 2 mappings

Case (iv) $f(1)4 \Rightarrow f(2) = 1 \Rightarrow$ 1 mapping

f(5) & f(6) both have 6 mappings each

Number of functions = $(4 + 3 + 2 + 1) \times 6 \times 6 = 360$

212. Let a, b, c be three distinct positive real numbers such that

$(2a)^{\log_e a} = (bc)^{\log_e b}$ and $b^{\log_e 2} = a^{\log_e c}$. Then 6a + 5bc is equal to _______.

JEE MAIN-10.04.2023, Shift-I

Ans. (Bonus) : $(2a)^{\ln a} = (bc)^{\ln b}$: $2a > 0, bc > 0$

$\ln a (\ln 2 + \ln a) = \ln b (\ln b + \ln c)$ … (i)

$\ln 2 . \ln b = \ln c . \ln a$ … (ii)

Let, $\ln 2 = \alpha$, $\ln a = x$, $\ln b = y$, $\ln c = z$

Then equation (ii) becomes - $\alpha y = xz$

$\Rightarrow \alpha = \dfrac{xz}{y}$

and (i) be $x(\alpha + x) = y(y + z)$

$x\left(\dfrac{xz}{y} + x\right) = y(y + z)$

$x^2 (z + y) = y^2 (y + z)$

$y + z = 0$ or $x^2 = y^2 \Rightarrow x = -y$

$\ln b + \ln c = 0$ or $\ln a + \ln b = 0$

$\ln bc = 0$ or $\ln ab = 0$

$bc = 1$ or $ab = 1$

$bc = 1$ or $ab = 1$

$(1)\text{if } bc = 1 \Rightarrow (2a)^{\ln a} = 1 \begin{cases} \to a = 1 \\ \to a = 1/2 \end{cases}$

$(a, b, c) = \left(\dfrac{1}{2}, \lambda, \dfrac{1}{\lambda}\right), \lambda \neq 1, 2, \dfrac{1}{2}$

Then,

$6a + 5bc = 3 + 5 = 8$

(II) $(a, b, c) = \left(\lambda, \dfrac{1}{\lambda}, \dfrac{1}{2}\right), \lambda \neq 1, 2, \dfrac{1}{2}$

In this situation infinite answer are possible

So, Bonus.

213. If $f(x) = \dfrac{(\tan 1^\circ)x + \log_e(123)}{x\log_e(1234) - (\tan 1^\circ)}, x > 0$, then the least value of $f(f(x)) + f\left(\left(\dfrac{4}{x}\right)\right)$ is:

(a) 2 (b) 4

(c) 8 (d) 0

JEE MAIN-10.04.2023, Shift-I

Ans. (b) : Given,

$f(x) = \dfrac{(\tan 1)x + \log 123}{x\log 1234 - \tan 1}$

Let $A = \tan 1$, $B = \log 123$, $C = \log 1234$

$f(x) = \dfrac{Ax + B}{xC - A}$

$f(f(x)) = \dfrac{A\left(\dfrac{Ax + B}{Cx - A}\right) + B}{C\left(\dfrac{Ax + B}{Cx - A}\right) - A}$

$= \dfrac{A^2x + AB + xBC - AB}{ACx + BC - ACx + A^2}$

$= \dfrac{x(A^2 + BC)}{(A^2 + BC)} = x$

$$f(f(x)) = x$$

$$f\left(f\left(\frac{4}{x}\right)\right) = \frac{4}{x}$$

$$f(f(x)) + f\left(f\left(\frac{4}{x}\right)\right)$$

$\because$ AM $\geq$ GM

$$x + \frac{4}{x} \geq 4$$

214. Let [α] denote the greatest integer $\leq \alpha$. Then $\left[\sqrt{1}\right]+\left[\sqrt{2}\right]+\left[\sqrt{3}\right]+...\left[\sqrt{120}\right]$ is equal to _____.

JEE MAIN-13.04.2023, Shift-II

Ans. (825) : $S = \left[\sqrt{1}\right]+\left[\sqrt{2}\right]+\left[\sqrt{3}\right]+....+\left[\sqrt{120}\right]$

$$\left[\sqrt{1}\right] = [1] = 1$$

$$\left[\sqrt{2}\right] = [1.414] = 1$$

$$\left[\sqrt{3}\right] = [1.732] = 1$$

$$\left[\sqrt{1}\right] \to \left[\sqrt{3}\right] = 1\times 3$$

$$\left[\sqrt{4}\right] \to \left[\sqrt{8}\right] = 2\times 5$$

$$\left[\sqrt{9}\right] \to \left[\sqrt{15}\right] = 3\times 7$$

$$\left[\sqrt{100}\right] \to \left[\sqrt{120}\right] = 10\times 21$$

$$S = 1\times3 + 2\times5 + 3\times7 + ... + 10\times21$$

$$= \sum_{r=1}^{10} r(2r+1)$$

$$= 2\sum_{r=1}^{10} r^2 + \sum_{r=1}^{10} r$$

$$= \frac{2\times10\times11\times21}{6} + \frac{10\times11}{2}$$

$= 770 + 55$

$= 825$

215. Let A = {x ∈ ℝ : [x + 3] + [x + 4] ≤ 3},

$B = \left\{x \in \mathbb{R} : 3^x\left(\sum_{r=1}^{\infty}\frac{3}{10^r}\right)^{x-3} < 3^{-3x}\right\}$, where [t] denotes greatest integer function. Then,

(a) $A \subset B, A \neq B$

(b) $A \cap B = \phi$

(c) $A = B$

(d) $B \subset C, A \neq B$

JEE MAIN-06.04.2023, Shift-I

Ans. (c) :

A = {x ∈ ℝ : [x + 3] + [x + 4] ≤ 3},

[x] + 3 + [x] + 4 ≤ 3

2[x]+ 7 ≤ 3

2[x]≤ – 4

[x]≤ – 1

A ⇒ x∈ (–∞, –1)

$$B = \left\{x \in R : 3^x\left(\sum_{r=1}^{\infty}\frac{3}{10^r}\right)^{x-3} < 3^{-3x}\right\}$$

$$3^x\left(\sum_{r=1}^{\infty}\frac{3}{10^r}\right)^{x-3} < 3^{-3x} \qquad ...(i)$$

$$\sum_{r=1}^{\infty}\frac{3}{10^r} = \frac{3}{10} + \frac{3}{10^2} + \frac{3}{10^3} + ... + \frac{3}{10^{\infty}}$$

$$= \frac{3}{10}\left(1 + \frac{1}{10} + \frac{1}{10^2} + ...\right)$$

$$= \frac{3}{10}\left(\frac{1}{1-\frac{1}{10}}\right) = \frac{3}{10}\times\frac{10}{9} = \frac{1}{3}$$

From equation (i)

$$3^x\left(\sum_{r=1}^{\infty}\frac{3}{10^r}\right)^{x-3} < 3^{-3x}$$

$$3^x\left(\frac{1}{3}\right)^{x-3} < 3^{-3x}$$

$$(3)^{x-x+3} < 3^{-3x}$$

$$3^3 < 3^{-3x}$$

3< – 3x

x < – 1

B ⇒ x < –1

Hence, A = B

216. Suppose f is a function satisfying f(x + y) = f(x) + f(y) for all x, y ∈ N and f(1) = $\frac{1}{5}$. If $\sum_{n=1}^{m}\frac{f(n)}{n(n+1)(n+2)} = \frac{1}{12}$, then m is equal to ________.

JEE MAIN-29.01.2023, Shift-I

Ans. (10) :

f(x + y) = f(x) + f(y)

⇒ f(x) = kx

$$f(1) = \frac{1}{5}$$

f(1) = k.1

$$\Rightarrow \frac{1}{5} = k$$

$$\therefore f(x) = \frac{1}{5}x$$

$$\sum_{n=1}^{m}\frac{f(n)}{n(n+1)(n+2)}=\frac{1}{5}\sum_{n=1}^{m}\frac{n}{n(n+1)(n+2)}=\frac{1}{12}$$

$$\Rightarrow \frac{1}{5}\sum_{n=1}^{m}\frac{1}{(n+1)(n+2)}=\frac{1}{12}$$

$$\sum_{n=1}^{m}\left(\frac{1}{n+1}-\frac{1}{n+2}\right)=\frac{5}{12}$$

$$\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)+\left(\frac{1}{4}-\frac{1}{5}\right)+.....+\left(\frac{1}{m+1}-\frac{1}{m+2}\right)=\frac{5}{12}$$

$$\frac{m+2-2}{2(m+2)}=\frac{5}{12}$$

$$\frac{m}{2(m+2)}=\frac{5}{12}$$

12m = 10 (m + 2)
12m = 10m + 20
12m – 10m = 20
2m = 20
m = 10

217. The equation $x^2 - 4x + [x] + 3 = x[x]$, where [x] denotes the greatest integer function, has:
(a) a unique solution in $(-\infty,\infty)$
(b) exactly two solutions in $(-\infty,\infty)$
(c) a unique solution in $(-\infty, 1)$
(d) no solution

JEE MAIN-24.01.2023, Shift-I

Ans. (a) : Given equation is
$x^2 - 4x + [x] + 3 = x[x]$......(i)
where, [x] is greatest integer function.
Now, Equation (i) can be written --
$x^2 - x[x] - x - 3x + [x] + 3 = 0$
$x^2 - x[x] - x + [x] - 3x + 3 = 0$
$x^2 - x[x] - (x-[x]) - 3(x-1) = 0$
$\{x(x-[x]) - 1(x-[x])\} - 3(x-1) = 0$
$(x-[x])(x-1) - 3(x-1) = 0$
$(x-1)(x-[x]-3) = 0$
when $x - 1 = 0$ or when $x - [x] - 3 = 0$
$\Rightarrow x = 1$ $\qquad \Rightarrow x - [x] = 3$
$\Rightarrow \{x\} = 3$, $\{\because \{x\} = x - [x]\}$
Which is not possible.
So, x = 1 in Solution of equation (i)
Therefore, the equation $x^2 - 4x + [x] + 3 = x[x]$ has a unique solution in $(-\infty,\infty)$.
Thus option (a) is correct answer.

218. Let x and y be distinct integers where $1 \le x \le 25$ and $1 \le y \le 25$. Then, the number of ways of choosing x and y, such that x + y is divisible by 5, is _____.

JEE MAIN-25.01.2023, Shift-I

Ans. (120) : $x + y = 5\lambda$
Cases:

x	y	Number of ways
5λ	5λ	20
$5\lambda+1$	$5\lambda+4$	25
$5\lambda+2$	$5\lambda+3$	25
$5\lambda+3$	$5\lambda+2$	25
$5\lambda+4$	$5\lambda+1$	25

Total = 120

219. The number of functions $f: \{1,2,3,4\} \to \{a \in Z \mid |a| \le 8\}$ satisfying $f(n) + \frac{1}{n}f(n+1) = 1, \forall n \in \{1,2,3\}$ is
(a) 2
(b) 3
(c) 4
(d) 1

JEE MAIN-25.01.2023, Shift-II

Ans. (a) : Given,
$f(n) + \frac{1}{n}.f(n+1) = 1$
n. f (n) + f (n + 1) = n
If, n = 1
f (1) + f (2) = 1 ....(i)
If, n = 2
2f (2) + f (3) = 2 ...(i)
If, n = 3
3.f(3) + f (4) = 3 ... (iii)
From equation (i), we get-
2 f (1) + 2f (2) = 2 ... (iv)
On subtracting equation (iv) from (ii), we get-
f (3) – 2 f (1) = 0
f (3) = 2f (1) ... (v)
In equation (iii), we get
3. (2f (1)) + f (4) = 3
6 f(1) + f (4) = 3
f (4) = 3 – 6 f(1)
Now, $-8 \le f(4) \le 8$
$-8 \le 3 - 6f(1) \le 8$
$\frac{-5}{6} \le f(1) \le \frac{11}{6}$
$\therefore$ f(1) = 0, 1
Case-I f (1) = 0, f (2) = 1
f (3) = 0, $\Rightarrow$ f (4) = 3
Case-II f (1) = 1, f (2) = 0
f (3) = 2, $\Rightarrow$ f (4) = –1
The number of possible function is 2.

220. Let $f(x) = 2x^n + \lambda$, $\lambda \in R$, $m \in N$, and $f(4) = 133$, $f(5) = 255$. Then the sum of all the positive integer divisors of $(f(3) - f(2))$ is
(a) 61 (b) 58
(c) 59 (d) 60

JEE MAIN-25.01.2023, Shift-II

Ans. (d) : Given function,

$f(x) = 2x^n + \lambda, \lambda \in R$

$f(4) = 133$

$f(5) = 255$

$133 = 2.4^n + \lambda$ ……(i)

$255 = 2.5^n + \lambda$ ……(ii)

On subtracting equation (i) from (ii), we get–

$122 = 2(5^n - 4^n)$

$61 = 5^n - 4^n$

here, n = 3

From equation (i), we get–

$133 = 2.4^3 + \lambda$

$= 2.64 + \lambda$

$133 = 128 + \lambda$

$\Rightarrow \lambda = 5$

$f(x) = 2\times3 + 5$

$\Rightarrow f(3) = 2.3^3 + 5 = 2.27 + 5 = 54 + 5 = 59$

$f(2) = 2.2^3 + 5 = 2.8 + 5 = 21$

$f(3) - f(2) = 59 - 21 = 38$

$= 2 \times 19$

Sum of all the positive integers

divisors = 2 + 19 + 38 + 1

= 60

221. If $f(x) = \frac{2^{2x}}{2^{2x}+2}, x \in R$, **then** $f\left(\frac{1}{2023}\right) + f\left(\frac{2}{2023}\right) + ... + f\left(\frac{2022}{2023}\right)$ **is equal to**

(a) 2010

(b) 2011

(c) 1011

(d) 1010

JEE MAIN-24.01.2023, Shift-II

Ans. (c) : Given,

$$f(x) = \frac{4^x}{4^x+2}$$

$$f(1-x) = \frac{4^{1-x}}{4^{1-x}+2}$$

$$\Rightarrow f(x) + f(1-x) = \frac{4^x}{4^x+2} + \frac{4^{1-x}}{4^{1-x}+2}$$

$$= \frac{4^x}{4^x+2} + \frac{4}{4+2.4^x}$$

$$= \frac{4^x}{4^x+2} + \frac{2}{2+4^x} = \frac{4^x+2}{4^x+2} = 1$$

$$\Rightarrow f\left(\frac{1}{2023}\right) + f\left(\frac{2}{2023}\right) + f\left(\frac{3}{2023}\right) + ... + f\left(\frac{2022}{2023}\right)$$

$$\therefore \quad f\left(\frac{1}{2023}\right) + f\left(\frac{2022}{2023}\right) = 1$$

$$f\left(\frac{2}{2023}\right) + f\left(\frac{2021}{2023}\right) = 1$$

$\vdots$

$$f\left(\frac{1011}{2023}\right) + f\left(\frac{1012}{2023}\right) = 1$$

$\Rightarrow$ 1 + 1 + 1 + ... (1011 times) = 1011

222. Let $x = (8\sqrt{3}+13)^{13}$ **and** $y = (7\sqrt{2}+9)^9$. **If [t] denotes the greatest integer ≤ t, then**

(a) [x] + [y] is even

(b) [x] is odd but [y] is even

(c) [x] and [y] are both odd

(d) [x] is even but [y] is odd

JEE MAIN-30.01.2023, Shift-II

Ans. (a) : Given, $x = (8\sqrt{3}+13)^{13}$ $y = (7\sqrt{2}+9)^9$

Now,

Let, $I + f = (8\sqrt{3}+13)^{13}$ where I = Integral part

$f' = (8\sqrt{3}-13)^{13}$ f = fractional part

$\Rightarrow I + f - f' = (8\sqrt{3}+13)^{13} - (8\sqrt{3}-13)^{13}$

$\Rightarrow I + f - f' = [{}^{13}C_0 (8\sqrt{3})^{13} + {}^{13}C_1 (8\sqrt{3})^{12}(13)^1 + {}^{13}C_2 (8\sqrt{3})^{11}(13)^2 + {}^{13}C_3 (8\sqrt{3})^{10}(13)^3 + ...+ {}^{13}C_{13} (8\sqrt{3})^{13}(13)^{13}] - [{}^{13}C_0 (8\sqrt{3})^{13} - {}^{13}C_1 (8\sqrt{3})^{12}(13)^1 + {}^{13}C_2 (8\sqrt{3})^{11}(13)^2 - {}^{13}C_3 (8\sqrt{3})^{10}(13)^3 + {}^{13}C_4 (8\sqrt{3})^9 (13)^4 .... {}^{13}C_{13}(13)^{13}]$

$I + f - f' = 2[{}^{13}C_1 (8\sqrt{3})^{12}(13)^1 + {}^{13}C_5 (8\sqrt{3})^8 (13)^5 + ... + {}^{13}C_{13}(13)^{13}]$

I + f – f' = 2x Integer

$\Rightarrow$ I is even or [x] is even

Also,

$y = (7\sqrt{2}+9)^9 = I + f$

$f' = (7\sqrt{2}-9)^9$

$I + f - f' = (7\sqrt{2}+9)^9 - (7\sqrt{2}-9)^9$

$I + f - f' = 2[{}^9C_1 (7\sqrt{2})^8 (9)^1 + {}^9C_3 (7\sqrt{2})^6 (9)^3 + {}^9C_5 (7\sqrt{2})^4 (9)^5 + {}^9C_7 (7\sqrt{2})^2 (9)^7 + {}^9C_9 (7\sqrt{2})^0 (9)^9]$

I + f – f' = 2 × Integer = even

$\Rightarrow$ [y] is even

223. If the functions f(x) = $\frac{x^3}{3}+2bx+\frac{ax^2}{2}$ and g(x) = $\frac{x^3}{3}+ax+bx^2$, a ≠ 2b have a common extreme point, then a + 2b + 7 is equal to:

(a) 4
(b) 6
(c) 3
(d) $\frac{3}{2}$

JEE MAIN-30.01.2023, Shift-II

Ans. (b) : $f(x)=\frac{x^3}{3}+2bx+\frac{ax^2}{2}$, $g(x)=\frac{x^3}{3}+ax+bx^2$

for extreme point find derivative of f(x) & g(x)

$f'(x) = x^2 + 2b + ax = 0$

$g'(x) = x^2 + a + 2bx = 0$

∵ Both f(x) & g(x) have common extreme points:

⇒ f'(x) = g'(x)

⇒ $x^2 + 2b + ax = x^2 + a + 2bx$

⇒ x = 1

∴ At x = 1 both f(x) & g(x) have common extreme point.

Also x = 1 will satisfies f'(x) & g'(x)

Now, f'(1) = 1 + 2b + a = 0

⇒ 2b + a+ 1 + 6 = 0 + 6

2b + a + 7 = 6

224. If $e^x = y + \sqrt{1+y^2}$ then the value of y is

(a) $\frac{1}{2\left(e^x+e^{-x}\right)}$
(b) $\frac{1}{2\left(e^x-e^{-x}\right)}$
(c) $e^x - e^{x/2}$
(d) none of these

SRMJEEE-2014

Ans. (d) : Given,

$$e^x = y + \sqrt{1+y^2}$$

Then, find y = ?

$$\therefore \quad e^x = y+\sqrt{1+y^2} \qquad ....(i)$$

Reciprocal of equation

$$\frac{1}{e^x}=\frac{1}{y+\sqrt{1+y^2}} \qquad .....(ii)$$

Then, for y subtract equation (i) from equation (ii), we get -

$$\frac{1}{e^x}-e^x=\frac{1}{\left(y+\sqrt{1+y^2}\right)}-\left(y+\sqrt{1+y^2}\right)$$

$$\frac{1}{e^x}-e^x=\frac{1-\left(y+\sqrt{1+y^2}\right)\left(y+\sqrt{1+y^2}\right)}{\left(y+\sqrt{1+y^2}\right)}$$

$$\frac{1-e^x.e^x}{e^x}=\frac{1-\left[y^2+y\sqrt{1+y^2}+y\sqrt{1+y^2}+\left(1+y^2\right)\right]}{\left(y+\sqrt{1+y^2}\right)}$$

$$\frac{1-e^{2x}}{e^x}=\frac{1-\left[1+2y^2+2y\sqrt{1+y^2}\right]}{\left(y+\sqrt{1+y^2}\right)}$$

$$\frac{1-e^{2x}}{e^x}=-\frac{2y^2+2y\sqrt{1+y^2}}{\left(y+\sqrt{1+y^2}\right)}$$

$$\frac{1-e^{2x}}{e^x}=-\frac{2y\left(y+\sqrt{1+y^2}\right)}{\left(y+\sqrt{1+y^2}\right)}$$

$$\frac{1-e^{2x}}{e^x}=-2y$$

$$y=\frac{e^{-x}-e^x}{-2}$$

$$y=\frac{e^x-e^{-x}}{2}$$

225. If $f(x)=\frac{x}{x-1}$, f(3x) in terms of f(x) is

(a) $\frac{3f(x)}{3f(x)-1}$
(b) $\frac{3f(x)}{3f(x)-3}$
(c) $\frac{3f(x)}{2f(x)+1}$
(d) 3f(x)-1

SRMJEEE-2015

Ans. (c) : Given, $f(x)=\frac{x}{x-1}$

x.f(x) – f(x) = x

xf(x) – x = f(x)

x [f(x) –1] = f(x)

$$x=\frac{f(x)}{f(x)-1} \qquad ....(1)$$

Then, $f(3x)=\frac{3x}{3x-1}$ .....(2)

Put the value of x by equation (i) in equation (ii), we get

$$f(3x)=\frac{3\times\frac{f(x)}{f(x)-1}}{3\times\frac{f(x)}{f(x)-1}-1}$$

$$f(3x)=\frac{\frac{3f(x)}{f(x)-1}}{\frac{3f(x)-\{f(x)-1\}}{f(x)-1}}$$

$$f(3x)=\frac{3f(x)}{3f(x)-f(x)+1}$$

$$f(3x)=\frac{3f(x)}{2f(x)+1}$$

226. for $f(x)=[x]$, where $[x]$ is the greatest integer function, which of the following is true, for every $x \in R$.

