YOUTH COMPETITION TIMES

VOLUME V

Vector & 3-D geometry Chapterwise Solved Papers

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In the event of any dispute, the judicial area will be Prayagraj.

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Syllabus for JEE (Main) - 2024

Syllabus for JEE Main Paper-1 (B.E./B.Tech.)

MATHEMATICS

UNIT 1: SETS, RELATIONS, AND FUNCTIONS: Sets and their representation: Union, intersection, and complement of sets and their algebraic properties; Power set; Relation, Type of relations, equivalence relations, functions; one-one, into and onto functions, the composition of functions.

UNIT 2: COMPLEX NUMBERS AND QUADRATIC EQUATIONS: Complex numbers as ordered pairs of reals, Representation of complex numbers in the form a + ib and their representation in a plane, Argand diagram, algebra of complex number, modulus, and argument (or amplitude) of a complex number, Quadratic equations in real and complex number system and their solutions Relations between roots and co-efficient, nature of roots, the formation of quadratic equation with given roots.

UNIT 3: MATRICES AND DETERMINATS: Matrices, algebra of matrices, type of matrices, determinants, and matrices of order two and three, evaluation of determinants, area of triangles using determinants, Adjoint, and evaluation of inverse of a square matrix using determinants and, Test of consistency and solution of simultaneous linear equations in two or three variables using matrices.

UNIT 4: PERMUTATIONS AND COMBINATIONS: The fundamental principle of counting, permutation as an arrangement and combination as section, Meaning of P (n, r) and C(n, r), simple applications.

UNIT 5: BINOMIAL THEOREM AND ITS SIMPLE APPLICATIONS: Binomial theorem for a positive integral index, general term and middle term, and simple applications.

UNIT 6: SEQUENCE AND SERIES: Arithmetic and Geometric progressions, insertion of arithmetic, geometric means between two given numbers, Relation between A.M and G.M.

UNIT 7: LIMIT, CONTINUITY, AND DIFFERENTIABILITY: Real-valued functions, algebra of functions, polynomials, rational, trigonometric, logarithmic, and exponential functions, inverse function. Graphs of simple functions. Limits, continuity, and differentiability. Differentiation of the sum, difference, product, and quotient of two functions. Differentiation of trigonometric, inverse trigonometric, logarithmic, exponential, composite, and implicit functions; derivatives of order up to two, Applications of derivatives: Rate of change of quantities, monotonic-Increasing and decreasing functions, Maxima and minima of functions of one variable.

UNIT 8: INTEGRAL CALCULAS: Integral as an anti-derivative, Fundamental integral involving algebraic, trigonometric, exponential, and logarithmic functions. Integrations by substitution, by parts, and by partial functions. Integrations by substitution, by parts, and by partial functions. Integration using trigonometric identities.

Evaluation of simple integrals of the type

$$\int \frac{dx}{x^2+a^2},\ \int \frac{dx}{\sqrt{x^2 \pm a^2}},\ \int \frac{dx}{a^2-x^2},\ \int \frac{dx}{\sqrt{a^2-x^2}},\ \int \frac{dx}{ax^2+bx+c},\ \int \frac{dx}{\sqrt{ax^2+bx+c}},\ \int \frac{(px+q)dx}{ax^2+bx+c},\ \int \frac{(px+q)dx}{\sqrt{ax^2+bx+c}},$$

$$\int \sqrt{a^2 \pm x^2}\, dx,\ \int \sqrt{x^2-a^2}\,dx$$

The fundamental theorem of calculus, properties of definite integrals. Evaluation of definite integrals, determining areas of the regions bounded by simple curves in standard form.

UNIT 9 : DIFFERENTIAL EQUATION : Ordinary differential equations, their order, and degree, the solution of differential equation by the method of separation of variables, solution of a homegeneous and linear differential equation of the type

$$\frac{dy}{dx} + p(x)y = q(x)$$

UNIT 10 : CO-ORDINATE GEOMETRY : Cartesian system of rectangular coordinates in a plane, distance formula, sections formula, locus, and its equation, the slope of a line, parallel and perpendicular lines, intercepts of a line on the co-ordinate axis.

Straight line : Various forms of equations of a line, intersection of lines, angles between two lines, conditions for concurrence of three lines, the distance of a point form a line, co-ordinate of the centroid orthocentre, and circumcentre of a triangle.

Circle, conic sections : A standard form of equations of a circle, the general form of the equation of a circle, its radius and central, equation of a circle when the endpoints of a diameter are given, points of intersection of a line and a circle with the centre at the origin and sections of conics, equations of conic sections (parabola, ellipse, and hyperbola) in standard forms.

UNIT 11 : THREE DIMENSIONAL GEOMETRY : Coordinates of a point in space, the distance between two points, section formula, directions ratios, and direction consines, and the angle between two intersecting lines. Skew lines, the shortest distance between them, and its equation. Equations of a line

UNIT 12: VECTOR ALGEBRA: Vectors and scalars, the addition of vectors, components of a vector in two dimensions and three-dimensional space, scalar and vector products.

UNIT 13: STATISTICS AND PROBABILITY: Measures of discretion; calculation of mean, median, mode of grouped and ungrouped data calculation of standard deviation, variance, and mean deviation for grouped and ungrouped data.

Probability: Probability of an event, addition and multiplication theorems of probability, Baye's theorem, probability distribution of a random variate.

UNIT 14: TRIGONOMETRY : Trigonometrical identities and trigonometrical funtions, inverse trigonometrical functions, and their properties.

All India Engineering Entrance Examination & JEE-Main Previous Years Papers Analysis Chart

Sl No	Exam	Proposed Year		Total Question
Joint Entrance Examination (JEE) Main				
1.	NTA JEE Main (April Session)	April 2024	24 Paper	720
2.	NTA JEE Main (January Session)	January 2024	20 Paper	600
3.	NTA JEE Main	15.04.2023	Shift-I	30
4.	NTA JEE Main	13.04.2023	Shift-I	30
5.	NTA JEE Main	13.04.2023	Shift-II	30
6.	NTA JEE Main	12.04.2023	Shift-I	30
7.	NTA JEE Main	11.04.2023	Shift-I	30
8.	NTA JEE Main	11.04.2023	Shift-II	30
9.	NTA JEE Main	10.04.2023	Shift-I	30
10.	NTA JEE Main	10.04.2023	Shift-II	30
11.	NTA JEE Main	08.04.2023	Shift-I	30
12.	NTA JEE Main	08.04.2023	Shift-II	30
13.	NTA JEE Main	06.04.2023	Shift-I	30
14.	NTA JEE Main	06.04.2023	Shift-II	30
15.	NTA JEE Main	01.02.2023	Shift-I	30
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17.	NTA JEE Main	24.01.2023	Shift-I	30
18.	NTA JEE Main	24.01.2023	Shift-II	30
19.	NTA JEE Main	25.01.2023	Shift-I	30
20.	NTA JEE Main	25.01.2023	Shift-II	30
21.	NTA JEE Main	29.01.2023	Shift-I	30
22.	NTA JEE Main	29.01.2023	Shift-II	30
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24.	NTA JEE Main	30.01.2023	Shift-II	30
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29.	NTA JEE Main	28.07.2022	Shift-I	30
30.	NTA JEE Main	28.07.2022	Shift-II	30
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32.	NTA JEE Main	27.07.2022	Shift-II	30
33.	NTA JEE Main	26.07.2022	Shift-I	30
34.	NTA JEE Main	26.07.2022	Shift-II	30
35.	NTA JEE Main	25.07.2022	Shift-I	30
36.	NTA JEE Main	25.07.2022	Shift-II	30
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38.	NTA JEE Main	29.06.2022	Shift-II	30
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40.	NTA JEE Main	28.06.2022	Shift-II	30
41.	NTA JEE Main	27.06.2022	Shift-I	30
42.	NTA JEE Main	27.06.2022	Shift-II	30
43.	NTA JEE Main	26.06.2022	Shift-I	30
44.	NTA JEE Main	26.06.2022	Shift-II	30
45.	NTA JEE Main	25.06.2022	Shift-I	30

46.	NTA JEE Main	25.06.2022	Shift-II	30
47.	NTA JEE Main	24.06.2022	Shift-I	30
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56.	NTA JEE Main	26.08.2021	Shift-II	30
57.	NTA JEE Main	27.07.2021	Shift-I	30
58.	NTA JEE Main	27.07.2021	Shift-II	30
59.	NTA JEE Main	25.07.2021	Shift-I	30
60.	NTA JEE Main	25.07.2021	Shift-II	30
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74.	NTA JEE Main	25.02.2021	Shift-II	30
75.	NTA JEE Main	24.02.2021	Shift-I	30
76.	NTA JEE Main	24.02.2021	Shift-II	30
77.	NTA JEE Main	06.09.2020	Shift-I	30
78.	NTA JEE Main	06.09.2020	Shift-II	30
79.	NTA JEE Main	05.09.2020	Shift-I	30
80.	NTA JEE Main	05.09.2020	Shift-II	30
81.	NTA JEE Main	04.09.2020	Shift-I	25
82.	NTA JEE Main	04.09.2020	Shift-II	25
83.	NTA JEE Main	03.09.2020	Shift-I	30
84.	NTA JEE Main	03.09.2020	Shift-II	30
85.	NTA JEE Main	02.09.2020	Shift-I	25
86.	NTA JEE Main	02.09.2020	Shift-II	25
87.	NTA JEE Main	09.01.2020	Shift-I	30
88.	NTA JEE Main	09.01.2020	Shift-II	30
89.	NTA JEE Main	08.01.2020	Shift-I	30
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91.	NTA JEE Main	07.01.2020	Shift-I	30
92.	NTA JEE Main	07.01.2020	Shift-II	30
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98.	NTA JEE Main	09.04.2019	Shift-II	30
99.	NTA JEE Main	08.04.2019	Shift-I	30
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107.	NTA JEE Main	09.01.2019	Shift-I	30
108.	NTA JEE Main	09.01.2019	Shift-II	30
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111.	JEE Main	15.04.2018	Shift-II	30
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113.	JEE Main	09.04.2017		30
114.	JEE Main	08.04.2017		30
115.	JEE Main	02.04.2017		30
116.	JEE Main	2016		30
117.	JEE Main	2015		30
118.	JEE Main	2014		30
119.	JEE Main	2013		30
120.	AIEEE	2012		30
121.	AIEEE	2011		30
122.	AIEEE	2010		30
123.	AIEEE	2009		30
124.	AIEEE	2008		30
	AIEEE	2007		30
125.	AIEEE	2006		30
126.	AIEEE	2005		30
127.	AIEEE	2004		30
128.	AIEEE	2003		30
129.	AIEEE	2002		30
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130.	ASSAM-CEE	2023		40
131.	ASSAM-CEE	2022		40
132.	ASSAM-CEE	2021		40
133.	ASSAM-CEE	2020		40
134.	ASSAM-CEE	2019		40
135.	ASSAM-CEE	2018		40
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136.	A.P. EAPCET	15.05.2023	Shift-I	80
137.	A.P. EAPCET	15.05.2023	Shift-II	80
138.	A.P. EAPCET	16.05.2023	Shift-I	80
139.	A.P. EAPCET	16.05.2023	Shift-II	80
140.	A.P. EAPCET	17.05.2023	Shift-I	80
141.	A.P. EAPCET	17.05.2023	Shift-II	80
142.	A.P. EAPCET	18.05.2023	Shift-I	80
143.	A.P. EAPCET	18.05.2023	Shift-II	80
144.	A.P. EAPCET	19.05.2023	Shift-I	80
145.	A.P. EAMCET	04.07.2022	Shift-I	80
146.	A.P. EAMCET	04.07.2022	Shift-II	80

147.	A.P. EAMCET	05.07.2022	Shift-I	80
148.	A.P. EAMCET	05.07.2022	Shift-II	80
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150.	A.P. EAMCET	06.07.2022	Shift-II	80
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152.	A.P. EAMCET	07.07.2022	Shift-II	80
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155.	A.P. EAMCET	07.09.2021	Shift-I	80
156.	A.P. EAMCET	23.08.2021	Shift-I	80
157.	A.P. EAMCET	23.08.2021	Shift-II	80
158.	A.P. EAMCET	19.08.2021	Shift-II	80
159.	A.P. EAMCET	20.08.2021	Shift-I	80
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197.	A.P. EAMCET	2011		80
198.	A.P. EAMCET	2010		80

199.	A.P. EAMCET	2009		80
200.	A.P. EAMCET	2008		80
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206.	A.P. EAMCET	2002		80
207.	A.P. EAMCET	2001		80
208.	A.P. EAMCET	2000		80
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228.	AMU	2012		50
229.	AMU	2011		50
230.	AMU	2010		70
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232.	AMU	2008		70
233.	AMU	2007		70
234.	AMU	2006		70
235.	AMU	2005		70
236.	AMU	2004		70
237.	AMU	2003		70
238.	AMU	2002		100
239.	AMU	2001		100
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240.	BCECE	2018		50
241.	BCECE	2017		50
242.	BCECE	2016		50
243.	BCECE	2015		50
244.	BCECE	2014		50
245.	BCECE	2013		50
246.	BCECE	2012		50
247.	BCECE	2011		50
248.	BCECE	2010		50

