YOUTH COMPETITION TIMES

VOLUME II CALCULUS

Chapterwise Solved Papers

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In the event of any dispute, the judicial area will be Prayagraj.

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Syllabus for JEE (Main) - 2024

Syllabus for JEE Main Paper-1 (B.E./B.Tech.)

MATHEMATICS

UNIT 1: SETS, RELATIONS, AND FUNCTIONS: Sets and their representation: Union, intersection, and complement of sets and their algebraic properties; Power set; Relation, Type of relations, equivalence relations, functions; one-one, into and onto functions, the composition of functions.

UNIT 2: COMPLEX NUMBERS AND QUADRATIC EQUATIONS: Complex numbers as ordered pairs of reals, Representation of complex numbers in the form a + ib and their representation in a plane, Argand diagram, algebra of complex number, modulus, and argument (or amplitude) of a complex number, Quadratic equations in real and complex number system and their solutions Relations between roots and co-efficient, nature of roots, the formation of quadratic equation with given roots.

UNIT 3: MATRICES AND DETERMINATS: Matrices, algebra of matrices, type of matrices, determinants, and matrices of order two and three, evaluation of determinants, area of triangles using determinants, Adjoint, and evaluation of inverse of a square matrix using determinants and, Test of consistency and solution of simultaneous linear equations in two or three variables using matrices.

UNIT 4: PERMUTATIONS AND COMBINATIONS: The fundamental principle of counting, permutation as an arrangement and combination as section, Meaning of P (n, r) and C(n, r), simple applications.

UNIT 5: BINOMIAL THEOREM AND ITS SIMPLE APPLICATIONS: Binomial theorem for a positive integral index, general term and middle term, and simple applications.

UNIT 6: SEQUENCE AND SERIES: Arithmetic and Geometric progressions, insertion of arithmetic, geometric means between two given numbers, Relation between A.M and G.M.

UNIT 7: LIMIT, CONTINUITY, AND DIFFERENTIABILITY: Real-valued functions, algebra of functions, polynomials, rational, trigonometric, logarithmic, and exponential functions, inverse function. Graphs of simple functions. Limits, continuity, and differentiability. Differentiation of the sum, difference, product, and quotient of two functions. Differentiation of trigonometric, inverse trigonometric, logarithmic, exponential, composite, and implicit functions; derivatives of order up to two, Applications of derivatives: Rate of change of quantities, monotonic-Increasing and decreasing functions, Maxima and minima of functions of one variable.

UNIT 8: INTEGRAL CALCULAS: Integral as an anti-derivative, Fundamental integral involving algebraic, trigonometric, exponential, and logarithmic functions. Integrations by substitution, by parts, and by partial functions. Integrations by substitution, by parts, and by partial functions. Integration using trigonometric identities.

Evaluation of simple integrals of the type

$\int \frac{dx}{x^2+a^2}$, $\int \frac{dx}{\sqrt{x^2 \pm a^2}}$, $\int \frac{dx}{a^2-x^2}$, $\int \frac{dx}{\sqrt{a^2-x^2}}$, $\int \frac{dx}{ax^2+bx+c}$, $\int \frac{dx}{\sqrt{ax^2+bx+c}}$, $\int \frac{(px+q)dx}{ax^2+bx+c}$, $\int \frac{(px+q)dx}{\sqrt{ax^2+bx+c}}$,

$\int \sqrt{a^2 \pm x^2}\, dx$, $\int \sqrt{x^2-a^2} dx$

The fundamental theorem of calculus, properties of definite integrals. Evaluation of definite integrals, determining areas of the regions bounded by simple curves in standard form.

UNIT 9 : DIFFERENTIAL EQUATION : Ordinary differential equations, their order, and degree, the solution of differential equation by the method of separation of variables, solution of a homegeneous and linear differential equation of the type

$\frac{dy}{dx} + p(x)y = q(x)$

UNIT 10 : CO-ORDINATE GEOMETRY : Cartesian system of rectangular coordinates in a plane, distance formula, sections formula, locus, and its equation, the slope of a line, parallel and perpendicular lines, intercepts of a line on the co-ordinate axis.

Straight line : Various forms of equations of a line, intersection of lines, angles between two lines, conditions for concurrence of three lines, the distance of a point form a line, co-ordinate of the centroid orthocentre, and circumcentre of a triangle.

Circle, conic sections : A standard form of equations of a circle, the general form of the equation of a circle, its radius and central, equation of a circle when the endpoints of a diameter are given, points of intersection of a line and a circle with the centre at the origin and sections of conics, equations of conic sections (parabola, ellipse, and hyperbola) in standard forms.

UNIT 11 : THREE DIMENSIONAL GEOMETRY : Coordinates of a point in space, the distance between two points, section formula, directions ratios, and direction consines, and the angle between two intersecting lines. Skew lines, the shortest distance between them, and its equation. Equations of a line

UNIT 12: VECTOR ALGEBRA: Vectors and scalars, the addition of vectors, components of a vector in two dimensions and three-dimensional space, scalar and vector products.

UNIT 13: STATISTICS AND PROBABILITY: Measures of discretion; calculation of mean, median, mode of grouped and ungrouped data calculation of standard deviation, variance, and mean deviation for grouped and ungrouped data.

Probability: Probability of an event, addition and multiplication theorems of probability, Baye's theorem, probability distribution of a random variate.

UNIT 14: TRIGONOMETRY : Trigonometrical identities and trigonometrical funtions, inverse trigonometrical functions, and their properties.

All India Engineering Entrance Examination & JEE-Main Previous Years Papers Analysis Chart

Sl No	Exam	Proposed Year		Total Question
Joint Entrance Examination (JEE) Main				
1.	NTA JEE Main (April Session)	April 2024	24 Paper	720
2.	NTA JEE Main (January Session)	January 2024	20 Paper	600
3.	NTA JEE Main	15.04.2023	Shift-I	30
4.	NTA JEE Main	13.04.2023	Shift-I	30
5.	NTA JEE Main	13.04.2023	Shift-II	30
6.	NTA JEE Main	12.04.2023	Shift-I	30
7.	NTA JEE Main	11.04.2023	Shift-I	30
8.	NTA JEE Main	11.04.2023	Shift-II	30
9.	NTA JEE Main	10.04.2023	Shift-I	30
10.	NTA JEE Main	10.04.2023	Shift-II	30
11.	NTA JEE Main	08.04.2023	Shift-I	30
12.	NTA JEE Main	08.04.2023	Shift-II	30
13.	NTA JEE Main	06.04.2023	Shift-I	30
14.	NTA JEE Main	06.04.2023	Shift-II	30
15.	NTA JEE Main	01.02.2023	Shift-I	30
16.	NTA JEE Main	01.02.2023	Shift-II	30
17.	NTA JEE Main	24.01.2023	Shift-I	30
18.	NTA JEE Main	24.01.2023	Shift-II	30
19.	NTA JEE Main	25.01.2023	Shift-I	30
20.	NTA JEE Main	25.01.2023	Shift-II	30
21.	NTA JEE Main	29.01.2023	Shift-I	30
22.	NTA JEE Main	29.01.2023	Shift-II	30
23.	NTA JEE Main	30.01.2023	Shift-I	30
24.	NTA JEE Main	30.01.2023	Shift-II	30
25.	NTA JEE Main	31.01.2023	Shift-I	30
26.	NTA JEE Main	31.01.2023	Shift-II	30
27.	NTA JEE Main	29.07.2022	Shift-I	30
28.	NTA JEE Main	29.07.2022	Shift-II	30
29.	NTA JEE Main	28.07.2022	Shift-I	30
30.	NTA JEE Main	28.07.2022	Shift-II	30
31.	NTA JEE Main	27.07.2022	Shift-I	30
32.	NTA JEE Main	27.07.2022	Shift-II	30
33.	NTA JEE Main	26.07.2022	Shift-I	30
34.	NTA JEE Main	26.07.2022	Shift-II	30
35.	NTA JEE Main	25.07.2022	Shift-I	30
36.	NTA JEE Main	25.07.2022	Shift-II	30
37.	NTA JEE Main	29.06.2022	Shift-I	30
38.	NTA JEE Main	29.06.2022	Shift-II	30
39.	NTA JEE Main	28.06.2022	Shift-I	30
40.	NTA JEE Main	28.06.2022	Shift-II	30
41.	NTA JEE Main	27.06.2022	Shift-I	30
42.	NTA JEE Main	27.06.2022	Shift-II	30
43.	NTA JEE Main	26.06.2022	Shift-I	30
44.	NTA JEE Main	26.06.2022	Shift-II	30
45.	NTA JEE Main	25.06.2022	Shift-I	30
46.	NTA JEE Main	25.06.2022	Shift-II	30
47.	NTA JEE Main	24.06.2022	Shift-I	30
48.	NTA JEE Main	24.06.2022	Shift-II	30

49.	NTA JEE Main	01.09.2021	Shift-I	30
50.	NTA JEE Main	01.09.2021	Shift-II	30
51.	NTA JEE Main	31.08.2021	Shift-I	30
52.	NTA JEE Main	31.08.2021	Shift-II	30
53.	NTA JEE Main	27.08.2021	Shift-I	30
54.	NTA JEE Main	27.08.2021	Shift-II	30
55.	NTA JEE Main	26.08.2021	Shift-I	30
56.	NTA JEE Main	26.08.2021	Shift-II	30
57.	NTA JEE Main	27.07.2021	Shift-I	30
58.	NTA JEE Main	27.07.2021	Shift-II	30
59.	NTA JEE Main	25.07.2021	Shift-I	30
60.	NTA JEE Main	25.07.2021	Shift-II	30
61.	NTA JEE Main	22.07.2021	Shift-I	30
62.	NTA JEE Main	22.07.2021	Shift-II	30
63.	NTA JEE Main	20.07.2021	Shift-I	30
64.	NTA JEE Main	20.07.2021	Shift-II	30
65.	NTA JEE Main	18.03.2021	Shift-I	30
66.	NTA JEE Main	18.03.2021	Shift-II	30
67.	NTA JEE Main	17.03.2021	Shift-I	30
68.	NTA JEE Main	17.03.2021	Shift-II	30
69.	NTA JEE Main	16.03.2021	Shift-I	30
70.	NTA JEE Main	16.03.2021	Shift-II	30
71.	NTA JEE Main	26.02.2021	Shift-I	30
72.	NTA JEE Main	26.02.2021	Shift-II	30
73.	NTA JEE Main	25.02.2021	Shift-I	30
74.	NTA JEE Main	25.02.2021	Shift-II	30
75.	NTA JEE Main	24.02.2021	Shift-I	30
76.	NTA JEE Main	24.02.2021	Shift-II	30
77.	NTA JEE Main	06.09.2020	Shift-I	30
78.	NTA JEE Main	06.09.2020	Shift-II	30
79.	NTA JEE Main	05.09.2020	Shift-I	30
80.	NTA JEE Main	05.09.2020	Shift-II	30
81.	NTA JEE Main	04.09.2020	Shift-I	25
82.	NTA JEE Main	04.09.2020	Shift-II	25
83.	NTA JEE Main	03.09.2020	Shift-I	30
84.	NTA JEE Main	03.09.2020	Shift-II	30
85.	NTA JEE Main	02.09.2020	Shift-I	25
86.	NTA JEE Main	02.09.2020	Shift-II	25
87.	NTA JEE Main	09.01.2020	Shift-I	30
88.	NTA JEE Main	09.01.2020	Shift-II	30
89.	NTA JEE Main	08.01.2020	Shift-I	30
90.	NTA JEE Main	08.01.2020	Shift-II	30
91.	NTA JEE Main	07.01.2020	Shift-I	30
92.	NTA JEE Main	07.01.2020	Shift-II	30
93.	NTA JEE Main	12.04.2019	Shift-I	30
94.	NTA JEE Main	12.04.2019	Shift-II	30
95.	NTA JEE Main	10.04.2019	Shift-I	30
96.	NTA JEE Main	10.04.2019	Shift-II	30
97.	NTA JEE Main	09.04.2019	Shift-I	30
98.	NTA JEE Main	09.04.2019	Shift-II	30
99.	NTA JEE Main	08.04.2019	Shift-I	30
100.	NTA JEE Main	08.04.2019	Shift-II	30
101.	NTA JEE Main	12.01.2019	Shift-I	30
102.	NTA JEE Main	12.01.2019	Shift-II	30
103.	NTA JEE Main	11.01.2019	Shift-I	30

104.	NTA JEE Main	11.01.2019	Shift-II	30
105.	NTA JEE Main	10.01.2019	Shift-I	30
106.	NTA JEE Main	10.01.2019	Shift-II	30
107.	NTA JEE Main	09.01.2019	Shift-I	30
108.	NTA JEE Main	09.01.2019	Shift-II	30
109.	JEE Main	16.04.2018		30
110.	JEE Main	15.04.2018	Shift-I	30
111.	JEE Main	15.04.2018	Shift-II	30
112.	JEE Main	08.04.2018		30
113.	JEE Main	09.04.2017		30
114.	JEE Main	08.04.2017		30
115.	JEE Main	02.04.2017		30
116.	JEE Main	2016		30
117.	JEE Main	2015		30
118.	JEE Main	2014		30
119.	JEE Main	2013		30
120.	AIEEE	2012		30
121.	AIEEE	2011		30
122.	AIEEE	2010		30
123.	AIEEE	2009		30
124.	AIEEE	2008		30
	AIEEE	2007		30
125.	AIEEE	2006		30
126.	AIEEE	2005		30
127.	AIEEE	2004		30
128.	AIEEE	2003		30
129.	AIEEE	2002		30
ASSAM-CEE				
130.	ASSAM-CEE	2023		40
131.	ASSAM-CEE	2022		40
132.	ASSAM-CEE	2021		40
133.	ASSAM-CEE	2020		40
134.	ASSAM-CEE	2019		40
135.	ASSAM-CEE	2018		40
Andhra Pradesh EAMCET/EAPCET				
136.	A.P. EAPCET	15.05.2023	Shift-I	80
137.	A.P. EAPCET	15.05.2023	Shift-II	80
138.	A.P. EAPCET	16.05.2023	Shift-I	80
139.	A.P. EAPCET	16.05.2023	Shift-II	80
140.	A.P. EAPCET	17.05.2023	Shift-I	80
141.	A.P. EAPCET	17.05.2023	Shift-II	80
142.	A.P. EAPCET	18.05.2023	Shift-I	80
143.	A.P. EAPCET	18.05.2023	Shift-II	80
144.	A.P. EAPCET	19.05.2023	Shift-I	80
145.	A.P. EAMCET	04.07.2022	Shift-I	80
146.	A.P. EAMCET	04.07.2022	Shift-II	80
147.	A.P. EAMCET	05.07.2022	Shift-I	80
148.	A.P. EAMCET	05.07.2022	Shift-II	80
149.	A.P. EAMCET	06.07.2022	Shift-I	80
150.	A.P. EAMCET	06.07.2022	Shift-II	80
151.	A.P. EAMCET	07.07.2022	Shift-I	80
152.	A.P. EAMCET	07.07.2022	Shift-II	80
153.	A.P. EAMCET	08.07.2022	Shift-I	80
154.	A.P. EAMCET	08.07.2022	Shift-II	80
155.	A.P. EAMCET	07.09.2021	Shift-I	80

156.	A.P. EAMCET	23.08.2021	Shift-I	80
157.	A.P. EAMCET	23.08.2021	Shift-II	80
158.	A.P. EAMCET	19.08.2021	Shift-II	80
159.	A.P. EAMCET	20.08.2021	Shift-I	80
160.	A.P. EAMCET	20.08.2021	Shift-II	80
161.	A.P. EAMCET	19.08.2021	Shift-I	80
162.	A.P. EAMCET	19.08.2021	Shift-II	80
163.	A.P. EAMCET	05.10.2021	Shift-II	80
164.	A.P. EAMCET	25.08.2021	Shift-I	80
165.	A.P. EAMCET	25.08.2021	Shift-II	80
166.	A.P. EAMCET	24.08.2021	Shift-I	80
167.	A.P. EAMCET	24.08.2021	Shift-II	80
168.	A.P. EAMCET	22.09.2020	Shift-I	80
169.	A.P. EAMCET	22.09.2020	Shift-II	80
170.	A.P. EAMCET	23.09.2020	Shift-I	80
171.	A.P. EAMCET	21.09.2020	Shift-I	80
172.	A.P. EAMCET	21.09.2020	Shift-II	80
173.	A.P. EAMCET	18.09.2020	Shift-I	80
174.	A.P. EAMCET	18.09.2020	Shift-II	80
175.	A.P. EAMCET	17.09.2020	Shift-I	80
176.	A.P. EAMCET	17.09.2020	Shift-II	80
177.	A.P. EAMCET	07.10.2020	Shift-I	80
178.	A.P. EAMCET	20.04.2019	Shift-I	80
179.	A.P. EAMCET	20.04.2019	Shift-II	80
180.	A.P. EAMCET	21.04.2019	Shift-I	80
181.	A.P. EAMCET	21.04.2019	Shift-II	80
182.	A.P. EAMCET	22.04.2019	Shift-I	80
183.	A.P. EAMCET	22.04.2019	Shift-II	80
184.	A.P. EAMCET	23.04.2019	Shift-I	80
185.	A.P. EAMCET	22.04.2018	Shift-I	80
186.	A.P. EAMCET	22.04.2018	Shift-II	80
187.	A.P. EAMCET	23.04.2018	Shift-I	80
188.	A.P. EAMCET	23.04.2018	Shift-II	80
189.	A.P. EAMCET	24.04.2018	Shift-I	80
190.	A.P. EAMCET	24.04.2018	Shift-II	80
191.	A.P. EAMCET	2017		80
192.	A.P. EAMCET	2016		80
193.	A.P. EAMCET	2015		80
194.	A.P. EAMCET	2014		80
195.	A.P. EAMCET	2013		80
196.	A.P. EAMCET	2012		80
197.	A.P. EAMCET	2011		80
198.	A.P. EAMCET	2010		80
199.	A.P. EAMCET	2009		80
200.	A.P. EAMCET	2008		80
201.	A.P. EAMCET	2007		80
202.	A.P. EAMCET	2006		80
203.	A.P. EAMCET	2005		80
204.	A.P. EAMCET	2004		80
205.	A.P. EAMCET	2003		80
206.	A.P. EAMCET	2002		80
207.	A.P. EAMCET	2001		80
208.	A.P. EAMCET	2000		80
209.	A.P. EAMCET	1999		80
210.	A.P. EAMCET	1998		80

211.	A.P. EAMCET	1997		80
212.	A.P. EAMCET	1996		80
213.	A.P. EAMCET	1995		80
214.	A.P. EAMCET	1994		80
215.	A.P. EAMCET	1993		80
216.	A.P. EAMCET	1992		80
217.	A.P. EAMCET	1991		80
AMU (Aligarh Muslim University)				
218.	AMU	2023		50
219.	AMU	2022		50
220.	AMU	2021		50
221.	AMU	2019		50
222.	AMU	2018		50
223.	AMU	2017		50
224.	AMU	2016		50
225.	AMU	2015		50
226.	AMU	2014		50
227.	AMU	2013		50
228.	AMU	2012		50
229.	AMU	2011		50
230.	AMU	2010		70
231.	AMU	2009		70
232.	AMU	2008		70
233.	AMU	2007		70
234.	AMU	2006		70
235.	AMU	2005		70
236.	AMU	2004		70
237.	AMU	2003		70
238.	AMU	2002		100
239.	AMU	2001		100
(Bihar) BCECE				
240.	BCECE	2018		50
241.	BCECE	2017		50
242.	BCECE	2016		50
243.	BCECE	2015		50
244.	BCECE	2014		50
245.	BCECE	2013		50
246.	BCECE	2012		50
247.	BCECE	2011		50
248.	BCECE	2010		50
249.	BCECE	2009		50
250.	BCECE	2008		50
251.	BCECE	2007		50
252.	BCECE	2006		50
253.	BCECE	2005		50
254.	BCECE	2004		50
255.	BCECE	2003		50
BITSAT				
256.	BITSAT	2023		40
257.	BITSAT	2022		40
258.	BITSAT	2021		40
259.	BITSAT	2019		40
260.	BITSAT	2018		40
261.	BITSAT	2017		40
262.	BITSAT	2016		40

263.	BITSAT	2015		40
264.	BITSAT	2014		40
265.	BITSAT	2013		40
266.	BITSAT	2012		40
267.	BITSAT	2011		40
268.	BITSAT	2010		40
269.	BITSAT	2009		40
270.	BITSAT	2008		40
271.	BITSAT	2007		40
272.	BITSAT	2006		40
273.	BITSAT	2005		40
Chhattisgarh-PET				
274.	Chhattisgarh-PET	2023		100
275.	Chhattisgarh-PET	2022		100
276.	Chhattisgarh-PET	2021		100
277.	Chhattisgarh-PET	2020		100
278.	Chhattisgarh-PET	2019		100
279.	Chhattisgarh-PET	2018		100
280.	Chhattisgarh-PET	2017		100
281.	Chhattisgarh-PET	2016		100
282.	Chhattisgarh-PET	2015		100
283.	Chhattisgarh-PET	2014		100
284.	Chhattisgarh-PET	2013		100
285.	Chhattisgarh-PET	2012		100
286.	Chhattisgarh-PET	2011		100
287.	Chhattisgarh-PET	2010		100
288.	Chhattisgarh-PET	2009		100
289.	Chhattisgarh-PET	2008		100
290.	Chhattisgarh-PET	2007		100
291.	Chhattisgarh-PET	2006		100
292.	Chhattisgarh-PET	2005		100
293.	Chhattisgarh-PET	2004		100
COMEDK				
294.	COMEDK-JEE	2023		60
295.	COMEDK-JEE	2022		60
296.	COMEDK-JEE	2021		60
297.	COMEDK-JEE	2020		60
298.	COMEDK-JEE	2019		60
299.	COMEDK-JEE	2018		60
300.	COMEDK-JEE	2017		60
301.	COMEDK-JEE	2016		60
302.	COMEDK-JEE	2015		60
303.	COMEDK-JEE	2014		60
304.	COMEDK-JEE	2013		60
305.	COMEDK-JEE	2012		60
306.	COMEDK-JEE	2011		60
Gujarat Common Entrance Test (GUJCET)				
307.	GUJCET	2023		40
308.	GUJCET	2022		40
309.	GUJCET	2021		40
310.	GUJCET	2020		40
311.	GUJCET	2019		40
312.	GUJCET	2018		40
313.	GUJCET	2017		40
314.	GUJCET	2016		40

315.	GUJCET	2015		40
316.	GUJCET	2014		40
317.	GUJCET	2011		40
318.	GUJCET	2010		40
319.	GUJCET	2009		40
320.	GUJCET	2008		40
321.	GUJCET	2007		40
HIMACHAL PRADESH-CET				
322.	HP-CET	2018		60
J & K-CET				
323.	J & K-CET	2020		75
324.	J & K-CET	2019		75
325.	J & K-CET	2018		75
326.	J & K-CET	2017		75
327.	J & K-CET	2016		75
328.	J & K-CET	2015		75
329.	J & K-CET	2014		75
330.	J & K-CET	2013		75
331.	J & K-CET	2012		75
332.	J & K-CET	2011		75
333.	J & K-CET	2010		75
334.	J & K-CET	2009		75
335.	J & K-CET	2008		75
336.	J & K-CET	2007		75
337.	J & K-CET	2006		75
338.	J & K-CET	2005		75
339.	J & K-CET	2004		75
340.	J & K-CET	2003		75
Jharkhand (JCECE)				
341.	JCECE	2019		50
342.	JCECE	2018		50
343.	JCECE	2017		50
344.	JCECE	2016		50
345.	JCECE	2015		50
346.	JCECE	2014		50
347.	JCECE	2013		50
348.	JCECE	2012		50
349.	JCECE	2011		50
350.	JCECE	2010		50
351.	JCECE	2009		50
352.	JCECE	2008		50
353.	JCECE	2007		50
354.	JCECE	2006		50
355.	JCECE	2005		50
356.	JCECE	2004		50
357.	JCECE	2003		50
358.	JCECE	2002		50
359.	JCECE	2001		50
Jamia Millia Islamia				
360.	Jamia Millia Islamia	2015		60
361.	Jamia Millia Islamia	2014		60
362.	Jamia Millia Islamia	2013		60
363.	Jamia Millia Islamia	2012		60
364.	Jamia Millia Islamia	2011		60
365.	Jamia Millia Islamia	2010		60

366.	Jamia Millia Islamia	2009		60
367.	Jamia Millia Islamia	2008		60
368.	Jamia Millia Islamia	2007		60
369.	Jamia Millia Islamia	2006		60
370.	Jamia Millia Islamia	2005		60
371.	Jamia Millia Islamia	2004		60
Kerala-KEAM				
372.	Kerala KEAM	2023		60
373.	Kerala KEAM	2022		60
374.	Kerala KEAM	2021		60
375.	Kerala KEAM	2020		60
376.	Kerala KEAM	2019		60
377.	Kerala KEAM	2018		60
378.	Kerala KEAM	2017		60
379.	Kerala KEAM	2016		60
380.	Kerala KEAM	2015		60
381.	Kerala KEAM	2014		60
382.	Kerala KEAM	2013		60
383.	Kerala KEAM	2012		60
384.	Kerala KEAM	2011		60
385.	Kerala KEAM	2010		60
386.	Kerala KEAM	2009		60
387.	Kerala KEAM	2008		60
388.	Kerala KEAM	2007		60
389.	Kerala KEAM	2006		60
390.	Kerala KEAM	2005		60
391.	Kerala KEAM	2004		60
Karnataka-CET (KCET)				
392.	Karnataka-CET	2023		60
393.	Karnataka-CET	2022		60
394.	Karnataka-CET	2021		60
395.	Karnataka-CET	2020		60
396.	Karnataka-CET	2019		60
397.	Karnataka-CET	2018		60
398.	Karnataka-CET	2017		60
399.	Karnataka-CET	2016		60
400.	Karnataka-CET	2015		60
401.	Karnataka-CET	2014		60
402.	Karnataka-CET	2013		60
403.	Karnataka-CET	2012		60
404.	Karnataka-CET	2011		60
405.	Karnataka-CET	2010		60
406.	Karnataka-CET	2009		60
407.	Karnataka-CET	2008		60
408.	Karnataka-CET	2007		60
409.	Karnataka-CET	2006		60
410.	Karnataka-CET	2005		60
411.	Karnataka-CET	2004		60
412.	Karnataka-CET	2003		60
413.	Karnataka-CET	2002		60
414.	Karnataka-CET	2001		60
415.	Karnataka-CET	2000		60
Kishore Vaigyanik Protsahan Yojana (KVPY)				
416.	KVPY-SB-SX	2023		15
417.	KVPY-SB-SX	2022		15

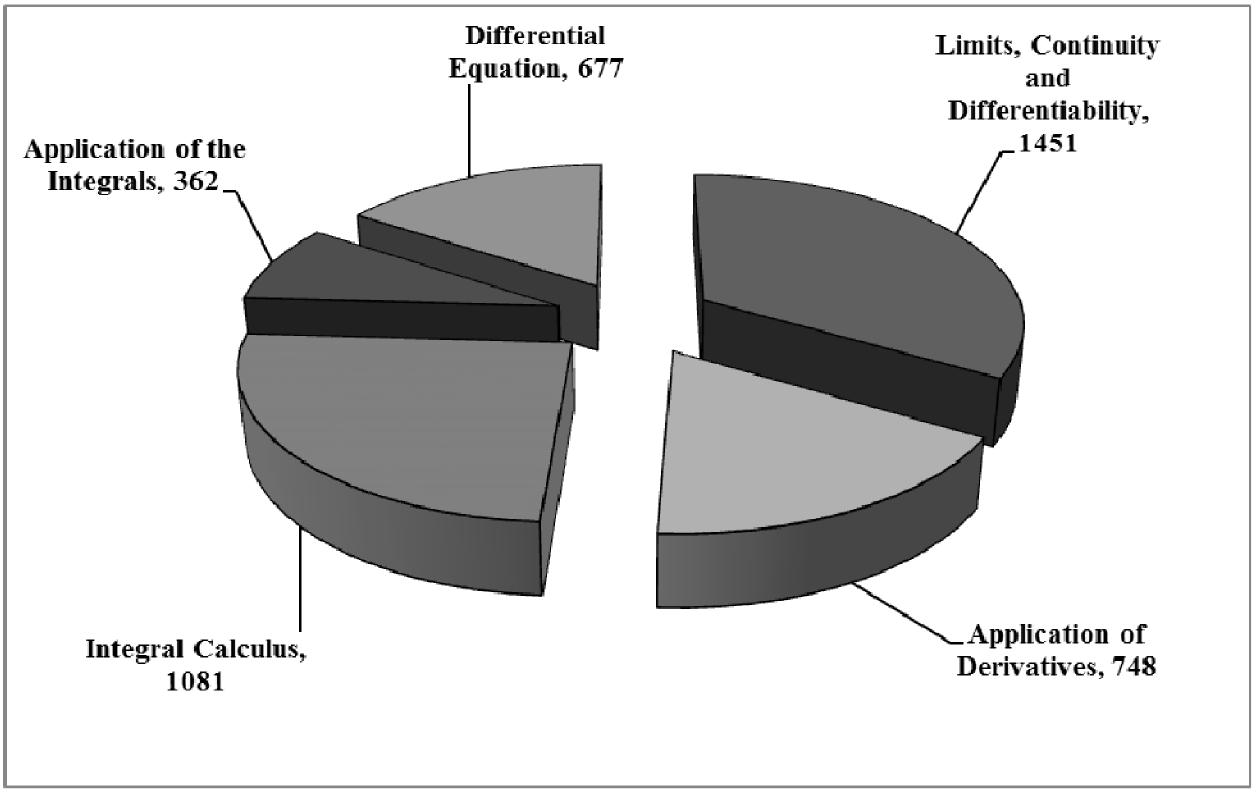
418.	KVPY-SB-SX	2021		15
419.	KVPY-SA	2021		15
420.	KVPY-SA	2020		15
421.	KVPY-SB-SX	2018		15
422.	KVPY-SA	2017		15
423.	KVPY-SB-SX	2016		15
424.	KVPY-SB-SX	2015		15
425.	KVPY-SA	2014		15
426.	KVPY-SB-SX	2013		15
427.	KVPY-SA	2012		15
428.	KVPY-SA	2009		15
429.	KVPY-SB-SX	2009		15
Madhya Pradesh Pre Engineering Test (MPPET)				
430.	MPPET	2013		50
431.	MPPET	2012		50
432.	MPPET	2009		50
433.	MPPET	2008		50
Manipal-UGET				
434.	Manipal	2023		50
435.	Manipal	2022		50
436.	Manipal	2021		50
437.	Manipal	2020		50
438.	Manipal	2019		50
439.	Manipal	2018		50
440.	Manipal	2017		50
441.	Manipal	2016		50
442.	Manipal	2015		50
443.	Manipal	2014		50
444.	Manipal	2013		50
445.	Manipal	2012		50
446.	Manipal	2011		50
447.	Manipal	2010		50
448.	Manipal	2009		50
449.	Manipal	2008		50
(Maharashtra) MHT-CET				
450.	MHT-CET	2022	All Shifts	500
451.	MHT-CET	2021	All Shifts	500
452.	MHT-CET	13.10.2020	Shift-I	100
453.	MHT-CET	13.10.2020	Shift-II	100
454.	MHT-CET	14.10.2020	Shift-I	100
455.	MHT-CET	14.10.2020	Shift-II	100
456.	MHT-CET	15.10.2020	Shift-I	100
457.	MHT-CET	15.10.2020	Shift-II	100
458.	MHT-CET	16.10.2020	Shift-I	100
459.	MHT-CET	16.10.2020	Shift-II	100
460.	MHT-CET	19.10.2020	Shift-I	100
461.	MHT-CET	19.10.2020	Shift-II	100
462.	MHT-CET	20.10.2020	Shift-I	100
463.	MHT-CET	20.10.2020	Shift-II	100
464.	MHT-CET	02.05.2019	Shift-I	100
465.	MHT-CET	02.05.2019	Shift-II	100

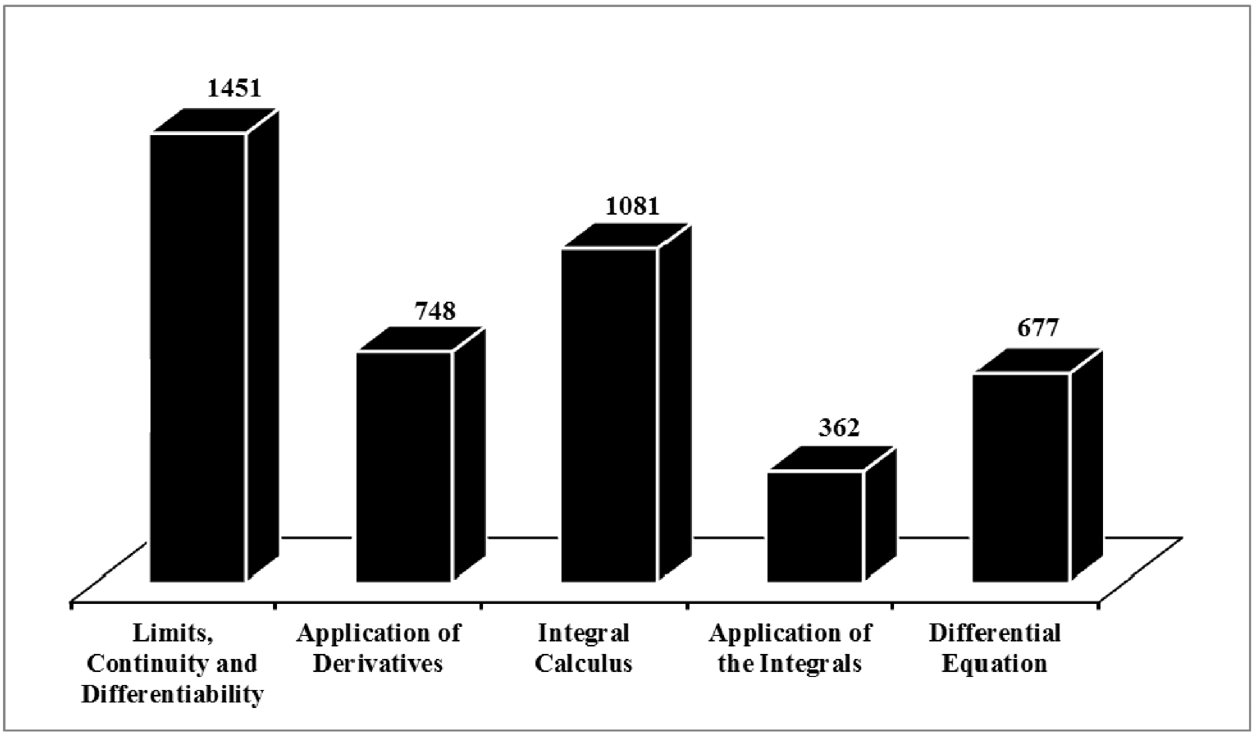
466.	MHT-CET	03.05.2019		100
467.	MHT-CET	2018		100
468.	MHT-CET	2017		100
469.	MHT-CET	2016		100
470.	MHT-CET	2015		100
471.	MHT-CET	2014		100
472.	MHT-CET	2013		100
473.	MHT-CET	2012		100
474.	MHT-CET	2011		100
475.	MHT-CET	2010		100
476.	MHT-CET	2009		100
477.	MHT-CET	2008		100
478.	MHT-CET	2007		100
479.	MHT-CET	2006		100
480.	MHT-CET	2005		100
481.	MHT-CET	2004		100
Rajasthan PET				
482.	Rajasthan PET	2012		40
483.	Rajasthan PET	2011		40
484.	Rajasthan PET	2010		40
485.	Rajasthan PET	2009		40
486.	Rajasthan PET	2008		40
487.	Rajasthan PET	2007		40
488.	Rajasthan PET	2006		40
489.	Rajasthan PET	2005		40
490.	Rajasthan PET	2004		40
491.	Rajasthan PET	2003		40
492.	Rajasthan PET	2002		40
493.	Rajasthan PET	2001		40
SCRA				
494.	SCRA	2015		60
495.	SCRA	2014		60
496.	SCRA	2013		60
497.	SCRA	2012		60
498.	SCRA	2010		60
499.	SCRA	2009		60
SRM-JEEE				
500.	SRM-JEEE	2022		40
501.	SRM-JEEE	2021		40
502.	SRM-JEEE	2020		40
503.	SRM-JEEE	2019		40
504.	SRM-JEEE	2018		40
505.	SRM-JEEE	2016		40
506.	SRM-JEEE	2015		40
507.	SRM-JEEE	2014		40
508.	SRM-JEEE	2013		40
509.	SRM-JEEE	2012		40
510.	SRM-JEEE	2011		40
511.	SRM-JEEE	2010		40
512.	SRM-JEEE	2009		40
513.	SRM-JEEE	2008		40

514.	SRM-JEEE	2007		40
Telangana EAMCET				
515.	TS-EAMCET	12.05.2023	Shift-I	80
516.	TS-EAMCET	12.05.2023	Shift-II	80
517.	TS-EAMCET	13.05.2023	Shift-I	80
518.	TS-EAMCET	13.05.2023	Shift-II	80
519.	TS-EAMCET	14.05.2023	Shift-I	80
520.	TS-EAMCET	14.05.2023	Shift-II	80
521.	TS-EAMCET	18.07.2022	Shift-I	80
522.	TS-EAMCET	18.07.2022	Shift-II	80
523.	TS-EAMCET	19.07.2022	Shift-I	80
524.	TS-EAMCET	19.07.2022	Shift-II	80
525.	TS-EAMCET	20.07.2022	Shift-I	80
526.	TS-EAMCET	20.07.2022	Shift-II	80
527.	TS-EAMCET	06.08.2021	Shift-I	80
528.	TS-EAMCET	06.08.2021	Shift-II	80
529.	TS-EAMCET	05.08.2021	Shift-I	80
530.	TS-EAMCET	05.08.2021	Shift-II	80
531.	TS-EAMCET	04.08.2021	Shift-I	80
532.	TS-EAMCET	04.08.2021	Shift-II	80
533.	TS-EAMCET	09.09.2020	Shift-I	80
534.	TS-EAMCET	09.09.2020	Shift-II	80
535.	TS-EAMCET	10.09.2020	Shift-I	80
536.	TS-EAMCET	10.09.2020	Shift-II	80
537.	TS-EAMCET	11.09.2020	Shift-I	80
538.	TS-EAMCET	11.09.2020	Shift-II	80
539.	TS-EAMCET	14.09.2020	Shift-I	80
540.	TS-EAMCET	14.09.2020	Shift-II	80
541.	TS-EAMCET	03.05.2019	Shift-I	80
542.	TS-EAMCET	03.05.2019	Shift-II	80
543.	TS-EAMCET	04.05.2019	Shift-I	80
544.	TS-EAMCET	04.05.2019	Shift-II	80
545.	TS-EAMCET	06.05.2019	Shift-I	80
546.	TS-EAMCET	05.05.2018	Shift-I	80
547.	TS-EAMCET	05.05.2018	Shift-II	80
548.	TS-EAMCET	02.05.2018	Shift-I	80
549.	TS-EAMCET	04.05.2018	Shift-II	80
550.	TS-EAMCET	07.05.2018	Shift-I	80
551.	TS-EAMCET	24.04.2017	Shift-I	80
552.	TS-EAMCET	2016		80
553.	TS-EAMCET	2015		80
554.	TS-EAMCET	2014		80
Tripura JEE				
555.	Tripura JEE	2023		50
556.	Tripura JEE	2022		50
557.	Tripura JEE	2021		50
558.	Tripura JEE	2019		50
(Uttar Pradesh) UPTU/UPSEE				
559.	UPTU/UPSEE	2020		50
560.	UPTU/UPSEE	2019		50
561.	UPTU/UPSEE	2018		50

562.	UPTU/UPSEE	2017		50
563.	UPTU/UPSEE	2016		50
564.	UPTU/UPSEE	2015		50
565.	UPTU/UPSEE	2014		50
566.	UPTU/UPSEE	2013		50
567.	UPTU/UPSEE	2012		50
568.	UPTU/UPSEE	2011		50
569.	UPTU/UPSEE	2010		50
570.	UPTU/UPSEE	2009		50
571.	UPTU/UPSEE	2008		50
572.	UPTU/UPSEE	2007		50
573.	UPTU/UPSEE	2006		50
574.	UPTU/UPSEE	2005		50
575.	UPTU/UPSEE	2004		50
VITEEE				
576.	VITEEE	2023		40
577.	VITEEE	2022		40
578.	VITEEE	2021		40
579.	VITEEE	2020		40
580.	VITEEE	2019		40
581.	VITEEE	2018		40
582.	VITEEE	2017		40
583.	VITEEE	2016		40
584.	VITEEE	2015		40
585.	VITEEE	2014		40
586.	VITEEE	2013		40
587.	VITEEE	2012		40
588.	VITEEE	2011		40
589.	VITEEE	2010		40
590.	VITEEE	2009		40
591.	VITEEE	2008		40
592.	VITEEE	2007		40
593.	VITEEE	2006		40
WEST BENGAL				
594.	West Bengal	2023		30
595.	West Bengal	2022		30
596.	West Bengal	2021		30
597.	West Bengal	2020		30
598.	West Bengal	2019		30
599.	West Bengal	2018		30
600.	West Bengal	2017		30
601.	West Bengal	2016		30
602.	West Bengal	2015		30
603.	West Bengal	2014		30
604.	West Bengal	2013		30
605.	West Bengal	2012		30
606.	West Bengal	2011		30
607.	West Bengal	2010		30
608.	West Bengal	2009		30
609.	West Bengal	2008		30
			Total	**36020**

Trend Analysis of previous year paper of IIT JEE Mathematics through Bar graph and Pie chart.





01. Limits, Continuity and Differentiability

A. Limits of Standard, Indeterminate form and Limit using Expansion and L′ Hospital's Rule

1. Let a > 0 be a root of the equation $2x^2 + x - 2 = 0$.

If $\lim\limits_{x\to\frac{1}{a}} \dfrac{16\left(1-\cos\left(2+x-2x^2\right)\right)}{\left(1-ax^2\right)} = \alpha + \beta\sqrt{17}$,

Where $\alpha, \beta \in Z$ then $\alpha + \beta$ is equal to ______.

JEE MAIN-05.04.2024, Shift-II

Ans. (170) :

Root of equation $2x^2 + x - 2 = 0$ $\nearrow a = \dfrac{-1+\sqrt{17}}{4}$, $\searrow b = \dfrac{-1-\sqrt{17}}{4}$

Then, root of equation $2x^2 - x - 2 = 0$ $\nearrow \dfrac{1}{a}$, $\searrow \dfrac{1}{b}$

$$\lim_{x\to\frac{1}{a}} 16.\frac{\left(1-\cos 2\left(x-\frac{1}{a}\right)\left(x-\frac{1}{b}\right)\right)}{4\left(x-\frac{1}{b}\right)^2} \times \frac{4\left(x-\frac{1}{b}\right)^2}{a^2\left(x-\frac{1}{a}\right)^2}$$

$$\lim_{x\to\frac{1}{a}} 16.\frac{\left(1-\cos 2\left(x-\frac{1}{a}\right)\left(x-\frac{1}{b}\right)\right)}{4\left(x-\frac{1}{b}\right)^2} \times \frac{4\left(x-\frac{1}{b}\right)^2}{a^2\left(x-\frac{1}{a}\right)^2}$$

$$= 16 \times \frac{2}{a^2}\left(\frac{1}{a}-\frac{1}{b}\right)^2$$

$$= \frac{32}{a^2}\left(\frac{17}{4}\right) = \frac{17.8}{a^2} = \frac{17\times 8\times 16}{\left(-1+\sqrt{17}\right)^2}$$

$$= \frac{136}{256}\left(18+2\sqrt{7}\right).16$$

$$= 153 + 17\sqrt{17}$$

Now comparing given equation, we get–

$$= \alpha + \beta\sqrt{17}$$

$$\alpha = 153 \text{ and } \beta = 17$$

Hence, $\alpha + \beta = 153 + 17 = 170$

2. If the function $f(x) = \dfrac{\sin 3x + \alpha \sin x - \beta \cos 3x}{x^3}$, $x \in R$, is continuous at x = 0, then f(0) is equal to:

(a) 2
(b) –2
(c) 4
(d) –4

JEE MAIN-05.04.2024, Shift-I

Ans. (d) : Given function,

$$f(x) = \frac{\sin 3x + \alpha \sin x - \beta \cos 3x}{x^3}$$

Function f (x) is continuous at x = 0

Then,

$$\lim_{x\to 0} f(x) = f(0) \quad [\because \text{ f(x)continuous at x = 0}]$$

$$= \lim_{x\to 0} \frac{\sin 3x + \alpha \sin x - \beta \sin 3x}{x^3}$$

For limit are exist when $\beta = 0$

$$= \lim_{x\to 0} \frac{\sin 3x + \alpha \sin x}{x^3}$$

$$= \lim_{x\to 0} \frac{3\sin x - 4\sin^3 x + \alpha \sin x}{x^3}$$

$$= \lim_{x\to 0} \frac{(3+\alpha)\sin x - 4\sin^3 x}{x^3}$$

For the finite solution,

$3 + \alpha = 0$

$\alpha = -3$

$$= \lim_{x\to 0} \frac{-4\sin^3 x}{x^3}$$

$=$ We know that $-\lim\limits_{x\to 0} \dfrac{\sin x}{x} = 1$

$= -4 \times 1 = -4$

3. If $\lim\limits_{x\to 1} \dfrac{(5x+1)^{1/3} - (x+5)^{1/3}}{(2x+3)^{1/2} - (x+4)^{1/2}} = \dfrac{m\sqrt{5}}{n(2n)^{2/3}}$, where gcd(m, n) = 1, then 8m + 12n is equal to____

JEE MAIN-04.04.2024, Shift-I

Ans. = 100 :

Since, the limit is in 0/0 form.

So, using L -Hospital rule-

$$\lim_{x\to 1} \frac{\frac{1}{3}(5x+1)^{-2/3}5 - \frac{1}{3}(x+5)^{-2/3}}{\frac{1}{2}(2x+3)^{-1/2}.2 - \frac{1}{2}(x+4)^{-1/2}}$$

$$\lim_{x\to1}\frac{\frac{1}{3}(5x+1)^{\frac{-2}{3}}\times5-\frac{1}{3}(x+5)^{\frac{-2}{3}}}{2\times\frac{1}{2}(2x+3)^{\frac{-1}{2}}-\frac{1}{2}(x+4)^{\frac{-1}{2}}}$$

$$\Rightarrow \frac{8}{3}\frac{\sqrt{5}}{(\alpha\times3)^{2/3}}=\frac{m(\sqrt{5})}{n(2n)^{2/3}}$$

Thus, m= 8, n= 3

$\therefore$ 8m + 12n = 100

4. Let $\lim_{n\to\infty}\left(\frac{n}{\sqrt{n^4+1}}-\frac{2n}{(n^2+1)\sqrt{n^4+1}}+\frac{n}{\sqrt{n^4+16}}-\frac{8n}{(n^2+4)\sqrt{n^4+16}}+......+\frac{n}{\sqrt{n^4+n^4}}-\frac{2n.n^2}{\sqrt{n^4+n^4}}\right)$ be $\frac{\pi}{k}$, using only the principal values of the inverse trigonometric functions. Then k^2 is equal to ______.

JEE MAIN-09.04.2024, Shift-I

Ans. (32) : $\sum_{r=1}^{\infty}\frac{n}{\sqrt{n^4+r^4}}-\frac{2nr^2}{(n^2+r^2)\sqrt{n^4+r^4}}$

$$\sum_{r=1}^{\infty}\frac{\frac{1}{n}}{\sqrt{1+\left(\frac{r}{n}\right)^4}}-\frac{2\left(\frac{1}{n}\right)\left(\frac{r}{n}\right)^2}{\left(1+\left(\frac{r}{n}\right)^2\right)\sqrt{1+\left(\frac{r}{n}\right)^4}}$$

$$\Rightarrow \int_0^1\frac{dx}{\sqrt{1+x^4}}-\frac{2x^2dx}{(1+x^2)\sqrt{1+x^4}}$$

$$\Rightarrow \int_0^1\frac{1-x^2}{(1+x^2)\sqrt{1+x^4}}dx$$

$$\Rightarrow \int_0^1\frac{\frac{1}{x^2}-1}{\left(x+\frac{1}{x}\right)\sqrt{\left(x+\frac{1}{x}\right)^2-2}}dx$$

$$\Rightarrow -\int_0^1\frac{1-\frac{1}{x^2}}{\left(x+\frac{1}{x}\right)\sqrt{\left(x+\frac{1}{x}\right)^2-2}}dx$$

Put, $x+\frac{1}{x}=t\Rightarrow1-\frac{1}{x^2}=dt$

$$\Rightarrow -\int_\infty^2\frac{dt}{t\sqrt{t^2-2}}$$

$$\Rightarrow -\int_\infty^2\frac{tdt}{t^2\sqrt{t^2-2}}$$

Let, $t^2-2=\alpha^2$

$t\,dt=\alpha\,d\alpha$

$$\Rightarrow -\int_\infty^{\sqrt{2}}\frac{\alpha d\alpha}{(\alpha^2+2)\alpha}$$

$$\Rightarrow -\int_\infty^{\sqrt{2}}\frac{d\alpha}{\alpha^2+2}$$

$$\Rightarrow \frac{-1}{\sqrt{2}}\tan^{-1}\frac{\alpha}{\sqrt{2}}\Big]_\infty^{\sqrt{2}}$$

$$\Rightarrow \frac{-1}{\sqrt{2}}\{\tan^{-1}1\}+\frac{1}{\sqrt{2}}\tan^{-1}\infty$$

$$\Rightarrow \frac{1}{\sqrt{2}}\left\{\frac{\pi}{2}-\frac{\pi}{4}\right\}$$

$$\Rightarrow \frac{\pi}{4\sqrt{2}}=\frac{\pi}{k}$$

So, $k=4\sqrt{2}$

$k^2=32$

5. $\lim_{x\to0}\frac{e-(1+2x)^{\frac{1}{2x}}}{x}$ is equal to :

(a) e (b) $\frac{-2}{e}$

(c) 0 (d) $e-e^2$

JEE MAIN-09.04.2024, Shift-II

Ans. (a) : $\lim_{x\to0}\frac{e-(1+2x)^{\frac{1}{2x}}}{x}$ ....(i)

$\lim_{x\to0}(x+1)^{\frac{1}{x}}=e\left[1-\frac{x}{2}+\frac{11}{24}x^2\right]$ putting $x=2x$

$$\lim_{x\to0}(1+2x)^{\frac{1}{2x}}=e\left[1-\frac{2x}{2}+\frac{11}{24}(2x)^2\right]$$

$$\lim_{x\to0}(1+2x)^{\frac{1}{2x}}=e\left[1-x+\frac{11}{24}(2x)^2\right] \quad .... (ii)$$

Value of equation (ii) put in equation (i)

$$\lim_{x\to0}\frac{e-e\left[1-x+\frac{11\times4x^2}{24}\right]}{x}$$

$$\lim_{x\to0}\left(e-\frac{11}{24}x\times e\right)=e$$

6. The value of $\lim_{x\to0}2\left(\frac{1-\cos x\sqrt{\cos2x}\sqrt[3]{\cos3x}....\sqrt[10]{\cos x10x}}{x^2}\right)$ is ________.

JEE MAIN-08.04.2024, Shift-I

Ans. (55) :

$$\lim_{x\to0}2\left(\frac{1-\left(1-\frac{x^2}{2!}\right)\left(1-\frac{4x^2}{2!}\right)\left(1-\frac{9x^2}{2!}\right)....\left(1-\frac{100x^2}{2!}\right)}{x^2}\right)$$

By expansion

$$\lim_{x\to0}2\left(\frac{1-\left(1-\frac{x^2}{2}\right)\left(1-\frac{1}{2}.\frac{4x^2}{2}\right)\left(1-\frac{1}{3}.\frac{9x^2}{2}\right)....\left(1-\frac{1}{10}.\frac{100x^2}{2}\right)}{x^2}\right)$$

$$\lim_{x\to0}2\left(\frac{1-\left(1-\frac{x^2}{2}\right)\left(1-\frac{2x^2}{2}\right)\left(1-\frac{3x^2}{2}\right)....\left(1-\frac{10x^2}{2}\right)}{x^2}\right)$$

$$\lim_{x\to0}\frac{2\left(1-1+x^2\left(\frac{1}{2}+\frac{2}{2}+\frac{3}{2}+...+\frac{10}{2}\right)\right)}{x^2}$$

$$2\left(\frac{1}{2}+\frac{2}{2}+\frac{3}{2}+....+\frac{10}{2}\right)$$

$$1+2+ ....+10 = \frac{10\times11}{2}=55$$

7. Let $f(x)$ $\begin{matrix} x \quad 1, x \text{ is even} \\ 2x, \quad x \text{ is odd,}\end{matrix}$ $x \quad N$. If for some $a \quad N, f(f(f(a))) \quad 21$, then $\lim_{x \quad a} \quad \frac{|x|^3}{a} \quad \frac{x}{a}$,

where [t] denotes the greatest integer less than or equal to t, is equal to:

(a) 169 (b) 121

(c) 225 (d) 144

JEE MAIN-01.02.2024, Shift-II

Ans. (d) :

$$f(x) \quad \begin{matrix} x \quad 1, \quad x \text{ is even} \\ 2x, \quad x \text{ is odd}\end{matrix}$$

$$f\left(f\left(f\left(a\right)\right)\right)=21$$

Case -I If a is even.

$f(a) = a - 1 = \text{odd}$

$f\left(f\left(a\right)\right) = 2\ (a - 1) = \text{even}$

$f\left(f\left(f\left(a\right)\right)\right) = 2a - 3 = 21$

$\Rightarrow 2a = 24$

$a = 12$

Case -II

If a is odd

$f(a) = 2a = \text{even}$

$f\left(f\left(a\right)\right) = 2a - 1 = \text{odd}$

$f\left(f\left(f\left(a\right)\right)\right) = 4a - 2 = 21$

It is not possible.

Hence, a = 12

Now,

$$\lim_{x\to12^-}\left(\frac{|x|^3}{12}-\left[\frac{x}{12}\right]\right)$$

$$\lim_{x\to12^-}\frac{|x|^3}{12}-\lim_{x\to12^-}\left[\frac{x}{12}\right]$$

$= 144 - 0$

$= 144$

8. Let {x} denote the fractional part of x and $f(x) \quad \frac{\cos^{\ 1}(1 \quad \{x\}^2)\sin^{\ 1}(1 \quad \{x\})}{\{x\} \quad \{x\}^3}$, x 0.