(a) $[x]+1=x$ (b) $[x]+1>x$

(c) $[x]+1\le x$ (d) $[x]+1<x$

MHT-CET 20

Ans. (b) : Given,

$f(x) = [x]$

We know that, the greatest integer function is also known as the step function. Greatest integer function is a function that gives the greatest integer less than or equal to a given number.

It means,

$[x] = n$, where, $n \le x < n + 1$ and 'n' is an integer.

Ex. $[5.2] = 5$ as, $5 \le 5.2 < 6$

and $[-5.3] = -6$, as $-6 \le -5.3 < -5$

Since, $x = [x] + \{x\} \Rightarrow \{x\} = x - [x]$

Where $[x]$ = Greatest integer function

$\{x\}$ = Fractional part

Then, $0 \le \{x\}<1$

So, $0 \le x - [x] < 1$

$0 + [x] \le x < 1 + [x]$

$[x] \le x < [x] + 1$

Hence, $[x] + 1 > x$

227. If $f(x)=x^2-3x+4$ and $f(x)=f(2x+1)$, then $x =$

(a) $-1,\frac{3}{2}$ (b) $-1,\frac{2}{3}$

(c) $1,\frac{2}{3}$ (d) $1,\frac{3}{2}$

MHT-CET 20

Ans. (b) : Given, $f(x) = x^2 - 3x + 4$

And $f(x) = f(2x +1)$

Then from -

$f(x) = f(2x +1)$

$x^2 - 3x + 4 = (2x +1)^2 -3 (2x +1) + 4$

$x^2- 3x + 4 = (4x^2 +4x + 1) - 6x -3 + 4$

$x^2 -3x + 4 = 4x^2 + 4x + 1 - 6x + 1$

$x^2 - 3x + 4 = 4x^2 - 2x + 2$

$4x^2 - 2x + 2 - x^2 + 3x - 4 = 0$

$3x^2 + x - 2 = 0$

$3x^2 + 3x - 2x - 2 = 0$

$3x (x + 1) -2(x + 1) = 0$

$(x + 3) (3x - 2) = 0$

So, $(x + 1) = 0$ or $(3x - 2) = 0$

$x = -1$ or $3x - 2 = 0$

$x = \frac{2}{3}$

Hence, $x = -1,\frac{2}{3}$

228. If $f(x)=ax^2+bx+2$ and $f(1)=4, f(3)=38$, then $a-b=$

(a) 8 (b) 2

(c) –2 (d) 15

MHT-CET 20

Ans. (a) : Given

$f(x) = ax^2 + bx + 2$

And $f(1) = 4$ $f(3) = 38$

Then, find $a -b$ = ?

$\therefore$ $f(1) = a\times (1)^2 + b \times 1 + 2$

$f(1) = a + b + 2$

$4 = a + b + 2$

$a + b = 4 - 2$

$a + b = 2$ .... (i)

And, $f(3) = a \times 3^2 + b \times 3 + 2$.

$38 = 9a + 3b + 2$

$9a + 3b + 2 = 38$

$9a + 3b = 36$

$3a + b = 12$ ….(ii)

On solving equation (i) and equation (ii), we get –

$a = 5, b = -3$

So, $a - b = 8$

229. If $\frac{\log x}{a-b}=\frac{\log y}{b-c}=\frac{\log z}{c-a}$, then xyz is equal to :

(a) 0 (b) 1

(c) –1 (d) 2

Ans. (b) : Given,

$$\frac{\log x}{a-b}=\frac{\log y}{b-c}=\frac{\log z}{c-a}$$

Let, $\frac{\log x}{a-b}=\frac{\log y}{b-c}=\frac{\log z}{c-a}=1$

Then, $\log x = (a - b) \times 1$

$\log x = a - b \Rightarrow x = e^{a-b}$

$\log y = (b - c) \times 1$

$\log y = b - c$

$y = e^{b-c}$

And, $\log z = 1. (c - a)$

$\log z = (c - a)$

$z = e^{c-a}$

So, $xyz = e^{a-b} \times e^{b-c} \times e^{c-a}$

$xyz = e^{a-b+b-c+c-a} = e^0 = 1$

230. A is a set having 6 distinct elements. The number of distinct functions from A to A which are not bijections is

(a) $6! - 6$ (b) $6^6 - 6$

(c) $6^6 - 6!$ (d) $6!$

Karnataka CET 2018

Ans. (c) : Given, A is a set having 6 distinct elements

Then, total number of distinct function from A to A = 6

And the total number of bijections (one-one not) from A to A = 6 !

So, the number of distinct functions from A to A which are not bijections is $6^6 - 6!$

231. If $f(x)=\sqrt{\log_{10}x^2}$. The set of all values of x for which f(x) is real, is

(a) $[-1, 1]$ (b) $[-1, \infty]$
(c) $(-\infty,1]$ (d) $(-\infty,-1]\cup[1, \infty]$

VITEEE-2010

Ans. (d) : $f(x)=\sqrt{\log_{10}x^2}$ is real, if

$\log_{10}x^2 \geq 0$

$x^2 \geq 1$

$x < -1$ and $x > 1$

$x \in (-\infty,-1]\cup[1,\infty)$

232. The value of $[(\log_b a)(\log_c b)(\log_a c)]$ is

(a) abc (b) log abc
(c) 0 (d) 1

UPSEE-2016

Ans. (d) : Given,

$[(\log_b a)(\log_c b)(\log_a c)]$

Then, $\log_b a \times \log_c b \times \log_a c$

$= \frac{\log_k a}{\log_k b}\times\frac{\log_k b}{\log_k c}\times\frac{\log_k c}{\log_k a}$

$= 1$

233. If $p=\frac{1}{\log_3\pi}+\frac{1}{\log_4\pi}+1$, then

(a) $2.5 < p < 3$ (b) $p > 3$
(c) $1.5 < p < 2$ (d) $2 < p < 2.5$

UPSEE-2016

Ans. (b) : Given, $p = \frac{1}{\log_3\pi}+\frac{1}{\log_4\pi}+1$

$p = \log_\pi 3 + \log_\pi 4 + 1$

$p = \log_\pi(3 \times 4) + 1$

$p = \log_\pi (12) + 1$

We know that –

$12 > (\pi^2) = (3.14)^2 = 9.8596$

Then, $12 > \pi^2$

$\log_\pi 12 > \log_\pi \pi^2$

$\log_\pi 12 > 2$

So, $p > 3$

234. The graph of the function y = f(x) is symmetrical about the line *x* = 2. Then,

(a) $f(x+2)=f(x-2)$
(b) $f(2+x)=f(2-x)$
(c) $f(x)=f(-x)$
(d) $f(x)=-f(-x)$

UPSEE-2010

Ans. (b) : Given,
The graph of the function y = f(x)
Is symmetrical about the line x = 2.
We know,
A function g(x) is symmetrical about y-axis means x = 0, we can write as -

$g(x) = g(-x)$

It is also written as -

$g(0 + x) = g(0 - x)$

So, function y = f(x) which is symmetrical about the line x = 2.
Then can be written as -
$f(2 + x) = f(2 - x)$

235. The number of solutions of $\log_4(x-1) = \log_2(x-3)$ is

(a) 3 (b) 1
(c) 2 (d) 0

UPSEE -2008

Ans. (b) : Given,

$\log_4(x-1) = \log_2(x-3)$

${}_4\log_4(x-1) = {}_4\log_2(x-3)$

$(x-1) = (2)^{2\log_2(x-3)}$ $[\because {}_a\log_a(x) = x]$

$(x-1) = 2^{\log_2(x-3)^2}$

$(x-1) = (x-3)^2$ $[\because x\log a = \log a^x]$

$x - 1 = x^2 + 9 - 6x$

$x^2 - 7x + 10 = 0$

$x^2 - 2x - 5x + 10 = 0$

$x(x-2) - 5(x-2) = 0$

$(x-2)(x-5) = 0$

When, x = 2

$\log_2(x-3) = \log_2(2-3) = \log_2(-1)$

$\log_2(-1)$ is not possible since, log does not have – ve value.
So, The number of solutions of $\log_4(x-1) = \log_2(x-3)$ is 1.

236. If a and b are positive integers such that (a^2-b^2) is a prime number, then

(a) $a^2-b^2=a+b$ (b) $a^2-b^2=a-b$
(c) $a^2+b^2=a-b$ (d) $a^2+b^2=a+b$

JCECE-2017

Ans. (a) : Given, a and b are positive integer such that (a^2-b^2) is a prime number.
Let a = 3, b = 2 are positive integer.
Then, $a^2 - b^2 = 3^2 - 2^2 = 9 - 4 = 5$

$a^2 - b^2 = 5$ and $a + b = 5$

Where, $a^2 - b^2 = 5$ is a prime number.
So, $a^2 - b^2 = a + b$

237. The number of integral solutions of the equation $\{x + 1\} + 2x = 4[x + 1] - 6$, is

(a) 0 (b) 1
(c) 2 (d) 3

JCECE-2016

Ans. (b) : Given, $\{x + 1\} + 2x = 4[x + 1] - 6$
We know –

$x = \{x\} + [x]$

$\{x\} = x - [x]$

Then, $x + 1 - [x + 1] + 2x = 4[x + 1] - 6$

$3x + 1 = 5[x + 1] - 6$

$3x = 5\{[x] + 1\} - 6 - 1$

$3x = 5[x] + 5 - 7$

$3x = 5[x] - 2$ .....(i)

Again we put, $x = \{x\} + [x]$

$3\{[x] + \{x\}\} = 5[x] - 2$

$3\{x\} = 2[x] - 2$

Since, $0 \le \{x\} < 1$

$0 \le 3\{x\} < 3$

And $0 \le 2[x] - 2 < 3$

$2 \le 2[x] < 5$

$1 \le [x] < \frac{5}{2}$

$\therefore$ $[x] = 1, 2$

Then, from equation (i), we get -

$[x] = 1 \Rightarrow x = 1$

$[x] = 2 \Rightarrow x = \frac{8}{3}$

So, x = 1 is the only integral equation.

238. The period of the function $f(x) = \frac{|\sin x| - |\cos x|}{|\sin x + \cos x|}$ is

(a) $\frac{\pi}{2}$ (b) 2π
(c) π (d) None of these

JCECE-2015

Ans. (c) : Given,

$$f(x) = \frac{|\sin x| - |\cos x|}{|\sin x + \cos x|}$$

$$f(x + \pi) = \frac{|\sin(\pi + x) - |\cos(\pi + x)||}{|\sin(x + \pi) + \cos(x + \pi)|}$$

$$f(x + \pi) = \frac{|\sin x| - |\cos x|}{|-\sin x - \cos x|}$$

$$f(x + \pi) = \frac{|\sin x| - |\cos x|}{|\sin x + \cos x|}$$

Here, we observe that

$f(x + \pi) = f(x), x \in R$.

So, f(x) is periodic with period π.

239. If $f(x) = \sqrt[n]{x^m}, n \in N$ is an even function, then m is

(a) even integer (b) Odd integer
(c) any integer (d) f(x) even is not possible

JCECE-2013

Ans. (a) **:** Given,

$f(x) = \sqrt[n]{x^m}, n \in N$ is an even function.

We know for even function $f(x) = f(-x)$

$$\sqrt[n]{x^m} = \sqrt[n]{(-x)^m}$$

$$x^m = (-x)^m$$

So, m is even integer.

240. Let the functions f, g, h are defined from the set of real numbers R to R such that

$f(x) = x^2 - 1$, $g(x) = \sqrt{(x^2 + 1)}$ and

$h(x) = \begin{cases} 0, & \text{if } x < 0 \\ x, & \text{if } x \ge 0, \end{cases}$ **then ho(fog) (x) is defined by**

(a) x (b) x^2
(c) 0 (d) None of these

JCECE-2008

Ans. (b) : Given,

$f(x) = x^2 - 1$, $g(x) = \sqrt{(x^2 + 1)}$

And $h(x) = \begin{cases} 0, & \text{if } x < 0 \\ x, & \text{if } x \ge 0 \end{cases}$

Then, $ho(fog)(x) = h\{f(g(x)\}$

$$= h\left\{f\left(\sqrt{x^2 + 1}\right)\right\}$$

$$= h\left\{\left(\sqrt{x^2 + 1}\right)^2 - 1\right\}$$

$$= h\{x^2 + 1 - 1\}$$

$$= h\{x^2\}$$

$$= x^2$$

So, ho(fog)(x) is defined by x^2

241. Let f(x) = x – [x] for all real number, where [x] is the integral part of x, then $\int_{-1}^{1} f(x)dx$ is equal to:

(a) 1 (b) 2
(c) 0 (d) 1/2

JCECE-2003

Ans. (a) : Given,

$f(x) = x - [x]$ $\forall$ real number

$$\int_{-1}^{1} f(x)dx = \int_{-1}^{1} (x - [x])dx$$

$$= \int_{-1}^{0} (x - [x])dx + \int_{-1}^{1} (x - [x])dx$$

$$= \int_{-1}^{0} (x + 1)dx + \int_{0}^{1} x\, dx$$

$$= \left\{\frac{x^2}{2} + x\right\}_{-1}^{0} + \left\{\frac{x^2}{2}\right\}_{0}^{1}$$

$$= \left\{0 - \left(\frac{1}{2} - 1\right)\right\} + \frac{1}{2}$$

$$= \frac{1}{2} + \frac{1}{2}$$

$$= 1.$$

242. **If $2f(x)-3f\left(\frac{1}{x}\right)=x^2, x\neq 0$ then what is f(2) equal to?**

(a) $\frac{3}{4}$ (b) $-\frac{3}{4}$

(c) $\frac{5}{4}$ (d) $-\frac{7}{4}$

SCRA-2009

Ans. (d): $2f(x)-3f\left(\frac{1}{x}\right)=x^2, x\neq 0$ ....(i)

Replace x by 1/x, we get-

$$2f\left(\frac{1}{x}\right)-3f(x)=\frac{1}{x^2} \quad ....(ii)$$

Multiply by 2 in eq^n. (i) and 3 in eq^n. (2), we get-

$$4f(x)-6f\left(\frac{1}{x}\right)=2x^2$$

$$6f\left(\frac{1}{x}\right)-9f(x)=3/x^2$$

$$-5f(x)=2x^2+3/x^2$$

$$f(x)=-\frac{1}{5}\left[2x^2+3/x^2\right]$$

Now,

$$f(2)=-\frac{1}{5}\left[2(2)^2+\frac{3}{(2)^2}\right]$$

$$f(2)=-\frac{1}{5}\left[8+\frac{3}{4}\right]$$

$$f(2)=-\frac{1}{5}\left(\frac{35}{4}\right)$$

$$f(2)=-\frac{7}{4}$$

243. **The solution of |x – 2| < 5 is all the real numbers satisfying**

(a) $-2<x<5$ (b) $-3<x<7$

(c) $-5<x<7$ (d) $-3<x<5$

SCRA-2012

Ans. (b) : Given,

$$|x-2|<5$$

$\Rightarrow -5<x-2<5$

$\Rightarrow -3<x<7$

244. **If A = {–2, –1, 0, 1, 2 } and f : A → Z, f(x) = x^2 – 2x – 3, then what is the pre-image (s) of –3 ?**

(a) 0 only (b) 2 only

(c) 0, 2 (d) Φ

SCRA-2012

Ans. (c) : Given, $f(x) = x^2 - 2x - 3$

Find pre-image at x = –3

$\because$ $f: A\to Z$

$\therefore$ $x^2-2x-3=-3$

or $x^2-2x=0$

$x(x-2)=0$

$\Rightarrow$ x = 0, 2

245. **The function f(x) = sin x + cos x will be**

(a) an even function (b) an odd function

(c) a constant function (d) None of these

CG PET- 2009

Ans. (d) : The function f(x) = sin x + cos x

$$=\sqrt{2}\left(\frac{1}{\sqrt{2}}\sin x+\frac{1}{\sqrt{2}}\cos x\right)$$

$$=\sqrt{2}\left(\sin x\cos\frac{\pi}{4}+\cos x\sin\frac{\pi}{4}\right)$$

$$=\sqrt{2}\sin\left(x+\frac{\pi}{4}\right)$$

But, $\sin\left(x+\frac{\pi}{4}\right)$ is a periodic function.

So,option (d) is correct.

246. **If $\log_{10^4}x=y$, then $\log_{10^8}x^4$ is equal to -**

(a) $\frac{2}{3}y$ (b) 3y

(c) 4y (d) 2y

CG PET- 2013

Ans. (d): Given,

$$\log_{10^4}x=y$$

$$x=(10^4)^y$$

$$x=10^{4y}$$

Then, $\log_{10^8}x^4=\frac{4}{8}\log_{10}x$

$$=\frac{4}{8}\log_{10}10^{4y}$$

$$=\frac{4y\times 4}{8}$$

$$=2y$$

247. **If for all x, y∈N, there exists a function f(x) satisfying $f(x+y)=f(x)\times f(y)$ such that $f(1)=3$ and $\sum_{x=1}^{n}f(x)=120$, then value of n will be**

(a) 4 (b) 5

(c) 6 (d) None of these

CG PET- 2015

Ans. (a) : Given, for x, y∈N,

$$f(x+y)=f(x)\cdot f(y)$$

Then function will be of the form-

$$f(x)=a^x, \text{ where } a\in N \quad [\because a\neq 1]$$

$\because$ $f(1)=3$

$\Rightarrow$ $f(1)=a^1=3$

$\Rightarrow$ a = 3

$\therefore$ Function is $f(x)=3^x$

Now, $\sum_{x=1}^{n}f(x)=120$

$\Rightarrow \quad \sum_{x=1}^{n} 3^x = 120$

$\Rightarrow \quad 3 + 3^2 + 3^3 + \ldots + 3^n = 120$

$\Rightarrow \quad \frac{3(3^n - 1)}{3-1} = 120$

$\Rightarrow \quad 3^n = 1 + \frac{120 \times 2}{3}$

$\Rightarrow \quad 3^n = 81 = 3^4$

So, $\quad n = 4$

248. If f : R → R be the signum function and g : R → R be the greatest integer function, then $\sin\left\{\pi\left((fog)\left(\frac{1}{2}\right)\right)\right\}$ is equal to

(a) 1 (b) $\frac{\sqrt{3}}{2}$

(c) 0 (d) $\frac{1}{\sqrt{2}}$

CG PET- 2016

Ans. (c) : We have,

$f(x) = \text{sgm}(x)$

And, $g(x) = [x]$

Now, $fog\left(\frac{1}{2}\right) = f\left(g\left(\frac{1}{2}\right)\right) = f\left(\left[\frac{1}{2}\right]\right)$

$= f(0) \quad [\because [0.5] = 0]$

$= \text{sgm}(0) = 0$

$[\because \text{sgm}(0) = 0]$

Now, $\sin\left[\pi\left\{fog\left(\frac{1}{2}\right)\right\}\right] = \sin(\pi \times 0) = \sin 0^\circ = 0$

249. The function $f(x) = \tan\pi x - x + [x]$ has period

(a) 1 (b) π

(c) 2π (d) None of these

CG PET- 2018

Ans. (d) : Given function is $f(x) = \tan\pi x - x + [x]$

Since, $[x]$ is not periodic function.

$\therefore$ $f(x)$ is a non-periodic function.

250. Let p and q be two real numbers such that p + q = 3 and $p^4 + q^4 = 369$. Then $\left(\frac{1}{p} + \frac{1}{q}\right)^{-2}$ is equal to ______.

JEE Main-26.06.2022, Shift-II

Ans. (4) : Given, $p + q = 3$ and $p^4 + q^4 = 369$

$\left(\frac{1}{p} + \frac{1}{q}\right)^{-2} = \left(\frac{p+q}{pq}\right)^{-2} = \frac{(pq)^2}{(p+q)^2}$

Now, $p^4 + q^4 = 369$

$(p^2 + q^2)^2 - 2(qp)^2 = 369$

$\left[(p+q)^2 - 2pq\right]^2 - 2(pq)^2 = 369$

$\left[(9 - 2pq)^2 - 2(pq)^2\right] = 369$

$81 + 4(pq)^2 - 36(pq) - 2(pq)^2 = 369$

$2(pq)^2 - 36(pq) + 81 = 369$

$\therefore \quad 2(pq)^2 - 36(pq) - 288 = 0$

Or $\quad (pq)^2 - 18(pq) - 144 = 0$

Or $\quad (pq - 24)(pq + 6) = 0$

$\Rightarrow pq = 24, -6$

So, $\left(\frac{1}{p} + \frac{1}{q}\right)^{-2} = \frac{(-6)^2}{9}$

$= \frac{36}{9} = 4$

251. The graph of the function
$y = \cos x \cos(x+2) - \cos^2(x+1)$ is a

(a) straight line passing through the point $(0, -\sin^2 1)$ and parallel to x-axis

(b) straight line passing through the origin

(c) parabola with vertex $(0, -\sin^2 1)$

(d) None of the above

SCRA-2014

Ans. (a): Given,

$y = \cos x \cos(x+2) - \text{Cos}^2(x+1)$

$= \cos(x + 1 - 1)\cos(x + 1 + 1) - \cos^2(x + 1)$

$= \cos^2(x + 1) - \sin^2 1 - \cos^2(x + 1)$

$= -\sin^2 1$

Which represent a straight line parallel to x-axis with $y = -\sin^2 1$ for all x and so also for $x = \frac{\pi}{2}$.

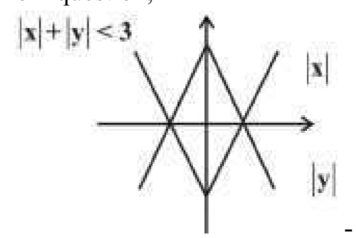
252. How many integral points are there within the graph of $|x| + |y| < 3$?