249.	BCECE	2009		50
250.	BCECE	2008		50
251.	BCECE	2007		50
252.	BCECE	2006		50
253.	BCECE	2005		50
254.	BCECE	2004		50
255.	BCECE	2003		50
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256.	BITSAT	2023		40
257.	BITSAT	2022		40
258.	BITSAT	2021		40
259.	BITSAT	2019		40
260.	BITSAT	2018		40
261.	BITSAT	2017		40
262.	BITSAT	2016		40
263.	BITSAT	2015		40
264.	BITSAT	2014		40
265.	BITSAT	2013		40
266.	BITSAT	2012		40
267.	BITSAT	2011		40
268.	BITSAT	2010		40
269.	BITSAT	2009		40
270.	BITSAT	2008		40
271.	BITSAT	2007		40
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275.	Chhattisgarh-PET	2022		100
276.	Chhattisgarh-PET	2021		100
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278.	Chhattisgarh-PET	2019		100
279.	Chhattisgarh-PET	2018		100
280.	Chhattisgarh-PET	2017		100
281.	Chhattisgarh-PET	2016		100
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286.	Chhattisgarh-PET	2011		100
287.	Chhattisgarh-PET	2010		100
288.	Chhattisgarh-PET	2009		100
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297.	COMEDK-JEE	2020		60

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301.	COMEDK-JEE	2016		60
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328.	J & K-CET	2015		75
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330.	J & K-CET	2013		75
331.	J & K-CET	2012		75
332.	J & K-CET	2011		75
333.	J & K-CET	2010		75
334.	J & K-CET	2009		75
335.	J & K-CET	2008		75
336.	J & K-CET	2007		75
337.	J & K-CET	2006		75
338.	J & K-CET	2005		75
339.	J & K-CET	2004		75
340.	J & K-CET	2003		75
Jharkhand (JCECE)				
341.	JCECE	2019		50
342.	JCECE	2018		50
343.	JCECE	2017		50
344.	JCECE	2016		50
345.	JCECE	2015		50

346.	JCECE	2014		50
347.	JCECE	2013		50
348.	JCECE	2012		50
349.	JCECE	2011		50
350.	JCECE	2010		50
351.	JCECE	2009		50
352.	JCECE	2008		50
353.	JCECE	2007		50
354.	JCECE	2006		50
355.	JCECE	2005		50
356.	JCECE	2004		50
357.	JCECE	2003		50
358.	JCECE	2002		50
359.	JCECE	2001		50
Jamia Millia Islamia				
360.	Jamia Millia Islamia	2015		60
361.	Jamia Millia Islamia	2014		60
362.	Jamia Millia Islamia	2013		60
363.	Jamia Millia Islamia	2012		60
364.	Jamia Millia Islamia	2011		60
365.	Jamia Millia Islamia	2010		60
366.	Jamia Millia Islamia	2009		60
367.	Jamia Millia Islamia	2008		60
368.	Jamia Millia Islamia	2007		60
369.	Jamia Millia Islamia	2006		60
370.	Jamia Millia Islamia	2005		60
371.	Jamia Millia Islamia	2004		60
Kerala-KEAM				
372.	Kerala KEAM	2023		60
373.	Kerala KEAM	2022		60
374.	Kerala KEAM	2021		60
375.	Kerala KEAM	2020		60
376.	Kerala KEAM	2019		60
377.	Kerala KEAM	2018		60
378.	Kerala KEAM	2017		60
379.	Kerala KEAM	2016		60
380.	Kerala KEAM	2015		60
381.	Kerala KEAM	2014		60
382.	Kerala KEAM	2013		60
383.	Kerala KEAM	2012		60
384.	Kerala KEAM	2011		60
385.	Kerala KEAM	2010		60
386.	Kerala KEAM	2009		60
387.	Kerala KEAM	2008		60
388.	Kerala KEAM	2007		60
389.	Kerala KEAM	2006		60
390.	Kerala KEAM	2005		60
391.	Kerala KEAM	2004		60
Karnataka-CET (KCET)				
392.	Karnataka-CET	2023		60
393.	Karnataka-CET	2022		60
394.	Karnataka-CET	2021		60

395.	Karnataka-CET	2020		60
396.	Karnataka-CET	2019		60
397.	Karnataka-CET	2018		60
398.	Karnataka-CET	2017		60
399.	Karnataka-CET	2016		60
400.	Karnataka-CET	2015		60
401.	Karnataka-CET	2014		60
402.	Karnataka-CET	2013		60
403.	Karnataka-CET	2012		60
404.	Karnataka-CET	2011		60
405.	Karnataka-CET	2010		60
406.	Karnataka-CET	2009		60
407.	Karnataka-CET	2008		60
408.	Karnataka-CET	2007		60
409.	Karnataka-CET	2006		60
410.	Karnataka-CET	2005		60
411.	Karnataka-CET	2004		60
412.	Karnataka-CET	2003		60
413.	Karnataka-CET	2002		60
414.	Karnataka-CET	2001		60
415.	Karnataka-CET	2000		60
Kishore Vaigyanik Protsahan Yojana (KVPY)				
416.	KVPY-SB-SX	2023		15
417.	KVPY-SB-SX	2022		15
418.	KVPY-SB-SX	2021		15
419.	KVPY-SA	2021		15
420.	KVPY-SA	2020		15
421.	KVPY-SB-SX	2018		15
422.	KVPY-SA	2017		15
423.	KVPY-SB-SX	2016		15
424.	KVPY-SB-SX	2015		15
425.	KVPY-SA	2014		15
426.	KVPY-SB-SX	2013		15
427.	KVPY-SA	2012		15
428.	KVPY-SA	2009		15
429.	KVPY-SB-SX	2009		15
Madhya Pradesh Pre Engineering Test (MPPET)				
430.	MPPET	2013		50
431.	MPPET	2012		50
432.	MPPET	2009		50
433.	MPPET	2008		50
Manipal-UGET				
434.	Manipal	2023		50
435.	Manipal	2022		50
436.	Manipal	2021		50
437.	Manipal	2020		50
438.	Manipal	2019		50
439.	Manipal	2018		50
440.	Manipal	2017		50

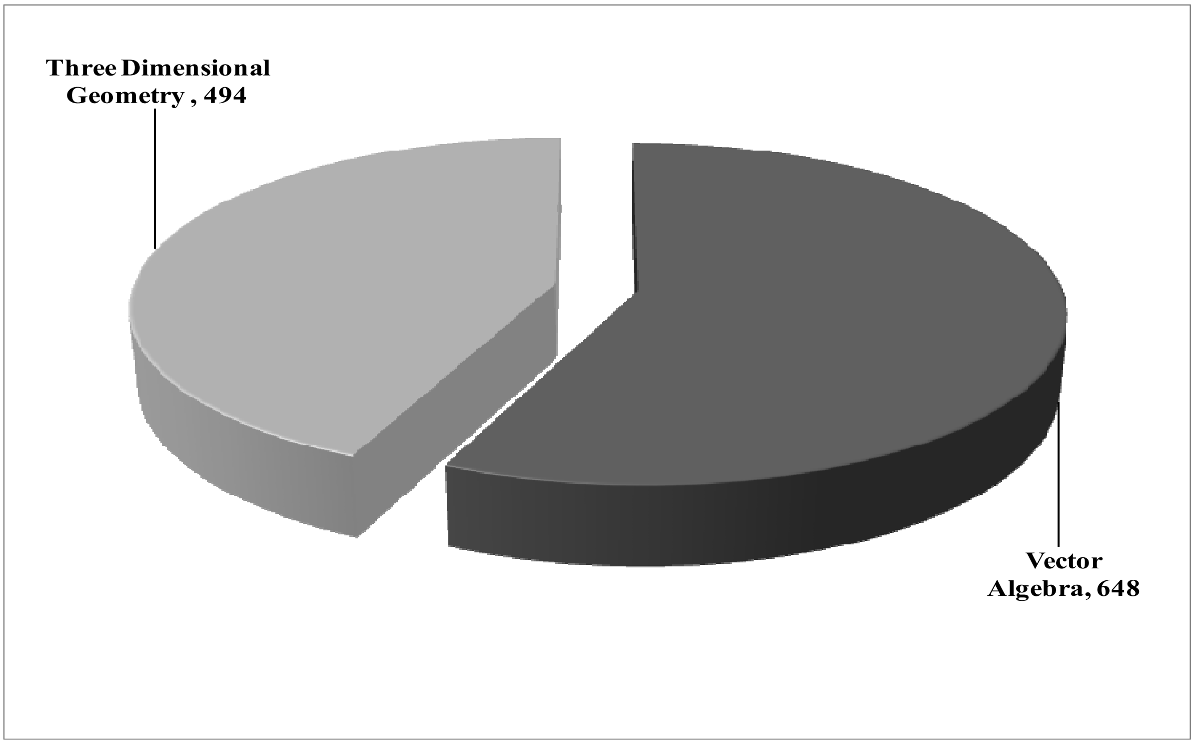
441.	Manipal	2016		50
442.	Manipal	2015		50
443.	Manipal	2014		50
444.	Manipal	2013		50
445.	Manipal	2012		50
446.	Manipal	2011		50
447.	Manipal	2010		50
448.	Manipal	2009		50
449.	Manipal	2008		50
(Maharashtra) MHT-CET				
450.	MHT-CET	2022	All Shifts	500
451.	MHT-CET	2021	All Shifts	500
452.	MHT-CET	13.10.2020	Shift-I	100
453.	MHT-CET	13.10.2020	Shift-II	100
454.	MHT-CET	14.10.2020	Shift-I	100
455.	MHT-CET	14.10.2020	Shift-II	100
456.	MHT-CET	15.10.2020	Shift-I	100
457.	MHT-CET	15.10.2020	Shift-II	100
458.	MHT-CET	16.10.2020	Shift-I	100
459.	MHT-CET	16.10.2020	Shift-II	100
460.	MHT-CET	19.10.2020	Shift-I	100
461.	MHT-CET	19.10.2020	Shift-II	100
462.	MHT-CET	20.10.2020	Shift-I	100
463.	MHT-CET	20.10.2020	Shift-II	100
464.	MHT-CET	02.05.2019	Shift-I	100
465.	MHT-CET	02.05.2019	Shift-II	100
466.	MHT-CET	03.05.2019		100
467.	MHT-CET	2018		100
468.	MHT-CET	2017		100
469.	MHT-CET	2016		100
470.	MHT-CET	2015		100
471.	MHT-CET	2014		100
472.	MHT-CET	2013		100
473.	MHT-CET	2012		100
474.	MHT-CET	2011		100
475.	MHT-CET	2010		100
476.	MHT-CET	2009		100
477.	MHT-CET	2008		100
478.	MHT-CET	2007		100
479.	MHT-CET	2006		100
480.	MHT-CET	2005		100
481.	MHT-CET	2004		100
Rajasthan PET				
482.	Rajasthan PET	2012		40
483.	Rajasthan PET	2011		40

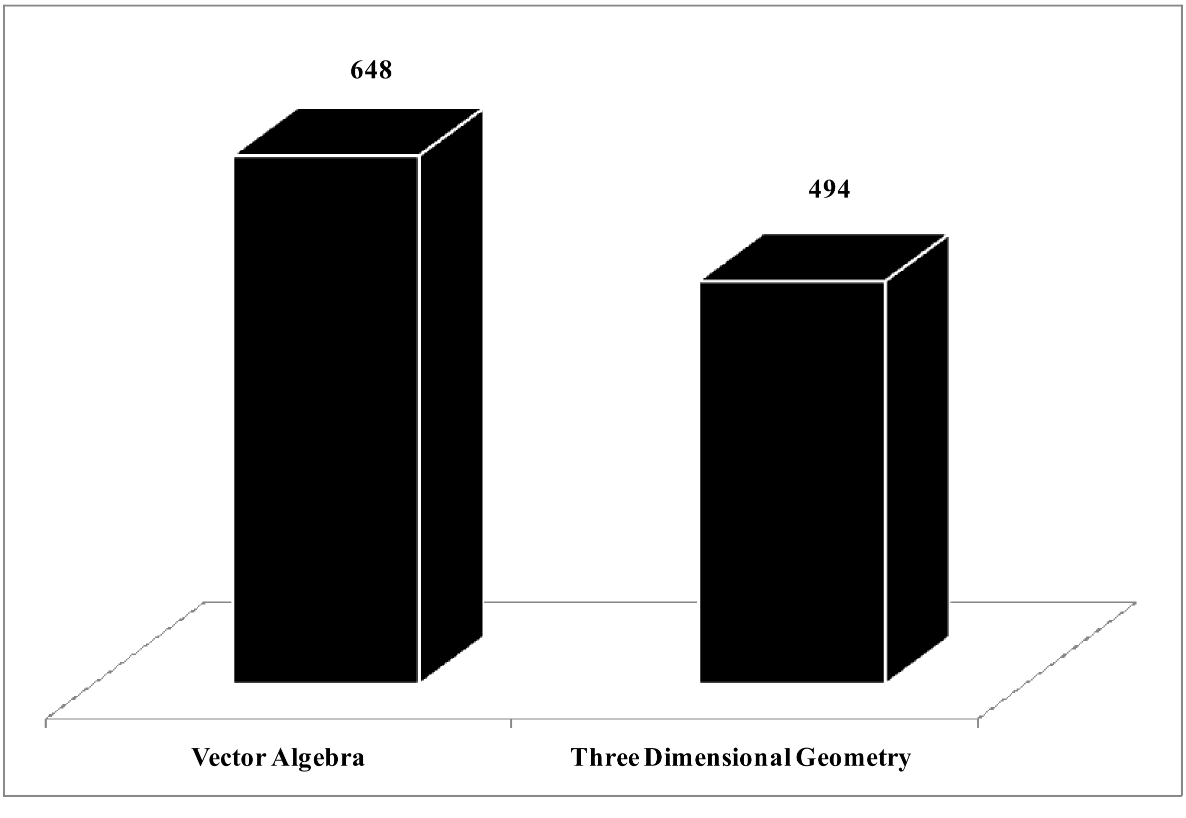
484.	Rajasthan PET	2010		40
485.	Rajasthan PET	2009		40
486.	Rajasthan PET	2008		40
487.	Rajasthan PET	2007		40
488.	Rajasthan PET	2006		40
489.	Rajasthan PET	2005		40
490.	Rajasthan PET	2004		40
491.	Rajasthan PET	2003		40
492.	Rajasthan PET	2002		40
493.	Rajasthan PET	2001		40
SCRA				
494.	SCRA	2015		60
495.	SCRA	2014		60
496.	SCRA	2013		60
497.	SCRA	2012		60
498.	SCRA	2010		60
499.	SCRA	2009		60
SRM-JEEE				
500.	SRM-JEEE	2022		40
501.	SRM-JEEE	2021		40
502.	SRM-JEEE	2020		40
503.	SRM-JEEE	2019		40
504.	SRM-JEEE	2018		40
505.	SRM-JEEE	2016		40
506.	SRM-JEEE	2015		40
507.	SRM-JEEE	2014		40
508.	SRM-JEEE	2013		40
509.	SRM-JEEE	2012		40
510.	SRM-JEEE	2011		40
511.	SRM-JEEE	2010		40
512.	SRM-JEEE	2009		40
513.	SRM-JEEE	2008		40
514.	SRM-JEEE	2007		40
Telangana EAMCET				
515.	TS-EAMCET	12.05.2023	Shift-I	80
516.	TS-EAMCET	12.05.2023	Shift-II	80
517.	TS-EAMCET	13.05.2023	Shift-I	80
518.	TS-EAMCET	13.05.2023	Shift-II	80
519.	TS-EAMCET	14.05.2023	Shift-I	80
520.	TS-EAMCET	14.05.2023	Shift-II	80
521.	TS-EAMCET	18.07.2022	Shift-I	80
522.	TS-EAMCET	18.07.2022	Shift-II	80
523.	TS-EAMCET	19.07.2022	Shift-I	80
524.	TS-EAMCET	19.07.2022	Shift-II	80
525.	TS-EAMCET	20.07.2022	Shift-I	80