If L and R respectively denotes the left hand limit and the right hand limit of f(x) at x = 0, then $\frac{32}{^2}(L^2 \quad R^2)$ is equal to______.

JEE MAIN-01.02.2024, Shift-I

Ans. (18) : $\because$ $\{x\} = x - [x]$

If $x\to 0^+$, $\{x\} = x$

If $x\to 0^-$, $\{x\} = x-(-1) = x+1$

We have,

$$f(x)=\frac{\cos^{-1}(1-\{x\}^2)\sin^{-1}(1-\{x\})}{\{x\}-\{x\}^3}$$

Finding RHL-

$$\lim_{x \quad 0} f(x) \quad \lim_{h \quad 0} f(0 \quad h)$$

$$\lim_{h \quad 0} f(h)$$

$$=\frac{\cos^{-1}(1-\{h\}^2)\sin^{-1}(1-\{h\})}{\{h\}-\{h\}^3}$$

$$=\lim_{h\to0}\frac{\cos^{-1}(1-h^2)\sin^{-1}(1-h)}{h(1-h^2)}$$

$$\lim_{h \quad 0}\frac{\cos^{\ 1}(1 \quad h^2)}{h} \quad \frac{\sin^{\ 1}1}{1}$$

$$=\frac{\pi}{2}\lim_{h\to0}\frac{\cos^{-1}\left(1-h^2\right)}{h}$$

Let,

$$\cos^{\ 1}(1 \quad h^2) \qquad \cos \quad 1 \quad h^2$$

$$\frac{}{2}\lim_{\ 0}\frac{}{\sqrt{1 \quad \cos}}$$

$$\frac{}{2}\lim_{\ 0}\frac{1}{\sqrt{\frac{1 \quad \cos}{2}}}$$

$$\frac{}{2}\frac{1}{\sqrt{1/2}}$$

$$R \quad \frac{}{\sqrt{2}}$$

For LHL -

$$L \quad \lim_{x \quad 0} f(x)$$

$$\lim_{h \quad 0} f(\quad h)$$

$$= \lim_{h\to 0}\frac{\cos^{-1}(1-\{-h\}^2)\sin^{-1}(1-\{-h\})}{\{-h\}-\{-h\}^3}$$

$$= \lim_{h\to 0}\frac{\cos^{-1}(1-(-h+1)^2)\sin^{-1}(1-(-h+1))}{(-h+1)-(-h+1)^3}$$

$$[\because \{-x\} = 1-\{x\}, x \notin I]$$

$$\lim_{h \quad 0}\frac{\cos \ ^1(\quad h^2 \quad 2h)\sin \ ^1 h}{(1 \quad h) \ 1 \quad (1 \quad h)^2}$$

$$\lim_{h \quad 0} \ \overline{\ \ } \ 2 \ \frac{\sin \ ^1 h}{1 \quad (1 \quad h)^2}$$

$$\frac{}{2}\lim_{h \quad 0}\frac{\sin \ ^1 h}{h^2 \quad 2h}$$

$$\frac{}{2}\lim_{h \quad 0}\frac{\sin \ ^1 h}{h} \quad \frac{1}{h \quad 2}$$

$\Rightarrow$ $$L \quad \frac{}{4}$$

$\Rightarrow$ $$\frac{32}{2}(L^2 \quad R^2) \quad \frac{32}{2} \quad \frac{^2}{2} \quad \frac{^2}{16} \quad 16 \quad 2 \quad 18$$

9. Let the slope of the line 45x + 5y + 3 = 0 be $27r_1+\frac{9r_2}{2}$ for some r_1, $r_2 \in R$. Then $\lim_{x\to 3}\left(\int_3^x \frac{8t^2}{\frac{3r_2x}{2}-r_2x^2-r_1x^3-3x}dt\right)$ is equal to _____.

JEE MAIN-29.01.2024, Shift-II

Ans. :(12) Given,

Lines- 45x +5y +3 = 0

Here slope = –9

According to the question,

$$27r_1 \quad \frac{9r_2}{2} \quad 9 \quad 6r_1 \quad r_2 \quad 2$$

Now,

$$= \lim_{x\to 3}\frac{\int_3^x 8t^2 dt}{\frac{3r_2x}{2}-r_2x^2-r_1x^3-3x}$$

$$= \lim_{x\to 3}\frac{8x^2}{\frac{3r_2}{2}-2r_2x-3r_1x^2-3}$$

$$= \frac{72}{\frac{3r_2}{2}-6r_2-27r_1-3}$$

$$= \frac{9\times 8}{\frac{-9r_2}{2}-27r_1-3}$$

$$= \frac{-24}{\frac{3r_2}{2}+9r_1+1}$$

$$= \frac{-24}{3\left(3r_1+\frac{r_2}{2}\right)+1} = \frac{-24}{-3+1} = 12$$

10. $\lim_{x\to\frac{\pi}{2}}\left(\frac{1}{\left(x-\frac{\pi}{2}\right)^2}\int_{x^3}^{\left(\frac{x}{2}\right)^3}\cos\left(t^{\frac{1}{3}}\right)dt\right)$ is equal to

(a) $\frac{3\pi}{8}$ (b) $\frac{3\pi^2}{4}$

(c) $\frac{3\pi}{4}$ (d) $\frac{3\pi^2}{8}$

JEE MAIN-29.01.2024, Shift-I

Ans. (d) : Given,

$$= \lim_{x\to\frac{\pi}{2}}\frac{\int_{x^3}^{\left(\frac{x}{2}\right)^3}\cos\left(t^{\frac{1}{3}}\right)}{\left(x-\frac{\pi}{2}\right)^2} \qquad \left(\because \frac{0}{0}\text{form}\right)$$

Using L' hospital rule, we get-

$$\lim_{x\to\frac{\pi}{2}}\frac{\cos\left(\frac{\pi}{2}\right)\times 0-\cos(x).3x^2}{2\left(x-\frac{\pi}{2}\right)}$$

$$= \lim_{x\to\frac{\pi}{2}}\frac{-\cos x.3x^2}{2\left(x-\frac{\pi}{2}\right)}$$

$$= \lim_{x\to\frac{\pi}{2}}\frac{\sin\left(x-\frac{\pi}{2}\right)}{\left(x-\frac{\pi}{2}\right)}\times\frac{3x^2}{2}$$

$$= 1\times\lim_{x\to\frac{\pi}{2}}\frac{3x^2}{2}$$

$$= \frac{3\pi^2}{8}$$

11. If a $\lim_{x \quad 0}\frac{\sqrt{1 \quad \sqrt{1 \quad x^4}} \quad \sqrt{2}}{x^4}$ and b $\lim_{x \quad 0}\frac{\sin^2 x}{\sqrt{2} \quad \sqrt{1 \quad \cos x}}$, then the value of ab^3 is:

(a) 30 (b) 36

(c) 25 (d) 32

JEE MAIN-27.01.2024, Shift-I

Ans. (d) : Given

$$a=\lim_{x\to0}\frac{\sqrt{1+\sqrt{1+x^4}}-\sqrt{2}}{x^4}\cdot\frac{1}{\sqrt{1+\sqrt{1+x^4}}+\sqrt{2}}$$

$$a\quad \lim_{x\quad 0}\frac{x^4}{x^4}\frac{1}{\sqrt{1\quad x^4}\quad 1}\cdot\frac{1}{\sqrt{1\quad \sqrt{1\quad x^4}}\quad \sqrt{2}}$$

$$\frac{1}{2}\cdot\frac{1}{2\sqrt{2}}\qquad a\qquad \frac{1}{4\sqrt{2}}$$

After rationalization,

$$b=\lim_{x\to0}\frac{\left(\frac{\sin^2 x}{x^2}\right).x^2}{\frac{(2-1-\cos x).x^2}{x^2}}.(\sqrt{2}+\sqrt{1+\cos x})$$

$$b\quad \frac{1}{\frac{1}{2}}.2\sqrt{2}\qquad b\quad 4\sqrt{2}$$

$$\text{Now, } ab^3\quad \frac{1}{4\sqrt{2}}.(4\sqrt{2})^3\quad 32$$

12. If $\lim\limits_{x\to0}\dfrac{3+\alpha\sin x+\beta\cos x+\log_e(1-x)}{3\tan^2 x}=\dfrac{1}{3}$, then $2\alpha-\beta$ is equal to.

(a) 2
(b) 7
(c) 5
(d) 1

JEE MAIN-27.01.2024, Shift-II

Ans. (c) : Given, $\lim\limits_{x\to0}\dfrac{3+\alpha\sin x+\beta\cos x+\log_e(1-x)}{3\tan^2 x}=\dfrac{1}{3}$

By using expansion-

$$\frac{3+\alpha\left[x-\frac{x^3}{3!}+.....\right]+\beta\left[1-\frac{x^2}{2!}+....\right]+\left[-x-\frac{x^2}{2}-....\right]}{3\frac{\tan^2 x}{x^2}\times x^2}=\frac{1}{3}$$

$$\lim_{x\to0}\frac{3+\alpha[x]+\beta\left[1-\frac{x^2}{2}\right]+\left(-x-\frac{x^2}{2}\right)}{3x^2}=\frac{1}{3}$$

$$\Rightarrow\lim_{x\to0}\frac{(3+\beta)+(\alpha-1)x+\left(\frac{-\beta}{2!}-\frac{1}{2}\right)x^2}{3x^2}=\frac{1}{3}$$

$\Rightarrow 3+\beta=0$

$\beta=-3$

$\Rightarrow \alpha-1=0$

$\alpha=1$

Then, $2\alpha-\beta=2+3=5$

13. Let $f:\left[-\frac{\pi}{2},\frac{\pi}{2}\right]\to R$ be a differentiable function such that $f(0)=\frac{1}{2}$. If the $\lim\limits_{x\to0}\dfrac{x\int_0^x f(t)dt}{e^{x^2}-1}=\alpha$, then $8\alpha^2$ is equal to :

(a) 16
(b) 2
(c) 1
(d) 4

JEE MAIN-30.01.2024, Shift-I

Ans. (b) : We have,

$$\lim_{x\quad 0}\frac{x\quad \overset{x}{\underset{0}{}}f(t)dt}{e^{x^2}\quad 1}$$

$$=\lim_{x\to0}\frac{x\int_0^x f(t)dt}{\frac{e^{x^2}-1}{x^2}\times x^2}$$

$$=\lim_{x\to0}\frac{x\int_0^x f(t)dt}{x^2}\qquad\left(\because\frac{0}{0}\text{form}\right)$$

$$\left(\because\frac{e^{x^2}-1}{x^2}=1\right)$$

Applying L' Hospital rule -

$$=\lim_{x\to0}\frac{xf(x)+\int_0^x f(t)dt.1}{2x}$$

$$=\lim_{x\to0}\frac{f(x)}{2}+\lim_{x\to0}\frac{\int_0^x f(t)dt}{2x}$$

$$=\frac{f(0)}{2}+\lim_{x\to0}\frac{f(x)}{2}$$

$$=\frac{f(0)}{2}+\frac{f(0)}{2}$$

$$=f(0)$$

$$=\frac{1}{2}$$

$\therefore\qquad \alpha=\dfrac{1}{2}$

So, $\qquad 8\alpha^2=8\times\dfrac{1}{4}=2$

14. Let a be the sum of all coefficients in the expansion of $(1-2x+2x^2)^{2023}(3-4x^2+2x^3)^{2024}$ and $b=\lim\limits_{x\to0}\left(\dfrac{\int_0^x\frac{\log(1+t)}{t^{2024}+1}dt}{x^2}\right)$. If the equations $cx^2+dx+e=0$ and $2bx^2+ax+4=0$ have a common root, where c, d, e $\in\mathbb{R}$, then d : c : e equals

(a) 4 : 1 : 4
(b) 1 : 1 : 4
(c) 2 : 1 : 4
(d) 1 : 2 : 4

JEE MAIN-31.01.2024, Shift-I

Ans. (b) : We have,

$(1-2x+2x)^{2023}\ (3-4x^2+2x^3)^{2024}$

Put x = 1

For getting sum of all coefficient a = 1

$$b=\lim_{x\to0}\left(\frac{\int_0^x \frac{\ln(1+t)}{t^{2024}+1}dt}{x^2}\right) \qquad \left(\because \frac{0}{0}\text{form}\right)$$

Using L- hospital's rule, we get-

$$=\lim_{x\to0}\frac{\frac{\ln(1+x)}{x^{2024}+1}}{2x}=\frac{1}{2}$$

$cx^2 + dx + e = 0$

$2bx^2 + ax + 4 = 0$

Have a common root

$cx^2 + dx + e = 0$

$x^2 + x + 4 = 0 \rightarrow (D < 0)$

Hence both root are common,

$$\frac{c}{1}=\frac{d}{1}=\frac{e}{4}$$

$c : d : e = 1 : 1 : 4 \Rightarrow d : c : e = 1 : 1 : 4$

15. $\lim\limits_{x\to0}\dfrac{e^{2|\sin x|}-2|\sin x|-1}{x^2}$

(a) is equal to 1 (b) does not exist

(c) is equal to –1 (d) is equal to 2

JEE MAIN-31.01.2024, Shift-I

Ans. (d) : Given,

$$\lim_{x\to0}\frac{e^{2|\sin x|}-2|\sin x|-1}{x^2}$$

It is even function.

So, LHL = RHL.

$$\text{Now, RHL} = \lim_{x\to0}\frac{e^{2\sin x}-2\sin x-1}{x^2}$$

$$=\lim_{x\to0}\frac{e^{2\sin x}.2\cos x-2\cos x}{2x}$$

$$=\lim_{x\to0}\cos x\left(\frac{e^{2\sin x}-1}{2\sin x}\right)\times\frac{2\sin x}{x}$$

$$=1\times1\times2=2$$

16. **If** $\lim\limits_{x\quad 0}\dfrac{ax^2e^x\quad b\log_e(1\quad x)\quad cxe^{\quad x}}{x^2\sin x}\quad 1$**, then** $16(a^2\quad b^2\quad c^2)$ **is equal to –**

JEE MAIN-31.01.2024, Shift-II

Ans. : (81)

$$\lim_{x\quad 0}\frac{ax^2e^x\quad b\log_e(1\quad x)\quad cxe^{\quad x}}{x^2\sin x}\quad 1$$

$$\lim_{x\to0}\frac{ax^2\left(1+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+...\right)-b\left(x-\frac{x^2}{2}+\frac{x^3}{3}-...\right)+cx\left(1-x+\frac{x^2}{2!}-\frac{x^3}{3!}+...\right)}{x^2.x\frac{\sin x}{x}}$$

$$\lim_{x\to0}\frac{(-b+c)x+x^2\left(a+\frac{b}{2}-c\right)+\left(a-\frac{b}{3}+\frac{c}{2}\right)x^3+...}{x^3}=1$$

For limit to be 1, we must have.

$$-b+c=0\ \ \&\ \ \ a+\frac{b}{2}-c=0$$

$$c=b\ \ \ \&\ a-\frac{b}{2}=0 \qquad \{\because\ c=b\}$$

$$\Rightarrow\ a=\frac{b}{2}$$

And,

$$a\quad\frac{b}{3}\quad\frac{c}{2}\quad 1$$

$$\frac{b}{2}\quad\frac{b}{3}\quad\frac{b}{2}\quad 1 \qquad \frac{3b\quad 2b\quad 3b}{6}=1$$

$4b = 6$

$$b=\frac{3}{2}$$

$a = b/2$

$$a=\frac{3}{2\times2}=\frac{3}{4}$$

$$a=\frac{3}{4},\ \ c=b=\frac{3}{2}$$

So,

$$16(a^2\quad b^2\quad c^2)$$

$$=\ 16\ \ \frac{9}{16}\quad\frac{9}{4}\quad\frac{9}{4}$$

$$=16\left(\frac{9+36+36}{16}\right)=81$$

17. $\lim\limits_{x\to0}\left(\left(\dfrac{(1-\cos^2(3x)}{\cos^3(4x)}\right)\left(\dfrac{\sin^3(4x)}{(\log_e(2x+1)^5}\right)\right)$ **is equal to**

(a) 24 (b) 9

(c) 18 (d) 15

JEE MAIN-08.04.2023, Shift-I

Ans. (c) : Given,

$$\lim_{x\to0}\left\{\frac{(1-\cos^2(3x)}{\cos^3(4x)}\right\}\left\{\frac{\sin^3(4x)}{(\log_e(2x+1)^5}\right\}$$

$$\lim_{x\to0}\left\{\frac{\sin^2(3x)}{\cos^3(4x)}\ \frac{\sin^3(4x)}{(\log_e(2x+1)^5}\right\}$$

$$\lim_{x\to0}\frac{\left(\frac{\sin^2 3x}{3x}\right)^2\left(\frac{\sin 4x}{4x}\right)^3(3x)^2(4x)^3}{\cos^3(4x).\left\{\frac{\log(1+2x)}{2x}\right\}^5.(2x)^5}$$

$$\lim_{x\to0}\left\{\frac{(1)^2(1)^1(3x)^2(4x)^3}{1.(2x)^5}\right\}$$

$$\frac{9.64}{32}=18$$

18. Let $S = \left\{z \in C - \{i, 2i\} : \frac{z^2 + 8iz - 15}{z^2 - 3iz - 2} \in R\right\}$ If $\alpha - \frac{13}{11}i \in S$, $a \in R - \{0\}$, then $242\alpha^2$ is equal to ....

JEE MAIN-11.04.2023, Shift-II

Ans. (1680) : $\left(\frac{z^2+8iz-15}{z^2-3iz-2}\right) \in R$

$\Rightarrow 1 + \frac{(11iz-13)}{(z^2-3iz-2)} \in R$

Put $Z = \alpha - \frac{13}{11}i$

$\Rightarrow (z^2 - 3iz - 2)$ is imaginary

Put $z = x + iy$

$\Rightarrow (x^2 - y^2 + 2xyi - 3ix + 3y - 2) \in$ Imaginary

$Re\ (x^2-y^2+3y-2+(2xy-3x)i = 0$

$\Rightarrow x^2-y^2+3y-2 = 0$

$x^2 = y^2 - 3y + 2$

$x^2 = (y-1)(y-2) \ \therefore z = \alpha - \frac{13}{11}i$

Put $x = \alpha$, $y = \frac{-13}{11}$

$\alpha^2 = \left(\frac{-13}{11} - 1\right)\left(\frac{-13}{11} - 2\right)$

$\alpha^2 = \frac{(24 \times 35)}{121}$

$242\alpha^2 = 48 \times 35 = 1680$

19. If $\lim_{x\to 0} \frac{e^{ax} - \cos(bx) - \frac{cxe^{-cx}}{2}}{1 - \cos(2x)} = 17$, then $5a^2 + b^2$ is equal to

(a) 76 (b) 72

(c) 64 (d) 68

JEE MAIN-13.04.2023, Shift-II

Ans. (d) :

$$\lim_{x\to 0} \frac{e^{ax} - \cos bx - \frac{cxe^{-cx}}{2}}{1 - \cos 2x} = 17$$

On expansion-

$$\lim_{x\to 0} \frac{\left(1 + ax + \frac{(ax)^2}{2!} + ...\right) - \left(1 - \frac{(bx)^2}{2!} + ...\right) - \frac{cx}{2}\left(1 - cx + \frac{(cx)^2}{2!}\right)}{\left(\frac{2\sin^2 x}{(x)^2}\right) \times (x)^2}$$

$$\lim_{X\to 0} \frac{x\left(a - \frac{c}{2}\right) + x^2\left(\frac{a^2}{2} + \frac{b^2}{2} + \frac{c^2}{2}\right)}{2x^2} = 17 \quad \left\{\because \frac{\sin x}{x} = 1\right\}$$

For limit to be exist

$a - \frac{c}{2} = 0 \Rightarrow c = 2a$

$$\Rightarrow \frac{\left(\frac{a^2}{2} + \frac{b^2}{2} + \frac{c^2}{2}\right)x^2}{2x^2} = 17$$

$$\Rightarrow \frac{\left(\frac{a^2}{2} + \frac{b^2}{2} + \frac{c^2}{2}\right)}{2} = 17$$

$\Rightarrow \frac{a^2}{2} + \frac{b^2}{2} + \frac{4a^2}{2} = 34$

$\Rightarrow 5a^2 + b^2 = 68$

20. If $\alpha > \beta > 0$ are the roots of the equation $ax^2 + bx + 1 = 0$, and

$\lim_{x\to\frac{1}{\alpha}} \left(\frac{1 - \cos(x^2 + bx + a)}{2(1 - ax)^2}\right)^{\frac{1}{2}} = \frac{1}{k}\left(\frac{1}{\beta} - \frac{1}{\alpha}\right)$, then k is equal to

(a) β (b) 2α

(c) 2β (d) α

JEE MAIN-08.04.2023, Shift-II

Ans. (b) : Given,

$\therefore ax^2 + bx + 1 =$

$a(x - \alpha)(x - \beta)$

$\therefore \alpha\beta = \frac{1}{a}$, $\alpha + \beta = \frac{-b}{a}$

$\therefore x^2 + bx + a = a(1 - \alpha x)(1 - \beta x)$

$$\therefore \lim_{x\to\frac{1}{\alpha}} \left\{\frac{1 - \cos(x^2 + bx + a)}{2(1 - ax)^2}\right\}^{\frac{1}{2}}$$

$$\Rightarrow \lim_{x\to\frac{1}{\alpha}} \left\{\frac{1 - \cos(1 - \alpha x)(1 - \beta x)}{2\{a(1 - \alpha x)(1 - \beta x)\}^2}.a^2(1 - \beta x)^2\right\}^{\frac{1}{2}}$$

$$= \left[\frac{1}{2}.\frac{1}{2}a^2\left(1 - \frac{\beta}{\alpha}\right)^2\right]^{\frac{1}{2}}$$

$$= \frac{1}{2}\frac{1}{\alpha\beta}\left(1 - \frac{\beta}{\alpha}\right) = \frac{1}{2}\left(\frac{1}{\alpha\beta} - \frac{1}{\alpha^2}\right)$$

$$\frac{1}{2\alpha}\left(\frac{1}{\beta} - \frac{1}{\alpha}\right) = \frac{1}{k}\left(\frac{1}{\beta} - \frac{1}{\alpha}\right)$$

$\boxed{k = 2\alpha}$

21. $\max_{0\le x\le\pi} \left\{x - 2\sin x\cos x + \frac{1}{3}\sin 3x\right\} =$

(a) 0 (b) π

(c) $\frac{5\pi + 2 + 3\sqrt{3}}{6}$ (d) $\frac{\pi + 2 - 3\sqrt{3}}{6}$

JEE MAIN-13.04.2023, Shift-I

Ans. (c) :

$$f(x) = x - \sin 2x + \frac{1}{3}\sin 3x$$

$$f'(x) = 1 - 2\cos 2x + \cos 3x = 0$$

$$f'(x) = 4\cos^3 x - 4\cos^2 x - 3\cos x + 3 = 0$$

$\Rightarrow \quad x = \frac{5\pi}{6}, \frac{\pi}{6}$

$\therefore \quad f''(x) = 4\sin 2x - 3\sin 3x$

$\Rightarrow \quad f''\left(\frac{5\pi}{6}\right) < 0$

$\Rightarrow \quad \left(\frac{5\pi}{6}\right)$ is point of local maxima

$\therefore \quad f\left(\frac{5\pi}{6}\right) = \frac{5\pi}{6} + \frac{\sqrt{3}}{2} + \frac{1}{3} = \frac{5\pi + 2 + 3\sqrt{3}}{6}$

22. Among

$(S_1) : \lim_{x\to\infty} \frac{1}{n^2}(2+4+6+....+2n) = 1$

$(S_2) : \lim_{x\to\infty} \frac{1}{n^{16}}(1^{15}+2^{15}+3^{15}+....+n^{15}) = \frac{1}{16}$

(a) Only (S1) is true
(b) Both (S1) and (S2) are true
(c) Both (S1) and (S2) are false
(d) Only (S2) is true

JEE MAIN-13.04.2023, Shift-I

Ans. (b) :

$$S_1 : \lim_{x\to\infty} \frac{n(n+1)}{n^2} = 1 \Rightarrow \text{True}$$

$$S_2 : \lim_{x\to\infty} \frac{1}{n^{16}}\left(\sum r^{15}\right) = \lim_{x\to\infty} \frac{1}{n}\sum\left(\frac{r}{n}\right)^{15}$$

Let, $\frac{r}{n} = x$

$$\frac{1}{n}dr = dx$$

$$= \int_0^1 x^{15}dx = \frac{1}{16} \Rightarrow \text{True}$$

Thus both the statements given above are true.

23. Let $f(x) = \frac{x}{(1+x^n)^{\frac{1}{n}}}, x \in \mathbb{R} - \{-1\}$, $n \in \mathbb{N}$, $n > 2$.

If $f^n(x)$ = n (fofof ….up to n times) (x), then

$\lim_{x\to\infty} \int_0^1 x^{n-2}\left(f^n(x)\right)dx$ is equal to ____:

JEE MAIN-06.04.2023, Shift-II

Ans. (0) : Given,

$$f(x) = \frac{x}{\left(1+x^n\right)^{1/n}}$$

$$x \in R - \{1\},\ n \in N,\ n > 2$$

$$f^n(x) = (\text{fofo......upto n times})(x)$$

Then, $\lim_{n\to\infty} \int_0^1 x^{n-2}\left(f^n(x)\right)dx$

Now, $f(f(x)) = \frac{x}{\left(1+2x^n\right)^{1/n}}$

$$f(f(f(x))) = \frac{x}{\left(1+3x^n\right)^{1/n}}$$

Similarly, $f^n(x) = \frac{x}{\left(1+nx^n\right)^{1/n}}$

Therefore, $\lim_{n\to\infty} \int \frac{x^{n-2}.x}{\left(1+nx^n\right)^{1/n}}dx$

$$\lim_{n\to\infty} \int \frac{x^{n-1}}{\left(1+nx^n\right)^{1/n}}dx$$

Let $\quad 1 + nx^n = t$

$n^2.x^{n-1}dx = dt$

$$x^{n-1}dx = \frac{dt}{n^2}$$

$\therefore \quad \lim_{n\to\infty} \frac{1}{n^2}\int_1^{1+n} \frac{dt}{t^{1/n}}$

$\Rightarrow \quad \lim_{n\to\infty} \frac{1}{n^2}\left[\frac{t^{1-\frac{1}{n}}}{1-\frac{1}{n}}\right]_1^{1+n}$

$\Rightarrow \quad \lim_{n\to\infty} \frac{1}{n(n-1)}\left((1+n)^{\frac{n-1}{n}} - 1\right)$

Again let $n = \frac{1}{h}$

$$\lim_{h\to\infty} \frac{\left(1+\frac{1}{h}\right)^{1-h} - 1}{\frac{1}{h}\left(\frac{1-h}{h}\right)}$$

Using series expansion = 0

24. $\lim_{x\to\infty}\left\{\left(2^{\frac{1}{2}} - 2^{\frac{1}{3}}\right)\left(2^{\frac{1}{2}} - 2^{\frac{1}{5}}\right)....\left(2^{\frac{1}{2}} - 2^{\frac{1}{2n+1}}\right)\right\}$ is equal to

(a) $\frac{1}{\sqrt{2}}$
(b) $\sqrt{2}$
(c) 1
(d) 0

JEE MAIN-06.04.2023, Shift-II

Ans. (d) :

$$\left(2^{\frac{1}{2}}-2^{\frac{1}{3}}\right)^n<\left(2^{\frac{1}{2}}-2^{\frac{1}{3}}\right).....\left(2^{\frac{1}{2}}-2^{\frac{1}{n-1}}\right)<\left(2^{\frac{1}{1}}-2^{\frac{1}{2n+1}}\right)^n$$

$$\lim_{n\to\infty}\left(2^{\frac{1}{2}}-2^{\frac{1}{3}}\right)^n<L<\lim_{n\to\infty}\left(2^{\frac{1}{2}}-2^{\frac{1}{2n-1}}\right)$$

$L=0$

25. Let x = 2 be a root of the equation $x^2 + px + q = 0$ and

$$f(x)=\begin{cases}\dfrac{1-\cos\left(x^2-4px+q^2+8q+16\right)}{(x-2p)^4}, & x\neq 2\\ 0, & x=2p\end{cases}$$

Then $\lim\limits_{x\to 2p^+}[f(x)]$

where [.] denotes greatest integer function, is

(a) –1
(b) 1
(c) 2
(d) 0

JEE MAIN-29.01.2023, Shift-I

Ans. (d) :
Given, Let x = 2 be a root of the equation
$x^2 + px + q = 0$
put x = 2
4 + 2p + q = 0
– 2p = 4 + q

$$f(x)=\begin{cases}\dfrac{1-\cos\left(x^2-4px+q^2+8q+16\right)}{(x-2p)^4}, & x\neq 2\\ 0, & x=2P\end{cases}$$

$$\lim_{x\to 2p^+}\left[\frac{1-\cos\left(x^2-4px+(q+4)^2\right)}{(x-2p)^4}\right]$$

$$\lim_{x\to 2p^+}\left[\frac{1-\cos\left(x^2-4px+4p^2\right)}{(x-2p)^4}\right]$$

$$\lim_{x\to 2p^+}\left[\frac{1-\cos(x-2p)^2}{(x-2p)^4}\right]$$

$$\lim_{x\to 2p^+}\left[\frac{1-\cos(x-2p)^2}{(x-2p)^4}\right]\quad\left\{\lim_{x\to 0}\frac{1-\cos\theta}{\theta^2}=\frac{1}{2}\right.$$

$$\lim_{x\to 2p^+}\left[\frac{1}{2}\right]=0$$

26. $\lim\limits_{t\to 0}\left(1^{\frac{1}{\sin^2 t}}+2^{\frac{1}{\sin^2 t}}+....n^{\frac{1}{\sin^2 t}}\right)^{\sin^2 t}$ is equal to

(a) $\dfrac{n(n+1)}{2}$
(b) n^2
(c) $n^2 + n$
(d) n

JEE MAIN-24.01.2023, Shift-I

Ans. (d) : Given

$$\lim_{t\to 0}\left[1^{\frac{1}{\sin^2 t}}+2^{\frac{1}{\sin^2 t}}+..........+n^{\frac{1}{\sin^2 t}}\right]^{\sin^2 t}$$

$$=\lim_{t\to 0}\left[1^{\text{cosec}^2 t}+2^{\text{cosec}^2 t}+..........+n^{\text{cosec}^2 t}\right]^{\sin^2 t}$$

$$=\lim_{t\to 0}\left(n^{\text{cosec}^2 t}\right)^{\sin^2 t}\left[\left(\frac{1}{n}\right)^{\text{cosec}^2 t}+\left(\frac{2}{n}\right)^{\text{cosec}^2 t}+..........+1\right]^{\sin^2 t}$$

$$=\lim_{t\to 0} n[0+0+..........+1]$$

$= n\times 1$
$= n$

27. The value of

$$\lim_{x\to\infty}\frac{1+2-3+4+5-6+.........+(3n-2)+(3n-1)-3n}{\sqrt{2n^4+42+3}-\sqrt{n^4+5n+4}}$$

is:

(a) $\dfrac{\sqrt{2+1}}{2}$
(b) $3\left(\sqrt{2}+1\right)$
(c) $\dfrac{3}{2\sqrt{6}}$
(d) $\dfrac{3}{2}\left(\sqrt{2}+1\right)$

JEE MAIN-25.01.2023, Shift-I

Ans. (d) : Given,

$$\lim_{x\to\infty}\frac{1+2-3+4+5-6+.....+(3n-2)(3n-1)-3n}{\sqrt{2n^4+4n+3}-\sqrt{n^4+5n+4}}$$

$$\lim_{x\to\infty}\frac{(1+2-3)+(4+5-6)+(7+8-9)+.....+[(3n-2)+(3n-1)-3n]}{\sqrt{2n^4+4n+3}-\sqrt{n^4+5n+4}}$$

$$\lim_{x\to\infty}\frac{0+3+6+9+....3n\text{ terms}}{\sqrt{2n^4+4n+3}-\sqrt{n^4+5n+4}}$$

$$\lim_{x\to\infty}\frac{3n(n-1)/2}{\sqrt{(2n^4+4n+3)}-\sqrt{n^4+5n+4}}$$

$$=\frac{3}{2(\sqrt{2}-1)}=\frac{3}{2}(\sqrt{2}+1)$$

28. The set of all values of a for which $\lim\limits_{x\to a}([x-5]-[2x+2])=0$, where [a] is equal to-

(a) (–7.5, –6.5]
(b) [–7.5, –6.5)
(c) [–7.5, –6.5]
(d) (–7.5, –6.5)

JEE MAIN-24.01.2023, Shift-II

Ans. (d) :

$$\lim_{x\to a}([x-5]-[2x+2])=0$$

$$\lim_{x\to a}\ ([x]-5-[2x]-2)=0$$

$$\lim_{x\to a}([x]-[2x])=7 \qquad ...(i)$$

Let, $a\in\left[n,\ n+\dfrac{1}{2}\right]$,

then $n-2n=7\Rightarrow n=-7$

$\Rightarrow a \in [-7, -6.5]$

Let, $a \in \left[n+\frac{1}{2}, n+1\right]$,

then $n-(2n+1) = 7 \Rightarrow n = -8$

$\Rightarrow a \in [-7.5 - 7)$

but limit doesn't exists at $a = -7.5$

Hence, $a \in (-7.5, 6.5)$

$\Rightarrow [a] - [2a] = 7$ $\{[x + I] = [x] + I\}$

check option,

Let, $a = \frac{-15}{2}$

$\Rightarrow [-7.5] - [-15] = 7$

$\Rightarrow -8 + 15 = 7$

$\Rightarrow 7 = 7$

Thus, option (d) is correct.

29. $\lim\limits_{x\to\infty} \frac{\left(\sqrt{3x+1}+\sqrt{3x-1}\right)^6 + \left(\sqrt{3x+1}-\sqrt{3x-1}\right)^6}{\left(x+\sqrt{x^2-1}\right)^6 + \left(x-\sqrt{x^2-1}\right)^6} x^3$

(a) is equal to 27 (b) is equal to $\frac{27}{2}$

(c) is equal to 9 (d) does not exist

JEE MAIN-31.01.2023, Shift-II

Ans. (a) : Given,

$$\lim_{x\to\infty} \frac{\left(\sqrt{3x+1}+\sqrt{3x-1}\right)^6 + \left(\sqrt{3x+1}-\sqrt{3x-1}\right)^6}{\left(x+\sqrt{x^2-1}\right)^6 + \left(x-\sqrt{x^2-1}\right)^6} x^3$$

$$\lim_{x\to\infty} \frac{x^3.x^3\left[\sqrt{3+\frac{1}{x}}+\sqrt{3-\frac{1}{x}}\right]^6 + \left[\sqrt{3+\frac{1}{x}}-\sqrt{3-\frac{1}{x}}\right]}{x^6\left[\left(1+\sqrt{1-\frac{1}{x^2}}\right)^6 + \left(1-\sqrt{1-\frac{1}{x^2}}\right)^6\right]}$$

$$= \frac{\left[\sqrt{3+0}+\sqrt{3-0}\right]^6 + \left[\sqrt{3+0}-\sqrt{3-0}\right]^6}{\left(1+\sqrt{1-0}\right)^6 + \left(1-\sqrt{1-0}\right)^6}$$

$$= \frac{\left(2\sqrt{3}\right)^6 + 0}{(2)^6 + 0} = \left(\sqrt{3}\right)^6 = 27$$

30. $\lim\limits_{n\to\infty}\left[\frac{1}{1+n}+\frac{1}{2+n}+\frac{1}{3+n}+...+\frac{1}{2n}\right]$ **is equal to**

(a) $\log_e\left(\frac{3}{2}\right)$ (b) $\log_e 2$

(c) $\log_e\left(\frac{2}{3}\right)$ (d) 0

JEE MAIN- 01.02.2023, Shift-I

Ans. (b) : $\lim\limits_{n\to\infty}\left[\frac{1}{1+n}+\frac{1}{2+n}+\frac{1}{3+n}+...+\frac{1}{2n}\right]$

$$= \lim_{n\to\infty}\left[\frac{1}{1+n}+\frac{1}{2+n}+\frac{1}{3+n}+...+\frac{1}{n+n}\right]$$

$$= \lim_{n\to\infty}\frac{1}{n}\left[\frac{1}{1+\frac{1}{n}}+\frac{1}{1+\frac{2}{n}}+\frac{1}{1+\frac{3}{n}}+...+\frac{1}{1+\frac{n}{n}}\right]$$

$$= \lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{n}\left(\frac{1}{1+\frac{r}{n}}\right)$$

Let $\frac{r}{n} = x$

$\frac{1}{n} = dx$

$r = 1$

$x = \lim\limits_{x\to\infty}\frac{1}{n} = 0$

$r = n$

$x = \lim\limits_{x\to\infty}\frac{n}{n} = 1$

$$= \int_0^1 \frac{1}{(1+x)}dx = \left[\log(1+x)\right]_0^1$$

$= \log 2 - \log(1)$

$= \log 2$

Hence option (b) is correct.

31. $\sum\limits_{r=1}^{n}(2r-1) = x$ **, then**

$\lim\limits_{n\to 0}\left[\frac{1^3}{x^2}+\frac{2^3}{x^2}+\frac{3^3}{x^2}+......+\frac{n^3}{x^2}\right] =$

(a) $\frac{1}{2}$ (b) 1 (c) $\frac{1}{4}$ (d) 4

Karnataka CET-2019

Ans. (c) : Given, $\sum\limits_{r=1}^{n}(2r-1) = x$

$2\sum\limits_{r=1}^{n} r - \sum 1 = x$

$2\left[\frac{n(n+1)}{2}\right] - n$

$[n(n+1)] - n = x \Rightarrow n^2 + n - n = x \Rightarrow n^2 = x$

$x^2 = n^4$

$$\lim_{n\to 0}\left[\frac{1^3}{x^2}+\frac{2^3}{x^2}+\frac{3^3}{x^2}+........+\frac{n^3}{x^2}\right]$$

$$= \lim_{n\to 0}\left[\frac{1}{x^2}(1^3+2^3+3^3+.........+n^3)\right]$$

$$= \lim_{n\to 0}\left[\frac{1}{n^4}\times\left\{\frac{n(n+1)}{2}\right\}^2\right] = \frac{1}{4}\lim_{n\to 0}\left[\frac{(n+1)^2}{n^2}\right]$$

$$= \frac{1}{4}\lim_{n\to 0}\left[\frac{n^2+1+2n}{n^2}\right] = \frac{1}{4}\lim_{n\to 0}\left[1+\frac{1}{n^n}+\frac{2}{n}\right]$$

$$= \frac{1}{4}\left[1+\frac{1}{0}+\frac{2}{0}\right] = \frac{1}{4}$$

32. The value of $\lim\limits_{x\to 0}\frac{|x|}{x}$ is

(a) 1 (b) –1 (c) 0 (d) Does not exist

Karnataka CET-2018

Ans. (d) : Given, $\lim\limits_{x\to 0}\frac{|x|}{x}$

$$\text{LHL} = \lim_{h\to 0^-}\frac{|0-h|}{0-h} \Rightarrow \lim_{h\to 0^-}\frac{h}{-h} = -1$$

$$\text{RHL} = \lim_{h\to 0^+}\frac{|0+h|}{0+h} \Rightarrow \lim_{h\to 0^+}\frac{h}{h} = 1$$

So, LHL $\neq$ RHL

$\therefore$ Limit does not exist at x = 0

33. If the function f(x) satisfies $\lim\limits_{x\to 1}\frac{f(x)-2}{x^2-1} = \pi$, then $\lim\limits_{x\to 1} f(x) =$

(a) 1 (b) 2 (c) 0 (d) 3

Karnataka CET-2014

Ans. (b) : Given, $\lim\limits_{x\to 1}\frac{f(x)-2}{x^2-1} = \pi$

$$\frac{\lim\limits_{x\to 1}(f(x)-2)}{\lim\limits_{x\to 1}x^2-1} = \pi$$

$$\lim_{x\to 1}(f(x)-2) = \pi\lim_{x\to 1}(x^2-1)$$

$$\lim_{x\to 1}f(x) - \lim_{x\to 1}2 = \pi(1-1)$$

$$\lim_{x\to 1}f(x) - 2 = 0$$

$$\lim_{x\to 1}f(x) = 2$$

34. $\lim\limits_{x\to 0}\frac{\log_e(1+x)}{3^x-1} =$

(a) $\log_e 3$ (b) 0 (c) $\log_3 e$ (d) 1

Karnataka CET-2013

Ans. (c) : Given, $\lim\limits_{x\to 0}\frac{\log_e(1+x)}{3^x-1}$

$$= \lim_{x\to 0}\frac{\frac{\log_e(1+x)}{x}}{\frac{3^x-1}{x}} = \frac{\lim\limits_{x\to 0}\frac{\log_e(1+x)}{x}}{\lim\limits_{x\to 0}\left(\frac{3^x-1}{x}\right)}$$

($\because$ using formula, $\lim\limits_{x\to 0}\left(\frac{a^x-1}{x}\right) = \log_e a$)

$$= \frac{1}{\log_e 3} = \log_3 e$$

35. $\mathop{\mathrm{Lt}}\limits_{x\to 0}\frac{x\,2^x - x}{1-\cos x} =$

(a) $\frac{1}{2}$ (b) 2log2 (c) log2 (d) $\frac{1}{2}\log 2$

Karnataka CET-2012

Ans. (b) : Given, $\lim\limits_{x\to 0}\frac{x2^x - x}{1-\cos x} = \lim\limits_{x\to 0}\frac{x(2^x-1)}{1-\cos x}$

$$= \lim_{x\to 0}\frac{x^2\left(\frac{2^x-1}{x}\right)}{2\sin^2\left(\frac{x}{2}\right)} \quad \left(\because \cos x = 1-\sin^2\left(\frac{x}{2}\right)\right)$$

$$= \lim_{x\to 0}\frac{x^2\left(\frac{2^x-1}{x}\right)}{\frac{2\sin^2\left(\frac{x}{2}\right)}{\left(\frac{x}{2}\right)^2}\times\left(\frac{x}{2}\right)^2} = \lim_{x\to 0}\frac{2\left(\frac{2^x-1}{x}\right)}{\frac{\sin^2\left(\frac{x}{2}\right)}{\left(\frac{x}{2}\right)^2}}$$

$$\left(\text{using}, \lim_{x\to 0}\frac{a^x-1}{x} = \log a\right)$$

= 2log2

36. $\lim\limits_{x\to\infty}\frac{3.2^{x+1}-4.5^{x+1}}{5.2^x+7.5^x} =$

(a) $\frac{-20}{7}$ (b) 0 (c) $\frac{3}{5}$ (d) $\frac{-4}{7}$

Karnataka CET-2009

Ans. (a) : Given, $\lim\limits_{x\to\infty}\frac{3.2^{x+1}-4.5^{x+1}}{5.2^x+7.5^x}$

$$= \lim_{x\to\infty}\frac{3\times 2\times 2^x - 4\times 5\times 5^x}{5.2^x+7.5^x} = \lim_{x\to\infty}\frac{6.2^x-20.5^x}{5.2^x+7.5^x}$$

$$= \lim_{x\to\infty}\frac{6.\left(\frac{2}{5}\right)^x - 20}{5.\left(\frac{2}{5}\right)^x + 7} = \frac{0-20}{0+7} = -\frac{20}{7}$$

37. $\lim\limits_{n\to\infty}\left\{n\sin\frac{2\pi}{3n}.\cos\frac{2\pi}{3n}\right\} =$

(a) $\frac{2\pi}{3}$ (b) $\frac{\pi}{6}$ (c) $\frac{\pi}{3}$ (d) 1

Karnataka CET-2010

Ans. (a) : Given, $\lim\limits_{n\to\infty}\left\{n.\sin\frac{2\pi}{3n}.\cos\frac{2\pi}{3n}\right\}$

$$= \frac{1}{2}\lim_{n\to\infty}\left\{n.2\sin\frac{2\pi}{3n}.\cos\frac{2\pi}{3n}\right\} = \frac{1}{2}\lim_{n\to\infty}\left\{n.\sin\frac{4\pi}{3n}\right\}$$

$$= \frac{1}{2}\lim_{n\to\infty}\left\{\frac{n.\sin\frac{4\pi}{3n}}{\frac{4\pi}{3n}}\times\frac{4\pi}{3n}\right\} = \frac{4\pi}{6}\lim_{n\to\infty}\left\{\frac{\sin 4\frac{\pi}{3n}}{\frac{4\pi}{3n}}\right\}$$

$$\left[\because \lim_{x\to\infty}\frac{\sin x}{x} = 1\right]$$

$$= \frac{4\pi}{6}\times 1 = \frac{2\pi}{3}$$

38. $\lim\limits_{x\to a}\dfrac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}}=$

(a) $\dfrac{2}{3}$ (b) $\dfrac{2}{\sqrt{3}}$ (c) $\dfrac{3\sqrt{3}}{2}$ (d) $\dfrac{2}{3\sqrt{3}}$

Karnataka CET-2011

Ans. (d) : Given, $\lim\limits_{x\to a}\dfrac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}}=$

$$\lim_{x\to a}\sqrt{a+2x}-\sqrt{3x}\cdot\frac{\sqrt{a+2x}+\sqrt{3x}}{\sqrt{a+2x}+\sqrt{3x}}\cdot\frac{1}{\sqrt{3a+x}-2\sqrt{x}}\cdot\frac{\sqrt{3a+x}+2\sqrt{x}}{\sqrt{3a+x}+2\sqrt{x}}$$

$$=\lim_{x\to a}\frac{[(a+2x)-3x]}{[(3a+x)-4x]}\cdot\frac{\sqrt{3a+x}+2\sqrt{x}}{\sqrt{a+2x}+\sqrt{3x}}$$

$$=\lim_{x\to a}\frac{(a-x)}{3(a-x)}\times\frac{\sqrt{3a+x}+2\sqrt{x}}{\sqrt{a+2x}+\sqrt{3x}}$$

$$=\lim_{x\to a}\frac{1}{3}\frac{\sqrt{3a+x}+2\sqrt{x}}{\sqrt{a+2x}+\sqrt{3x}}$$

$$=\frac{1}{3}\times\frac{\sqrt{3a+a}+2\sqrt{a}}{\sqrt{a+2a}+\sqrt{3a}}=\frac{1}{3}\times\frac{\sqrt{4a}+2\sqrt{a}}{\sqrt{3a}+\sqrt{3a}}$$

$$=\frac{1}{3}\times\frac{2\sqrt{a}+2\sqrt{a}}{2\sqrt{3a}}=\frac{1}{3}\times\frac{4\sqrt{a}}{2\sqrt{3a}}=\frac{2}{3\sqrt{3}}$$

39. $\lim\limits_{x\to\infty}x\sin\left(\dfrac{2}{x}\right)$ is equal to

(a) 2 (b) $\dfrac{1}{2}$ (c) ∞ (d) 0

Karnataka CET-2008

Ans. (a) : Given, $\lim\limits_{x\to\infty}\ x\sin\left(\dfrac{2}{x}\right)$

$$=\lim_{x\to\infty}\frac{x\sin\left(\frac{2}{x}\right)}{\left(\frac{2}{x}\right)}\times\left(\frac{2}{x}\right)\ =2\lim_{x\to\infty}\frac{\sin\left(\frac{2}{x}\right)}{\left(\frac{2}{x}\right)}\ =2\times1=2$$

40. $\lim\limits_{x\to1}\dfrac{\tan(x^2-1)}{x-1}$ is equal to

(a) 2 (b) $\dfrac{1}{2}$ (c) –2 (d) $\dfrac{-1}{2}$

Karnataka CET-2007

Ans. (a) : Given, $\lim\limits_{x\to1}\dfrac{\tan(x^2-1)}{x-1}$

$$=\lim_{x\to1}\frac{\tan(x^2-1)}{x-1}\times\frac{x+1}{x+1}=\lim_{x\to1}\frac{\tan(x^2-1)}{x^2-1}\times(x+1)$$

$$=\lim_{x\to1}\frac{\tan(x^2-1)}{x^2-1}\times\lim_{x\to1}(x+1)=1\times(1+1)=2$$

41. **The value of** $\lim\limits_{x\to0}\dfrac{5^x-5^{-x}}{2x}=$

(a) log5 (b) 0 (c) 1 (d) 2 log5

Karnataka CET-2006

Ans. (a) : Given,

$$\lim_{x\to0}\frac{5^x-5^{-x}}{2x}$$

From L– Hospital's's rule-

$$=\lim_{x\to0}\frac{5^x-5^{-x}}{2x}=\lim_{x\to0}\frac{5^x\log5-5^{-x}\log5\times(-1)}{2}$$

$$=\lim_{x\to0}\frac{5^x\log5+5^{-x}\log5}{2}=\frac{\log5+\log5}{2}=\log5$$

42. $\lim\limits_{x\to0}\dfrac{a^{\sin x}-1}{b^{\sin x}-1}$ **is equal to :**

(a) a/b (b) b/a (c) $\dfrac{\log a}{\log b}$ (d) $\dfrac{\log b}{\log a}$

Karnataka CET-2000

Ans. (c) : Given, $\lim\limits_{x\to0}\dfrac{a^{\sin x}-1}{b^{\sin x}-1}$

Applying L'-Hospital's's rule,

$$=\lim_{x\to0}\frac{a^{\sin x}\log a.\cos x-0}{b^{\sin x}\log b.\cos x-0}$$

$$=\lim_{x\to0}\frac{a^{\sin x}\log a.\ \cos x}{b^{\sin x}\log b.\ \cos x}=\frac{a^0\log a.\ \cos0}{b^0\log b.\ \cos0}=\frac{\log a}{\log b}$$

43. $\lim\limits_{x\to1}\dfrac{1+\log x-x}{1-2x+x^2}=$

(a) 1 (b) –1 (c) zero (d) –1/2

Karnataka CET-2000

Ans. (d) : Given, $\lim\limits_{x\to1}\dfrac{1+\log x-x}{1-2x+x^2}$

Applying L'-Hospital's rule,

$$\lim_{x\to1}\frac{1+\log x-x}{1-2x+x^2}=\lim_{x\to1}\frac{\frac{1}{x}-1}{-2+2x}$$

$$\lim_{x\to1}\frac{(1-x)}{2x(x-1)}=\lim_{x\to1}\frac{-(x-1)}{2x(x-1)}$$

$$\lim_{x\to1}\left(-\frac{1}{2x}\right)=-\frac{1}{2}$$

44. **If** $\lim\limits_{x\to\infty}\left[\dfrac{x^3+1}{x^2+1}-(ax+b)\right]=2$**, then :**

(a) a = 1 and b = 1 (b) a = 1 and b = –1
(c) a = 1 and b = –2 (d) a = 1 and b = 2

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Ans. (c) : Given, $\lim_{x\to\infty}\left[\frac{x^3+1}{x^2+1}-(ax+b)\right]=2$

$$\lim_{x\to\infty}\left[\frac{x^3+1-ax^3-bx^2-ax-b}{x^2+1}\right]=2$$

$$\lim_{x\to\infty}\left[\frac{x^3(1-a)-bx^2-ax-b+1}{x^2+1}\right]=2$$

For the infinite limit exist, the coefficient of x^3 must be zero.

$\therefore$ $1-a=0$

$a=1$

$$\lim_{x\to\infty}\left[\frac{-bx^2-x-b+1}{x^2+1}\right]=2$$

$$\lim_{x\to\infty}\left[\frac{-b\left(x^2+1\right)-x+1}{x^2+1}\right]=2$$

$$\lim_{x\to\infty}\left[\frac{-b(x^2+1)}{x^2+1}-\frac{x}{x^2+1}+\frac{1}{x^2+1}\right]=2$$

$$\lim_{x\to\infty}\left[-b-\frac{x}{x^2+1}+\frac{1}{x^2+1}\right]=2$$

$-b-0+0=2$

$b=-2$

45. If f(x) = $\begin{vmatrix} \sin x & \cos x & \tan x \\ x^3 & x^2 & x \\ 2x & 1 & 1 \end{vmatrix}$, then $\lim_{x\to0}\frac{f(x)}{x^2}$ is :

(a) –1 (b) 3 (c) 1 (d) zero

Karnataka CET-2002

Ans. (c) : Given, f(x) = $\begin{vmatrix} \sin x & \cos x & \tan x \\ x^3 & x^2 & x \\ 2x & 1 & 1 \end{vmatrix}$

$f(x) = \sin x\,(x^2-x) - \cos x.\,(x^3-2x^2) + \tan x\,(x^3-2x^3)$

$f(x) = x\,(x-1)\sin x - x^2\,(x-2)\cos x - x^3\tan x$

$$\frac{f(x)}{x^2}=\frac{(x-1)\sin x}{x}-(x-2)\cos x-x\tan x\,.$$

$$=\sin x-\frac{\sin x}{x}-(x-2)\cos x-x\tan x$$

$$\lim_{x\to0}\frac{f(x)}{x^2}=\lim_{x\to0}\left[\sin x-\frac{\sin x}{x}-(x-2)\cos x-x\tan x\right]$$

$= \sin 0 - 1 - (0-2)\cos 0° - 0 = 0 - 1 + 2$

$$\lim_{x\to0}\frac{f(x)}{x^2}=1$$

46. $\lim_{\theta\to\frac{\pi}{2}}\frac{\frac{\pi}{2}-\theta}{\cot\theta}$

(a) 0 (b) –1 (c) 1 (d) ∞

Karnataka CET-2004

Ans. (c) : Given, $\lim_{\theta\to\frac{\pi}{2}}\frac{\frac{\pi}{2}-\theta}{\cot\theta}$

Applying L'-Hospital's's rule,

$$\lim_{\theta\to\frac{\pi}{2}}\frac{\frac{\pi}{2}-\theta}{\cot\theta}=\lim_{\theta\to\frac{\pi}{2}}\frac{0-1}{-\text{cosec}^2\theta}=\lim_{\theta\to\frac{\pi}{2}}\frac{1}{\text{cosec}^2\theta}$$

$$=\lim_{\theta\to\frac{\pi}{2}}\frac{1}{\left(\text{cosec}\frac{\pi}{2}\right)^2}=\frac{1}{1^2}=1$$

47. Consider the following statements :

Statement 1 : $\lim_{x\to1}\frac{ax^2+bx+c}{cx^2+bx+a}$ is 1

(where $a+b+c\neq0$)

Statement 2 : $\lim_{x\to-2}\frac{\frac{1}{x}+\frac{1}{2}}{x+2}$ is $\frac{1}{4}$

(a) Only statement 2 is true
(b) Only statement 1 is true
(c) Both statements 1 and 2 are true
(d) Both statements 1 and 2 are false

Karnataka CET-2021

Ans. (b) : Statement -I

$$\lim_{x\to1}\frac{ax^2+bx+c}{cx^2+bx+a}=\frac{a\times1^2+b\times1+c}{c\times1^2+b\times1+a}=\frac{a+b+c}{a+b+c}=1$$

Statement-II

$$\lim_{x\to-2}\frac{\frac{1}{x}+\frac{1}{2}}{x+2}=\lim_{x\to-2}\frac{(2+x)}{2x(x+2)}$$

$$=\lim_{x\to-2}\frac{1}{2x}=\frac{1}{2\times(-2)}=-\frac{1}{4}$$

$\lim_{x\to1}\frac{ax^2+bx+c}{cx^2+bx+a}$ is 1 and $\lim_{x\to1-2}\frac{\frac{1}{x}+\frac{1}{2}}{x+2}$ is not $\frac{1}{4}$

So, statement I is correct and statement II is not correct.
Hence option (b) is correct.