(a) 13 (b) 15

(c) 21 (d) 24

SCRA-2015

Ans. (a) : From question,



Integral points are

(–2,0), (–1,0), (0,0), (1,0), (2,0), (0,1), (0,2)

(0,–1), (0,–2), (1,1), (–1, 1), (1,–1), (–1,1)

So, total integral points are 13.

253. **Let f(x) = 2x + $\tan^{-1}$ x and g(x) = $\log_e\left(\sqrt{1+x^2}+x\right)$, x ∈ [0, 3]. Then**

(a) min f'(x) = 1 + max g'(x)
(b) there exist $0 < x_1 < x_2 < 3$ such that f(x) < g(x), ∀ x ∈ (x_1,x_2)
(c) max f(x) > max g(x)
(d) there exists x ∈ [0, 3] such that f '(x) < g' (x)

JEE Main-01.02.2023, Shift-I

Ans. (c) : Given, $f(x) = 2x + \tan^{-1}x$

And, $g(x) = \log_e\left(\sqrt{1+x^2}+x\right),\ x \in [0,3]$

Then, $f'(x) = 2 + \dfrac{1}{1+x^2}$

And, $g'(x) = \dfrac{1}{\sqrt{1+x^2}+x}\left(\dfrac{2x}{2\sqrt{1+x^2}}+1\right)$

$$g'(x) = \frac{x+\sqrt{1+x^2}}{x+\sqrt{1+x^2}} \times \frac{1}{\sqrt{1+x^2}}$$

$$g'(x) = \frac{1}{\sqrt{1+x^2}}$$

Since, both does not have critical values.

$f(0) = 0, f(3) = 6 + \tan^{-1}3$

g(0) = 0, g(3) = $\log\left(\sqrt{10}+3\right)$

Consider –

h(x) = f(x) – g(x)

∴ $h'(x) > 0\ \forall\ x \in (0,3)$

So, h(x) is increasing function.

Hence, max f(x) > max g(x)

254. **The function *f* (x) = $\sqrt{\dfrac{1}{\sqrt{x}} - \sqrt{x+1}}$ is defined for**

(a) $0 < x \le \dfrac{\sqrt{5}-1}{2}$ (b) $\dfrac{-1-\sqrt{5}}{2} < x < 0$

(c) $0 < x < \dfrac{\sqrt{3}-1}{2}$ (d) $\dfrac{-1-\sqrt{3}}{2} < x < 0$

AMU-2006

Ans. (a) : Function $\sqrt{\dfrac{1}{x} - \sqrt{x+1}}$

$\sqrt{x} \ne 0$

$x \ne 0,\qquad x+1 \ge 0$

$x > 0,\qquad x \ge -1$

$\Rightarrow\quad \dfrac{1}{\sqrt{x}} - \sqrt{x+1} \ge 0$

$\dfrac{1}{\sqrt{x}} \ge \sqrt{x+1}$

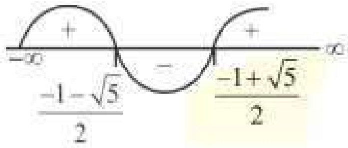
$\dfrac{1}{x} \ge x+1$

$x^2 + x - 1 \le 0$

$\Rightarrow\quad x = \dfrac{-1 \pm \sqrt{1-4\times1\times(-1)}}{2}$

$\Rightarrow\quad x = \dfrac{-1 \pm \sqrt{1+4}}{2}$

$x = \dfrac{-1+\sqrt{5}}{2}, \dfrac{-1-\sqrt{5}}{2}$



$x \in \left(\dfrac{-1-\sqrt{5}}{2}, \dfrac{-1+\sqrt{5}}{2}\right)$

∴ $\quad 0 < x \le \dfrac{\sqrt{5}-1}{2}$

255. **If $f(x) = \dfrac{x}{x-1}$ then $\dfrac{f(a)}{f(a+1)}$ is equal to**

(a) $f(a^2)$ (b) $f\left(\dfrac{1}{a}\right)$

(c) $f(-a)$ (d) $f\left[\dfrac{-a}{a-1}\right]$

AMU-2002

Ans. (a) : Given,

$$f(x) = \frac{x}{x-1}$$

Then, $\dfrac{f(a)}{f(a+1)} = \dfrac{\frac{a}{a-1}}{\frac{a+1}{a+1-1}}$

$= \dfrac{a}{a-1} \times \dfrac{a}{a+1}$

$= \dfrac{a^2}{a^2-1}$

∴ $\dfrac{f(a)}{f(a+1)} = f(a^2)$

256. **Consider a function f: N → R, satisfying f(1) + 2f (2) + 3f(3)+......+ xf(x) = x(x + 1) f(x) : x ≥ 2 with f(1) = 1. Then $\dfrac{1}{f(2022)} + \dfrac{1}{f(2028)}$ is equal to**

(a) 8400 (b) 8100
(c) 8200 (d) 8000

JEE Main-29.01.2023, Shift-II

Ans. (b) : Given, a function f : N → R, satisfying –

f(1) + 2f (2) + 3f(3) + ……+ xf(x) = x(x + 1)f (x)

Replace x by x + 1, we get –

x(x + 1) f(x) + (x + 1)f (x + 1) = (x + 1) (x + 2)f(x + 1)

$$\frac{x}{f(x+1)}+\frac{1}{f(x)}=\frac{(x+2)}{f(x)}$$

$xf(x) = (x + 1)f(x + 1) = \frac{1}{2}, x \geq 2$

$f(2) = \frac{1}{4},\ f(3)=\frac{1}{6}$

Now, $f(2022)=\frac{1}{4044}$

$f(2028) = \frac{1}{4056}$

So, $\frac{1}{f(2022)}+\frac{1}{f(2028)}=4044+4056$

$= 8100.$

257. Let f : R → R be a function defined $f(x)=\frac{2e^{2x}}{e^{2x}+e}$.
Then $f\left(\frac{1}{100}\right)+f\left(\frac{2}{100}\right)+f\left(\frac{3}{100}\right)+......+f\left(\frac{99}{100}\right)$ is equal to______.

JEE Main-27.06.2022, Shift-I

Ans. (99) : Given, a function f : R → R is defined on –

$$f(x)=\frac{2e^{2x}}{e^{2x}+e} \quad .....(i)$$

Replace (x) by (1 – x), we get –

$$f(1-x) = \frac{2e^{2(1-x)}}{e^{2(1-x)}+e}$$

$$f(1-x) = \frac{2e^{2-2x}}{e^{2-2x}+e} \quad ....(ii)$$

On adding equation (i) and equation (ii), we get –

$$f(x) + f(1-x) = \frac{2e^{2x}}{e^{2x}+e}+\frac{2e^{2-2x}}{e^{2-2x}+e}$$

$$= \frac{2e^{2x}}{e^{2x}+e}+\frac{2e^{2}\times e^{-2x}}{e^{2}\times e^{-2x}+e}$$

$$= 2\left[\frac{e^{2x}}{e^{2x}+e}+\frac{e^{2}}{e^{2}+e^{2x+1}}\right]$$

$$= 2\left[\frac{e^{2x-1}}{e^{2x-1}+1}+\frac{1}{1+e^{2x-1}}\right]$$

$$= 2$$

So, $f\left(\frac{1}{100}\right)+f\left(\frac{2}{100}\right)+f\left(\frac{3}{100}\right)+.....+f\left(\frac{99}{100}\right)$

$$=\left\{f\left(\frac{1}{100}\right)+f\left(\frac{99}{100}\right)\right\}+\left\{f\left(\frac{2}{100}\right)+f\left(\frac{98}{100}\right)\right\}$$

$$+ ......+ \left\{f\left(\frac{49}{100}\right)+f\left(\frac{51}{100}\right)\right\}+f\left(\frac{1}{2}\right)$$

$$= \{2+2+2+.......49\text{times}\}+\frac{2e}{e+e}$$

$= 98 + 1$

$= 99.$

258. Let A = {n ∈ N : H.C.F. (n, 45) = 1} and Let B = {2k : k ∈{1, 2,....,100}}. Then the sum of all the elements of A ∩ B is_____.

JEE Main-26.06.2022, Shift-I

Ans. (5264) : Given, A = {n∈N : H. C. F. (n, 45) = 1}

And, B = {2k : k∈ {1, 2, 3 …….100}

Since, $45 = 3^2 \times 5$

Then, A must be a set that does not consist of either 3 multiples or 5 multiples.

⇒ A = {1, 2, 4, 7, 8, 11, 13 ……}

And, B = {2, 4, 6, …….200}

So, A ∩ B = {1, 2, 4, 7, 8, 11, 13, 14, ……} ∩ {2, 4, 6, 8,……..,200}

⇒ A ∩ B = {2, 4, 8, 14, ……,200}

Since, find the sum of the element in A ∩ B.

Then,

⇒ [2 + 4 + 8 + 14 + ………+ 200]

⇒ 2 [1 + 2 + 4 + 7+ ……….+ 100]

⇒ 2 [sum of the natural number up to 100 – sum of multiples (3, 5)]

$$\Rightarrow 2\left[\frac{100\times101}{2}-\frac{3(33\times34)}{2}-\frac{5\times20\times21}{2}+\frac{15\times6\times7}{2}\right]$$

⇒ 2[5050 – 3 (561) – 5(210) + 15 × 21]

⇒ 2 [5050 – 1683 – 1050 + 315]

⇒ 2 × 2632 = 5264.

259. The remainder when $(2021)^{2023}$ is divided by 7 is :

(a) 1 (b) 2
(c) 5 (d) 6

JEE Main-26.06.2022, Shift-I

Ans. (c) : Given, $(2021)^{2023}$

$= (7\times 288+5)^{2023}$

Here, 7 × 288 goes to 0 because 288 is a multiple of 7.

So, 5^{2023}

$= (7-2)^{2023}$

$= (-2)^{2023}$

$= -1\times2^{1}(2^{3})^{674}$

$= -1\times 2(7+1)^{674}$

$= -2(1+7)^{674}$

$= -2+7$

$= 5.$

260. The absolute minimum value, of the function $f(x) = |x^2 - x + 1| + [x^2 - x + 1]$, where [t] denotes the greatest integer function, in the interval [–1, 2], is

(a) $\frac{3}{2}$ (b) $\frac{1}{4}$

(c) $\frac{5}{4}$ (d) $\frac{3}{4}$

JEE Main-31.01.2023, Shift-II

Ans. (d) : Given, $f(x) = |x^2 - x + 1| + [x^2 - x + 1]$, $x \in [-1, 2]$

Let, $y = g(x) = x^2 - x + 1$

$= (x - 1/2)^2 + \frac{3}{4}$

Since, $|x^2 - x + 1|$ and $[x^2 - x + 2]$

Then, both have minimum value at $x = \frac{1}{2}$

So, absolute minimum of $f(x) = \frac{3}{4} + 0$

$= \frac{3}{4}$.

261. The total number of functions, f : {1, 2, 3, 4} – {1, 2, 3, 4, 5, 6} such that f(1) + f(2) = f(3), is equal to:

(a) 60 (b) 90

(c) 108 (d) 126

JEE Main-25.07.2022, Shift-I

Ans. (b) : Given,

A = {1, 2, 3, 4}

B = {1, 2, 3, 4, 5, 6}

Here f(3) can be 2, 3, 4, 5, 6

Then, f(3) = 2, (f(1), f(2)) → (1, 1) → 6 cases

f(3) = 3, (f(1), f(2)) → (1, 2), (2, 1)

→ 2 × 6 = 12 cases

f(3) = 4, (f(1), f(2)) → (1, 3), (3, 1), (2, 2)

→3

6 = 18 cases

f(3) = 5, (f(1), f(2)) → (1, 4), (4, 1), (2, 3), (3, 2)

→ 4 × 6 = 24 cases

f(3) = 6, (f(1)) , f(2)) → (1, 5), (5, 1), (2, 4), (4, 2), (3, 3)

→ 5 × 6 = 30 cases

Total number of cases = 6 + 12 + 18 + 24 + 30 = 90

262. Let f : N → R be a function such that f (x + y) = 2f (x) f (y) for natural numbers x and y. If f(1) = 2, then the value of α for which

$$\sum_{k=1}^{10} f(\alpha+k) = \frac{512}{3}\left(2^{20}-1\right)$$

holds, is

(a) 2 (b) 3

(c) 4 (d) 6

JEE Main-25.06.2022, Shift-I

Ans. (c) : Given,

$f : N \to R$, $f(x + y) = 2 f(x) f(y)$ ....(i)

$f(1) = 2$

$\sum_{k=1}^{10} f(\alpha+k) = 2f(\alpha)\sum_{k=1}^{10} f(k)$

$= 2f(\alpha)\{f(1) + f(2) + ... + f(10)\}$ ....(ii)

Form equation (i),

$f(2) = 2 f^2(1) = 2^3$

$f(3) = 2 f(2) f(1) = 2^5$

……………………..

……………………..

……………………

$f(10) = 2^9 f^{10}(1) = 2^{19}$

∴ $f(\alpha) = 2^{2\alpha-1}$; $\alpha \in N$

Form equation (ii)

$\sum_{k=1}^{10} f(\alpha+k) = 2\left(2^{2\alpha-1}\right)\left(2 + 2^3 + 2^5 + ... + 2^{19}\right)$

$\frac{512}{3}\left(2^{20}-1\right) = 2^{2\alpha}\left(2.\frac{\left(2^{20}-1\right)}{3}\right)$

$\frac{512}{3}\left(2^{20}-1\right) = \frac{2^{2\alpha+1}}{3}\left(2^{20}-1\right)$

Comparing both side, we get–

$2^{2\alpha+1} = 512$

$2^{2\alpha+1} = 2^9$

$2\alpha + 1 = 9$

$2\alpha = 8$

Hence, $\alpha = 4$

263. The remainder when 3^{2022} is divided by 5 is

(a) 1 (b) 2

(c) 3 (d) 4

JEE Main-24.06.2022, Shift-I

Ans. (d) : Given, 3^{2022}

$= (3^2)^{1011}$

$= (9)^{1011}$

$= (10 - 1)^{1011}$

$= {}^{1011}C_0 \cdot 10^{1011} - {}^{1011}C_1 \cdot 10^{1010} + ...+ {}^{1011}C_{1010}\, 10^1 - {}^{1011}C_{1011}$

= 10k – 1, where k = integer

= 10k – 1 – 4 + 4

= 10 k – 5 + 4

= 5 (2 k – 1) + 4

So, when it is divided by 5, remainder will be '4'

264. Let $f(x) = ax^2 + bx + c$ be such that f(1) = 3, f(–2) = λ and f(3) = 4. If f(0) + f(1) + f(–2) + f(3) = 14 then λ is equal to

(a) –4 (b) $\frac{13}{2}$

(c) $\frac{23}{2}$ (d) 4

JEE Main-28.07.2022, Shift-II

Ans. (d) : Given, $f(x) = ax^2 + bx + c$

Then, $f(1) = a + b + c = 3$ ….. (i)

$f(-2) = 4a - 2b + c = \lambda$ …… (ii)

$f(3) = 9a + 3b + c = 4$ ……. (iii)

$\because$ $f(0) + f(1) + f(-2) + f(3) = 14$

$\therefore$ $c + 3 + \lambda + 4 = 14$

$c + \lambda = 7$

$\lambda = 7 - c$

Solving (i) and (ii):-

$$\begin{array}{r} 2a+2b+2c=6 \\ 4a-2b+c=\lambda \\ \hline 6a+3c=6+\lambda \end{array}$$

From (ii) and (iii):-

$$\begin{array}{r} 12a-6b+3c=3\lambda \\ 18a+6b+2c=8 \\ \hline 30a+5c=3\lambda+8 \end{array}$$

Now, we have–

$6a + 3c = 6 + \lambda$ …..(iv)

$30a + 5c = 3\lambda + 8$ …..(v)

Solving (iv) and (v), we get –

$$\begin{array}{r} 30a+15c=30+5\lambda \\ 30a+5c=8+3\lambda \\ \hline 10c=22+2\lambda \end{array}$$

$\therefore$ $c = \frac{22}{10} + \frac{\lambda}{5}$

Then, $\lambda = 7 - \frac{22}{10} - \frac{\lambda}{5}$

Or $\frac{6}{5}\lambda = \frac{70-22}{10} = \frac{48}{10}$

So, $\lambda = \frac{48}{10} \times \frac{5}{6} = \frac{8}{2} = 4$

265. If $x^2 + y^2 + z^2 \neq 0$, $x = cy + bz$, $y = az + cx$ and $z = bx + ay$, then $a^2 + b^2 + c^2 + 2abc$ is equal to

(a) 1 (b) 2

(c) $a + b + c$ (d) $ab + bc + ca$

AP EAMCET-2002

Ans. (a) : Given, $x^2 + y^2 + z^2 \neq 0$

And the system of equation–

$x - cy - bz = 0$

$cx - y + az = 0$

$bx + ay - z = 0$

This can be written as:–

$$\begin{bmatrix} 1 & -c & -b \\ c & -1 & a \\ b & a & -1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

We should have, $\begin{vmatrix} 1 & -c & -b \\ c & -1 & a \\ b & a & -1 \end{vmatrix} = 0$

$\Rightarrow$ $1(1 - a^2) + c(-c - ab) - b(ac + b) = 0$

Or $1 - a^2 - c^2 - abc - abc - b^2 = 0$

So, $a^2 + b^2 + c^2 + 2abc = 1$

266. The least number among $\sqrt[3]{4}$, $\sqrt[4]{5}$, $\sqrt[4]{7}$ and $\sqrt[3]{8}$ is

(a) $\sqrt[3]{8}$ (b) $\sqrt[4]{7}$

(c) $\sqrt[3]{4}$ (d) $\sqrt[4]{5}$

AP EAMCET-2002

Ans. (d) : Given, the numbers

$4^{\frac{1}{3}}, 5^{\frac{1}{4}}, 7^{\frac{1}{4}}, 8^{\frac{1}{3}}$

Then, LCM of (3, 4) = 12

Now, $\left(4^{\frac{1}{3}}\right)^{12} = 4^4 = 256$

$\left(5^{\frac{1}{4}}\right)^{12} = 5^3 = 125$

$\left(7^{\frac{1}{4}}\right)^{12} = 7^3 = 343$

$\left(8^{\frac{1}{3}}\right)^{12} = 8^4 = 4096$

So, we see that 125 is least number.

Hence, $\sqrt[4]{5}$ is least number.

267. If log 2 = a, log 3 = b, log 7 = c and $6^x = 7^{x+4}$ then x is equal to

(a) $\frac{4b}{c+a-b}$ (b) $\frac{4c}{a+b-c}$

(c) $\frac{4b}{c-a-b}$ (d) $\frac{4a}{a+b-c}$

AP EAMCET-2002

Ans. (b) : Given,

$\log 2 = a$, $\log 3 = b$, $\log 7 = c$ and $6^x = 7^{x+4}$

Taking log both the sides of the above condition–

$\log 6^x = \log 7^{x+4}$

$\Rightarrow$ $x \log(2 \times 3) = (x + 4) \log 7$

$\Rightarrow$ $x[\log 2 + \log 3] = (x+4).\log 7$

$\Rightarrow$ $x[a+b] = (x+4).c$

$\Rightarrow$ $x[a+b] = xc + 4c$

Or $x[a+b-c] = 4c$

$x = \frac{4c}{a+b-c}$

268. If $x > 0$, then $\frac{x}{1+x} - \log(1+x)$

(a) is less than zero

(b) is greater than zero

(c) is equal to zero

(d) takes all the real values

AP EAMCET-22.04.2018, Shift-II

Ans. (a) : Given, $x > 0$ then

$\frac{x}{1+x} - \log(1+x)$

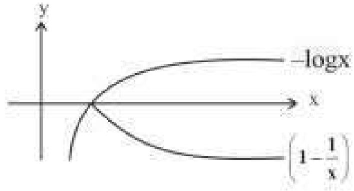
$$= \left[1-\frac{1}{1+x}\right]-\log(1+x)$$

Let, 1 + x = x.

Then,

$$\left(1-\frac{1}{x}\right)-\log x\ ,\ x>0$$

We will know by the graph–



So, for x > 0

$$\log x > \left(1-\frac{1}{x}\right)$$

$$\therefore \quad 1-\frac{1}{x}-\log x<0$$

Or $$\frac{x}{1+x}-\log(1+x)<0$$

So, $\dfrac{x}{1+x}-\log(1+x)$ is less than zero.

269. The value of x satisfying $\log_2(3x-2) = \log_{1/2} x$ is

(a) 1 (b) $-\frac{1}{3}$

(c) –1 (d) $\frac{1}{3}$

AMU-2011

Ans. (a) : $\log_2(3x-2)=\log_{\frac{1}{2}} x$

$$\log_2(3x-2)=\frac{1}{\log_x\left(\frac{1}{2}\right)}=\frac{1}{-\log_x 2}$$

$\log_2(3x-2) = -\log_2 x$

$\log_2(3x-2) + \log_2 x = 0$

$\log_2\{x(3x-2)\} = 0$

$x(3x-2) = 2^0$

$x(3x-2) = 1$

$3x^2 - 2x - 1 = 0$

$(x-1)(3x+1) = 0$

$x = 1, \frac{-1}{3}$

But $x = \frac{-1}{3}$ is not possible.

So x = 1 is the only one solution.