526.	TS-EAMCET	20.07.2022	Shift-II	80
527.	TS-EAMCET	06.08.2021	Shift-I	80
528.	TS-EAMCET	06.08.2021	Shift-II	80
529.	TS-EAMCET	05.08.2021	Shift-I	80
530.	TS-EAMCET	05.08.2021	Shift-II	80
531.	TS-EAMCET	04.08.2021	Shift-I	80
532.	TS-EAMCET	04.08.2021	Shift-II	80
533.	TS-EAMCET	09.09.2020	Shift-I	80
534.	TS-EAMCET	09.09.2020	Shift-II	80
535.	TS-EAMCET	10.09.2020	Shift-I	80
536.	TS-EAMCET	10.09.2020	Shift-II	80
537.	TS-EAMCET	11.09.2020	Shift-I	80
538.	TS-EAMCET	11.09.2020	Shift-II	80
539.	TS-EAMCET	14.09.2020	Shift-I	80
540.	TS-EAMCET	14.09.2020	Shift-II	80
541.	TS-EAMCET	03.05.2019	Shift-I	80
542.	TS-EAMCET	03.05.2019	Shift-II	80
543.	TS-EAMCET	04.05.2019	Shift-I	80
544.	TS-EAMCET	04.05.2019	Shift-II	80
545.	TS-EAMCET	06.05.2019	Shift-I	80
546.	TS-EAMCET	05.05.2018	Shift-I	80
547.	TS-EAMCET	05.05.2018	Shift-II	80
548.	TS-EAMCET	02.05.2018	Shift-I	80
549.	TS-EAMCET	04.05.2018	Shift-II	80
550.	TS-EAMCET	07.05.2018	Shift-I	80
551.	TS-EAMCET	24.04.2017	Shift-I	80
552.	TS-EAMCET	2016		80
553.	TS-EAMCET	2015		80
554.	TS-EAMCET	2014		80
Tripura JEE				
555.	Tripura JEE	2023		50
556.	Tripura JEE	2022		50
557.	Tripura JEE	2021		50
558.	Tripura JEE	2019		50
(Uttar Pradesh) UPTU/UPSEE				
559.	UPTU/UPSEE	2020		50
560.	UPTU/UPSEE	2019		50
561.	UPTU/UPSEE	2018		50
562.	UPTU/UPSEE	2017		50
563.	UPTU/UPSEE	2016		50
564.	UPTU/UPSEE	2015		50
565.	UPTU/UPSEE	2014		50
566.	UPTU/UPSEE	2013		50
567.	UPTU/UPSEE	2012		50
568.	UPTU/UPSEE	2011		50

569.	UPTU/UPSEE	2010		50
570.	UPTU/UPSEE	2009		50
571.	UPTU/UPSEE	2008		50
572.	UPTU/UPSEE	2007		50
573.	UPTU/UPSEE	2006		50
574.	UPTU/UPSEE	2005		50
575.	UPTU/UPSEE	2004		50
VITEEE				
576.	VITEEE	2023		40
577.	VITEEE	2022		40
578.	VITEEE	2021		40
579.	VITEEE	2020		40
580.	VITEEE	2019		40
581.	VITEEE	2018		40
582.	VITEEE	2017		40
583.	VITEEE	2016		40
584.	VITEEE	2015		40
585.	VITEEE	2014		40
586.	VITEEE	2013		40
587.	VITEEE	2012		40
588.	VITEEE	2011		40
589.	VITEEE	2010		40
590.	VITEEE	2009		40
591.	VITEEE	2008		40
592.	VITEEE	2007		40
593.	VITEEE	2006		40
WEST BENGAL				
594.	West Bengal	2023		30
595.	West Bengal	2022		30
596.	West Bengal	2021		30
597.	West Bengal	2020		30
598.	West Bengal	2019		30
599.	West Bengal	2018		30
600.	West Bengal	2017		30
601.	West Bengal	2016		30
602.	West Bengal	2015		30
603.	West Bengal	2014		30
604.	West Bengal	2013		30
605.	West Bengal	2012		30
606.	West Bengal	2011		30
607.	West Bengal	2010		30
608.	West Bengal	2009		30
609.	West Bengal	2008		30
			Total	**36020**

Trend Analysis of previous year paper of IIT JEE Mathematics through Bar graph and Pie chart.





01. Vector Algebra

A. Distance, Position and section formula of vector

1. Let O be the origin and the position vector of the point P be $-\hat{i}-2\hat{j}+3\hat{k}$. If the position vectors of the A, B and C are $-2\hat{i}+\hat{j}-3\hat{k}, 2\hat{i}+4\hat{j}-2\hat{k}$ and $-4\hat{i}+2\hat{j}-\hat{k}$ respectively, then the projection of the vector $\overrightarrow{OP}$ on a vector perpendicular to the vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$ is:

(a) $\frac{10}{3}$ (b) $\frac{8}{3}$
(c) $\frac{7}{3}$ (d) 3

JEE MAIN-10.04.2023, Shift-I

Ans. (d) : Position vector of the point P(–1, –2, 3), A(–2, 1, –3) B(2, 4, –2), and C(–4, 2, –1)

projection of the vector $\overrightarrow{OP}$ on a vector perpendicular to the vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$ $=\overrightarrow{OP}.\frac{\overrightarrow{AB}\times\overrightarrow{AC}}{\left|\left(\overrightarrow{AB}\times\overrightarrow{AC}\right)\right|}$

$$\overrightarrow{AB}\times\overrightarrow{AC}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 4 & 3 & 1\\ -2 & 1 & 2\end{vmatrix}$$

$=\hat{i}(5)-\hat{j}(8+2)+\hat{k}(4+6)$

$=5\hat{i}-10\hat{j}+10\hat{k}$

Now

$$\overrightarrow{OP}.\frac{\overrightarrow{AB}\times\overrightarrow{AC}}{\left|\left(\overrightarrow{AB}\times\overrightarrow{AC}\right)\right|}=\left(-\hat{i}-2\hat{j}+3\hat{k}\right).\frac{\left(5\hat{i}-10\hat{j}+10\hat{k}\right)}{\sqrt{(5)^2+(-10)^2+(10)^2}}$$

$$=\frac{-5+20+30}{\sqrt{25+100+100}}$$

$$=\frac{45}{\sqrt{225}}=\frac{45}{15}=3$$

2. Let the vectors $\vec{a},\vec{b},\vec{c}$ represent three coterminous edges of a parallelepiped of volume V. Then the volume of the parallelepiped, whose coterminous edges are represented by $\vec{a},\vec{b},+\vec{c}$ and $\vec{a}+2\vec{b}+3\vec{c}$ is equal to:

(a) 2 V (b) 6 V (c) 3 V (d) V

JEE MAIN-06.04.2023, Shift-II

Ans. (d) :

$$\left[\vec{a},\vec{b}+\vec{c},\vec{a}+2\vec{b}+3\vec{c}\right]$$

Let, $V=\left[\vec{a}\ \vec{b}\ \vec{c}\right]$

$$V_1=\begin{vmatrix}1 & 0 & 0\\ 0 & 1 & 1\\ 1 & 2 & 3\end{vmatrix}\left[\vec{a}\ \vec{b}\ \vec{c}\right]$$

$V_1=(3-2)\left[\vec{a}\ \vec{b}\ \vec{c}\right]=V$

3. Let $\vec{a}=5\hat{i}-\hat{j}-3\hat{k}$ and $\vec{b}=\hat{i}+3\hat{j}+5\hat{k}$ be two vectors. Then which one of the following statements is TRUE?

(a) Projection of $\vec{a}$ on $\vec{b}$ is $\frac{-17}{\sqrt{35}}$ and the direction of the projection vector is opposite to the direction of $\vec{b}$.

(b) Projection of $\vec{a}$ on $\vec{b}$ is $\frac{17}{\sqrt{35}}$ and the direction the projection vector is opposite to the direction of $\vec{b}$.

(c) Projection of $\vec{a}$ on $\vec{b}$ is $\frac{17}{\sqrt{35}}$ and the direction of the projection vector is same as of $\vec{b}$.

(d) Projection of $\vec{a}$ on $\vec{b}$ is $\frac{-17}{\sqrt{35}}$ and the direction of the projection vector is same as of $\vec{b}$.

JEE MAIN-01.02.2023, Shift-II

Ans. (*) : Given,

$\vec{a}=5\hat{i}-\hat{j}-3\hat{k}$

and $\vec{b}=\hat{i}+3\hat{j}+5\hat{k}$

Projection of $\vec{a}$ on $\vec{b}=\vec{a}.\hat{b}$

$$=\frac{\vec{a}.\vec{b}}{|\vec{b}|}$$

$$=\frac{-13}{\sqrt{1+9+25}}$$

$$=\frac{-13}{\sqrt{35}}$$

and projection vector is in opposite direction of $\vec{b}$.

4. If $\vec{a}=2\hat{i}+3\hat{j}+\hat{k}$, $\vec{b}=4\hat{i}+5\hat{j}+3\hat{k}$ and $\vec{c}=6\hat{i}+\hat{j}+5\hat{k}$ are the position vectors of the vertices of triangle ABC respectively, then the position vector of the intersection of the medians of the triangle ABC is

(a) $2\hat{i}+3\hat{j}+3\hat{k}$ (b) $4\hat{i}+3\hat{j}+3\hat{k}$
(c) $5\hat{i}+3\hat{j}+3\hat{k}$ (d) $3\hat{i}+3\hat{j}+4\hat{k}$

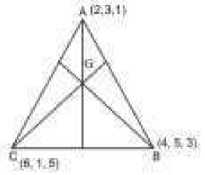
MHT CET-2020

Ans. (b) : Given,
Position vector,

$\vec{a} = 2\hat{i} + 3\hat{j} + \hat{k}$

$\vec{b} = 4\hat{i} + 5\hat{j} + 3\hat{k}$

And, $\vec{c} = 6\hat{i} + \hat{j} + 5\hat{k}$



Intersection median of triangle ABC is G which is centroid of triangle.