48. If

f(x) = $\begin{vmatrix} \cos x & 1 & 0 \\ 0 & 2\cos x & 3 \\ 0 & 1 & 2\cos x \end{vmatrix}$, then $\lim_{x\to\pi}$ f(x) =

(a) –1 (b) 1 (c) 0 (d) 3

Karnataka CET-2021

Ans. (a) : Given, f (x) = $\begin{vmatrix} \cos x & 1 & 0 \\ 0 & 2\cos x & 3 \\ 0 & 1 & 2\cos x \end{vmatrix}$

$f(x) = \cos x(4\cos^2 x - 3)$

$f(x) = 4\cos^3 x - 3\cos x$

$$\lim_{x\to\pi}f(x)=\lim_{x\to\pi}4\cos^3x-3\cos x$$

$= 4(\cos\pi)^3 - 3(-1) = -4 + 3 = -1$

49. $\lim\limits_{n\to\infty}\dfrac{n(2n+1)^2}{(n+2)(n^2+3n-1)}=$

(a) 0 (b) 2 (c) 4 (d) ∞

MHT CET-2021

Ans. (c) :

$$\lim_{n\to\infty}\frac{n(2n+1)^2}{(n+2)(n^2+3n-1)}=\lim_{n\to\infty}\frac{n\left[n\left(2+\frac{1}{n}\right)\right]^2}{\left[n\left(1+\frac{2}{n}\right)\right]\left[n^2\left(1+\frac{3}{n}-\frac{1}{n^2}\right)\right]}$$

$$=\lim_{n\to\infty}\frac{n^3\left(2+\frac{1}{n}\right)^2}{n^3\left(1+\frac{2}{n}\right)\left(1+\frac{3}{n}-\frac{1}{n^2}\right)}$$

$$=\frac{(2+0)^2}{(1+0)(1+0-0)}=\frac{4}{1}=4$$

50. **If $\lim\limits_{x\to2}=\dfrac{x^n-2^n}{x-2}=80$, where n is a positive integer, then n =**

(a) 3 (b) 5 (c) 2 (d) None of these

MHT CET-2021

Ans. (b) : $\lim\limits_{x\to2}\dfrac{x^n-2^n}{x-2}=80$

Using L'-Hospital's rule

$$\lim_{x\to2}\frac{nx^{n-1}-0}{1-0}=80$$

$n\cdot2^{n-1}=80$

$n\cdot2^{n-1}=5\times16$

$n\cdot2^{n-1}=5\times2^4$

Comparing both sides, we get $n=5$

51. $\lim\limits_{x\to1}=\dfrac{x^3-1}{x^2+5x-6}=.....$

(a) 0 (b) $\dfrac{3}{7}$ (c) $\dfrac{1}{2}$ (d) $-\dfrac{1}{6}$

MHT CET-2021

Ans. (b) : Given, $\lim\limits_{x\to1}\dfrac{x^3-1}{x^2+5x-6}$

$$=\lim_{x\to1}\frac{(x-1)(x^2+x+1)}{(x+6)(x-1)}=\frac{1+1+1}{1+6}=\frac{3}{7}$$

52. $\lim\limits_{x\to\frac{\pi}{2}}=\dfrac{1+\cos 2x}{(\pi-2x)^2}=.....$

(a) 1 (b) 2 (c) 3 (d) $\dfrac{1}{2}$

MHT CET-2021

Ans. (d) : Given, $L=\lim\limits_{x\to\frac{\pi}{2}}\dfrac{1+\cos2x}{(\pi-2x)^2}$

Put $x=\dfrac{\pi}{2}+h$ as $x\to\dfrac{\pi}{2},h\to0$

$$\therefore L=\lim_{h\to0}\frac{1+\cos2\left(\frac{\pi}{2}+h\right)}{\left[\pi-2\left(\frac{\pi}{2}+h\right)\right]^2}=\lim_{h\to0}\frac{1+\cos(\pi+2h)}{(-2h)^2}$$

$$L=\lim_{h\to0}\frac{1-\cos 2h}{4h^2}=\lim_{h\to0}\frac{2\sin^2 h}{4h^2}=\frac{1}{2}\lim_{h\to0}\left(\frac{\sin h}{h}\right)^2=\frac{1}{2}(1)^2=\frac{1}{2}$$

53. $\lim\limits_{x\to0}\left(\dfrac{\sin(x+a)+\sin(a-x)-2\sin a}{x\sin x}\right)=$

(a) $\sin a$ (b) $\cos a$ (c) $-\sin a$ (d) $\dfrac{1}{2}\cos a$

MHT CET-2021

Ans. (c) : Given, $\lim\limits_{x\to0}\left[\dfrac{\sin(x+a)+\sin(a-x)-2\sin a}{x\sin x}\right]$

$$\left[\because\ \sin C+\sin D=2\sin\left(\frac{C+D}{2}\right).\cos\left(\frac{C-D}{2}\right)\right]$$

$$=\lim_{x\to0}\left[\frac{2\sin\left(\frac{2a}{2}\right)\cdot\cos\left(\frac{2x}{2}\right)-2\sin a}{x\sin x}\right]$$

$$=\lim_{x\to0}\frac{2\sin a\cdot\cos x-2\sin a}{x\sin x}$$

$$=\lim_{x\to0}\frac{-2\sin a(1-\cos x)}{x\sin x}=\lim_{x\to0}\frac{-2\sin a\left(2\sin^2\frac{x}{2}\right)}{x\sin x}$$

$$=\lim_{x\to0}\frac{(-4\sin a)\left(\frac{\sin^2\frac{x}{2}}{x^2}\right)}{\left(\frac{x\sin x}{x^2}\right)}=\lim_{x\to0}\frac{(-4\sin a)\left(\frac{\sin\frac{x}{2}}{\frac{x}{2}}\times\frac{1}{2}\right)^2}{\frac{\sin x}{x}}$$

$$=\frac{(-4\sin a)\left(1\times\frac{1}{2}\right)^2}{1}=-4\sin a\times\frac{1}{4}=-\sin a$$

54. **The value of $\lim\limits_{x\to2}\dfrac{x^3-8}{x^2-4}$ is**

(a) 3 (b) $\dfrac{3}{2}$ (c) 1 (d) 0

MHT CET-2021

Ans. (a) :

$$\lim_{x\to 2}\frac{x^3-8}{x^2-4}=\lim_{x\to 2}\frac{x^3-2^3}{x^2-2^2}=\lim_{x\to 2}\frac{\left(\frac{x^3-2^3}{x-2}\right)}{\left(\frac{x^2-2^2}{x-2}\right)}$$

$$\left[\because \lim_{x\to a}\frac{x^n-a^n}{x-a}=n(a)^{n-1}\right]$$

As $x\to 2, x\neq 2, x-2\neq 0$

$$=\frac{3(2)^{3-1}}{2(2)^{2-1}}=\frac{3.2^2}{2.2}=3$$

55. $\lim_{x\to 0}\frac{\log_e(1+x)}{3^x-1}=$

(a) $\log_e 3$ (b) 0 (c) 1 (d) $\log_3 e$

MHT CET-2021

Ans. (d) : $\lim_{x\to 0}=\frac{\log_e(1+x)}{3^x-1}$

Applying L'-Hospital's's rule,

$$\lim_{x\to 0}\frac{\frac{1}{1+x}}{3^x\log e(3-0)}=\frac{\frac{1}{1+0}}{3^0\log_e 3}=\frac{1}{\log_e 3}=\log_3 e$$

56. The value of $\lim_{x\to 7}\frac{2-\sqrt{x-3}}{x^2-49}$

(a) $\frac{2}{9}$ (b) $\frac{-2}{49}$ (c) $\frac{1}{56}$ (d) $\frac{-1}{56}$

MHT CET-2021

Ans. (d) : Given,

$$\lim_{x\to 7}\frac{2-\sqrt{x-3}}{x^2-49}=\lim_{x\to 7}\frac{2-\sqrt{x-3}}{(x-7)(x+7)}\times\frac{2+\sqrt{x-3}}{2+\sqrt{x-3}}$$

$$=\lim_{x\to 7}\frac{2^2-(x-3)}{(x-7)(x+7)\left(2+\sqrt{x-3}\right)}$$

$$=\lim_{x\to 7}\frac{(7-x)}{(x-7)(x+7)\left(2+\sqrt{x-3}\right)}=\lim_{x\to 7}\frac{-1}{(x+7)\left(2+\sqrt{x-3}\right)}$$

$$=\frac{-1}{(7+7)\left(2+\sqrt{7-3}\right)}=\frac{-1}{(14)(2+2)}=\frac{-1}{56}$$

57. The value of $\lim_{x\to 0}\frac{2}{x}\log(1+x)$ **is equal to**

(a) e (b) e^2 (c) $\frac{1}{2}$ (d) 2

MHT CET-2021

Ans. (d) : Given, $\lim_{x\to 0}\frac{2}{x}\log(1+x)$

$$=2\lim_{x\to 0}\log(1+x)^{\frac{1}{x}}\qquad\left[\because \lim_{x\to 0}\log(1+x)^{1/x}=\log_e e\right]$$

$$=2\log_e e=2(1)=2$$

58. $\lim_{x\to 0}\frac{x\times 2^x-x}{1-\cos x}=$

(a) 0 (b) log4 (c) log2 (d) None of these

MHT CET-2021

Ans. (b) : Given, $\lim_{x\to 0}\frac{x\cdot 2^x-x}{1-\cos x}$

$$=\lim_{x\to 0}\frac{x\left(2^x-1\right)}{1-\cos x}=\lim_{x\to 0}\frac{x\left(2^x-1\right)}{2\sin^2\frac{x}{2}}$$

$$=\lim_{x\to 0}\frac{\left[\frac{x\left(2^x-1\right)}{x^2}\right]}{\left[\frac{2\sin^2\frac{x}{2}}{x^2}\right]}=\frac{1}{2}\lim_{x\to 0}\frac{\left(\frac{2^x-1}{x}\right)}{\left(\frac{\sin\frac{x}{2}}{\frac{x}{2}}\times\frac{1}{2}\right)^2}$$

$$\left\{\because \lim_{x\to 0}\frac{a^x-1}{x}=\log a\right\}$$

$$=\frac{1}{2}\times\frac{\log 2}{\left(1\times\frac{1}{2}\right)^2}=\frac{1}{2}\times\frac{\log 2}{\frac{1}{4}}=2\log 2=\log 2^2=\log 4$$

59. If a, b, c and d are positive, then $\lim_{x\to\infty}\left(1+\frac{1}{a+bx}\right)^{c+dx}$ **is equal to**

(a) $e^{d/b}$ (b) $e^{c/a}$ (c) $e^{(c+d)/(a+b)}$ (d) e

MHT CET-2011

Ans. (a) : Given, $\lim_{x\to\infty}\left(1+\frac{1}{a+bx}\right)^{c+dx}$

$$=\lim_{x\to\infty}\left[\left(1+\frac{1}{a+bx}\right)^{a+bx}\right]^{\frac{c+dx}{a+bx}}$$

$$\left[\because \lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x=e\right]$$

$$=\left[\lim_{x\to\infty}\left(1+\frac{1}{a+bx}\right)^{a+bx}\right]^{\lim_{x\to\infty}\frac{x\left(\frac{c}{x}+d\right)}{x\left(\frac{a}{x}+b\right)}}=e^{\frac{(0+d)}{((0+b))}}=e^{\frac{d}{b}}$$

60. $\lim_{x\to 0}\frac{15^x-5^x-3^x+1}{1-\cos 2x}=$

(a) $\frac{1}{2}\log(15)$ (b) $\frac{1}{2}(\log 5)(\log 3)$

(c) $\frac{1}{2}\log(8)$ (d) $(\log 5)(\log 3)$

MHT CET-2009

Ans. (b) : Given,

$$\lim_{x\to 0}\frac{15^x-5^x-3^x+1}{1-\cos 2x}$$

$$=\lim_{x\to 0}\frac{5^x.3^x-5^x-3^x+1}{1-\cos 2x}=\lim_{x\to 0}\frac{5^x\left(3^x-1\right)-1\left(3^x-1\right)}{2\sin^2 x}$$

$$=\lim_{x\to 0}\frac{\left(5^x-1\right)\left(3^x-1\right)}{2\sin^2 x}=\lim_{x\to 0}\frac{\left[\frac{\left(5^x-1\right)\left(3^x-1\right)}{x^2}\right]}{\left(\frac{2\sin^2 x}{x^2}\right)}$$

$$=\lim_{x\to 0}\frac{\left(\frac{5^x-1}{x}\right)\left(\frac{3^x-1}{x}\right)}{2\left(\frac{\sin x}{x}\right)^2}=\frac{(\log 5)(\log 3)}{2(1)^2}=\frac{1}{2}(\log 5)(\log 3)$$

61. $\lim\limits_{x\to 1}\left[\log(ex)\right]^{\frac{1}{\log x}}=$

(a) $1-e$ (b) e^2 (c) e (d) 0

MHT CET-2009

Ans. (c) : Given, $\lim\limits_{x\to 1}\left[\log(ex)\right]^{\frac{1}{\log x}}$

$$=\lim_{x\to 1}\left[\log e+\log x\right]^{\frac{1}{\log x}}=\lim_{x\to 1}\left(1+\log x\right)^{\frac{1}{\log x}}$$

$$=\lim_{\log x\to 0}\left(1+\log x\right)^{\frac{1}{\log x}}\quad .....\text{as } x\to 1, \log x\to 0$$

$= e$

62. $\lim\limits_{n\to\infty}\frac{1+2+3+...+n}{3n^2}=?$

(a) $\frac{1}{6}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) $\frac{1}{4}$

MHT CET-2007

Ans. (a) : Given, $\lim\limits_{n\to\infty}\frac{1+2+3+...+n}{3n^2}$

$$=\lim_{n\to\infty}\frac{\left[\frac{n(n+1)}{2}\right]}{3n^2}=\lim_{n\to\infty}\frac{n(n+1)}{2\left(3n^2\right)}$$

$$=\lim_{n\to\infty}\frac{(n+1)}{6n}=\frac{1}{6}\lim_{n\to\infty}\left(1+\frac{1}{n}\right)=\frac{1}{6}(1+0)=\frac{1}{6}$$

63. $\lim\limits_{x\to 0}e^{\log x}$

(a) 1 (b) ∞ (c) 0 (d) Not defined

MHT CET-2007

Ans. (c) : Given, $\lim\limits_{x\to 0}e^{\log x}=\lim\limits_{x\to 0}x=0$

64. $\lim\limits_{x\to 0}\left(\frac{1+2x}{1-2x}\right)^{1/x}=$

(a) e (b) e^2 (c) e^3 (d) e^4

MHT CET-2007

Ans. (d) : Given, $\lim\limits_{x\to 0}\left(\frac{1+2x}{1-2x}\right)^{1/x}$

$$=\lim_{x\to 0}\frac{(1+2x)^{\frac{1}{x}}}{(1-2x)^{\frac{1}{x}}}=\lim_{x\to 0}\frac{\left[(1+2x)^{\frac{1}{2x}}\right]^2}{\left[(1-2x)^{-\frac{1}{2x}}\right]^{-2}}=\frac{e^2}{e^{-2}}=e^4$$

65. $\lim\limits_{x\to 0}\frac{\sqrt{a+x}-\sqrt{a-x}}{4x}=$

(a) $\frac{1}{4a}$ (b) $\frac{1}{4\sqrt{a}}$ (c) $\frac{1}{2\sqrt{a}}$ (d) None of these

MHT CET-2005

Ans. (b) : Given, $\lim\limits_{x\to 0}\frac{\sqrt{a+x}-\sqrt{a-x}}{4x}$

$$=\lim_{x\to 0}\frac{\sqrt{a+x}-\sqrt{a-x}}{4x}\times\frac{\sqrt{a+x}+\sqrt{a-x}}{\sqrt{a+x}+\sqrt{a-x}}$$

$$=\lim_{x\to 0}\frac{(a+x)-(a-x)}{4x\left(\sqrt{a+x}+\sqrt{a-x}\right)}=\lim_{x\to 0}\frac{2x}{4x\left(\sqrt{a+x}+\sqrt{a-x}\right)}$$

$$=\lim_{x\to 0}\frac{1}{2\left(\sqrt{a+x}+\sqrt{a-x}\right)}=\lim_{x\to 0}\frac{1}{2\left(\sqrt{a+x}+\sqrt{a-x}\right)}$$

$$=\frac{1}{2\left(\sqrt{a+0}+\sqrt{a-0}\right)}=\frac{1}{2\left(2\sqrt{a}\right)}=\frac{1}{4\sqrt{a}}$$

66. $\lim\limits_{x\to 0}\frac{8\sin x-x\cos x}{3\tan x+x^2}$

(a) $\frac{7}{3}$ (b) $\frac{7}{4}$ (c) $\frac{8}{3}$ (d) $\frac{8}{4}$

MHT CET-2005

Ans. (a) : $\lim\limits_{x\to 0}\frac{8\sin x-x\cos x}{3\tan x+x^2}$

Dividing Numerator and Denominator by x

$$=\lim_{x\to 0}\frac{\left(\frac{8\sin x-x\cos x}{x}\right)}{\left(\frac{3\tan x+x^2}{x}\right)}$$

$$=\lim_{x\to 0}\frac{8\left(\frac{\sin x}{x}\right)-\cos x}{3\left(\frac{\tan x}{x}\right)+x}=\frac{8\times 1-\cos 0}{3\times 1+0}=\frac{8(1)-1}{3(1)+0}=\frac{7}{3}$$

67. $\lim\limits_{x\to 2}\frac{x^5-32}{x^3-8}=$

(a) $\frac{20}{3}$ (b) $\frac{1}{5}$ (c) 10 (d) $\frac{14}{5}$

MHT CET-2006

Ans. (a) : Given,

$$\lim_{x\to 2}\frac{x^5-32}{x^3-8}=\lim_{x\to 2}\frac{x^5-2^5}{x^3-2^3}=\lim_{x\to 2}\frac{\left(\frac{x^5-2^5}{x-2}\right)}{\left(\frac{x^3-2^3}{x-2}\right)}$$

$(x\to 2, x\neq 2 \therefore x-2\neq 0)$

$$=\frac{\lim_{x\to 2}\frac{x^5-2^5}{x-2}}{\lim_{x\to 2}\frac{x^3-2^3}{x-2}}=\frac{5(2)^4}{3(2)^2}=\frac{5(2)^2}{3}=\frac{20}{3}$$

68. $\lim_{x\to\frac{\pi}{6}}\frac{\cot^2 x-3}{\operatorname{cosec} x-2}=$

(a) 2 (b) 3 (c) 4 (d) 5

MHT CET-2006

Ans. (c) : Given, $\lim_{x\to\frac{\pi}{6}}\frac{\cot^2 x-3}{\operatorname{cosec} x-2}=\lim_{x\to\frac{\pi}{6}}\frac{(\operatorname{cosec}^2 x-1-3)}{\operatorname{cosec} x-2}$

$$=\lim_{x\to\frac{\pi}{6}}\frac{\operatorname{cosec}^2 x-4}{\operatorname{cosec} x-2}$$

$$=\lim_{x\to\frac{\pi}{6}}\frac{(\operatorname{cosec} x-2)(\operatorname{cosec} x+2)}{(\operatorname{cosec} x-2)}$$

$$=\operatorname{cosec}\frac{\pi}{6}+2=2+2=4$$

69. $\lim_{x\to 0}\frac{a^x+a^{-x}-2}{x^2}$ **equals**

(a) $2\log a$ (b) $(\log a)^2$ (c) $\log a^2$ (d) $\log 2a$

MHT CET-2006, 2004

Ans. (b) : Given,

$$\lim_{x\to 0}\frac{a^x+a^{-x}-2}{x^2}=\lim_{x\to 0}\frac{a^x+\frac{1}{a^x}-2}{x^2}=\lim_{x\to 0}\frac{(a^x)^2+1-2a^x}{a^x.x^2}$$

$$=\lim_{x\to 0}\frac{(a^x-1)^2}{a^x.x^2}=\lim_{x\to 0}\frac{1}{a^x}\left(\frac{a^x-1}{x}\right)^2=\frac{1}{a^0}(\log a)^2=(\log a)^2$$

70. $\lim_{x\to\infty} x^{1/x}=$

(a) 1 (b) ∞ (c) 0 (d) none of these

COMEDK-2011

Ans. (a) : Let, $y=\lim_{x\to\infty}x^{1/x}$ ....(i)

Taking log in equation (i) on both side, we get –

$$\log\ y=\lim_{x\to\infty}\frac{1}{x}\log x$$

$$\log\ y=\lim_{x\to\infty}\frac{\log x}{x}$$

Applying L'-Hospital's's Rule, we get log y =

$$\log y=\lim_{x\to\infty}\frac{\frac{1}{x}}{1}=\lim_{x\to\infty}\frac{1}{x}$$

$$\log y=\frac{1}{\infty}$$

$\log y=0$

$y=e^0$

$y=1$

71. $\lim_{x\to\infty}\left(\frac{x^3}{3x^2-4}-\frac{x^2}{3x+2}\right)$ **is equal to**

(a) $\frac{-1}{4}$ (b) $\frac{-1}{2}$ (c) 0 (d) $\frac{2}{9}$

COMEDK-2020, VITEEE-2017

Ans. (d) : Given,

$$\lim_{x\to\infty}\left(\frac{x^3}{3x^2-4}-\frac{x^2}{3x+2}\right)$$

$$=\lim_{x\to\infty}\left(\frac{x^3(3x+2)-x^2(3x^2-4)}{(3x^2-4)(3x+2)}\right)$$

$$=\lim_{x\to\infty}\left(\frac{3x^4+2x^3-3x^4+4x^2}{9x^3+6x^2-12x-8}\right)$$

$$=\lim_{x\to\infty}\left(\frac{2x^3+4x^2}{9x^3+6x^2-12x-8}\right)$$

$$=\lim_{x\to\infty}\left(\frac{2+\frac{4}{x}}{9+\frac{6}{x}-\frac{12}{x^2}-\frac{8}{x^3}}\right)=\frac{2+0}{9+0-0-0}=\frac{2}{9}$$

72. $\underset{x\to\infty}{\text{Lt}}\left(\frac{x^2+5x+3}{x^2+x+3}\right)^{1/x}$ **is**

(a) e^4 (b) e^2 (c) e^3 (d) 1

SRM JEEE-2019

Ans. (d) : Given, $\lim_{x\to\infty}\left(\frac{x^2+5x+3}{x^2+x+3}\right)^{1/x}=\lim_{x\to\infty}\left(\frac{\frac{x^2+5x+3}{x^2}}{\frac{x^2+x+3}{x^2}}\right)^{1/x}$

$$=\lim_{x\to\infty}\left(\frac{1+\frac{5}{x}+\frac{3}{x^2}}{1+\frac{1}{x}+\frac{3}{x^2}}\right)^{1/x}=\left(\frac{1+0+0}{1+0+0}\right)^{1/\infty}=1^0=1$$

73. **Find** $\lim_{x\to\infty}\left(\frac{x}{2+x}\right)^{2x}$

(a) e^{-4} (b) e^4 (c) ∞ (d) 0

SRM JEEE-2019

Ans. (a) : Given, $\lim_{x\to\infty}\left(\frac{x}{2+x}\right)^{2x}$

We know that.

$$\lim_{x\to a} f(x)^{g(x)} = \lim_{e^{x\to a}}[f(x)-1]g(x)\,]$$

$$\lim_{x\to\infty}\left(\frac{x}{2+x}\right)^{2x} = \lim_{e^{x\to\infty}}\left[\frac{x}{2+x}-1\right]2x$$

$$= \lim_{e^{x\to\infty}}\left[\frac{x-2-x}{2+x}\right]2x = \lim_{e^{x\to\infty}}\left(\frac{-2}{2+x}\right)2x$$

$$= \lim_{e^{x\to\infty}}\left[\frac{-4x}{2+x}\right] = \lim_{e^{x\to\infty}}\left[\frac{-4}{\frac{2}{x}+1}\right] = e^{\left(\frac{-4}{0+1}\right)} = e^{-4}$$

74. If $\lim_{x\to1}\frac{x+x^2+x^3+.......+x^n-n}{x-1}=5050$, then n equals

(a) 10 (b) 100 (c) 150 (d) 200

SRM JEEE-2012

Ans. (b) : Given, $\lim_{x\to1}\frac{x+x^2+x^3+......+x^n-n}{x-1}=5050$

Applying L–Hospital's's rule,

$$\lim_{x\to1}\frac{1+2x+3x^2+......+nx^{n-1}}{1}=5050$$

$1+2+3+.....+n=5050$

$\frac{n(n+1)}{2}=5050$

$n^2+n-10100=0$

$(n+101)(n-100)=0$

$n=-101, 100 \quad (\because n\neq-101)$

$n=100$

75. The value of $\lim_{x\to\infty}\left(\frac{x^2+6}{x^2-6}\right)$ is given by

(a) 0 (b) 1 (c) –1 (d) ∞

SRM JEEE-2011

Ans. (b) : Given, $\lim_{x\to\infty}\left(\frac{x^2+6}{x^2-6}\right)$ $\left(\frac{\infty}{\infty}\text{form}\right)$

Using L' Hospital's rule, we get–

$$\lim_{x\to\infty}\frac{2x}{2x}=1$$

76. If $\lim_{x\to a}\frac{a^x-x^a}{x^x-a^a}=-1$, then the value of a is

(a) 1 (b) 0 (c) e (d) none of therse

AMU-2015

SRM JEEE-2013

Ans. (a) : Given, $\lim_{x\to a}\frac{a^x-x^a}{x^x-a^a}=-1$

$$\lim_{x\to a}\frac{a^x-x^a}{x^x-a^a}\quad\left(\frac{0}{0}\text{form}\right)$$

[Using L– Hospital's's rule]

$$\lim_{x\to a}\frac{a^x\log a-a\,x^{a-1}}{x^x[\log x+1]-0}=-1$$

$$\frac{a^a\log a-a\,a^{a-1}}{a^a[\log a+1]}=-1 \Rightarrow \frac{a^a[\log a-1]}{a^a[1+\log a]}=-1$$

$$\frac{\log a-1}{1+\log a}=-1 \Rightarrow \log a-1=-\log a-1$$

$2\log a=0 \Rightarrow \log a=0 \Rightarrow a=e^0=1$

77. $\lim_{x\to1}\frac{1+\log x-x}{1-2x+x^2}=$

(a) 1 (b) –1 (c) 0 (d) $-\frac{1}{2}$

SRM JEEE-2013

Ans. (d) : Given, $\lim_{x\to1}\frac{1+\log x-x}{1-2x+x^2}$

Using L' Hospital's rule,

$$=\lim_{x\to1}\frac{1/x-1}{-2+2x}$$

Again, using L' Hospital's rule,

$$\lim_{x\to1}\frac{-1/x^2}{2}=\frac{-1}{2}$$

78. $\lim_{x\to\infty}\left(1+\frac{1}{x}+\frac{1}{x^2}\right)^{2x}$ is equal to

(a) e^2 (b) e (c) 2e (d) $2e^2$

SRM JEEE-2015

Ans. (a) : Given, $\lim_{x\to\infty}\left(1+\frac{1}{x}+\frac{1}{x^2}\right)^{2x}$ (1^∞ form)

$$\left[\because \lim_{x\to a} f(x)^{g(x)} = e^{\lim_{x\to a}[f(x)-1]g(x)}\right]$$

$$\lim_{x\to\infty}\left(1+\frac{1}{x}+\frac{1}{x^2}\right)^{2x} = e^{\lim_{x\to\infty}\left(1+\frac{1}{x}+\frac{1}{x^2}-1\right)2x} = e^{\lim_{x\to\infty}\left(2+\frac{2}{x}\right)} = e^2$$

79. $\lim_{x\to0}\frac{\log(3+x)-\log(3-x)}{x}$ is equal to

(a) 0 (b) –1/3 (c) 2/3 (d) –2/3

SRM JEEE-2016, JECE-2006

Ans.(c): Given,

$$\lim_{x\to0}\frac{\log(3+x)-\log(3-x)}{x}\quad\left(\frac{0}{0}\text{ form}\right)$$

Applying L'-Hospital's rule,

$$\lim_{x\to0}\frac{\left(\frac{1}{3+x}\right)-\left(\frac{-1}{3-x}\right)}{1}=\frac{1}{3}+\frac{1}{3}=\frac{2}{3}$$

80. If f(2) = 4 and f'(2) = 4, then

$$\lim_{x\to 2}\left(\frac{xf(2)-2f(x)}{x-2}\right) \text{ is equal to}$$

(a) 2 (b) –2 (c) –4 (d) 3

SRM JEEE-2015, CG PET-2010

VITEEE-2006, AMU-2005

Ans. (c) : Given, f(2) = 4 & f '(2) = 4

$$\lim_{x\to 2}\left[\frac{xf(2)-2f(x)}{x-2}\right] \qquad \left[\frac{0}{0}\text{ form}\right]$$

Using L'- Hospital's's rule,

$$=\lim_{x\to 2}\left(\frac{f(2)-2f'(x)}{1}\right) = f(2)-2f'(2)$$

$$= 4-2\times 4 \quad = 4-8 = -4$$

81. The limit of $\lim_{x\to 0}\left(\frac{\log_e(1+x)}{x^2}+\frac{x-1}{x}\right)$

(a) is equal to $\frac{1}{2}$ (b) is equal to $-\frac{1}{2}$

(c) is equal to 2 (d) does not exist

BITSAT-2019

Ans. (a) :

$$=\lim_{x\to 0}\left[\frac{\log(1+x)}{x^2}+\frac{x-1}{x}\right]=\lim_{x\to 0}\left[\frac{\log(1+x)}{x^2}+1-\frac{1}{x}\right]$$

We know that,

$$\log(1+x) = x-\frac{x^2}{2}+\frac{x^3}{3}-.........$$

$$\frac{\log(1+x)}{x^2}=\frac{x-\frac{x^2}{2}+\frac{x^3}{3}-.........}{x^2}$$

$$\frac{\log(1+x)}{x^2}=\frac{1}{x}-\frac{1}{2}+\frac{x}{3}-...........$$

$$\frac{\log(1+x)}{x^2}+1-\frac{1}{x}=\frac{1}{x}-\frac{1}{2}+\frac{x}{3}-...........+1-\frac{1}{x}$$

$$=1-\frac{1}{2}+\frac{x}{3}-.......... \qquad ....(i)$$

$$\lim_{x\to 0}\left[\frac{\log(1+x)}{x^2}+1-\frac{1}{x}\right]=\lim_{x\to 0}\left[1-\frac{1}{2}...........\right]$$

$$\lim_{x\to 0}\left[\frac{\log(1+x)}{x^2}+\frac{x-1}{x}\right]=1-\frac{1}{2}+0 \quad (\because \text{ from eq}^n.\text{ (i)})$$

$$=\frac{1}{2}$$

82. $\lim_{n\to\infty}\left[\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+....+\frac{1}{n(n+1)}\right]$ **is equal to**

(a) 1 (b) –1 (c) 0 (d) None of these

VITEEE-2010

Ans. (a) : Given, $\lim_{n\to\infty}\left[\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+....+\frac{1}{n(n+1)}\right]$

$$=\lim_{n\to\infty}\left[\left(1-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)+....+\left(\frac{1}{n}-\frac{1}{n+1}\right)\right]$$

$$=\lim_{n\to\infty}\left(1-\frac{1}{n+1}\right)=\lim_{n\to\infty}\frac{n}{n+1}$$

$$=\lim_{n\to\infty}\frac{n}{n\left(1+\frac{1}{n}\right)}=\lim_{n\to\infty}\frac{1}{\left(1+\frac{1}{n}\right)}=1$$

83. $\lim_{x\to\infty}\left(\frac{x+5}{x+2}\right)^{x+3}$ **equals**

(a) e (b) e^2 (c) e^3 (d) e^5

VITEEE-2009, JCECE-2006, 2004

Ans. (c) : Given, $\lim_{x\to\infty} f(x)^{g(x)} = e^{\lim_{x\to\infty} g(x)[f(x)-1]}$

Now, $$\lim_{x\to\infty}\left(\frac{x+5}{x+2}\right)^{x+3}=e^{\lim_{x\to\infty}(x+3)\left(\frac{x+5}{x+2}-1\right)}$$

$$=e^{\lim_{x\to\infty}(x+3)\left(\frac{x+5-x-2}{x+2}\right)}=e^{3\lim_{x\to\infty}\left(\frac{x+3}{x+2}\right)}$$

$$=e^{3\lim_{x\to\infty}\left(\frac{x\left(1+\frac{3}{x}\right)}{x\left(1+\frac{2}{x}\right)}\right)} \quad =e^{3\left(\frac{1+\frac{1}{\infty}}{1+\frac{2}{\infty}}\right)}$$

$$=e^{3\left(\frac{1+0}{1+0}\right)}=e^{3\times 1} \quad =e^3$$

84. $\lim_{x\to\pi/4}\frac{\tan x-1}{\cos 2x}$ **is equal to**

(a) 1 (b) 0 (c) –2 (d) –1

VITEEE-2015

Ans. (d) : Given, $\lim_{x\to\pi/4}\frac{\tan x-1}{\cos 2x}$

$$=\lim_{h\to 0}\frac{\tan\left(\frac{\pi}{4}+h\right)-1}{\cos 2\left(\frac{\pi}{4}+h\right)} \qquad \left[\because x=\frac{\pi}{4}+h\right]$$

$$=\lim_{h\to 0}\frac{\left(\frac{1+\tan h}{1-\tan h}\right)-1}{\cos\left(\frac{\pi}{2}+2h\right)}=\lim_{h\to 0}\frac{1+\tan h-1+\tan h}{-\sin 2h(1-\tan h)}$$

$$=\lim_{h\to 0}\frac{-2\tan h}{2\sin h\cos h(1-\tan h)}$$

$$=\lim_{h\to 0}\frac{-1}{\cos^2 h(1-\tan h)}=\frac{-1}{(\cos 0^o)^2(1-\tan 0^o)}$$

$$=\frac{-1}{1(1-0)}=-1$$

85. **If f : R → R be such that f(1) = 3 and f' (1) = 6. Then, $\lim_{x\to 0}\left\{\frac{f(1+x)}{f(1)}\right\}^{1/x}$ equals to**

(a) 1 (b) $e^{1/2}$ (c) e^2 (d) e^3

VITEEE-2013

Ans. (c) : Given, $\lim_{x\to 0}\left\{\frac{f(1+x)}{f(1)}\right\}^{1/x}$

Let, $y=\left(\frac{f(1+x)}{f(1)}\right)^{1/x}$

Taking log on both sides

$$\log y=\frac{1}{x}\left[\log f(1+x)-\log f(1)\right]$$

$$\lim_{x\to 0}\log y=\lim_{x\to 0}\frac{1}{x}\left[\log f(1+x)-\log f(1)\right]$$

Using L' Hospital's rule –

$$\lim_{x\to 0}\log y=\lim_{x\to 0}\left[\frac{1}{f(1+x)}f'(1+x)\right]$$

$$\lim_{x\to 0}\log\ y=\frac{f'(1)}{f(1)}=\frac{6}{3}$$

$$\log\left(\lim_{x\to 0}y\right)=2$$

$$\lim_{x\to 0}y=e^2$$

86. **The value of $\lim_{x\to\infty}\left(\frac{\pi}{2}-\tan^{-1}x\right)^{1/x}$ is**

(a) 0 (b) 1 (c) –1 (d) e

VITEEE-2012

Ans. (b) : Given, $\lim_{x\to\infty}\left(\frac{\pi}{2}-\tan^{-1}x\right)^{1/x}$

Let, $y=\lim_{x\to\infty}\left(\frac{\pi}{2}-\tan^{-1}x\right)^{1/x}$

Taking log on both sides, we get–

$$\log y=\lim_{x\to\infty}\frac{1}{x}\log\left(\frac{\pi}{2}-\tan^{-1}x\right)\quad(\because \log a^b=b\log a)$$

Using L-Hospital's's rule–

$$\log y=\lim_{x\to\infty}\frac{\left(-\frac{1}{1+x^2}\right)}{\frac{\pi}{2}-\tan^{-1}x}\Rightarrow \log y=\lim_{x\to\infty}\frac{\frac{2x}{\left(1+x^2\right)^2}}{-\left(\frac{1}{1+x^2}\right)}$$

Again using L-Hospital's's rule

$$\log y=\lim_{x\to\infty}\frac{-2x}{1+x^2}\Rightarrow \log y=0$$

$$y=e^0\Rightarrow y=1$$

87. **Evaluate $\lim_{x\to 2}\frac{\sqrt{(x+7)}-3\sqrt{(2x-3)}}{\sqrt[3]{(x+6)}-2\sqrt[3]{(3x-5)}}$.**

(a) $\frac{17}{9}$ (b) $\frac{17}{18}$ (c) $\frac{34}{23}$ (d) $\frac{26}{7}$

VITEEE-2019

Ans. (c) : Given, $L=\lim_{x\to 2}\frac{\sqrt{(x+7)}-3\sqrt{(2x-3)}}{\sqrt[3]{(x+6)}-2\sqrt[3]{(3x-5)}}$ $\left(\frac{0}{0}\text{form}\right)$

Let x – 2 = t such that when

x →2, t → 0, Then

$$L=\lim_{t\to 0}\frac{\sqrt{(t+2+7)}-3\sqrt{2(t+2)-3}}{\sqrt[3]{(t+2+6)}-2\sqrt[3]{3(t+2)-5}}$$

$$L=\lim_{t\to 0}\frac{(t+9)^{\frac{1}{2}}-3(2t+1)^{\frac{1}{2}}}{(t+8)^{\frac{1}{3}}-2(3t+1)^{\frac{1}{3}}}$$

It is also $\frac{0}{0}$ form

$$L=\lim_{t\to 0}\frac{\left[9\left(\frac{t}{9}+1\right)\right]^{1/2}-3(2t+1)^{1/2}}{\left[8\left(\frac{t}{8}+1\right)\right]^{1/3}-2(3t+1)^{1/3}}$$

$$L=\frac{3}{2}\lim_{t\to 0}\frac{\left(1+\frac{t}{9}\right)^{\frac{1}{2}}-(2t+1)^{\frac{1}{2}}}{\left(1+\frac{t}{8}\right)^{\frac{1}{3}}-(3t+1)^{\frac{1}{3}}}$$

It is also $\frac{0}{0}$ form

$$=\frac{3}{2}\lim_{t\to 0}\frac{\frac{1}{2}\times\frac{t}{9}-(2t)\frac{1}{2}}{\frac{t}{8}\times\frac{1}{3}-(3t)\frac{1}{3}}=\frac{3\left(\frac{1}{18}-1\right)}{2\left(\frac{1}{24}-1\right)}=\frac{34}{23}$$

88. **The value of $\lim_{x\to 0}\frac{x^3\cot x}{1-\cos x}$ is**

(a) 1 (b) –2 (c) 2 (d) 0

VITEEE-2018

Ans. (c) : Given,

$$\lim_{x\to 0}\frac{x^3\cot x}{1-\cos x}=\lim_{x\to 0}\left(\frac{x^3\cot x}{1-\cos x}\times\frac{1+\cos x}{1+\cos x}\right)$$

$$=\lim_{x\to 0}\left(\frac{x^3\cot x(1+\cos x)}{1-\cos^2 x}\right)$$

$$\because \sin^2\theta+\cos^2\theta=1$$

$$=\lim_{x\to 0}\left(\frac{x^3\cot x(1+\cos x)}{\sin^2 x}\right)$$

$$\because \cot\theta=\frac{\cos\theta}{\sin\theta}$$

$$= \lim_{x\to 0}\left(\frac{x}{\sin x}\right)^3 \times \lim_{x\to 0}\cos x \times \lim_{x\to 0}(1+\cos x)$$

$$= \lim_{x\to 0}\left(\frac{1}{\frac{\sin x}{x}}\right)^3 \times \cos 0^o \times (1+\cos 0^o)$$

$= 1 \times 1\ (1+1) = 2$

$\therefore \quad \lim_{x\to 0}\frac{x^3 \cot x}{1-\cos x} = 2$

89. The value of $\lim_{x\to\frac{\pi}{2}}\frac{\sin(\cos x)\cos x}{\sin x - \operatorname{cosec} x}$ is

(a) ∞ (b) 1 (c) 0 (d) –1

UPSEE-2011

Ans. (d) : Given, $\lim_{x\to\frac{\pi}{2}}\frac{\sin(\cos x).\cos x}{\sin x - \operatorname{cosec} x}$

$$= \lim_{x\to\frac{\pi}{2}}\frac{\sin(\cos x).\cos x}{\sin x - \operatorname{cosec} x}\times\frac{\cos x}{\cos x}$$

$$= \lim_{x\to\frac{\pi}{2}}\frac{\sin(\cos x)}{\cos x} = \lim_{x\to\frac{\pi}{2}}\frac{\cos^2 x}{\sin x - \frac{1}{\sin x}}$$

$$= 1\times\lim_{x\to\frac{\pi}{2}}\frac{\cos^2 x\cdot\sin x}{\sin^2 x - 1} = \lim_{x\to\frac{\pi}{2}}\frac{\cos^2 x\cdot\sin x}{-\cos^2 x}$$

$$= \lim_{x\to\frac{\pi}{2}}(-\sin x) = -\sin\frac{\pi}{2} = -1$$

90. $\lim_{x\to 0}(\operatorname{cosec} x)^{1/\log x}$ is equal to

(a) 0 (b) 1 (c) $\frac{1}{e}$ (d) None of these

UPSEE-2010

Ans. (c) : Given,

$\lim_{x\to 0}(\operatorname{cosec} x)^{1/\log x}$

Let, $\quad y = \lim_{x\to 0}(\operatorname{cosec} x)^{1/\log x}$

Taking log on both side.

$$\log y = \lim_{x\to 0}\frac{1}{\log x}\log(\operatorname{cosec} x)$$

$$\log y = \lim_{x\to 0}\frac{\log(\operatorname{cosec} x)}{\log x}$$

Using L-Hospital's rule

$$\log y = \lim_{x\to 0}\frac{\frac{1}{\operatorname{cosec} x}.(-\operatorname{cosec} x.\cot x)}{1/x}$$

$$\log y = \lim_{x\to 0} -x\cot x$$

$$\log y = \lim_{x\to 0}\frac{-x}{\tan x}$$

$\log y = -1 \Rightarrow y = e^{-1} = 1/e$

91. $\lim_{x\to 0}\left\{\frac{1+\tan x}{1+\sin x}\right\}^{\operatorname{cosec} x}$ is equal to

(a) $\frac{1}{e}$ (b) 1 (c) e (d) e^2

UPSEE -2008

Ans. (b) : Given.

$$\lim_{x\to 0}\left\{\frac{1+\tan x}{1+\sin x}\right\}^{\operatorname{cosec} x}$$

$$= \left[\because \lim_{x\to 0}[f(x)]^{g(x)} = e^{\lim_{x\to 0}[f(x)-1]g(x)}\right]$$

$$= e^{\lim_{x\to 0}\left(\left(\frac{1+\tan x}{1+\sin x}\right)-1\right)\operatorname{cosec} x} = e^{\lim_{x\to 0}\left[\frac{\tan x - \sin x}{1+\sin x}\right]\operatorname{cosec} x}$$

$$= e^{\lim_{x\to 0}\ \sin x\left(\frac{\sec x - 1}{1+\sin x}\right)\times\frac{1}{\sin x}} = e^{\lim_{x\to 0}\left(\frac{\sec x - 1}{1+\sin x}\right)} = e^{\left(\frac{\sec 0 - 1}{1+\sin 0.}\right)}$$

$$= e^{\left(\frac{1-1}{1+0}\right)} = e^0 = 1$$

92. If *f* be a function such that *f*(9) = 9 and *f*′(9) = 3, then $\lim_{x\to 9}\frac{\sqrt{f(x)}-3}{\sqrt{x}-3}$ is equal to

(a) 9 (b) 3 (c) 1 (d) None of these

UPSEE -2008

BCECE-2010

Ans. (b) : Given,

f (9) = 9 and f′ (9) = 3

$$\lim_{x\to 9}\frac{\sqrt{f(x)}-3}{\sqrt{x}-3}$$

Using L-Hospital's rule,

$$= \lim_{x\to 9}\frac{\frac{1}{2}[f(x)]^{\frac{1}{2}-1}f'(x)-0}{\frac{1}{2}x^{\left(\frac{1}{2}-1\right)}-0} = \lim_{x\to 9}\frac{[f(x)]^{\frac{-1}{2}}.f'(x)}{x^{\frac{-1}{2}}}$$

$$= \lim_{x\to 9}\frac{\sqrt{x}f'(x)}{\sqrt{f(x)}} = \frac{\sqrt{9}f'(9)}{\sqrt{f(9)}} = \frac{3\times 3}{\sqrt{9}} = \frac{3\times 3}{3} = 3$$

93. The value of $\lim_{x\to\infty}\left(\frac{x^2+bx+4}{x^2+ax+5}\right)$ is

(a) $\frac{b}{a}$ (b) 0 (c) 1 (d) $\frac{4}{5}$

JCECE-2008

UPSEE-2007

Ans. (c) : Given,

$$\lim_{x\to\infty}\left(\frac{x^2+bx+4}{x^2+ax+5}\right) = \lim_{x\to\infty}\left(\frac{1+\frac{b}{x}+\frac{4}{x^2}}{1+\frac{a}{x}+\frac{5}{x^2}}\right) = \frac{1+0+0}{1+0+0} = 1$$

94. $\lim\limits_{x\to\infty}\dfrac{(2x-3)(3x-4)}{(4x-5)(5x-6)}$ **is equal to**

(a) $\dfrac{1}{10}$ (b) 0 (c) $\dfrac{1}{5}$ (d) $\dfrac{3}{10}$

Kerala CEE-2018

UPSEE-2007

Ans. (d) : Given,

$$\lim_{x\to\infty}\frac{(2x-3)(3x-4)}{(4x-5)(5x-6)}=\lim_{x\to\infty}\frac{6x^2-17x+12}{20x^2-49x+30}$$

$$=\lim_{x\to\infty}\frac{6-\frac{17}{x}+\frac{12}{x^2}}{20-\frac{49}{x}+\frac{12}{x^2}}=\frac{6-0+0}{20-0+0}=\frac{6}{20}=\frac{3}{10}$$

95. **The value of** $\lim\limits_{x\to0}\dfrac{(1+x)^{1/2}-1}{(1+x)^{1/2}-1}$

(a) 3/2 (b) 2/3 (c) 0 (d) None of these

UPSEE-2018

Ans. (d) : Given,

$$\lim_{x\to0}\frac{(1+x)^{1/2}-1}{(1+x)^{1/2}-1}$$

Using L-Hospital's rule

$$=\lim_{x\to0}\frac{\frac{1}{2\sqrt{1+x}}-0}{\frac{1}{2\sqrt{1+x}}-0}=\lim_{x\to0}1\ =1$$

96. **The** $\lim\limits_{y\to a}\left\{\left(\sin\dfrac{y-a}{2}\right).\left(\tan\dfrac{\pi y}{2a}\right)\right\}$ **is**

(a) $\dfrac{2a}{\pi}$ (b) $\dfrac{a}{\pi}$ (c) $-\dfrac{a}{\pi}$ (d) $\dfrac{a}{2\pi}$

UPSEE-2017

Ans. (c) : Given,

$$\lim_{y\to a}\left\{\left(\sin\frac{y-a}{2}\right).\left(\tan\frac{\pi y}{2a}\right)\right\}=\lim_{y\to a}\frac{\sin\left(\frac{y-a}{2}\right)}{\cot\left(\frac{\pi y}{2a}\right)}$$

Using L– Hospital's rule

$$=\lim_{y\to a}\frac{\cos\left(\frac{y-a}{2}\right)\times\frac{1}{2}}{-\operatorname{cosec}^2\left(\frac{\pi y}{2a}\right)\times\frac{\pi}{2a}}=\frac{\cos0\times\frac{1}{2}}{-\left(\operatorname{cosec}\frac{\pi}{2}\right)^2\times\frac{\pi}{2a}}$$

$$=\frac{1\times\frac{1}{2}}{-1\times\frac{\pi}{2a}}=-\frac{a}{\pi}$$

97. **The value of the following expression is** $\lim\limits_{n\to\infty}\dfrac{1}{n^3}\left(1^2+2^2+3^2+......n^2\right)$

(a) $\dfrac{1}{2}$ (b) $\dfrac{2}{3}$ (c) $\dfrac{1}{3}$ (d) $\dfrac{1}{6}$

UPSEE-2016

Ans. (c) : Given,

$$\lim_{n\to\infty}\frac{1}{n^3}\left(1^2+2^2+3^2+.........n^2\right)$$

$$=\lim_{n\to\infty}\frac{1}{n^3}\times\frac{n(n+1)(2n+1)}{6}=\lim_{n\to\infty}\frac{(n+1)(2n+1)}{6n^2}$$

$$=\lim_{n\to\infty}\frac{2n^2+3n+1}{6n^2}=\lim_{n\to\infty}\frac{2+\frac{3}{n}+\frac{1}{n^2}}{6}=\frac{2+0+0}{6}=\frac{1}{3}$$

98. **The value of** $\lim\limits_{x\to0}\dfrac{1-\cos^3x}{x\sin x\ \cos x}$ **is**

(a) $\dfrac{2}{5}$ (b) $\dfrac{3}{5}$ (c) $\dfrac{3}{2}$ (d) $\dfrac{3}{4}$

JCECE-2015

Ans. (c) : Given,

$$\lim_{x\to0}\frac{1-\cos^3x}{x\sin x.\cos x}=\lim_{x\to0}\frac{2\left(1-\cos^3x\right)}{x.\sin2x}$$

Using L– Hospital's

$$=\lim_{x\to0}\frac{2\left[0-3\cos^2x.(-\sin x)\right]}{x.\cos2x.2+\sin2x.1}=\lim_{x\to0}\frac{6\sin x.\cos^2x}{2x\cos2x+\sin2x}$$

Again using L– Hospital's

$$=\lim_{x\to0}\frac{6\left[\sin x.2\cos x.(-\sin x)+\cos^2x.\cos x\right]}{\left[2x.(-\sin2x).2\right]+\cos2x.1]+\cos2x.2}$$

$$=\lim_{x\to0}\frac{6[-2\sin^2x\cos x+\cos^3x]}{-4x\sin2x+2\cos2x+2\cos2x}$$

$$=\lim_{x\to0}\frac{6\left(-2\sin^2x\cos x+\cos^3x\right)}{-4x\sin2x+4\cos2x}=\frac{6(0+1)}{0+4}=\frac{6}{4}=\frac{3}{2}$$

99. $\lim\limits_{x\to0}\dfrac{2^x-1}{(1+x)^{1/2}-1}$ **is equal to**

(a) log2 (b) log4 (c) $\log\sqrt{2}$ (d) None of these

JCECE-2014

Ans. (b) : Given,

$$\lim_{x\to0}\frac{2^x-1}{(1+x)^{1/2}-1}$$

Using L- Hospital's rule

$$=\lim_{x\to0}\frac{2^x.\log2-0}{\frac{1}{2}(1+x)^{-\frac{1}{2}}-0}=\lim_{x\to0}2^{x+1}\log2.(1+x)^{1/2}$$

$$=\lim_{x\to0}2^{x+1}\sqrt{1+x}.\log2\ =2^{0+1}\ \sqrt{1+0}\ .\ \log2$$

= 2log2 =log4

100. The value of $\lim\limits_{x\to 0}$ $(\cos x + a\sin bx)^{1/x}$ is

(a) 1 (b) ab (c) e^{ab} (d) $e^{b/a}$

JCECE-2011

Ans. (c) : Given,

$\lim\limits_{x\to 0}$ $(\cos x + a\sin bx)^{1/x}$

If $\lim\limits_{x\to a} f(x) = \lim\limits_{x\to a} g(x) = 0$ such that $\lim\limits_{x\to a}\dfrac{f(x)}{g(x)}$ exists.

$$\left[\therefore \lim_{x\to a}[1+f(x)]^{\frac{1}{g(x)}} = e^{\lim\limits_{x\to a}\frac{f(x)}{g(x)}}\right]$$

Let us add and subtract 1 to the given expression.

$$= \lim_{x\to 0}(1+\cos x + a\sin bx - 1)^{1/x}$$

Here, f (x) = cosx + a sin bx – 1

and g(x) = x

$$= e^{\lim\limits_{x\to 0}\left[\frac{\cos x + a\sin bx - 1}{x}\right]} = e^{\lim\limits_{x\to 0}\left[\frac{a\sin bx}{x} - \left(\frac{1-\cos x}{x}\right)\right]}$$

$$= e^{\lim\limits_{x\to 0}\left[\frac{ab\sin bx}{bx} - \frac{2\sin^2 x/2}{x}\right]} = e^{\lim\limits_{x\to 0}\left[\frac{ab\sin bx}{bx} - 0\right]} = e^{ab}$$

101. $\lim\limits_{x\to 1}\dfrac{1+\log x - x}{1-2x+x^2}$ is equal to

(a) 1 (b) –1 (c) 0 (d) $-\dfrac{1}{2}$

JCECE-2010

Ans. (d) : Given,

$$\lim_{x\to 1}\frac{1+\log x - x}{1-2x+x^2}$$

Using L-Hospital's rule

$$= \lim_{x\to 1}\frac{0+\frac{1}{x}-1}{-2+2x} = \lim_{x\to 1}\frac{(1-x)}{-2x(1-x)}$$

$$= \lim_{x\to 1}\frac{1}{-2x} = \frac{-1}{2\times 1} = -\frac{1}{2}$$

102. $\lim\limits_{x\to 2^+}\dfrac{|x-2|}{x-2}$ is equal to

(a) –1 (b) 1 (c) 2 (d) –2

JCECE-2009

Ans. (b) : Given,

$$\lim_{x\to 2^+}\frac{|x-2|}{x-2}$$

$$\because \{|x-2| = x-2 \text{ for } x > 2\}$$

$$= \lim_{x\to 2^+}\frac{(x-2)}{(x-2)} = \lim_{x\to 2^+} 1 = 1$$

103. $\lim\limits_{x\to 1}\dfrac{x^8-2x+1}{x^4-2x+1}$ equals

(a) 3 (b) 0 (c) –3 (d) 1

JCECE-2007

Ans. (a) : Given

$$= \lim_{x\to 1}\frac{x^8-2x+1}{x^4-2x+1}$$

Using L– Hospital's's rule,

$$= \lim_{x\to 1}\frac{8x^7-2}{4x^3-2} = \frac{8\times 1^7-2}{4\times 1^3-2} = \frac{8-2}{4-2} = \frac{6}{2} = 3$$

104. $\lim\limits_{x\to\frac{\pi}{6}}\dfrac{\sin 2x}{\sin x}$ is equals to

(a) $\sqrt{3}$ (b) $\dfrac{1}{\sqrt{3}}$ (c) 2 (d) $\dfrac{1}{2}$

JCECE-2007

Ans. (a) : Given,

$$\lim_{x\to\frac{\pi}{6}}\frac{\sin 2x}{\sin x} = \frac{\sin\left(2\times\frac{\pi}{6}\right)}{\sin\frac{\pi}{6}} = \frac{\sin\frac{\pi}{3}}{\sin\frac{\pi}{6}} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$$

105. $\lim\limits_{x\to\infty}\sqrt{\dfrac{x-\sin x}{x+\cos^2 x}}$ is equal to

(a) –1 (b) 0 (c) $\dfrac{1}{2}$ (d) 1

Kerala CEE-2018

JCECE-2007

Ans. (d) : Given,

$$\lim_{x\to\infty}\sqrt{\frac{x-\sin x}{x+\cos^2 x}} = \lim_{x\to\infty}\sqrt{\frac{x\left(1-\frac{\sin x}{x}\right)}{x\left(1+\frac{\cos^2 x}{x}\right)}}$$

$$= \lim_{x\to\infty}\sqrt{\frac{1-\frac{\sin x}{x}}{1+\frac{\cos^2 x}{x}}} = \sqrt{\frac{1-0}{1+0}} = 1$$

106. For x ∈ R, $\lim\limits_{x\to\infty}\left(\dfrac{x-3}{x+2}\right)^x$ is equal to:

(a) e (b) e^{-1} (c) e^{-5} (d) e^5

JCECE-2006

Ans. (c) : Given

$$\lim_{x\to\infty}\left(\frac{x-3}{x+2}\right)^x \left[\because \lim_{x\to\infty}[f(x)]^{g(x)} = e^{\lim\limits_{x\to\infty}[f(x)-1]g(x)}\right]$$

$$= \lim_{x\to\infty}\left(\frac{x-3}{x+2}\right)^x = e^{\lim\limits_{x\to\infty}\left[\frac{x-3}{x+2}-1\right]x}$$

$$= e^{\lim\limits_{x\to\infty}\left[\frac{x-3-x-2}{x+2}\right]x} = e^{\lim\limits_{x\to\infty}\left(\frac{-5x}{x+2}\right)} = e^{\lim\limits_{x\to\infty}\left(\frac{-5}{1+\frac{2}{x}}\right)} = e^{\left(\frac{-5}{1+0}\right)} = e^{-5}$$

107. $\lim\limits_{x\to0}\frac{\sqrt{1-\cos 2x}}{\sqrt{2}x}$ **is equal to:**

(a) λ (b) –1 (c) zero (d) does not exist

APEAMET-2021, CGPET-2006, JCECE-2006

Ans. (d) : Given, $\lim\limits_{x\to0}\frac{\sqrt{1-\cos 2x}}{\sqrt{2}x}$

$=\lim\limits_{x\to0}\frac{\sqrt{1-(1-2\sin^2 x)}}{\sqrt{2}x}=\lim\limits_{x\to0}\frac{\sqrt{2\sin^2 x}}{\sqrt{2}x}$

$=\lim\limits_{x\to0}\frac{\pm\sqrt{2}\sin x}{\sqrt{2}\ x}=\lim\limits_{x\to0}\pm\frac{\sin x}{x}$

for $x\to0^+=\lim\limits_{x\to0^+}\frac{\sin x}{x}=1$

for $x\to0^-=\lim\limits_{x\to0^-}\frac{-\sin x}{x}=-1$

Hence, Limit does not exist.