270. Let f = {(0, –1), (–1, –3), (2, 3), (3, 5)} be a function from Z to Z defined by f(x) = ax + b. Then

(a) a = 1, b = –2 (b) a = 2, b = 1

(c) a = 2, b = –1 (d) a = 1, b = 2

AMU-2011

Ans. (c) : Given,

f = {(0, –1), (–1, –3), (2, 3), (3, 5)} defined by f(x) = ax + b

y = ax + b

For ordered pair (0, –1)

–1 = a(0) + b

⇒ b = –1

For ordered pair (–1, –3)

–3 = – a + b

–3 = – a – 1

a = 2

So, a = 2 and b = –1

271. If $f(x)=\frac{x-1}{x+1}$, then $f(2x)$ is

(a) $\frac{f(x)+1}{f(x)+3}$ (b) $\frac{3f(x)+1}{f(x)+3}$

(c) $\frac{f(x)+3}{f(x)+1}$ (d) $\frac{f(x)+3}{3f(x)+1}$

AMU-2010

Ans. (b) : Given, $f(x)=\frac{x-1}{x+1}$

$$\Rightarrow \frac{-f(x)-1}{f(x)-1}=x \Rightarrow x=\frac{f(x)+1}{1-f(x)}$$

$$\text{Now, } f(2x)=\frac{2x-1}{2x+1}=\frac{2.\left[\frac{f(x)+1}{1-f(x)}\right]-1}{2.\left[\frac{f(x)+1}{1-f(x)}\right]+1}=\frac{3f(x)+1}{f(x)+3}$$

272. Let A = {–1, 0, 1, 2}, B = {–4, –2, 0, 2} and f, g : A → B be functions defined by f (x) = x^2 – x and $g(x)=2\left|x-\frac{1}{2}\right|-1$. Then

(a) f = g (b) f = 2g

(c) g = 2f (d) none of these

AMU-2010

Ans. (a) : Since, f (x) and g(x) has same domain and codomain A and B and $f(1) = (1)^2 - 1 = 0$

$$g(1)=2\left|1-\frac{1}{2}\right|-1=2\times\frac{1}{2}-1=0$$

$f(1) = 0 = g(1), f(0) = 0 = g(0)$

$f(-1) = 2 = g(-1), f(2) = 2 = g(2)$

A = {–1, 0, 1, 2}

B = {–4, –2, 0, 2}

So, the functions are equal (f = g)

273. If $f(x) = \frac{5^x}{5+5^x}$ then $f\left(\frac{1}{20}\right)+f\left(\frac{2}{20}\right)+....+f\left(\frac{39}{20}\right)$ is

(a) 20 (b) $\frac{29}{2}$

(c) $\frac{19}{2}$ (d) $\frac{39}{2}$

AMU-2021

Ans. (d) : Given, $f(x) = = \frac{5^X}{5^X+5}$ ....(i)

Replace x by 2 – x, we get –

$$f(2-x) = \frac{5^{2-x}}{5^{2-x}+5}$$

$$f(2-x) = \frac{\frac{5^2}{5^x}}{\frac{5^2}{5^x}+5}$$

$$f(2-x) = \frac{5}{5+5^x} \quad ....(ii)$$

On adding equation (i) and equation (ii), we get –

$$f(x) + f(2-x) = \frac{5^x}{5^x+5}+\frac{5}{5^x+5}$$
$$= 1$$

Let, $x = \frac{1}{20}$

Then, $f(x) = f\left(\frac{1}{20}\right)$

$$f(2-x) = f\left(2-\frac{1}{20}\right)$$

$$f(2-x) = f\left(\frac{39}{20}\right)$$

Then, $f\left(\frac{1}{20}\right)+f\left(\frac{39}{20}\right)=1$

Because, f(x) + f(2 – x) = 1

Similarly, $f\left(\frac{2}{20}\right)+f\left(\frac{38}{20}\right)=1$

. .
. .
. .
. .

$$f\left(\frac{19}{20}\right)+f\left(\frac{21}{20}\right)=1$$

And, $f\left(\frac{20}{20}\right)=f(1)=\frac{5^1}{5^1+5}=\frac{1}{2}$

So, $f\left(\frac{1}{20}\right)+f\left(\frac{2}{20}\right)+f\left(\frac{3}{20}\right)+....+f\left(\frac{39}{20}\right)$

$= \{1 + 1+ 1+ ......19 \text{ times}\} + \frac{1}{2}$

$= 19 \times 1 + \frac{1}{2}$

$= \frac{39}{2}$.

274. If f : Z → Z is defined by f(x) = $x^9 - 11x^8 - 2x^7 + 22x^6 + x^4 - 12x^3 + 11x^2 + x - 3 \ \forall \ x \in Z$, then f(11) =

(a) 7 (b) 8

(c) 6 (d) 9

AP EAPCET-25.08.2021, Shift-II

Ans. (b) : Given,

If, $f: Z \to Z$ is defined by $f(x) = x^9 - 11x^8 - 2x^7 + 22x^6 + x^4 - 12x^3 + 11x^2 + x - 3 \ \forall \ x \in Z$

Then, $f(11) = 11^9 - 11(11)^8 - 2(11)^7 + 22(11)^6 + (11)^4 - 12(11)^3 + 11(11)^2 + 11 - 3$

f(11) = 8

275. If $1^4 + 2^4 + 3^4 + ...+n^4 = f(n)(1^2 + 2^2 + ...+ n^2)$, $\forall n \in N$ then f(4) =

(a) $\frac{58}{5}$ (b) $\frac{57}{5}$

(c) $\frac{59}{5}$ (d) $\frac{56}{5}$

AP EAPCET-24.08.2021, Shift-II

Ans. (c): Given,

$$f(n) = \frac{1^4+2^4+3^4+..........+n^4}{1^2+2^2+3^2+.........+n^2}$$

Then, $f(4) = \frac{1^4+2^4+3^4+4^4}{1^2+2^2+3^2+4^2}$

$$= \frac{1+16+81+256}{1+4+9+16}$$
$$= \frac{354}{30}$$
$$= \frac{59}{5}$$

276. If $12^{4+2x^2} = \left(24\sqrt{3}\right)^{3x^2-2,}$, then x =

(a) $\pm\sqrt{\frac{13}{12}}$ (b) $\pm\sqrt{\frac{14}{5}}$

(c) $\pm\sqrt{\frac{12}{13}}$ (d) $\pm\sqrt{\frac{5}{14}}$

AP EAMCET-2016

Ans. (b) : Given,

$$12^{4+2x^2} = \left(24.\sqrt{3}\right)^{3x^2-2}$$

$$12^{4+2x2} = 12^{3x^2-2}.\ 2^{3x^2-2}\ 3^{\frac{(3x^2-2)}{2}}$$

$$12^{4+2x^2-3x^2+2} = 2^{3x^2-2}\ .\ 3^{\frac{(3x^2-2)}{2}}$$

$(4\times3)^{6-x^2} = 2^{3x^2-2}.\ 3^{\frac{3x^2-2}{2}}$

$2^{12-2x^2}.3^{6-x^2} = 2^{3x^2-2}\ 3^{\frac{3x^2-2}{2}}$

$\Rightarrow 12-2x^2 = 3x^2-2$ (By comparing the power of 2)

$\Rightarrow 5x^2 = 14$

$x^2 = \frac{14}{5}$

So, $x = \pm\sqrt{\frac{14}{5}}$

277. $\frac{\mathbf{x^4}}{\mathbf{x^3-3x+2}}$ **is a……………**

(a) Proper fraction (b) Improper fraction
(c) Mixed fraction (d) Not a fraction

AP EAMCET-17.09.2020, Shift-I

Ans. (b) : $\left(\frac{x^4}{x^3-3x+2}\right)$ is a improper fraction since (degree of numerator $\geq$ degree of denominator) for a improper fraction.

278. Assuming $|x|$ to be so small, that x^2 and higher powers of x can be neglected, then

$\frac{\sqrt{1+x}+(1-x)^{3/2}}{(1+x)+\sqrt{1+x}} =$

(a) $1+\frac{5x}{4}$ (b) $1-\frac{5x}{4}$
(c) $1+\frac{4x}{5}$ (d) $1-\frac{4x}{5}$

AP EAMCET-17.09.2020, Shift-I

Ans. (b) : Given, $|x|$ is very small, x^2 is negligible –

$$\frac{\sqrt{1+x}+(1-x)^{3/2}}{(1+x)+\sqrt{1+x}} = \frac{1+\frac{1}{2}x+1-\frac{3}{2}x}{1+x+1+\frac{x}{2}}$$

$$= \frac{2-x}{2+\frac{3x}{2}} = \frac{4-2x}{4+3x} = \frac{(4-2x)(4-3x)}{(16-9x^2)}$$

$$= \frac{16-20x+6x^2}{16-9x^2} = \frac{16-20x}{16}$$

($\because$ x^2 negligible)

$$= 1-\frac{5x}{4}$$

279. If $4^x - 3^{x-\frac{1}{2}} = 3^{x+\frac{1}{2}} - 2^{2x-1}$ then the value of x is

(a) 7/2 (b) 5/2
(c) 1/2 (d) 3/2

AP EAMCET-04.07.2021, Shift-I

Ans. (d) : $4^x - 3^{x-1/2} = 3^{x+1/2} - 2^{2x-1}$

$4^x - 3^x\times3^{-1/2} = 3^x\times3^{1/2} - 2^{2x}\times2^{-1}$

$2^{2x} + 2^{2x}\times2^{-1} = 3^x\times3^{1/2} + 3^x3^{-1/2}$

$2^{2x}\left(1+\frac{1}{2}\right) = 3^x\left(\sqrt{3}+\frac{1}{\sqrt{3}}\right)$

$2^{2x}\left(\frac{3}{2}\right) = 3^x\left(\frac{3+1}{\sqrt{3}}\right)$

$\frac{2^{2x}}{3^x} = \frac{4}{\sqrt{3}}\times\frac{2}{3}$

$\frac{2^{2x}}{3^x} = \frac{8}{3\sqrt{3}}$

$\frac{4^x}{3^x} = \frac{4^{\frac{3}{2}}}{3^{\frac{3}{2}}}$

$\left(\frac{4}{3}\right)^x = \left(\frac{4}{3}\right)^{3/2}$

On comparing both side, we get-

$x = \frac{3}{2}$

280. If f : R $\rightarrow$ R is defined by f (x) = 7 + cos (5x + 3) for x $\in$ R, then the period of f is

(a) 2π (b) π
(c) $\frac{\pi}{5}$ (d) $\frac{2\pi}{5}$

AP EAMCET-2011

Ans. (d) : Given,
f : R $\rightarrow$ R is defined by f(x) = 7 + cos (5x + 3) for x $\in$R
Then, if adding a constant term to a function shifts the graph above but does not change the period of the function.
$\therefore$ The period of the function is same as that of the period of function cos (5x + 3).
Since, the period of cos(x) is 2π and the period of cos(nx) would be $\frac{2\pi}{n}$.
So, the period of f(x) is $\frac{2\pi}{5}$.

281. If $x = \sqrt{\frac{2+\sqrt{3}}{2-\sqrt{3}}}$, then $x^2(x-4)^2$ is equal to:

(a) 7 (b) 4
(c) 2 (d) 1

AP EAMCET-2006

Ans. (d) : Given, $x = \sqrt{\frac{(2+\sqrt{3})}{(2-\sqrt{3})}}$

On simplifying, we gets –

$x = \sqrt{\frac{(2+\sqrt{3})(2+\sqrt{3})}{(2-\sqrt{3})(2+\sqrt{3})}}$

$$x = \sqrt{\frac{\left(2+\sqrt{3}\right)^2}{4-3}}$$

$x = 2+\sqrt{3}$

Then, $x - 4 = -2 + \sqrt{3}$

$\therefore$ x (x – 4) = (2 + $\sqrt{3}$) (– 2 + $\sqrt{3}$)

= 3 – 4

= –1

So, $x^2 (x-4)^2 = (-1)^2 = 1$

282. If $f(x) = \log\left(x+\sqrt{x^2+1}\right)$, then f(x) is

(a) even function (b) odd function
(c) periodic function (d) none of these

AMU-2005, 2004
AIEEE-2003

Ans. (b) : Given, f(x) = log $\left(x+\sqrt{x^2+1}\right)$

We know that, for odd function-

f(–x) = –f(x) ⇒ f(x) + f(–x) = 0

For even function-

f(–x) = f(x) ⇒ f(x) + f(–x) = 2f(x)

We have,

$$f(-x) = \log\left(-x+\sqrt{(-x)^2+1}\right)$$

$$f(-x) = \log\left(-x+\sqrt{x^2+1}\right)$$

$$f(x)+f(-x) = \log\left(x+\sqrt{x^2+1}\right)+\log\left(-x+\sqrt{x^2+1}\right)$$

$$= \log\left(x+\sqrt{x^2+1}\right)\left(-x+\sqrt{x^2+1}\right)$$

$$= \log\left(\left(\sqrt{x^2+1}\right)^2 - x^2\right)$$

$= \log(x^2 - 1 - x^2)$

= log (1)

= 0

Hence it is odd function

283. If f : R → R, such that $f(x) = \frac{e^x + e^{-x}}{e^x - e^{-x}}$, then f is

(a) an odd function
(b) a neither even nor odd function
(c) an even function
(d) a periodic function

MHT-CET 2020

Ans. (a) : Given,

$$f(x) = \frac{e^x + e^{-x}}{e^x - e^{-x}}$$

$$f(x) = \frac{e^x + \frac{1}{e^x}}{e^x - \frac{1}{e^x}}$$

$$f(x) = \frac{e^{2x}+1}{e^{2x}-1} \qquad \ldots (i)$$

And $f(-x) = \frac{e^{-x}+e^{x}}{e^{-x}-e^{x}}$

$$f(-x) = \frac{\frac{1}{e^x}+e^x}{\frac{1}{e^x}-e^x}$$

$$f(-x) = \frac{1+e^{2x}}{1-e^{2x}}$$

$$f(-x) = -\left(\frac{e^{2x}+1}{e^{2x}-1}\right)$$

f(–x) = –f(x) [From equation (i)]

$\therefore$ f(x) is odd function

284. If f(x) = 2x^2, find $\frac{f(3.8)-f(4)}{3.8-4}$

(a) 156 (b) 0.156
(c) 1.56 (d) 15.6

Karnataka CET-2015

Ans. (d) : Given, f(x) = 2x^2

Then find $\frac{f(3.8)-f(4)}{(3.8-4)} = ?$

So, $\frac{f(3.8)-f(4)}{3.8-4} = \frac{2\times(3.8)^2 - 4^2\times 2}{3.8-4}$

$$= \frac{2\left[(3.8)^2 - 4^2\right]}{(3.8-4)}$$

$$= \frac{-2\left[4^2-(3.8)^2\right]}{-(4-3.8)}$$

$$= \frac{2(4-3.8)(4+3.8)}{(4-3.8)} = 2 \times 7.8 = 15.6$$

285. If f(x) = cos (log$_e$ x), then

$f(x)f(y) - \frac{1}{2}\left[f\left(\frac{y}{x}\right) + f(xy)\right]$ has the value

(a) 1 (b) 1/2
(c) –2 (d) 0

COMEDK-2019

Ans. (d) : Given,

f(x) = cos (log$_e$x), $f\left(\frac{y}{x}\right) = \cos\left(\log_e \frac{y}{x}\right)$,

F(xy) = cos (log$_e$xy)

$$\therefore f(x)f(y) - \frac{1}{2}\left[f\left(\frac{y}{x}\right)+f(xy)\right]$$

$$= \cos(\log_e x)\cos(\log_e y) - \frac{1}{2}\left[\cos\left(\log_e \frac{y}{x}\right)+\cos(\log_e xy)\right]$$

$$= \cos(\log_e x)\cos(\log_e y) - \frac{1}{2}\left[\cos\left(\log_e y - \log_e x\right)+\cos(\log_e x + \log_e y)\right]$$

$$= \cos(\log_e x)\cos(\log_e y) - \frac{1}{2}\left[2\cos\left(\log_e x\right)\cos\left(\log_e y\right)\right]$$

= cos (log$_e$x) cos (log$_e$y) – cos (log$_e$y)cos (log$_e$x) = 0

286. $x = \frac{1}{2}\left(\sqrt{3} + \frac{1}{\sqrt{3}}\right)$, **then** $\frac{\sqrt{x^2-1}}{x - \sqrt{x^2-1}}$ **is equal to**

(a) 1 (b) 2

(c) 3 (d) $\frac{1}{2}$

AP EAMCET-2005

Ans. (a) : Given, $x = \frac{1}{2}\left(\sqrt{3} + \frac{1}{\sqrt{3}}\right)$

Then, $\frac{\sqrt{x^2-1}}{x-\sqrt{x^2-1}} = \frac{\sqrt{x^2-1}\left(x+\sqrt{x^2-1}\right)}{\left(x-\sqrt{x^2-1}\right)\left(x+\sqrt{x^2-1}\right)}$

$= \frac{\sqrt{x^2-1}\left(x+\sqrt{x^2-1}\right)}{x^2-x^2+1}$

$= = \sqrt{x^2-1}\left(x+\sqrt{x^2-1}\right)$

We have, $x = \frac{1}{2} \times \frac{4}{\sqrt{3}} = \frac{2}{\sqrt{3}}$

$\therefore\ x^2 - 1 = \frac{4}{3} - 1 = \frac{1}{3}$

$\therefore \sqrt{x^2-1}\left(x+\sqrt{x^2-1}\right) = \frac{1}{\sqrt{3}}\left(\frac{2}{\sqrt{3}} + \frac{1}{\sqrt{3}}\right) = \frac{1\times 3}{\sqrt{3}\times\sqrt{3}} = 1$

287. **If F is function such that F (0) = 2, F (1) = 3, F (x + 2) = 2F (x) – F (x + 1) for x ≥ 0, then F (5) is equal to**

(a) –7 (b) –3

(c) 17 (d) 13

VITEEE-2010

Ans. (d) : Given,

F (0) = 2, F (1) = 3

$F(x+2) = 2F(x) - F(x-1)$ ...(i)

Putting x = 0, we get –

$F(2) = 2F(0) - F(1)$

$F(2) = 2(2) - 3 \quad \{\because F(0) = 2,\ F(1) = 3\}$

$F(2) = 4 - 3$

$F(2) = 1$

Putting x = 1, in equation (i) we get –

$F(3) = 2\ F(1) - F(2)$

$F(3) = 2(3) - 1 \quad \{\because F(1) = 3, F(2) = 1\}$

F (3) = 5

Putting x = 2, in equation (i) we get –

$F(4) = 2F(2) - F(3)$

$F(4) = 2(1) - 5 \quad \{\because F(2) = 1,\ F(3) = 5\}$

F (4) = –3

Putting x = 3, in equation (i) we get –

$F(5) = 2F(3) - F(4)$

$F(5) = 2(5) + 3 \quad \{\because F(3) = 5, F(4) = -3\}$

F(5) = 13

288. **If** $\log_{27}(\log_3 x) = \frac{1}{3}$**, then the value of x is**

(a) 3 (b) 6

(c) 9 (d) 27

AP EAMCET-2004

Ans. (d) : Given,

$\log_{27}(\log_3 x) = \frac{1}{3}$

$\therefore\ \log_3 x = (27)^{1/3} = 3$

So, $x = 3^3 = 27$

289. **Let f be an odd function defined on the real number such that f(x) = 3 sin x + 4 cos x, for x ≥ 0 then f(x) for x < 0 is**

(a) –3 sin x + 4 cos x (b) –3 sin x – 4 cos x

(c) 3sin x + 4 cos x (d) 3 sin x – 4 cos x

UPSEE-2017

Ans. (d) : Given, f be an odd function defined on the real number such that f(x) = 3 sin x + 4 cos x for x ≥ 0.

Then, f (x) = 3sin x + 4 cos x

Since, f is odd function.

Then, f (–x) = –f(x), x ≥ 0

f(–x) = 3 sin (–x) + 4 cos (–x)

f(–x) = – 3 sin x + 4 cos x $\left[\begin{array}{l}\because\ \cos(-\theta) = \cos\theta \\ \quad\ \sin(-\theta) = -\sin\theta\end{array}\right]$

f(–x) = – (3 sin x – 4 cos x)

So, comparing f(–x) = –f(x)

–f(x) = – (3 sin x – 4 cos x)

Hence, for odd function f (x) for x < 0 is 3 sin x – 4 cos x.

290. **If the real valued function** $f(x) = \frac{a^x - 1}{x^n(a^x+1)}$ **is even, then n is equal to**

(a) 2 (b) $\frac{2}{3}$

(c) $\frac{1}{4}$ (d) 3

UPSEE-2013

Ans. (d) : Given function, f(x) = $\frac{a^x - 1}{x^n\left(a^x+1\right)}$ is even.

We know that for even function.

f(–x) = f(x)

$$\frac{a^{-x}-1}{(-x)^n\left(a^{-x}+1\right)} = \frac{\left(a^x-1\right)}{\left(x^n\right)\left(a^x+1\right)}$$

$$\frac{1-a^x}{(-1)^n x^n (a^x+1)} = \frac{(a^x-1)}{x^n(a^x+1)}$$

$$\frac{1-a^x}{(-1)^n x^n (a^x+1)} = \frac{-(1-a^x)}{x^n(a^x+1)}$$

$(-1)^n = -1$

So, it satisfies n = 3 is odd.

Hence, n = 3

291. The number of reflexive relations of a set with four elements is equal to:

(a) 2^{16} (b) 2^{12}
(c) 2^8 (d) 2^4

UPSEE-2004

Ans. (d) : Given,

Set A with four element

We know that, total number of reflexive relations of a set with n elements = 2^n

So, total number of reflexive relations of a set with 4 elements = 2^4

292. If $f(x)=\frac{1-x}{1+x}, x\neq 0,-1$ and $\alpha = f(f(x))+f(f(1/x))$, then

(a) $\alpha > 2$ (b) $\alpha < -2$
(c) $|\alpha| > 2$ (d) $\alpha = 2$

JCECE-2015

Ans. (c) : Given, $f(x) = \frac{1-x}{1+x}, x \neq 0, -1$

Let, $\alpha = f(f(x)) + f\left(f\left(\frac{1}{x}\right)\right)$

Then, $f\{f(x)\} = f\left(\frac{1-x}{1+x}\right)$

$$f\{f(x)\} = f\{f(x)\} = \frac{1-\frac{1-x}{1+x}}{1+\frac{1-x}{1+x}} \qquad \left[\because f(x) = \frac{1-x}{1+x}\right]$$

$$f(f(x)) = \frac{1+x-1+x}{1+x+1-x}$$

$$f(f(x)) = \frac{2x}{2}$$

$$f(f(x)) = x$$

And, $f\left(\frac{1}{x}\right) = \frac{1-\frac{1}{x}}{1+\frac{1}{x}}$

$$f\left(\frac{1}{x}\right) = \frac{x-1}{x+1}$$

$$\therefore \quad f\left\{f\left(\frac{1}{x}\right)\right\} = \frac{1-\frac{x-1}{x+1}}{1+\frac{x-1}{x+1}}$$

$$f\left\{f\left(\frac{1}{x}\right)\right\} = \frac{x+1-x+1}{x+1+x-1}$$

$$f\left(f\left(\frac{1}{x}\right)\right) = \frac{2}{2x} = \frac{1}{x}$$

Now, $\alpha = f\{f(x)\} + \left\{f\left(\frac{1}{x}\right)\right\}$

$$\alpha = x + \frac{1}{x}$$

$$|\alpha| = \left|x + \frac{1}{x}\right| \geq 2$$

Hence, $|\alpha| \geq 2$.