Centroid of triangle $(G) = \dfrac{\vec{a} + \vec{b} + \vec{c}}{3}$

$= \dfrac{2\hat{i} + 3\hat{j} + \hat{k} + 4\hat{i} + 5\hat{j} + 3\hat{k} + 6\hat{i} + \hat{j} + 5\hat{k}}{3}$

$= \dfrac{12\hat{i} + 9\hat{j} + 9\hat{k}}{3} = 4\hat{i} + 3\hat{j} + 3\hat{k}$

5. If the position vectors of the vertices, A, B, C of a triangle ABC are $4\hat{i} + 7\hat{j} + 8\hat{k}$, $2\hat{i} + 3\hat{j} + 4\hat{k}$ and $2\hat{i} + 5\hat{j} + 7\hat{k}$ respectively, then the position vector of the point where bisector of angel A meets BC is

(a) $\frac{1}{3}(6\hat{i} + 11\hat{j} + 15\hat{k})$ (b) $\frac{1}{4}(8\hat{i} + 14\hat{j} + 19\hat{k})$

(c) $\frac{1}{2}(4\hat{i} + 8\hat{j} + 11\hat{k})$ (d) $\frac{1}{3}(6\hat{i} + 13\hat{j} + 18\hat{k})$

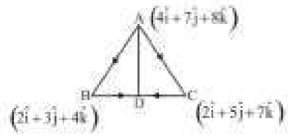
MHT CET-2020

Ans. (d) : We have,
The position vector,

$\vec{a} = 4\hat{i} + 7\hat{j} + 8\hat{k}$

$\vec{b} = 2\hat{i} + 3\hat{j} + 4\hat{k}$

$\vec{c} = 2\hat{i} + 5\hat{j} + 7\hat{k}$



$\overrightarrow{AB} = (\vec{b} - \vec{a}) = (2\hat{i} + 3\hat{j} + 4\hat{k}) - (4\hat{i} + 7\hat{j} + 8\hat{k})$

$= -2\hat{i} - 4\hat{j} - 4\hat{k}$

$|\overrightarrow{AB}| = \sqrt{4 + 16 + 16} = 6$

$\overrightarrow{AC} = (\vec{c} - \vec{a})$

$\overrightarrow{AC} = (2\hat{i} + 5\hat{j} + 7\hat{k}) - (4\hat{i} + 7\hat{j} + 8\hat{k})$

$\overrightarrow{AC} = -2\hat{i} - 2\hat{j} - \hat{k}$

$|\overrightarrow{AC}| = \sqrt{4 + 4 + 1} = 3$

Then AB divides BC in the ratio AB : AC
Position vector, Of D

$= \dfrac{|\overrightarrow{AB}|(2\hat{i} + 5\hat{j} + 7\hat{k}) + |\overrightarrow{AC}|(2\hat{i} + 3\hat{j} + 4\hat{k})}{|\overrightarrow{AB}| + |\overrightarrow{AC}|}$

$= \dfrac{6(2\hat{i} + 5\hat{j} + 7\hat{k}) + 3(2\hat{i} + 3\hat{j} + 4\hat{k})}{6 + 3}$

$= \dfrac{(12\hat{i} + 30\hat{j} + 42\hat{k} + 6\hat{i} + 9\hat{j} + 12\hat{k})}{6 + 3}$

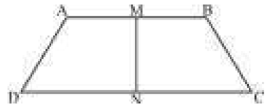
$= \dfrac{(18\hat{i} + 39\hat{j} + 54\hat{k})}{6 + 3} = \dfrac{1}{3}(6\hat{i} + 13\hat{j} + 18\hat{k})$

6. In a quadrilateral ABCD, M and N are the midpoints of the sides AB and CD respectively. If $\overrightarrow{AD} + \overrightarrow{BC} = t\overrightarrow{MN}$, then t =

(a) $\frac{1}{2}$ (b) $\frac{3}{2}$ (c) 2 (d) 4

MHT CET-2020

Ans. (c) : Let $\vec{a}, \vec{b}, \vec{c}\ \vec{d}, \vec{m}, \vec{n}$ be the position vectors of A, B, C, D, M, N, respectively. M and N are the midpoints of AB and CD respectively.



$\vec{m} = \dfrac{\vec{a} + \vec{b}}{2}$,

$\vec{a} + \vec{b} = 2\vec{m}$

$\vec{n} = \dfrac{\vec{c} + \vec{d}}{2}$

$\vec{c} + \vec{d} = 2\vec{n}$

$\overrightarrow{AD} = (\vec{d} - \vec{a}), \overrightarrow{BC} = (\vec{c} - \vec{b})$

Given,

$\overrightarrow{AD} + \overrightarrow{BC} = t(\overrightarrow{MN})$

$(\vec{d} - \vec{a}) + (\vec{c} - \vec{b}) = t(\vec{n} - \vec{m})$

$\vec{d} - \vec{a} + \vec{c} - \vec{b} = t(\vec{n} - \vec{m})$

$(\vec{d} + \vec{c}) - (\vec{a} + \vec{b}) = t(\vec{n} - \vec{m})$

$2\vec{n} - 2\vec{m} = t(\vec{n} - \vec{m})$

$2(\vec{n} - \vec{m}) = t(\vec{n} - \vec{m})$

So, $t = 2$

7. The perimeter of the triangle whose vertices have the position vectors $\hat{i} + \hat{j} + \hat{k}, 5\hat{i} + 3\hat{j} - 3\hat{k}$ and $2\hat{i} + 5\hat{j} + 9\hat{k}$ is

(a) $(\sqrt{15} - \sqrt{157})$units (b) $(\sqrt{15} + \sqrt{157})$units

(c) $(15 - \sqrt{157})$units (d) $(15 + \sqrt{157})$units

MHT CET-2020

Ans. (d) : Given the position vector,

$\vec{a} = \hat{i} + \hat{j} + \hat{k}, \quad \vec{b} = 5\hat{i} + 3\hat{j} - 3\hat{k}, \quad \vec{c} = 2\hat{i} + 5\hat{j} + 9\hat{k}$

Let, A, B, C be the vertices of the triangle

$\overrightarrow{AB} = \vec{b} - \vec{a} = (5\hat{i} + 3\hat{j} - 3\hat{k}) - (\hat{i} + \hat{j} + \hat{k})$

$\therefore \quad \overrightarrow{AB} = 4\hat{i} + 2\hat{j} - 4\hat{k}$

$|\overrightarrow{AB}| = \sqrt{16 + 4 + 16}$

$|\overrightarrow{AB}| = 6$

$\overrightarrow{BC} = \vec{c} - \vec{b}$

$\overrightarrow{BC} = (2\hat{i} + 5\hat{j} + 9\hat{k}) - (5\hat{i} + 3\hat{j} - 3\hat{k})$

$\overrightarrow{BC} = -3\hat{i} + 2\hat{j} + 12\hat{k}$

$\left|\overrightarrow{BC}\right| = \sqrt{9+4+144}$

$\left|\overrightarrow{BC}\right| = \sqrt{157}$

$\overrightarrow{AC} = \vec{c} - \vec{a} = \left(2\hat{i} + 5\hat{j} + 9\hat{k}\right) - \left(\hat{i} + \hat{j} + \hat{k}\right)$

$\overrightarrow{AC} = \hat{i} + 4\hat{j} + 8\hat{k}$

$\left|\overrightarrow{AC}\right| = \sqrt{1+16+64}$

$\left|\overrightarrow{AC}\right| = \sqrt{81} = 9$

So, perimeter of triangle $= \left|\overrightarrow{AB}\right| + \left|\overrightarrow{BC}\right| + \left|\overrightarrow{AC}\right|$

$= 6 + \sqrt{157} + 9$

$= 15 + \sqrt{157}$ units.

8. If the origin is the centroid of the triangle whose vertices are A (2, p, –3), B (q, –2, 5) and C (–5, 1, r) then

(a) $p = 1, q = 3, r = 2$ (b) $p = 1, q = 3, r = -2$

(c) $p = 1, q = -3, r = -2$ (d) $p = -1, q = 3, r = -2$

MHT CET-2020

Ans. (b) : Given,

Vertices A (2, p, – 3) , B (q, – 2, 5) and C (– 5, 1, r)

$\vec{a} = 2\hat{i} + p\hat{j} - 3\hat{k}, \vec{b} = q\hat{i} - 2\hat{j} + 5\hat{k}, \vec{c} = -5\hat{i} + \hat{j} + r\hat{k}$

We know that,

$\vec{G} = 0\hat{i} + 0\hat{j} + 0\hat{k}$

Centroid of triangle,

$$G = \frac{\vec{a} + \vec{b} + \vec{c}}{3}$$

$\vec{a} + \vec{b} + \vec{c} = \vec{0}$

$\left(2\hat{i} + p\hat{j} - 3\hat{k}\right) + \left(q\hat{i} - 2\hat{j} + 5\hat{k}\right) + \left(-5\hat{i} + \hat{j} + r\hat{k}\right) = 0\hat{i} + 0\hat{j} + 0\hat{k}$

$\left(2 + q - 5\right)\hat{i} + \left(p - 2 + 1\right)\hat{j} + \left(-3 + 5 + r\right)\hat{k} = 0\hat{i} + 0\hat{j} + 0\hat{k}$

Then, $2 + q - 5 = 0$

$q = 3$

$-3 + 5 + r = 0$

$r = -2$

$p - 2 + 1 = 0$

$p = 1$

So, p = 1, q = 3 and r = – 2

9. If $A(0,4,0), B(0,0,3)$ and $C(0,4,3)$ are the vertices of ΔABC, then its incentre is,

(a) $(0,3,2)$ (b) $(3,0,2)$

(c) $(0,2,3)$ (d) $(2,0,3)$

MHT CET-2020

Ans. (a) : Let, $\vec{a}, \vec{b}, \vec{c}$ be the position vector.

Given, A (0, 4, 0), B (0, 0, 3), C (0, 4, 3) are the vertices of ΔABC.

$\vec{a} = 4\hat{j}, \vec{b} = 3\hat{k}, \vec{c} = 4\hat{j} + 3\hat{k}$

$\overrightarrow{BC} = \vec{c} - \vec{b} = \left(4\hat{j} + 3\hat{k}\right) - 3\hat{k}$

$\overrightarrow{BC} = 4\hat{j}$

$\left|\overrightarrow{BC}\right| = \sqrt{16} = 4$

$\overrightarrow{AC} = \vec{c} - \vec{a} = \left(4\hat{j} + 3\hat{k}\right) - 4\hat{j}$

$\overrightarrow{AC} = 3\hat{k}$

$\left|\overrightarrow{AC}\right| = \sqrt{9} = 3$

$\overrightarrow{AB} = \vec{b} - \vec{a} = 3\hat{k} - 4\hat{j}$

$\left|\overrightarrow{AB}\right| = \sqrt{16+9} = 5$

We know that,

$$\vec{I} = \frac{\left|\overrightarrow{BC}\right|\vec{a} + \left|\overrightarrow{AC}\right|\vec{b} + \left|\overrightarrow{AB}\right|\vec{c}}{\left|\overrightarrow{BC}\right| + \left|\overrightarrow{AC}\right| + \left|\overrightarrow{AB}\right|}$$

$$\vec{I} = \frac{4\left(4\hat{j}\right) + 3\left(3\hat{k}\right) + 5\left(4\hat{j} + 3\hat{k}\right)}{4+3+5}$$

$$\vec{I} = \frac{16\hat{j} + 9\hat{k} + 20\hat{j} + 15\hat{k}}{12} = \frac{36\hat{j} + 24\hat{k}}{12}$$

$\vec{I} = 3\hat{j} + 2\hat{k}$

So, incentre of triangle is $\vec{I}$ (0, 3, 2).

10. If a, b, c are lengths of the sides BC, CA, AB respectively of ΔABC and H is any point in the plane of ΔABC such that a $a\overrightarrow{AH} + b\overrightarrow{BH} + c\overrightarrow{CH} = \vec{0}$, then H is the

(a) Incetnre of ΔABC

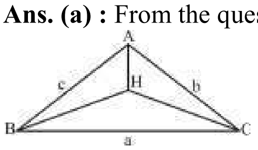
(b) Orthocentre of ΔABC

(c) Circumcentre of ΔABC

(d) Centroid of ΔABC

MHT CET-2020

Ans. (a) : From the question,



Consider H to be origin.

Then position vector of the vertices A, B, C are $\vec{a}, \vec{b}, \vec{c}$ respectively.

$a\overrightarrow{AH} + b\overrightarrow{BH} + c\overrightarrow{CH} = \vec{0}$

$a\vec{a} + b\vec{b} + c\vec{c} = 0$

$\frac{a\vec{a} + b\vec{b} + c\vec{c}}{a+b+c} = 0,$

Which is position vector of in centre.

Hence, H is in centre of ΔABC.

11. If $\overrightarrow{AB} = 3\hat{i} + 5\hat{j} + 4\hat{k}$, $\overrightarrow{AC} = 5\hat{i} - 5\hat{j} + 2\hat{k}$ represent the sides of triangle ABC, then the length of median through A is

(a) 6 units (b) 5 units

(c) $\sqrt{6}$ units (d) $\sqrt{5}$ units

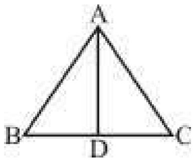
MHT CET-2020

Ans. (b) : Given,

$\overrightarrow{AB} = 3\hat{i} + 5\hat{j} + 4\hat{k}$

$\overrightarrow{AC} = 5\hat{i} - 5\hat{j} + 2\hat{k}$

Let $\overrightarrow{AD}$ is median



Position vector of,

$$\overrightarrow{AD} = \frac{\overrightarrow{AB} + \overrightarrow{AC}}{2}$$

$$\overrightarrow{AD} = \frac{\left(3\hat{i} + 5\hat{j} + 4\hat{k}\right) + \left(5\hat{i} - 5\hat{j} + 2\hat{k}\right)}{2}$$

$$\overrightarrow{AD} = \frac{(3+5)\hat{i} + (5-5)\hat{j} + (4+2)\hat{k}}{2}$$

$$\overrightarrow{AD} = \frac{8\hat{i} + 6\hat{k}}{2}$$

$$\overrightarrow{AD} = 4\hat{i} + 3\hat{k}$$

$$\left|\overrightarrow{AD}\right| = \sqrt{16+9} = \sqrt{25} = 5$$

Hence, length of median is 5 units.