108. $\lim\limits_{x\to a}\frac{\log(x-a)}{\log(e^x-e^a)}$ **is equal to :**

(a) 0 (b) 1 (c) a (d) does not exist

JCECE-2005

Ans. (b) : Given,

$\lim\limits_{x\to a}\frac{\log(x-a)}{\log(e^x-e^a)}$

Using L– Hospital's rule

$=\lim\limits_{x\to a}\frac{\left(\frac{1}{x-a}\right)}{\frac{1}{e^x-e^a}.e^x}=\lim\limits_{x\to a}\frac{e^x-e^a}{e^x(x-a)}$

Again using L-Hospital's rule

$=\lim\limits_{x\to a}\frac{e^x-0}{e^x(1-0)+(x-a)e^x}=\lim\limits_{x\to a}\frac{e^x}{e^x+e^x(x-a)}$

$=\lim\limits_{x\to a}\frac{e^x}{e^x(1+x-a)}=\frac{1}{1+a-a}=1$

109. **The value of** $\lim\limits_{x\to1}(1-x)\tan\left(\frac{\pi}{2}x\right)$**:**

(a) 3π/4 (b) 2π/3 (c) 2/π (d) π/4

JCECE-2004

Ans. (c) : Given, $\lim\limits_{x\to1}(1-x)\ \tan\left(\frac{\pi}{2}x\right)$

$=\lim\limits_{h\to0}\left[1-(1-h)\ \tan\frac{\pi}{2}(1-h)\right]$

$=\lim\limits_{h\to0}h\tan\left(\frac{\pi}{2}-\frac{\pi h}{2}\right)=\lim\limits_{h\to0}h\ \cot\frac{\pi h}{2}$

$=\lim\limits_{h\to0}\frac{h}{\tan\frac{\pi h}{2}}=\lim\limits_{h\to0}\frac{1}{\frac{\tan\frac{\pi h}{2}}{\frac{\pi h}{2}}\times\frac{\pi}{2}}$

$\therefore\quad=\frac{1}{1\times\frac{\pi}{2}}=\frac{2}{\pi}\qquad\left\{\because\lim\limits_{x\to0}\frac{\tan x}{x}=1\right\}$

110. $\lim\limits_{x\to0}x\sec x$ **is**

(a) 1 (b) 0 (c) ∞ (d) None of these

JCECE-2019

Ans. (b): Given,

$\lim\limits_{x\to0}x\sec x$

$=\lim\limits_{x\to0}x\frac{1}{\cos x}=\frac{0}{\cos 0^0}=\frac{0}{1}=0$

Hence, the value of $\lim\limits_{x\to0}x\ \sec x$ is 0.

111. **If** $\lim\limits_{x\to0}\frac{\{(a-n)nx-\tan x\}\sin nx}{x^2}=0$**, where n is a non-zero real number, then a is equal to**

(a) 0 (b) $\frac{n+1}{n}$ (c) n (d) $n+\frac{1}{n}$

JCECE-2016

Ans. (d) : Given,

$\lim\limits_{x\to0}\frac{\{(a-n)nx-\tan x\}\sin nx}{x^2}=0$

$\lim\limits_{x\to0}\frac{\sin nx}{x}\frac{[(a-n)nx-\tan x]}{x}=0$

$\lim\limits_{x\to0}n.\frac{\sin nx}{nx}\left[(a-n)n-\frac{\tan x}{x}\right]=0$

$n\times1[(a-n)n-1]=0$

$n[(a-n)n-1]=0$

$(a-n)n-1=0$

$(a-n)\ n=1$

$a-n=\frac{1}{n}\Rightarrow a=n+\frac{1}{n}$

112. $\lim\limits_{x\to0}\frac{\tan x-\sin x}{\sin^3 x}$ **is equal to**

(a) $\frac{1}{2}$ (b) 0 (c) 1 (d) 2

BCECE-2018

Ans. (a) : Given,

$=\lim\limits_{x\to0}\frac{\frac{\sin x}{\cos x}-\sin x}{\sin^3 x}=\lim\limits_{x\to0}\frac{\frac{\sin x-\sin x.\cos x}{\cos x}}{\sin^3 x}$

$=\lim\limits_{x\to0}\frac{\sin x(1-\cos x)}{\cos x\ .\ \sin^3 x}=\lim\limits_{x\to0}\frac{(1-\cos x)}{\cos x\ .\ \sin^2 x}$

$$= \lim_{x\to0} \frac{1-\cos x}{\cos x\left(1-\cos^2 x\right)}$$
$$= \lim_{x\to0} \frac{(1-\cos x)}{\cos x(1-\cos x)(1+\cos x)}$$
$$= \lim_{x\to0} \frac{1}{\cos x(1+\cos x)} = \frac{1}{1(1+1)} = \frac{1}{2}$$

113. $\lim\limits_{x\to\infty} \dfrac{\cot^{-1}\left(\sqrt{x+1}-\sqrt{x}\right)}{\sec^{-1}\left\{\left(\dfrac{2x+1}{x-1}\right)^x\right\}}$ **is equal to**

(a) 1 (b) 0 (c) $\frac{\pi}{2}$ (d) Non-existent

BCECE-2017

Ans. (a) : Given,
$$\lim_{x\to\infty} \frac{\cot^{-1}\left(\sqrt{x+1}-\sqrt{x}\right)}{\sec^{-1}\left\{\left(\frac{2x+1}{x-1}\right)^x\right\}}$$
$$= \lim_{x\to\infty} \frac{\cot^{-1}\left[\frac{\sqrt{x+1}-\sqrt{x}}{\sqrt{x+1}+\sqrt{x}}\times\left(\sqrt{x+1}+\sqrt{x}\right)\right]}{\sec^{-1}\left\{\left(\frac{2+\frac{1}{x}}{1-\frac{1}{x}}\right)^x\right\}}$$
$$= \lim_{x\to\infty} \frac{\cot^{-1}\left[\frac{x+1-x}{\sqrt{x+1}+\sqrt{x}}\right]}{\sec^{-1}\left\{\left(\frac{2+\frac{1}{x}}{1-\frac{1}{x}}\right)^x\right\}} = \frac{\lim\limits_{x\to\infty}\cot^{-1}\left(\frac{1}{\sqrt{x+1}+\sqrt{x}}\right)}{\lim\limits_{x\to\infty}\sec^{-1}\left\{\left(\frac{2+\frac{1}{x}}{1-\frac{1}{x}}\right)^x\right\}}$$
$$= \frac{\cot^{-1}(0)}{\sec^{-1}(\infty)} = \frac{\pi/2}{\pi/2} = 1$$

114. The value of $\lim\limits_{x\to0}\dfrac{\int_0^{x^2}\sec^2 t\,dt}{x\sin x}$ **is**

(a) 0 (b) 3 (c) 2 (d) 1

BCECE-2017 / SCRA-2010

Ans. (d) : Given,
$$\lim_{x\to0}\frac{\int_0^{x^2}\sec^2 t\,dt}{x\sin x} = \lim_{x\to0}\frac{[\tan t]_0^{x^2}}{x\sin x} = \lim_{x\to0}\frac{\tan x^2}{x\sin x}$$
$$= \lim_{x\to0} = \frac{\tan x^2}{x^2\frac{\sin x}{x}} = \lim_{x\to0}\frac{\tan x^2}{x^2}.\frac{1}{\lim\limits_{x\to0}\frac{\sin x}{x}} = 1\times1 = 1$$

115. The value of $\lim\limits_{x\to0}\dfrac{(1-\cos 2x)\sin 5x}{x^2\sin 3x}$ **equal to**

(a) $\frac{10}{3}$ (b) $\frac{3}{10}$ (c) $\frac{6}{5}$ (d) $\frac{5}{6}$

BCECE-2016

Ans. (a): Given,
$$\lim_{x\to0}\frac{(1-\cos 2x)\sin 5x}{x^2\sin 3x}$$
$$= \lim_{x\to0}\frac{\left[1-\left(1-2\sin^2 x\right)\right]\sin 5x\times\frac{5x}{5x}}{x^2\sin 3x\times\frac{3x}{3x}}$$
$$= \lim_{x\to0}\frac{\frac{2\sin^2 x}{x^2}.\frac{\sin 5x}{5x}.5x}{\frac{\sin 3x}{3x}\times3x} = \frac{10}{3}\lim_{x\to0}\frac{\left(\frac{\sin x}{x}\right)^2.\frac{\sin 5x}{5x}}{\frac{\sin 3x}{3x}}$$
$$= \frac{10}{3}\times\frac{1\times1}{1} = \frac{10}{3}$$

116. The value of $\lim\limits_{x\to0}\dfrac{e^x+\log(1+x)-(1-x)^{-2}}{x^2}$ **is equal to**

(a) 0 (b) –3 (c) –1 (d) Infinity

BCECE-2016

Ans. (b) : Given,
$$\lim_{x\to0}\frac{e^x+\log(1+x)-(1-x)^{-2}}{x^2}$$
Using L– Hospital's's rule,
$$= \lim_{x\to0}\frac{e^x+\frac{1}{1+x}+2(1-x)^{-3}(-1)}{2x}$$
$$= \lim_{x\to0}\frac{e^x+(1+x)^{-1}-2(1-x)^{-3}}{2x}$$
Again using L– Hospital's's rule,
$$= \lim_{x\to0}\frac{e^x-(1+x)^{-2}+6(1-x)^{-4}(-1)}{2}$$
$$= \lim_{x\to0}\frac{e^x-(1+x)^{-2}-6(1-x)^{-4}}{2}$$
$$= \frac{e^0-(1+0)-6(1-0)}{2} = \frac{1-1-6}{2} = \frac{-6}{2} = -3$$

117. $\lim\limits_{x\to0}\dfrac{(1-\cos 2x)(3+\cos x)}{x\tan 4x}$ **is equal to**

(a) 4 (b) 3 (c) 2 (d) $\frac{1}{2}$

BCECE-2015

Ans. (c): Given, $\lim\limits_{x\to0}\dfrac{\left[1-\left(1-2\sin^2 x\right)\right](3+\cos x)}{x\tan 4x}$
$$= \lim_{x\to0}\frac{2\sin^2 x(3+\cos x)}{x.\frac{\tan 4x}{4x}.4x}$$
$$= \lim_{x\to0}\frac{2\sin^2 x}{x^2}.\lim_{x\to0}\frac{(3+\cos x)}{4}\lim_{x\to0}\frac{1}{\frac{\tan 4x}{4x}}$$
$$= 2\times(1)^2.\frac{(3+\cos 0^\circ)}{4}.\frac{1}{1} = 2\times\frac{(3+1)}{4} = 2\times1 = 2$$

118. If f'(2) = 6, f'(1) = 4, then $\lim_{h\to 0}\frac{f(2h+2+h^2)-f(2)}{f(h+h^2+1)-f(1)}$ is equal to

(a) 3 (b) $-\frac{3}{2}$ (c) $\frac{3}{2}$ (d) Does not exist

BCECE-2014

Ans. (a) : Given, $f'(2)=6,\ f'(1)=4$

$$\lim_{h\to 0}\frac{f(2h+2+h^2)-f(2)}{f(h+h^2+1)-f(1)}$$

Using L-Hospital's rule,

$$=\lim_{h\to 0}\frac{f'(2h+2+h^2).(2+2h)-0}{f'(h+h^2+1)(1+2h)-0}$$

$$=\lim_{h\to 0}\frac{f'(0+2+0).(2+0)}{f'(0+0+1)(1+0)}=\frac{f'(2)2}{f'(1).1}=\frac{6\times 2}{4}=3$$

119. $\lim_{x\to\infty}\left(\frac{x+a}{x+b}\right)^{x+b}$ is

(a) 1 (b) e^{b-a} (c) e^{a-b} (d) e^b

BCECE-2012

Ans. (c) : Given, $\lim_{x\to\infty}\left(\frac{x+a}{x+b}\right)^{x+b}$

$$\left\{\because \lim_{x\to\infty}[f(x)]^{g(x)}=e^{\lim_{x\to\infty}[f(x)-1]g(x)}\right\}$$

$$\lim_{x\to\infty}\left(\frac{x+a}{x+b}\right)^{x+b}=e^{\lim_{x\to\infty}\left(\frac{x+a}{x+b}-1\right)(x+b)}$$

$$=e^{\lim_{x\to\infty}\left(\frac{a-b}{x+b}\right)(x+b)}=e^{\lim_{x\to\infty}(a-b)}=e^{a-b}$$

120. $\lim_{x\to 0}\frac{1-\cos^3 x}{x\sin x\cos x}$ is equal to

(a) $\frac{2}{5}$ (b) $\frac{3}{5}$ (c) $\frac{3}{2}$ (d) $\frac{3}{4}$

BCECE-2009

Ans.(c) :

Given, $\lim_{x\to 0}\frac{1-\cos^3 x}{x\sin x.\cos x}=\lim_{x\to 0}\frac{2(1-\cos^3 x)}{x.2\sin x.\cos x}$

$$=\lim_{x\to 0}\frac{2(1-\cos^3 x)}{x.\sin 2x}=\lim_{x\to 0}\frac{2-2\cos^3 x}{x\sin 2x}$$

Applying L– Hospital's's rule,

$$=\lim_{x\to 0}\frac{0-6\cos^2 x(-\sin x)}{x\cos 2x(2)+\sin 2x(1)}=\lim_{x\to 0}\frac{6\cos^2 x\sin x}{2x\cos 2x+\sin 2x}$$

This is $\frac{0}{0}$ form

Applying L– Hospital's's rule,

$$=\lim_{x\to 0}\frac{6[\cos^2 x.\cos x+\sin x.2\cos x(-\sin x)]}{2[x(-\sin 2x).2+\cos 2x]+\cos 2x.2}$$

$$=\lim_{x\to 1}\frac{6[\cos^3 x-2\sin^2 x.\cos x]}{-4x\sin 2x+2\cos 2x+2\cos 2x}$$

$$=\lim_{x\to 0}\frac{6(\cos^3 x-2\sin^2 x.\cos x)}{-4x\sin 2x+4\cos 2x}$$

$$=\lim_{x\to 0}\frac{6(\cos^3 x-2\sin^2 x.\cos x)}{4(\cos 2x-x\sin 2x)}=\frac{6(\cos 0^\circ-0)}{4(\cos 0^\circ-0)}$$

$$=\frac{6\times 1}{4\times 1}=\frac{3}{2}$$

121. $\lim_{x\to\infty}\left[\sqrt{x^2+2x-1}-x\right]$ is equal to

(a) ∞ (b) $\frac{1}{2}$ (c) 4 (d) 1

BCECE-2008

Ans.(d) : $\lim_{x\to\infty}\left[\sqrt{x^2+2x-1}-x\right]$

$$=\lim_{x\to\infty}\frac{\sqrt{x^2+2x-1}-x}{\sqrt{x^2+2x-1}+x}\times\left(\sqrt{x^2+2x-1}+x\right)$$

$$=\lim_{x\to 0}\frac{x^2+2x-1-x^2}{\sqrt{x^2+2x-1}+x}=\lim_{x\to\infty}\frac{2x-1}{\sqrt{x^2+2x+1}+x}$$

$$=\lim_{x\to\infty}\frac{x\left(2-\frac{1}{x}\right)}{x\left(\sqrt{1+\frac{2}{x}+\frac{1}{x^2}}+1\right)}=\frac{2-0}{\sqrt{1+0}+1}=\frac{2}{1+1}=\frac{2}{2}=1$$

122. If 0 < p < q, then $\lim_{n\to\infty}(q^n+p^n)^{1/n}$ is equal to

(a) e (b) p (c) q (d) 0

BCECE-2008

Ans. (c) : Given, $0<p<q$

$\frac{p}{q}<1$

$$=\lim_{n\to\infty}(q^n+p^n)^{1/n}=\lim_{n\to\infty}\left[\left\{q^n\left(1+\frac{p^n}{q^n}\right)\right\}^{1/n}\right]=\lim_{n\to\infty}q\left[1+\left(\frac{p}{q}\right)^n\right]^{1/n}$$

$= q\,(1+0)^{1/n}$ $\left\{\because \frac{p}{q}<1\right\}$

$= q$

123. The value of

$$\lim_{\alpha\to 0}\frac{\operatorname{coses}^{-1}(\sec\alpha)+\cot^{-1}(\tan\alpha)+\cot^{-1}\cos(\sin^{-1}\alpha)}{\alpha}\ \textbf{is}$$

(a) 0 (b) –1 (c) –2 (d) 1

BCECE-2007

Ans. (c) :

$$\lim_{\alpha\to 0}\frac{\text{cosec}^{-1}(\sec\alpha)+\cot^{-1}(\tan\alpha)+\cot^{-1}\cos(\sin^{-1}\alpha)}{\alpha}$$

$$=\lim_{\alpha\to 0}\frac{\text{cosec}^{-1}\left[\text{cosec}\left(\frac{\pi}{2}-\alpha\right)\right]+\cot^{-1}\left[\cot\left(\frac{\pi}{2}-\alpha\right)\right]+\cos^{-1}\cos\left(\cos^{-1}\sqrt{1-\alpha^2}\right)}{\alpha}$$

$$=\lim_{\alpha\to 0}\frac{\frac{\pi}{2}-\alpha+\frac{\pi}{2}-\alpha+\cot^{-1}\sqrt{1-\alpha^2}}{\alpha}$$

Using L-Hospital's's rule,

$$=\lim_{\alpha\to 0}\frac{-2-\frac{1}{1+1-\alpha^2}\left[\frac{1}{2\sqrt{1-\alpha^2}}(-2\alpha)\right]}{1}$$

$$=\lim_{\alpha\to 0}\ -2-\frac{1}{2-\alpha^2}\left(\frac{\alpha}{\sqrt{1-\alpha^2}}\right)\ =\ -2-\frac{1}{2-0}\times 0\ =-2$$

124. $\lim\limits_{x\to 0}\dfrac{2\sin^2 3x}{x^2}$ **is equal to :**

(a) 0 (b) 1 (c) 18 (d) 36

BCECE-2004

Ans. (c) : Given, $\lim\limits_{x\to 0}\dfrac{2\sin^2 3x}{x^2}$

Using L-Hospital's rule,

$$=\lim_{x\to 0}\frac{4\sin 3x.\cos 3x.3}{2x}\ =\lim_{x\to 0}\frac{6\sin 3x\ .\cos 3x}{x}$$

Again apply L-Hospital's rule,

$$=\lim_{x\to 0}\frac{6[\sin 3x.(-\sin 3x).3+\cos 3x.\cos 3x.3]}{1}$$

$$=\lim_{x\to 0}\frac{18\left[-\sin^2 3x+\cos^2 3x\right]}{1}$$

$$=\lim_{x\to 0}18\left(\cos^2 3x-\sin^2 3x\right)$$

$$=\lim_{x\to 0}\ 18\cos 6x\qquad\left\{\because \cos^2 x-\sin^2 x=\cos 2x\right\}$$

$= 18\cos 0^\circ = 18\times 1 = 18$

125. $\lim\limits_{x\to 0}\dfrac{e^{1/x}}{e^{1/x+1}}$ **is equal to :**

(a) 0 (b) 1 (c) does not exist (d) none of these

BCECE-2004

Ans. (d) : Given,

$$\lim_{x\to 0}\frac{e^{1/x}}{e^{1/x+1}}=\lim_{x\to 0}\frac{e^{1/x}}{e^{1/x}.e}\ =\ \frac{1}{e}$$

126.The value of $\lim\limits_{x\to\infty}\dfrac{\sqrt{1+x^4}-(1+x^2)}{x^2}$ **is equal to :**

(a) 0 (b) –1 (c) 2 (d) none of these

BCECE-2004

Ans. (a) : Given,

$$\lim_{x\to\infty}\frac{\sqrt{1+x^4}-(1+x^2)}{x^2}$$

Apply L'-Hospital's rule,

$$=\lim_{x\to\infty}\frac{\frac{1}{2}(1+x^4)^{-1/2}4x^3-2x}{2x}\ =\lim_{x\to\infty}\frac{4x^3}{4x\sqrt{1+x^4}}-\frac{2x}{2x}$$

$$=\lim_{x\to\infty}\frac{x^2}{\sqrt{1+x^4}}-1\lim_{x\to\infty}\frac{x^2}{x^2\sqrt{\frac{1}{x^4}+1}}-1$$

$$=\lim_{x\to\infty}\frac{1}{\sqrt{\frac{1}{x^4}+1}}-1=\frac{1}{\sqrt{0+1}}-1\ =1-1=0$$

127. If $\lim\limits_{x\to 1}\dfrac{x^2-ax+b}{(x-1)}=5$**, then (a + b) is equal to**

(a) –4 (b) –3 (c) –7 (d) 7

MHT CET-2022

Ans. (c) : We have,

$$\lim_{x\to 1}\frac{x^2-ax+b}{x-1}=5\qquad ......(i)$$

On applying L- Hospital rule, we get –

$$\lim_{x\to 1}\frac{2x-a}{1}=5$$

at $x = 1$

$2\times 1-a=5$

$a=2-5$

$a=-3$

Value $a = -3$ substituting equation (i)-

$$\lim_{x\to 1}\frac{x^2-(-3x)+b}{x-1}=5$$

$1+3+b=0$

$b=-4$

$(a+b)=-3-4=-7$

128. $\lim\limits_{x\to 0}\dfrac{2x}{|x|+x^2}=$

(a) –2 (b) Limit does not exists

(c) Limit exists (d) 2

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Ans. (b) : Given,

$$\lim_{x\to 0}\frac{2x}{|x|+x^2}$$

LHL

$$\lim_{x\to 0^-}\frac{2x}{|x|+x^2}\ =2\frac{x}{-x+x^2}\ =\frac{2x}{x^2+x}=\frac{2}{x-1}$$

RHL

$$\lim_{x\to 0^+}\frac{2x}{|x|+x^2}\ =2\lim_{x\to 0^+}\frac{x}{|x|+x^2}\ =2\cdot\frac{x}{x+x^2}=\frac{2}{x+1}$$

$\because$ LHL. $\neq$ RHL

Hence, limit does not exists,

129. $\lim\limits_{x\to 0}\dfrac{27^x-9^x-3^x+1}{\sqrt{5}-\sqrt{4+\cos x}}=$

(a) $8\sqrt{5}(\log 3)^2$ (b) $\sqrt{5}(\log 3)^2$

(c) $8\sqrt{5}\log 3$ (d) $16\sqrt{5}\log 3$

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Ans. (a) : Given,

$$\lim_{x\to 0}\frac{27^x-9^x-3^x+1}{\sqrt{5}-\sqrt{4+\cos x}}$$

On applying L Hospital's rule, we get –

$$=\lim_{x\to 0}\frac{27^x\log 27-9^x\log 9-3^x\log 3}{\frac{-1}{2\sqrt{4+\cos x}}.(-\sin x)}$$

$$=\lim_{x\to 0}\frac{2\log 3\left(3\cdot 27^x-2\cdot 9^x-3^x\right)\sqrt{4+\cos x}}{\sin x}$$

Again applying L- Hospital's rule,

$$=\frac{\lim\limits_{x\to 0}\left(2\log 3\cdot 3\cdot 27^x\log 27-2\cdot 9^x\log 9-3^x\cdot\log 3\right)\sqrt{4+\cos x}+2\log 3\left(3\cdot 27^x-2\cdot 9^x-3^x\right)\frac{1}{2\sqrt{4+\cos}}(-\sin x)}{\cos x}$$

$$=\frac{2\log 3[3\log 27-2\log 9-\log 3]\left(\sqrt{4+1}\right)+2\log 3[3-2-1]\frac{-\sin 0}{2\sqrt{4+1}}}{1}=$$

$2\log 3[9\log 3-4\log 3-\log 3]\ \sqrt{5}+0$

$=2\sqrt{5}(\log 3\times 4\log 3)=8\sqrt{5}(\log 3)^2$

130. $\lim\limits_{x\to 2}\left(\dfrac{5x-8}{8-3x}\right)^{\frac{3}{2x-4}}=$

(a) $e^{3/2}$ (b) e^6 (c) e^2 (d) $e^{5/2}$

MHT CET-2022

Ans. (b) : Given,

$$\lim_{x\to 2}\left(\frac{5x-8}{8-3x}\right)^{\frac{3}{2x-4}}$$

Let, $x-2=h\Rightarrow x=h+2$

at $x=2$, $h=0$

$$\therefore\quad \lim_{h\to 0}\left[\frac{5(2+h)-8}{8-3(2+h)}\right]^{\frac{3}{2h}}$$

$$=\lim_{h\to 0}\left[\frac{10+5h-8}{8-6-3h}\right]^{\frac{3}{2h}}=\lim_{h\to 0}\left[\frac{2+5h}{2-3h}\right]^{3/2h}$$

$$=\lim_{h\to 0}\left[\frac{1+5/2h}{1-3h/2}\right]^{3/2h}=\frac{\lim\limits_{h\to 0}\left[\left(1+\frac{5h}{2}\right)^{\frac{2}{5h}}\right]^{\frac{5}{2}\times\frac{3}{2}}}{\lim\limits_{x\to\infty}\left[\left(1-\frac{3h}{2}\right)^{\frac{-2}{3h}}\right]^{\frac{-3}{2}\times\frac{3}{2}}}$$

$$\left[\because \lim_{x\to 0}(1+x)^{\frac{1}{x}}=e\right]\qquad =\frac{e^{15/4}}{e^{-9/4}}=e^{\frac{24}{4}}=e^6$$

131. $\lim\limits_{x\to 1}\dfrac{2^{2x-2}-2^x+1}{\sin^2(x-1)}$

(a) $2\log 2$ (b) $\frac{1}{2}(\log 2)^2$ (c) $2(\log 2)^2$ (d) $(\log 2)^2$

MHT CET-2022

Ans. (d) : We have, $\lim\limits_{x\to 1}\dfrac{2^{2x-2}-2^x+1}{\sin^2(x-1)}$

Let, $x=1+h\Rightarrow x-1=h$

at $x=1$, $h\to 0$

$$\therefore\ \lim_{h\to 0}\frac{2^{2h}-2^{1+h}+1}{\sin^2 h}=\lim_{h\to 0}\frac{\left(2^h-1\right)^2}{\sin^2 h}$$

$$=\lim_{h\to 0}\left[\frac{\left(\frac{2^h-1}{h}\right)^2}{\frac{\sin^2 h}{h^2}}\right]$$

$$=\frac{(\log 2)^2}{(1)^2}\qquad\left[\because \lim\frac{a^x-1}{x}=\log a\right]$$

$=(\log 2)^2$

132. **Let $f(x)=5-|x-2|$ and $g(x)=|x+1|$, $x\in R$. If f(x) attains maximum value at α and g(x) attains minimum value at β them $\lim\limits_{x\to-\alpha\beta}\dfrac{(x-1)(x^2-5x+6)}{(x^2-6x+8)}$ is equal to**

(a) $-\frac{1}{2}$ (b) $\frac{3}{2}$ (c) $-\frac{3}{2}$ (d) $\frac{1}{2}$

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Ans. (d) : Given, $f(x)=5-|x-2|$

f(x) is maximum at

$|x-2|=0$

$x=2=\alpha$

$g(x)=|x+1|$

g(x) is minimum at

$|x+1|=0\Rightarrow x=-1=\beta$

$$\therefore\ \lim_{x\to-\alpha\beta}\frac{(x-1)(x^2-5x+6)}{x^2-6x+8}=\lim_{x\to 2}\frac{(x-1)(x^2-5x+6)}{x^2-6x+8}$$

$$=\lim_{x\to 2}\frac{(x-1)(x^2-3x-2x+6)}{x^2-4x-2x+8}$$

$$=\lim_{x\to 2}\frac{(x-1)[x(x-3)-2(x-3)]}{x(x-4)-2(x-4)}$$

$$=\lim_{x\to 2}\frac{(x-1)(x-2)(x-3)}{(x-2)(x-4)}$$

$$=\lim_{x\to 2}\frac{(x-1)(x-3)}{(x-4)}=\frac{(2-1)(2-3)}{2-4}=\frac{-1}{-2}=\frac{1}{2}$$

133. $\lim\limits_{x\to 0}\dfrac{\sin^2 x}{\sqrt{2}-\sqrt{1+\cos x}}$ **equals**

(a) 4 (b) $4\sqrt{2}$ (c) $\sqrt{2}$ (d) $2\sqrt{2}$

MHT CET-2022

Ans. (b) : We have,

$$\lim_{x\to 0}\frac{\sin^2 x}{\sqrt{2}-\sqrt{1+\cos x}}=\lim_{x\to 0}\frac{\sin^2 x}{\sqrt{2}-\sqrt{2\cos^2\frac{x}{2}}}$$

$$=\lim_{x\to 0}\frac{\sin^2 x}{\sqrt{2}\left(1-\cos\frac{x}{2}\right)}=\lim_{x\to 0}\frac{\sin^2 x}{2\sqrt{2}\sin^2\frac{x}{4}}$$

$$=\lim_{x\to 0}\frac{1}{2\sqrt{2}}\left[\frac{\sin^2 x}{x^2}\times\frac{(x/4)^2}{\sin^2\frac{x}{4}}\right]\times 16=\frac{16}{2\sqrt{2}}=4\sqrt{2}$$

144. $\lim\limits_{x\to\infty}\left[\dfrac{8x^2+5x+3}{2x^2-7x-5}\right]^{\frac{4x+3}{8x-1}}=$

(a) 2 (b) $\sqrt{2}$ (c) 4 (d) $\dfrac{1}{2}$

MHT CET-2022

Ans. (a) : We have,

$$\lim_{x\to\infty}\left[\frac{8x^2+5x+3}{2x^2-7x-5}\right]^{\frac{4x+3}{8x-1}}$$

$$\left[\lim_{x\to\infty}\left(\frac{8x^2+5x+3}{2x^2-7x-5}\right)\right]^{\lim\limits_{x\to\infty}\frac{4x+3}{8x-1}}$$

$\because$ $$\lim_{x\to\infty}\frac{8x^2+5x+3}{2x^2-7x-5}=\lim_{x\to\infty}\frac{\frac{8x^2+5x+3}{x^2}}{\frac{2x^2-7x-5}{x^2}}$$

Dividing numerator and denominator by x^2

$$=\lim_{x\to\infty}\left[\frac{8+5/x+3/x^2}{2-7/x-5/x^2}\right]=\frac{8+0+0}{2-0-0}=4$$

$\because$ $$\lim_{x\to\infty}\frac{4x+3}{8x-1}=\lim_{x\to\infty}\left(\frac{4+3/x}{8-1/x}\right)=\frac{4+0}{8-0}=\frac{1}{2}$$

$\therefore$ $$\lim_{x\to\infty}\left[\frac{8x^2+5x+3}{2x^2-7x-5}\right]^{\frac{4x+3}{8x-1}}=4^{1/2}=\left(2^2\right)^{\frac{1}{2}}=2$$

135. $\lim\limits_{n\to\infty}n\left(\sqrt{n^2+9}-n\right)=$

(a) $\dfrac{9}{2}$ (b) $\dfrac{9}{\sqrt{2}}$ (c) 9 (d) $\dfrac{9}{4}$

MHT CET-2022

Ans. (a) : Given,

$$\lim_{n\to\infty}n\left(\sqrt{n^2+9}-n\right)$$

$$=\lim_{n\to\infty}n\left(\sqrt{n^2+9}-n\right)\cdot\frac{\left(\sqrt{n^2+9}+n\right)}{\left(\sqrt{n^2+9}+n\right)}$$

$$=\lim_{n\to\infty}\frac{n\left(n^2+9-n^2\right)}{\sqrt{n^2+9}+n}=\lim_{n\to\infty}\frac{9n}{\sqrt{n^2+9}+n}$$

$$=\lim_{n\to\infty}\frac{\frac{9n}{\sqrt{n^2+9}+n}}{n}=\lim_{n\to\infty}\frac{9}{\sqrt{\frac{n^2+9}{n^2}}+1}$$

$$=\lim_{n\to\infty}\frac{9}{\sqrt{1+\frac{9}{n^2}}+1}=\frac{9}{\sqrt{1+0}+1}=\frac{9}{2}$$

136. $\lim\limits_{x\to 3}\dfrac{5^{x-3}-4^{x-3}}{\sin(x-3)}=$

(a) $\log\left(\dfrac{5}{4}\right)$ (b) $\dfrac{\log 5}{\log 4}$ (c) $\dfrac{\log 5}{4}$ (d) log 5 – 4

MHT CET-2021

Ans. (a) : Given,

$$\lim_{x\to 3}\frac{5^{x-3}-4^{x-3}}{\sin(x-3)}\text{ while (0/0) form}$$

Apply L. Hospital's rule i.e., Differentiating numerator and denominator. We get,

$$\lim_{x\to 3}\frac{5^{x-3}\cdot\log 5-4^{x-3}\cdot\log 4}{\cos(x-3)}$$

$$=\frac{\log 5-\log 4}{1}=\log\left(\frac{5}{4}\right)$$

137. **If** $\lim\limits_{x\to 5}\dfrac{x^k-5^k}{x-5}=500$**, then the value of k, where** $k\in N$ **is**

(a) 6 (b) 5 (c) 4 (d) 3

MHT CET-2021

J&K CET-2010

Ans. (c) : Given,

$$\lim_{x\to 5}\frac{x^k-5^k}{x-5}=500$$

L.H.S. Differentiating numerator and denominator, we get L. Hospital rule

$$\lim_{x\to 5}\frac{x^k-5^k}{x-5}=\lim_{x\to 5}\frac{k\cdot x^{k-1}-0}{1-0}$$

$$=\lim_{x\to 5}k\cdot x^{k-1}$$

$$=k\cdot 5^{k-1}=500$$

Let us check the options.

Put, $k=6$

$6\cdot 5^{6-1}=6\cdot 5^5\neq 500$

Put, $k=5$

$5\cdot 5^{5-1}=5\cdot 5^4\neq 500$

Put, $k=4$

$4\cdot 5^3=4\cdot 125=500$

$\therefore$ $k=4$ is the solution.

138. $\lim\limits_{x\to 1}\dfrac{(2x-3)(\sqrt{x}-1)}{2x^2+x-3}=$

(a) $\dfrac{1}{10}$ (b) $\dfrac{-1}{5}$ (c) $\dfrac{1}{5}$ (d) $\dfrac{-1}{10}$

MHT CET-2021

Ans. (d) : Let,

$$y=\lim_{x\to 1}\frac{(2x-3)(\sqrt{x}-1)}{2x^2+x-3}$$

Here,

$2x^2 + x - 3 = 2x^2 + 3x - 2x - 3$
$= x(2x+3) - 1(2x+3)$
$= (2x+3)(x-1)$

$$\therefore \quad y=\lim_{x\to 1}\frac{(2x-3)(\sqrt{x}-1)}{(2x+3)(\sqrt{x}-1)(\sqrt{x}+1)}$$

$$y=\lim_{x\to 1}\frac{(2x-3)}{(2x+3)(\sqrt{x}+1)}$$

$$y=\frac{(2-3)}{(2+3)(1+1)}=\frac{-1}{10}$$

139. If $a=\lim\limits_{n\to\infty}\dfrac{1+2+3+.....+n}{n^2}$ and $b=\lim\limits_{n\to\infty}\dfrac{1^2+2^2+3^2+.....+n^2}{n^3}$, then

(a) 3a = 2b (b) 2a = 3b (c) a = b (d) a = 2b

MHT CET-2021

Ans. (b) : Given,

$$a=\lim_{n\to\infty}\frac{(1+2+3+...+n)}{n^2}=\lim_{n\to\infty}\frac{n(n+1)/2}{n^2}$$

$$=\lim_{n\to\infty}\frac{1}{2}(1+1/n)=\frac{1}{2}(1+0)=\frac{1}{2}$$

$$b=\lim_{n\to\infty}\frac{1^2+2^2+3^2+....+n^2}{n^3}$$

$$=\lim_{n\to\infty}\frac{n(n+1)(2n+1)}{6\cdot n^3}$$

$$=\lim_{n\to\infty}\frac{1}{6}\left[\left(\frac{n+1}{n}\right)\cdot\frac{(2n+1)}{n}\right]$$

$$=\lim_{n\to\infty}\frac{1}{6}[(1+1/n)(2+1/n)]=1/6\,[1]\times[2]=1/3$$

$$\therefore \quad \frac{a}{b}=\frac{1/2}{1/3}=\frac{3}{2}$$

$\therefore \quad 2a = 3b$

140. $\lim\limits_{t\to 0}\dfrac{\sin 2t}{8t^2+4t}$ is equal to

(a) $\dfrac{1}{2}$ (b) $\dfrac{2}{5}$ (c) $\dfrac{1}{6}$ (d) $\dfrac{1}{3}$ (e) 1

Kerala CEE-2021

Ans. (a) : Let,

$$y=\lim_{t\to 0}\frac{\sin 2t}{8t^2+4t}$$

$$y=\lim_{t\to 0}\frac{\sin 2t}{4t(2t+1)}$$

$$y=\frac{1}{2}\lim_{t\to 0}\frac{\sin 2t}{2t}\cdot\frac{1}{(2t+1)}$$

Since $t\to 0$, we also see that $2t\to 0$

$\therefore$ Taking of 2t as x

$$\lim_{x\to 0}\left(\frac{\sin x}{x}\right)=1$$

$$\lim_{t\to 0}\left(\frac{\sin 2t}{2t}\right)=1$$

$$=\frac{1}{2}\cdot\frac{1}{(0+1)}=\frac{1}{2}$$

141. $\lim\limits_{x\to 0}\dfrac{x}{\sqrt{9-x}-3}$ is equal to

(a) 6 (b) 3 (c) –3 (d) –6 (e) 0

Kerala CEE-2021

Ans. (d) : $y=\lim\limits_{x\to 0}\dfrac{x}{\sqrt{9-x}-3}$

$$y=\lim_{x\to 0}\left(\frac{x}{\sqrt{9-x}-3}\right)\times\left(\frac{\sqrt{9-x}+3}{\sqrt{9-x}+3}\right)$$

$$y=\lim_{x\to 0}\left(\frac{x\left(\sqrt{9-x}+3\right)}{9-x-9}\right)$$

$$y=\lim_{x\to 0}\left(\frac{x\left(\sqrt{9-x}+3\right)}{-x}\right)$$

$$y=\frac{\sqrt{9-0}+3}{-1}$$

$y=-6$

142. Let $f(x)=\begin{cases}3x+2, & \text{if } x<-2\\ x^2-3x-1, & \text{if } x\ge -2\end{cases}$. Then $\lim\limits_{x\to -2^-}f(x)$ and $\lim\limits_{x\to -2^+}f(x)$ are respectively

(a) –4, 3 (b) 6, 3 (c) –6, 3 (d) –4, 9 (e) 9, – 4

Kerala CEE-2021

Ans. (d) : We have,

$$f(x)=\begin{cases}3x+2, & \text{if } x<-2\\ x^2-3x-1, & \text{if } x\ge -2\end{cases}$$

$$\lim_{x\to -2^-}f(x)=\lim_{h\to 0}f(-2-h)$$

$$=\lim_{h\to 0}\left[3(-2-h)+2\right]=\lim_{h\to 0}\left[-6-3h+2\right]$$

$$=\lim_{h\to -2}f(x)-4-3\times 0=-4$$

$$=\lim_{h\to -2^+}f(x)=\lim_{h\to -0}f(-2+h)$$

$$=\lim_{h\to 0}\left[(-2+h)^2-3(-2+h)-1\right]$$

$$=\lim_{h\to 0}\left[4+h^2-4h+6-3h-1\right]$$

$$=\lim_{h\to -2^+}f(x)\left[10+0-4\times 0-3\times 0-1\right]$$

$$\lim_{h\to -2^+}=9$$

143. $\lim\limits_{x\to -3}\frac{x^2+16x+39}{2x^2+7x+3}$ is equal to

(a) 2 (b) $\frac{8}{3}$ (c) $\frac{-8}{3}$ (d) – 2 (e) 0

Kerala CEE-2021

Ans. (d) : $y=\lim\limits_{x\to -3}\left(\frac{x^2+16x+39}{2x^2+7x+3}\right)$

$$y=\lim_{x\to -3}\left(\frac{(x+3)(x+13)}{(x+3)(2x+1)}\right)$$

$$y=\lim_{x\to -3}\left(\frac{x+13}{2x+1}\right)$$

$$y=\frac{-3+13}{-6+1}\Rightarrow y=-2$$

144. $\lim\limits_{x\to 0}\frac{\sin(t^2)}{t\sin(5t)}$ is equal to

(a) 5 (b) 25 (c) $\frac{1}{25}$ (d) $\frac{1}{5}$ (e) 0

Kerala CEE-2022

Ans. (d) : Let,

$$y=\lim_{t\to 0}\left[\frac{\sin(t^2)}{t\sin(5t)}\right]$$

$$y=\lim_{t\to 0}\left[\frac{\sin(t^2)}{5t.t\frac{\sin(5t)}{5t}}\right]$$

$$y=\frac{1}{5}\left[\frac{\lim\limits_{t\to 0}\left(\frac{\sin t^2}{t^2}\right)}{\lim\limits_{t\to 0}\frac{\sin(5t)}{5t}}\right] \qquad \left[\because \lim_{x\to 0}\left(\frac{\sin x}{x}\right)=1\right]$$

$\therefore \quad y=\frac{1}{5}\times\frac{1}{1} \Rightarrow y=\frac{1}{5}$

145. If $f(x)=\begin{cases} x^2\sin\left(\frac{\pi}{6}x\right) \text{for } x\le -3 \\ x\cos\left(\frac{\pi}{3}x\right) \text{for } x> -3\end{cases}$,

then the value of $\lim_{x\to -3^+} f(x)$ is equal to

(a) 3 (b) –3 (c) 9 (d) –9 (e) 0

Kerala CEE-2022

Ans. (a) : Given,

$$\lim_{x\to -3^+} f(x)=\lim_{h\to 0} f(-3+h) \qquad (\because x>-3)$$

$\therefore \quad =\lim\limits_{h\to 0} f(-3+h)=\lim\limits_{x\to 0}(-3+h)\cos\left[\frac{\pi}{3}(-3+h)\right]$

$$\lim_{h\to 0} f(-3+h)=(-3+0)\cos\left[\frac{\pi}{3}(-3+0)\right]$$

$= -3\cos(-\pi) \quad [\cos(-\theta)=\cos\theta]$

$= 3\cos\pi = 3\times 1$

$\therefore \lim\limits_{x\to -3^+} f(x)=3$

146. $\lim_{t\to 0}\left(\frac{(2t-3)(t-2)}{t}-\frac{3(t+2)}{t}\right)$ is equal to

(a) 10 (b) –10 (c) –7 (d) 7 (e) 5

Kerala CEE-2022

Ans. (b) : Let,

$$y=\lim_{t\to 0}\left[\frac{(2t-3)(t-2)}{t}-\frac{3(t+2)}{t}\right]$$

$$y=\lim_{t\to 0}\left[\frac{2t^2-4t-3t+6-3t-6}{t}\right]$$

$$y=\lim_{t\to 0}\left[\frac{2t^2-10t}{t}\right] \Rightarrow y=\lim_{t\to 0}[2t-10]$$

$y = 2\times 0-10 \Rightarrow y=-10$

147. $\lim\limits_{x\to 0}\frac{x^{100}\sin 7x}{(\sin x)^{101}}$ is equal to

(a) 7 (b) $\frac{1}{7}$ (c) 14 (d) 1 (e) 0

Kerala CEE-2020

Ans. (a) : Let, $y=\lim\limits_{x\to 0}\left(\frac{x^{100}\sin(7x)}{(\sin x)^{101}}\right)$

$$y=\lim_{x\to 0}\left(\frac{x^{100}}{x^{101}}\right)\left[\frac{\sin 7x}{\left(\frac{\sin x}{x}\right)^{101}}\right]$$

$$y=\lim_{x\to 0}7\left(\frac{\sin 7x}{7x}\right)\times\frac{1}{\left(\frac{\sin x}{x}\right)^{101}}$$

$\because \quad \lim\limits_{x\to 0}\left(\frac{\sin\theta}{\theta}\right)=1$

as $\quad x\to 0$ so, $7x\to 0$

$\therefore \quad y=7$

148. $\lim\limits_{x\to 3}\frac{e^{x-3}-x+1}{x^2-\log(x-2)}$ is equal to

(a) $\frac{-1}{3}$ (b) $\frac{-2}{9}$ (c) $\frac{-1}{2}$ (d) $\frac{-1}{4}$ (e) $\frac{-1}{9}$

Kerala CEE-2020

Ans. (e) : Let,

$$y=\lim_{x\to 3}\left[\frac{e^{x-3}-x+1}{x^2-\log(x-2)}\right]\Rightarrow y=\left[\frac{e^{3-3}-3+1}{(3)^2-\log(3-2)}\right]$$

$$y=\frac{e^0-3+1}{9-\log 1}\Rightarrow y=\frac{1-3+1}{9-0}\Rightarrow y=\frac{-1}{9}$$

149. $\lim\limits_{x\to 0}\frac{1+x-e^x}{x^2}=$

(a) $\frac{1}{2}$ (b) $\frac{-1}{2}$ (c) 1 (d) –1 (e) 0

Kerala CEE-2018

Ans. (b) : Let, $y=\lim_{x\to0}\left(\frac{1+x-e^x}{x^2}\right)$

$\because \quad e^x = 1+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+......$

$\therefore \quad y=\lim_{x\to0}\left[\frac{(1+x)-\left(1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+.....\right)}{x^2}\right]$

$y=\lim_{x\to0}\left[\frac{-x^2\left(\frac{1}{2!}+\frac{x}{3!}+.....\right)}{x^2}\right]$

$y=-\frac{1}{2!}, \qquad y=-\frac{1}{2}$

150. $\lim_{x\to0}\frac{\left(\sqrt{1+2x}\right)-1}{x}=$

(a) 0 (b) –1 (c) $\frac{1}{2}$ (d) 1 (e) $\frac{-1}{2}$

Kerala CEE-2018

Ans. (d) : We have, $\lim_{x\to0}\frac{\left(\sqrt{1+2x}\right)-1}{x}$

ON applying L' – Hospital's rule, we get –

$\lim_{x\to0}\frac{\frac{1}{2\sqrt{1+2x}}\cdot2-0}{1}$

$\lim_{x\to0}\frac{1}{\sqrt{1+2x}}$

Putting limit, x = 0

$\frac{1}{\sqrt{1+2(0)}}=1$

151. $\lim_{x\to0}\frac{1-\cos mx}{1-\cos nx}=$

(a) $\frac{m^2}{n^2}$ (b) $\frac{n^2}{m^2}$ (c) ∞ (d) –∞ (e) 0

Kerala CEE-2018

Ans. (a) : We have, $\lim_{x\to0}\frac{1-\cos mx}{1-\cos nx}$

We know that,

$\cos 2x = 1 - 2\sin^2 x$

$\therefore \quad 2\sin^2\frac{x}{2}=1-\cos\frac{x}{2}$

Now, $\lim_{x\to0}\frac{1-\cos mx}{1-\cos nx}=\lim_{x\to0}\left\{\frac{2\sin^2\frac{mx}{2}}{2\sin^2\frac{nx}{2}}\right\}$

$=\lim_{x\to0}\left[\left\{\frac{\sin\frac{mx}{2}}{\frac{mx}{2}}\right\}^2\cdot\frac{m^2x^2}{4}\cdot\frac{1}{\left\{\frac{\sin\frac{nx}{2}}{\frac{nx}{2}}\right\}^2}\cdot\frac{4}{n^2x^2}\right]$

$=\frac{m^2}{n^2}\times1=\frac{m^2}{n^2}$

152. If f(9) = f '(9) = 0, then $\lim_{x\to9}\frac{\sqrt{f(x)}-3}{\sqrt{x}-3}$ is equal to

(a) 0 (b) f(0) (c) f '(3) (d) f (9) (e) 1

Kerala CEE-2017

Ans. (a) : Given, $\lim_{x\to9}\frac{\sqrt{f(x)}-3}{\sqrt{x}-3}$ f(9) = f '(9) = 0

Applying L' Hospital rule,

$=\lim_{x\to9}\frac{\frac{f'(x)}{2\sqrt{f(x)}}}{\frac{1}{2\sqrt{x}}}=\lim_{x\to9}\frac{\sqrt{x}f'(x)}{\sqrt{f(x)}}$

$=\frac{\sqrt{9}f'(9)}{\sqrt{f(9)}}=\frac{3\times0}{0}=0$

153. The value of $\lim_{n\to\infty}\frac{{}^nC_3-{}^nP_3}{n^3}$ is equal to

(a) $\frac{-5}{6}$ (b) $\frac{5}{6}$ (c) $\frac{1}{6}$ (d) $-\frac{1}{6}$ (e) $\frac{2}{3}$

Kerala CEE-2016

Ans. (a) : $\lim_{n\to\infty}\left(\frac{{}^nC_3-{}^nP_3}{n^3}\right)=\lim_{n\to\infty}\left(\frac{{}^nC_3-{}^nC_3 3!}{n^3}\right)$

$=\lim_{n\to\infty}\left[\frac{{}^nC_3(1-3!)}{n^3}\right]=-5\lim_{n\to\infty}\left[\frac{n(n-1)(n-2)}{1\times2\times3n^3}\right]$

$=\frac{-5}{6}\lim_{n\to\infty}\left[1\left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)\right]=\frac{-5}{6}$