293. The function $f(x) = \log\left(x+\sqrt{x^2+1}\right)$ is:

(a) even function
(b) odd function
(c) neither even nor odd
(d) periodic function

BCECE(Engg.)-2008
BCECE-2006
JCECE-2004

Ans. (b) : Given,

$$f(x) = \log\left(x+\sqrt{x^2+1}\right)$$

Then check function is –

$$f(-x) = \log\left(-x+\sqrt{(-x)^2+1}\right)$$

$$f(-x) = \log\left(\sqrt{1+x^2}-x\right)$$

$$f(-x) = \log\left\{\left(\sqrt{1+x^2}-x\right)\times\frac{\sqrt{1+x^2}+x}{\sqrt{1+x^2}+x}\right\}$$

$$f(-x) = \log\left\{\left(\frac{1+x^2-x^2}{\sqrt{1+x^2}+x}\right)\right\}$$

$$f(-x) = \log\left\{\left(\frac{1}{\sqrt{1+x^2}+x}\right)\right\}$$

$$f(-x) = \log 1 - \log\left\{\sqrt{1+x^2}+x\right\}$$

$$f(-x) = 0 - \log\left\{x+\sqrt{1+x^2}\right\}$$

$$f(-x) = -\log\left\{x+\sqrt{1+x^2}\right\}$$

$$f(-x) = -f(x)$$

So, $f(x) = \log\left\{x+\sqrt{1+x^2}\right\}$ is an odd function.

294. **If $f(x)=\left(\frac{1}{x}\right)^x$, then the maximum value of f(x) is:**

(a) e (b) $(e)^{1/e}$

(c) $\left(\frac{1}{e}\right)^e$ (d) none of these

JCECE-2004

Ans. (b) : Given,

$$f(x) = \left(\frac{1}{x}\right)^x \quad \ldots(i)$$

Then, let $y=\left(\frac{1}{x}\right)^x$

Taking log both sides, we get –

$$\log y = \log\left(\frac{1}{x}\right)^x$$

$$\log y = x\log\left(\frac{1}{x}\right) \qquad (\because \log a^m = m\log a)$$

Differentiating both side, w.r.t. x, we get –

$$\frac{1}{y}\frac{dy}{dx}=x\cdot\frac{1}{\frac{1}{x}}\cdot\frac{-1}{x^2}+\log\left(\frac{1}{x}\right)\cdot 1$$

$$\frac{1}{y}\frac{dy}{dx}=x^2\times\left(-\frac{1}{x^2}\right)+\log\left(\frac{1}{x}\right)\cdot 1$$

$$\frac{1}{y}\frac{dy}{dx}=-1+\log\frac{1}{x}$$

$$\frac{dy}{dx}=y\left(-1+\log\frac{1}{x}\right)$$

$$\frac{dy}{dx}=\left(\frac{1}{x}\right)^x\left(-1+\log\frac{1}{x}\right)$$

We know that, for maximum value, $\frac{dy}{dx}=0$

$$0=\left(\frac{1}{x}\right)^x\left(-1+\log\frac{1}{x}\right)$$

$$\log_e\frac{1}{x}=1$$

$$\frac{1}{x}=e^1$$

$$x=\frac{1}{e}$$

Putting the value of $x=\frac{1}{e}$ in equation (i), we get –

$$f(x) = \left(\frac{1}{1/e}\right)^{\frac{1}{e}} = (e)^{\frac{1}{e}}$$

295. **If $f(x)=\log\left(\frac{1+x}{1-x}\right)$, then $f\left(\frac{2x}{1+x^2}\right)$ will be equal to:**

(a) $2f(x^2)$ (b) $f(x^2)$

(c) 2f(2x) (d) 2f(x)

JCECE-2004

Ans. (d) : Given,

$$f(x)=\log\left(\frac{1+x}{1-x}\right)$$

Then, $$f\left(\frac{2x}{1+x^2}\right)=\log\left(\frac{1+\frac{2x}{1+x^2}}{1-\frac{2x}{1+x^2}}\right)$$

$$f\left(\frac{2x}{1+x^2}\right)=\log\left(\frac{1+x^2+2x}{1+x^2-2x}\right)$$

$$f\left(\frac{2x}{1+x^2}\right)=\log\left\{\frac{(1+x)^2}{(1-x)^2}\right\}$$

$$f\left(\frac{2x}{1+x^2}\right)=\log\left\{\frac{(1+x)}{(1-x)}\right\}^2$$

$$f\left(\frac{2x}{1+x^2}\right)=2\log\left\{\frac{(1+x)}{(1-x)}\right\}$$

$$f\left(\frac{2x}{1+x^2}\right)=2f(x)$$

296. **The value of [sin x] + [1 + sin x] + [2 + sin x] in $x\in\left[\pi,\frac{3\pi}{2}\right]$ can be ([.] is the greatest integer function) can be**

(a) 0 (b) 1

(c) 2 (d) 3

BCECE-2018

Ans. (a) : Given,

$$x\in\left[\pi,\frac{3\pi}{2}\right]$$

Then, the value of [sin x] + [1 + sin x] + [2 + sin x] = ?

Where, [·] is the greatest integer function.

Then, from $x\in\left[\pi,\frac{3\pi}{2}\right]$-

$$-1\le \sin x\le 0$$

So, [1+ sin x] = 0 and [sin x] = –1

Hence,

[sin x] + [1+ sin x] + [2 + sin x] = –1 + 0 + 2 + [sin x]

= – 1 + 0 + 2 – 1

= – 2 + 2

[sin x] + [1 + sin x] + [2 + sin x] = 0

297. **The function $f(x) = 2\cos 5x + 3\sin\sqrt{5}x$ is**

(a) a periodic function with period 2π

(b) a periodic function with period $\frac{2\pi}{5}$

(c) a periodic function with period $\frac{2\pi}{\sqrt{5}}$

(d) not a periodic function

BCECE-2018

Ans. (d) : Given,

$f(x) = 2\cos 5x + 3\sin\sqrt{5}x$

Differentiate on both side with respect to x, we get–

$f'(x) = -2\sin 5x \times 5 + 3\cos\sqrt{5}x \times \sqrt{5}$

$f'(x) = -10\sin 5x + 3\sqrt{5}\cos\sqrt{5}x$

Then, $2\cos 5x$, $3\sin\sqrt{5}x$ are periodic function with periods $\frac{2\pi}{5}$ and $\frac{2\pi}{\sqrt{5}}$.

But $\frac{2\pi}{5}$ and $\frac{2\pi}{\sqrt{5}}$ have no common multiple.

So, $f(x) = 2\cos 5x + 3\sin\sqrt{5}x$ is not periodic function.

298. **If a, b, c are positive real numbers, then $\frac{1}{\log_{ab}abc}+\frac{1}{\log_{bc}abc}+\frac{1}{\log_{ca}abc} =$**

(a) 0 (b) 1

(c) 2 (d) 3

BCECE-2017

Ans. (c) : Given,

a, b, c are positive real number,

Then, $\frac{1}{\log_{ab}abc}+\frac{1}{\log_{bc}abc}+\frac{1}{\log_{ca}abc}$

$=\frac{1}{\frac{\log abc}{\log ab}}+\frac{1}{\frac{\log abc}{\log bc}}+\frac{1}{\frac{\log abc}{\log ca}} \quad \left(\because \log_b a = \frac{\log a}{\log b}\right)$

$=\frac{\log ab}{\log abc}+\frac{\log bc}{\log abc}+\frac{\log ca}{\log abc}$

$=\frac{\log ab+\log bc+\log ca}{\log abc}$

$=\frac{\log(ab\times bc\times ca)}{\log abc}$

$=\frac{\log(a^2b^2c^2)}{\log abc}$

$=\frac{\log(abc)^2}{\log(abc)}$

$=\frac{2\log(abc)}{\log(abc)}$

$= 2$

299. **The number of functions f, from the set $A = \{x \in N : x^2 - 10x + 9 \le 0\}$ to the set $B = \{n^2 : n \in N\}$ such that $f(x) \le (x-3)^2 + 1$, for every $x \in A$, is _______.**

JEE Main-27.07.2022, Shift-II

Ans. (1440) : Given,

$(x^2 - 10x + 9) \le 0$

$(x-1)(x-9) \le 0$

$x \in [1, 9]$

$A = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

Now,

$f(x) \le (x-2)^2 + 1$

$x = 1 : f(1) \le 5 \Rightarrow 1^2, 2^2$

$x = 2 : f(2) \le 2 \Rightarrow 1^2$

$x = 3 : f(3) \le 1 \Rightarrow 1^2$

$x = 4 : f(4) \le 2 \Rightarrow 1^2$

$x = 5 : f(5) \le 5 \Rightarrow 1^2, 2^2$

$x = 6 : f(6) \le 10 \Rightarrow 1^2, 2^2, 3^2$

$x = 7 : f(7) \le 17 \Rightarrow 1^2, 2^2, 3^2, 4^2$

$x = 8 : f(8) \le 26 \Rightarrow 1^2, 2^2, 3^2, 4^2, 5^2$

$x = 9 : f(9) \le 37 \Rightarrow 1^2, 2^2, 3^2, 4^2, 5^2, 6^2$

Total number of function = 2(6!) = 2(720) = 1440

300. **The number of functions f: {1, 2, 3, 4} → {a ∈ Z: |a| ≤ 8} satisfying $f(n) + \frac{1}{n}f(n+1) = 1, \forall n \in \{1, 2, 3\}$ is**

(a) 2 (b) 1

(c) 4 (d) 3

JEE Main-25.01.2023, Shift-II

Ans. (a) : Given,

$f(n) + \frac{1}{n}f(n+1) = 1$

$n.f(n) + f(n+1) = 1$

When $n = 1$

$f(1) + f(2) = 1 \quad \text{…(i)}$

When $n = 2$

$2f(2) + f(3) = 2 \quad \text{…(ii)}$

When $n = 3$

$3f(3) + f(4) = 3 \quad \text{…(iii)}$

Now, multiple by 2 in equation (i), we get –

$2f(1) + 2f(2) = 2 \quad \text{…(iv)}$

On subtracting equation (iv) from (ii), we get –

$f(3) - 2f(1) = 0$

$f(3) = 2f(1) \quad \text{…(iv)}$

Now, putting the value in equation (iii), we get–

$3[2f(1)] + f(4) = 3$

$6f(1) + f(4) = 3$

$f(4) = 3 - 6f(1)$

Therefore, $-8 \le f(4) \le 8$

$-8 \le 3 - 6f(1) \le 8$

$-11 \le -6f(1) \le 5$

$$-\frac{5}{6} \le f(1) \le \frac{11}{6}$$

f (1) = 0, 1

Case – I : f (1) = 0, f(2) = 1

f(3) = 0, f(4) = 3

Case – II: f(1) = 1, f(2) = 0

f(3) = 2, f(4) = –3

There can be 2 function such that like this.

301. $\frac{\sqrt{8+\sqrt{28}}+\sqrt{8-\sqrt{28}}}{\sqrt{8+\sqrt{28}}-\sqrt{8-\sqrt{28}}}$ **is equal to**

(a) 2 (b) 7

(c) $\sqrt{7}$ (d) $\sqrt{2}$

AP EAMCET-2001

Ans. (c) : Given,

$$\frac{\left(\sqrt{8+\sqrt{28}}\right)+\sqrt{8-\sqrt{28}}}{\sqrt{8+\sqrt{28}}-\sqrt{8-\sqrt{28}}} \times \frac{\left(\sqrt{8+\sqrt{28}}\right)+\sqrt{8-\sqrt{28}}}{\sqrt{8+\sqrt{28}}+\sqrt{8-\sqrt{28}}}$$

$$= \frac{8+\sqrt{28}+8-\sqrt{28}+2\sqrt{8+\sqrt{28}}.\sqrt{8-\sqrt{28}}}{8+\sqrt{28}-\left(8-\sqrt{28}\right)}$$

$$= \frac{16+2\sqrt{64-28}}{2\sqrt{28}} = \frac{8+\sqrt{36}}{\sqrt{28}} = \frac{8+6}{\sqrt{28}} = \frac{14}{\sqrt{28}}$$

$$= \frac{7}{\sqrt{7}} = \sqrt{7}$$

302. If the periods of the functions sin(ax + b) and tan(cx + d) are respectively $\frac{4}{7}$ and $\frac{2}{5}$, then sin(|a| + |c|) + cos (|a| – |c|) =

(a) –1 (b) 0

(c) 1 (d) 2

AP EAMCET-22.04.2019, Shift-II

Ans. (a) : We know that,

Period of sin(ax + b) = $\left|\frac{2\pi}{a}\right|$

Period of tan (ax + b) = $\frac{\pi}{|a|}$

$$\therefore \frac{2\pi}{|a|} = \frac{4}{7}$$

$$\frac{\pi}{|c|} = \frac{2}{5}$$

$$|a| = \frac{7\pi}{2} \text{ and } |c| = \frac{5\pi}{2}$$

$$\therefore \sin(|a| + |c|) + \cos(|a| - |c|) = \sin\left(\frac{7\pi}{2}+\frac{5\pi}{2}\right) + \cos\left(\frac{7\pi}{2}-\frac{5\pi}{2}\right)$$

= sin(6π) + cos(π) = 0 + (–1) = –1

303. If $x+\log_{10}\left(1+2^x\right) = x\log_{10}5+\log_{10}6$, then the value of x is

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$

(c) 1 (d) 2

WB JEE-2018

Ans. (c) : Given, $x + \log_{10}(1+2^x) = x \log_{10} 5 + \log_{10}6$

$$\log_{10} 10^x + \log_{10}(2^x+1) = \log_{10} 5^x + \log_{10} 6$$

$$\Rightarrow \log_{10}\left(10^x\left(2^x+1\right)\right) = \log_{10}\left(5^x.6\right)$$

$$10^x\left(2^x+1\right) = 5^x.6$$

$$2^x\left(2^x+1\right) = 6 \quad \left(\because 5^x \ne 0\right)$$

let $2^x = t$

$$t(t+1) = 6$$

$$t^2+t-6=0$$

$$t^2+3t-2t-6=0$$

$$(t+3)(t-2)=0$$

$$t = -3,\ t = 2$$

$$2^x = 2$$

$$x = 1$$

304. Consider the function f(x) = cosx^2 . Then,

(a) f is of period 2π (b) f is of period $\sqrt{2\pi}$

(c) f is not periodic (d) f is of period π

WB JEE-2019

Ans. (c) : We have,

$$f(x) = \cos x^2$$

Let T be the period of f(x). Then

$$f(x+T) = f(x)$$

$$\Rightarrow \quad \cos(x+T)^2 = \cos x^2$$

But there is no value of T for which

$$\cos(x+T)^2 = \cos x^2$$

$\therefore$ f(x) is not periodic

305. **Which of the following is an even function?**

(a) $\sqrt{x}$ (b) $x^2+\sin^2 x$

(c) $\sin^3 x$ (d) None of these

COMEDK 2017

Ans. (b) : Let f(x) =$x^2 + \sin^2 x$, then f(–x) = f(x). Therefore, f(x)=$x^2 + \sin^2 x$ is an even function.

306. If $\log_2 6+\frac{1}{2x} = \log_2\left(2^{\frac{1}{x}}+8\right)$ then the values of x are

(a) $\frac{1}{4},\frac{1}{3}$ (b) $\frac{1}{4},\frac{1}{2}$

(c) $-\frac{1}{4},\frac{1}{2}$ (d) $\frac{1}{3},-\frac{1}{2}$

WB JEE-2019

Ans. (b) : We have,

$$\log_2 6+\frac{1}{2x}=\log_2\left(2^{\frac{1}{x}}+8\right)$$

$$\log_2\left(2^{\frac{1}{x}}+8\right)-\log_2 6=\frac{1}{2x}$$

$$\log_2\left(\frac{2^{\frac{1}{x}}+8}{6}\right)=\frac{1}{2x}$$

$$\frac{2^{\frac{1}{x}}+8}{6}=2^{\frac{1}{2x}}$$

$$2^{\frac{1}{x}}+8=6.2^{\frac{1}{2x}}$$

Let $y=2^{\frac{1}{2x}}$

$\Rightarrow$ $y^2+8=6y$

$\Rightarrow$ $y^2-6y+8=0$

$\Rightarrow$ $(y-4)(y-2)=0$

$\Rightarrow$ $y=4,2$

$\Rightarrow$ $2^{\frac{1}{2x}}=4$ and $2^{\frac{1}{2x}}=2$

$\Rightarrow$ $\frac{1}{2x}=2$ and $\frac{1}{2x}=1$

$\Rightarrow$ $x=\frac{1}{4},\frac{1}{2}$

307. Let f(x) = sin x + cos ax be periodic function. Then,

(a) a is any real number
(b) a is any irrational number
(c) a is rational number
(d) a = 0

WB JEE-2020

Ans. (c) : Given,
f(x) = sinx + cosax

$\because$ Period of sinx = $\frac{2\pi}{1}$

Period of cos ax = $\frac{2\pi}{a}$

Hence period of f(x) = L.C.M of $\left\{\frac{2\pi}{1},\frac{2\pi}{a}\right\}$

$=\frac{\text{L.C.M. of }(2\pi,2\pi)}{\text{H.C.F. of }\{1,a\}}=\frac{2\pi}{k}$

Where k = H.C.F of 1 and a

$\therefore$ $\frac{1}{k}$ = integer = q (say) ≠ 0 and $\frac{a}{k}$

Integer = p cosy

$\therefore$ $\frac{\frac{a}{k}}{\frac{1}{k}}=\frac{p}{q}$

$a=\frac{p}{q}$

a = rational number

308. If a and b are arbitrary positive real numbers, then the least possible value of $\frac{6a}{5b}+\frac{10b}{3a}$ is

(a) 4
(b) $\frac{6}{5}$
(c) $\frac{10}{3}$
(d) $\frac{68}{15}$

WB JEE-2020

Ans. (a) : We know that,

AM ≥ GM

$$\frac{6a}{5b}+\frac{10b}{3a}\ge 2\sqrt{\frac{6a}{5b}\times\frac{10b}{3a}}$$

$$\frac{6a}{5b}+\frac{10b}{3a}\ge 2\times 2$$

$$\frac{6a}{5b}+\frac{10b}{3a}\ge 4$$

309. The period of the function f(x) = |sinx| – |cosx|

(a) $\pi/2$
(b) π
(c) 2π
(d) None of these

BITSAT-2016

Ans. (b) : Given, the period of the function is-

$f(x)=|\sin x|-|\cos x|$

$\therefore f(x+\pi)=|\sin(x+\pi)|-|\cos(x+\pi)|$

$\Rightarrow f(x+\pi)=|\sin x|-|\cos x|$

$\Rightarrow f(x+\pi)=|\sin x|-|\cos x|=f(x)$, for all $x\in R$

$\Rightarrow f(x+\pi)=f(x)$ for all $x\in R$

So, f(x) is periodic with period π.

310. If $f(x)=\cot^{-1}\left(\frac{x^x-x^{-x}}{2}\right)$, then $f'(1)$ is equal to

(a) –1
(b) 1
(c) log 2
(d) –log 2

UPSEE -2008

Ans. (a) : Given,

$$f(x)=\cot^{-1}\left(\frac{x^x-x^{-x}}{2}\right)$$

Firstly, differentiate the x^x and x^{-x}

Let, x^x = u and x^{-x} = v

Taking log on both side, we get –

log u = x log x and log v = – x log x

Then, differentiate –

$$\frac{1}{u}\frac{du}{dx} = x.\frac{1}{x} + \log x$$

And, $$\frac{1}{v}\frac{dv}{dx} = -x.\frac{1}{x} - \log x$$

$$\frac{du}{dx} = u(1+\log x) \text{ and } \frac{dv}{dx} = v(-1-\log x)$$

$$\frac{du}{dx} = x^x(1+\log x) \text{and} \frac{dv}{dx} = -x^{-x}(1+\log x)$$

So, $$f'(x) = \frac{-1}{1+\left(\frac{x^x - x^{-x}}{2}\right)^2}\frac{d}{dx}\left(\frac{x^x - x^{-x}}{2}\right)$$

$$f'(x) = \frac{-4}{x^{2x}+x^{-2x}+4-2}\left[\frac{1}{2}\left[\left(x^x + x^{-x}\right)(1+\log x)\right]\right]$$

$$f'(x) = \frac{-4}{x^{2x}+x^{-2x}+2}\left[\frac{1}{2}\left[\left(x^x + x^{-x}\right)(1+\log x)\right]\right]$$

Hence, $f'(1) = \frac{-2}{1+1+2}\left[2\{1+\log 1\}\right]$

$$f'(1) = \frac{-4}{4}$$

$$f'(1) = -1$$

311. If $2\log(x+1) - \log(x^2-1) = \log 2$, then x =

(a) only 3 (b) –1 and 3
(c) only – 1 (d) 1 and 3

WB JEE-2020

Ans. (a) : Given,
2 log (x + 1) – log (x^2 – 1) = log 2
log $(x + 1)^2$ – log (x^2 – 1) = log 2

$$\log\left|\frac{(x+1)^2}{x^2-1}\right| = \log 2$$

$$\frac{(x+1)^2}{(x^2-1)} = 2$$

$$\frac{(x+1)(x+1)}{(x+1)(x-1)} = 2$$

$$\frac{x+1}{x-1} = 2$$

x + 1 = 2x – 2
x = 3

312. If x = log $_{0.1}$ 0.001, y = log$_9$81, then $\sqrt{x-2\sqrt{y}}$ is equal to.

(a) $3-\sqrt{2}$ (b) $\sqrt{3}-2$
(c) $\sqrt{2}-1$ (d) $\sqrt{2}-2$

AP EAMCET-2001

Ans. (c) : Given, x = $\log_{0.1}$ 0.001, y = $\log_9$ 81
On simplifying, we get –

$x = \log_{0.1} 0.001 = \log_{10^{-1}} 10^{-3} = \frac{-3}{-1}\log_{10} 10 = 3$.