12. If $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ are the position vector of the points A, B, C, D respectively such that $3\vec{a} - \vec{b} + 2\vec{c} - 4\vec{d} = \vec{0}$, then the position vector of the point of intersection of the line segments AC and BD is

(a) $\frac{\vec{a}+\vec{b}}{2}$ (b) $\frac{3\vec{a}+\vec{c}}{4}$

(c) $\frac{\vec{b}+3\vec{d}}{4}$ (d) $\frac{\vec{b}+4\vec{d}}{5}$

MHT CET-2020

Ans. (d) : Given,

$3\vec{a} - \vec{b} + 2\vec{c} - 4\vec{d} = 0$

Rearranging the term in equation,

$3\vec{a} + 2\vec{c} = \vec{b} + 4\vec{d}$

$$\therefore \quad \frac{3\vec{a}+2\vec{c}}{3+2} = \frac{\vec{b}+4d}{1+4} \Rightarrow \text{So,} \quad \frac{3\vec{a}+2\vec{c}}{5} = \frac{\vec{b}+4\vec{d}}{5}$$

13. If $\vec{a}, \vec{b}, \vec{c}$ are the position vectors of the points $A(1,3,0), B(2,5,0), C(4,2,0)$ respectively and $\vec{c} = t_1\vec{a} + t_2\vec{b}$, then value of t_1t_2 =

(a) 160 (b) –16 (c) 16 (d) –160

MHT CET-2020

Ans. (d) : Given,

$$\vec{c} = 4\hat{i} + 2\hat{j} + 0\hat{k}$$

$$\vec{a} = \hat{i} + 3\hat{j} + 0\hat{k}$$

$$\vec{b} = 2\hat{i} + 5\hat{j} + 0\hat{k}$$

From given conditions,

$\vec{c} = t_1\vec{a} + t_2\vec{b}$

$\left(4\hat{i} + 2\hat{j} + 0\hat{k}\right) = t_1\left(\hat{i} + 3\hat{j} + 0\hat{k}\right) + t_2\left(2\hat{i} + 5\hat{j} + 0\hat{k}\right)$

$\therefore \quad = (t_1 + 2t_2)\hat{i} + (3t_1 + 5t_2)\hat{j}$

$\therefore t_1 + 2t_2 = 4$ and $3t_1 + 5t_2 = 2$

Solving, we get $t_2 = 10$ and $t_1 = -16$

So, $t_1t_2 = -160$

14. A ≡ (2, 3, –2) and B ≡ (4, 1, –2) are two vertices of ΔABC. P, Q and R are the midpoints of AB, BC and AC respectively. The coordinates of R are $\left(\frac{5}{2}, \frac{5}{2}, -\frac{7}{2}\right)$. Then the centroid of ΔPQR is

(a) $(3,-2,-3)$ (b) $(3,2,-3)$

(c) $(3,2,3)$ (d) $\left(\frac{3}{2}, -\frac{7}{2}, \frac{3}{2}\right)$

MHT CET-2019

Ans. (b) : Let,

$\vec{a}, \vec{b}, \vec{c}, \vec{p}, \vec{q}, \vec{r}$ be the position vectors of A, B, C, P, Q, R respectively.

$$\therefore \quad \vec{a} = 2\hat{i} + 3\hat{j} - 2\hat{k},$$

$$\vec{b} = 4\hat{i} + \hat{j} - 2\hat{k},$$

$$\vec{r} = \frac{5}{2}\hat{i} + \frac{5}{2}\hat{j} - \frac{7}{2}\hat{k}$$

Consider, $\vec{c} = x\hat{i} + y\hat{j} + z\hat{k}$

Then, from question –

$$\frac{\vec{a}+\vec{c}}{2} = \vec{r}$$

$$\frac{\left(2\hat{i}+3\hat{j}-2\hat{k}\right)+\left(x\hat{i}+y\hat{j}+z\hat{k}\right)}{2} = \left(\frac{5}{2}\hat{i}+\frac{5}{2}\hat{j}-\frac{7}{2}\hat{k}\right)$$

$$(2+x)\hat{i} + (3+y)\hat{j} + (z-2)\hat{k} = 5\hat{i} + 5\hat{j} - 7\hat{k}$$

On comparing both side, we get -

x = 3, y = 2, z = –5

Then, $\vec{c} = 3\hat{i} + 2\hat{j} - 5\hat{k}$

As the centroid of the triangle formed by joining the mid points of the sides of a given triangle coincides with the centroid of the given triangle.

∴ Centroid

$$\text{of}\,\Delta PQR = \left(\frac{2+4+3}{3}, \frac{3+1+2}{3}, \frac{-2-2-5}{3}\right) = (3,2,-3)$$

15. If P (1, 2, 3), R(4, 5, –1) are the vertices and G(2, 3, –1) is the centroid of ΔPQR, then coordinates of midpoint of PQ are

(a) $(1,2,1)$ (b) $(1,2,2)$

(c) $(1,-2,-1)$ (d) $(1,2,-1)$

MHT CET-2019

Ans. (d) : Let,

$\vec{p}, \vec{q}, \vec{r}$ and $\vec{g}$ be the position vectors of P, Q, R and G respectively.

Given,

$$\vec{p} = \hat{i} + 2\hat{j} + 3\hat{k}$$

$$\vec{r} = 4\hat{i} + 5\hat{j} - \hat{k}$$

$\vec{g} = 2\hat{i} + 3\hat{j} - \hat{k}$

We know that,
Centroid,

$$\vec{g} = \frac{\vec{p} + \vec{q} + \vec{r}}{3}$$

$3\left(2\hat{i} + 3\hat{j} - \hat{k}\right) = \left(\hat{i} + 2\hat{j} + 3\hat{k}\right) + \vec{q} + \left(4\hat{i} + 5\hat{j} - \hat{k}\right)$

$6\hat{i} + 9\hat{j} - 3\hat{k} = 5\hat{i} + 7\hat{j} + 2\hat{k} + \vec{q}$

$\vec{q} = \hat{i} + 2\hat{j} - 5\hat{k}$

Co-ordinates of mid-point P and Q is,

$$= \left(\frac{1+1}{2}, \frac{2+2}{2}, \frac{3-5}{2}\right) = (1, 2, -1)$$

Hence, the mid-point of PQ (1, 2, –1).

16. If A, B, C and D are (3, 7, 4), (5, –2, 3), (–4, 5, 6) and $(1,2,3)$ respectively, then the volume of the parallelopiped with AB, AC and AD as the coterminus edges, is (in cubic units)

(a) 92 (b) 94 (c) 91 (d) 93

MHT CET-2019

Ans. (a) : Given,

$A = (3,7,4), B = (5,-2,3), C = (-4,5,6), D = (1,2,3)$

Let $\vec{a}, \vec{b}, \vec{c}, \vec{d}$ be the position vectors of A, B, C, D respectively.

$\vec{a} = 3\hat{i} + 7\hat{j} + 4\hat{k}$

$\vec{b} = 5\hat{i} - 2\hat{j} + 3\hat{k}$

$\vec{c} = -4\hat{i} + 5\hat{j} + 6\hat{k}$

$\vec{d} = \hat{i} + 2\hat{j} + 3\hat{k}$

$\overrightarrow{AB} = \vec{b} - \vec{a}$

$\overrightarrow{AB} = \left(5\hat{i} - 2\hat{j} + 3\hat{k}\right) - \left(3\hat{i} + 7\hat{j} + 4\hat{k}\right)$

$\overrightarrow{AB} = 2\hat{i} - 9\hat{j} - \hat{k}$

$\overrightarrow{AD} = \vec{d} - \vec{a}$

$\overrightarrow{AD} = \left(\hat{i} + 2\hat{j} + 3\hat{k}\right) - \left(3\hat{i} + 7\hat{j} + 4\hat{k}\right)$

$\overrightarrow{AD} = -2\hat{i} - 5\hat{j} - \hat{k}$

$\overrightarrow{AC} = \vec{c} - \vec{a}$

$\overrightarrow{AC} = \left(-4\hat{i} + 5\hat{j} + 6\hat{k}\right) - \left(3\hat{i} + 7\hat{j} + 4\hat{k}\right)$

$\overrightarrow{AC} = -7\hat{i} - 2\hat{j} + 2\hat{k}$

Volume of parallelepiped,

$$\left|\overrightarrow{AB}\ \overrightarrow{AC}\ \overrightarrow{AD}\right| = \begin{vmatrix} 2 & -9 & -1 \\ -7 & -2 & 2 \\ -2 & -5 & -1 \end{vmatrix}$$

$$= \left|2(2+10) + 9(7+4) - 1(35-4)\right| = |24 + 99 - 31| = |92|$$

= 92 cubic units

17. If G (3, –5, r) is centroid of triangle ABC where A (7, –8, 1), $B(p,q,5)$ and $C(q+1, 5p, 0)$ are vertices of a triangle then values of p, q, r are respectively

(a) –4,5,4 (b) 6,5,4
(c) –3,4,3 (d) –2,3,2

MHT CET-2019

Ans. (d) : Let,

$\vec{a}, \vec{b}, \vec{c}, \vec{g}$ be the position vectors of A, B, C and G respectively.

$\vec{a} = 7\hat{i} - 8\hat{j} + \hat{k}$,

$\vec{b} = p\hat{i} + q\hat{j} + 5\hat{k}$

And, $\vec{c} = (q+1)\hat{i} + 5p\hat{j} + 0\hat{k}$

$\vec{g} = 3\hat{i} - 5\hat{j} + r\hat{k}$

By centroid formula,

$$\vec{g} = \frac{\vec{a} + \vec{b} + \vec{c}}{3}$$

$3\vec{g} = \vec{a} + \vec{b} + \vec{c}$

$3\left(3\hat{i} - 5\hat{j} + r\hat{k}\right) = \left(7\hat{i} - 8\hat{j} + \hat{k}\right) + \left(p\hat{i} + q\hat{j} + 5\hat{k}\right)$
$\quad + \left[(q+1)\hat{i} + 5p\hat{j}\right]$

$9\hat{i} - 15\hat{j} + 3r\hat{k} = (8+p+q)\hat{i} + (-8+q+5p)\hat{j} + (6)\hat{k}$

By equality of vectors,

$8+p+q = 9, -8+q+5p = -15$ and $3r = 6$

$\therefore q = 1-p \Rightarrow -8+1-p+5p = -15 \Rightarrow 4p - 7 = -15 \Rightarrow p = -2$

$\therefore q = 1-(-2) = 3$ and $6 = 3r \Rightarrow r = 2$

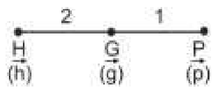
So, (p,q,r) = (–2, 3, 2)

18. If $G(\vec{g}), H(\vec{h})$ and $P(\vec{p})$ are centroid, orthocentere and circumcentere of a triangle and $x\vec{p} + y\vec{h} + z\vec{g} = 0$, then $(x,y,z) =$

(a) 1,1,–2 (b) 2,1,–3 (c) 1,3,–4 (d) 2,3,–5

MHT CET-2016

Ans. (b) :



Centroid (G) = $\vec{g}$

Orthocenter (H) = $\vec{h}$

Circumcenter (P) = $\vec{p}$

We know that,
Section formula,

$$\vec{g} = \frac{2\vec{p} + 1\cdot\vec{h}}{2+1}$$

$3\vec{g} = 2\vec{p} + \vec{h}$

$2\vec{p} + \vec{h} - 3\vec{g} = 0$ ....(i)

$x\vec{p} + y\vec{h} + z\vec{g} = 0$ ....(ii)

On comparing equation (i) and (ii), we get–

x = 2, y = 1, z = – 3

So, (x, y, z) = (2, 1, – 3)

19. If P is orthocenter, Q is circumcentre and G is centroid of ΔABC, then $\overrightarrow{QP} =$

(a) $3\overrightarrow{QG}$ (b) $2\overrightarrow{QG}$ (c) $\overrightarrow{QG}$ (d) $4\overrightarrow{QG}$

MHT CET-2012

Ans. (a) : Let,
$\vec{p}$ and $\vec{g}$ be the position vectors of P and G with respect to the circumcentre Q.

$\overrightarrow{QP} = \vec{p}$ and $\overrightarrow{QG} = \vec{g}$

We know that,
G divides segment $\overrightarrow{QP}$ internally ratio of 1:2

$$\frac{m\vec{a}+n\vec{b}}{m+n} = \vec{g}$$

$\therefore \; \vec{g} = \frac{1.\vec{p}+2\vec{q}}{1+2} = \frac{\vec{p}}{3}$ ($\because$ q = 0 where q is reference point)

$\therefore \; \vec{p} = 3\vec{g}$

$\overrightarrow{QP} = 3\overrightarrow{QG}$

20. If the position vectors of the vertices A, B and C are $6\hat{i}, 6\hat{j}$ and $\hat{k}$ respectively with respect to origin O, the volume of the tetrahedron OABC is

(a) 6 (b) 3 (c) $\frac{1}{6}$ (d) $\frac{1}{3}$

MHT CET-2012

Ans. (a) : Given,

$\overrightarrow{OA} = 6\hat{i} = 6\hat{i}+0\hat{j}+0\hat{k}$

$\overrightarrow{OB} = 6\hat{j} = 0\hat{i}+6\hat{j}+0\hat{k}$

$\overrightarrow{OC} = \hat{k} = 0\hat{i}+0\hat{j}+\hat{k}$

The position vectors of A, B, C, with respect to origin respectively.

$$\left[\overrightarrow{OA} \;\; \overrightarrow{OB} \;\; \overrightarrow{OC}\right] = \begin{vmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 1 \end{vmatrix} = 6(6-0)-0+0 = 36$$

Now, volume of tetrahedron

$$= \frac{1}{6}\left[\overrightarrow{OA} \;\; \overrightarrow{OB} \;\; \overrightarrow{OC}\right] = \frac{1}{6}(36) = 6$$

21. If $A(x,2,8), B(3,y,4)$ and $C(4,1,z)$ are vertices of ΔABC and $G(2,1,5)$ is the centroid then the values of x, y and z are respectively

(a) $(1,0,2)$ (b) $(-1,0,2)$
(c) $(1,0,3)$ (d) $(-1,0,3)$

MHT CET-2010

Ans. (d) : Let,
$\vec{a}, \vec{b}, \vec{c}, \vec{g}$ be the position vectors of A, B, C and G respectively.