154. The value of $\lim_{x\to0}\frac{\cot4x}{\operatorname{cosec}3x}$ is equal to

(a) $\frac{4}{3}$ (b) $\frac{3}{4}$ (c) $\frac{2}{3}$ (d) $\frac{3}{2}$ (e) 0

Kerala CEE-2016

Ans. (b) : Given, $\lim_{x\to0}\frac{\cot4x}{\operatorname{cosec}3x}$

$=\lim_{x\to0}\left(\frac{\sin3x}{\tan4x}\right)=\lim_{x\to0}\frac{\frac{\sin3x}{3x}\times3x}{\frac{\tan4x}{4x}\times4x}=\lim_{x\to0}\frac{3x}{4x}=\frac{3}{4}$

155. **If $f(x) = 3x^2 - 7x + 5$, then $\lim_{x\to 0}\frac{f(x)-f(0)}{x}$ is equal to**

(a) 6 (b) –7 (c) 7 (d) –6 (e) 5

Kerala CEE-2015

Ans. (b) : Given,

$f(x) = 3x^2 - 7x + 5$

$f(0) = 3 \times 0 - 7 \times 0 + 5 = 5$

$$\lim_{x\to 0}\frac{f(x)-f(0)}{x} = \lim_{x\to 0}\frac{3x^2-7x+5-5}{x}$$

$$= \lim_{x\to 0}\frac{3x^2-7x}{x}$$

L' Hospital rule,

$$= \lim_{x\to 0}\frac{6x-7}{1} = -7$$

156. **$\lim_{x\to\infty}\left(\frac{x^2}{3x-2}-\frac{x}{3}\right)$ is equal to**

(a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{-2}{3}$ (d) $\frac{-2}{9}$ (e) $\frac{2}{9}$

Kerala CEE-2015

Ans. (e) : Let,

$$y = \lim_{x\to\infty}\left(\frac{x^2}{3x-2}-\frac{x}{3}\right) = \lim_{x\to\infty}\frac{3x^2-3x^2+2x}{3(3x-2)}$$

$$y = \lim_{x\to\infty}\frac{2x}{3(3x-2)}$$

$$y = \lim_{x\to\infty}\frac{2x}{3(3-2/x)x}$$

$$y = \frac{2}{3(3-0)} = \frac{2}{9}$$

157. **The value of $\lim_{y\to\infty}\left[y\sin\left(\frac{1}{y}\right)-\frac{1}{y}\right]$ is equal to**

(a) 1 (b) ∞ (c) –1 (d) 0

(e) $-\infty$

Kerala CEE-2015

Ans. (a) : $\lim_{y\to\infty}\left[y\sin(1/y)-1/y\right]$

$$= \lim_{y\to\infty}\left(\frac{\sin(1/y)}{1/y}\right) - \lim_{y\to\infty}(1/y) = 1 - 0 = 1$$

158. **$\lim_{x\to 0}\left(\frac{10\sin 9x}{9\sin 10x}\right)\left(\frac{8\sin 7x}{7\sin 8x}\right)\left(\frac{6\sin 5x}{5\sin 6x}\right)\left(\frac{4\sin 3x}{3\sin 4x}\right)\left(\frac{\sin x}{\sin 2x}\right)$ is equal to**

(a) $\frac{63}{256}$ (b) $\frac{1}{6}$ (c) $\frac{6}{5}$ (d) $\frac{1}{2}$ (e) $\frac{256}{63}$

Kerala CEE-2015

Ans. (d) :

$$\lim_{x\to 0}\left[\frac{10\sin 9x}{9\sin 10x}\right]\left[\frac{8\sin 7x}{7\sin 8x}\right]\left[\frac{6\sin 5x}{5\sin 6x}\right]\left[\frac{4\sin 3x}{3\sin 4x}\right]\left[\frac{\sin x}{\sin 2x}\right]$$

$$= \lim_{x\to 0}\left[\frac{\frac{\sin 9x}{9x}}{\frac{\sin 10x}{10x}}\right]\left[\frac{\frac{\sin 7x}{7x}}{\frac{\sin 8x}{8x}}\right]\left[\frac{\frac{\sin 5x}{5x}}{\frac{\sin 6x}{6x}}\right]\left[\frac{\frac{\sin 3x}{3x}}{\frac{\sin 4x}{4x}}\right]\left[\frac{\frac{\sin x}{x}}{\frac{\sin 2x}{2x}}\times\frac{1}{2}\right] = \frac{1}{2}$$

159. **$\lim_{x\to 0}\frac{e^{x^2}-\cos x}{x^2}$ is equal to**

(a) $\frac{3}{2}$ (b) $\frac{1}{2}$ (c) 1 (d) $-\frac{3}{2}$ (e) $-\frac{1}{2}$

Kerala CEE-2013

Ans. (c) : Let, $y = \lim_{x\to 0}\frac{e^{x^2}-\cos x}{x^2}$

$$y = \lim_{x\to 0}\frac{\left(1+\frac{x^2}{1!}+\frac{x^4}{2!}+....\infty\right)-\left(1-\frac{x^2}{2!}+\frac{x^4}{4!}-....\infty\right)}{x^2}$$

$$y = \lim_{x\to 0}\frac{x^2+\left(\frac{1}{2!}-\frac{1}{4!}\right)x^4+.....}{x^2}$$

$$y = \lim_{x\to 0}\ 1+\left(\frac{1}{2}-\frac{1}{24}\right)x^2+....$$

$$y = 1+0 = 1$$

160. **$\lim_{x\to 3}\frac{\sqrt{x}-\sqrt{3}}{\sqrt{x^2-9}}$ is equal to**

(a) 1 (b) 3 (c) $\sqrt{3}$ (d) $-\sqrt{3}$ (e) 0

Kerala CEE-2013

Ans. (e) :

$$\lim_{x\to 3}\frac{\sqrt{x}-\sqrt{3}}{\sqrt{x^2-9}} = \lim_{x\to 3}\frac{\sqrt{x}-\sqrt{3}}{\sqrt{(x-3)(x+3)}}\times\frac{\sqrt{x}+\sqrt{3}}{\sqrt{x}+\sqrt{3}}$$

$$= \lim_{x\to 3}\frac{x-3}{\sqrt{(x-3)(x+3)}}\times\frac{1}{\sqrt{x}+\sqrt{3}}$$

$$= \lim_{x\to 3}\frac{\sqrt{x-3}}{\sqrt{x+3}}\times\frac{1}{\sqrt{x}+\sqrt{3}} = 0$$

161. **The value of $\lim_{x\to 0}\frac{\log(1+2x)}{x}$ is equal to**

(a) 1 (b) 2 (c) 3 (d) $\frac{3}{2}$ (e) $\frac{1}{2}$

Kerala CEE-2012

Ans. (b) : $\lim_{x\to 0}\frac{\log(1+2x)}{x} = \lim_{x\to 0}\frac{2\log(1+2x)}{2x}$

Let, $y = 2x$

$$\therefore \lim_{y\to 0}\frac{2\log(1+y)}{y} \quad \left[\because \lim_{y\to 0}\frac{\log(1+y)}{y} = 1\right]$$

$= 1 \times 2 = 2$

162. $\lim\limits_{x\to 0^-}\dfrac{1}{3-2^{1/x}}$ **is equal to**

(a) 0 (b) 1 (c) $\dfrac{1}{2}$ (d) $\dfrac{1}{3}$ (e) $-\infty$

Kerala CEE-2012

Ans. (d) : $\lim\limits_{x\to 0^-}\dfrac{1}{3-2^{\frac{1}{x}}}$

Let $x = 0 - h$

$\therefore \lim\limits_{h\to 0}\dfrac{1}{3-2^{\frac{1}{0-h}}}=\lim\limits_{h\to 0}\dfrac{1}{3-2^{\frac{-1}{h}}}$

$=\dfrac{1}{3-2^{-1/0}}=\dfrac{1}{3-2^{-\infty}}=\dfrac{1}{3-0}=\dfrac{1}{3}$

163. $\lim\limits_{k\to\infty}\left(\dfrac{1^3+2^3+3^3+....+k^3}{k^4}\right)$ **is equal to**

(a) 0 (b) 2 (c) $\dfrac{1}{3}$ (d) ∞ (e) $\dfrac{1}{4}$

Kerala CEE-2011

Ans. (e) : $\lim\limits_{k\to\infty}\left(\dfrac{1^3+2^3+3^3+.....+k^3}{k^4}\right)$

$=\lim\limits_{k\to\infty}\left(\dfrac{k^2(k+1)^2}{4}\times\dfrac{1}{k^4}\right)$

$\left[\because 1^3+2^3+.....+k^3=\left[\dfrac{k(k+1)}{2}\right]^2\right]$

$=\lim\limits_{k\to\infty}\left(\dfrac{k^4(1+1/k)^2}{4}\times\dfrac{1}{k^4}\right)=\dfrac{(1+0)^2}{4}=\dfrac{1}{4}$

164. $\lim\limits_{x\to 2}\dfrac{x^{100}-2^{100}}{x^{77}-2^{77}}$ **is equal to**

(a) $\dfrac{100}{77}$ (b) $\dfrac{100}{77}(2^{22})$ (c) $\dfrac{100}{77}(2^{21})$

(d) $\dfrac{100}{77}(2^{23})$ (e) $\dfrac{100}{77}(2^{24})$

Kerala CEE-2011

Ans. (d) : $\lim\limits_{x\to 2}\dfrac{x^{100}-2^{100}}{x^{77}-2^{77}}=\lim\limits_{x\to 2}\dfrac{x^{100}-2^{100}}{x-2}\times\dfrac{x-2}{x^{77}-2^{77}}$

$=\lim\limits_{x\to 2}\left(\dfrac{x^{100}-2^{100}}{x-2}\right)\times\dfrac{1}{\lim\limits_{x\to 2}\left(\dfrac{x^{77}-2^{77}}{x-2}\right)}$

$=100\times 2^{99}\times\dfrac{1}{77\times 2^{76}}=\dfrac{100}{77}\times 2^{23}$

165. $\lim\limits_{x\to 0}\dfrac{(1+2x)^{10}-1}{x}$ **is equal to**

(a) 5 (b) 10 (c) 15 (d) 20 (e) 0

Kerala CEE-2011

Ans. (d) : $\lim\limits_{x\to 0}\dfrac{(1+2x)^{10}-1}{x}$

By L– Hospital's rule,

$\lim\limits_{x\to 0}\dfrac{10(1+2x)^9\times 2-0}{1}=\lim\limits_{x\to 0}\dfrac{20(1+2x)^9}{1}$

$=20\times(1+0)^9=20$

166. $\lim\limits_{x\to\infty}\left(\dfrac{x^3}{3x^2-4}-\dfrac{x^2}{3x+2}\right)$ **is equal to**

(a) $-\dfrac{1}{4}$ (b) $-\dfrac{1}{2}$ (c) 0 (d) $\dfrac{2}{9}$ (e) $-\dfrac{6}{5}$

Kerala CEE-2010

Ans. (d) : Given,

$=\lim\limits_{x\to\infty}\left(\dfrac{x^3}{3x^2-4}-\dfrac{x^2}{3x+2}\right)$

$=\lim\limits_{x\to\infty}\left(\dfrac{2(x^3+2x^2)}{9x^3+6x^2-12x-8}\right)$

$=\lim\limits_{x\to\infty}\left(\dfrac{2(1+2/x)}{9+\dfrac{6}{x}-\dfrac{12}{x^2}-\dfrac{-8}{x^3}}\right)=\dfrac{2\times 1}{9}=\dfrac{2}{9}$

167. $\lim\limits_{x\to 0}\left(\dfrac{x}{\sqrt{1+x}-\sqrt{1-x}}\right)$ **is equal to**

(a) 0 (b) 1 (c) 2 (d) –1 (e) –2

Kerala CEE-2010

Ans. (b) : Given,

$\lim\limits_{x\to 0}\dfrac{x}{\sqrt{1+x}-\sqrt{1-x}}=\lim\limits_{x\to 0}\dfrac{x}{\sqrt{1+x}-\sqrt{1-x}}\times\dfrac{\sqrt{1+x}+\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}$

$=\lim\limits_{x\to 0}\dfrac{x\left(\sqrt{1+x}+\sqrt{1-x}\right)}{1+x-1+x}=\lim\limits_{x\to 0}\dfrac{x\left(\sqrt{1+x}+\sqrt{1-x}\right)}{2x}$

$=\lim\limits_{x\to 0}\dfrac{\sqrt{1+x}+\sqrt{1-x}}{2}=\dfrac{2}{2}=1$

168. The value of $\lim\limits_{n\to\infty}\left\{\dfrac{1}{na}+\dfrac{1}{na+1}+\dfrac{1}{na+2}+.......+\dfrac{1}{nb}\right\}$ **is**

(a) log (ab) (b) log (a/b) (c) log (b/a) (d) –log(a/b)

Manipal UGET-2013

Ans. (c) :

$\lim\limits_{n\to\infty}\left[\dfrac{1}{na}+\dfrac{1}{na+1}+\dfrac{1}{na+2}+........\dfrac{1}{nb}\right]$

$\lim\limits_{n\to\infty}\dfrac{1}{n}\left[\dfrac{1}{a}+\dfrac{1}{a+\frac{1}{n}}+\dfrac{1}{a+\frac{2}{n}}+........+\dfrac{1}{b}\right]$

$$= \lim_{n\to\infty} \frac{1}{n}\left[\frac{1}{a}+\frac{1}{a+\frac{1}{n}}+ \; .... + \frac{1}{a+(b-a)}\right]$$

$$= \lim_{n\to\infty} \sum_{r=0}^{n(b-a)} \frac{1}{na+r} = \lim_{n\to\infty} \frac{1}{n}\sum_{r=0}^{n(b-a)} \frac{1}{a+\left(\frac{r}{n}\right)}$$

Let $\frac{r}{n} = x$

at $r = 0, \quad x = 0$

$r = (b-a)n, \quad x = (b-a)$

So,

$$= \int_0^{(b-a)} \frac{1}{a+x}dx = \left[\ell n\,|a+x|\right]_0^{(b-a)}$$

$$= \ell n\,|a+b-a| - \ell n\,|a| = \ell n\left(\frac{b}{a}\right)$$

169. Let $f(xy) = f(x).f(y)$ for all $x, y \in R$. If $f'(1) = 2$ and $f(4) = 4$, then $f'(4)$ equal to

(a) 4 (b) 1 (c) $\frac{1}{2}$ (d) 8

Manipal UGET-2013

Ans. (d) :

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0}\frac{f(x)f\left(1+\frac{h}{x}\right)-f(x)}{h}$$

$$f'(x) = \frac{f(x)}{x}(f'(1))$$

$$\int\frac{df(x)}{f(x)} = \int\frac{2}{x}dx \qquad [\because f'(1) = 2]$$

$\log f(x) = 2\log x + c$

Given,

$f(2) = 4$

$\Rightarrow \log 4 = 2\log 2 + c$

$\Rightarrow c = 0$

$\Rightarrow \log f(x) = 2\log x$

$\Rightarrow \log f(x) = \log x^2$

$f(x) = x^2$

$f'(x) = 2x$

$f'(4) = 2\times 4 \Rightarrow f'(4) = 8$

170. The value of $\lim_{x\to\infty}\left\{...\frac{x}{x+\frac{\sqrt[3]{x}}{x+\frac{\sqrt[3]{x}}{x+\sqrt[3]{x}}}}\right\}$ is

(a) 0 (b) 1 (c) 2 (d) 1/2

Manipal UGET-2013

Ans. (b) : Given that-

$$\lim_{x\to\infty}\left\{\frac{x}{x+\frac{\sqrt[3]{x}}{x+\frac{\sqrt[3]{x}}{x+\sqrt[3]{x}}}}\right\}$$

$$\text{Let } y = \frac{x}{x+\frac{\sqrt[3]{x}}{x+\sqrt[3]{x}...\text{infinity}}}$$

$$y = \frac{x}{x+\frac{1}{x^{2/3}}.\frac{x}{x+\sqrt[3]{x}....\infty}}$$

$$y = \frac{x}{x+\frac{y}{x^{2/3}}}$$

$$\Rightarrow \quad y = \frac{x^{5/3}}{x^{5/3}+y}$$

$$\Rightarrow \quad y^2 + \left(x^{5/3}\right).y - x^{5/3} = 0$$

$$\therefore \quad y = \frac{-x^{5/3}\pm\sqrt{x^{10/3}+4x^{5/3}}}{2}$$

$$y = \frac{-x^{5/3}+\sqrt{x^{10/3}+4x^{5/3}}}{2} \quad (\because y > 0)$$

$$= \frac{4x^{5/3}}{2\left(\sqrt{x^{10/3}+4x^{5/3}}+x^{5/3}\right)} = \frac{2}{\sqrt{\left(1+\frac{4}{x^{5/3}}\right)}+1}$$

$$\therefore \lim_{x\to\infty} y = \frac{2}{\sqrt{1+0}+1} = \frac{2}{2} = 1$$

171. The value of $\lim_{x\to 1}\frac{\left[\sum_{K=1}^{100}x^K\right]-100}{x-1}$ is

(a) –5050 (b) 0 (c) 5050 (d) None of these

Manipal UGET-2013

Ans. (c) :

$$\lim_{x\to 1}\frac{\left[\sum_{K=1}^{100}x^K\right]-100}{(x-1)}$$

$$= \lim_{x\to 1}\frac{\left(x+x^2+x^3+.....+x^{100}\right)-100}{(x-1)}$$

$$= \lim_{x\to 1}\frac{(x-1)+\left(x^2-1\right)+\left(x^3-1\right)+...+\left(x^{100}-1\right)}{(x-1)}$$

$$= \lim_{x\to 1}\left\{\left(\frac{x-1}{x-1}\right)+\left(\frac{x^2-1}{x-1}\right)+\left(\frac{x^3-1}{x-1}\right)+...+\left(\frac{x^{100}-1}{x-1}\right)\right\}$$

$$= \lim_{x\to 1}\left(\frac{x-1}{x-1}\right)+\lim_{x\to 1}\left(\frac{x^2-1}{x-1}\right)+....+\lim_{x\to 1}\left(\frac{x^{100}-1}{x-1}\right)$$

Applying L. Hospital Rule-

$$= \lim_{x\to1}\left(\frac{1}{1}\right)+\lim_{x\to1}\left(\frac{2x}{1}\right)+............+\lim_{x\to1}\left(\frac{100x^{99}}{1}\right)$$

= 1 + 2 + 3 + ………+100

$$= \Sigma\ 100 = \frac{100\times(100+1)}{2} \qquad \left\{\because\ \Sigma n = \frac{n(n+1)}{2}\right\}$$

$$= \frac{100\times101}{2} = 50\times101 = 5050$$

172. $\lim\limits_{x\to0}\left(1^{\operatorname{cosec}^2 x}+2^{\operatorname{cosec}^2 x}....+n^{\operatorname{cosec}^2 x}\right)^{\sin^2 x}$ **is equal to**

(a) 1 (b) $\frac{1}{n}$ (c) n (d) 0

Manipal UGET-2014

Ans. (c) : Given, $\lim\limits_{x\to0}\left(1^{\operatorname{cosec}^2 x}+2^{\operatorname{cosec}^2 x}....+n^{\operatorname{cosec}^2 x}\right)^{\sin^2 x}$

Let,

y = $\operatorname{cosec}^2 x$

Required limit = $\lim\limits_{y\to\infty}(1^y+2^y+....n^y)^{1/y}$ (0/∞ form)

$$\Rightarrow \quad = \lim_{y\to\infty}(n^y)^{1/y}$$

$$\Rightarrow \quad n\left[\left(\frac{1}{n}\right)^y+\left(\frac{2}{n}\right)^y+....+\left(\frac{n-1}{n}\right)^y+1\right]^{1/y}$$

$$\Rightarrow \quad \lim_{y\to\infty} n\left[\left(\frac{1}{n}\right)^y+\left(\frac{2}{n}\right)^y+.....+\left(\frac{n-1}{n}\right)^y+1\right]^{1/y}$$

$$= n.1^o = n.1 = n$$

173. $\lim\limits_{x\to\frac{\pi}{2}}\dfrac{\left(1-\tan\frac{x}{2}\right)(1-\sin x)}{\left(1+\tan\frac{x}{2}\right)(\pi-2x)^3}$ **is equal to**

(a) $\frac{1}{8}$ (b) 0 (c) $\frac{1}{32}$ (d) ∞

Manipal UGET-2014

Ans. (c) :

Put $x=\frac{\pi}{2}-h$ as $x\to\frac{\pi}{2}, h\to0$

∴ Given,

$$= \lim_{h\to0}\frac{1-\tan\left(\frac{\pi}{4}-\frac{h}{2}\right)(1-\cos h)}{1+\tan\left(\frac{\pi}{4}-\frac{h}{2}\right)(2h)^3}$$

$$= \lim_{h\to0}\tan\left(\frac{h}{2}\right)\lim_{h\to0}\frac{2.\sin^2(h/2)}{8h^3}$$

$$= \lim_{h\to0}\frac{1}{4}\left(\frac{\tan h/2}{2\times\frac{h}{2}}\right)\times\lim_{x\to0}\left(\frac{\sin\frac{h}{2}}{\frac{h}{2}}\right)^2\times\frac{1}{4} = 1/32$$

174. $\lim\limits_{x\to0}\dfrac{e^{\sin x}-1}{x}$ **is equal to**

(a) 0 (b) e (c) 1 (d) Does not exist

Manipal UGET-2014

Ans. (c) : $\lim\limits_{x\to0}\dfrac{e^{\sin x}-1}{x}$

$$= \lim_{x\to0}\left[\frac{\left(e^{\sin x}-1\right)}{\sin x}\times\frac{\sin x}{x}\right] = \lim_{x\to0}\frac{e^{\sin x}-1}{\sin x}\times\lim_{x\to0}\frac{\sin x}{x}$$

$$= 1\times1 = 1$$

175. **If** $f(x)=\dfrac{\sin\left(e^{x-2}-1\right)}{\log(x-1)}$**, them** $\lim\limits_{x\to2} f(x)$ **is given by**

(a) –2 (b) –1 (c) 0 (d) 1

Manipal UGET-2015

SCRA-2010

Ans. (d) : Given,

$$\lim_{x\to2}\frac{\sin\left(e^{x-2}-1\right)}{\log(x-1)},\ \left(\frac{0}{0}\right)\text{form}$$

$$\Rightarrow \quad \lim_{x\to2}\frac{\cos\left(e^{x-2}-1\right).e^{x-2}}{\frac{1}{(x-1)}} \text{ [using L-hospital rule]}$$

$$\Rightarrow \quad \frac{\cos 0^o.e^o}{1} = 1$$

176. $\lim\limits_{x\to-1}\left(\dfrac{x^4+x^2+x+1}{x^2-x+1}\right)^{\frac{1-\cos(x+1)}{(x+1)^2}}$ **is equal to**

(a) 1 (b) $\sqrt{\frac{2}{3}}$ (c) $\sqrt{\frac{3}{2}}$ (d) $e^{1/2}$

Manipal UGET-2015

Ans. (b) :

$$\lim_{x\to-1}\left(\frac{x^4+x^2+x+1}{x^2-x+1}\right)^{\frac{1-\cos(x+1)}{(x+1)^2}}$$

$$\Rightarrow \quad \lim_{x\to-1}\left(\frac{x^4+x^2+x+1}{x^2-x+1}\right)^{\lim\limits_{x\to-1}\frac{1-\cos(x+1)}{(x+1)^2}}$$

$$\Rightarrow \quad \left(\frac{2}{3}\right)^{\lim\limits_{x\to-1}\frac{2\sin^2\frac{x+1}{2}}{(x+1)^2}}$$

$$\Rightarrow \quad \left(\frac{2}{3}\right)^{\lim\limits_{x\to-1}\frac{2\sin^2\left(\frac{x+1}{2}\right)}{4\frac{(x+1)^2}{4}}} \Rightarrow \left(\frac{2}{3}\right)^{1/2} = \sqrt{\frac{2}{3}}$$

177. If $A_i = \frac{x-a_i}{|x-a_i|}$, i = 1, 2, 3, ..., n and $a_1<a_2<a_3< ...< a_n$, then $\lim_{x\to a_m}(A_1A_2A_3...A_n)$ where $1 \le m \le n$

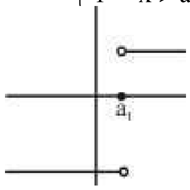
(a) is equal to $(-1)^m$ (b) is equal to $(-1)^{m+1}$
(c) is equal to $(-1)^{n-m}$ (d) does not exist

SCRA-2009

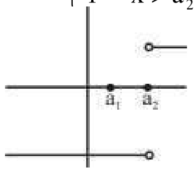
Ans. (d): $A_i = \frac{x-a_i}{|x-a_i|}$ $i = 1,2,3....n$ θ

And, $a_1< a_2 <a_3......<a_n$ then

$$A_1 = \begin{Bmatrix} -1 & x<a_1 \\ 1 & x>a_1 \end{Bmatrix}$$



$$A_2 = \begin{Bmatrix} -1 & x<a_2 \\ 1 & x>a_2 \end{Bmatrix}$$



$$A_3 = \begin{Bmatrix} -1 & x<a_3 \\ 1 & x>a_3 \end{Bmatrix}$$

$$\begin{array}{l|l} A_1A_2..........A_n & 1, \text{ if } x>a_n \\ 1\ \ 1.........1 & -1,\ x\in(a_{n-1}, an) \\ A_1A_2..........\underset{(-1)}{A_{n-1}}\ \underset{(-1)}{A_n} & 1,\ x\in(a_{n-2}, a_{n-1}) \end{array}$$

$(-1)^n$ $x<a_1$

Now, $(-1)^{n-1}$ $x\in(a_1,a_2)$

$\vdots$

1 $x\in(a_n,\infty)$

$f = A_1A_2........A_n$

a_i = limit does not exist for any (a_i)

178. If $f(x) = \lim_{n\to\infty}\frac{\ln(2+x)-x^{2n}\sin x}{1+x^{2n}}$ then what is $\lim_{x\to 1} f(x)$ equal to?

(a) ln3 (b) –sin1 (c) 0 (d) Limit does not exist

SCRA-2009

Ans.(b): $\lim_{n\to\infty}\frac{l n(2+x)-x^{2n}\sin x}{1+x^{2n}}$

$$n\to\infty\begin{Bmatrix} 1 & \text{if } x=1 \\ \infty & \text{if } |x|>1 \\ 0 & \text{if } |x|<1 \end{Bmatrix}$$

If a function is continuous at x=1

LHL= RHL = f(1)

f(1–h) = f(1 + h) = f(1)

$$f(x) = \lim_{n\to 0}\frac{\ln(2+x)-x^{2n}\sin x}{1+x^{2n}} \Rightarrow f(1) = \frac{\ln 3-\sin 1}{2}$$

LHL

$$f(1-h) = \lim_{h\to 0}\lim_{n\to\infty}\frac{\log(2+1-h)-(1-h)^{2n}\sin(1-h)}{1+(1+h)^{2n}}$$

$$\Rightarrow \lim_{h\to 0}\frac{\log(3-h)-0}{1+0}$$

$$\lim_{x\to 1^-} f(x) = \log 3$$

R.H.L.

$$f(1+h) = \lim_{h\to 0}\lim_{n\to\infty}\frac{\log(2+1+h)-(1+h)^{2n}\sin(1+h)}{1+(1-h)^{2n}}$$

$$= \lim_{h\to 0}\lim_{n\to\infty}\frac{(1+h)^{2n}\left[\frac{\log(3+h)}{(1+h)^{2n}}-\sin(1+h)\right]}{(1+h)^{2n}\left(\frac{1}{(1+h)^{2n}}+1\right)}$$

$$= \lim_{h\to 0}\left[\frac{\frac{\log(3+h)}{\infty}-\sin(1+h)}{\left(\frac{1}{\infty}+1\right)}\right]$$

$$= \left[\frac{0-\sin 1}{1}\right] \Rightarrow \lim_{x=1+} f(x) = -\sin(1)$$

179. What is $\lim_{x\to 2}\frac{\sin\left(e^{x-2}-1\right)}{\ell n(x-1)}$ equal to ?

(a) 0 (b) 1 (c) –1 (d) –2

SCRA-2010, WBJEE-2009

Ans. (b) : $\lim_{x\to 2}\frac{\sin\left(e^{x-2}-1\right)}{\log(x-1)}$ $\frac{0}{0}$ form.

Applying L' Hospital Rule –

$$= \lim_{x\to 2}\frac{\cos\left(e^{x-2}-1\right)\cdot e^0}{\frac{1}{1}} = \frac{\cos o\cdot 1}{1} = \frac{1}{1} = 1$$

180. Let $f(t) = \begin{vmatrix} \cos t & t & 1 \\ 2\sin t & t & 2t \\ \sin t & t & t \end{vmatrix}$, then what is $\lim_{t\to 0}\frac{f(t)}{t^2}$ equal to ?

(a) 0 (b) –1 (c) 2 (d) 3

SCRA-2010

Ans. (a) : $f(t) = \begin{vmatrix} \cos t & t & 1 \\ 2\sin t & t & 2t \\ \sin t & t & t \end{vmatrix}$

$$f(t) = t\begin{vmatrix} \cos t & 1 & 1 \\ 2\sin t & 1 & 2t \\ \sin t & 1 & t \end{vmatrix}$$

$R_1 \to R_1 - R_3$

$R_2 \to R_2 - R_3$

$$f(t) = t\begin{vmatrix} \cos t-\sin t & 0 & 1-t \\ \sin t & 0 & t \\ \sin t & 1 & t \end{vmatrix}$$

$f(t)=t\left[\left(0+(1-t)\sin t+0\right)-\right.$
$\left.\left(0+t(\cos t-\sin t)+0\right)+0\right]$
$f(t)=t[\sin t-t\sin t-t\cos t+t\sin t]$
$f(t)=t(\sin t-t\cos t)$
$\lim\limits_{t\to0}\frac{f(t)}{t^2}=\lim\limits_{t\to0}\frac{t(\sin t-t\cos t)}{t^2}\frac{0}{0}$ form
Applying L' Hospital Rule :-
$=\lim\limits_{t\to0}\frac{(\sin t-t\cos t)+t(\cos t-\cos t+t\sin t)}{2t}\frac{0}{0}$ form
$=\lim\limits_{t\to0}\frac{\cos t-\cos t+t\sin t+2t\sin t+t^2\cos t}{2}=\frac{0}{2}=0$

181. $\lim\limits_{x\to\infty}\sum\limits_{r=1}^{n}\frac{1}{n}e^{r/n}$ is equal to

(a) e (b) e – 1 (c) 1 – e (d) e + 1

CG PET- 2006, 2005

Ans. (b) : Given,
$$\lim_{x\to\infty}\sum^{n}\frac{1}{n}e^{r/n}$$
Let, $\frac{r}{n}=x$ $x=0,\quad x=1$
$$\frac{1}{n}dr=dx$$
$$\int_0^1 e^x\,dx$$
$$\left[e^x\right]_0^1\Rightarrow\left[e^1-e^0\right]=e-1$$

182. The value of $\lim\limits_{x\to2}\frac{3^{x/2}-3}{3^x-9}$ is

(a) 0 (b) $\frac{1}{3}$ (c) $\frac{1}{6}$ (d) log 3

CG PET- 2005

Ans. (c) : Given, $\lim\limits_{x\to2}\frac{3^{x/2}-3}{3^x-9}$
It can be written as –
$$\lim_{x\to2}\frac{3^{x/2}-3}{3^x-3^2}\Rightarrow\lim_{x\to2}\frac{3^{x/2}-3}{\left(3^{x/2}-3\right)\left(3^{x/2}+3\right)}$$
$$\lim_{x\to2}\frac{1}{\left(3^{x/2}+3\right)}\Rightarrow\frac{1}{3^{2/2}+3}=\frac{1}{6}$$

183. The value of $\lim\limits_{x\to0}\frac{(1-\cos2x)\sin5x}{x^2\sin3x}$ is

(a) $\frac{10}{3}$ (b) $\frac{3}{10}$ (c) $\frac{6}{5}$ (d) $\frac{5}{6}$

CG PET- 2005

Ans. (a) : We have,
$$\lim_{x\to0}\frac{(1-\cos2x)\sin5x}{x^2\sin3x}$$
$$\lim_{x\to0}\frac{2\sin^2x.\sin5x}{x^2\sin3x}$$
We know that,
$$\lim_{x\to0}\frac{\sin x}{x}=1$$
$$\lim_{x\to0}=\frac{2\left(\frac{\sin^2x}{x^2}\right).\left(\frac{\sin5x}{5x}\right)\times5x}{\left(\frac{\sin3x}{3x}\right)\times3x}$$
$$\lim_{x\to0}\frac{2\ 1.1\times5x}{1.3x}=\frac{2\times5}{3}=\frac{10}{3}$$

184. $\lim\limits_{x\to a}\frac{x^{-1}-a^{-1}}{x-a}$ is equal to

(a) $\frac{1}{a}$ (b) $-\frac{1}{a}$ (c) $\frac{1}{a^2}$ (d) $-\frac{1}{a^2}$

CG PET- 2006

Ans. (d) : Given, $\lim\limits_{x\to a}\frac{x^{-1}-a^{-1}}{x-a}$
Which is form of ,
Applying L – hospital's rule –
$$\lim_{x\to a}\frac{-x^2}{1}\Rightarrow\lim_{x\to a}-\frac{1}{x^2}\Rightarrow-\frac{1}{a^2}$$

185. $\lim\limits_{x\to\infty}\frac{(2x-3)(3x-4)}{(4x-5)(5x-6)}$ is equal to

(a) 0 (b) $\frac{1}{10}$ (c) $\frac{1}{5}$ (d) $\frac{3}{10}$

CG PET- 2006

Ans. (d) : Given, $\lim\limits_{x\to\infty}\frac{(2x-3)(3x-4)}{(4x-5)(5x-6)}$
$$\lim_{x\to\infty}\frac{\left(2-\frac{3}{4}\right)\left(3-\frac{4}{x}\right)}{\left(4-\frac{5}{x}\right)\left(5-\frac{6}{x}\right)}=\frac{6}{20}=\frac{3}{10}$$

186. If $\in R$, then $\lim\limits_{x\to\infty}\left(\frac{x-3}{x+2}\right)^x$ is equal to

(a) e (b) e^{-1} (c) e^{-5} (d) e^5

CG PET-2006

Ans. (c) : Given, $\lim\limits_{x\to\infty}\left(\frac{x-3}{x+2}\right)^x$
$$\lim_{x\to\infty}\left[\left(1-\frac{5}{x+2}\right)^{\frac{x+2}{-5}\times\frac{-5}{x+2}\times x}\right]$$
$$e^{\lim\limits_{x\to\infty}}\left(\frac{5}{x+2}\right)^{-x}$$
$$e^{\lim\limits_{x\to\infty}}\left(\frac{-x.5}{x\left(1+\frac{2}{x}\right)}\right),\ e^{5\lim}\frac{-5}{1+0},\ e^{-5}$$

187. If $\lim\limits_{x\to0}\frac{\log(3+x)-\log(3-x)}{x}=k$, then the value of k will be

(a) 0 (b) $-\frac{1}{3}$ (c) $\frac{2}{3}$ (d) $-\frac{2}{3}$

CG PET- 2006

Ans. (c) : We have,

$$\lim_{x\to 0}\frac{\log(3+x)-\log(3-x)}{x}=k$$

Applying L-Hospital's rule,

$$\lim_{x\to 0}\frac{\frac{1}{3+x}+\frac{1}{3-x}}{1}\Rightarrow \frac{1}{3}+\frac{1}{3}=\frac{2}{3}$$

188. $\lim_{h\to 0}\frac{\sqrt{x+h}-\sqrt{x}}{h}$ **is equal to**

(a) $\frac{1}{2\sqrt{x}}$ (b) $\frac{1}{\sqrt{x}}$ (c) $2\sqrt{x}$ (d) $\sqrt{x}$

CG PET- 2007

Ans. (a) : $\lim_{h\to 0}\frac{\sqrt{x+h}-\sqrt{x}}{h}$

$$\lim_{h\to 0}\frac{\sqrt{x+h}-\sqrt{x}}{h}\times\frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}\Rightarrow\lim_{h\to 0}\frac{(x+h)-x}{h\left(\sqrt{x+h}+\sqrt{x}\right)}$$

$$\lim_{h\to 0}\frac{h}{h\left(\sqrt{x+h}+\sqrt{x}\right)}\Rightarrow\frac{1}{\sqrt{x}+\sqrt{x}}\Rightarrow\frac{1}{2\sqrt{x}}$$

189. $\lim_{\theta\to 0}\frac{5\theta\cos\theta-2\sin\theta}{3\theta+\tan\theta}$ **is equal to**

(a) 3/4 (b) –3/4 (c) 0 (d) None of these

CG PET- 2007

Ans. (a) : We have, $\lim_{\theta\to 0}\frac{5\theta\cos\theta-2\sin\theta}{3\theta+\tan\theta}$

$$\lim_{\theta\to 0}\frac{\left(\frac{5\theta\cos\theta}{\theta}-\frac{2\sin\theta}{\theta}\right)\theta}{\left(\frac{3\theta}{\theta}+\frac{\tan\theta}{\theta}\right)\theta}\qquad\left(\frac{0}{0}\text{ from}\right)$$

$$\lim_{\theta\to 0}\frac{5\cos\theta-\frac{2\sin\theta}{\theta}}{3+\frac{\tan\theta}{\theta}}$$

We know that, $\lim_{x\to 0}\cos x\to 1$

$$\lim_{x\to 0}\frac{\sin x}{x}\to 1$$

$$\lim_{x\to 0}\frac{\tan x}{x}\to 1$$

$$\frac{\lim_{\theta\to 0}5\cos\theta-\lim_{\theta\to 0}\frac{2\sin\theta}{\theta}}{3+\lim_{\theta\to 0}\frac{\tan\theta}{\theta}}\Rightarrow\frac{5\times 1-2\times 1}{3+1}=\frac{3}{4}$$

190. $\lim_{x\to 2^+}\frac{|x-2|}{x-2}$ **is equal to**

(a) –1 (b) 1 (c) 2 (d) –2

CG PET- 2008

Ans. (b) : Given,

$$\lim_{x\to 2^+}\frac{|x+2|}{x-2}$$

$$\{|x-2|=x-2\text{ for }x>2\}$$

$$\lim_{x\to 2^+}\frac{x-2}{x-2}=1$$

191. The value of $\lim_{x\to 1}(1-x)\tan\left(\frac{\pi x}{2}\right)$ **will be**

(a) $\frac{\pi}{2}$ (b) $\frac{2}{\pi}$ (c) 2π (d) π

CG PET- 2009

Ans. (b) : Given, $\lim_{x\to 1}(1-x)\tan\frac{\pi x}{2}$

Let, $x-1=y \quad x=y+1$

$x\to 1 \quad y\to 0$

$$\lim_{y\to 0}(-y)\tan\left[\frac{\pi}{2}(y+1)\right]\Rightarrow\lim_{y\to 0}y\tan\left(\frac{\pi}{2}+\frac{\pi y}{2}\right)$$

$$\lim_{y\to 0}y\cot\left(\frac{\pi}{2}y\right)\Rightarrow\lim_{y\to 0}\frac{y}{\tan\left(\frac{\pi}{2}y\right)}$$

$$\lim_{y\to 0}\frac{y\times\left(\frac{\pi}{2}\right)}{\tan\left(\frac{\pi y}{2}\right)}\times\frac{1}{\left(\frac{\pi}{2}\right)}\Rightarrow 1\times\frac{2}{\pi}=\frac{2}{\pi}$$

192. $\lim_{x\to 0}\frac{x.2^x-x}{1-\cos x}$ **equals**

(a) log2 (b) $\frac{1}{2}$log2 (c) 2log2 (d) None of these

CG PET- 2010

Ans. (c) : $\lim_{x\to 0}\frac{x2^x-x}{1-\cos x}$

Applying L-Hospital's rule $\left(\frac{0}{0}\right)$ from

$$\lim_{x\to 0}\frac{x.2^x\log 2+2^x-1}{+\sin x}$$

Again using L-Hospital's rule

$$\lim_{x\to 0}\frac{\log 2\left(2^x+x2^x\log 2\right)+2^x\log 2}{\cos x}$$

$$\lim_{x\to 0}\frac{2.2^x\log 2+x2^x\log 2^2}{\cos x}$$

$$\frac{2\cdot 2^0\log 2+0}{\cos 0}=\frac{2\times 1\times\log 2}{1}=2\log 2$$

193. $\lim_{x\to 5}\frac{x-5}{|x-5|}$ **equals**

(a) 2 (b) 0 (c) –2 (d) None of these

CG PET- 2011

Ans. (d) : We have,

$$\lim_{x\to 5}\frac{x-5}{|x-5|}$$

Now, R.H.S $\lim_{x\to 5^+}\frac{x-5}{x-5}$ $\quad |x-5|=x-5, x>5$

$\Rightarrow$ 1

L.H.S

$$\lim_{x\to 5^-}\frac{x-5}{-(x-5)}$$

$\Rightarrow$ –1

Hence, L.H.S ≠ R.H.S

Limit does not exist.

194. $\lim\limits_{x\to\frac{\pi}{3}}\dfrac{\sin\left(\frac{\pi}{3}-x\right)}{2\cos x-1}$ **is equal to**

(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{3}}$ (c) $\frac{-1}{\sqrt{3}}$ (d) $\frac{2}{\sqrt{3}}$

CG PET- 2011

Ans. (b) : $\lim\limits_{x\to\pi/3}\dfrac{\sin\left(\frac{\pi}{3}-x\right)}{2\cos x-1}$ $\quad\frac{0}{0}$ from

Applying L-Hospital's rule we get,

$$\lim_{x\to\pi/3}\frac{\cos\left(\frac{\pi}{3}-x\right)(-1)}{-2\sin x}\Rightarrow\lim_{x\to\pi/3}\frac{\cos\left(\frac{\pi}{3}-x\right)}{2\sin x}$$

$$\frac{1}{2\frac{\sqrt{3}}{2}}=\frac{1}{\sqrt{3}}$$

195. **If** $\lim\limits_{x\to0}\dfrac{x(1+a\cos x)-b\sin x}{x^3}=1$**, then**

(a) $a=-\frac{5}{2},\ b=-\frac{1}{2}$ (b) $a=-\frac{3}{2},\ b=-\frac{1}{2}$

(c) $a=-\frac{3}{2},\ b=-\frac{5}{2}$ (d) $a=-\frac{5}{2},\ b=-\frac{3}{2}$

CG PET- 2012

Ans. (d) :

$\lim\limits_{x\to0}\dfrac{x(1+a\cos x)-b\sin x}{x^3}=1$ $\quad\frac{0}{0}$ from

Applying L-Hospital's rule

$$\lim_{x\to0}\frac{1+a\cos x-ax\sin x-b\cos x}{3x^2}$$

Again applying L-Hospital's rule

$$\lim_{x\to0}\frac{-a\sin x-a\sin x-ax\cos x+b\sin x}{6x}$$

$$\lim_{x\to0}\frac{-2a\sin x-ax\cos x+b\sin x}{6x}$$

$$\lim_{x\to0}\frac{-2a}{6}\frac{\sin x}{x}-\lim_{x\to0}\frac{ax\cos x}{6x}+\lim_{x\to0}\frac{b\sin x}{6x}$$

$$\frac{-2a}{6}-\frac{a}{6}+\frac{b}{6}=1$$

$$\frac{-a}{2}+\frac{b}{6}=1 \qquad \text{....(i)}$$

And, $1+a-b=0$ ....(ii)

We get, $a=\frac{-5}{2}$ and $b=\frac{-3}{2}$

196. **If α and β are roots of $ax^2+bx+c=0$, then** $\lim\limits_{x\to\alpha}\dfrac{1-\cos(ax^2+bx+c)}{(x-\alpha)^2}$ **is equal to**

(a) $\frac{a^2}{2}(\alpha-\beta)^2$ (b) $-\frac{a^2}{2}(\alpha-\beta)^2$ (c) 0 (d) 1

CG PET- 2013

Ans. (a) : Given, α and β are roots of $ax^2+bx+c=0$

$\lim\limits_{x\to\alpha}\dfrac{1-\cos(ax^2+bx+c)}{(x-\alpha)^2}$ $\quad\frac{0}{0}$ from

Applying L-Hospital's rule

$$\lim_{x\to\alpha}\frac{\sin(ax^2+bx+c).(2ax+b)}{(x-\alpha)^2}$$

$$\lim_{x\to\alpha}\frac{\sin(a(x-\alpha)(x-\beta))(2ax+b)}{2(x-\alpha)(x-\beta).a}\times a(x-\beta)$$

$$\lim_{x\to\alpha}\frac{1}{2}(2ax+b)(x-\beta).a$$

$$\lim_{x\to\alpha}\frac{a^2}{2}\left(2x+\frac{b}{a}\right)(x-\beta)$$

$$\Rightarrow\quad\frac{a^2}{2}\left(2\alpha+\frac{b}{a}\right)(\alpha-\beta)$$

$$\frac{a^2}{2}(2\alpha-\alpha-\beta)(\alpha-\beta)$$

$$\frac{a^2}{2}(\alpha-\beta)(\alpha-\beta)\qquad\left\{\begin{array}{l}\alpha+\beta=-\frac{b}{a}\\ \alpha.\beta=\frac{b}{a}\end{array}\right\}$$

$$\frac{a^2}{2}(\alpha-\beta)^2$$

197. $\lim\limits_{x\to0}\dfrac{\int_0^{x^2}\sin\sqrt{t}\,dt}{x^3}$ **is equal to**

(a) $\frac{2}{3}$ (b) $\frac{1}{3}$ (c) 0 (d) ∞

CG PET- 2014

Ans. (a) :

$$\lim_{x\to0}\frac{\int_0^{x^2}\sin\sqrt{t}\,dt}{x^3}$$

Applying L-Hospital's rule we get, $\quad\frac{0}{0}$ from

$$\lim_{x\to0}\frac{\frac{d}{dx}\int_0^{x^2}\sin\sqrt{t}\,dt}{\frac{d}{dx}\cdot x^3}$$

By lebinitz rule,

$$\lim_{x\to0}\frac{\sin\sqrt{x^2}.2^x-0}{3x^2}$$

$$\lim_{x\to0}\frac{\sin x.2}{3x}$$

$$\frac{2}{3}\lim_{x\to0}\frac{\sin x}{x}=\frac{2}{3}$$

198. **If** $\lim\limits_{x\to\infty}\sum\dfrac{\log(n+r)-\log n}{n}=2\left(\log2-\frac{1}{2}\right)$**, then** $\lim\limits_{x\to\infty}\dfrac{1}{n^\lambda}\left[(n+1)^\lambda(n+2)^\lambda...(n+n)^\lambda\right]^{1/n}$ **is equal to**

(a) $\frac{4\lambda}{e}$ (b) $\left(\frac{4}{e}\right)^\lambda$ (c) $\left(\frac{4}{e}\right)^{\frac{1}{\lambda}}$ (d) $\left(\frac{e}{4}\right)^\lambda$

CG PET- 2014

Ans. (b) : Given,

$$\lim_{x\to\infty}\sum_{r=1}^{n}\frac{\log(r+n)-\log n}{n}=2\left(\log 2-\frac{1}{2}\right)$$

Now,

$$\lim_{n\to\infty}\frac{1}{n^{\lambda}}\left[(n+1)^{\lambda}(n+2)^{\lambda}......(n+n)^{\lambda}\right]^{1/n}$$

$$\lim_{n\to\infty}\frac{1}{n^{\lambda}}\left[n^{\lambda}\left(n+\frac{1}{n}\right).n^{\lambda}\left(1+\frac{2}{n}\right)^{\lambda}.........n^{\lambda}\left(2^{k}\right)\right]^{1/n}$$

$$\lim_{n\to\infty}\frac{1}{n^{\lambda}}\left[n^{\lambda}\left(n+\frac{1}{n}\right)^{\lambda}\left(1+\frac{2}{n}\right)^{\lambda}......n^{\lambda}\left(2^{k}\right)\right]^{1/n}$$

$$\lim_{x\to\infty}\frac{1}{n^{\lambda}}n\left[\left(n+\frac{1}{n}\right)^{\lambda}\left(1+\frac{2}{n}\right)^{\lambda}......\left(2^{k}\right)\right]^{1/n}$$

$$\lim_{x\to\infty}\left[\left(n+\frac{1}{n}\right)^{\lambda}\left(1+\frac{2}{n}\right)^{\lambda}......\left(2^{k}\right)\right]^{1/n}$$

Let,

$$L=\lim_{x\to\infty}\sum_{r=1}^{n}\left[\left(1+\frac{r}{n}\right)^{\lambda}\right]^{1/n}$$

$$\log\quad L=\frac{1}{n}\sum_{r=0}^{n}\log\left(1+\frac{r}{n}\right)$$

$$\lim_{x\to\infty}\lambda\sum_{r=0}^{n}\frac{\log(n+r)-\log n}{n}$$

$\Rightarrow\quad \lambda\times 2\,(\log2-1/2)$

$\Rightarrow\quad 2\lambda\log2-\lambda$

$$\log=\log2^{2\lambda}-\log e^{\lambda}=\log\left(\frac{4}{e}\right)^{\lambda}$$

199. If $\lim\limits_{x\to 0}\dfrac{\sin(\sin x)-\sin x}{ax^3+bx^5+c}=\dfrac{-1}{12}$, then

(a) $a=2,\ b\in R,\ c=0$
(b) $a=-2,\ b\in R,\ c=0$
(c) $a=1,\ b\in R,\ c=0$
(d) $a=-1,\ b\in R,\ c=0$

CG PET- 2014

Ans. (a) : $\lim\limits_{x\to0}\dfrac{\sin(\sin x)-\sin x}{ax^3+bx^5+c}=\dfrac{-1}{12}$

We know that,

$$\sin x=x-\frac{x^3}{3!}+\frac{x^5}{5!}$$

$$\sin(\sin x)=\sin x-\frac{(\sin x)^3}{3!}+\frac{(\sin x)^5}{5!}$$

$$\lim_{x\to0}\frac{\sin x-\frac{(\sin x)^3}{3!}+\frac{(\sin x)^5}{5!}......\sin x}{ax^3+bx^5+c}$$

$$\lim_{x\to0}\frac{-\sin^3 x\left(\frac{1}{3!}-\frac{(\sin x)^2}{5!}+.....\right)}{x^3\left(a+bx^2\right)}=\frac{-1}{12}$$

$$-\frac{1}{6.a}=\frac{-1}{12}$$

$a=2$

200. Let $f(x)$ be differentiable on the interval $(0,\infty)$ such that $f(1)=1$ and $\lim\limits_{x\to x}\dfrac{t^2f(x)-x^2f(t)}{t-x}=1$ for each $x>0$. Then, $f(x)$ is equal to

(a) $\dfrac{1}{3x}+\dfrac{2}{3}x^2$ (b) $-\dfrac{x}{3}+\dfrac{4x^2}{3}$ (c) $-\dfrac{1}{x}$ (d) $-\dfrac{1}{x}+\dfrac{2}{x^2}$

CG PET- 2014

Ans. (a) : Given,

$$\lim_{t\to x}\frac{t^2(x)-x^2f(t)}{t-x}=1$$

Appling L-Hospital Rule

$$\lim_{t\to x}\frac{2tf(x)-x^2f'(t)}{1}=1$$

$2xf(x)-x^2f'(x)=1$

$x^2f'(x)-2x\,f(x)+1=0$

Let, $\quad y=f(x)\quad \dfrac{dy}{dx}=f'(x)$

Equation become,

$$X^2\frac{dy}{dx}-2xy+1=0$$

$$\frac{dy}{dx}-\frac{2}{x}y=-\frac{1}{x^2}$$

Which is a linear differential equation

$$\text{I.F}=e^{\int pdx}=e^{\int\frac{2}{x}dx}$$

$$=e^{\log x^{-2}}=\frac{1}{x^2}$$

For solution

$$y.\text{I.F}=\int a.\text{I.F}dx+C$$

$$yx^{-2}=\int x^{-2}\left(-\frac{1}{x^2}\right)dx+C$$

$$\frac{y}{x^2}=-\int\left(-\frac{1}{x^4}\right)+C\Rightarrow\frac{y}{x^2}=\frac{1}{3x^3}+C$$

$$y=f(x)==\frac{1}{3x}+Cx^2$$

When, $\quad x=1\quad y=f(1)=1$

$$1=\frac{1}{3}+C\Rightarrow C=\frac{2}{3}$$

$$f(x)=\frac{1}{3x}+\frac{2}{3}x^2$$

201. $\lim\limits_{x\to2}\dfrac{2-\sqrt{2+x}}{2^{1/3}-(4-x)^{1/3}}$ is equal to

(a) $2\cdot3^{-1/2}$ (b) $3\cdot2^{-4/3}$
(c) $-3\cdot2^{-4/3}$ (d) None of the above

CG PET- 2015

Ans. (c) : Given, $\lim\limits_{x\to 2}\dfrac{2-\sqrt{2+x}}{2^{1/3}-(4-x)^{1/3}}$ $\left[\dfrac{0}{0}\text{form}\right]$

$$=\lim_{x\to 2}\frac{0-\dfrac{1}{2\sqrt{2+x}}}{0-\dfrac{1}{3}\cdot\dfrac{(-1)}{(4-x)^{2/3}}}$$

$$=\lim_{x\to 2}\frac{-3(4-x)^{2/3}}{2\sqrt{2+x}}=\frac{-3}{2}\times\frac{(4-2)^{2/3}}{\sqrt{2+2}}$$

$$=\frac{-3}{2}\times\frac{2^{2/3}}{2}$$

$$=-3\cdot 2^{-4/3}$$

202. **The value of** $\lim\limits_{x\to 0}\dfrac{\sin x^4-x^4\cos x^4+x^{20}}{x^4\left(e^{2x^4}-1-2x^4\right)}$ **equal to**

(a) 0 (b) $-\dfrac{1}{6}$ (c) $\dfrac{1}{6}$ (d) Does not exist

CG PET- 2016

Ans. (c) : $\lim\limits_{x\to 0}\dfrac{\sin x^4-x^4\cos x^4+x^{20}}{x^4\left(e^{2x^4}-1-2x^4\right)}$

Using L' Hospital Rule

$$=\lim_{x\to 0}\frac{4x^7\sin x^4+20x^{19}}{8e^{2x^4}x^7-16x^7+4e^{2x^4}-4x^3}$$

$$=\lim_{x\to 0}\frac{4x^4\left(x^4\sin x^4+5x^{16}\right)}{4x^3\left(2e^{2x^4}x^4-4x^4+e^{2x^4}-1\right)}$$

Using L'-Hospital Rule

$$=\lim_{x\to 0}\frac{4x^3\sin x^4+4x^7\cos x^4+80x^{15}}{16e^{2x^4}x^7+16e^{2x^4}x^3-16x^3}$$

$$=\lim_{x\to 0}\frac{4x^3\left(\sin x^4+x^4\cos x^4+20x^{12}\right)}{4x^3\left(4e^{2x^4}+4e^{2x^4}-4\right)}$$

Using L'-Hospital Rule

$$=\lim_{x\to 0}\frac{4x^3\cos x^4+4x^3\cos x^4-4x^7\sin x^4+240x^{11}}{32e^{2x^4}\cdot x^7+48e^{2x^4}\cdot x^3}$$

$$=\lim_{x\to 0}\frac{4x^3\left(2\cos x^4-x^4\sin x^4+60x^{11}\right)}{4x^3\left(8e^{2x^4}\cdot x^4+12e^{2x^4}\right)}$$

$$=\lim_{x\to 0}\frac{2\cos x^4-x^4\sin x^4+60x^{11}}{e^{2x^4}\left(8x^4+12\right)}$$

$$=\frac{2\cos(0)^4-(0)^4\sin(0)^4+60(0)^{11}}{e^{2(0)2}\left[8(0)^4+12\right]}=\frac{2}{12}=\frac{1}{6}$$

203. $\lim\limits_{x\to 1}\dfrac{\log x}{x-1}$ **is equal to**

(a) –1 (b) 0 (c) e (d) 1

CG PET- 2018

Ans. (d) : Given, $\lim\limits_{x\to 1}\dfrac{\log x}{x-1}$

$$=\lim_{x\to 1}\frac{1/x}{1}[\text{using L'Hospital Rule}]=1$$

204. $\lim\limits_{x\to 0}\dfrac{\cos(\sin x)-\cos x}{x^4}$ **is equal to:**

(a) $\dfrac{1}{3}$ (b) $\dfrac{1}{4}$ (c) $\dfrac{1}{6}$ (d) $\dfrac{1}{12}$

JEE Main-26.06.2022, Shift-II

Ans. (c) : Given, $\lim\limits_{x\to 0}\dfrac{\cos(\sin x)-\cos x}{x^4}$

We know that,

$$\cos A-\cos B=-2\sin\left(\frac{A+B}{2}\right).\sin\left(\frac{A-B}{2}\right)$$

$$\cos(\sin x)-\cos x=-2\sin\left(\frac{\sin x+x}{2}\right).\sin\left(\frac{\sin x-x}{2}\right)$$

$$\lim_{x\to 0}\frac{-2\sin\left(\dfrac{\sin x+x}{2}\right)\cdot\sin\left(\dfrac{\sin x-x}{2}\right)}{x^4}$$

$$\lim_{x\to 0}\frac{-2\sin\left(\dfrac{\sin x+x}{2}\right)}{x^2}\cdot\frac{\sin\left(\dfrac{\sin x-x}{2}\right)}{x^2}$$

$$\lim_{x\to 0}\frac{-2\sin\left(\dfrac{\sin x+x}{2}\right)}{\dfrac{\sin x+x}{2}}\cdot\frac{\sin x+x}{2x^2}\cdot\frac{\sin\left(\dfrac{\sin x-x}{2}\right)}{\dfrac{\sin x-x}{2}}\cdot\frac{\sin x-x}{2x^2}$$

$$\lim_{x\to 0}-2\cdot 1.\frac{\sin x+x}{2x^2}\cdot 1.\frac{\sin x-x}{2x^2}$$

$$\lim_{x\to 0}-2\left[\frac{\sin x+x}{2x^2}\right]\left[\frac{\sin x-x}{2x^2}\right]$$

$$\lim_{x\to 0}-\frac{1}{2x^4}\left[\sin^2 x-x^2\right] \qquad \frac{0}{0}\text{from}$$

Applying L-Hospital's rule we get,

$$\lim_{x\to 0}-\left[\frac{2\sin x\cos x-2x}{8x^3}\right]$$

$$\lim_{x\to 0}\left[\frac{2x-\sin 2x}{8x^3}\right] \qquad \frac{0}{0}\text{from}$$

Again using L-Hospital's rule,

$$\lim_{x\to 0}\frac{2-2\cos 2x}{24x^2} \qquad \text{from }\frac{0}{0}$$

$$\lim_{x\to 0}\left[\frac{0-2(-2)\sin 2x}{48x}\right]\Rightarrow\lim_{x\to 0}\left[\frac{4\sin x}{48x}\right]$$

$$\lim_{x\to 0}\frac{1}{6}\left\{\frac{\sin 2x}{2x}\right\}$$

$$\frac{1}{6}\lim_{x\to 0}\frac{\sin 2x}{2x} \qquad \left\{\because\ \lim_{x\to 0}\frac{\sin x}{x}=1\right\}$$

$$\frac{1}{6}$$

205. If $\lim_{x\to0}\frac{\alpha e^x+\beta e^{-x}+\gamma\sin x}{x\sin^2 x}=\frac{2}{3}$, where α, β, γ ∈ R, then which of the following is NOT correct?