And, y = $\log_9 9^2 = 2\log_9 9 = 2$

So, $\sqrt{x-2\sqrt{y}} = \sqrt{3-2\sqrt{2}}$

$= \sqrt{\left(\sqrt{2}-1\right)^2} = \sqrt{2}-1$

313. If $x = \frac{2}{3+\sqrt{7}}$ then $(x – 3)^2$ is equal to

(a) 1 (b) 3
(c) 6 (d) 7

EAMCET-2000,1996

Ans. (d) : Given, $x = \frac{2}{3+\sqrt{7}}$

$$= \frac{2\left(3-\sqrt{7}\right)}{\left(3+\sqrt{7}\right)\left(3-\sqrt{7}\right)} = \frac{2\left(3-\sqrt{7}\right)}{9-7} = 3-\sqrt{7}$$

$\therefore$ x – 3 = $-\sqrt{7}$
So, $(x – 3)^2$ = 7

314. If f is any function, then $\frac{1}{2}\left[f(x)+f(-x)\right]$ is always :

(a) odd
(b) even
(c) neither even nor odd
(d) one-one

BCECE-2005

Ans. (b): Given,
f is any function.
And let g(x) = $\frac{1}{2}\left[f(x)+f(-x)\right]$
We know that –
For even, f(–x) = f(x)
And for odd, f(–x) = – f(x)
Then, g(–x) = $\frac{1}{2}\left[f(-x)+f(x)\right]$
We see that, g(–x) = g(x)
Hence, the given function is always even.

315. Let p (x) = ax^2 + bx, q (x) = lx^2 + mx + n, with p (1) – q (1) = 0, p (2) –q(2) = 1 and p (3) – q(3) = 4, then p (4) – q(4) equals to

(a) 0 (b) 5
(c) 6 (d) 9

Rajasthan PET-2011

Ans. (d) : We have,
p(x) = ax^2 + bx and q(x) = lx^2 + mx + n
Now, calculate–
p(1) –q(1) = 0
$a(1)^2$ + b (1) – [l (1^2 + m (1) + n] =0
a + b – l – m – n = 0
(a – l) + (b – m) – n = 0 ...(i)
Similarly, calculate –
p(2) –q(2) = 1
4(a – l) + 2 (b – m) – n = 1 ...(ii)

And,

$p(3) - q(3) = 4$

$9(a - l) + 3(b - m) - n = 4$ ….(iii)

Assume,

$(a - l) = u$ and $(b - m) = v$

So,

From (i), $u + v - n = 0$ ...(iv)

From (ii), $4u + 2v - n = 1$ ...(v)

From (iii), $9u + 3v - n = 4$ …(vi)

Subtracting (iv) from (v), we have –

$3u + v = 1$ ...(vii)

Subtracting (v) from (vi), we get–

$5u + v = 3$ …(viii)

Subtracting (vii) from (viii), we get–

$2u = 2 \Rightarrow u = 1$

Now, from (vii) , we get –

$v = -2$

And, from (iv,) we get-

$n = -1$

Now, calculate-

$$\begin{aligned} p(4) - q(4) &= 16(a - l) + 4(b - m) - n \\ &= 16u + 4v - n \\ &= 16 \times 1 + 4(-2) + 1 \\ &= 16 - 8 + 1 \\ &= 9 \end{aligned}$$

316. f (x) is real valued function such that 2f(x) + 3f(–x) = 15 – 4x for all x ∈ R. Then f(2) =

(a) –15 (b) 22

(c) 11 (d) 0

WB JEE-2021

Ans. (c): Given that,

$2f(x) + 3f(-x) = 15 - 4x$ …(i)

Replacing x by (–x) we get–

$2f(-x) + 3f(x) = 15 - 4(-x)$

$\Rightarrow 3f(x) + 2f(-x) = 15 + 4x$ … (ii)

On solving equation (i) and (ii) we get –

$4f(x) + 6f(-x) = 30 - 8x$

$9f(x) + 6f(-x) = 45 + 12x$

$- \quad - \quad - \quad -$

$-5f(x) = -15 - 20x$

$$f(x) = \frac{(-5)(3+4x)}{-5} = 3 + 4x$$

Now, putting the value of x = 2 we get–

$f(2) = 3 + 4 \times 2$

$= 3 + 8 = 11$

Hence option (c) is correct.

317. Consider the real valued function h : {0,1,2…..100} → R such that h (0) = 5, h (100)=20 and satisfying h(p) = $\frac{1}{2}$ {h(p + 1) + h(p – 1)} for every p = 1, 2….99. Then the value of h (1) is

(a) 5.15 (b) 5.5

(c) 6 (d) 6.15

WB JEE-2021

Ans. (a): Given,

$$h(p) = \frac{1}{2}[h(p+1) + h(p-1)]$$

Here, h (p –1), h(p), h(p + 1) are in A.P.

Therefore,

$h(100) = h(0) + 99d$

$$d = \frac{20-5}{99}$$

$$d = \frac{15}{99}$$

Now, $h(1) = h(0) + d$

$$h(1) = 5 + \frac{15}{99}$$

$$h(1) = \frac{5 \times 99 + 15 \times 1}{99}$$

$$h(1) = \frac{495+15}{99} = \frac{510}{99}$$

$h(1) = 5.15$

318. Let A = {1, 2, 3, 4, 5} and B = {1, 2, 3, 4, 5, 6}. Then the number of function f : A → B satisfying f(1) + f(2) = f(4) – 1 is equal to ______

JEE Main-11.04.2023, Shift-II

Ans. (360) : Given,

A = { 1, 2, 3, 4, 5}

And, B = {1, 2, 3, 4, 5, 6}

Now,

$f(1) + f(2) + 1 = f(4) \le 6$

$f(1) + f(2) \le 5$

C – I

$f(1) = 1 \Rightarrow f(2) = 1, 2, 3, 4 \Rightarrow$ 4 mappings

C–II

$f(1) = 2 \Rightarrow f(2) = 1, 2, 3, \Rightarrow$ 3 mappings

C. II

$f(1) = 3 \Rightarrow f(2) = 1, 2 \Rightarrow$ 2 mapping

C.IV

$f(1) = 4 \Rightarrow f(2) = 1 \Rightarrow$ 1 mapping

And f (5) and f(6) both have 6 and 6 mapping.

Hence, the number of function = $(4 + 3 + 2 + 1) \times 6 \times 6$

$= 10 \times 36$

$= 360$

319. If $f(x) = \frac{2x-3}{(x-2)(x-3)}$ is a valued function then the value that f (x) does not take is

(a) –10 (b) 2

(c) 1 (d) –2

TS EAMCET-19.07.2022, Shift-II

Ans. (d) : Let, y = f(x) = $\frac{2x-3}{(x-2)(x-3)}$

$(2x-3) = y(x^2-5x+6)$

$yx^2-(2+5y)x+(6y+3)=0$

x is real $\Rightarrow b^2-4ac \ge 0$

$(2+5y)^2-4(6y+3)\ge 0$

$y^2+8y+4\ge 0$

$y^2+8y+16-12\ge 0$

$(y+4)^2-\left(\sqrt{12}\right)^2\ge 0$

$\left(y+4-\sqrt{12}\right)\left(y+4+\sqrt{12}\right)\ge 0$

$y = \left[-\left(4+\sqrt{12}\right), -\left(4-\sqrt{12}\right)\right]$

Hence, y does not take –2.

320. Match the functions of List-I with their nature in List-II and choose the correct option.

List - I	List - II
A) $f: R\to R$ defined by $f(x)=\cos(112x-37)$	I) Injection but not surjection
B) $f: A\to B$ defined by $f(x)=x\|x\|$ when $A=[-2,2]$ & $B=[-4,4]$	II) Surjection but not injection
C) $f: R\to R$ defined by $f(x)=(x-2)(x-3)(x-5)$	III) Bijection
D) $f: N\to N$ defined by $f(n)=n+1$	IV) Neither injection nor surjection
	V) Composite function

The correct match is

	A	B	C	D
(a)	II	II	III	V
(b)	IV	I	II	III
(c)	IV	III	II	V
(d)	IV	III	II	V

TS EAMCET-03.05.2019, Shift-I

Ans. (d) : Given,

f(x) = cos (112x – 37)

Let, g(x) = cos x, h(x) = 112x – 37

f(x) = g(h(x)) is composite function.

$f(x) = \begin{cases} x^2 \text{ if } x\ge 0 \\ -x^2 \text{ if } x<0\end{cases}$

Since, f is both one-one and onto

$\therefore$ f(x) = (x – 1) (x – 2)(x – 3)

1, 2, 3, $\in$ R.

f(1) = f(2) = f(3)

f is not one-one

co-domain = range

f is onto but not one-one

f(n) = n + 1

$\therefore$ f is one-one.

As f does not have any pre image.

Hence, f is not onto.

321. Let R be the set of all real number.

Statement I: The function $f:\left(-\frac{\pi}{2},\frac{\pi}{2}\right)\to R$ defined by f(x) = sec x + tan x is a one-one function.

Statement II: The function $f:[0,\infty)\to R$ defined by $f(x) = x^2$ is a one –one function.

Which of the above statement is (are) true?

(a) Statement I is true, but Statement II is false
(b) Statement II is true, but Statement I is false
(c) Both Statement I and Statement II are true
(d) Both Statement I and Statement II are false

TS EAMCET-18.07.2022, Shift-II

Ans. (c) : Given,

$f:\left(-\frac{\pi}{2},\frac{\pi}{2}\right)\to R$

f(x) = sec x + tan x

$= \frac{1+\tan^2\frac{x}{2}}{1-\tan^2\frac{x}{2}}+\frac{2\tan\frac{x}{2}}{1-\tan^2\frac{x}{2}}$

$= \frac{\left(1+\tan\frac{x}{2}\right)^2}{1-\tan^2\frac{x}{2}} = \frac{1+\tan\frac{x}{2}}{1-\tan\frac{x}{2}} = \tan\left(\frac{\pi}{4}+\frac{x}{2}\right)$

$f'(x) = \sec^2\left(\frac{\pi}{4}+\frac{x}{2}\right).\frac{1}{2} > 0 \ \forall\ x = \in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

$\therefore$ f(x) is an increasing function.

Hence, f(x) is one- one function.

f: $[0,\infty)\to R$, $f(x) = x^2$

$f'(x) = 2x \ge 0\ \forall\ x\in[0,\infty)$

Both Statements I and II are true.

322. The number of functions f from {1, 2, 3, ... 20} onto {1, 2, 3, ... 20}such that f(k) is a multiple of 3, whenever k is a multiple of 4, is

(a) (15)! × 6!
(b) $5^6\times 15$
(c) 5! × 6!
(d) $6^5\times(15)!$

JEE Main 11.01.2019 Shift - II

Ans. (a) : Let, the multiple of 3 is f (k).

f(k) = (3, 6, 9, 12, 15, 18)

for k = 4, 8, 12, 16, 20

For these k we have 6.5.4.3.2 ways = 6!

For other numbers we have 15! ways.

So total = 15! 6!.

323. If $f: R\to R$ is defined as $f(x) = \frac{3^x+3^{-x}}{2}$, $\forall x,\in R$ and it satisfies f(x + y) + f(x – y) = a f(x) f(y), then a =

(a) 2 (b) 1 (c) 4 (d) 8

TS EAMCET-04.08.2021, Shift-I

Ans. (a): Let, $f(x) = \frac{K^x + K^{-x}}{2}, \left(K \in R^+\right)$, then

We know that, f(x + y) + f(x – y) = 2f(x) f(y)

So, by camping f (x + y) + f(x – y) = af(x) f(y)

The above condign, then a = 2

324. The number of integral values of x satisfying $9x - 2 < (x+2)^2 < 12x - 3$ is

(a) not finite (b) 3

(c) 4 (d) 5

TS EAMCET-14.09.2020, Shift-I

Ans. (b) : We have,

$$9x - 2 < (x+2)^2 < 12x - 3$$

Case I: $9x - 2 < x^2 + 4x + 4$

$x^2 - 5x + 6 > 0 \Rightarrow (x-3)(x-2) > 0$

$x \in (-\infty, 2) \cup (3, \infty)$ ...(i)

Case II: $x^2 + 4x + 4 < 12x - 3$

$x^2 - 8x + 7 < 0 \Rightarrow (x-7)(x-1) < 0$

$x \in (1, 7)$ ...(ii)

From equation (i) and (ii) $x \in (1,2) \cup (3,7)$

Number of integral value of x is 3 i.e., {4, 5, 6}.

325. The number of non-constant functions f from X = {0, 1, 2} to Y = {1, 2, 3, 4, 5, 6, 7, 8} such that f(i) ≤ f(j) for i, j ∈ X and i < j is

(a) 120 (b) 92

(c) 56 (d) 112

TS EAMCET-03.05.2019, Shift-II

Ans. (d) : Given,

Sets X = {0,1,2} and Y = { 1, 2, 3, 4, 5, 6, 7, 8} and non – constant function f : X →Y, such that

f(i) ≤ f (j) i, j ∈ X and i < j.

Now following two cases are possible

Case – I

Let range of 'f ' are a, b, c ∈ y, that a < b < c

means function is strictly increasing.

Then number of way of selection 3 distinct number a_1 b_2 and c is ${}^8C_3 = 56$

Case – II

If a = b < c or a < b = c

Now number of ways of selecting two numbers

a = b, c or a, b = c is ${}^8C_2$ and since two elements are identical which we can make in ${}^2C_1$ ways. So,

number of ways to make such combination is ${}^8C_2 \times {}^2C_1$ = 56

So, required number of non – constant functions are

= 56 + 56 = 112.

326. If the function *f* : [a, b]→ defined by $f(x) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1+\sin x & 1 \\ 1+\cos x & 1 & 1 \end{bmatrix}$ is one-one and onto, then

(a) $a = \frac{-\pi}{4}, b = \frac{\pi}{6}$ (b) $a = \frac{-\pi}{2}, b = \frac{\pi}{2}$

(c) $a = \frac{-\pi}{6}, b = \frac{\pi}{4}$ (d) $a = \pi, b = \pi$

TS EAMCET-04.05.2019, Shift-II

Ans. (a) : Given.

$$f(x) = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+\sin x & 1 \\ 1+\cos x & 1 & 1 \end{vmatrix}$$

On applying $C_3 \rightarrow C_3 - C_1$ and $C_2 \rightarrow C_2 - C_1$ we get –

$$f(x) = \begin{vmatrix} 1 & 0 & 0 \\ 1 & \sin x & 0 \\ 1+\cos x & -\cos x & -\cos x \end{vmatrix}$$

f(x) = –sinx . cosx

f(x) = – sin x cos x

$f(x) = \frac{-\sin 2x}{2}$

= $\frac{-\sqrt{3}}{4} \le -\frac{-\sin 2x}{2} \le \frac{1}{2}$

= $-\frac{\sqrt{3}}{2} \le -\sin 2x \le 1$

= $-1 \le \sin 2x \le \frac{\sqrt{3}}{2}$

= $2x = -\frac{\pi}{2}$

$x = \frac{-\pi}{4}$

$2x = \frac{\pi}{3}$

$x = \frac{\pi}{6}$

So, $-\frac{\pi}{4} \le x \le \frac{\pi}{6}$

327. If f :R →R is defined as f (x) = x – [x] + 3, ∀ x π R, then f is

(a) Not a function

(b) A periodic function with period π

(c) A periodic function with period 1

(d) An invertible function

AP EAMCET-23.09.2020, Shift-I

Ans. (c) : Given,

f (x) = x – [x] + 3

= x – (x – {x}) + 3 [∵[x] = x – {x}]

f(x) = {x} + 3

∴ f(x) is periodic function with period 1.

328. If $5^x = (0.5)^y = 1000$, then $\frac{1}{x} - \frac{1}{y}$ is equal to

(a) 1 (b) $\frac{1}{2}$

(c) $\frac{1}{3}$ (d) $\frac{1}{4}$

EAMCET-2000

Ans. (c) : Given,
$5^x = (0.5)^y = 1000$
Then, $5^x = 1000 \Rightarrow x \log 5 = 3$

$x = \frac{3}{\log 5}$

$\frac{1}{x} = \frac{\log 5}{3}$

And, $(0.5)^y = 1000 \Rightarrow y\log(0.5) = \log 10^3 = 3$

$\therefore \ y = \frac{3}{\log \frac{5}{10}}$

$y = \frac{3}{\log 5 - 1}$

$\Rightarrow \frac{1}{y} = \frac{\log 5 - 1}{3} = \frac{\log 5}{3} - \frac{1}{3}$

So, $\frac{1}{x} - \frac{1}{y} = \frac{\log 5}{3} - \frac{\log 5}{3} + \frac{1}{3} = \frac{1}{3}$

329. If $x = 7 + 4\sqrt{3}$ and $xy = 1$, then $\frac{1}{x^2} + \frac{1}{y^2}$ is equal to

(a) 64 (b) 134

(c) 194 (d) $\frac{1}{49}$

EAMCET-1999

Ans. (c) : Given, $x = 7 + 4\sqrt{3}$ and $xy = 1$

$\therefore \ \frac{1}{x} = \frac{1}{7+4\sqrt{3}} = \frac{7-4\sqrt{3}}{(7+4\sqrt{3})(7-4\sqrt{3})}$

$= \frac{7-4\sqrt{3}}{49-48} = 7 - 4\sqrt{3}$

And, $\frac{1}{y} = 7 + 4\sqrt{3}$

$\therefore \ \frac{1}{x^2} + \frac{1}{y^2} = (7-4\sqrt{3})^2 + (7+4\sqrt{3})^2 = 49 + 48 + 49 + 48 = 97 \times 2 = 194.$

330. $\log_8 128$ is equal to

(a) $\frac{7}{3}$ (b) $\frac{3}{7}$ (c) $\frac{1}{16}$ (d) 16

EAMCET-1999

Ans. (a) : Given,

$$\log_8 128 = \frac{\log_2 128}{\log_2 8} = \frac{\log_2 2^7}{\log_2 2^3} = \frac{7\log_2 2}{3\log_2 2} = \frac{7}{3}$$

331. If $x = 2\sqrt{2} + \sqrt{7}$, then $x + \frac{1}{x}$ is equal to

(a) $2\sqrt{2}$ (b) $4\sqrt{2}$

(c) 8 (d) $\sqrt{7}$

EAMCET-1998

Ans. (b) : Given,
$x = 2\sqrt{2} + \sqrt{7}$

$\therefore \ \frac{1}{x} = \frac{1}{2\sqrt{2}+\sqrt{7}} \times \frac{(2\sqrt{2}-\sqrt{7})}{(2\sqrt{2}-\sqrt{7})}$

$\frac{1}{x} = \frac{2\sqrt{2}-\sqrt{7}}{8-7} = 2\sqrt{2} - \sqrt{7}$

$\therefore \ x + \frac{1}{x} = 2\sqrt{2} + \sqrt{7} + 2\sqrt{2} - \sqrt{7} = 4\sqrt{2}$

332. $0.0001 < n < 0.001$, then

(a) $-4 < \log n < -3$ (b) $-3 < \log n < -2$

(c) $-2 < \log n < -1$ (d) $-5 < \log n < -4$

EAMCET-1996

Ans. (a) : Given $0.0001 < n < 0.001$
Taking log both side, we get –
$\log 0.0001 < \log n < \log 0.001$
or $\log 10^{-4} < \log n < \log 10^{-3}$
or $-4 < \log n < -3$

333. If $\log 2 + \frac{1}{2}\log a + \frac{1}{2}\log b = \log(a+b)$, then

(a) $a = b$ (b) $a = -b$

(c) $a = 2, b = 0$ (d) $a = 10, b = 1$

EAMCET-1995

Ans. (a) : Given,

$\log 2 + \frac{1}{2}\log a + \frac{1}{2}\log b = \log(a+b)$

or

$\log 2 + \log \sqrt{a} + \log \sqrt{b} = \log(a+b)$

or $\log 2.\sqrt{ab} = \log a + b$

$\therefore$ $a + b = 2.\sqrt{ab}$

or $(\sqrt{a} - \sqrt{b})^2 = 0$

So, $a = b$

334. If $\log_2 x + \log_x 2 = \frac{10}{3} = \log_2 y + \log_y 2$ and $x \neq y$, then $x + y$ is equal to

(a) $2^{1/3} + 3^2$ (b) $2^3 + 2^{1/3}$

(c) $3^{1/3} + 2^3$ (d) None of these

EAMCET-1994

Ans. (b) : Given,

$\log_2 x + \log_x 2 = \frac{10}{3}$

Or $\log_2 x + \frac{1}{\log_2 x} = \frac{10}{3}$

Let, $\log_2 x = t$

$\therefore \quad t + \frac{1}{t} = \frac{10}{3}$

$\Rightarrow \quad t^2 + 1 = \frac{10}{3}.t$

$\Rightarrow \quad 3t^2 - 10t + 3 = 0$

$\Rightarrow \quad (3t-1)(t-3) = 0$

$\Rightarrow \quad t = 3, \frac{1}{3}$

Take t = 3, we get

$\log_2 x = 3 \Rightarrow x = 2^3$

$\log_2 x = \frac{1}{3} \Rightarrow x = 2^{1/3}$

So, $x = 2$

$y = 2^{1/3}$

$\because \quad x \neq y.$

Hence, $(x + y) = 2^3 + 2^{1/3}$

335. If $x = \sqrt{7+4\sqrt{3}}$, then $x + \frac{1}{x}$ is equal to

(a) 4 (b) 6

(c) 2 (d) 3

EAMCET-1994

Ans. (a) : Given,

$x = \sqrt{7+4\sqrt{3}}$

$x = \sqrt{\left(2+\sqrt{3}\right)^2}$

$= 2+\sqrt{3}$

$\frac{1}{x} = \frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}} = 2-\sqrt{3}$