$\therefore \; \vec{a} = x\hat{i}+2\hat{j}+8\hat{k}$

$\vec{b} = 3\hat{i}+y\hat{j}+4\hat{k}$

$\vec{c} = 4\hat{i}+\hat{j}+z\hat{k}$

$\vec{g} = 2\hat{i}+\hat{j}+5\hat{k}$

As G is the centroid of $\Delta ABC, \vec{g} = \frac{\vec{a}+\vec{b}+\vec{c}}{3}$ i.e.

$3\vec{g} = \vec{a}+\vec{b}+\vec{c}$

$3(2\hat{i}+\hat{j}+5\hat{k}) = (x\hat{i}+2\hat{j}+8\hat{k})+(3\hat{i}+y\hat{j}+4\hat{k})+(4\hat{i}+\hat{j}+z\hat{k})$

$6\hat{i}+3\hat{j}+15\hat{k} = (x+3+4)\hat{i}+(2+y+1)\hat{j}+(8+4+z)\hat{k}$

$= (x+7)\hat{i}+(y+3)\hat{j}+(z+12)\hat{k}$

$x+7=6$ i.e. $x=-1, y+3=3$ i.e. $y=0$ and $z+12=15$ i.e. $z=3$

So, (x, y, z) = (–1, 0, 3)

22. If $2\vec{a}+3\vec{b}-5\vec{c}=0$, then the ratio in which C divided AB is

(a) 2 : 3 internally (b) 2 : 3 externally
(c) 3 : 2 internally (d) 3 : 2 externally

MHT CET-2009

Ans. (c) : Given,

$2\vec{a}+3\vec{b}-5\vec{c}=0$

$2\vec{a}+3\vec{b}=5\vec{c}$

$$\frac{2\vec{a}+3\vec{b}}{5} = \vec{c}, \qquad \frac{2\vec{a}+3\vec{b}}{3+2} = \vec{c}$$

$$\frac{2\left(\vec{a}+\frac{3}{2}\vec{b}\right)}{2\left(\frac{3}{2}+1\right)} = \vec{c},$$

$$\frac{\left(\vec{a}+\frac{3}{2}\vec{b}\right)}{\left(1+\frac{3}{2}\right)} = \vec{c} \qquad \text{....(i)}$$

We know that,

$$c = \frac{a+\lambda b}{1+\lambda} \qquad \text{....(ii)}$$

From comparing equation (i) & (ii), we get –

$$\lambda = \frac{3}{2}$$

Hence, the point C divides AB internally in the ratio of 3:2.

23. Let A (3,5,6) and B (4,6,–3). Find ratio in which yz plane is dividing AB.

(a) 3 : 4 externally (b) 3 : 4 internally
(c) 4 : 3 externally (d) 4 : 3 internally

MHT CET-2008

Ans. (a) : Given,

$A = (3,5,6)$ and $B = (4,6,-3)$

Let, yz-plane divide AB in the ratio of k : 1 at point P.
x coordinate of point P will be 0.
($x_1 = 3, \quad x_2 = 4$)

$$P = \frac{kx_2+x_1}{k+1},$$

$$0 = \frac{k(4)+1(3)}{k+1}$$

$4k+3=0$

$\frac{k}{1} = \frac{-3}{4}$

k : 1 = – 3 : 4

$\because$ Externally ratio is negative.

Hence, YZ-plane divides the segment AB externally in the ratio of 3 : 4.

24. For 3 points $A(\vec{a}), B(\vec{b}), C(\vec{c})$ if $3\vec{a}+2\vec{b}-5\vec{c}=0$, then

(a) Point C divides AB externally in ratio 3 : 2
(b) 3 Points from ΔABC
(c) C is not mid-point of AB
(d) C divides AB internally in ratio 2 : 3

MHT CET-2007

Ans. (d) : Given that,

$3\vec{a}+2\vec{b}-5\vec{c}=0$

$5\vec{c}=3\vec{a}+2\vec{b}$

$\vec{c} = \frac{3\vec{a}+2\vec{b}}{5} = \frac{2\vec{b}+3\vec{a}}{2+3}$

$\vec{c} = \frac{2\vec{b}+3\vec{a}}{2+3}$

$$c = \frac{3\left(\frac{2}{3}\vec{b}+\vec{a}\right)}{3\left(\frac{2}{3}+1\right)} \quad \ldots(i)$$

$$c = \frac{a+\lambda b}{\lambda+1} \quad \ldots(ii)$$

Comparing equation (i) & (ii), we get-

$\lambda = \frac{2}{3}$

This shows that the point C divides segment AB internally in the ratio 2 : 3.

25. If the position vectors of the vertices A, B, C of a triangle ABC are $7\hat{j}+10\hat{k}, -\hat{i}+6\hat{j}+6\hat{k}$ and $-4\hat{i}+9\hat{j}+6\hat{k}$ respectively, the triangle is :

(a) equilateral
(b) isosceles
(c) scalene
(d) right angled and isosceles also

BITSAT-2008

Ans. (d) : Given,

The position vectors of the vertices,

$\overrightarrow{OA} = 7\hat{j}+10\hat{k}, \quad \overrightarrow{OB} = -\hat{i}+6\hat{j}+6\hat{k},$

$\overrightarrow{OC} = -4\hat{i}+9\hat{j}+6\hat{k}$

$\overrightarrow{AB} = \overrightarrow{OB}-\overrightarrow{OA}$

$\overrightarrow{AB} = (-\hat{i}+6\hat{j}+6\hat{k})-(7\hat{j}+10\hat{k})$

$\overrightarrow{AB} = -\hat{i}-\hat{j}-4\hat{k}$

$|\overrightarrow{AB}| = \sqrt{1+1+16} = 3\sqrt{2}$

$\overrightarrow{BC} = \overrightarrow{OC}-\overrightarrow{OB}$

$\overrightarrow{BC} = (-4\hat{i}+9\hat{j}+6\hat{k})-(-\hat{i}+6\hat{j}+6\hat{k})$

$\overrightarrow{BC} = -3\hat{i}+3\hat{j}$

$|\overrightarrow{BC}| = \sqrt{(-3)^2+(3)^2}$

$\overrightarrow{BC} = \sqrt{18} = 3\sqrt{2}$

$\overrightarrow{CA} = \overrightarrow{OA}-\overrightarrow{OC}$

$\overrightarrow{CA} = (7\hat{j}+10\hat{k})-(-4\hat{i}+9\hat{j}+6\hat{k})$

$\overrightarrow{CA} = 4\hat{i}-2\hat{j}+4\hat{k}$

$|\overrightarrow{CA}| = \sqrt{(4)^2+(-2)^2+(4)^2}$

$|\overrightarrow{CA}| = \sqrt{36} = 6$

$3\sqrt{2}$, $3\sqrt{2}$ & 6 are sides of a right angled Δ.

$|AB|^2+|BC|^2 = |AC|^2$

$\because (3\sqrt{2})^2+(3\sqrt{2})^2 = 36$

Hence, the ΔABC is a right-angled and isosceles also.

26. If the area of the parallelogram with $\vec{a}$ and $\vec{b}$ as two adjacent sides is 15 sq. units then the area of the parallelogram having $3\vec{a}+2\vec{b}$ and $\vec{a}+3\vec{b}$ as two adjacent sides in sq. units is

(a) 45 (b) 75 (c) 105 (d) 120

Karnataka CET-2006,2021

Ans. (c) : Given,

Area of parallelogram (A) = 15 sq. units

Sides $(3\vec{a}+2\vec{b}), (\vec{a}+3\vec{b})$

We know that,

Area of parallelogram (A) = $|a \times b|$

$= |(3\vec{a}+2\vec{b})\times(\vec{a}+3\vec{b})|$

$= |7(\vec{a}\times\vec{b})| = 7|\vec{a}\times\vec{b}|$

$(A) = 7\times 15 = 105$ square units

Hence, the required of parallelogram = 105 square units.

27. The diagonals of a parallelogram are the vectors $3\hat{i}+6\hat{j}-2\hat{k}$ and $-\hat{i}-2\hat{j}-8\hat{k}$, then the length of the shorter side of parallelogram is

(a) $2\sqrt{3}$ (b) $\sqrt{14}$ (c) $\sqrt{29}$ (d) $4\sqrt{3}$

Karnataka CET-2021

Ans. (c) : Given,
Diagonals of a parallelogram,

$$\vec{d}_1 = 3\hat{i}+6\hat{j}-2\hat{k}$$

$$\vec{d}_2 = -\hat{i}-2\hat{j}-8\hat{k}$$

We know that,

$$\vec{a} = \frac{1}{2}\left(\vec{d}_1+\vec{d}_2\right)$$

$$\vec{a} = \frac{1}{2}\left\{\left(3\hat{i}+6\hat{j}-2\hat{k}\right)+\left(-\hat{i}-2\hat{j}-8\hat{k}\right)\right\}$$

$$\vec{a} = \frac{2\hat{i}+4\hat{j}-10\hat{k}}{2}, \quad \vec{a} = \hat{i}+2\hat{j}-5\hat{k}$$

$$|\vec{a}| = \sqrt{1+4+25}, \; |\vec{a}| = \sqrt{30}$$

$$\vec{b} = \frac{\vec{d}_1-\vec{d}_2}{2}, \; \vec{b} = \frac{\left(3\hat{i}+6\hat{j}-2\hat{k}\right)-\left(-\hat{i}-2\hat{j}-8\hat{k}\right)}{2}$$

$$\vec{b} = \frac{4\hat{i}+8\hat{j}+6\hat{k}}{2}, \quad \vec{b} = 2\hat{i}+4\hat{j}+3\hat{k}$$

$$|\vec{b}| = \sqrt{4+16+9}$$

So, $|\vec{b}| = \sqrt{29}$

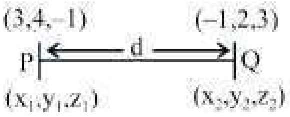
28. Let (3, 4, –1) and (–1, 2, 3) be the end points of a diameter of a sphere. Then, the radius of the sphere is equal to

(a) 2 units (b) 3 units
(c) 6 units (d) 7 units

COMEDK-2017

Ans. (b) : Let,
P(3,4,–1) and Q(–1, 2, 3) be the end points of the diameter of a sphere.

$\therefore$ Length of diameter = PQ



Since, $d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$

Then, $d = \sqrt{(-1-3)^2+(2-4)^2+(3+1)^2}$

$$d = \sqrt{16+4+16}$$

$$d = \sqrt{16+4+16} = \sqrt{36}$$

d = 6 units

So, radius $= \frac{d}{2} = \frac{6}{2} = 3$units

29. Let $\vec{r}$ be the position vector of a point P(x,y,z), where x, y and z are natural numbers and $\vec{a} = \vec{i}+\vec{j}+\vec{k}$. What is the total number of possible positions of point P for which $\vec{r}.\vec{a}$ =10?

(a) 18 (b) 36 (c) 66 (d) 72

SCRA-2009

Ans. (b) : The position vector of a point P is

$$\vec{r} = x\hat{i}+y\hat{j}+z\hat{k}$$

Where, x,y,z $\in$ N
and $\vec{a} = \hat{i}+\hat{j}+\hat{k}$ $\vec{r}.\vec{a} = 10$

$$(x\hat{i}+y\hat{i}+z\hat{k})\cdot(\hat{i}+\hat{j}+\hat{k}) = 10$$

x + y + z=10
Here, x,y,z are natural number so, x,y,z can not be negative and zero as well so at least x,y,z should be 1.
So, value of x,y,z vary from 2 to 8
So, combination of values become ${}^7C_3+1$

$$\text{Passible value} = \frac{\lfloor 7}{\lfloor 4 \lfloor 3}+1$$

$$= \frac{7\times6\times5}{3\times2\times1}+1 = 35+1 = 36$$

30. The position vector of a point R which divides the line joining two points P and Q whose position vectors are $\hat{i}+2\hat{j}-\hat{k}$ and $-\hat{i}+\hat{j}-\hat{k}$ respectively, in the ratio 2 : 1 externally is

(a) $-3\hat{i}-\hat{k}$ (b) $3\hat{i}+\hat{k}$
(c) $2\hat{i}+\hat{j}-\hat{k}$ (d) none of these

AMU-2010

Ans. (a) : Given,

$$P = \hat{i}+2\hat{j}-\hat{k} \text{ and } Q = -\hat{i}+\hat{j}-\hat{k}$$

$$R = \frac{2.\left(-\hat{i}+\hat{j}-\hat{k}\right)-1\left(\hat{i}+2\hat{j}-\hat{k}\right)}{2-1}$$

[$\because$ Externally division]

$$R = -2\hat{i}+2\hat{j}-2\hat{k}-\hat{i}-2\hat{j}+\hat{k}$$

$$R = -3\hat{i}-\hat{k}$$

31. The points having position vectors $60\hat{i}+3\hat{j}, 40\hat{i}-8\hat{j}, a\hat{i}-52\hat{j}$, are collinear, if a =

(a) –20 (b) –40 (c) 20 (d) 40

CG PET- 2018

Ans. (b) : Given that,

$$\overrightarrow{OA} = 60\hat{i}+3\hat{j}$$

$$\overrightarrow{OB} = 40\hat{i}-8\hat{j}$$

$$\overrightarrow{OC} = a\hat{i}-52\hat{j}$$

$$\overrightarrow{AB} = -20\hat{i}-11\hat{j}$$

$$\overrightarrow{AC} = (a-60)\hat{i}-55\hat{j}$$

$\therefore$ Vector are collinear.