(a) $\alpha^2+\beta^2+\gamma^2=6$
(b) $\alpha\beta+\beta\gamma+\gamma\alpha+1=0$
(c) $\alpha\beta^2+\beta\gamma^2+\gamma\alpha^2+3=0$
(d) $\alpha^2-\beta^2+\gamma^2=4$

JEE Main-29.07.2022, Shift-I

Ans. (c) : We have,

$$\lim_{x\to0}\frac{\alpha e^x+\beta e^{-x}+\gamma\sin x}{x\sin^2 x}=\frac{2}{3}$$

$$\lim_{x\to0}\frac{\alpha e^x+\beta e^{-x}+\gamma\sin x}{x^3\left[\frac{\sin^2 x}{x^2}\right]}=\frac{2}{3}$$

$$\lim_{x\to0}\frac{\alpha e^x+\beta e^{-x}+\gamma\sin x}{x^3}=\frac{2}{3}$$

$\alpha+\beta=0$ ....(i)

Using L – Hospital's rule,

$$\lim_{x\to0}\frac{\alpha e^x-\beta e^{-x}+\gamma\cos x}{3x^2}=\frac{2}{3}$$

$\alpha-\beta+\gamma=0$ ....(ii)

$$\lim_{x\to0}\frac{\alpha e^x+\beta e^{-x}-\gamma\sin x}{6x}=\frac{2}{3}$$

∴ $\alpha+\beta=0$

Again applying L- Hospital's rule,

$$\lim_{x\to0}\frac{\alpha e^x-\beta e^{-x}-\gamma\cos x}{6}=\frac{2}{3}$$

$$\frac{\alpha-\beta-\gamma}{6}=\frac{2}{3}$$

$\alpha-\beta-\gamma=4$ ....(iii)

From (ii) and (iii), we get –

$\alpha-\beta+\gamma=0$
$\alpha-\beta-\gamma=4$
$2\alpha-2\beta=4$
$\alpha-\beta=2$ ....(iv)

Now (i) and (iv), we get–

$\alpha+\beta=0$
$\alpha-\beta=2$
$\alpha-\beta=0$
$\alpha-\beta=2$
$2\alpha=2$
$\alpha=1$

Then, $\beta=-1$

From equation (iii),

$\alpha-\beta-r=4$
$1+1-\gamma=4$
$-\gamma=2\Rightarrow\ \gamma=-2$

By option (c), $\alpha\beta^2+\beta\gamma^2+\gamma\alpha^2+3=0$

$(1)(-1)^2+(-1)(-2)^2+(-2)(1)^2+3=0$

$1-4-2+3=0\Rightarrow -6+4=0\Rightarrow -2\neq0$

206. What is $\lim_{x\to\infty}\frac{\sqrt{1-\cos(2x-2)}}{x-1}$ equal to?

(a) $\sqrt{2}$ (b) $-\sqrt{2}$ (c) 0 (d) Limit does not exist

SCRA-2014

Ans. (d) : $\lim_{x\to1}\frac{\sqrt{1-\cos(2x-2)}}{x-1}=\lim_{x\to1}\frac{\sqrt{2\sin^2(x-1)}}{x-1}$

$=\lim_{x\to1}\frac{\sqrt{2}|\sin(x-1)|}{(x-1)}$

R.H.L. $=\lim_{x\to1^+}\frac{\sqrt{2}\sin(x-1)}{(x-1)}$

R.H.L. $=\sqrt{2}$

L.H.L. $=\lim_{x\to1^-}\frac{-\sqrt{2}\sin(x-1)}{(x-1)}$

L.H.L. $=-\sqrt{2}$

So, L.H.L ≠ R.H.L.

Limit does not exists.

207. If f(a) = 2, f'(a) = 1, g(a) = – 1, g'(a) = 2, then what is $\lim_{x\to a}\left[\frac{g(x)f(a)-g(a)f(x)}{x-a}\right]$ equal to?

(a) –5 (b) $\frac{1}{5}$ (c) 5 (d) $-\frac{1}{5}$

SCRA-2015

Ans. (c) : f(a) = 2, f'(a)=1, g(a) = –1, g'(a)=2

$\lim_{x\to a}\left[\frac{g(x)f(a)-g(a)f(x)}{x-a}\right]\ \frac{0}{0}$ form

= f(a) g'(a)–g(a)f'(a) = (2) (2) – (–1) (1) = 4 + 1=5

208. $\lim_{x\to\infty}\left(\frac{1}{1+n}+\frac{1}{2+n}+........+\frac{1}{2n}\right)$ is equal to

(a) $\log_e 2$ (b) $\log_e\left(\frac{2}{3}\right)$ (c) 0 (d) $\log_e\left(\frac{3}{2}\right)$

JEE Main-01.02.2023, Shift-I

Ans. (a) : Given, $\lim_{x\to\infty}\left(\frac{1}{1+n}+\frac{1}{2+n}+........\frac{1}{2n}\right)$

$$\lim_{x\to\infty}\left(\frac{1}{r+n}\right)\Rightarrow\lim_{x\to\infty}\frac{1}{n}\left(\frac{1}{1+\frac{r}{n}}\right)$$

Let,

$\frac{r}{n}=x$ $x\to0,\ x\to1$

$\frac{1}{n}dr=dx$

$$\lim_{x\to\infty}\int_0^1\frac{1}{1+x}dx$$

$(\log(x+1))_0^1=\log(1+1)-\log1=\log_e 2$

209. $\lim\limits_{n\to\infty}\left(\dfrac{n^2}{(n^2+1)(n+1)}+\dfrac{n^2}{(n^2+4)(n+2)}+\dfrac{n^2}{(n^2+9)(n+3)}+\dfrac{n^2}{(n^2+n^2)(n+n)}\right)$ **is equal to**

(a) $\dfrac{\pi}{8}+\dfrac{1}{4}\log_e 2$ (b) $\dfrac{\pi}{4}+\dfrac{1}{8}\log_e 2$

(c) $\dfrac{\pi}{4}-\dfrac{1}{8}\log_e 2$ (d) $\dfrac{\pi}{8}+\dfrac{1}{8}\log_e \sqrt{2}$

JEE Main-24.06.2022, Shift-II

Ans. (a) : Given,

$$\lim_{n\to\infty}\left[\frac{n^2}{(n^2+1)(n+1)}+\frac{n^2}{(n^2+4)(n+2)}+\frac{n^2}{(n^2+9)(n+3)}+\frac{n^2}{(n^2+n^2)(n+n)}\right]$$

$$\lim_{n\to\infty}\sum_{r=1}^{n}\frac{n^2}{(n^2+r^2)(n+r)}$$

$$\lim_{n\to\infty}\sum_{r=1}^{n}\frac{n^2}{n^3\left(1+\frac{r^2}{n^2}\right)\left(1+\frac{r}{n}\right)}$$

$$\lim_{n\to 0}\sum_{r=\infty}^{n}\frac{1}{n}\left(\frac{1}{\left(1+\frac{r^2}{n^2}\right)\left(1+\frac{r}{n}\right)}\right)$$

Let, $\dfrac{r}{n}=x$

$\dfrac{1}{n}dr=dx \quad x\to 0,\ x\to 1$

$$\int_0^1\frac{dx}{(1+x^2)(1+x)}$$

$$\frac{1}{2}\int_0^1\frac{dx}{1+x}+\frac{1}{2}\int_0^1\frac{1-x}{x^2+1}dx$$

$$\left[\frac{1}{2}\log(x+1)+\frac{1}{2}\tan^{-1}x\right]_0^1-\frac{1}{4}\left[\log(x^2+1)\right]_0^1$$

$$=\frac{1}{2}\left(\log 2+\frac{\pi}{4}\right)-\frac{1}{4}\log 2$$

$$=\frac{1}{2}\left(\frac{1}{2}\log 2+\frac{\pi}{4}\right)=\frac{1}{4}\log_e 2+\frac{\pi}{8}$$

210. If for p ≠ q ≠ 0, then function $f(x)=\dfrac{\sqrt[7]{p(729+x)}-3}{\sqrt[3]{729+qx}-9}$ **is continuous at x = 0, then :**

(a) $7pq\ f(0)-1=0$ (b) $63q\ f(0)-p^2=0$

(c) $21q\ f(0)-p^2=0$ (d) $7pq\ f(0)-9=0$

JEE Main-27.07.2022, Shift-II

Ans. (b) : $f(0)=\lim\limits_{x\to 0}f(x)$

Limit should be $\dfrac{0}{0}$ form

So, $\sqrt[7]{p.729}-3=0 \Rightarrow p.3^6=3^7 \Rightarrow p=3$

Now, $f(0)=\lim\limits_{x\to 0}\dfrac{\sqrt[7]{3(3^6+x)}-3}{\sqrt[3]{3^6+qx}-9}$

$$=\lim_{x\to 0}\frac{3\left[\left(1+\frac{x}{3^6}\right)^{1/7}-1\right]}{9\left[\left(1+\frac{qx}{3^6}\right)^{1/3}-1\right]}=\frac{3}{9}\times\frac{\frac{1}{7.3^6}}{\frac{q}{3.36}}$$

$$f(0)=\frac{1}{3}\times\frac{3}{7q}=\frac{1}{7q}$$

$7qf(0)-1=0$

$7.p^2.qf(0)-p^2=$ (from option)

$63\ qf(0)-p^2=0$

211. The value of

$$\lim_{n\to\infty}\frac{1+2-3+4+5-6+......(3n-2)(3n-1)-3n}{\sqrt{2n^4+4n+3}-\sqrt{n^4+5n+4}}$$

is :

(a) $3(\sqrt{2}+1)$ (b) $\dfrac{3}{2}(\sqrt{2}+1)$ (c) $\dfrac{\sqrt{2}+1}{2}$ (d) $\dfrac{3}{2\sqrt{2}}$

JEE Main-25.01.2023, Shift-I

Ans. (b) : Given,

$$\lim_{n\to\infty}\frac{1+2-3+4+5-6+......(3n-2)(3n-1)-3n}{\sqrt{2n^4+4n+3}-\sqrt{n^4+5n+4}}$$

$$\lim_{n\to\infty}\sum_{r=1}^{n}\left(\frac{(3r-1)+(3r-2)-3r}{\sqrt{2n^4+4n+3}-\sqrt{n^4+5n+4}}\right)$$

$$\lim_{n\to\infty}\sum_{r=1}^{n}\frac{(3r-3)}{\sqrt{2n^4+4n+3}-\sqrt{n^4+5n+4}}$$

$$\lim_{n\to\infty}\frac{\frac{3n(n+1)}{2}-3}{n^2\left[\sqrt{2+\frac{4}{n^3}+\frac{3}{n^4}}-\sqrt{1+\frac{5}{n^3}+\frac{4}{n^4}}\right]}$$

$$\lim_{n\to\infty}\frac{\frac{3}{2}\left(1+\frac{1}{n}\right)-\frac{3}{n^2}}{\sqrt{2+\frac{4}{n^3}+\frac{3}{n^4}}-\sqrt{1+\frac{5}{n^3}+\frac{4}{n^4}}}$$

$$=\frac{\frac{3}{2}(1+0)-0}{\sqrt{2+0+0}-\sqrt{1}}=\frac{3}{2(\sqrt{2}-1)}$$

$$=\frac{3}{2(\sqrt{2}-1)}\times\frac{\sqrt{2}+1}{\sqrt{2}+1}=\frac{3}{2}(\sqrt{2}+1)$$

212. Let y = y(t) be a solution of the differential equal $\frac{dy}{dt}+\alpha y=\gamma e^{-\beta t}$ where, $\alpha > 0$, $\beta > 0$ and $\gamma > 0$. Then $\lim\limits_{t\to\infty} y(t)$

(a) is – 1 (b) is 0 (c) is 1 (d) does not exist

JEE Main-25.01.2023, Shift-II

Ans. (b) : Given differential equation –

$$\frac{dy}{dx}+\alpha y=\gamma e^{-\beta t}$$

For integrating factor.

$$\text{I.F.} = e^{\int \alpha\, dt}=e^{\alpha t}$$

For general solution,

$$ye^{\alpha t}=\int \gamma e^{-\beta t}.e^{\alpha t}\,dt$$

$$ye^{\alpha t}=\int \gamma^{(\alpha-\beta)t}\,dt$$

$$ye^{\alpha t}=\gamma\frac{e^{(\alpha-\beta)t}}{\alpha-\beta}+c$$

$$y=\gamma\frac{e^{-\beta t}}{\alpha-\beta}+\frac{c}{e^{\alpha t}}$$

Now,

$$\lim_{t\to\infty} y(t)=\gamma\frac{1}{e^{\beta t}(\alpha-\beta)}+\frac{c}{e^{\alpha t}}=\frac{\gamma}{\infty}+\frac{c}{\infty}=\gamma.0+c.0=0$$

213. $\lim\limits_{n\to\infty}\frac{3}{n}\left[4+\left(2+\frac{1}{n}\right)^2+\left(2+\frac{2}{n}\right)^2+......+\left(3-\frac{1}{n}\right)^2\right]$ is equal to

(a) $\frac{19}{3}$ (b) 12 (c) 0 (d) 19

JEE Main-30.01.2023, Shift-II

Ans. (d) : Given,

$$\lim_{n\to\infty}\frac{3}{n}\left[4+\left(2+\frac{1}{n}\right)^2+\left(2+\frac{2}{n}\right)^2+.....\left(3-\frac{1}{n}\right)^2\right]$$

$$\lim_{n\to\infty}\frac{3}{n}\sum_{r=0}^{n-1}\left(2+\frac{r}{n}\right)^2$$

Let, $\frac{r}{n}=x \qquad \frac{1}{n}dr=dx$

$$\lim_{n\to\infty}3\int_0^1(2+x)^2\,dx$$

$$3\left[\frac{(2+x)^3}{3}\right]_0^1$$

$(2+1)^3-(2+0)^3$

$27-8=19$

214. Let f, g and h be the real valued functions defined on R as

$$f(x)=\begin{cases}\frac{x}{|x|}, & x\neq 0\\ 1 & x=0\end{cases},\quad g(x)=\begin{cases}\frac{\sin(x+1)}{x+1}, & x\neq -1\\ 1, & x=-1\end{cases}$$

and h(x) = 2[x] – f(x) ; where [x] is the greatest integer ≤ x. Then the value of $\lim\limits_{x\to 1} g(h(x-1))$ is

(a) 1 (b) 0 (c) sin(1) (d) –1

JEE Main-30.01.2023, Shift-II

Ans. (a) : Given, $f(x)=\left\{\frac{x}{|x|}, x\neq 0\right\}$

And, h(x) = 2 [x] – f(x)

L.H.L.

$$\lim_{h\to 0^-} g(h(1-h-1))$$

$$\lim_{h\to 0^-} g\left(2[-h]-\frac{-h}{|h|}\right)$$

$$\lim_{h\to 0^-} g(2(-1)+1)$$

$$\lim_{h\to 0^-} g(-1)=1$$

R.H.L

$$\lim_{h\to 0^+} g(h(1+h-1))$$

$$\lim_{h\to 0}[2h-f(h)]$$

$$\lim_{h\to 0^+} g(0-1)=1$$

215. Let $\beta=\lim\limits_{x\to 0}\frac{\alpha x-(e^{3x}-1)}{\alpha x(e^{3x}-1)}$ for some $\alpha\in R$. Then the value of $\alpha+\beta$ is :

(a) $\frac{14}{5}$ (b) $\frac{3}{2}$ (c) $\frac{5}{2}$ (d) $\frac{7}{2}$

JEE Main-26.07.2022, Shift-II

Ans. (c) : Given,

$$\beta=\lim_{x\to 0}\frac{\alpha x-(e^{3x}-1)}{\alpha x(e^{3x}-1)}$$

We know that,

$$e^x=1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+.........$$

$$e^{3x}=1+3x+\frac{(3x)^2}{2!}+\frac{(3x)^3}{3!}$$

$$\beta=\lim_{x\to 0}\frac{\alpha x-\left(1+3x+\frac{(3x)^2}{2!}+\frac{(3x)^3}{3!}+........-1\right)}{\alpha x\left(\frac{e^{3x}-1}{3x}\right).3x}$$

$$\beta=\lim_{x\to 0}\frac{\alpha x-3x-\frac{9x^2}{2!}+.......}{3\alpha x^2\left(\frac{e^{3x}-1}{3x}\right)}$$

$$\beta=\lim_{x\to 0}\frac{(\alpha-3)x-\frac{9x^2}{2!}}{3\alpha x^2}$$

For limit existence $\alpha-3=0 \Rightarrow \alpha=3$

Now, $\beta=\lim\limits_{x\to 0}-\frac{9}{2!}\frac{x^2}{\times 3\alpha x^2} \Rightarrow \frac{-9}{2\times 3\alpha}=-\frac{1}{2}$

Now, $\alpha=3$ and $\beta=-\frac{1}{2}$

Then, $\alpha+\beta=3-\frac{1}{2}=\frac{5}{2}$

216. $\lim\limits_{x\to 0}\frac{48}{x^4}\int_0^x \frac{t^3}{t^6+1}dt$ **is equal to ______.**

JEE Main-30.01.2023, Shift-I

Ans. (12) : Given, $\lim\limits_{x\to 0}\frac{48}{x^4}\int_0^x \frac{t^3}{t^6+1}dt$ $\quad \frac{0}{0}$ from

Now applying, $\lim\limits_{x\to 0} 48\frac{\int_0^x \frac{t^3}{t^6+1}dt}{x^4}$

Appling L' Hospital rule,

$$\lim_{x\to 0} 48\frac{\frac{x^3}{x^6+1}-0}{4x^3} \Rightarrow \lim_{x\to 0}\frac{48}{4(x^6+1)}$$

$$\lim_{x\to 0}\frac{12}{x^6+1} = 12$$

217. **The value of** $\lim\limits_{x\to 1}\frac{(x^2-1)\sin^2(\pi x)}{x^4-2x^3+2x-1}$ **is equal to :**

(a) $\frac{\pi^2}{6}$ (b) $\frac{\pi^2}{3}$ (c) $\frac{\pi^2}{2}$ (d) π^2

JEE Main-29.06.2022, Shift-II

Ans. (d) : Given,

$$\lim_{x\to 1}\frac{(x^2-1)\sin^2(\pi x)}{x^4-2x^3+2x-1}$$

$$\lim_{x\to 1}\frac{(x^2-1)\sin^2\pi x}{(x^4-1)-(2x^3-2x)}$$

$$\lim_{x\to 1}\frac{(x^2-1)\sin^2\pi x}{(x^2-1)(x^2+1)-2x(x^2-1)}$$

$$\lim_{x\to 1}\frac{(x^2-1)\sin^2\pi x}{(x^2-1)(x^2+1-2x)} = \lim_{x\to 1}\frac{\sin^2\pi x}{(x-1)^2}$$

$$\lim_{x\to 1}\frac{2\sin\pi x\cos\pi x.\pi}{2(x-1)}$$

$$\lim_{x\to 1}\frac{\pi\sin 2\pi x}{2(x-1)}$$

Again L.H. Rule

$$\lim_{x\to 1}\frac{2\pi^2\cos 2\pi x}{2} = \pi^2$$

218. **Let a be an integer such that** $\lim\limits_{x\to 7}\frac{18-[1-x]}{[x-3a]}$ **exists, where [t] is greatest integer ≤ t. Then a is equal to:**

(a) –6 (b) –2 (c) 2 (d) 6

JEE Main-27.06.2022, Shift-I

Ans. (a) : Given, $\lim\limits_{x\to 7}\frac{18-[1-x]}{[x]-3a}$

Now L.H.L

$$\lim_{x\to 7^-}\frac{18-[1-x]}{[x]-3a} = \frac{18-[1-7]}{[7]-3a} = \frac{18-(-6)}{6-3a} = \frac{24}{6-3a}$$

Now, for R.H.L

$$\lim_{x\to 7^+}\frac{18-[1-x]}{[x]-3a} = \frac{18-(-7)}{[7^+]-3a} = \frac{25}{7-3a}$$

Now, L.H.L = R.H.L.

$$\frac{24}{6-3a} = \frac{25}{7-3a}$$

$24(7 - 3a) = 25\,[6 - 3a]$

$168 - 72a = 150 - 75a$

$18 = -3a \Rightarrow a = -6$

219. $\lim\limits_{x\to \frac{1}{\sqrt{2}}}\frac{\sin(\cos^{-1}x)-x}{1-\tan(\cos^{-1}x)}$ **is equal to:**

(a) $\sqrt{2}$ (b) $-\sqrt{2}$ (c) $\frac{1}{\sqrt{2}}$ (d) $-\frac{1}{\sqrt{2}}$

JEE Main-26.06.2022, Shift-I

Ans. (d) : Given, $\lim\limits_{x\to \frac{1}{\sqrt{2}}}\frac{\sin(\cos^{-1}x)-x}{1-\tan(\cos^{-1}x)}$

$\cos\theta = x,\ \cos^{-1}x = \theta$

$$\tan\theta = \frac{\sqrt{1-x^2}}{x}$$

$$\sin\theta = \sqrt{1-x^2}$$

$$\lim_{x\to \frac{1}{\sqrt{2}}}\frac{\sin\left(\sin^{-1}\sqrt{1-x^2}\right)-x}{1-\tan\left(\tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)\right)}$$

$$\lim_{x\to \frac{1}{\sqrt{2}}}\frac{\sqrt{1-x^2}-x}{1-\frac{\sqrt{1-x^2}}{x}} \Rightarrow \lim_{x\to \frac{1}{\sqrt{2}}}\frac{\sqrt{1-x^2}-x}{\frac{x-\sqrt{1-x^2}}{x}}$$

$$\lim_{x\to \frac{1}{\sqrt{2}}}(-x) = -\frac{1}{\sqrt{2}}$$

220. **Let [f] denote the greatest integer ≤ t and {t} denote the fractional part of t. Then integral value of α for which the left hand limit of the function** $f(x) = [1 + x] + \frac{\alpha^{2[x]+\{x\}}+[x]-1}{2[x]+\{x\}}$ **at x = 0 equal to** $\alpha-\frac{4}{3}$ **is**

JEE Main-27.06.2022, Shift-II

Ans. (3) : Given, $f(x)=[1+x]+\dfrac{\alpha^{2[x]+\{x\}}+[x]-1}{2[x]+\{x\}}$

Now, Left hand limit,

$\lim\limits_{x\to0^-} f(x)=\alpha-\dfrac{4}{3}$

$0 \le x < 1$, than [x]

is 0 and $-1 \le x < 0$

Then [x]

is -1

$\dfrac{\alpha^{-1}-2}{-1}=\alpha-\dfrac{4}{3} \Rightarrow \dfrac{1}{\alpha}-2=\alpha-\dfrac{4}{3}$

$3\alpha^2-10\alpha+\beta=0$

$(\alpha-3)(3\alpha-1)=0$

$\alpha = 3$ and $\dfrac{1}{3}$

α is an integer.

Hence, $\alpha = 3$

221. $\lim\limits_{x\to\infty}\dfrac{\left(\sqrt{3x+1}+\sqrt{3x-1}\right)^6+\left(\sqrt{3x+1}-\sqrt{3x-1}\right)^6}{\left(x+\sqrt{x^2-1}\right)^6+\left(x-\sqrt{x^2-1}\right)^6}x^3$

(a) is equal to $\dfrac{27}{2}$ (b) is equal to 9

(c) does not exist (d) is equal to 27

JEE Main-31.01.2023, Shift-II

Ans. (d) : Given,

$$\lim_{x\to\infty}\left(\frac{x^3\left(\sqrt{3x+1}+\sqrt{3x-1}\right)^6+\left(\sqrt{3x+1}-\sqrt{3x-1}\right)^6}{\left(x+\sqrt{x^2-1}\right)^6+\left(x-\sqrt{x^2-1}\right)^6}\right)$$

$$\lim_{x\to\infty}\left(\frac{x^3\left(\sqrt{3+\frac{1}{x}}+\sqrt{3-\frac{1}{x}}\right)^6+\left(\sqrt{3+\frac{1}{x}}-\sqrt{3-\frac{1}{x}}\right)^6}{x^6\left(1+\sqrt{1-\frac{1}{x^2}}\right)^6+\left(1-\sqrt{1-\frac{1}{x^2}}\right)^6}\right)x^3$$

$$\lim_{x\to\infty}\frac{\left(\sqrt{3+\frac{1}{x}}+\sqrt{3-\frac{1}{x}}\right)^6+\left(\sqrt{3+\frac{1}{x}}-\sqrt{3-\frac{1}{x}}\right)^6}{\left(1+\sqrt{1-\frac{1}{x^2}}\right)^6+\left(\sqrt{1-\frac{1}{x^2}}\right)^6}$$

$$=\frac{\left(\sqrt{3+0}+\sqrt{3-0}\right)^6+\left(\sqrt{3+0}-\sqrt{3-0}\right)^6}{(1+1)^6+(1-1)^6}$$

$$=\frac{\left(2\sqrt{3}\right)^6}{(2)^6}=27$$

222. If $\lim\limits_{n\to\infty}\dfrac{(n+1)^{k-1}}{n^{k+1}}$ [(nk +1) + (nk + 2) + ….. + (nk + n)] = 33. $\lim\limits_{n\to\infty}\dfrac{1}{n^{k+1}}$. $[1^k + 2^k + 3^k + …. + n^k]$, then the integral value of k is equal to _______.

JEE Main-25.07.2022, Shift-I

Ans. (5) : Given,

LHS $\lim\limits_{n\to\infty}\dfrac{1}{n^{k+1}}\dfrac{(n+1)^{k-1}}{n^{k+1}}[nk.n+1+2+....+n]$

$=\lim\limits_{n\to\infty}\dfrac{(n+1)^{k-1}}{n^{k+1}}.\left[n^2k+\dfrac{n(n+1)}{2}\right]$

$$\lim_{n\to\infty}\frac{(n+1)^{k-1}.n^2\left[k+\dfrac{\left(1+\frac{1}{n}\right)}{2}\right]}{n^{k+1}}$$

$$=\lim_{n\to\infty}\left(1+\frac{1}{n}\right)\left(k+\frac{\left(1+\frac{1}{n}\right)}{2}\right)=\left(k+\frac{1}{2}\right)$$

RHS

$=\lim\limits_{n\to\infty}\dfrac{1}{n^{k+1}}(1^k+2^k+....+n^k)=\dfrac{1}{k+1}$

LHS = RHS

$= k+\dfrac{1}{2}=33.\dfrac{1}{k+1}=(2k+1)(k+1)=66$

$=(k-5)(2k+13)=0$

$=k=5$ or $-\dfrac{13}{2}$

223. If $\lim\limits_{n\to\infty}\left(\sqrt{n^2-n-1}+n\alpha+\beta\right)=0$ then 8 (α + β) is equal to :

(a) 4 (b) –8 (c) –4 (d) 8

JEE Main-25.07.2022, Shift-I

Ans. (c) : $\lim\limits_{n\to\infty}\left(1-\dfrac{n+1}{n^2}\right)^{\frac{1}{2}}+\alpha n+\beta=0$

$$\lim_{n\to\infty}n\left\{1-\frac{1}{2}\left(\frac{n+1}{n^2}\right)+\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2!}\left(\frac{n+1}{n^2}\right)^2+......\right\}+\alpha n+\beta=0$$

$\lim\limits_{n\to\infty}n-\dfrac{1}{2}+\dfrac{1}{n}+....+n\alpha+\beta=0$

$\alpha=-1,\beta=\dfrac{1}{2}$

$8(\alpha+\beta)=-4$

224. $\lim\limits_{n\to\infty}\dfrac{(n!)^{1/n}}{n}$ **equals**

(a) e (b) e^{-1} (c) 1 (d) None of these

AMU-2015

Ans. (b) : $f(x)=\lim\limits_{n\to\infty}\dfrac{(n!)^{1/n}}{n} \Rightarrow f(x)=\lim\limits_{n\to\infty}\left(\dfrac{n!}{n^n}\right)^{1/n}$

$y=\lim\limits_{n\to\infty}\left(\dfrac{1}{n}.\dfrac{2}{n}.\dfrac{3}{n}......\dfrac{n}{n}\right)^{1/n}$

Taking log on both side we get -

$$\log y=\lim_{x\to\infty}\frac{1}{n}\sum_{r=1}^{n}\log\frac{r}{n} \Rightarrow \log y=\int_0^1\log x\,dx$$

$$\log y=[x\log x-x]_0^1 \Rightarrow \log y=-1 \Rightarrow y=e^{-1}$$

225. $\lim\limits_{x\to\frac{\pi}{4}}\dfrac{8\sqrt{2}-(\cos x+\sin x)^7}{\sqrt{2}-\sqrt{2}\sin 2x}$ **is equal to**

(a) 14 (b) 7 (c) $14\sqrt{2}$ (d) $7\sqrt{2}$

JEE Main-25.07.2022, Shift-II

Ans. (a) : $\lim\limits_{x\to\frac{\pi}{4}}\dfrac{8\sqrt{2}-(\cos x+\sin x)^7}{\sqrt{2}-\sqrt{2}\sin 2x}\ \left\{\dfrac{0}{0}\text{from}\right\}$

Now, using L-Hospital's rule

$$\lim_{x\to\frac{\pi}{4}}\frac{-7(\cos x+\sin x)^6(-\sin x+\cos x)}{-\sqrt{2}.2\cos 2x}$$

$$\lim_{x\to\frac{\pi}{4}}\frac{-7(\cos x+\sin x)^6(\cos x-\sin x)}{-2\sqrt{2}\cos 2x}$$

$$\frac{-7\left(\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}\right)^6}{-2\sqrt{2}}\lim_{x\to\frac{\pi}{4}}\frac{\cos x-\sin x}{\cos 2x}$$

$$\frac{7(\sqrt{2})^6}{2\sqrt{2}}\lim_{x\to\frac{\pi}{4}}\frac{-\sin x-\cos x}{-(\sin 2x).2}=\frac{7(\sqrt{2})^6}{2\sqrt{2}}\times\frac{\sin\frac{\pi}{4}+\cos\frac{\pi}{4}}{2.\sin\frac{2\pi}{4}}$$

$$\frac{7\times 8}{2\sqrt{2}}\times\left[\frac{\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}}{2}\right]=\frac{56}{4\sqrt{2}}\times\frac{2}{\sqrt{2}}=14$$

226. $\lim\limits_{x\to\infty}\dfrac{1}{2^n}\left[\dfrac{1}{\sqrt{1-\frac{1}{2^n}}}+\dfrac{1}{\sqrt{1-\frac{2}{2^n}}}+\dfrac{1}{\sqrt{1-\frac{3}{2^n}}}+..+\dfrac{1}{\sqrt{1-\frac{2^n-1}{2^n}}}\right]$ **is equal to**

(a) $\frac{1}{2}$ (b) 1 (c) 2 (d) –2

JEE Main-25.07.2022, Shift-II

Ans. (c) : Given,

$$\lim_{x\to\infty}\frac{1}{2^n}\frac{1}{\sqrt{1-\frac{1}{2^n}}}+\frac{1}{\sqrt{1-\frac{2}{2^n}}}+\frac{1}{\sqrt{1-\frac{3}{2^n}}}+..+\frac{1}{\sqrt{1-\frac{2^n-1}{2^n}}}$$

Let, $2^n = p$, when $n\to\infty$,then $p\to\infty$

$$\lim_{p\to\infty}\frac{1}{p}\left\{\frac{1}{\sqrt{1-\frac{1}{p}}}+\frac{1}{\sqrt{1-\frac{2}{p}}}+\frac{1}{\sqrt{1-\frac{3}{p}}}+----+\frac{1}{\sqrt{1-\frac{p-1}{p}}}\right\}$$

$$\lim_{p\to\infty}\frac{1}{p}\left(\sum_{r=1}^{p-1}\frac{1}{\sqrt{1-\frac{r}{p}}}\right)=\int_0^1\frac{dx}{\sqrt{1-x}}=\int_0^1\frac{dx}{\sqrt{x}}$$

$$\because\ \int_0^a f(x)dx=\int_0^a f(a-x)dx\ =\ \left(2x^{1/2}\right)_0^1=2$$

227. $\lim\limits_{x\to\infty}\left(\dfrac{2+\sin x}{x^2+3}\right)=$

(a) 0 (b) 1 (c) –1 (d) ∞

APEAPCET-20.08.2021, Shift-I

Ans. (a):

$\lim\limits_{x\to\infty}\left(\dfrac{2+\sin x}{x^2+3}\right)=$

Since,

$\sin x\in[-1, 1]$

Hence,

When $x\to\infty$, then the function is of $\frac{\infty}{\infty}$form.

$$\lim_{x\to\infty}\left(\frac{2+\sin x}{x^2+3}\right)=\frac{2+\text{value in between}(-1,1)}{\infty+3}$$

$=0$

228. If [.] here denotes the greatest integer function, $\lim\limits_{x\to0}x^7\left[\dfrac{1}{x^3}\right]=$

(a) 1 (b) 0 (c) –1 (d) Does not exist

APEAPCET-20.08.2021, Shift-I

Ans. (b): Given,

$$\lim_{x\to0}x^7\left(\frac{1}{x^3}\right)$$

RHL:- Let x = 0 + h where $h\to0$

$$\lim_{h\to0}h^7\left[\frac{1}{(0+h)^3}\right]=0$$

LHL:-

Let x = 0 – h, where $h\to0$

$$\lim_{h\to0}(-h)^7\left[\left(\frac{1}{(0-h)^3}\right)\right]=0$$

∵ LHL = RHL

$$\therefore\lim_{x\to0}x^7\left[\frac{1}{x^3}\right]=0$$

229. If $\lim\limits_{x\to0}\{1+x\log(1+a^2)\}^{1/x}=2a\sin^2\theta$, **a > 0 and** $\theta\in R$**, then ______**

(a) $\theta=n\pi\pm\frac{\pi}{2}.(n\in z)$ (b) $\theta=n\pi\pm\frac{\pi}{3}.(n\in z)$

(c) $\theta=n\pi+\frac{\pi}{2}.(n\in z)$ (d) $\theta=n\pi\pm\frac{\pi}{4}.(n\in z)$

APEAPCET-20.08.2021, Shift-I

Ans. (a) : Given,

$$\lim_{x\to0}\left\{1+x\log(1+a)^2\right\}^{1/x}=2a\sin^2\theta,\ a>0$$

L.H.S,

$$\lim_{x\to0}\left\{1+x\log(1+a^2)\right\}^{1/x}\quad\left[(\infty)^\infty\text{ form}\right]$$

$$=e^{\lim\limits_{x\to0}\frac{1}{x}\left\{1+x\log(1+a^2)-1\right\}}$$

$$=e^{\lim\limits_{x\to0}\log(1+a^2)}=(1+a^2)$$

So,

$(1 + a^2) = 2a\sin^2\theta$

$a^2 - 2a\sin^2\theta + 1 = 0$

$$a=\frac{2\sin^2\theta\pm\sqrt{4\sin^4\theta-4}}{2}\Rightarrow a=\frac{2\sin^2\theta\pm2\sqrt{\sin^4\theta-1}}{2}$$

$$a=\sin^2\theta\pm\sqrt{\sin^4\theta-1}$$

$\because a>1$

$\sin^4\theta=1$

$\sin^2\theta=1=\sin^2\frac{\pi}{2}$

$\theta=n\pi\pm\frac{\pi}{2}$

230. $\lim\limits_{n\to\infty} n\left[\frac{1}{(3n^2+8n+4)}+\frac{1}{3n^2+16n+16}+....+\frac{1}{15n^2}\right]=$

(a) $\frac{1}{2}\log\frac{9}{5}$ (b) $\frac{1}{4}\log\frac{9}{5}$ (c) $2\log\frac{9}{5}$ (d) $\frac{1}{4}\log\frac{5}{9}$

AP EAMCET-22.04.2018, Shift-II

Ans. (b) : We have,

$$\lim_{n\to\infty} n\left[\frac{1}{3n^2+8n+4}+\frac{1}{3n^2+16n+16}.....\frac{1}{15n^2}\right]$$

$$\lim_{n\to\infty} n\times\frac{1}{n^2}\left[\frac{1}{3+\frac{8r}{n}+\frac{4r^2}{n^2}}\right]\Rightarrow\lim_{n\to\infty}\frac{1}{n}\left[\frac{1}{3+\frac{8r}{n}+\frac{4r^2}{n^2}}\right]$$

Let,

$\frac{r}{n}\to x$

$\frac{1}{n}\to dx$

$$\int_0^1\frac{1}{4x^2+8x+3}dx=\int_0^1\frac{1}{(2x+2)^2-1}dx$$

$$=\int_0^1\frac{1}{(2x+2+1)(2x+2-1)}dx$$

$$=\frac{1}{2}\int_0^1\left(\frac{1}{2x+1}-\frac{1}{(2x+3)}\right)dx$$

$$=\frac{1}{4}\left[\log(2x+1)-\log(2x+3)\right]_0^1$$

$$=\frac{1}{4}[\log3-\log5]+\frac{1}{4}\log3$$

$$=\frac{1}{4}[2\log3-\log5]=\frac{1}{4}[\log9-\log5]=\frac{1}{4}\log\frac{9}{5}$$

231. $\lim\limits_{x\to\frac{\pi}{2}}\frac{1+\cos2x}{\cot3x(3^{\sin2x}-1)}=$

(a) $\frac{1}{3\log9}$ (b) $\frac{2}{3\log3}$ (c) $\frac{1}{3\log3}$ (d) $\frac{3}{\log3}$

AP EAMCET-22.04.2018, Shift-II

Ans. (c) : Given,

$$\lim_{x\to\frac{\pi}{2}}\frac{1+\cos2x}{\cot3x\left(3^{\sin2x}-1\right)}=\lim_{x\to\frac{\pi}{2}}\frac{2\cos^2x}{\frac{\cos3x}{\sin3x}\left[3^{\sin2x}-1\right]}$$

Let, $x\to\frac{\pi}{2}-h$

$$=\lim_{h\to0}\frac{2\cos^2\left(\frac{\pi}{2}-h\right)}{\frac{\cos3\left(\frac{\pi}{2}-h\right)}{\sin3\left(\frac{\pi}{2}-h\right)}\left[3^{\sin2\left(\frac{\pi}{2}-h\right)}-1\right]}$$

$$=\lim_{h\to0}\frac{2\sin^2h}{\frac{-\sin3h}{-\cos3h}\left[3^{\sin2h}-1\right]}$$

$$=\lim_{h\to0}\frac{\frac{2\sin^2h}{h^2}\times h^2\times\cos3h}{\frac{\sin3h}{3h}\times3h\left(\frac{3^{\sin2h}-1}{\sin2h}\right)\frac{\sin^2 2h}{2h}\times2h}$$

We know,

$$\lim_{h\to0}\frac{\sin x}{x}=1$$

And, $\lim\limits_{x\to0}\frac{a^x-1}{a}=\log a=\frac{2\times1}{(1\times3)\log3\times1\times2}=\frac{1}{3\log3}$

232. $\lim\limits_{n\to\infty} n^{-nk}$

$$\left\{(n+1)\left(n+\frac{1}{2}\right)\left(n+\frac{1}{2^2}\right)....\left(n+\frac{1}{2^{k-1}}\right)\right\}^n=$$

(a) 2 (b) $e^{2\left(1-\frac{1}{2^k}\right)}$ (c) $2\left(1-\frac{1}{2^k}\right)$ (d) e^2

AP EAMCET-22.04.2018, Shift-II

Ans. (b) : Given,

$$\lim_{n\to\infty} n^{-nk}\left\{(n+1)\left(n+\frac{1}{2}\right)\left(n+\frac{1}{2^2}\right).....\left(n+\frac{1}{2^{k-1}}\right)\right\}^n$$

$$\lim_{n\to\infty}\left[\frac{(n+1)\left(n+\frac{1}{2}\right)\left(n+\frac{1}{n^2}\right)+.....\left(n+\frac{1}{2^{k-1}}\right)}{n^{nk}}\right]^n$$

$$\lim_{n\to\infty}\left[\frac{(n+1)}{n^k}\left(\frac{n+\frac{1}{2}}{n^k}\right)\left(\frac{n+\frac{1}{2^2}}{n^k}\right).....\frac{\left(n+\frac{1}{2^{k-1}}\right)}{n^k}\right]^n$$

$$\lim_{n\to\infty}\left(\left(1+\frac{1}{n^{k-1}}\right)^n\left(1+\frac{1}{2n^{k-1}}\right)^n\left(1+\frac{1}{2^2n^{k-1}}\right)^n.....1+\frac{1}{2^{n-1}n^{k-1}}\right)$$

We know that,

$$\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x=e$$

$$=\lim_{n\to\infty}e^{\left\{1+\frac{1}{2^{k-1-1}}\frac{1}{4^{k+1}}\frac{1}{2^{k-1}}\right\}}=e^{2\left(1-\frac{1}{2^k}\right)}$$

233. If $\lim\limits_{x\to 0}\frac{ae^x - b\cos x + ce^{-x}}{x\sin x}$ =2, then
(a) a =1 , b = 2, c = 1 (b) a = 1, b = 1, c = 2
(c) a = 2, b = 1, c = 1 (d) a = b = c = 1

AMU-2011

Ans. (a) : $\lim\limits_{x\to 0}\frac{ae^x - b\cos x + ce^{-x}}{x\sin x} = 2$

$$\lim_{x\to 0}\frac{a\left(1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+...\right)-b\left(1-\frac{x^2}{2!}+\frac{x^4}{4!}.....\right)+c\left(1-x+\frac{x^2}{2!}-\frac{x^3}{3!}+....\right)}{x^2\left(\frac{\sin x}{x}\right)}$$

$$\lim_{x\to 0}\frac{(a-b+c)+(a-c)x+\left(\frac{a}{2!}+\frac{b}{2!}+\frac{c}{2!}\right)x^2+....}{x^2}$$

For the limit to exist and evaluate the limit to 2
a – b + c = 0 and a – c = 0

and $\frac{a}{2!}+\frac{b}{2!}+\frac{c}{2!}=2$

so, a – b + c = 0 ……..(i)
a – c = 0 ……..(ii)
a + b + c = 4 ……..(iii)

Adding equation (i) and (iii)
2(a + c) = 4
a + c = 2
Now, a + c = 2
and a – c = 0
2a = 2
a = 1
Then c = 1 and b = 2

234. If the function f (x) satisfies $\lim\limits_{x\to 1}\frac{f(x)-3}{x^2-1}=\pi$, then $\lim\limits_{x\to 1} f(x)$ is
(a) 1 (b) 2 (c) 3 (d) π

AMU-2010

Ans. (c) : Given, $\lim\limits_{x\to 1}\frac{f(x)-3}{x^2-1}=\pi$,

$$\frac{\lim\limits_{x\to 1}(f(x)-3)}{\lim\limits_{x\to 1}(x^2-1)}=\pi$$

$$\lim_{x\to 1} f(x) - \lim_{x\to 1} 3 = \pi\lim_{x\to 1}(x^2-1)$$

$$\lim_{x\to 1} f(x) - 3 = \pi(0) \Rightarrow \lim_{x\to 1} f(x) - 3 = 0$$

$$\lim_{x\to 1} f(x) = 3$$

235. $\lim\limits_{x\to 1}\frac{x^{15}-1}{x^{10}-1}$ =
(a) 2/3 (b) 3/2 (c) 1 (d) does not exist

COMEDK-2020, AMU-2010

Ans. (b) :

$$\lim_{x\to 1}\frac{x^{15}-1}{x^{10}-1}=\lim_{x\to 1}\frac{x^{15}-1}{x-1}\times\lim_{x\to 1}\frac{x-1}{x^{10}-1}$$

$$=\frac{15(1)^{14}}{10(1)^9}=\frac{15}{10}=\frac{3}{2}\quad\left(\because \frac{x^n}{x-a}=na^{n-1}\right)$$

Hence, $\lim\limits_{x\to 1}\frac{x^{15}-1}{x^{10}-1}=\frac{3}{2}$

236. $\lim\limits_{x\to 0}\frac{\tan(x)+4\tan(2x)-3\tan(3x)}{x^2\tan(x)}$ =
(a) 8 (b) –8 (c) 16 (d) –16

APEAPCET- 23.08.2021, Shift-2

Ans. (d) : Given,

$$\lim_{x\to 0}\frac{\tan(x)+4\tan(2x)-3\tan(3x)}{x^2\tan(x)}\quad\left(\frac{0}{0}\text{form}\right)$$

$$\tan x = x + \frac{x^3}{3}+\frac{2x^5}{15}+.......$$

$$=\lim_{x\to 0}\frac{\left(x+\frac{x^3}{3}\right)+4\left(2x+\frac{(2x)^3}{3}\right)-3\left(3x+\frac{(3x)^3}{3}\right)}{x^2\left(x+\frac{x^3}{3}\right)}$$

$$=\lim_{x\to 0}\frac{x(1+8-9)+\left(\frac{1}{3}+4\times\frac{(2)^3}{3}-3\times\frac{(3)^3}{3}\right)x^3}{x^3\left(1+\frac{x^2}{3}\right)}$$

$$=\frac{1+32-81}{3}=-\frac{48}{3}=-16$$

237. $\lim\limits_{x\to 0}\frac{8}{x^8}\left(1-\cos\left(\frac{x^2}{2}\right)-\cos\left(\frac{x^2}{4}\right)+\cos\left(\frac{x^2}{2}\right)\cos\left(\frac{x^2}{4}\right)\right)$ =
(a) 1/4 (b) 1/8 (c) 1/16 (d) 1/32

APEAPCET- 23.08.2021, Shift-2

Ans. (d) : Given,

$$\lim_{x\to 0}\frac{8}{x^8}\left\{\left(1-\cos\frac{x^2}{2}\right)-\cos\frac{x^2}{4}\left(1-\cos\frac{x^2}{2}\right)\right\}$$

$$=\lim_{x\to 0}\frac{8}{x^8}\left[\left(1-\cos\frac{x^2}{2}\right)\left(1-\cos\frac{x^2}{4}\right)\right]$$

$$=\lim_{x\to 0}\frac{8}{x^8}\left[2\sin^2\frac{x^2}{4}\times 2\sin^2\frac{x^2}{8}\right]=\lim_{x\to 0}8\times 4\left[\frac{\sin^2\frac{x^2}{4}}{x^4}\times\frac{\sin^2\frac{x^2}{8}}{x^4}\right]$$

$$=\lim_{x\to 0}32\left[\left(\frac{\sin x^2/4}{x^2}\right)^2\times\left(\frac{\sin\frac{x^2}{8}}{x^2}\right)^2\right]$$

$$=\lim_{x\to 0}\frac{32}{4^2\times 8^2}\left(\frac{\sin\frac{x^2}{4}}{\frac{x^2}{4}}\right)^2\times\left(\frac{\sin\frac{x^2}{8}}{\frac{x^2}{8}}\right)^2=\frac{1}{32}$$

238. $\lim_{n\to\infty}\frac{n(2n+1)^2}{(n+2)(n^2+3n-1)}=$

(a) 0 (b) 4 (c) 2 (d) ∞

AP EAPCET-25.08.2021, Shift-II

Ans. (b) : Given,

$$\lim_{n\to\infty}\frac{n(2n+1)^2}{(n+2)(n^2+3n-1)}$$

$$\lim_{n\to\infty}\frac{(2+1/n)^2}{(1+2/n)(1+3/n-7/n^2)}=\frac{(2+0)^2}{(1+0)(1+0+0)}=4$$

239. **Let** $L=\lim_{x\to0}\frac{a-\sqrt{a^2-x^2}-\frac{x^2}{4}}{x^4}, a>0$. **If L is finite, then**

(a) a = 2 (b) a = 1 (c) $a=\frac{1}{3}$ (d) None of these

AMU-2014

Ans. (a) : Given, $L=\lim_{x\to0}\frac{a-\sqrt{a^2-x^2}-\frac{x^2}{4}}{x^4}, a>0$

$$\Rightarrow \lim_{x\to0}\frac{a-a\left(1-\frac{x^2}{a^2}\right)^{\frac{1}{2}}-\frac{x^2}{4}}{x^4}$$

$$\Rightarrow \lim_{x\to0}\frac{a-a\left(1-\frac{1}{2}\frac{x^2}{a^2}-\frac{1}{8}\frac{x^4}{a^4}+....\right)-\frac{x^2}{4}}{x^4}$$

We know that

$$(1-x)^{\frac{1}{2}}=1-\frac{x}{2}+\frac{\frac{1}{2}\left(\frac{1}{2}-1\right)}{2}x^2-\frac{\frac{1}{2}\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-1\right)}{3!}x^3$$

$$\lim_{x\to0}\frac{x^2\left(\frac{1}{2a}-\frac{1}{4}\right)+a\left(\frac{x^4}{8a^4}+....\right)}{x^4}$$

as limit exist we have, $\frac{1}{2a}-\frac{1}{4}=0 \Rightarrow a=2$

240. $\lim_{n\to\infty}\sum_{r=1}^{n}\cot^{-1}\left(r^2+\frac{3}{4}\right)=$

(a) $\cot^{-1}2$ (b) $\cot^{-1}\frac{1}{3}$ (c) $\tan^{-1}2$ (d) $\tan^{-1}\frac{1}{3}$