$\therefore \; x + \frac{1}{x} = 2+\sqrt{3}+2-\sqrt{3} = 4$

336. If x > 0 and $\log_4(x^3 + x^2) - \log_4(x + 1) = 2$, then value of x is

(a) 4 (b) 64

(c) 8 (d) 2

EAMCET-1993

Ans. (a) : Given x > 0 then

$\log_4(x^3 + x^2) - \log_4(x+1) = 2$. this can be written as

$\log_4 x^2(x+1) - \log_4(x+1) = 2$

or $\quad \log_4 x^2 + \log_4(x+1) - \log_4(x+1) = 2$

$\log_4 x^2 = 2$

$x^2 = 4^2$

$\therefore x = 4$

337. The real value(s) of x which satisfy $\left(5+2\sqrt{6}\right)^{x^2-3} + \left(5-2\sqrt{6}\right)^{x^2-3} = 10$ is/are

(a) $2, -\sqrt{2}$ (b) $\pm 2, \pm\sqrt{2}$

(c) $2, \sqrt{2}$ (d) $-2, -\sqrt{2}$

EAMCET-1992

Ans. (b) : Given,

$\left(5+2\sqrt{6}\right)^{x^2-3} + \left(5-2\sqrt{3}\right)^{x^2-3} = 10$

Let $t = \left(5+2\sqrt{6}\right)^{x^2-3}$ then

$\frac{1}{t} = \left(5-2\sqrt{6}\right)^{x^2-3}$

Then we have

$t + \frac{1}{t} = 10$

or $\quad t^2 + 1 = 10t$

or $\quad t^2 + 1 = 10t$

or $\quad t = \frac{10 \pm \sqrt{100-4}}{2}$

$= \left(5 \pm 2\sqrt{6}\right)$

Taking + sign we get

$\left(5+2\sqrt{6}\right)^{x^2-3} = \left(5+2\sqrt{6}\right)$

$\therefore\; x^2 - 3 = 1 \quad$ or $\quad x^2 = 4$

or $\quad x = \pm 2$

Again

$\left(5+2\sqrt{6}\right)^{x^2-3} = \left(5-2\sqrt{6}\right)$

$= \frac{1}{5+2\sqrt{6}}$

$= \left(5+2\sqrt{6}\right)^{-1}$

$\therefore \quad x^2 - 3 = -1$

or $\quad x^2 = 2$

or $\quad x = \pm\sqrt{2}$

338. The value of $\frac{\log_3 5 \times \log_{25} 27 \times \log_{49} 7}{\log_{81} 3}$ is

(a) 1 (b) 6

(c) $\frac{2}{3}$ (d) 3

WB JEE-2010

Ans. (d) : Given,

$\frac{\log_3^5 \times \log_{25}^{27} \times \log_{49}^7}{\log_{81}^3}$

$= \frac{\log_3^5 \times \frac{\log_3^{27}}{\log_3^{25}} \times \frac{\log_7^7}{\log_7^{49}}}{\frac{\log_3^3}{\log_3^{81}}}$

$= \frac{\log_3^5 \times \frac{3}{2\log_3^5} \times \frac{1}{2}}{\frac{1}{4}}$

$= \frac{3}{4} \times \frac{4}{1} = 3$

339. The value of $\left(\frac{1}{\log_3 12}+\frac{1}{\log_4 12}\right)$ is

(a) 0 (b) $\frac{1}{2}$

(c) 1 (d) 2

WB JEE-2009

Ans. (c) : Given,

$$\frac{1}{\log_3 12}+\frac{1}{\log_4 12}$$

$$=\frac{1}{\log_3 3\times 4}+\frac{1}{\log_4 4\times 3}$$

$$=\frac{1}{\log_3 3+\log_3 4}+\frac{1}{\log_4 4+\log_4 3}$$

$$=\frac{1}{1+\log_3 4}+\frac{1}{1+\log_4 3}$$

$$=\frac{1}{1+\frac{1}{\log_4 3}}+\frac{1}{1+\frac{1}{\log_4 3}}$$

$$=\frac{\log_4 3}{1+\log_4 3}+\frac{1}{1+\log_4 3}=\frac{1+\log_4 3}{1+\log_4 3}=1$$

340. The even function of the following is

(a) $f(x)=\frac{a^x+a^{-x}}{a^x-a^{-x}}$

(b) $f(x)=\frac{a^x+1}{a^x-1}$

(c) $f(x)=x\cdot\frac{a^x-1}{a^x+1}$

(d) $f(x)=\log_2\left(x+\sqrt{x^2+1}\right)$

WB JEE-2011

Ans. (c) : A function is even function if

f(x) = f(–x)

Let as consider–

$$f(x)=x\,.\,\frac{(a^x-1)}{(a^x+1)}$$

$$\therefore\ f(-x)=\frac{-x\left[a^{-x}-1\right]}{\left[a^{-x}+1\right]}$$

$$=-x\left[\frac{1-a^x}{1+a^x}\right]=x.\left[\frac{a^x-1}{a^x+1}\right]=f(x)$$

341. If $\log_3 x+\log_3 y=2+\log_3 2$ and $\log_3(x+y)=2$, then

(a) x = 1, y = 8 (b) x = 8, y = 1

(c) x = 3, y = 6 (d) x = 9, y = 3

WB JEE-2011

Ans. (c) : Given,

$\log_3 x+\log_3 y=2+\log_3 2$

or $\log_3 x+\log_3 y-\log_3 2=2$

or $\log_3 [xy/2]=2$

$\therefore\ \frac{xy}{2}=3^2=9$

$\therefore\ xy=18$ .....(i)

We known,

$(x-y)^2=(x+y)^2-4xy$

$=9^2-4.18$

$=81-72=9$

$\therefore\ x-y=\pm 3$ $x-y=-3$

$\therefore\ x+y=9$ $x+y=9$

$2x=12$ $2x=6$

$x=6$ $x=3$

$\therefore\ y=3$ $y=6$

342. If $\log_7 2=\lambda$, then the value of $\log_{49}(28)$ is

(a) $(2\lambda+1)$ (b) $(2\lambda+3)$

(c) $\frac{1}{2}(2\lambda+1)$ (d) $2(2\lambda+1)$

WB JEE-2011

Ans. (c) : Given,

$\log_7 2=\lambda$

Now, the value of –

$$\log_{49} 28=\frac{\log_7 28}{\log_7 49}=\frac{\log_7(7\times 4)}{\log_7 7^2}$$

$$=\frac{\log_7 7+\log_7 4}{2\log_7 7}\qquad \{\because \log_a a=1\}$$

$$=\frac{1+\log_7 4}{2}=\frac{1+2\log_7 2}{2}$$

$$=\frac{1+2\lambda}{2}=\frac{1}{2}(2\lambda+1)$$

343. In the function $f(x)=\frac{a^x+a^{-x}}{2},(a>2)$ then f(x+y) +f (x–y) is equal to

(a) $f(x)-f(y)$ (b) $f(y)$

(c) $2f(x)f(y)$ (d) $f(x)f(y)$

AP EAMCET-19.08.2021, Shift-I

Ans. (c): Given,

$$f(x)=\frac{a^x+a^{-x}}{2},\ a>2$$

$$f(x+y)+f(x-y)=\frac{a^{x+y}+a^{-(x+y)}}{2}+\frac{a^{(x-y)}+a^{-(x-y)}}{2}$$

$$=\frac{a^x\cdot a^y+a^{-x}\cdot a^{-y}+a^x\cdot a^{-y}+a^{-x}\cdot a^y}{2}$$

$$=\frac{a^x\left(a^y+a^{-y}\right)+a^{-x}\left(a^y+a^{-y}\right)}{2}$$

$$=\frac{\left(a^y+a^{-y}\right)\left(a^x+a^{-x}\right)}{2}$$

$$= \frac{(a^y + a^{-y})(a^x + a^{-x}) \times 2}{2 \times 2} \left[f(x) = \frac{a^x + a^{-x}}{2},\ f(y) = \frac{a^y + a^{-y}}{2} \right]$$
$$= 2 \cdot f(x) \cdot f(y)$$

344. The solution of the equation $\log_{101}\log_7\left(\sqrt{x+7}+\sqrt{x}\right) = 0$ is

(a) 3 (b) 7
(c) 9 (d) 49

WB JEE-2014

Ans. (c) : Given,

$$\log_{101}\log_7\left(\sqrt{x+7}+\sqrt{x}\right) = 0$$
$$\log_7\left(\sqrt{x+7}+\sqrt{x}\right) = 1$$
$$\sqrt{x+7}+\sqrt{x} = 7$$

Squaring both the sides, we get –

$$x + 7 + x + 2.\ \sqrt{x}.\sqrt{x+7} = 49$$

or, $2\sqrt{x}.\sqrt{x+7} = 49 - 7 - 2x$

$= 42 - 2x$

$2\sqrt{x}.\sqrt{x+7} = 49 - 7 - 2x$

Again squaring, we get –

$x(x + 7) = (21)^2 + x^2 - 42x$

or, $x^2 + 7x = (21)^2 + x^2 - 42x$

$\therefore$ $49x = 21 \times 21$

$\therefore$ $x = \frac{21 \times 21}{7 \times 7} = 3 \times 3 = 9$

345. Consider the non-constant differentiable function f of one variable which obeys the relation $\frac{f(x)}{f(y)} = f(x-y)$. If f' (0) = p and f' (5) = q, then f' (– 5) is

(a) $\frac{p^2}{q}$ (b) $\frac{q}{p}$

(c) $\frac{p}{q}$ (d) q

WB JEE-2017

Ans. (a) : We have,

$$\frac{f(x)}{f(y)} = f(x-y)$$

$\Rightarrow$ $f(x) = a^{kx}$

$\therefore$ $f'(x) = ka^{ka} \log a$

Again, $f'(0) = P$

$\Rightarrow$ $ka^o \log a = P$

$\Rightarrow$ $k \log a = P$

Also, $f-(5) = q$

$ka^{sk} \log a = q$

$a^{sk} P = q$

$a^{sk} = \frac{q}{p}$

Now, $f'(-5) = ka^{-sk} \log a$

$$= \frac{k \log a}{a^{sk}}$$
$$= \frac{p}{\left(\frac{q}{p}\right)} = \frac{p^2}{q}$$

346. If $(a^4 - 2a^2b^2 + b^4)^{x-1} = (a - b)^{2x} (a + b)^{-2}$, a > 0 b > 0, then x =

(a) $\frac{\log a}{\log b}$ (b) $\frac{\log b}{\log a}$

(c) $\frac{\log(a+b)}{\log|a-b|}$ (d) $\frac{\log|a-b|}{\log(a+b)}$

COMEDK-2018

Ans. (d) : $(a^2 - b^2)^{2(x-1)} = (a-b)^{2x}(a+b)^{-2}$

$\Rightarrow (a+b)^{2x-2}.(a-b)^{2x-2} = (a-b)^{2x}(a+b)^{-2}$

$\Rightarrow (a+b)^{2x} = (a-b)^2$

$\Rightarrow (a+b)^x = |a-b|$

$\Rightarrow x = \frac{\log|a-b|}{\log(a+b)}.$

347. If $(4)^{\log_9 3} + (9)^{\log_2 4} = (10)^{\log_x 83}$, then x is equal to

(a) 10 (b) 4
(c) –10 (d) –4

COMEDK-2012

Ans. (a) : $(4)^{\log_{3^2} 3} + (9)^{\log_{2^2} 2} = (10)^{\log_x 83}$

$\Rightarrow (4)^{1/2} + 9^2 = (10)^{\log_x 83} \Rightarrow (83)^1 = (83)^{\log_x 10}$

[Using $x \log_a y = y \log_a x$]

$\Rightarrow 1 = \log_x 10 \Rightarrow x = 10$

348. If $\log_2\left(9^{x-1}+7\right) - \log_2\left(3^{x-1}+1\right) = 2$, then values of x are

(a) 1, 2 (b) 0, 2
(c) 0, 1 (d) 1, 4

Karnataka CET-2012

Ans. (a) : Given,

$$\log_2\left(9^{x-1}+7\right) - \log_2\left(3^{x-1}+1\right) = 2$$
$$= \log_2 \frac{9^{x-1}+7}{\left(3^{x-1}+1\right)}$$
$$= \frac{9^{x-1}+7}{3^{x-1}+1} = 2^2 = 4$$
$$= 9^{x-1} + 7 = 4.\ 3^{x-1} + 4$$
$$= 3^{2(x-1)} - 4 \times 3^{(x-1)} + 3 = 0$$

Let, $3^{x-1} = t$ so, we get –

$t^2 - 3t - t + 3 = 0$

$t(t - 3) - 1(t - 3) = 0$

$(t - 3)(t - 1) = 0$

$\Rightarrow$ $t = 1, t = 3$

Take, $t = 3 \Rightarrow 3^{x-1} = 3$

$x - 1 = 1, x = 2$

Take, $t = 1 \Rightarrow 3^{x-1} = 1$
$x - 1 = 0,\ x = 1$
$\therefore$ The value of x are 1, 2

349. If $f(x) = e^x g(x)$, $g(0) = 2$, $g'(0) = 1$, then $f(0)$ is equal to

(a) 1 (b) 3
(c) 2 (d) 0

Jamia Millia Islamia-2010

Ans. (b) : We have,
$f(x) = e^x g(x)$
Now differentiating, we get–
$f'(x) = e^x g'(x) + g(x) e^x$
$f'(x) = e^x(g(x) + g'(x))$
$f'(0) = e^0(g(0) + g'(0))$
$g(0) = 2$ and $g'(0) = 1$
$f'(0) = e^0(2 + 1)$
$f'(0) = 1(3)$
$f'(0) = 3$

350. If $f(x) = |\log_e|x||$, then $f_0(x)$ equals

(a) $\dfrac{1}{|x|}, x \neq 0$

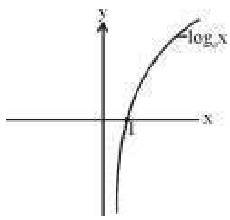
(b) $\dfrac{1}{x}$ for $|x| > 1$ and $\dfrac{-1}{x}$ for $|x| < 1$

(c) $\dfrac{-1}{x}$ for $|x| > 1$ and $\dfrac{1}{x}$ for $|x| < 1$

(d) $\dfrac{1}{x}$ for $x > 0$ and $-\dfrac{1}{x}$ for $x < 0$

Jamia Millia Islamia-2009

Ans. (b) : $f(x) = |\log_e|x||$
Let, $y = f(x) = |\log_e|x||$
Thus, for $x > 1$
$f(x) = \log_e x$
$f'(x) = \dfrac{1}{x}$ …. (i)



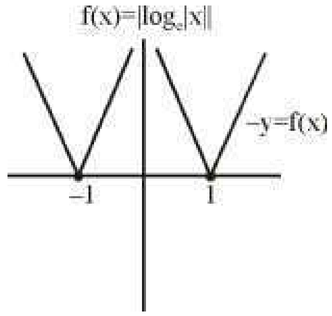
For, $x < -1$
$f(x) = \log_e(-x)$
$f(x) = \log_e(-x)$

$f'(x) = -\dfrac{1}{x}(-1) = \dfrac{1}{x}$ ….(ii)

For, $x \in (0, 1)$ or $0 < x < 1$
$f(x) = -\log_e x$ $\{\because -\log_e x \ \ x \in (0, 1)\}$
$f'(x) = -\dfrac{1}{x}$ ….(iii)

For, $x \in (-1, 0)$
$f(x) = -\log(-x)$
$f'(x) = -\left(-\dfrac{1}{x}(-1)\right)$
$f'(x) = -\dfrac{1}{x}$ ….(iv)

Then,



$$f'(x) = \begin{cases} \dfrac{1}{x} & x > 1 \\ \dfrac{1}{x} & x < -1 \\ -\dfrac{1}{x} & 0 < x < 1 \\ -\dfrac{1}{x} & -1 < x < 0 \end{cases}$$

$$f'(x) = \begin{cases} \dfrac{1}{x} & |x| > 1 \\ -\dfrac{1}{x} & |x| < 1 \end{cases}$$

351. If $f(x + 2y, x - 2y) = xy$, then $f(x, y)$ equals

(a) $\dfrac{x^2 - y^2}{8}$ (b) $\dfrac{x^2 - y^2}{4}$

(c) $\dfrac{x^2 + y^2}{4}$ (d) $\dfrac{x^2 - y^2}{2}$

Jamia Millia Islamia-2009

Ans. (a) : We have,
$f(x + 2y, x - 2y) = xy$
Let, $x + 2y = U$
And, $x - 2y = V$
Now adding, we get–
$2x = U + V$
$x = \dfrac{U+V}{2}$
And subtracting, we get–
$4y = U - V$
$y = \dfrac{U+V}{4}$
Then $f(x, y) = \left(\dfrac{U+V}{2}\right)\left(\dfrac{U-V}{4}\right) = \dfrac{U^2 - V^2}{8}$
Hence, $f(x, y) = \dfrac{x^2 - y^2}{8}$

352. If f(x) is an odd periodic function with period 2, then f(4) equals

(a) 0 (b) 2
(c) 4 (d) –4

Jamia Millia Islamia-2008

Ans. (a) : Given,

$f(x)$ = odd periodic function

Period = 2

$\therefore \quad f(-x) = -f(x)$

$f(x+2) = f(x)$

$f(2) = f(0)$

$f(-2) = f(-2+2)$

$f(-2) = f(0)$

Now,

$f(0) = f(-2) = -f(2) = -f(0)$

$\therefore \quad 2f(0) = 0$

$f(0) = 0$

$\therefore f(4) = f(2) = f(0) = 0$

353. If $\log_4 2 + \log_4 4 + \log_4 x + \log_4 16 = 6$, then value of x is

(a) 64 (b) 4

(c) 8 (d) 32

Jamia Millia Islamia-2008

Ans. (d) : We have,

$\log_4 2 + \log_4 4 + \log_4 x + \log_4 16 = 6$

$\log_4 (2 \times 4 \times x \times 16) = 6$

$\log_4 (128x) = 6$

$4^3 \times 2x = 4^6$

$2x = 4^3$

$x = 32$

354. If $y = 3^{x-1} + 3^{-x-1}$ (x real), then the least value of y is

(a) 2 (b) 6

(c) 2/3 (d) None of these

Jamia Millia Islamia-2006

Ans. (c) : Given that,

$y = 3^{x-1} + 3^{-x-1}$

Now we know that–

A.M ≥ G.M

$$\frac{3^{x-1}+3^{-x-1}}{2} \ge \left(3^{x-1}.3^{-x-1}\right)^{\frac{1}{2}}$$

$$3^{x-1}+3^{-x-1} \ge 2\left(3^{x-1}.3^{-x-1}\right)^{\frac{1}{2}}$$

$$3^{x-1}+3^{-x-1} \ge \frac{2}{3}$$

355. If $f(x) = \cos(\log x)$, then $f(x)\, f(y) -\frac{1}{2}\left[f\left(\frac{x}{y}\right)+f(xy)\right]$ has the value

(a) –1 (b) $\frac{1}{2}$

(c) –2 (d) zero

Jamia Millia Islamia-2006

Ans. (d) : Given that,

$f(x) = \cos(\log x)$

Now, $f(x).\, f(y) -\frac{1}{2}\left[f\left(\frac{x}{y}\right)+f(xy)\right]$

$$\cos(\log x).\cos(\log y)-\frac{1}{2}\left[\cos\log\left(\frac{x}{y}\right)+\cos\log(xy)\right]$$

$= \cos(\log x).\cos(\log y)$

$$-\frac{1}{2}\left[\cos[\log x-\log y]+\cos[\log x+\log y]\right]$$

$= \cos(\log x).\cos(\log y)$

$$-\frac{1}{2}\left[2\cos\left[\left(\frac{\log x-\log y+\log x+\log y}{2}\right).\cos\left(\frac{\log x-\log y-\log x-\log y}{2}\right)\right]\right]$$

$$\Rightarrow \cos(\log x).\cos(\log y) - \cos.\left(\frac{2\log x}{2}\right)\cos\left(\frac{2\log y}{2}\right)$$

$\Rightarrow \cos(\log x).\cos(\log y) - \cos\log x.\cos\log y = 0$

356. The number of solution of $\log_4 (x-1) = \log_2 (x-3)$ is

(a) 3 (b) 1

(c) 2 (d) 0

Manipal UGET-2012

Manipal UGET-2011

Ans. (b) : Given that,

$\log_4 (x-1) = \log_2 (x-3) = \log_{4^{1/2}} (x-3)$

$\log_4(x-1) = 2\log_4(x-3)$

$\log_4 (x-1) = \log_4 (x-3)^2$

$(x-1) = (x-3)^2$

$x^2+9-6x = x-1$

$x^2+9-6x = x-1$

$x^2-7x+10 = 0$

$(x-2)(x-5) = 0$

$x = 2,5$

$x = 2$ or $x = 5$

Hence, $x = 5$

$\because \quad x = 2$ makes $\log(x-3)$ undefined.