$\therefore$ $\overrightarrow{AB} = \lambda\overrightarrow{AC}$

$$\frac{-20}{a-60} = \frac{-11}{-55}$$

$$\frac{-20}{a-60} = \frac{1}{5}$$

a – 60= – 100
a = –100+60
a = – 40

32. The position vectors of A and B are $\left(\hat{i}+\hat{j}+\hat{k}\right)$ and $\left(\frac{1}{3}\hat{j}+\frac{1}{3}\hat{k}\right)$. If 'B' divides the line AC in the ratio 2 : 1, then position vector of 'C' is

(a) $\left(\frac{1}{2},0,0\right)$ (b) $\left(0,\frac{1}{3},0\right)$

(c) $\left(\frac{-1}{2},\frac{-1}{2},0\right)$ (d) $\left(\frac{-1}{2},0,0\right)$

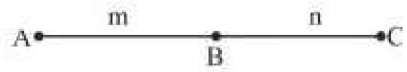
APEAPCET-20.08.2021, Shift-I

Ans. (d): Let, $OA=\left(\hat{i}+\hat{j}+\hat{k}\right)$

and $OB=\left(\frac{1}{3}\hat{j}+\frac{1}{3}\hat{k}\right)$

Given, ratio = 2 : 1 = m : n

So, m = 2, n = 1



$$\because OB=\frac{mOC+nOA}{m+n}=\frac{2OC+\left(\hat{i}+\hat{j}+\hat{k}\right)}{3}$$

$$3\left(\frac{1}{3}\hat{j}+\frac{1}{3}\hat{k}\right)=2(OC)+\left(\hat{i}+\hat{j}+\hat{k}\right)$$

$$\hat{j}+\hat{k}-\hat{i}-\hat{j}-\hat{k}=2.(OC)$$

$$\Rightarrow OC=-\frac{1}{2}\hat{i}+0\hat{j}+0\hat{k}$$

$$\therefore C=\left(\frac{-1}{2},0,0\right)$$

33. The locus of a point which is at a distance of 4 units from (3,–2) in xy-plane is_____

(a) $x^2+y^2+6x-4y+16=0$

(b) $x^2+y^2-6x-4y+3=0$

(c) $x^2+y^2-6x+4y-16=0$

(d) $x^2+y^2-6x+4y-3=0$

AP EAPCET-25.08.2021, Shift-II

Ans. (d) : Let, the point be (x,y)

Then, $\sqrt{(3-x)^2+(-2-y)^2}=4$

$$(3-x)^2+(-2-y)^2=16$$

$$9+x^2-6x+4+y^2+4y=16$$

$$x^2+y^2-6x+4y-3=0$$

34. The position vectors of the points A, B, C and D are $2\hat{i}+4\hat{k}, 5\hat{i}+3\sqrt{3}\hat{j}+4\hat{k}, -2\sqrt{3}\hat{j}+\hat{k}$ and $2\hat{i}+\hat{k}$ respectively. Then $\overrightarrow{CD}=$

(a) $\frac{2}{3}\overrightarrow{AB}$ (b) $\frac{1}{3}\overrightarrow{AB}$ (c) $\frac{3}{2}\overrightarrow{AB}$ (d) $\frac{2}{5}\overrightarrow{AB}$

J&K CET-2016

Ans. (a) : Given,

$$A=2\hat{i}+4\hat{k}$$

$$B=5\hat{i}+3\sqrt{3}\hat{j}+4\hat{k}$$

$$C=-2\sqrt{3}\hat{j}+\hat{k}$$

$$D=2\hat{i}+\hat{k}$$

$$\because \overrightarrow{AB}=\vec{B}-\vec{A}$$

$$\overrightarrow{AB}=5\hat{i}+3\sqrt{3}\hat{j}+4\hat{k}-2\hat{i}-4\hat{k}$$

$$\overrightarrow{AB}=3\hat{i}+3\sqrt{3}\hat{j}$$

$$\left|AB\right|=\sqrt{9+9\times 3}=\sqrt{9+27}=\sqrt{36}$$

$$AB=6$$

And, $\overrightarrow{CD}=\vec{D}-\vec{C}=2\hat{i}+\hat{k}+2\sqrt{3}\hat{j}-\hat{k}$

$$\overrightarrow{CD}=2\hat{i}+2\sqrt{3}\hat{j}$$

$$\left|\overrightarrow{CD}\right|=\sqrt{4+12}$$

$$CD=\sqrt{16}=4$$

Therefore,

$$\frac{CD}{AB}=\frac{4}{6}$$

$$CD=\frac{4}{6}\times AB$$

$$CD=\frac{4}{6}AB\Rightarrow\overrightarrow{CD}=\frac{2}{3}\overrightarrow{AB}$$

35. The distance of the point having position vector $-\hat{i}+2\hat{j}+6\hat{k}$ from the straight line passing through the point (2, 3, –4) and parallel to the vector, $6\hat{i}+3\hat{j}-4\hat{k}$ is

(a) $2\sqrt{13}$ (b) $4\sqrt{3}$ (c) 6 (d) 7

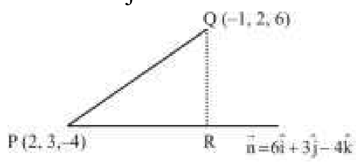
JEE Main 10.04.2019, Shift-II

Ans. (d) : Given that,

Position vector = $-\hat{i}+2\hat{j}+6\hat{k}$

And point = (2, 3, –4)

Parallel vector = $6\hat{i}+3\hat{j}-4\hat{k}$



$$\because \overrightarrow{PQ}=-3\hat{i}-\hat{j}+10\hat{k}$$

$$PQ=\sqrt{110}$$

$$\therefore PR=\left|\frac{\overrightarrow{PQ}\,\vec{n}}{|\vec{n}|}\right|=\frac{61}{\sqrt{61}}=\sqrt{61}$$

So, $RQ=\sqrt{PQ^2-PR^2}$

$$=\sqrt{110-61}=\sqrt{49}=7$$

36. Let A(3, 0, –1), B(2, 10, 6) and C(1, 2, 1) be the vertices of a triangle and M be the mid-point of AC. If G divides BM in the ratio 2 : 1 then cos (∠GOA) (0, being the origin) is equal to

(a) $\frac{1}{\sqrt{15}}$ (b) $\frac{1}{2\sqrt{15}}$ (c) $\frac{1}{\sqrt{30}}$ (d) $\frac{1}{6\sqrt{10}}$

JEE Main 10.04.2019, Shift-I

Ans. (a) : Given, vertices of

A = (3, 0, –1)

B = (2, 10, 6)

C = (1, 2, 1)

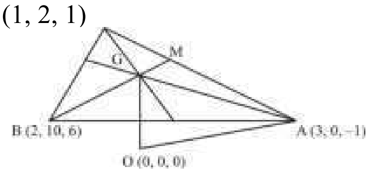
G divided BM in 2 : 1 and M is midpoint of AC

G is centroid of the given triangle

$$G=\left(\frac{3+2+1}{3},\frac{0+10+2}{3},\frac{-1+6+1}{3}\right)$$

$G = (2, 4, 2)$ ....(i)

C (1, 2, 1)



If θ be angle ∠GOA then

$$\overrightarrow{OA}\cdot\overrightarrow{OG}=\left|\overrightarrow{OA}\right|\cdot\left|\overrightarrow{OG}\right|\cos\theta \quad \text{.... (ii)}$$

Here,

$$\overrightarrow{OA}=\left(3\hat{i}-0\hat{i}\right)+\left(0\hat{j}-0\hat{j}\right)+\left(-\hat{k}-0\hat{k}\right)$$

$$\overrightarrow{OA}=3\hat{i}-\hat{k} \quad \text{....(iii)}$$

$$\overrightarrow{OG}=\left(2\hat{i}-0\hat{i}\right)+\left(4\hat{j}-0\hat{j}\right)+\left(2\hat{k}-0\hat{k}\right)$$

$$\overrightarrow{OG}=2\hat{i}+4\hat{j}+2\hat{k} \quad \text{....(iv)}$$

Now, from equation (ii), we get –

$$\overrightarrow{OA}\cdot\overrightarrow{OG}=\left|\overrightarrow{OA}\right|\cdot\left|\overrightarrow{OG}\right|\cos\theta$$

$$\left(3\hat{i}-\hat{k}\right)\cdot\left(2\hat{i}+4\hat{j}+2\hat{k}\right)=\sqrt{3^2+(-1)^2}\cdot\sqrt{2^2+4^2+2^2}\cos\theta$$

$$(6+0-2)=\sqrt{10}\cdot\sqrt{24}\cos\theta$$

$$4=\sqrt{2}\cdot\sqrt{5}\cdot2\sqrt{3}\cdot\sqrt{2}\cos\theta$$

$$4=\sqrt{2}\cdot2\sqrt{15}\cdot\sqrt{2}\cos\theta$$

$$4=4\sqrt{15}\cos\theta$$

$$\cos\theta=\frac{1}{\sqrt{15}}$$

37. For p > 0, a vector $v_2=2\hat{i}+(p+1)\hat{j}$ is obtained by rotating the vector $v_1=\sqrt{3}p\hat{i}+\hat{j}$ by an angle θ about origin in counter clockwise direction. If $\tan\theta=\frac{(\alpha\sqrt{3}-2)}{(4\sqrt{3}+3)}$, then the value of α is equal to

JEE Main 20.07.2021, Shift-II

Ans. (6) : Here, v_2 obtained by rotating v_1

$\therefore \quad |v_1|^2=|v_2|^2$

$3p^2 + 1 = 4 + (p + 1)^2$

$3p^2 + 1 = 4 + p^2 + 1 + 2p$

$2p^2 – 2p – 4 = 0$

$p^2 – p – 2 = 0$

$(p + 1) (p – 2) = 0$

$p = –1, p = 2$

Since, $p > 0$ given, then $p = –1$ is discarded.

Now, $\cos\theta=\frac{v_1v_2}{|v_1||v_2|}=\frac{4\sqrt{3}+3}{\sqrt{13}\cdot\sqrt{13}}=\frac{4\sqrt{3}+3}{13}$

$$\sin\theta=\sqrt{1-\cos^2\theta}=\sqrt{1-\frac{\left(4\sqrt{3}+3\right)^2}{(13)^2}}$$

$$\sin\theta=\frac{\sqrt{112-24\sqrt{3}}}{13}$$

$$\therefore \quad \tan\theta=\frac{\sin\theta}{\cos\theta}=\frac{\sqrt{112-24\sqrt{3}}}{4\sqrt{3}+3}$$

Given, $\tan\theta=\frac{\alpha\sqrt{3}-2}{4\sqrt{3}+3}$

So, $\sqrt{112-24\sqrt{3}}=\alpha\sqrt{3}-2$

$\Rightarrow \quad 6\sqrt{3}-2=\alpha\sqrt{3}-2$

$\therefore \quad \alpha = 6$

38. If C is the mid-point of AB and P is any point outside AB, then

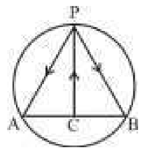
(a) PA + PB + PC = 0 (b) PA + PB + 2PC = 0
(c) PA + PB = PC (d) PA + PB = 2PC

AIEEE-2005

Ans. (d) : Applying triangle law of addition of vectors in triangles PAC and PBC,

We have,

$$\overrightarrow{PA}+\overrightarrow{AC}=\overrightarrow{PC} \text{ and } \overrightarrow{PB}+\overrightarrow{BC}=\overrightarrow{PC}$$



$$\overrightarrow{PA}+\overrightarrow{AC}+\overrightarrow{PB}+\overrightarrow{BC}=\overrightarrow{PC}+\overrightarrow{PC}$$

$$\overrightarrow{PA}+\overrightarrow{PB}+\left(\overrightarrow{AC}+\overrightarrow{BC}\right)=2\overrightarrow{PC}$$

$$\overrightarrow{PA}+\overrightarrow{PB}+\left(\overrightarrow{AC}-\overrightarrow{AC}\right)=2\overrightarrow{PC}$$

$$\overrightarrow{PA}+\overrightarrow{PB}=2\overrightarrow{PC}$$

39. A vector a has components 3p and 1 with respect to rectangular Cartesian system. This system is rotated through a certain angle about the origin in the counter clockwise sense. If, with respect to new system, a has components p + 1 and $\sqrt{10}$, then a value of p is equal to

(a) 1 (b) $-\frac{5}{4}$ (c) $\frac{4}{5}$ (d) –1

JEE Main 18.03.2021, Shift-I

Ans. (d) : Vector in initial position having component 3p and 1

$$\vec{a}_{\text{initial}}=(3p)\hat{i}+\hat{j}$$

After rotating vector become having component p + 1 and $\sqrt{10}$

$$\vec{a}_{\text{initial}}=(p+1)\hat{i}+\left(\sqrt{10}\right)\hat{j}$$

Equating magnitude

$$\therefore \quad \left|\vec{a}_{\text{initial}}\right|=\left|\vec{a}_{\text{final}}\right|$$

$(3p)^2 + 1 = (p + 1)^2 + \left(\sqrt{10}\right)^2$

$9p^2 + 1 = p^2 + 1 + 2p + 10$

$8p^2 – 2p – 10 = 0$

$4p^2 – p – 5 = 0$

$4p^2 – 5p + 4p – 5 = 0$

$$p(4p-5)+1(4p-5)=0$$
$$(4p-5)(p+1)=0$$
$$p=\frac{5}{4},-1$$
Hence, the value of p = –1.