AP EAMCET-06.07.2022, Shift-II

Ans. (c) : Given, $\lim_{n\to\infty}\sum_{r=1}^{n}\cot^{-1}\left(r^2+\frac{3}{4}\right)$

$$\lim_{n\to\infty}\sum_{r=1}^{n}\tan^{-1}\left(\frac{1}{r^2+\frac{3}{4}}\right)\Rightarrow\lim_{n\to\infty}\sum_{r=1}^{n}\tan^{-1}\left(\frac{1}{r^2-\frac{1}{4}+1}\right)$$

$$\lim_{n\to\infty}\sum_{r=1}^{n}\tan^{-1}\left(\frac{1}{1+\left(r+\frac{1}{2}\right)\left(r-\frac{1}{2}\right)}\right)$$

$$\lim_{n\to\infty}\sum_{r=1}^{n}\tan^{-1}\left(\frac{\left(r+\frac{1}{2}\right)-\left(r-\frac{1}{2}\right)}{1+\left(r+\frac{1}{2}\right)\left(r-\frac{1}{2}\right)}\right)$$

$$\lim_{n\to\infty}\sum_{r=1}^{n}\tan^{-1}\left(r+\frac{1}{2}\right)-\tan^{-1}\left(r-\frac{1}{2}\right)$$

$$\lim_{n\to\infty}\sum_{r=1}^{n}\left\{\tan^{-1}\left(n+\frac{1}{2}\right)-\tan^{-1}\left(\frac{1}{2}\right)\right\}=\frac{\pi}{2}-\tan^{-1}\left(\frac{1}{2}\right)$$

$\Rightarrow \tan^{-1}2$

241. **If n > 0 and** $\lim_{x\to\infty}\frac{((a-n)nx-\tan x)\sin x}{x^2}=0$**, then minimum value of a is**

(a) 1 (b) 2 (c) 3 (d) –1

AP EAMCET-06.07.2022, Shift-II

Ans. (b) : Given,

$$\lim_{x\to\infty}\frac{((a-n)nx-\tan x)\sin x}{x^2}=0$$

$$\lim_{x\to0}\frac{[(a-n)nx-\tan x]}{x}\lim_{x\to0}\frac{\sin nx}{nx}n=0$$

$$n\left[\lim_{x\to0}\frac{(a-n)nx}{x}-\lim_{x\to0}\frac{\tan x}{x}\right]\lim_{nx\to0}\frac{\sin x}{nx}=0$$

$\Rightarrow$ n[(a – n) n – 1]× 1 = 0

$\Rightarrow$ (a – n)n = 1

$\Rightarrow$ $a=\frac{1}{n}+n \Rightarrow a=\frac{n^2+1}{n}$

a is minimum when n is minimum

∴ the minimum value of n is 1

$$a=\frac{1+1}{1}=2$$

242. $\lim_{x\to0}\left(\frac{e^x-1}{x}\right)^{\frac{x}{x+1-e^x}}=$

(a) e (b) e^{-1} (c) e^2 (d) e^{-2}

AP EAMCET-06.07.2022, Shift-II

Ans. (b) : Given,

$$\lim_{x\to0}\left(\frac{e^x-1}{x}\right)^{\frac{x}{x+1-e^x}}$$

$$e^{\lim_{x\to0}\left(\frac{e^x-1}{x}-1\right)\left(\frac{x}{x+1-e^x}\right)}$$

$$e^{\lim_{x\to0}\left(\frac{e^x-1-x}{x}\right)\left(\frac{x}{x+1-e^x}\right)}$$

$$e^{\lim_{x\to0}\left(\frac{e^x-1-x}{x+1-e^x}\right)}$$

$$e^{\lim_{x\to0}-\left(\frac{-e^x+1+x}{x+1-e^x}\right)}=e^{-1}$$

243. $\lim\limits_{x\to 0}\dfrac{x^2(\tan 2x-2\tan x)^2}{(1-\cos 2x)^4}=$

(a) 4 (b) 2 (c) $\frac{1}{2}$ (d) $\frac{1}{4}$

AP EAMCET-21.04.2019, Shift-I

Ans. (d) : Given, $\lim\limits_{x\to 0}\dfrac{x^2(\tan 2x-2\tan x)^2}{(1-\cos 2x)^4}$

$$=\lim_{x\to 0}\frac{x^2\left[\left(2x+\frac{(2x)^3}{3}+\frac{2}{15}(2x)^5+...\right)-2\left(x+\frac{x^3}{3}+\frac{2}{15}x^5+...\right)\right]^2}{\left[1-\left(1-\frac{(2x)^2}{2!}+\frac{(2x)^4}{4!}-\frac{(2x)^6}{6!}+...\right)\right]^4}$$

$$=\lim_{x\to 0}\frac{4x^8\left[\left(\frac{4}{3}-\frac{1}{3}\right)+\frac{2}{15}(16x^2-x^2)+...\right]^2}{16x^8\left[1+\frac{x^2}{3}+....\right]^4}=\frac{4}{16}=\frac{1}{4}$$

244. $\lim\limits_{x\to\infty}\left(\dfrac{6x^2-\cos 3x}{x^2+5}-\dfrac{5x^3+3}{\sqrt{x^6+2}}\right)=$

(a) 11 (b) 0 (c) –1 (d) 1

AP EAMCET-21.04.2019, Shift-I

Ans. (a) : Given,

$$\lim_{x\to\infty}\left(\frac{6x^2-\cos 3x}{x^2+5}-\frac{5x^3+3}{\sqrt{x^6+2}}\right)$$

$$=\lim_{x\to\infty}\left(\frac{x^2\left(6-\frac{\cos 3x}{x^2}\right)}{x^2\left(1+\frac{5}{x^2}\right)}-\frac{x^3\left(5+\frac{3}{x^3}\right)}{|x^3|\sqrt{1+\frac{2}{x^6}}}\right)$$

$$=\lim_{x\to\infty}\left(\left(\frac{6-\frac{\cos 3x}{x^2}}{1+\frac{5}{x^2}}\right)-\frac{5+\frac{3}{x^3}}{(-1)\sqrt{1+\frac{2}{x^6}}}\right)$$

$$=\frac{6-0}{1+0}-5(-1)=6+5=11$$

245. $\lim\limits_{x\to\infty}\left[\dfrac{1}{n}+\dfrac{n^2}{(n+1)^3}+\dfrac{n^2}{(n+2)^3}+\dfrac{n^2}{(n+3)^3}+...+\dfrac{1}{125n}\right]=$

(a) $\frac{3}{8}$ (b) $\frac{15}{32}$ (c) $\frac{12}{25}$ (d) $\frac{35}{72}$

AP EAMCET-20.04.2019, Shift-II

Ans. (c) : Given,

$$\lim_{x\to\infty}\left[\frac{1}{n}+\frac{n^2}{(n+1)^3}+\frac{n^2}{(n+2)^3}+\frac{n^2}{(n+3)^3}+...+\frac{1}{125n}\right]=$$

It can be written as

$$\lim_{x\to\infty}\left[\frac{n^2}{(n+0)^2}+\frac{n^2}{(n+1)^2}+\frac{n^2}{(n+2)^2}+......\frac{n^2}{(n+4n)^3}\right]$$

$$\lim_{n\to\infty}\sum_{r=0}^{4n}\frac{n^2}{(n+r)^3}$$

$$\lim_{n\to\infty}\sum_{r=0}^{4n}\frac{n^2}{n^3\left(1+\frac{r}{n}\right)^3}\Rightarrow\lim_{n\to\infty}\sum_{r=0}^{4n}\frac{1}{n\left(1+\frac{r}{n}\right)^3}$$

Let,

$\frac{r}{n}=x$ When, $r=0$, $x\to 0$

$r=4n$, $x=4$

Then,

$\int_0^4\frac{1}{(1+x)^3}$ let, $1+x=t$

$dx=dt$

$t=1$ and $t=5$

$$=\int\frac{1}{t^3}dt=\left[\frac{t^{-2}}{-2}\right]_1^5$$

$$=-\frac{1}{2}\left[\frac{1}{25}-1\right]=-\frac{1}{2}\left[\frac{1-25}{25}\right]=-\frac{1}{2}\times\frac{-24}{25}=\frac{12}{25}$$

246. **By the definition of the definite integral, the value of**

$$\lim_{n\to\infty}\left(\frac{1}{\sqrt{n^2-1}}+\frac{1}{\sqrt{n^2-2^2}}+....+\frac{1}{\sqrt{n^2-(n-1)^2}}\right)\textbf{ is}$$

equal to

(a) π (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{6}$

AP EAMCET-2016

Ans. (b) : Given,

$$\lim_{n\to\infty}\left(\frac{1}{\sqrt{n^2-1}}+\frac{1}{\sqrt{n^2-2^2}}.......\frac{1}{\sqrt{n^2-(n-1)^2}}\right)$$

$$=\lim_{n\to\infty}\sum_{r=1}^{n-1}\frac{1}{\sqrt{n^2-r^2}}=\lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{n-1}\frac{1}{\sqrt{1-\left(\frac{r}{n}\right)^2}}$$

$$=\int_0^1\frac{dx}{\sqrt{1-x^2}}\qquad\left(\because\int\frac{dx}{\sqrt{1-x^2}}=\sin^{-1}x\right)$$

$$=\left[\sin^{-1}x\right]_0^1=\frac{\pi}{2}$$

247. $\lim\limits_{x\to 0}\dfrac{6^x-3^x-2^x+1}{x^2}=$

(a) $(\log_e 2)\log_e 3$ (b) $\log_e 5$ (c) $\log_e 6$ (d) 0

AP EAMCET-2016

Ans. (a) : Given,

$$\lim_{x\to 0}\frac{6^x-3^x-2^x+1}{x^2}=\lim_{x\to 0}\frac{3^x(2^x-1)-1(2^x-1)}{x^2}$$

$$=\lim_{x\to 0}\frac{(3^x-1)(2^x-1)}{x^2}=\lim_{x\to 0}\left(\frac{3^x-1}{x}\right)\cdot\left(\frac{2^x-1}{x}\right)$$

We know that $\lim\limits_{x\to 0}\dfrac{a^x-1}{x}=\log a$

$=\log_e 3.\ \log_e 2$

248. If $\lim_{x\to 0}\frac{\log(3+x)-\log(3-x)}{x}=K$, then K is equal to

(a) 2/5 (b) 2/3 (c) 1/2 (d) 5/2

AMU-2016

Ans. (b) : $\lim_{x\to 0}\frac{\log(3+x)-\log(3-x)}{x}$

$=\lim_{x\to 0}\frac{\log\left[3\left(1+\frac{x}{3}\right)\right]-\log\left[3\left[1-\frac{x}{3}\right]\right]}{x}$

$=\lim_{x\to 0}\frac{\log 3+\log\left(1+\frac{x}{3}\right)-\log 3-\log\left(1-\frac{x}{3}\right)}{x}$

($\because$ log m.n = log m + log n)

$=\lim_{x\to 0}\frac{\log\left(1+\frac{x}{3}\right)-\log\left(1-\frac{x}{3}\right)}{x}$

$=\lim_{x\to 0}\frac{\log\left(1+\frac{x}{3}\right)}{\frac{x}{3}\times 3}-\lim_{x\to 0}\frac{\log\left(1-\frac{x}{3}\right)}{-\frac{x}{3}\times(-3)}$

$=\frac{1}{3}-\frac{1}{(-3)}=\frac{2}{3}$

249. Let f : R → R be such that f(1) = 3, and f' (1) = 6. Then $\lim_{x\to 0}\left[\frac{f(1+x)}{f(1)}\right]^{\frac{1}{x}}=$

(a) 1 (b) $e^{\frac{1}{2}}$ (c) e^2 (d) e^3

AMU-2013

Ans. (c) : Let $y=\left[\frac{f(1+x)}{f(1)}\right]^{\frac{1}{x}}$

$\log y=\frac{1}{x}\log\left[\frac{f(1+x)}{f(1)}\right]=\frac{\log f(1+x)-\log f(1)}{x}$

$\lim_{x\to 0}\log y=\lim_{x\to 0}\frac{\log f(1+x)-\log 3}{x}=\lim_{x\to 0}\frac{f'(1+x)}{f(1+x)}$

$\frac{f'(1)}{f(1)}=\frac{6}{3}=2$

log y = 2

y = e^2

250. If f(x) is differentiable and strictly increasing function, then the value of $\lim_{x\to 0}\frac{f(x^2)-f(x)}{f(x)-f(0)}=$

(a) –1 (b) 0 (c) 1 (d) 2

AMU-2013

Ans. (a) : Given,

$\lim_{x\to 0}\frac{f(x^2)-f(x)}{f(x)-f(0)}=\lim_{x\to 0}\frac{f(x^2)-f(0)-\{f(x)-f(0)\}}{f(x)-f(0)}$

$=\lim_{x\to 0}\frac{f(x^2)-f(0)}{f(x)-f(0)}-1$

$=\lim_{x\to 0}\frac{f(x^2)-f(0)}{x^2}\times\frac{x^2}{f(x)-f(0)}-1$

$=f(0)\times\frac{1}{f(0)}\lim_{x\to 0}x-1=-1$

251. $\lim_{n\to\infty}\left(\frac{1}{1^5+n^5}+\frac{2^4}{2^5+n^5}+\frac{3^4}{3^5+n^5}+...+\frac{n^4}{n^5+n^5}\right)$

(a) $\frac{1}{5}\log 3$ (b) $\frac{1}{3}\log 5$ (c) $\frac{1}{2}\log 5$ (d) $\log\sqrt[5]{2}$

AP EAMCET-04.07.2021, Shift-I

Ans. (d) : Given,

$\lim_{n\to\infty}\left(\frac{1}{1^5+n^5}+\frac{2^4}{2^5+n^5}+\frac{3^4}{3^5+n^5}+......+\frac{n^4}{n^5+n^5}\right)$

Now, general form of the series,

$\lim_{n\to\infty}\sum_{r=1}^{n}\frac{r^4}{r^5+n^5}$

$\lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{n^5}\left(\frac{r^4}{\frac{r^5}{n^5}+1}\right)\Rightarrow\lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{n}\left(\frac{\left(\frac{r}{n}\right)^4}{1+\left(\frac{r}{n}\right)^5}\right)$

$\lim_{n\to\infty}\int_0^1\frac{x^4}{1+x^5}dx$

$\lim_{n\to\infty}\frac{1}{5}\left[\log(1+x^5)\right]_0^1$ Let, $\frac{r}{n}=x$

$=\frac{1}{5}[\log 2-0]$ $\frac{1}{n}=dx$

$=\frac{1}{5}\log 2$ x = 0, to 1

$\log(2)^{1/5}=\log\sqrt[5]{2}$

252. Let f : $R^+\to R^+$ be a function satisfying f(x) – x = λ (constant), ∀x ∈ R^+ and f(x f(y)) = f(xy) + x, ∀x, y ∈R^+.

Then, $\lim_{x\to 0}\frac{(f(x))^{1/3}-1}{(f(x))^{1/2}-1}=$

(a) 1/3 (b) 0 (c) 2/3 (d) 1

AP EAMCET-04.07.2021, Shift-I

Ans. (c) : Given, $f(xf(y)) = f(xy) + x$ ...(i)

Now, interchanging x and y them

f(y. f(x)) = f(yx) + y ...(ii)

Again replace x with f(x) in Equation (i)

We get,

f(f(x). f(y)) = f(y.f(x)) + f(x) ...(iii)

Therefore equation (i) – (iii)

f(f(x). f(y)) = f(xy) + y + f(x) ...(iv)

Again interchange x and y equation (iv)

We have

f(f(y). f(x)) = f(yx) + x + f(y) ...(v)

Equation (iv) and (v)

f(xy) + y + f(x) = f(yx) + x + f(y) ... (vi)

Suppose $f(x) - x = f(y) - y = \lambda$

Substitution $f(x) = \lambda + x$ in equation (i)

We have

$x.f(y) + \lambda = (xy + \lambda) + x$

$x.f(y) = xy + x$

Therefore

$x(y + \lambda) = xy + x$

$\lambda = x$

$\lambda = 1\ (x > 0)$

So, $f(x) = x + \lambda = x + 1$

Hence, $\lim_{x\to 0}\frac{(f(x))^{1/3}-1}{(f(x))^{1/2}-1} \Rightarrow \lim_{x\to 0}\frac{(x+1)^{1/3}-1}{(1+x)^{1/2}-1}$

$$\lim_{x\to 0}\frac{(x+1)^{1/3}-1}{1+x-1}\cdot\frac{1+x-1}{(1+x)^{1/2}-1} \Rightarrow \frac{1/3}{1/2}=\frac{2}{3}$$

253. $\lim_{x\to 8}\frac{\sqrt{1+\sqrt{1-x}}-2}{x-8}$ is equal to

(a) $\frac{3}{2}$ (b) $\frac{1}{4}$ (c) $\frac{1}{24}$ (d) $\frac{1}{12}$

AP EAMCET-2011

Ans. (c) : Given,

$$\lim_{x\to 8}\frac{\sqrt{1+\sqrt{1-x}}-2}{x-8} \quad \frac{0}{0}\text{from}$$

Using L- Hospital's rule

$$\lim_{x\to 8}\frac{\frac{1}{2\sqrt{1+\sqrt{1+x}}}\frac{d}{dx}\left(1+\sqrt{1+x}\right)-0}{1-0}$$

$$= \lim_{x\to 8}\frac{1}{2\sqrt{1\sqrt{1+x}}}\left(0+\frac{1}{2\sqrt{1+x}}\right)\frac{d}{dx}(1+x)$$

$$= \lim_{x\to 8}\frac{1}{2\sqrt{1+\sqrt{1+x}}}\times\frac{1}{2\sqrt{1+x}}(0+1)$$

$$= \frac{1}{2\sqrt{1+\sqrt{9}}}\times\frac{1}{2\sqrt{9}}\times 1 = \frac{1}{2\sqrt{1+3}}\times\frac{1}{2\times 3}\times 1$$

$$= \frac{1}{2\times 2}\times\frac{1}{6} = \frac{1}{24}$$

254. $\lim_{x\to 0}\frac{\tan x-\sin x}{x^2}$ is equal to

(a) 0 (b) 1 (c) $\frac{1}{2}$ (d) $-\frac{1}{2}$

AP EAMCET-2010

Ans. (a) : Given,

$$\lim_{x\to 0}\frac{\tan x-\sin x}{x^2}$$

Which is form of {0/0}

Now, applying L – Hospital's rule,

$$\lim_{x\to 0}\frac{\sec^2 x-\cos x}{2x} \quad \frac{0}{0}\text{from}$$

Again taking derivative of numerator and denominator i.e. by applying L- Hospital's rule

$$\lim_{x\to 0}\frac{2\sec x(\sec x.\tan x)-(-\sin x)}{2}$$

$$\lim_{x\to 0}\frac{2\sec^2 x.\tan x+\sin x}{2}$$

$$= \frac{2\sec^2(0).\tan(0)+\sin(0)}{2} = \frac{2\times 1\times 0+0}{2} = 0$$

255. $\lim_{x\to 0}\frac{e^x-e^{\sin x}}{2(x-\sin x)}$

(a) –1/2 (b) 1/2 (c) 1 (d) 3/2

AP EAMCET-2007

Ans. (b) : We have,

$$\lim_{x\to 0}\frac{e^x-e^{\sin x}}{2(x-\sin x)} = \lim_{x\to 0}e^{\sin x}\left(\frac{e^{x-\sin x}-1}{2(x-\sin x)}\right)$$

$$= \lim_{x\to 0}e^{\sin x}.\frac{1}{2}\times 1 \quad \left(\because \lim_{x\to 0}\frac{e^x-1}{x}=1\right)$$

$$= \frac{1}{2}\lim_{x\to 0}e^{\sin 0} = \frac{1}{2}.1 = \frac{1}{2}$$

256. $\lim_{x\to 0}\frac{\cos 4x-1}{x}$

(a) –2 (b) –4 (c) $-\frac{1}{2}$ (d) 0

AMU-2004

Ans. (d) : Give that,

$$\lim_{x\to 0} = \frac{\cos 4x-1}{x}$$

Using L-Hospital rule,

$$\lim_{x\to 0} = \frac{-4\sin 4x}{1} = -4[\sin 0] = 0$$

257. $\lim_{x\to 0}\frac{\alpha x-(e^{4x}-1)}{\alpha x(e^{4x}-1)} = \beta$ if limit exist then $2(\alpha+\beta)$ is

(a) –1 (b) –7 (c) 1 (d) 7

AMU-2021

Ans. (d) : Given,

$$\lim_{x\to0}\frac{\alpha x-\left(e^{4x}-1\right)}{\alpha x\left(e^{4x}-1\right)}=\beta \qquad \left[\frac{0}{0}\text{from}\right]$$

Using L-Hospital's rule we get,

$$\lim_{x\to0}\frac{\alpha-4e^{4x}}{\alpha x\left(4e^{4x}\right)+\alpha\left(e^{4x}-1\right)}$$

When $\alpha-4=0$

$\alpha=4$

$$\lim_{x\to0}\frac{4-4e^{4x}}{4\left(e^{4x}-1\right)+4x\left(4e^{4x}\right)}$$

$$\left\{\frac{0}{0}\right\}\text{from}$$

Again applying L-Hospital's rule

$$\lim_{x\to0}\frac{-e^{4x}.4}{4e^{4x}+4e^{4x}}=-\frac{1}{2}=\beta$$

$$\alpha=4 \text{ and } \beta=-\frac{1}{2}$$

Then, $2(\alpha+\beta)$

$$\Rightarrow \quad 2\left(4-\frac{1}{2}\right) \Rightarrow \quad 2\left(\frac{7}{2}\right)\Rightarrow 7$$

258. $\lim\limits_{x\to0}\dfrac{\sqrt{1+x\sin x}-\sqrt{\cos x}}{\tan^2 2x}=$

(a) 3 (b) $\frac{3}{2}$ (c) $\frac{3}{4}$ (d) $\frac{3}{16}$

AP EAMCET-24.04.2018, Shift-I

Ans. (d) : Given,

$$\lim_{x\to0}\frac{\sqrt{1+x\sin x}-\sqrt{\cos x}}{\tan^2 2x}$$

Now rationalizing we get –

$$\lim_{x\to0}\frac{\sqrt{1+x\sin x}-\sqrt{\cos x}}{\tan^2 2x}\times\frac{\sqrt{1+x\sin x}+\sqrt{\cos x}}{\sqrt{1+x\sin x}+\sqrt{\cos x}}$$

$$\lim_{x\to0}\frac{1+x\sin x-\cos x}{\tan^2 2x\left(\sqrt{1+x\sin x}+\sqrt{\cos x}\right)}$$

$$\lim_{x\to0}\frac{2\sin^2 x/2+x\sin x}{\tan^2 2x\times2}$$

$$\frac{1}{2}\lim_{x\to0}2.\frac{\sin^2 x/2+x.\sin x}{\tan^2 2x}$$

$$\frac{1}{2}\lim_{x\to0}\frac{\dfrac{2\sin^2 x/2}{\frac{x^2}{4}}\times\dfrac{x^2}{4}+x^2\dfrac{\sin x}{x}}{\dfrac{\tan^2 2x}{4x^2}\times4x^2}$$

$$\frac{1}{2}\lim_{x\to0}\frac{2\left(\dfrac{\sin x/2}{x/2}\right)^2\times\dfrac{x^2}{4}+x^2\dfrac{\sin x}{x}}{\left(\dfrac{\tan 2x}{2x}\right)^2\times4x^2}$$

$$\frac{1}{2}\lim_{x\to0}\frac{2\times1\times\frac{x^2}{4}+x^2\times1}{1\times4x^2}=\frac{1}{2}.\left[\frac{\frac{1}{2}+1}{4}\right]=\frac{1}{2}\times\frac{3}{2\times4}\Rightarrow\frac{3}{16}$$

259. $\lim\limits_{x\to0}\dfrac{\tan^3 x-\sin^3 x}{x^5}$ **is equal to**

(a) $\frac{5}{2}$ (b) $\frac{3}{2}$ (c) $\frac{3}{5}$ (d) $\frac{2}{5}$

AP EAMCET-2013

Ans. (b) : Given, $\lim\limits_{x\to0}\dfrac{\tan^3 x-\sin^3 x}{x^5}$

$$\lim_{x\to0}\frac{\tan^3 x\left(1-\cos^3 x\right)}{x^5}\Rightarrow\lim_{x\to0}\frac{\tan^3 x}{x^3}\left(\frac{1-\cos^3 x}{x^2}\right)$$

$$\lim_{x\to0}1.\left(\frac{1-\cos^3 x}{x^2}\right) \qquad \left\{\frac{0}{0}\right\}\text{from}$$

Applying L-Hospital's rule

$$\lim_{x\to0}\frac{-3\cos^2 x\left(-\sin x\right)}{2x}$$

$$\lim_{x\to0}\frac{3}{2}\cos^2 x.\frac{\sin x}{x}\Rightarrow\frac{3}{2}\times1\times1=\frac{3}{2}$$

260. $\lim\limits_{x\to0}x^2\sin\dfrac{\pi}{x}$ **is equal to**

(a) 1 (b) 0 (c) does not exist (d) ∞

AP EAMCET-2005

Ans. (b) : Given, $\lim\limits_{x\to0}x^2\sin\dfrac{\pi}{x}$

We know that, $\lim\limits_{x\to0}\dfrac{\sin x}{x}=1$

$$\lim_{x\to0}x^2\left(\frac{\sin\frac{\pi}{x}}{\frac{\pi}{x}}\right)\times\frac{\pi}{x}$$

$$\lim_{x\to0}x^2.1.\frac{\pi}{x}$$

$$\lim_{x\to0}\pi x \Rightarrow \pi\times0=0$$

261. **The value of** $\lim\limits_{x\to\infty}\dfrac{1}{n^3}\sum\limits_{k=1}^{n}\left(k^2x\right)$ **is**

(a) x (b) $\frac{x}{2}$ (c) $\frac{x}{3}$ (d) $\frac{x}{4}$

AP EAMCET-2004

Ans. (c) : $\lim_{x\to\infty}\frac{1}{n^3}\sum_{k=1}^{n}\left(k^2x\right)$

$$L=\lim_{n\to\infty}\frac{1}{n^3}\left[1^2x+2^2x+3^2x+4^2x......n^2x\right]$$

$$=\lim_{n\to\infty}\frac{1}{n^3}x\left[1^2+2^2+3^2+4^2......n^2\right]$$

$$=\lim_{n\to\infty}\frac{1}{n^3}x\left(\frac{n(n+1)(2n+1)}{6}\right)$$

$$=\lim_{n\to\infty}\frac{x}{n^3}n^3\left(\frac{\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)}{6}\right)=\lim_{n\to\infty}x\left(\frac{\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)}{6}\right)$$

$$\frac{x.(1+0)(2+0)}{6}\Rightarrow\frac{2x}{6}\Rightarrow\frac{x}{3}$$

262. $\lim_{x\to 0}\frac{\sin x\sin^{-1}x}{x^2}$ **is equal to**

(a) 0 (b) 1 (c) –1 (d) ∞

AP EAMCET-2001

Ans. (b) : $\lim_{x\to 0}\frac{\sin x.\sin^{-1}x}{x^2}$

It can be written as –

$$\lim_{x\to 0}\frac{\sin x}{x}.\frac{\sin^{-1}x}{x}\Rightarrow\lim_{x\to 0}\frac{\sin x}{x}.\lim_{x\to 0}\frac{\sin^{-1}x}{x}$$

$1.\lim_{x\to 0}\frac{\sin^{-1}x}{x}$ $\left\{\frac{0}{0}\right\}$from

Applying L-Hospital's rule,

$$\lim_{x\to 0}\frac{\frac{1}{\sqrt{1-x^2}}}{1}\Rightarrow\lim_{x\to 0}\frac{1}{\sqrt{1-0}}\Rightarrow 1$$

263. $\lim_{x\to 0}\left(\frac{x.10^x-x}{1-\cos x}\right)$ **is equal to**

(a) log 10 (b) 2 log 10 (c) 3 log 10 (d) 4 log 10

AP EAMCET-2001

Ans. (b) : We have, $\lim_{x\to 0}\frac{x10^x-x}{1-\cos x}$ $\left\{\frac{0}{0}\right\}$from

Appling L- Hospital's rule we get,

$$\lim_{x\to 0}\frac{10^x\log(x\log 10+1)+10^x(\log 10)}{\cos x}$$

$$\lim_{x\to 0}\frac{x.10^x\log 10+10^x-1}{\sin x}\Rightarrow\lim_{x\to 0}\frac{10^x[x\log 10+1]-1}{\sin x}$$

Applying L-Hospital's rule we get,

$$\lim_{x\to 0}\frac{10^x+[x\log 10+1]10^x\log 10}{\cos x}\Rightarrow\frac{1+\log 10}{1}$$

$\Rightarrow$ log10 + log10 $\Rightarrow$ 2 log 10

264. By the definition of the definite integral, the value of

$$\lim_{x\to\infty}\left(\frac{1^4}{1^5+n^5}+\frac{2^4}{2^5+n^5}+\frac{3^4}{3^5+n^5}+..+\frac{n^4}{n^5+n^5}\right)\text{is}$$

(a) log2 (b) $\frac{1}{5}\log 2$ (c) $\frac{1}{4}\log 2$ (d) $\frac{1}{3}\log 2$

AP EAMCET-2014

Ans. (b) :

$$\lim_{x\to\infty}\left(\frac{1^4}{1^5+n^5}+\frac{2^4}{2^5+n^5}+\frac{3^4}{3^5+n^5}+..+\frac{n^4}{n^5+n^5}\right)$$

Now general equation,

If $\lim_{n\to\infty}\sum_{n=1}^{n}\left(\frac{r^4}{r^5+n^5}\right)$ Let, $\frac{r}{n}=x,\ r=1,\ n=\infty$

$\lim_{n\to\infty}\sum_{r=1}^{n}\frac{r^4}{\left(\frac{r^5}{n^5}+1\right)}$ $\frac{1}{n}dr=dx,$

$\lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{n}\frac{\left(\frac{r}{n}\right)^4}{\left(\left(\frac{r}{n}\right)^5+1\right)}$ $\frac{1}{\infty}=0\quad x=0$

$f(x)=\frac{x^4}{x^5+1}$ $r=n\quad x=1$

$$\int_0^1 f(x)dx=\int_0^1\frac{x^4}{x^5+1}dx$$

$\frac{1}{5}\int_1^2\frac{1}{t}dt$ Let, $x^5+1=t$

$\frac{1}{5}[\log t]_1^2$ $5x^4\,dx=dt$

$\Rightarrow\frac{1}{5}[\log 2-\log 1]$ $x^4\,dx=\frac{1}{5}dt$

$\frac{1}{5}[\log 2-0]$ $\{\because\log 1=0\}$

$\frac{1}{5}\log 2$

265. If *f* (x) = xtan^{-1} x, then $\lim_{x\to 1}\frac{f(x)-f(1)}{x-1}$ **equal to**

(a) $\frac{\pi+3}{4}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi+1}{4}$ (d) $\frac{\pi+2}{4}$

AP EAMCET-2014

Ans. (d) : Given,

$$f(x)=x\tan^{-1}x$$

$\lim_{x\to 1}\frac{f(x)-f(1)}{x-1}$ $\left\{\frac{0}{0}\right\}$from

Applying L-Hospital's rule we get,

$$\lim_{x\to 1}\frac{f'(x)-0}{1-0}$$

$$\lim_{x\to 1}f'(x)=f'(1)$$

$$f'(x)=\frac{d}{dx}x\tan^{-1}x\Rightarrow f'(x)=\tan^{-1}x+x\times\frac{1}{1+x^2}$$

$$f'(1)=\tan^{-1}(1)+\frac{1}{2}\Rightarrow f'(1)=\frac{\pi}{4}+\frac{1}{2}\Rightarrow\frac{\pi+2}{4}$$

266. **If $\lim\limits_{x\to 0}\frac{\sqrt{1+x^2}-\sqrt{1-x+x^2}}{3^x-1}$ is equal to**

(a) $\frac{1}{\log_e 3}$ (b) $\log_e 9$ (c) $\frac{1}{\log_e 9}$ (d) $\log_e 3$

AP EAMCET-2014

Ans. (c) : $\lim\limits_{x\to 0}\frac{\sqrt{1+x^2}-\sqrt{1-x+x^2}}{3^x-1}$ $\left\{\frac{0}{0}\right\}$ fore

Applying L-Hospital's rule,

$$\lim_{x\to 0}\frac{\frac{1}{2\sqrt{1+x^2}}(2x)-\frac{1}{2\sqrt{1-x+x^2}}(-1+2x)}{3^x\log 3}$$

$$\lim_{x\to 0}\frac{\frac{x}{\sqrt{1+x^2}}-\frac{2x-1}{2\sqrt{1-x+x^2}}}{3^x\log 3}$$

$$\Rightarrow \frac{0-\left(-\frac{1}{2}\right)}{\log 3}\Rightarrow\frac{1}{2\log 3}\Rightarrow\frac{1}{\log 3^2}\Rightarrow\frac{1}{\log_e 9}$$

267. **If $g(x)=\frac{x}{[x]}$ for $x>2$, then $\lim\limits_{x\to 2}\frac{g(x)-g(2)}{x-2}=$**

(a) –1 (b) 0 (c) $\frac{1}{2}$ (d) 1

AP EAMCET-2015

Ans. (c) : Given, $g(x)=\frac{x}{[x]}$ for $x>2$

$$g(x)=\frac{x}{2}\qquad [x]=2, 2\le x<3$$

$$[x]=3, 3\le x<4$$

$$g(2)=\frac{2}{2}=1$$

$$\lim_{x\to 2}\frac{g(x)-g(2)}{x-2}=\lim_{x\to 2}\frac{\frac{x}{2}-1}{x-2}=\lim_{x\to 2}\frac{x-2}{2(x-2)}$$

$$\Rightarrow \frac{1}{2}$$

268. **$\lim\limits_{x\to\frac{\pi}{2}}\left(\frac{2x-\pi}{\cos x}\right)$ is equal to**

(a) 0 (b) $\frac{1}{2}$ (c) –2 (d) 5

AP EAMCET-2015

Ans. (c) : Given,

$$\lim_{x\to\pi/2}\left(\frac{2x-\pi}{\cos x}\right)$$

By Using L- Hospital's rule, $\left\{\frac{0}{0}\right\}$ from

$$L=\lim_{x\to\pi/2}\frac{2-0}{(-\sin x)}$$

$$-2\lim_{x\to\pi/2}\frac{1}{\sin x}=-2$$

269. **$\lim\limits_{n\to\infty}\frac{1^{99}+2^{99}+.....+n^{99}}{n^{100}}$ is equal to**

(a) $\frac{99}{100}$ (b) $\frac{1}{100}$ (c) $\frac{1}{99}$ (d) $\frac{1}{101}$

EAMCET-1994

Ans. (b) : Given,

$$\lim_{n\to\infty}\frac{1^{99}+2^{99}+3^{99}+.......n^{99}}{n^{100}}$$

We know that,

$$1^3+2^3+3^3+.......n^3=\left[\frac{n(n+1)}{2}\right]^2$$

$$\Rightarrow\quad \frac{n^4}{4}+\lambda_1 n^3+\lambda_2 n^2+......$$

$$1^\alpha+2^\alpha+3^\alpha+4^\alpha........n^\alpha=\frac{n^{\alpha+1}}{\alpha+1}+\lambda_1 n^\alpha+\lambda_2 n^{\alpha-1}+.....$$

$$\lim_{n\to\infty}\frac{1^{99}+2^{99}+3^{99}+......n^{99}}{n^{100}}$$

$$\lim_{n\to\infty}\frac{\frac{n^{100}}{100}+\lambda_1 n^{99}+\lambda_2 n^{98}+\lambda_3 n^{97}}{n^{100}}$$

$$\lim_{n\to\infty}\frac{1}{100}+\frac{\lambda_1}{n}+\frac{\lambda_2}{n^2}+\frac{\lambda_3}{n^3}+.........$$

$$\frac{1}{100}+\frac{\lambda_1}{\infty}+\frac{\lambda_2}{\infty}+\frac{\lambda_3}{\infty}+......=\frac{1}{100}$$

270. **Consider the function f(x) = $x\sin\frac{1}{x}, x\neq 0$ and f(0) = 0, then**

(a) it is continuous for all real values of x
(b) it is discontinuous everywhere
(c) f(x) exists and discontinuous at $x=\frac{\pi}{2}$
(d) None of the above

EAMCET-1994

Ans. (a) : We have,

$$f(x)=x\sin\frac{1}{x}\text{ for } x\neq 0$$

and f(0) = 0

For continuous function, L.H.L = R.H.L = f(a)

Now for L.H.L

$$\lim_{x\to 0^-}x\sin\frac{1}{x}$$

$$\lim_{h\to 0}(0-h)\sin\left(\frac{1}{0-h}\right)$$

$$\lim_{h\to 0}-h\sin\left(-\frac{1}{h}\right)$$

$$\lim_{h\to 0}h\sin\left(\frac{1}{h}\right)\qquad\{\because \sin(-\theta)=-\sin\theta\}$$

$$=h\sin(\infty)=0$$

R.H.L

$$\lim_{x\to 0^+} x\sin\frac{1}{x}$$

$$\lim_{h\to 0}(0+h)\sin\left(\frac{1}{0+h}\right)$$

$$\lim_{h\to 0} h\sin\frac{1}{h} = 0$$

and f(0) = 0. $\sin\left(\frac{1}{0}\right) = 0$

So, L.H.L = R.H.L = f(0)

271. $\lim_{n\to\infty}\frac{1}{n^2}\cdot\sum_{r=1}^{n} r\,e^{r/n}$ **is equal to**

(a) 0 (b) 1 (c) e (d) 2e

EAMCET-1992

Ans. (b) : We have, $\lim_{n\to\infty}\frac{1}{n^2}\sum_{r=1}^{n} r\,e^{r/n}$

$$\lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{n}\frac{r}{n}e^{r/n}$$

Let, $\frac{r}{n}=x$, $\frac{1}{n}dr=dx$

$$\int_0^1 x.e^x\,dx$$

$$x.e^x-\int 1e^x\,dx$$

$$\left[xe^x-e^x\right]_0^1 \Rightarrow \left[e^1-e^1+e^0\right]\Rightarrow e^0=1$$

272. If the function $f(x)=\frac{3x+4\tan x}{x}$ is continuous, then what is the value of f(x) at x = 0?

(a) 6 (b) 7 (c) 5 (d) None of these

EAMCET-1991

Ans. (b) : Given, $f(x)=\frac{3x+4\tan x}{x}$

$$f(x)=3\lim_{x\to 0}\frac{x}{x}+4\lim_{x\to 0}\frac{\tan x}{x} = 3.1 + 4.1 = 3 + 4 = 7$$

273. $\lim_{x\to 0}\frac{x-\sin x}{x+\cos^2 x}$ **is equal to**

(a) 0 (b) 1 (c) ∞ (d) does not exist

EAMCET-1991

Ans. (a) : Given, $\lim_{x\to 0}\frac{x-\sin x}{x+\cos^2 x} = \lim_{x\to 0}\frac{0-0}{0+1}=0$

274. $\lim_{n\to\infty}\left[\frac{n+1}{n^2+1^2}+\frac{n+2}{n^2+2^2}+\frac{n+3}{n^2+3^2}+....+\frac{1}{n}\right]$ **is equal to**

(a) $\frac{\pi}{4}+\frac{1}{2}\log 2$ (b) $\frac{\pi}{2}+\frac{1}{4}\log 2$

(c) $\frac{\pi}{4}+\frac{1}{4}\log 2$ (d) $\frac{\pi}{2}+\frac{1}{2}\log 2$

WBJEE-2017, 2009, EAMCET-1997

Ans. (a) : $\lim_{n\to\infty}\left[\frac{n+1}{n^2+1^2}+\frac{n+2}{n^2+2^2}+\frac{n+3}{n^2+3^2}+....+\frac{1}{n}\right]$

For general term,

$$\lim_{n\to\infty}\sum_{r=1}^{n}\left(\frac{n+r}{n^2+r^2}\right)=\lim_{n\to\infty}\sum_{r=1}^{n}\left(\frac{n\left(1+\frac{r}{n}\right)}{n^2\left(1+\left(\frac{r}{n}\right)^2\right)}\right)$$

$$\lim_{n\to\infty}\sum_{r=1}^{n}\left(\frac{1+r/n}{1+(r/n)^2}\right)$$

Let, $\frac{r}{n}=x$ $\frac{1}{n}dr=dx$

$$\int_0^1\frac{1+x}{1+x^2}dx$$

$$\int_0^1\left(\frac{1}{1+x^2}+\frac{x}{1+x^2}\right)dx$$

$$\int_0^1\frac{1}{1+x^2}dx+\int_0^1\frac{x}{1+x^2}dx$$

$$\left[\tan^{-1}x\right]_0^1+\frac{1}{2}\left[\log\left(1+x^2\right)\right]_0^1$$

$$\frac{\pi}{4}+\frac{1}{2}\log 2$$

275. $\lim_{n\to\infty}\left[\frac{1}{n+1}+\frac{1}{n+2}+....+\frac{1}{2n}\right]$ **is equal to**

(a) log 2 (b) log 3 (c) log 4 (d) $\frac{\pi}{2}$

EAMCET-1999

Ans. (a) : Given, $\lim_{n\to\infty}\left[\frac{1}{n+1}+\frac{1}{n+2}+........\frac{1}{2n}\right]$

Now,

General equation -

$$\lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{n+r} = \lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{\left(1+\frac{r}{n}\right)}.\frac{1}{n}$$

Let, $\frac{r}{n}=x,\ \frac{1}{n}dr=dx,\ x=0, x=1$

$$\int_0^1\frac{1}{1+x}dx$$

⇒ $\left[\log(1+x)\right]_0^1 \Rightarrow \log(1+1)-\log(1+0)$

⇒ log2 – 0 ⇒ log2

276. $\lim_{n\to\infty}\sum_{r=1}^{n}\left(\frac{1}{n}\right)\sqrt{\frac{n+r}{n-r}}$ **is equal to**

(a) $\frac{\pi+2}{2}$ (b) $\frac{\pi+2}{4}$ (c) $\frac{\pi+1}{2}$ (d) $\frac{\pi+2}{3}$

EAMCET-1999

Ans. (a) : Given, $\lim\limits_{n\to\infty}\sum\limits_{r=1}^{n}\left(\frac{1}{n}\right)\sqrt{\frac{n+r}{n-r}}$

$$\lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{n}\sqrt{\frac{(1+r/n)}{(1-r/n)}}$$

$$\lim_{n\to\infty}\sum_{r=1}^{n}\sqrt{\frac{1+x}{1-x}}\,dx$$

$$\int_0^1\frac{\sqrt{1+x}}{\sqrt{1-x}}dx \Rightarrow \int_0^1\frac{\sqrt{1+x}}{\sqrt{1-x}}\times\frac{\sqrt{1+x}}{\sqrt{1+x}}$$

$[\because \text{Let}, x=\sin\theta, \frac{dx}{d\theta}=\cos\theta]$

$$\int_0^1\frac{1+x}{\sqrt{1-x^2}}$$

$$\int_0^1\frac{1+\sin\theta}{\sqrt{1-\sin^2\theta}}.\cos\theta.d\theta$$

$$\int_0^1\frac{1+\sin\theta}{\cos\theta}.\cos\theta.d\theta$$

$$\int_0^1(1+\sin\theta)d\theta$$

$$[\theta-\cos\theta]_0^1$$

$$\left[\sin^{-1}x-\left(1-x^2\right)\right]_0^1$$

$$\left[\sin^{-1}x-1+x^2\right]_0^1$$

$$\sin^{-1}(1)-1+1-\left[\sin^{-1}(0)-1+0\right]$$

$$\frac{\pi}{2}-0+1=\frac{\pi}{2}+1=\frac{\pi+2}{2}$$

277. $\lim\limits_{x\to\infty}\frac{2x+7\sin x}{4x+3\cos x}$ **is equal to**

(a) -1 (b) 1 (c) $-\frac{1}{2}$ (d) $\frac{1}{2}$

EAMCET-1998

Ans. (d) : $\lim\limits_{x\to\infty}\frac{2x+7\sin x}{4x+3\cos x} = \lim\limits_{x\to\infty}\frac{2+7\frac{\sin x}{x}}{4+3\frac{\cos x}{x}}$

$\lim\limits_{x\to\infty}\frac{\cos x}{x}=0$ and $\lim\limits_{x\to\infty}\frac{\sin x}{x}=0$

$$\lim_{x\to\infty}\frac{2+7\times0}{4+3\times0}=\frac{2}{4}=\frac{1}{2}$$

278. $\lim\limits_{x\to\infty}\frac{1}{\sin^2x}-\frac{1}{\sinh^2x}$ **is equal to**

(a) $\frac{2}{3}$ (b) 0 (c) $\frac{1}{3}$ (d) $-\frac{2}{3}$

EAMCET-1998

Ans. (a) : Given, $\lim\limits_{x\to\infty}\frac{1}{\sin^2 x}-\frac{1}{\sinh^2 x}$

$$\lim_{x\to\infty}\frac{\sinh^2 x-\sin^2 x}{\sin^2 x.\sinh^2 x}$$

$$\lim_{x\to\infty}\frac{\left(x+\frac{x^3}{3!}+\frac{x^5}{5!}+.....\right)^2-\left(x-\frac{x^3}{3!}+\frac{x^5}{5!}....\right)^2}{\left(x-\frac{x^3}{3!}+\frac{x^5}{5!}....\right)^2\left(x+\frac{x^3}{3!}+\frac{x^5}{5!}.......\right)^2}$$

$$\lim_{x\to\infty}\frac{\left(x^2+\frac{x^6}{(3!)^2}+\frac{2x^4}{3!}\right)-\left(x^2+\frac{x^6}{(3!)^2}-\frac{2x^4}{3!}....\right)}{x^4\left(1-\frac{x^2}{3!}+....\right)\left(1+\frac{x^2}{3!}+.......\right)}$$

$$\lim_{x\to\infty}\frac{2.\frac{2x^4}{3!}}{x^4.1}=\frac{4}{6}=\frac{2}{3}$$

279. $\lim\limits_{x\to0}\frac{\sin ax}{\sin bx}$ **is equal to**

(a) $\frac{b}{a}$ (b) 0 (c) $\frac{a}{b}$ (d) 1

EAMCET-1998

Ans. (c) : $\lim\limits_{x\to0}\frac{\sin ax}{\sin bx}$

Applying L-Hospital's rule, $\left\{\frac{0}{0}\right\}$ form

$$\lim_{x\to0}\frac{a\cos ax}{b\cos bx}\Rightarrow\frac{a.1}{b.1}=\frac{a}{b}$$

280. $\lim\limits_{x\to0}\log\left|\frac{\log(1+x)}{x}\right|$ **is equal to**

(a) 0 (b) 1 (c) e (d) $\frac{1}{e}$

EAMCET-1998

Ans. (a) : Given,

$$\lim_{x\to0}\log\left|\frac{\log(1+x)}{x}\right|$$

We know that,

$$\log(1+x)=x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+......$$

$$\lim_{x\to0}\log\left|\frac{x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+.....}{x}\right|$$

$$\lim_{x\to0}\log\left|1-\frac{x}{2}+\frac{x^2}{3}-\frac{x^3}{4}.....\right|$$

$$\log|1-0+0...|$$

$\log1 = 0$

281. $\lim_{x\to 0}\frac{\sin|x|}{x}$ **is equal to**

(a) 1 (b) 0 (c) Positive infinity (d) Does not exist

WB JEE-2010

Ans. (d) : We have given, $\lim_{x\to 0}\frac{\sin|x|}{x}$

L.H.L. $\lim_{x\to 0}\frac{\sin(-x)}{x} \Rightarrow \lim_{x\to 0}-\frac{\sin x}{x} = -1$

R.H.L. $\lim_{x\to 0}\frac{\sin x}{x} = 1$

R.H.L ≠ L.H.L limit does not exist.

282. The value of $\underset{x\to 0}{\text{It}}\frac{\sin^2 x+\cos x-1}{x^2}$ **is**

(a) 1 (b) $\frac{1}{2}$ (c) $\frac{1}{2}$ (d) 0

WB JEE-2010

Ans. (b) : Given, $\lim_{x\to 0}\frac{\sin^2 x+\cos x-1}{x^2}$ $\left\{\frac{0}{0}\right\}$from

Applying L-Hospital's rule,

$$\lim_{x\to 0}\frac{2\sin x.\cos x-\sin x}{2x}$$

$$\lim_{x\to 0}\frac{\sin 2x-\sin x}{2x} \quad (\because \sin 2x = 2\sin x.\cos x)$$

Applying L-Hospital's rule,

$$\lim_{x\to 0}\frac{2\cos 2x-\cos x}{2}$$

$$\frac{2}{2}.\cos 0-\frac{1}{2}\cos 0 = 1-\frac{1}{2} = \frac{1}{2}$$

283. The value of $\underset{x\to 0}{\text{It}}\left(\frac{1+5x^2}{1+3x^2}\right)^{\frac{1}{x^2}}$ **is**

(a) e^2 (b) e (c) $\frac{1}{e}$ (d) $\frac{1}{e^2}$

WB JEE-2010

Ans. (a) : Given,

$$=\lim_{x\to 0}\left(\frac{1+5x^2}{1+3x^2}\right)^{1/x^2} = e^{\lim_{x\to 0}\left(\frac{1+5x^2}{1+3x^2}-1\right)^1\left(\frac{1}{x^2}\right)}$$

$$= e^{\lim_{x\to 0}\left(\frac{1+5x^2-1-3x^2}{1+3x^2}\right)\left(\frac{1}{x^2}\right)}$$

$$= e^{\lim_{x\to 0}\left(\frac{2x^2}{1+3x^2}\right)\left(\frac{1}{x^2}\right)} = e^{\lim_{x\to 0}\left(\frac{2}{1+3x^2}\right)}$$

$$= e^{\left(\frac{2}{1+0}\right)} = e^2$$

284. The value of $\lim_{x\to 1}\frac{x+x^2+.....+x^n-n}{x-1}$ **is**

(a) n (b) $\frac{n+1}{2}$ (c) $\frac{n(n+1)}{2}$ (d) $\frac{n(n-1)}{2}$

WB JEE-2011

Ans. (c) : We have,

$$\lim_{x\to 1}\frac{x+x^2+....+x^n-n}{x-1}$$

$$=\frac{1+1+1....+1-n}{1-1} = \frac{n-n}{0} = \left\{\frac{0}{0}\right\}\text{from}$$

Applying L-Hospital's rule,

$$=\lim_{x\to 1}\frac{1+2x+........+nx^{n-1}-0}{1-0} = \frac{1+2+.........+n}{1}$$

Sum of n term

$$=\frac{n(n+1)}{2}$$

285. $\lim_{x\to 0}\frac{\sin(\pi\sin^2 x)}{x^2} =$

(a) π^2 (b) 3π (c) 2π (d) π

WB JEE-2011

Ans. (d) : Given, $\lim_{x\to 0}\frac{\sin(\pi\sin^2 x)}{x^2}$

$$= \lim_{x\to 0}\pi\sin^2 x\times\frac{\sin(\pi\sin^2 x)}{\pi\sin^2 x}\times\frac{1}{x^2} = \pi\lim_{x\to 0}\frac{\sin^2 x}{x^2}$$

$$\pi\lim_{x\to 0}\left(\frac{\sin x}{x}\right)^2 = \pi$$

286. The value of $\lim_{n\to\infty}\frac{(n!)^{\frac{1}{n}}}{n}$ **is**

(a) 1 (b) $\frac{1}{e^2}$ (c) $\frac{1}{2e}$ (d) $\frac{1}{e}$

WB JEE-2012

Ans. (d) : Given, $\lim_{n\to\infty}\frac{(n!)^{\frac{1}{n}}}{n} = \lim_{n\to\infty}\left(\frac{n!}{n^n}\right)^{\frac{1}{n}}$

We know that,

$$\frac{n!}{n} = \frac{1\times 2\times 3\times ...\times n}{n\times n\times ....\times n...\times n}$$

$$\therefore \left\{\left(\frac{n!}{n^n}\right)\right\}^{\frac{1}{n}} = \left\{\frac{1}{n}.\frac{2}{n}.\frac{3}{n}.....\frac{r}{n}...\frac{n}{n}\right\}^{\frac{1}{n}}$$

$$\lim_{n\to\infty}\left\{\frac{n!}{n^n}\right\}^{\frac{1}{n}} = \lim_{n\to\infty}\left\{\frac{1}{n}.\frac{2}{n}.\frac{3}{n}....\frac{r}{n}...\frac{n^2}{n}\right\}^{\frac{1}{n}}$$

Consider, $A = \lim_{n\to\infty}\left\{\frac{n!}{n^n}\right\}^{\frac{1}{n}}$

Then, $A = \lim_{n\to\infty}\left\{\frac{1}{n}.\frac{2}{n}.\frac{3}{n}.....\frac{r}{n}....\frac{n^2}{n}\right\}^{\frac{1}{n}}$

$$\log A = \lim_{n\to\infty}\frac{1}{n}\sum\log\left(\frac{r}{n}\right) = \int_0^1\log x\,dx$$

$$= \left[x\log x-\int\frac{1}{x}.x.dx\right]_0^1$$

Integrating by parts we get –

$$[x\log x-x]_0^1 = -1$$

So, $A = e^{-1} = \frac{1}{e}$.