357. If $a = \log_2 3$, $b = \log_2 5$ and equal to $c = \log_7 2$, then $\log_{140} 63$ in terms of a, b, c is

(a) $\frac{2ac+1}{2a+abc+1}$ (b) $\frac{2ac+1}{2a+c+a}$

(c) $\frac{2ac+1}{2c+ab+a}$ (d) None of these

Manipal UGET-2012

Ans. (d) : Given that,

$a = \log_2 3$,

$b = \log_2 5$,

$c = \log_7 2$,

Now, $\log_{140} 63 = \log_{2^2\times5\times7}(3\times3\times7)$

$$= \frac{\log_2(3\times3\times7)}{\log_2(2^2\times5\times7)} = \frac{\log_2 3+\log_2 3+\log_2 7}{2\log_2 2+\log_2 5+\log_2 7}$$

$$= \frac{2a+\frac{1}{c}}{2+b+\frac{1}{c}} = \frac{2ac+1}{2c+bc+1}$$

358. If $\alpha \in \left[0, \frac{\pi}{2}\right)$, then $\sqrt{x^2+x} + \frac{\tan^2\alpha}{\sqrt{x^2+x}}$ is always greater than or equal to

(a) $2 \tan \alpha$ (b) 1
(c) 2 (d) $\sec^2 \alpha$

Manipal UGET-2012

Ans. (a) : Here, $\alpha \in \left[0, \frac{\pi}{2}\right) \Rightarrow \tan\alpha$ is $(+ve)$

As, we know, if a b > 0 $\Rightarrow \frac{a+b}{2} \ge \sqrt{ab}$

i.e., AM $\ge$ GM

$$\therefore \frac{\sqrt{x^2+x} + \frac{\tan^2\alpha}{\sqrt{x^2+x}}}{2} \ge \sqrt{\sqrt{x^2+x}\,\frac{\tan^2\alpha}{\sqrt{x^2+x}}}$$

($\because$ using $\ge$ GM)

$$\Rightarrow \sqrt{x^2+x} + \frac{\tan^2\alpha}{\sqrt{x^2+x}} \ge 2\tan\alpha$$

359. If $f(x) = \begin{cases} \frac{\sin[x]}{[x]}, & [x] \ne 0 \\ 0, & [x] = 0 \end{cases}$

Where [x] denotes the greatest integer less than or equal to x, then $\lim\limits_{x\to 0} f(x)$ equals

(a) 1 (b) 0
(c) –1 (d) None of these

Manipal UGET-2012

Ans. (d) : As,

$$f(x) = \begin{cases} \frac{\sin[x]}{[x]}, & [x] \ne 0 \\ 0, & [x] = 0 \end{cases}$$

$$\Rightarrow f(x) = \begin{cases} \frac{\sin[x]}{[x]}, & x \in R-[0,1) \\ 0, & 0 \le x < 1 \end{cases}$$

R.H.L, at x = 0

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0} \frac{\sin[0+h]}{[0+h]} = 0$$

L.H.L at x =0

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0} \frac{\sin[0-h]}{[0-h]}$$

$$= \lim_{h\to 0} \frac{\sin(-1)}{-1} = \sin 1$$

Hence, R.H.L R.H.L $\ne$ L.H. L

$\therefore$ Limit does not exist.

360. For which of the following values of 'x', does the function f(x) = log $\left[\left\{\sqrt{(25-x^2)}\right\}/(2-x)\right]$ have the real values?

(a) $-5 < x < 5$ (b) $-5 < x < 2$
(c) $x > -2$ (d) $x < 2$

J&K CET-2019

Ans. (b) : Given function,

$$f(x) = \log\left[\frac{\left\{\sqrt{25-x^2}\right\}}{2-x}\right]$$

$$f(x) = \log\left(\sqrt{25-x^2}\right) - \log(2-x)$$

$\sqrt{25-x^2} > 0$ and $2-x > 0$

$= 25 - x^2 > 0, -x > -2$

$= x^2 - 25 < 0, \; x < 2$

$= (x-5)(x+5) < 0 \quad x \in (-\infty, 2)$

$= x \in (-5, 5), \; x \in (-5, 2)$

$= -5 < x < 2$

361. Let function $f(x) = (x-1)^2 (x+1)^3$. Then which of the following is false?

(a) There exists a point where f(x) has a maximum value
(b) There exists a point where f(x) has a minimum value
(c) There exists a point where f(x) has neither maximum nor minimum value
(d) All of the above

J&K CET-2015

Ans. (d) : We have,

$f(x) = (x-1)^2 (x+1)^3$

Now, differentiating we get –

$f'(x) = (x-1)^2\, 3(x+1)^2 + (x+1)^3\, 2(x-1)$

For maxima or minima–

$f'(x) = 0$

$3(x-1)^2 (x+1)^2 + 2(x+1)^3 (x-1) = 0$

$(x-1)(x+1)^2 (5x-1) = 0$

$x = -1, 1, \frac{1}{5}$

Again differentiating w. r. t x we get–

$f''(x) = (5x^2 - 6x + 1)\, 2(x+1) + (x+1)^2 (10x-6)$

$= 2\left[5x^3 - x^2 - 5x + 1\right] + 2\left[5x^3 + 7x^2 - x - 3\right]$

$f''(x) = 2[10x^3 + 6x^2 - 6x - 2]$

at $x = -1$

$f''(x) = 0$

at $x = \frac{1}{5}$

$f''(x) < 0$

at $x = 1$

$f''(x) > 0$

At, x = – 1 f(x) has neither maximum nor minimum

at x = –1 f(x) has minimum value.

$x=\frac{1}{5}$ f(x) has maximum value.

362. The number of the solutions of the equation $5^{2x-1}+5^{x+1}=250$ is/are

(a) 0 (b) 1
(c) 2 (d) infinitely many

J&K CET-2015

Ans. (b) : We have given equation.

$5^{2x-1}+5^{x+1}=250$

$\frac{5^{2x}}{5}+5^x\cdot 5=250$

$5^{2x}+25\cdot 5^x=1250$

Let, $5^x=t$

$t^2+25t-1250=0$

$t=\frac{-25\pm\sqrt{(25)^2-4\times1\times(-1250)}}{2\times1}$

$t=\frac{-25\pm\sqrt{625+5000}}{2}$

$t=\frac{-25\pm 75}{2}$

$t=\frac{-25+75}{2}$ or $t=\frac{-25-75}{2}$

t = 25 and t = – 50

$5^x=5^2$ $5^x=-50$

x = 2

$5^x=-50$ can not express in 5^x

Hence, the number of solution is one.

363. If a function F is such that F(0) = 2, F (1) = 3, F(n + 2) = 2F(n) – F(n + 1) for n ≠ 0, then F(5) is equal to

(a) –7 (b) –3
(c) 7 (d) 13

J&K CET-2003

Ans. (d) : Given F(0) = 2, F(1) = 3

F(n + 2) = 2F(n) – F(n + 1)

Putting, n = 0

F(0 + 2) = 2 F(0) – F(0 + 1)

F(2) = 2 × 2 – 3

F(2) = 1

Put n = 1

F(3) = 2F (1) –F(2)

F(3) = 2 × 3 – 1

F(3) = 5

Put n = 2

F(2 + 2) = 2F(2) – F(3)

F(4) = 2 × 1 – 5

F(4) = –3

Put n = 3

F(5) = 2F(3) – F(4)

= 2 × 5 – (–3)

= 10 + 3

F(5) = 13

364. If $\log_2[\log_3\{\log_4(\log_5 x)\}]=0$, then the value of x is

(a) 5^{24} (b) 1
(c) 2^{25} (d) 5^{64}

J&K CET-2003

Ans. (d) : We have,

$\log_2[\log_3\{\log_4(\log_5 x\}]=0$

Using property $\log_a b=x$, $a^x=b$

$\log_3\{\log_4(\log_5 x)\}=2^0$

$\log_3\{\log_4(\log_5 x)\}=1$

$\log_4(\log_5 x)=3^1$

$\log_5 x=4^3$

$\log_5 x=64$

$x=5^{64}$

365. If $f(x)=ax^2+bx+c$ satisfies f(1) + 2f(2) = 0 and 2f(1) + f(2) = 0, then 3a + b =

(a) 2 (b) –1
(c) 0 (d) 1

AP EAMCET-06.07.2022, Shift-I

Ans. (c) : We have equation,

$f(x)=ax^2+bx+c$

f(1) + 2f(2) = 0

2f(1) + f(2) = 0

Now f(1) = a + b + c

f(2) = 4a + 2b + c

f(2) – f(1) = 3a + b

Now, f(1) + 2f(2) = 0

a + b + c + 2 (4a + 2b + c) = 0

2(a + b + c) + 4a + 2b + c = 0

Adding these equation,

3(a + b + c) + 3 (4a + 2b + c) = 0

3[a + b + c + 4a + 2b + c] = 0

3[5a + 3b + 2c] = 0

It can be written as

(a + b + c) – 2 (a + b + c) + 2 (4a + 2b + c)

– (4a + 2b + c)

As – (a + b + c) + 4a + 2b + c = 0

3a + b = 0

366. Let f be a function defined by $f(xy)=\frac{f(x)}{y}$ for all positive real numbers x and y If f(30) = 20, then f (40) =

(a) 10 (b) 15
(c) 25 (d) 17

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Ans. (b) : Given function

$f(xy)=\frac{f(x)}{y}$ .....(i)

Put x = 1

$f(y)=\frac{f(1)}{y}$

Put y = 30

$f(30) = \frac{f(1)}{30}$

$f(1) = f(30) \cdot 30$

$= 20 \times 30$

$f(1) = 600$

Put y = 40 in equation

$f(40) = \frac{f(1)}{40}$

$f(40) = \frac{600}{40}$

$f(40) = 15$

367. If $f(x) = \frac{1}{\sqrt{x+2\sqrt{2x-4}}} + \frac{1}{\sqrt{x-2\sqrt{2x-4}}}$ for x > 2, then $f(11)$ is equal to

(a) $\frac{7}{6}$ (b) $\frac{5}{6}$

(c) $\frac{6}{7}$ (d) $\frac{5}{7}$

EAMCET-2003

Ans. (c) : Given,

$f(x) = \frac{1}{\sqrt{x+2\sqrt{2x-4}}} + \frac{1}{\sqrt{x-2\sqrt{2x-4}}}$

Now find f(11)

Putting x = 11

$f(11) = \frac{1}{\sqrt{11+2\sqrt{2\times11-4}}} + \frac{1}{\sqrt{11-2\sqrt{2\times11-4}}}$

$= \frac{1}{\sqrt{11+2\sqrt{22-4}}} + \frac{1}{\sqrt{11-2\sqrt{22-4}}}$

$= \frac{1}{\sqrt{11+2\sqrt{18}}} + \frac{1}{\sqrt{11-2\sqrt{18}}}$

$= \frac{1}{\sqrt{9+2+6\sqrt{2}}} + \frac{1}{\sqrt{9+2-6\sqrt{2}}}$

$= \frac{1}{\sqrt{(3)^2+(\sqrt{2})^2+6\sqrt{2}}} + \frac{1}{\sqrt{(3)^2+(\sqrt{2})^2-6\sqrt{2}}}$

$= \frac{1}{\sqrt{(\sqrt{2}+3)^2}} + \frac{1}{\sqrt{(3)^2-(\sqrt{2})^2}}$

$= \frac{1}{3+\sqrt{2}} + \frac{1}{3-\sqrt{2}}$

$= \frac{3-\sqrt{2}+3+\sqrt{2}}{(3+\sqrt{2})(3-\sqrt{2})}$

$= \frac{6}{3^2-(\sqrt{2})^2}$

$= \frac{6}{9-2} = \frac{6}{7}$

368. If $e^{f(x)} = \frac{10+x}{10-x}$, $x \in (-10, 10)$ and $f(x) = kf\left(\frac{200x}{100+x^2}\right)$, then k is equal to

(a) 0.5 (b) 0.6

(c) 0.7 (d) 0.8

EAMCET-2003

Ans. (a) : We have given,

$e^{f(x)} = \frac{10+x}{10-x}$ $x \in (-10, 10)$

Taking log on both side

$\log e^{f(x)} = \log\left(\frac{10+x}{10-x}\right)$

$f(x) = \log\left(\frac{10+x}{10-x}\right)$

Given $f(x) = kf\left(\frac{200x}{100+x^2}\right)$

$\log\left(\frac{10+x}{10-x}\right) = k\log\left(\frac{10+\frac{200x}{100+x^2}}{10-\frac{200x}{100+x^2}}\right)$

$\log\left(\frac{10+x}{10-x}\right) = k\log\left(\frac{1000+10x^2+200x}{1000+10x^2-200x}\right)$

$= k\log\left(\frac{100+x^2+20x}{100+x^2-20x}\right)$

$= k\log\left(\frac{10+x}{10-x}\right)^2$

$\log\left(\frac{10+x}{10-x}\right) = 2k\log\left(\frac{10+x}{10-x}\right)$

1 = 2K

$k = \frac{1}{2}$

k = 0.5

369. If $\frac{\log x}{\log 5} = \frac{\log 36}{\log 6} = \frac{\log 64}{\log y}$, what are the values of x and y respectively?

(a) 8, 25 (b) 25, 8

(c) 8, 8 (d) 25, 25

Jamia Millia Islamia-2011

Ans. (b) : We have

$\frac{\log x}{\log 5} = \frac{\log 36}{\log 6} = \frac{\log 64}{\log y}$

Now we take

$\frac{\log x}{\log 5} = \frac{\log 36}{\log 6}$

$\frac{\log x}{\log 5} = \frac{\log 6^2}{\log 6}$

$$\frac{\log x}{\log 5} = \frac{2\log 6}{\log 6}$$

$\log x = 2\log 5$

$\log x = \log 25$

$x = 25$

Again we take

$$\frac{\log 36}{\log 6} = \frac{\log 64}{\log y}$$

$$\frac{2\log 6}{\log 6} = \frac{3\log 4}{\log y}$$

$$\frac{3\log 4}{\log y} = 2$$

$3\log 4 = 2\log y$

$\log 4^3 = \log y^2$

$y^2 = 64$

$y = \pm 8$

Hence, $x = 25$

$y = 8$

370. If $f(x) = \log_e\{\log x\}$, then f(x) at x = e, is

(a) e (b) –e

(c) e^2 (d) e^{-1}

Jamia Millia Islamia-2013

Ans. (d) : $f(x) = \log x\ (\log x) = \dfrac{\log(\log x)}{\log x}$

$$f(x) = \frac{\log x.\frac{1}{\log x}.\frac{1}{x} - \log(\log x).\frac{1}{x}}{(\log x)^2}$$

$$= \frac{1-\log(\log x)}{x(\log x)^2}$$

Now,

$$f(e) = = \frac{1-\log(\log e)}{e(\log e)^2}$$

$$= \frac{1-\log(1)}{e} = \frac{1}{e} = e^{-1}$$

371. Solve the equation, $3^{x^2-x} = 25 - 4^{x^2-x}$

(a) – 1 only (b) 2 only

(c) Both – 1 and 2 (d) No solution

AP EAMCET-21.09.2020, Shift-I

Ans. (c) : Given,

$$3^{x^2-x} = 25 - 4^{x^2-x}$$

$$3^{x^2-x} + 4^{x^2-x} = 25$$

$$3^{x^2-x} + 4^{x^2-x} = 3^2 + 4^2$$

Now comparing the coefficient

$$3^{x^2-x} = 3^2$$

$x^2 - x = 2$

$x^2 - x - 2 = 0$

$(x - 2)(x + 1) = 0$

$x = 2, -1$

And $4^{x^2-x} = 4^2$

$x^2 - x = 2$

$x^2 - x - 2 = 0$

$(x - 2)(x + 1) = 0$

$x = 2, -1$

Hence, there are two solutions.

372. The equivalent function of $\log x^2$ is

(a) 2 log x (b) 2 log |x|

(c) $|\log x^2|$ (d) $(\log x)^2$

CG PET-2021

Ans. (b) : Given,

$f(x) = \log x^2$

We know that

$$\sqrt{x^2} = |x|$$

Then $f(x) = 2\log|x|$

Hence equivalent formation of

$\log x^2 = 2\log|x|$

373. The number of real solutions of the equation $e^{4x} + 4e^{3x} - 58e^{2x} + 4e^x + 1 = 0$ is ______.

JEE Main-28.06.2022, Shift-I

Ans. (2) : $e^{4x} + 4e^{3x} - 58e^{2x} + 4e^x + 1 = 0$

Let $$f(x) = e^{2x}\left(e^{2x} + \frac{1}{e^{2x}} + 4\left(e^x + \frac{1}{e^x}\right) - 58\right)$$

Let $$t = e^x + \frac{1}{e^x}$$

$= h(t) = t^2 + 4t - 58 = 0$

$$t = \frac{-4 \pm \sqrt{16 + 4.58}}{2}$$

$$\frac{-4 \pm 2\sqrt{62}}{2}$$

$t_1 = -2 + 2\sqrt{62}$

$t_2 = -2 - 2\sqrt{62}$ (not possible)

$t \ge 2$

$$e^x + \frac{1}{e^x} = -2 + 2\sqrt{62}$$

$$e^{2x} - \left(-2 + 2\sqrt{62}\right)e^x + 1 = 0$$

$$\left(-2 + 2\sqrt{62}\right) - 4$$

$$4 + 4.62 - 8\sqrt{62} - 4$$

$$248 - 8\sqrt{62} > 0$$

$$\frac{-b}{2a} > 0$$

Both roots are positive 2 real roots.

374. If $f(x) = (a - x^n)^{1/n}$ where a > 0 and n is a positive integer, then $f[f(x)]$ is equal to

(a) x^3 (b) x^2

(c) x (d) None of these

Manipal UGET-2019

Ans. (c) : Given that-

$$f(x)=(a-x^n)^{1/n}$$

$$\therefore\ f[f(x)]=\left[a-\{f(x)^n\}\right]^{1/n}$$

$$=\left[a-(a-x^n)\right]^{1/n}$$

$$=\left[x^n\right]^{1/n}$$

$$= x$$

375. If $3^x + 2^{2x} \geq 5^x$, then the solution set for x is

(a) $(-\infty, 2]$ (b) $[2, \infty)$
(c) $[0, 2]$ (d) $\{2\}$

Manipal UGET-2019

Ans. (a) : Given that-

$3^x + 2^{2x} \geq 5^x$

$$\left(\frac{3}{5}\right)^x+\left(\frac{4}{5}\right)^x\geq 1$$

$(\sin\theta)^x + (\cos\theta)^x \geq 1$ (by triangle inquality)

$x \leq 2$

$\therefore$ solution set is $(-\infty, 2]$.

376. The period of

$$f(x)=\sin\left(\frac{\pi x}{n-1}\right)+\cos\left(\frac{\pi x}{n}\right), n\in Z, n>2 \text{ is}$$

(a) $2\pi n(n-1)$

(b) $4n(n-1)$

(c) $2n(n-1)$

(d) None of the above

Manipal UGET-2019

Ans. (c) : We have-

$$f(x)=\sin\left(\frac{\pi x}{n-1}\right)+\cos\left(\frac{\pi x}{n}\right), n\,\varepsilon\, z, n>2$$

Let,

$$f(x)=g(x)+p(x) \quad \forall\ n>2$$

$$\therefore \text{ Period of } g(x)=\frac{2\pi(n-1)}{\pi}=2(n-1)$$

$$\text{and period of } p(x)=\frac{2\pi n}{\pi}=2n$$

Period of f(x) = LCM of [p(x) and g(x)]

= 2n(n–1)

377. The number of real solutions of the equation $1+|e^x-1|=e^x(e^x-2)$ is

(a) 1 (b) 2
(c) 4 (d) 8

Manipal UGET-2019

Ans. (a) : Given that:-

$$1+|e^x-1|=e^x(e^x-2)$$

Adding 1 both sides:-

$$1+1+|e^x-1|=e^x(e^x-2)+1$$

$$1+1+|e^x-1|=e^{2x}-2e^x+1$$

$$2+|e^x-1|=(e^x-1)^2$$

$$\Rightarrow\ (e^x-1)^2-|e^x-1|-2=0$$

Let $(e^x-1)=y$

Then-

$y^2 - y - 2 = 0$

$(y-2)(y+1) = 0$

$y = 2, -1$

Now,

$y = 2$

$|e^x-1|=2$

$e^x-1=\pm 2$

$e^x=1\pm 2$

$e^x=3,-1$

$e^x=3$

$x=\log_e{}^3$

$\therefore$ There is one real solution of the equation.

378. If n be any integer, then n (n + 1) (2n +1) is:

(a) odd number
(b) integral multiple of 6
(c) perfect square
(d) does not necessarily have any of the foregoing proof

Manipal UGET-2019

Ans. (b) : Given that,

n(n+1)(2n+1)

For n = 1 $\rightarrow$ $1\times2\times3 = 6$

For n = 2 $\rightarrow$ $2\times3\times5 = 30$

6, 30 $\neq$ perfect square

So, n(n+1)(2n+1) always an integral multiple of even numbers.

So, it is a integral multiple of 6 (6, 30, 84 -------)

379. If $f(x)=\begin{cases}3x^2+12x-1, & -1\leq x\leq 2\\ 37-x, & 2\leq x\leq 3\end{cases}$, then

(a) $f(x)$ is decreasing on $[-1, 2]$
(b) $f'(2)$ does not exist
(c) $f(x)$ has the maximum value at x = 2
(d) None of the above

Manipal UGET-2017

Ans. (b) : In the interval $[-1, 2]$, $f'(x) = 6x + 12 > 0$

hence, f(x) is increasing in $[-1, 2]$

Now, f(x) being a polynomial in x_2 continuous in $-1\leq x<2$ and in $2<x\leq 3$ all check at x = 2

$$\lim_{h\to 0} f(2-h)=\lim_{h\to 0}3(2-h)^2+12(2-h)-1$$

$= 12 + 24 - 1 = 35$