40. If α, β, γ are distinct real numbers are α + β + γ ≠ 0, then the points with position vectors $\alpha\hat{i}+\beta\hat{j}+\gamma\hat{k}, \beta\hat{i}+\gamma\hat{j}+\alpha\hat{k}$ and $\gamma\hat{i}+\alpha\hat{j}+\beta\hat{k}$ are

(a) Collinear
(b) Vertices of a scalene triangle
(c) Vertices of isosceles triangle
(d) Vertices of an equilateral triangle

AP EAMCET-23.04.2019, Shift-I

Ans. (d) :
$$A=\alpha\hat{i}+\beta\hat{j}+\gamma\hat{k}$$
$$B=\beta\hat{i}+\gamma\hat{j}+\alpha\hat{k}$$
$$C=\gamma\hat{i}+\alpha\hat{j}+\beta\hat{k}$$
Therefore,
$$\overrightarrow{AB}=(\beta-\alpha)\hat{i}+(\gamma-\beta)\hat{j}+(\alpha-\gamma)\hat{k} \quad ....(i)$$
$$\overrightarrow{BC}=(\gamma-\beta)\hat{i}+(\alpha-\gamma)\hat{j}+(\beta-\alpha)\hat{k} \quad ....(ii)$$
$$\overrightarrow{CA}=(\alpha-\gamma)\hat{i}+(\beta-\alpha)\hat{j}+(\gamma-\beta)\hat{k} \quad ....(iii)$$
We know that,
The longitude of vector
$$\vec{a}=x\hat{i}+y\hat{j}+z\hat{k}$$
$$|\vec{a}|=\sqrt{x^2+y^2+k^2}$$
$$|\overrightarrow{AB}|=\sqrt{(\beta-a)^2+(\gamma-\beta)^2+(\alpha-\gamma)^2}$$
$$|\overrightarrow{BC}|=\sqrt{(\beta-\alpha)^2+(\gamma-\beta)^2+(\alpha-\gamma)^2}$$
$$|\overrightarrow{CA}|=\sqrt{(\beta-\alpha)^2+(\gamma-\beta)^2+(\alpha-\gamma)^2}$$
All the sides are equal in longitude
Hence, it forms an equilateral triangle

41. If the points having the position vectors $3\hat{i}-2\hat{j}-\hat{k}, 2\hat{i}+3\hat{j}-4\hat{k}, -\hat{i}+\hat{j}+2\hat{k}$ and $4\hat{i}+5\hat{j}+\lambda\hat{k}$ are coplanar, then λ=

(a) $-\frac{146}{17}$ (b) 8 (c) –8 (d) $\frac{146}{17}$

AP EAMCET-2017

Ans. (a) : Let, the given points are
$$A\left(3\hat{i}-2\hat{j}-\hat{k}\right), B\left(2\hat{i}+3\hat{j}-4\hat{k}\right)$$
$$C\left(-\hat{i}+\hat{j}+2\hat{k}\right) \text{and } D\left(4\hat{i}+5\hat{j}+\lambda\hat{k}\right)$$
$$AB=-\hat{i}+5\hat{j}-3\hat{k}$$
$$BC=-3\hat{i}-2\hat{j}+6\hat{k}$$
$$CD=5\hat{i}+4\hat{j}+(\lambda-2)\hat{k}$$
Since, these points are coplanar
$$\begin{vmatrix}-1 & 5 & -3\\ -3 & -2 & 6\\ 5 & 4 & \lambda-2\end{vmatrix}=0$$
$$-1(-2\lambda+4-24)-5[-3(\lambda-2)-30]-3(-12+10)=0$$
$$2\lambda+20+15(\lambda-2)+150+36-30=0$$
$$17\lambda+20-30+150+6=0$$
$$17\lambda+146=0 \Rightarrow \lambda=\frac{-146}{17}$$

42. If the position vectors of the points A, B, C, D are $7\hat{i}-4\hat{j}+7\hat{k}, \hat{i}-6\hat{j}+10\hat{k}, -\hat{i}-3\hat{j}+4\hat{k}, 5\hat{i}-\hat{j}+5\hat{k}$ respectively, then ABCD is

(a) parallelogram but not rhombus
(b) a square
(c) a quadrilateral which is not a parallelogram
(d) a rectangle

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Ans. (c) :
AB = position vector of B - position vector of A
$$=-6\hat{i}-2\hat{j}+3\hat{k}$$
$$|AB|=\sqrt{(-6)^2+(2)^2+3^2}=\sqrt{49}=7$$
BC = position vector of C - position vector of B
$$=-2\hat{i}+3\hat{j}-6\hat{k} \Rightarrow |BC|=\sqrt{4+9+36}=7$$
$$CD=6\hat{i}+2\hat{j}+\hat{k}$$
$$|CD|=\sqrt{36+4+1}=\sqrt{41}$$
$$DA=2\hat{i}-3\hat{j}+2\hat{k}$$
$$|DA|=\sqrt{4+9+4}=\sqrt{17}$$
$$AC=-8\hat{i}+\hat{j}-3\hat{k}=AB+BC$$
Hence, ABCD is quadrilateral which is not a parallelogram.

43. The distance of a point (2, 5, – 3) from the plane 6x – 3y + 2z – 4 = 0 is _______

(a) $\frac{13}{\sqrt{7}}$ (b) $\frac{5}{7}$ (c) $\frac{5}{\sqrt{7}}$ (d) $\frac{13}{7}$

GUJCET-2021

Ans. (d) : Given, point (2, 5, –3)
Plane 6x – 3y + 2z – 4 = 0
We know that-
$$D=\frac{|ax_1+by_1+cz_1+d|}{\sqrt{a^2+b^2+c^2}}$$
Here, a = 6, b = – 3, c = 2, d = – 4
$(2, 5, -3) \Rightarrow (x_1, y_1, z_1)$
$$D=\frac{|12-15+2\times(-3)-4|}{\sqrt{36+9+4}}$$
$$D=\frac{|12-15-6-4|}{\sqrt{49}}=\frac{|12-25|}{7}=\frac{13}{7}$$

44. The position vector of point A is (4, 2, –3). If p_1 is perpendicular distance of A from XY-plane and p_2 is perpendicular distance from Y-axis, then $p_1 + p_2$ =_____.

(a) 8 (b) 3 (c) 2 (d) 7

GUJCET-2017

Ans. (a) : Given, the position vector of point A is (4, 2,–3)

Then A(4, 2, –3) = $A\left(4\hat{i}+2\hat{j}-3\hat{k}\right)$

Since, p_1 is the perpendicular distance of A from xy – plane.

So, x = 0 & y = 0, $p_1 = |z|$

$\therefore\ p_1 = \left|0\hat{i}+0\hat{j}-3\hat{k}\right| \Rightarrow p_1 = \left|-3\hat{k}\right|$

$$p_1 = \sqrt{(-3)^2} = 3$$

And, p_2 is perpendicular distance from y –axis.

So, y = 0, $p_2 = |x + z|$

$\therefore p_2 = \left|4\hat{i}+0\hat{j}-3\hat{k}\right| = \sqrt{4^2+(-3)^2} = \sqrt{25} = 5$

Then, $p_1 + p_2 = 3 + 5 = 8$

45. What is the perpendicular distance between 2x + 2y – z + 1 = 0 and $x+y-\frac{z}{2}+2=0$?

(a) $\sqrt{5}$ (b) $\sqrt{2}$ (c) 2 (d) 1

GUJCET-2007

Ans. (d) : $2x + 2y - z + 1 = 0$ .... (1)

$2x + 2y - z + 4 = 0$

$$\text{Distance} = \left|\frac{d_1-d_2}{\sqrt{a^2+b^2+c^2}}\right|$$

$$= \left|\frac{1-4}{\sqrt{(2)^2+(2)^2+(-1)^2}}\right| = \left|\frac{-3}{\sqrt{4+4+1}}\right| = \left|\frac{-3}{\sqrt{9}}\right| = \frac{3}{3}$$

= 1

46. Find the distance between the planes $\vec{r}(2\hat{i}-\hat{j}+3\hat{k}) = 4$ and $\vec{r}(6\hat{i}-3\hat{j}+9\hat{k})+13 = 0$

(a) $\frac{5}{3\left(\sqrt{14}\right)}$ (b) $\frac{10}{3\left(\sqrt{14}\right)}$

(c) $\frac{25}{3\left(\sqrt{14}\right)}$ (d) None of these

GUJCET-2011

Ans. (c):

$\vec{r}\cdot(2\hat{i}-\hat{j}+3\hat{k}) = 4$ ...(i)

$\vec{r}\cdot(6\hat{i}-3\hat{j}+9\hat{k})+13 = 0$

$\vec{r}\cdot(2\hat{i}-\hat{j}+3\hat{k}) = \frac{-13}{3}$ ... (ii)

Clearly both planes are parallel.

Hence, distance between them is –

$$= \frac{4+\frac{13}{3}}{\sqrt{4+1+9}} = \frac{12+13}{3\sqrt{14}}$$

So, Distance $= \frac{25}{3\sqrt{14}}$

47. If $2\hat{i}-\hat{j}+\hat{k}, \hat{i}-3\hat{j}-5\hat{k}$ are the position vectors of the points A and B respectively, C divides AB in the ratio 2 : 3 and M is the mid-point of AB, then 5 (position vector of C) – 2 (position vector of M) =

(a) $5\hat{i}-5\hat{j}-3\hat{k}$ (b) $11\hat{i}-13\hat{j}-11\hat{k}$

(c) $5\hat{i}+5\hat{j}-3\hat{k}$ (d) $11\hat{i}+13\hat{j}-11\hat{k}$

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Ans. (a) : Let, $\vec{c}$ be the position vector C

Since, C divides AB in the ratio 2 : 3

$$\vec{c} = \frac{2\left(\hat{i}-3\hat{j}-5\hat{k}\right)+3\left(2\hat{i}-\hat{j}+\hat{k}\right)}{2+3}$$

$$= \frac{2\hat{i}-6\hat{j}-10\hat{k}+6\hat{i}-3\hat{j}+3\hat{k}}{5}$$

$$\vec{c} = \frac{8\hat{i}-9\hat{j}-7\hat{k}}{5}$$

M is midpoint of AB, $= \frac{2\hat{i}-\hat{j}+\hat{k}+\hat{i}-3\hat{j}-5\hat{k}}{2}$

$$= \frac{3\hat{i}-4\hat{j}-4\hat{k}}{2}$$

Now, $5\vec{c}-2\vec{m}$

$$= 5\times\frac{8\hat{i}-9\hat{j}-7\hat{k}}{5} - 2\times\frac{3\hat{i}-4\hat{j}-4\hat{k}}{2}$$

$$= 8\hat{i}-9\hat{j}-7\hat{k}-3\hat{i}+4\hat{j}+4\hat{k} = 5\hat{i}-5\hat{j}-3\hat{k}$$

48. If $3\hat{i}-5\hat{j}+2\hat{k}$, $7\hat{i}+2\hat{j}-4\hat{k}$, $\hat{i}-3\hat{j}+4\hat{k}$ and $-7\hat{i}-17\hat{j}+16\hat{k}$ are position vectors of the points A, B, C and D respectively, then the angle between AB and CD is

(a) 0 (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{2}$ (d) π

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Ans. (d) :

$$AB = \left(7\hat{i}+2\hat{j}-4\hat{k}\right)-\left(3\hat{i}-5\hat{j}+2\hat{k}\right)$$

$$= \left(4\hat{i}+7\hat{j}-6\hat{k}\right)$$

$$CD = \left(-7\hat{i}-17\hat{j}+16\hat{k}\right)-\left(\hat{i}-3\hat{j}+4\hat{k}\right)$$

$$= -8\hat{i}-14\hat{j}+12\hat{k}$$

$AB\cdot CD = -32 - 98 - 72 = -202$

$|AB| = \sqrt{4^2+7^2+6^2} = \sqrt{101}$

$|CD| = \sqrt{8^2+(14)^2+(12)^2} = \sqrt{404}$

Let, θ be the required angle,

$$\therefore\quad \cos\theta = \frac{AB\cdot CD}{|AB||CD|} = \frac{-202}{\sqrt{101}\sqrt{404}} = \frac{-202}{2\times101}$$

$\cos\theta = -1$

$\theta = \pi$

Hence, the angle between AB and CD is π

49. If a, b, c are distinct real numbers and P, Q, R are three points whose position vectors are respectively $a\hat{i}+b\hat{j}+c\hat{k}, b\hat{i}+c\hat{j}+a\hat{k}$ and $c\hat{i}+a\hat{j}+b\hat{k}$, then ∠QPR =