287. $\lim_{x\to 0}\frac{\pi^x-1}{\sqrt{1+x}-1}$

(a) does not exist (b) equals $\log_e(\pi^2)$

(c) equals 1 (d) lies between 10 and 11

WB JEE-2012

Ans. (b) : Given, $\lim_{x\to 0}\frac{\pi^x-1}{\sqrt{1+x}-1}$ $\left[\frac{0}{0}\text{ form}\right]$

Using L – Hospital rule, we get –

$$\lim_{x\to 0}\frac{\pi^x\log_e\pi}{\frac{1}{2\sqrt{1+x}}}$$

$= \lim_{x\to 0}2\sqrt{1+x}\left(\pi^x\log_e\pi\right) = 2\sqrt{1}\left(\pi^\circ\log_e\pi\right) = 2\log_e\pi$

$= \log_e\pi^2$

288. $\lim_{z\to 1}\frac{z^{(1/3)}-1}{z^{(1/6)}-1} =$

(a) – 1 (b) 1 (c) 2 (d) –2

AP EAMCET-19.08.2021, Shift-I

Ans. (c): Given, $\lim_{z\to 1}\frac{(z)^{1/3}-1}{(z)^{1/6}-1}$ $\left(\frac{0}{0}\right)$ form

$$=\lim_{z\to 1}\frac{\frac{(z)^{1/3}-1}{z-1}}{\frac{(z)^{1/6}-1}{z-1}}$$

$$=\frac{\lim_{z\to 1}\frac{z^{1/3}-(1)^{1/3}}{z-1}}{\lim_{z\to 1}\frac{z^{1/6}-(1)^{1/6}}{z-1}} \quad \left[\because \lim_{x\to a}\frac{x^n-a^n}{x-a}=na^{n-1}\right]$$

$$=\frac{\frac{1}{3}(1)^{1/3-1}}{\frac{1}{6}(1)^{(1/6-1)}} = \frac{6}{3}=2$$

289. $\lim_{n\to\infty}\frac{1+32+243+...+n^5}{n^6} =$

(a) $\frac{1}{5}$ (b) $\frac{1}{11}$ (c) $\frac{1}{6}$ (d) $\frac{1}{2}$

AP EAMCET-22.04.2019, Shift-II

Ans. (c) : Given,

$$\lim_{n\to\infty}\frac{1+32+243+.....+n^5}{n^6} = \lim_{n\to\infty}\frac{1^5+2^5+3^5+.....+n^5}{n^6}$$

$$= \lim_{n\to\infty}\frac{\sum_{r=1}^{n}r^5}{n^6} = \frac{1}{n}\lim_{n\to\infty}\sum_{r=1}^{n}\left(\frac{r}{n}\right)^5$$

Limit of a sum formula –

$$= \int_0^1 x^5 = \left[\frac{x^6}{6}\right]_0^1 = \frac{1}{6}$$

290. **The positive integer n for which** $\lim_{x\to 0}\frac{(\cos x-1)(\cos x-e^x)}{x^n}$ **exists and is finite, is**

(a) 4 (b) 3 (c) 2 (d) 1

AP EAMCET-22.04.2019, Shift-II

Ans. (b) : Given,

$$\lim_{x\to 0}\frac{(\cos x-1)(\cos x-e^x)}{x^n}$$

On using expansion of cos x and e^x, we get –

$$\lim_{x\to 0}\frac{\left\{1-\frac{x^2}{2}+\frac{x^4}{4!}...-1\right\}\left[\left\{1-\frac{x^2}{2!}+\frac{x^4}{4!}....\right\}-\left\{1+\frac{x}{1!}+.....\right\}\right]}{x^n}$$

$$= \lim_{x\to 0}\frac{\left\{\frac{-x^2}{2!}+\frac{x^4}{4!}......\right\}\left\{-x-x^2.....\right\}}{x^n}$$

$$= \lim_{x\to 0}\frac{x^3\left(\frac{1}{2}+\frac{1}{2}x+.....\right)}{x^n}$$

Is finite non-zero when n = 3

291. **Let t_n denotes the nth term of the infinite series** $\frac{1}{1!}+\frac{10}{2!}+\frac{21}{3!}+\frac{34}{4!}+\frac{49}{5!}+....$ **Then,** $\lim_{n\to\infty}t_n$ is

(a) e (b) 0 (c) e^2 (d) 1

WB JEE-2014

Ans. (b) : Given,

$$\frac{1}{1!}+\frac{10}{2!}+\frac{21}{3!}+\frac{34}{4!}+\frac{49}{5!}+.....$$

$$= \frac{1}{1!}+\frac{1+9}{2!}+\frac{1+9+11}{3!}+\frac{1+9+11+13}{4!}$$

A.P. 9, 11, 13

$$S=\frac{n}{2}[18+(n-1)2]=(8+n)n$$

$$t_n=\frac{1+n(n+8)}{(n+1)!},\ n\ge 0,\ n\in I$$

$$\lim_{n\to\infty}t_n=\lim_{n\to\infty}\left[\frac{1}{(n+1)!}+\frac{(n+1)+7}{(n+1)(n-1)!}\right]$$

$$\lim_{n\to\infty}t_n=\frac{1}{\infty}+\frac{1}{\infty}+\frac{7}{\infty}$$

$$\lim_{n\to\infty}t_n=0$$

292. **Applying Lagrange's Mean Value Theorem for a suitable function f(x) in [0,h], we have** $f(h)=f(0)+hf'(\theta h), 0<\theta<1$. **Then, for f(x)= cos x, the value of** $\lim_{h\to 0^+}\theta$ is

(a) 1 (b) 0 (c) 1/2 (d) 1/3

WB JEE-2014

Ans. (c) : From question, we know that in a lagrange mean value theorem there exist C ∈ (a, b) such that,

$$f^1(c)=\frac{f(b)-f(a)}{b-a}$$

Then, $f^1(\theta h)=\frac{f(h)-\cos 0}{h-0}$

$$-\sin(\theta h)=\frac{\cosh-\cos 0}{h-0}$$

[∵ f(x) = f(x)]

$$=\frac{\cosh-1}{h}=\frac{\left[1-\frac{h^2}{2}\right]-1}{h}$$

{neglecting higher power of h}

$$-\sin(\theta h)=\frac{\frac{-h^2}{2}}{h}\Rightarrow \sin(\theta h)=\frac{\frac{h}{2}}{1}$$

$$\theta h=\sin^{-1}\left(\frac{h}{2}\right)\Rightarrow \theta=\frac{\sin^{-1}\left(\frac{h}{2}\right)\times\frac{1}{2}}{h\times\frac{1}{2}}$$

So, $\lim\limits_{x\to 0^+}\theta=\frac{1}{2}\lim\limits_{n\to 0^+}\frac{\sin^{-1}\left(\frac{h}{2}\right)}{\left(\frac{h}{2}\right)}=\frac{1}{2}\times 1=\frac{1}{2}$

293. Let [x] denote the greatest integer less than or equal to x for any real number x. Then, $\lim\limits_{n\to\infty}\frac{\left[n\sqrt{2}\right]}{n}$ is equal to

(a) 0 (b) 2 (c) $\sqrt{2}$ (d) 1

WB JEE-2014

Ans. (c) : Given, [x] = greatest integer less than or equal to x for any real number x.

We know that,

x = 1 ≤ |x| ≤ x

Then, from question, we get –

$$n\sqrt{2}-1\le\left[n\sqrt{2}\right]\le n\sqrt{2}$$

Dividing by n we get,

$$\frac{n\sqrt{2}-1}{n}\le\frac{\left[n\sqrt{2}\right]}{n}\le\frac{n\sqrt{2}}{n}$$

$$=\frac{n\sqrt{2}}{n}-\frac{1}{n}\le\frac{\left[n\sqrt{2}\right]}{n}\le\frac{n\sqrt{2}}{n}=\sqrt{2}-\frac{1}{n}\le\frac{\left[n\sqrt{2}\right]}{n}\le\sqrt{2}$$

By applying limit, we get –

$$\lim_{n\to\infty}\left(\sqrt{2}-\frac{1}{n}\right)\le\lim_{n\to\infty}\frac{\left[x\sqrt{2}\right]}{n}\le\lim_{n\to\infty}\sqrt{2}$$

$$=\sqrt{2}-\frac{1}{\infty}\le\lim_{n\to\infty}\frac{\left[n\sqrt{2}\right]}{n}\le\sqrt{2}=\sqrt{2}\le\lim_{n\to\infty}\frac{\left[n\sqrt{2}\right]}{n}\le\sqrt{2}$$

So, $\lim\limits_{n\to\infty}\frac{\left[n\sqrt{2}\right]}{n}=\sqrt{2}$

294. If $\lim\limits_{x\to 0}\frac{2a\sin x-\sin 2x}{\tan^3 x}$ exists and is equal to 1, then the value of a is

(a) 2 (b) 1 (c) 0 (d) –1

WB JEE-2014

Ans. (b) : Given, $\lim\limits_{x\to 0}\frac{2a\sin x-\sin 2x}{\tan^3 x}\left[\frac{0}{0}\text{form}\right]$

$$=\lim_{x\to 0}\frac{2a\sin x-2\sin\cos x}{\sin^3/\cos^3 x}=\lim_{x\to 0}\frac{2\sin x\left[a-\cos x\right].\cos^3 x}{\sin^3 x}$$

$$=\lim_{x\to 0}\frac{2\cos^3\left[a-\cos x\right]}{\sin^2 x}=\frac{2(a-1)}{0}$$

= But given limit exists-

⇒ 2(a – 1) = 0 ⇒ a = 1

We now get $\frac{0}{0}$ form

On using 1 Hospital rule …

$$2=\lim_{x\to 0}\frac{\cos^3 x(\sin x)+(a-\cos x)(3\cos^2 x)}{2\sin x.\cos x}$$

$$=2=\lim_{x\to 0}\frac{\cos^2 x-(a-\cos x)(3\cos x)}{1}$$

= 1 – (1 – 1) (3) = 1 – 0 = 1

295. Let f : R → R be differentiable at x=0. If f(0)=0 and *f*'(0)=2, then the value of $\lim\limits_{x\to 0}\frac{1}{x}\left[f(x)+f(2x)+f(3x)+...+f(2015x)\right]$ is

(a) 2015 (b) 0 (c) 2015×2016 (d) 2015×2014

WB JEE-2015

Ans. (c) : Given, f(0) = 0 and f'(0) = 2

Then, $2=\lim\limits_{x\to 0}\frac{1}{x}\left[f(x)+f(2x)+f(3x)+...+f(2015x)\right]$

On Appling l- Hospital rule

$$=\lim_{x\to 0}\frac{\left[f(x)+2f(2x)+3f(3x)+...2015f(2015x)\right]}{1}$$

$$=\frac{2+2\times 2+3\times 2+....+2015\times 2}{1}$$

= 2[1 + 2 + 3 + …..+2015]

$$=\frac{2\times 2015\times 2016}{2}=2015\times 2016$$

296. $\lim\limits_{n\to 0}\frac{\sqrt{1}+\sqrt{2}+...+\sqrt{n}}{n\sqrt{n}}$ is equal to

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{2}{3}$ (d) 0

WB JEE-2015

Ans. (c) : Given,

$$\lim_{n\to\infty}\frac{\sqrt{1}+\sqrt{2}+.......+\sqrt{n}}{n\sqrt{n}}=\lim_{n\to\infty}\frac{1}{n}\sum_{k=1}^{n}\sqrt{\frac{k}{n}}$$

We know that,

$$\int^b f(x)dx=\lim_{n\to\infty}\frac{1}{n}\sum_{x-1}^{n}f\left(a+\frac{b-a}{n}\right)$$

Then, a = 0, b = 1 and f(x) = $\sqrt{x}$

So,

$$\int_0^1\sqrt{x}dx=\frac{2}{3}$$

297. **If $\lim_{x\to 0}\frac{axe^x - b\log(1+x)}{x^2} = 3$ then the values of a and b are, respectively**

(a) 2, 2 (b) 1, 2 (c) 2, 1 (d) 2, 0

WB JEE-2015

Ans. (a) : Given,

$$\lim_{x\to 0}\frac{axe^x - b\log(1+x)}{x^2} = 3 \qquad \left[\frac{0}{0}\text{form}\right]$$

On using L - Hospital rule, we get –

$$\lim_{x\to\infty}\frac{ae^x + axe^x - \frac{b}{1+x}}{2x} = 3 \qquad \left[\frac{0}{0}\text{from}\right]$$

$\Rightarrow a - b = 0 \Rightarrow a = b$

Again, using L - Hospital rule, we get –

$$\lim_{x\to 0}\frac{ae^x + ae^x + axe^x + \frac{b}{(1+x)^2}}{2} = 3$$

$$= \lim_{x\to 0}\frac{2ae^x + axe^x + \frac{b}{(1+x)^2}}{2} = 3$$

$$= \lim_{x\to 0} 2ae^x + axe^x + \frac{b}{(1+x)^2} = 6$$

$2a + b = 6 \qquad (\because a = b)$

$3a = 6$

$a = 2$

Then, $b = 2$

Hence, $a = 2,\ b = 2$

298. **Let $x_n = \left(1-\frac{1}{3}\right)^2\left(1-\frac{1}{6}\right)^2\left(1-\frac{1}{10}\right)^2 \ldots \left(1-\frac{1}{\frac{n(n+1)}{2}}\right)^2, n \ge 2$. Then, the value of $\lim_{x\to\infty} x_n$ is**

(a) 1/3 (b) 1/9 (c) 1/81 (d) 0

WB JEE-2015

Ans. (b) : Given,

$$x_n = \left(1-\frac{1}{3}\right)^2\left(1-\frac{1}{6}\right)^2\left(1-\frac{1}{10}\right)^2 \ldots \left(1-\frac{1}{n\frac{(n+1)}{2}}\right)^2, x \ge 2$$

$$\text{Then, } x_n = \left[\left(1-\frac{1}{3}\right)\left(1-\frac{1}{6}\right)\left(1-\frac{1}{10}\right)\ldots\left(1-\frac{2}{n(n+1)}\right)\right]^2$$

$$\Rightarrow x_n = \left[\prod_{n=2}^{n}\left(\frac{n^2+n-2}{n(n+1)}\right)\right]^2 = \left[\prod_{n=2}^{n}\left(\frac{(n+2)(n-1)}{n(n+1)}\right)\right]^2$$

$$= \left[\prod_{n=2}^{n}\left(\frac{n+2}{n+1}\right)\right]^2\left[\prod_{n=2}^{n}\left(\frac{n-1}{n}\right)\right]^2$$

$$\Rightarrow x_n = \left[\left(\frac{4}{3}.\frac{5}{4}.\frac{6}{5}.......\right)^2\left(\frac{1}{2}.\frac{2}{3}.\frac{3}{4}.....\frac{n-1}{n}\right)^2\right]$$

$$\Rightarrow x_n = \frac{1}{3^2}\left(\frac{n+2}{n}\right)^2 \Rightarrow x_n = \frac{1}{9}\left(1+\frac{2}{n}\right)^2$$

$$\Rightarrow \lim_{n\to\infty} x_n = \frac{1}{9}(1+0)^2$$

Hence, $\lim_{n\to\infty} x_n = \frac{1}{9}$

299. **The value of $\lim_{x\to 2}\int_2^x \frac{3t^2}{(x-2)}dt$ is**

(a) 10 (b) 12 (c) 8 (d) 16

WB JEE-2015

Ans. (b) : Given, $\lim_{x\to 2}\frac{\int_2^x 3t^2 dt}{x-2}$

Since x is not a variable for integral,

Therefore, $= \lim_{x\to 2}\frac{\frac{d}{dx}\int_2^x 3t^2 dt}{\frac{d}{dx}(x-2)} = \lim_{x\to 2}\frac{3x^2}{1} = 3\times(2)^2 = 12$

On using L-Hospital rule, since form of the limit is % form and to differentiate integral by Leibnitz rule.

300. **Let for all x > 0, f(x) = $\lim_{n\to\infty} n\left(x^{1/n} - 1\right)$, Then**

(a) $f(x) + f\left(\frac{1}{x}\right) = 1$ (b) f(xy) = f(y) + f(x)

(c) f(xy) = xf(y) + yf(x) (d) f(xy) = xf(x) + yf(y)

WB JEE-2017

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Ans. (b) : Given, f(x) = $\lim_{n\to\infty} n\left(x^{1/n} - 1\right), \forall x > 0$

Then, $\lim_{n\to\infty}\frac{\left(x^{1/n}-1\right)}{1/n} = \log x$

$\log x + \log\ \frac{1}{x} = 0$

logxy = logx + logy

$\therefore$ f(xy) = f(x) + f(y)

301. **If f '' (0) = k, k ≠ 0, then the value of $\lim_{x\to\infty}\frac{2f(x) - 3f(2x) + f(4x)}{x^2}$ is**

(a) k (b) 2k (c) 3k (d) 4k

WB JEE-2017

Ans. (c) : We have,

$$\lim_{x\to\infty}\frac{2f(x)-3f(x)+f(4x)}{x^2} \qquad \left[\frac{\infty}{\infty}\text{from}\right]$$

On using L – Hospital rule, we get –

$$= \lim_{x\to\infty}\frac{2f'(x)-3f'(2x).2+f'(4x).4}{2x}$$

$$= \lim_{x\to\infty}\frac{f'(x)-3f'(2x)+2f'(4x)}{x}$$

Again, using L – Hospital rule, we get –

$$= \lim_{x\to\infty}\frac{f''(x)-3f''(x).2+2f''(4x).4}{1}$$

$$= \lim_{x\to\infty}f''(x)-6f''(2x)+8f''(4x)$$

$$= f''(0)-6f''(0)+8f''(0)$$

$$= k - 6k + 8k = 3k \qquad [\because f''(0)=k, k\neq 0]$$

302. The value of

$$\lim_{n\to\infty}\frac{1}{n}\left\{\sec^2\frac{\pi}{4n}+\sec^2\frac{2\pi}{4n}+...+\sec^2\frac{n\pi}{4n}\right]\text{is}$$

(a) $\log_e 2$ (b) $\frac{\pi}{2}$ (c) $\frac{4}{\pi}$ (d) e

WB JEE-2018

Ans. (c) : Given,

$$\lim_{n\to\infty}\frac{1}{n}\left[\left\{\sec^2\frac{\pi}{4n}+\sec^2\frac{2\pi}{4n}+.....+\sec^2\frac{n\pi}{4n}\right\}\right]$$

$$= \lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{n}\sec^2\frac{r\pi}{4n}$$

When r = 1, $\frac{r}{n}=0$ and when, $n = r \Rightarrow \frac{r}{n}=1$

Then we get – $\int_0^1 \sec^2\frac{\pi x}{4}dx$

$$= \frac{4}{\pi}\left[\tan\frac{\pi x}{4}\right]_0^1 = \frac{4}{\pi}\left[\tan\frac{\pi}{4}-\tan 0\right] = \frac{4}{\pi}$$

303. Let $f(x) = 3x^{10}-7x^8+5x^6-21x^3+3x^2-7$. Then

$$\lim_{h\to 0}\frac{f(1-h)-f(1)}{h^3+3h}$$

(a) does not exist (b) is $\frac{50}{3}$ (c) is $\frac{53}{3}$ (d) is $\frac{22}{3}$

WB JEE-2018

Ans. (c) : Given, $f(x) = 3x^{10}-7x^8+5x^6-21x^3+3x^2-7$

and $L = \lim_{h\to 0}\frac{f(1-h)-f(1)}{h^3+3h}$

$$L = \lim_{h\to 0}\frac{f(1-h)-f(1)}{h(h^2+3)} = \lim_{h\to 0}\frac{f(1-h)-f(1)}{-h}\cdot\lim_{h\to 0}\frac{-1}{(h^2+3)}$$

$$L = f'(1)\left(-\frac{1}{3}\right) \qquad ......(i)$$

Since, $f(x) = 3x^{10}-7x^8+5x^6-21x^3+3x^2-7$

$f'(x) = 30x^9-56x^7+30x^5-63x^2+6x$

$f'(1) = -53$

On putting the value in (i), we get –

$$L = \frac{53}{3}$$

304. $$\lim_{n\to\infty}\left\{\frac{1}{n+m}+\frac{1}{n+2m}+\frac{1}{n+3m}+...+\frac{1}{n+nm}\right\}=$$

(a) $\frac{\log_e(m)}{m}$ (b) $\frac{\log_e(1+m)}{1+m}$

(c) $\frac{\log_e(1+m)}{m}$ (d) $\frac{\log_e(1+m)}{1-m}$

AP EAMCET-22.04.2019, Shift-I

Ans. (c) : Given,

$$\lim_{n\to\infty}\left\{\frac{1}{n+m}+\frac{1}{n+2m}+\frac{1}{n+3m}+.....+\frac{1}{n+nm}\right\}$$

$$P = \lim_{n\to\infty}\left\{\frac{1}{n+m}+\frac{1}{n+2m}+....+\frac{1}{n+nm}\right\}$$

$$P = \lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{n+rm} = \lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{n\left(1+\frac{r}{n}m\right)}$$

$$P = \lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{n}\left\{\frac{1}{1+\left(\frac{r}{n}\right)m}\right\} \Rightarrow P = \lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{n}\frac{1}{1+\left(\frac{r}{n}\right)m}$$

Where $h=\frac{1}{n}$, $xh = 1$

$\Rightarrow n\to\infty, n\to 0$

$$P = \int_0^1\frac{1}{1+xm}dx = \frac{1}{m}\int_0^1\frac{m}{1+xm}dx$$

$$P = \frac{1}{m}\left|\log(1+xm)\right|_0^1 \Rightarrow P = \frac{1}{m}\log(1+m)$$

So, $\lim_{n\to\infty}\left\{\frac{1}{n+m}+\frac{1}{x+2m}+\frac{1}{x+3m}+....+\frac{1}{n+nm}\right\}$

$$= \frac{\log_e(1+m)}{m}$$

305. Define

$$f(x) = \begin{cases}\dfrac{\sqrt{1+px}-\sqrt{1-px}}{x}, & \text{if } -1\le x<0\\ \dfrac{2x+1}{x-2}, & \text{if } 0\le x\le 1\end{cases}$$

If $\lim_{x\to 0} f(x)$ exists, then p =

(a) –1 (b) $-\frac{1}{2}$ (c) $\frac{1}{2}$ (d) 1

AP EAMCET-22.04.2019, Shift-I

Ans. (b) : Given,

$$f(x) = \begin{cases}\dfrac{\sqrt{1+px}-\sqrt{1-px}}{x}, \text{if} & -1\le x<0\\ \dfrac{2x+1}{x-2}, \quad \text{if} & 0\le x<1\end{cases}$$

We know that,

A function f (x) is said to be continuous at a point x = a of its domain if $\lim_{x\to a}f(x) = f(a)$

$$\lim_{x\to a^+}f(a+n) = \lim_{x\to a^-}f(a-n) = f(a)$$

Then $\lim_{x\to 0^-} f(x) = \lim_{n\to 0} f(0-n)$

$= \lim_{h\to 0}\frac{\sqrt{1-ph}-\sqrt{1+ph}}{-h}$

On rationatization method–

$\lim_{h\to 0}\frac{\left(\sqrt{1-ph}-\sqrt{1+ph}\right).\left(\sqrt{1-ph}+\sqrt{1+ph}\right)}{-h\left(\sqrt{1-ph}+\sqrt{1+ph}\right)}$

$= \lim_{h\to 0}\frac{2p}{\sqrt{1-ph}-\sqrt{1+ph}}$

We have, $\lim_{x\to 0} f(x) = \lim_{x\to 0^+} f(x)$

$= \lim_{h\to 0} f(-h) = \lim_{h\to 0} f(h) = \lim_{h\to 0}\frac{2p}{\sqrt{1-ph}-\sqrt{1+ph}}$

$= \lim_{h\to 0}\frac{2h+1}{(h-2)} \Rightarrow \frac{2p}{2} = \frac{-1}{2}$

$\Rightarrow p = -\frac{1}{2}$

306. If $f(x) = \begin{cases} 4x-5, & x \le 2 \\ x-k & x > 2 \end{cases}$, then the value of 'k' if $\lim_{x\to 2} f(x)$ may exist is equal to

(a) –1 (b) –2 (c) 1 (d) 2

AP EAMCET-22.09.2020, Shift-II

Ans. (a) : Given, $f(x) = \begin{cases} 4x-5 & x \le 2 \\ x-k & x > 2 \end{cases}$

Then, $\lim_{x\to 0} f(x) = \lim_{h\to 0}(2-h)$

$= \lim_{h\to 0}[4(2-h)-5] = \lim_{h\to 0} 3-4h = 8-5 = 3$

And, $\lim_{x\to 2^+} f(x) = \lim_{h\to 0} f(2+h) = \lim_{h\to 0}(2+h)-k = 2-k$

Since, $\lim_{x\to 0} f(x)$ exists

$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x)$

$3 = 2-k \Rightarrow k = -1$

So, $k = -1$

307. $\lim_{x\to 0}\frac{\left(1+\frac{x}{2}\right)^{5/7}-1}{x} =$

(a) $\frac{5}{7}$ (b) $\frac{10}{7}$ (c) $\frac{5}{14}$ (d) $\frac{5}{17}$

AP EAMCET-22.09.2020, Shift-II

Ans. (c) : Given, $\lim_{x\to 0}\frac{\left(1+\frac{x}{2}\right)^{5/7}-1}{x}$ $\left(\frac{0}{0}\text{form}\right)$

On using L - Hospital rule, we get–

$= \lim_{x\to 0}\frac{\frac{5}{7}\left(1+\frac{x}{2}\right)^{5/7-1}\frac{1}{2}}{1} = \lim_{x\to 0}\frac{5}{14}\left(1+\frac{x}{2}\right)^{-2/7} = \frac{5}{14}$

308. If $\lim_{x\to 3}\left(\frac{x^n-3^n}{x-3}\right) = 108$ and $n \in N$, then the value of 'n' is

(a) 3 (b) 6 (c) 5 (d) 4

AP EAMCET-22.09.2020, Shift-II

Ans. (d) : Given, $\lim_{x\to 3}\frac{x^n-3^n}{x-3} = 108$

We know that, $= \lim_{x\to a}\frac{-a^n-a^n}{x-a} = n.a^{n-1}$

Then $\lim_{x\to 3}\frac{x^n-3^n}{x-3} = n.3^{4-1} = 4.3^3$

So, $n = 4$

279. $\lim_{n\to\infty}\frac{n!}{(n+1)!-n!} =$

(a) 1 (b) –1 (c) 2 (d) 0

AP EAMCET-18.09.2020, Shift-I

Ans. (d) : Given, $\lim_{x\to\infty}\frac{n!}{(n+1)!-n!}$

$\lim_{x\to\infty}\frac{n!}{(n+1)n!-n!}$

$\lim_{x\to\infty}\frac{n!}{(n+1-1)n!} = \lim_{x\to\infty}\frac{1}{n} = \frac{1}{\infty} = 0$

310. If [x] denotes the greatest integer $\le$ x, then $\lim_{n\to\infty}\frac{1}{n^3}\left\{[1^2x]+[2^2x]+[3^3x]+...+[n^2x]\right\} =$

(a) $\frac{x}{2}$ (b) $\frac{x}{3}$ (c) $\frac{x}{6}$ (d) 0

AP EAMCET-22.04.2018, Shift-I

Ans. (b) : We have,

$\lim_{n\to\infty}\frac{1}{n^3}\left\{[1^2x]+[2^2x]+[3^3x]+...+[n^2x]\right\}$

$= \lim_{n\to\infty}\left\{\frac{\sum_{r=1}^{n}[r^2x]}{n^3}\right\} = \lim_{n\to\infty}\left\{\frac{\sum_{r=1}^{n} r^2x-\{r^2x\}}{n^3}\right\}$

Where, {} denotes fractional part function –

$\lim_{n\to\infty}\left\{\frac{x\frac{n(n+1)(2n+1)}{6}}{n^3}-\sum_{r=1}^{n}\frac{r^2x}{n^3}\right\} = x\frac{(1)(1)(2)x}{6}-0 = \frac{x}{3}$

311. Find the value of $\lim_{x\to 0}\frac{\sin(x^m)}{(\sin x)^n}$, given that n<m.

(a) 2 (b) 1 (c) 0 (d) ∞

AP EAMCET-21.09.2020, Shift-II

Ans. (c) : Given,

$= \lim_{x\to 0}\frac{\sin(x^m)}{(\sin x)^n} = \lim_{x\to 0}\frac{\sin(x^m)}{x^m}\frac{x^m}{x^n}\times\left(\frac{x}{\sin x}\right)^n = \lim_{x\to 0} x^{m-n}$

So, if n < m the value of limit is equal to zero.

312. $\lim_{x\to 1}\left((1-x)\tan\left(\frac{\pi x}{2}\right)\right) =$

(a) $\frac{1}{x}$ (b) $\frac{3}{\pi}$ (c) $\frac{4}{\pi}$ (d) $\frac{2}{\pi}$

AP EAMCET-21.09.2020, Shift-II

Ans. (d) : Given, $\lim_{x\to 1}\left((1-x)\tan\left(\frac{\pi x}{2}\right)\right)$

Let, t = 1 – x ⇒ x = 1 – t

So, x → 1 then t → 0

$= \lim_{t\to 0} t\tan\left(\frac{\pi}{2}(1-t)\right) = \lim_{t\to 0} t\tan\left(\frac{\pi}{2}-\frac{\pi}{2}t\right)$

$= \lim_{t\to 0} t\cot\frac{\pi}{2}t = \lim_{t\to 0}\frac{t}{\tan\frac{\pi}{2}t} = \lim_{t\to 0}\frac{2}{\pi}\left(\frac{\frac{\pi}{2}t}{\tan\frac{\pi}{2}t}\right) = \frac{2}{\pi}$

313. Let a = min {$x^2 + 2x + 3 : x \in R$} and

b = $\lim_{\theta\to 0}\frac{1-\cos\theta}{\theta^2}$. Then $\sum_{r=0}^{n} a^r b^{n-r}$ is

(a) $\frac{2^{n+1}-1}{3.2^n}$ (b) $\frac{2^{n+1}+1}{3.2^n}$ (c) $\frac{4^{n+1}-1}{3.2^n}$ (d) $\frac{1}{2}\left(2^n-1\right)$

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Ans. (c) : Let, f(x) = $x^2 + 2x + 3$

$A = f(x)_{min} = \frac{-D}{4a} = \frac{(-4+12)}{4} = \frac{8}{4} = 2$

And, b = $\lim_{x\to 0}\frac{1-\cos\theta}{\theta^2} = \lim_{x\to 0}\frac{1-1+2\sin^2\theta/2}{\theta^2}$

$= \lim_{x\to 0}\frac{2\cos\theta/2}{(\theta/2)^2\cdot 4} = \lim_{x\to 0}\frac{1}{2}\cdot\frac{\sin^2\theta/2}{(\theta/2)^2}$

$= \frac{1}{2}\lim_{x\to 0}\frac{\sin^2\theta/2}{(\theta/2)^2} = \frac{1}{2}, 1 = \frac{1}{2} \Rightarrow b = \frac{1}{2}$

Now, $\sum_{r=0}^{n} a^r\cdot b^{n-r} = \sum_{r=0}^{n}(2)^r\left(\frac{1}{2}\right)^{n-r} = \sum_{r=0}^{n}2^r\cdot(2)^{rn-r} = \sum_{r=0}^{n}2^{2r-n}$

$= 2^{-n}\sum_{r=0}^{n}2^{2r} = 2^{-n}\left(1+2^2+2^4+2^6+...+2^{2n}\right)$

$= 2^{-n}\left[\frac{1+(2^2)^{n+1}-1}{2^2-1}\right] \qquad \left\{\therefore s_n = \frac{a(r^n-1)}{r-1}\right\}$

$= 2^{-n}\left[\frac{4^{n+1}-1}{3}\right] = \frac{4^{n+1}-1}{3.2^n}$

314. $\lim_{x\to 0^+}\left(e^x + x\right)^{1/x}$

(a) Does not exist finitely (b) is 1 (c) is e^2 (d) is 2

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Ans. (c) : Let L = $\lim_{x\to 0^+}\left(e^x+x\right)^{1/x}$

$\Rightarrow ln L = \lim_{x\to 0^+}\frac{\log\left(e^x+x\right)^{1/x}}{x} \Rightarrow ln L = \lim_{x\to 0^+}\frac{\frac{1}{(e^x+x)}.e^x+1}{1}$

[using L' Hospita rule]

$\Rightarrow ln L = \lim_{x\to 0^+}\frac{e^x+1}{e^x+x} \Rightarrow ln L = 2 \Rightarrow L = e^2$

315. The limit of the interior angle of a regular polygon of n sides as n→∞ is

(a) π (b) $\frac{\pi}{3}$ (c) $\frac{3\pi}{2}$ (d) $\frac{2\pi}{3}$

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Ans. (a) : The limit of the interior angle of a regular polygon of n sides as n→∞ is π or 180^o.

316. $\lim_{x\to 0+}\left(x^n ln x\right), n > 0$

(a) does not exist (b) exists and is zero
(c) exists and is 1 (d) exists and is e^{-1}

WB JEE-2019

Ans. (b) : We have, $\lim_{x\to 0^+} x^n \ln x$

$= \lim_{x\to 0^+}\frac{ln x}{x^{-n}} \qquad \left[\frac{\infty}{\infty}\text{form}\right]$

On using L-Hospital's rule, we get –

$= \lim_{x\to 0^+}\frac{\frac{1}{x}}{-nx^{-n-1}} = \lim_{x\to 0^+}\frac{-1}{nx^{-n}} = 0\left[\because \frac{1}{x-n} = 0, \text{when } x = 0\right]$

317. Let $f(x) \equiv \frac{1}{3}x\sin x - (1-\cos x)$. The smallest positive integer k such that $\lim_{x\to 0}\frac{f(x)}{x^k} \neq 0$ is

(a) 4 (b) 3 (c) 2 (d) 1

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Ans. (c) : Given, $f(x) \equiv \frac{1}{3}x\sin x - (1-\cos x)$

So, $\lim_{x\to 0}\frac{f(x)}{x^k} \neq 0$

$= \lim_{x\to 0}\frac{\frac{x\sin x}{3}-1+\cos x}{x^k} \neq 0 \qquad \left(\frac{0}{0}\text{form}\right)$

$\lim_{x\to 0}\frac{\frac{\sin x}{3}+\frac{x\cos x}{3}-\sin x}{kx^{k-1}} \neq 0$

$\lim_{x\to 0}\frac{\frac{\cos x}{3}+\frac{\cos x}{3}-\frac{x\sin x}{3}-\cos x}{k(k-1)x^{k-2}} \neq 0$

If k = 2

$\lim_{x\to 0}\frac{f(x)}{x^k} \neq 0$

318. $\lim_{x\to 1}\left(\frac{1}{\ln x}-\frac{1}{(x-1)}\right)$

(a) Does not exist (b) 1 (c) $\frac{1}{2}$ (d) 0

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Ans. (c) : Given, $\lim_{x\to 1}\left(\frac{1}{ln x}-\frac{1}{(x-1)}\right)$

$\lim_{x\to 1}\frac{x-1-ln x}{ln(x)(x-1)} \qquad \left[\frac{0}{0}\text{form}\right]$

On applying L-Hospital rule, we get –

$= \lim_{x\to 1}\frac{1-\frac{1}{x}}{\frac{x-1}{x}+ln x} = \lim_{x\to 1}\frac{\frac{1}{x^2}}{\frac{1}{x^2}+\frac{1}{x}} = \lim_{x\to 1}\frac{1}{x+1} = \frac{1}{2}$

319. $\lim_{n\to\infty}\left\{\frac{\sqrt{n}}{\sqrt{(n^3)}}+\frac{\sqrt{n}}{\sqrt{(n+4)^3}}+\frac{\sqrt{n}}{\sqrt{(n+8)^3}}+....+\frac{\sqrt{n}}{\sqrt{[n+4(n-1)]^3}}\right\}$ **is**

(a) $\frac{5-\sqrt{5}}{10}$ (b) $\frac{5+\sqrt{5}}{10}$ (c) $\frac{2+\sqrt{3}}{2}$ (d) $\frac{2-\sqrt{3}}{2}$

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Ans. (a): Given, $\lim_{x\to\infty}\frac{\sqrt{n}}{\sqrt{n^3}}+\frac{\sqrt{n}}{\sqrt{(n+4)^3}}+.....+\frac{\sqrt{n}}{\sqrt{[n+4(n-1)]^3}}$

$$=\lim_{n\to\infty}\frac{\sqrt{n}}{\sqrt{n^3}}\left[1+\frac{1}{\sqrt{\left(1+\frac{4}{n}\right)^3}}+.....+\frac{1}{\sqrt{\left[n+\frac{4(n-1)}{n}\right]^3}}\right]$$

$$=\lim_{n\to\infty}\frac{\sqrt{n}}{n\sqrt{n}}\left[1+\frac{1}{\sqrt{\left(1+\frac{4}{n}\right)^3}}+.....+\frac{1}{\sqrt{\left[1+\frac{4(n-1)}{n}\right]^3}}\right]$$

$$I=\lim_{n\to\infty}\frac{1}{n}\sum_{r=0}^{n-1}\frac{1}{\sqrt{\left[1+4\left(\frac{r}{n}\right)\right]^3}}$$

Let, $\frac{r}{n}=x$

$\lim_{n\to\infty}\frac{1}{n}\to dx.$

$$I=\int_0^1\frac{1}{\sqrt{(1+4x)^3}}dx \Rightarrow I=\int_0^1\frac{1}{(1+4x)^{3/2}}dx=\int_0^1(1+4x)^{-3/2}dx$$

$$I=\left[\frac{(1+4x)^{-\frac{3}{2}+1}}{\left(-\frac{3}{2}+1\right)(4)}\right]_0^1 \Rightarrow I=\left[\frac{(1+4x)^{-\frac{3}{2}+1}}{\left(-\frac{3}{2}+1\right)(4)}\right]_0^1=-\frac{1}{2}\left[\frac{1}{(1+4x)^{\frac{1}{2}}}\right]_0^1$$

$$I=-\frac{1}{2}\left[\frac{1}{(1+4)^{\frac{1}{2}}}-\frac{1}{(1+0)^{1/2}}\right]=\frac{-1}{2}\left[\frac{1}{5^{1/2}}-1\right]$$

$$I=\frac{-1}{2}\left[\frac{1}{\sqrt{5}}-1\right]=\frac{1}{2}\left[1-\frac{1}{\sqrt{5}}\right]\Rightarrow I=\frac{1}{2}\left[1-\frac{\sqrt{5}}{5}\right]\Rightarrow I=\frac{5-\sqrt{5}}{10}.$$

Hence option (a) is correct.

320. The $\lim_{x\to\infty}\left(\frac{3x-1}{3x+1}\right)^{4x}$ **equals**

(a) 1 (b) 0 (c) $e^{-8/3}$ (d) $e^{-4/9}$

WB JEE-2021

Ans. (c): Given, $\lim_{x\to\infty}\left(\frac{3-\frac{1}{x}}{3+\frac{1}{x}}\right)^{4x}$

In the other form $\lim_{x\to\infty}f(x)^{g(x)}=1^{\infty}$

$$=e^{\lim_{x\to\infty}g(x)(f(x)-1)} = e^{\lim_{x\to\infty}4x\left(\frac{3-1/x}{3+1/x}-1\right)}$$

$$=e^{\lim_{x\to\infty}\left[4x\left(\frac{3x-1}{3x+1}-1\right)\right]} = e^{\lim_{x\to\infty}\left[4x\frac{3x-1-(3x+1)}{3x+1}\right]}$$

$$=e^{\lim_{x\to\infty}\frac{-8x}{3x+1}}=e^{\lim_{x\to\infty}\frac{-8}{3+\frac{1}{x}}}=e^{\frac{-8}{3}}\quad(\because \text{Put limit } x=\infty)$$

Hence option (c) is correct.

321. Let $S_n = \cot^{-1}2+\cot^{-1}8+\cot^{-1}18+\cot^{-1}32$ +……to n^{th} term. Then $\lim_{n\to\infty}S_n$ is

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{8}$

WB JEE-2021

Ans. (b): Let,

$S_n=\cot^{-1}2+\cot^{-1}8+\cot^{-1}18+\cot^{-1}32+\ldots\infty$

$S_n=\cot^{-1}2\cdot1^2+\cot^{-1}2\cdot2^2+\cot^{-1}2\cdot3^2+\cot^{-1}2\cdot4^2+\ldots\infty$

$$S_n=\left(\sum_{r=1}^{n}\cot^{-1}(2r^2)\right)=\left(\sum_{r=1}^{n}\tan^{-1}\left(\frac{1}{2r^2}\right)\right)$$

$$T_r=\tan^{-1}\left(\frac{1}{2r^2}\right)=\tan^{-1}\left(\frac{2}{4r^2}\right)=\tan^{-1}\left(\frac{(2r+1)-(2r-1)}{1+4r^2-1}\right)$$

$$=\tan^{-1}\left(\frac{(2r+1)-(2r-1)}{1+(2r+1)(2r-1)}\right)$$

$$\left(\because \tan^{-1}\left(\frac{x-y}{1+xy}\right)=\tan^{-1}x-\tan^{-1}y\right)$$

$T_r=\tan^{-1}(2r+1)-\tan^{-1}(2r-1)$

$$S_\infty=\lim_{n\to\infty}\sum_{r=1}^{n}\left[\tan^{-1}(2r+1)-\tan^{-1}(2r-1)\right]$$

$$=\sum_{r=1}^{\infty}\left[\tan^{-1}(2r+1)-\tan^{-1}(2r-1)\right]$$

$$=\tan^{-1}(\infty)-\tan^{-1}(1)=\frac{\pi}{2}-\frac{\pi}{4}=\frac{\pi}{4}$$

322. $\lim_{x\to0}\left(\frac{1}{x}\ln\sqrt{\frac{1+x}{1-x}}\right)$ **is**

(a) 1/2 (b) 0 (c) 1 (d) does not exist

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Ans. (c) : Given, $\lim_{x\to\infty}\frac{\ln\sqrt{\frac{1+x}{1-x}}}{x}$

Which is in determinant form

By substitution method,

Put, $x=\cos2\theta$

$2\theta=\cos^{-1}x \Rightarrow \theta=\frac{1}{2}(\cos^{-1}x)$

When, $x\to0$, then $\left(\theta\to\frac{\pi}{4}\right)$

$$=\lim_{\theta\to\frac{\pi}{4}}\frac{\ln(\cot^2\theta)^{\frac{1}{2}}}{\cos2\theta}=\lim_{\theta\to\frac{\pi}{4}}\frac{\ln(\cot\theta)}{\cos2\theta}=\lim_{\theta\to\frac{\pi}{4}}\frac{\ln|\cot\theta|}{\cos2\theta}$$

$$\lim_{\theta\to\frac{\pi}{4}}\frac{\frac{1}{\cot\theta}\times-\operatorname{cosec}^2\theta}{-(\sin2\theta)2}=\frac{2}{2}=1$$

323. $\lim\limits_{n\to\infty}\frac{1}{n}\left\{\begin{matrix}\sin^5\left(\frac{\pi}{6\pi}\right)+\sin^5\left(\frac{2\pi}{6n}\right)+\sin^5\left(\frac{3\pi}{6n}\right)+\\ ....+\sin^5\left(\frac{\pi}{2}\right)\end{matrix}\right\}=$

(a) $\frac{8}{15\pi}$ (b) $\frac{8}{5\pi}$ (c) $\frac{32}{5\pi}$ (d) $\frac{16}{5\pi}$

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Ans. (d) : Given,

$\lim\limits_{n\to\infty}\frac{1}{n}\left\{\sin^5\left(\frac{\pi}{6n}\right)+\sin^5\left(\frac{2\pi}{6n}\right)+\sin^5\left(\frac{3\pi}{6n}\right)+...+\sin^5\left(\frac{\pi}{2}\right)\right\}$

$=\lim\limits_{n\to\infty}\frac{1}{n}\left\{\sin^5\left(\frac{\pi}{6n}\right)+\sin^5\left(\frac{2\pi}{6n}\right)+\sin^5\left(\frac{3\pi}{6n}\right)+...+\sin^5\left(\frac{3n\pi}{6n}\right)\right\}$

$=\lim\limits_{n\to\infty}\frac{1}{n}\sum_{r=1}^{3n}\sin^5\left(\frac{r\pi}{6n}\right)=\int_0^3\sin^5\left(\frac{\pi}{6}x\right)dx$

Let, $\frac{\pi}{6}x=t$

For, upper limit at x = 3, $t=\frac{\pi}{2}$ and lower limit at x = 0,

t = 0 and dx = $\frac{6}{n}dt$

So, $\int_0^3\sin^5\left(\frac{\pi}{6}x\right)dt=\frac{\pi}{6}\int_0^{\pi/2}\sin^5(t)dt$

$=\frac{6}{\pi}\times\frac{4\times2}{5\times3\times1}=\frac{16}{5\pi}$

324. $\cos\left[\lim\limits_{x\to\infty}\frac{2\pi|x|+\pi x}{|x|-3x}+\lim\limits_{x\to\infty}\frac{\cos\left(\frac{\pi}{2}\cos^2x\right)}{x^2}\right]$

(a) 1 (b) –1 (c) 0 (d) $\frac{1}{\sqrt{2}}$

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Ans. (b) : Given, $\cos\left[\lim\limits_{x\to\infty}\frac{2\pi|x|+\pi x}{|x|-3x}+\lim\limits_{x\to 0}\frac{\cos\left(\frac{\pi}{2}\cos^2x\right)}{x^2}\right]$

$=\cos\left[\lim\limits_{x\to\infty}\frac{2\pi x+\pi x}{x-3x}+\lim\limits_{x\to 0}\frac{\cos\left(\frac{\pi}{2}(1-\sin^2x)\right)}{x^2}\right]$

$=\cos\left[\lim\limits_{x\to\infty}\frac{3\pi x}{-2x}+\lim\limits_{x\to 0}\frac{\sin\left(\frac{\pi}{2}\sin^2x\right)}{\left(\frac{\pi}{2}\sin^2x\right)}\times\frac{\frac{\pi}{2}\sin^2x}{x^2}\right]$

$=\cos\left[-\frac{3\pi}{2}+\frac{\pi}{2}\right]=\cos(-\pi)$ $(\because\cos(-\theta)=\cos\theta)$

$=\cos\pi=-1$

325. $\lim\limits_{x\to\infty}\frac{[6^2+12^2+18^2+.....+(6n)^2]^2}{[5+10+15+........+5n][2^3+4^3+6^3......8n^3]}=$

(a) $\frac{4}{5}$ (b) $\frac{144}{5}$ (c) $\frac{4}{25}$ (d) $\frac{144}{25}$

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Ans. (b) : Given,

$\lim\limits_{x\to\infty}\frac{[6^2+12^2+18^2+....+(6n^2)]^2}{[5+10+15+....+5n][2^3+4^3+6^3+....+8n^3]}$

$=\lim\limits_{x\to\infty}\frac{6^4[1^2+2^2+3^2+....+n^2]^2}{(5)[1+2+3+....+n](2)^3[1^3+2^3+3^3+....+n^3]}$

$=\lim\limits_{n\to\infty}\frac{6^4\left(\frac{n(n+1)(2n+1)}{6}\right)^2}{5\left(\frac{n(n+1)}{2}\right)\times2^3\times\left(\frac{n(n+1)}{2}\right)^2}$

$=\lim\limits_{n\to\infty}\frac{6^4\times\left[\frac{\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)}{6}\right]^2}{5\times\left[\frac{1+\frac{1}{n}}{2}\right]\times2^3\left[\frac{1+\frac{1}{n}}{2}\right]^2}=\frac{6^4\times\frac{1}{6^2}\times2^2}{5\times\frac{1}{2}\times2^3\times\frac{1}{2^2}}=\frac{144}{5}$

326. $\lim\limits_{n\to\infty}\sum_{r=1}^{n}\tan^{-1}\left(\frac{2r}{r^4+r^2+2}\right)=$

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$ (c) $\frac{-\pi}{4}$ (d) $\frac{-\pi}{2}$

AP EAMCET-23.04.2018, Shift-II

Ans. (a) : We can say that,

$r^4+r^2+1=(r^2-r+1)(r^2+r+1)$

So, $\tan^{-1}\left[\frac{2r}{1+(r^4+r^2+1)}\right]$

$\tan^{-1}(r^2+r+1)-\tan^{-1}(r^2-r+1)$

So, $\sum_{r=1}^{n}\tan^{-1}\left[\frac{2r}{r^4+r^2+2}\right]$

$=\sum_{r=1}^{n}\left\{\tan^{-1}(r^2+r+1)-\tan^{-1}(r^2-r+1)\right\}$

$=\tan^{-1}(r^2+r+1)-\tan^{-1}(1)$

$=\tan^{-1}\frac{x^2+r}{1+r^2+r+1}=\tan^{-1}\frac{x^2+r}{r^2+r+2}$

So, $\lim\limits_{n\to\infty}\sum_{r=1}^{n}\tan^{-1}\left(\frac{2r}{r^4+r^2+2}\right)=\lim\limits_{n\to\infty}\tan^{-1}\left(\frac{x^2+r}{x^2+r+2}\right)=\frac{\pi}{4}$

327. $\lim_{x\to\pi}\frac{1-\sin x/2}{\left(\cos\frac{x}{2}\right)\left(\cos\frac{x}{4}-\sin\frac{x}{4}\right)}=$

(a) $\frac{3}{\sqrt{2}}$ (b) $\frac{5}{\sqrt{2}}$ (c) $\frac{1}{\sqrt{2}}$ (d) 9

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Ans. (c) : We have,

$$\lim_{x\to\pi}\frac{1-\sin\frac{x}{2}}{\left(\cos\frac{x}{2}\right)\left(\cos\frac{x}{4}-\sin\frac{x}{4}\right)}$$

Let, $x=\pi+h$, $x\to\pi$, $h\to 0$

$$\lim_{h\to0}\frac{1-\sin\left(\frac{\pi}{2}+\frac{h}{2}\right)}{\cos\left(\frac{\pi}{2}+\frac{h}{2}\right)\left(\cos\left(\frac{\pi+h}{4}\right)-\sin\left(\frac{\pi+h}{4}\right)\right)}$$

$$=\lim_{h\to0}\frac{1-\cos\frac{h}{2}}{-\sin\frac{h}{2}\left(\cos\frac{\pi}{4}.\cos\frac{h}{4}-\sin\frac{\pi}{4}\sin\frac{h}{4}-\sin\frac{\pi}{4}\cos\frac{h}{4}-\cos\frac{\pi}{4}.\sin\frac{h}{4}\right)}$$

$$=\lim_{h\to0}\frac{\sqrt{2}\left(1-\cos\frac{h}{2}\right)}{-\sin\frac{h}{2}\left\{\cos\frac{h}{4}-\sin\frac{h}{4}-\cos\frac{h}{4}-\sin\frac{h}{4}\right\}}$$

$$=\lim_{h\to0}\frac{\sqrt{2}\left(1-\cos\frac{h}{2}\right)}{\sin\frac{h}{2}\left(-2\sin\frac{h}{4}\right)}=\lim_{h\to0}\frac{\sqrt{2}.2\sin^2\frac{h}{4}}{2.\sin\frac{h}{2}\times\sin\frac{h}{4}}$$

$$=\lim_{h\to0}\sqrt{2}\frac{\sin\frac{h}{4}}{\sin\frac{h}{2}}=\sqrt{2}\lim_{h\to0}\frac{\sin\frac{h}{4}.2\left(\frac{h}{2}\right)}{4\left(\frac{h}{4}\right).\sin\frac{h}{2}}$$

$$=\sqrt{2}\times\frac{2}{4}.\lim_{h\to0}\left[\frac{\sin\frac{h}{4}}{\frac{h}{4}}\right]\left[\frac{\frac{h}{2}}{\sin\frac{h}{2}}\right]=\sqrt{2}\times\frac{2}{4}\times1\times1=\frac{\sqrt{2}}{2}=\frac{1}{\sqrt{2}}$$

328. If

$$\lim_{x\to\infty}\left(1+\frac{p}{x}\right)^{qx}=e^9.\text{ where } p\,.\,q\in N \text{ then } p+q=$$

(a) 6 (b) 9 (c) 81 (d) 18

AP EAMCET-17.09.2020, Shift-II

Ans. (a) : Given,

$$\lim_{x\to\infty}\left(1+\frac{p}{x}\right)^{qx}=e^9$$

Let, $\frac{1}{x}=y$

When, $x\to\infty\Rightarrow y\to0$

$\therefore\ \lim_{y\to0}(1+py)^{q/y}=e^9$

$\left\{\lim_{y\to0}(1+py)^{q/y}\right\}=e^9$

$(e^p)^q=e^9$

$pq=9$

Let, we take $p=3$

$q=3$

And, $p+q=6$

329. $\lim_{\pi\to\infty}\frac{\sqrt{1}+\sqrt{2}+...+\sqrt{n}}{n^{3/2}}=$

(a) 0 (b) $\frac{2}{3}$ (c) 1 (d) $\frac{3}{2}$

AP EAMCET-21.04.2019, Shift-II

Ans. (b) : We have,

$$\lim\frac{\sqrt{1}+\sqrt{2}+....+\sqrt{n}}{n^{3/2}}$$

$$=\lim_{n\to\infty}\frac{\sqrt{1}+\sqrt{2}+....+\sqrt{n}}{n.\sqrt{n}}$$

$$=\lim_{n\to\infty}\frac{1}{n}\left[\sqrt{\frac{1}{n}}+\sqrt{\frac{2}{n}}+.....+\sqrt{\frac{n}{n}}\right]$$

$$=\sum_{r=1}^{n}\lim_{n\to\infty}\frac{1}{n}\sqrt{\frac{r}{n}}=\int_0^1\sqrt{x}\,dx$$

$$=\frac{2}{3}\left[x^{3/2}\right]_0^1=\frac{2}{3}$$

330. $\lim_{n\to\infty}\left(\frac{1}{3\cdot7}+\frac{1}{7\cdot11}+\frac{1}{11.15}+...+(\text{n terms})\right)=$

(a) $\frac{1}{12}$ (b) $\frac{1}{4}$ (c) $\frac{1}{3}$ (d) 0

AP EAMCET-21.04.2019, Shift-II

Ans. (a) : Given,

$$\lim_{n\to\infty}\left[\frac{1}{3.7}+\frac{1}{7.11}+\frac{1}{11.15}+....+(\text{n terms}\right]$$

$$=\lim_{n\to\infty}\frac{1}{4}\left[\frac{4}{3.7}+\frac{4}{7.11}+\frac{4}{11.15}+.....\right]$$

$$=\lim_{n\to\infty}\frac{1}{4}\left[\frac{1}{3}-\frac{1}{7}+\frac{1}{7}-\frac{1}{11}+\frac{1}{11}-\frac{1}{15}...-\frac{1}{n}+\frac{1}{n}-\frac{1}{n+4}\right]$$

$$=\lim_{n\to\infty}\frac{1}{4}\left[\frac{1}{3}-\frac{1}{n+4}\right]$$

$$=\lim_{x\to\infty}\frac{1}{12}-\lim_{n\to\infty}\frac{1}{4(n+4)}=\frac{1}{12}-\lim_{n\to\infty}\frac{1}{4(n+4)}$$

$$=\frac{1}{12}-\lim_{n\to\infty}\frac{1}{4n\left(1+\frac{4}{n}\right)}=\frac{1}{12}-0=\frac{1}{12}$$

331. $\lim_{n\to\infty}\left[\sqrt{x^2+ax+b}-x\right](a<0<b)$

(a) depends on both a and b

(b) depends only on b

(c) depends only on a

(d) depends not depend on a and b

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Ans. (c) : Given, $\lim_{x\to\infty}\sqrt{x^2+ax+b}-x$

$$=\lim_{x\to\infty}\left(\sqrt{x^2+ax+b}-x\right)\times\left[\frac{\sqrt{x^2+ax+b}+x}{\sqrt{x^2+ax+b}+x}\right]$$

[By rationalization]

$$=\lim_{x\to\infty}\frac{x^2+ax+b-x^2}{\sqrt{x^2+ax+b}+x}$$

$$=\lim_{x\to\infty}\frac{ax+b}{\sqrt{x^2+ax+b}+x}$$

$$=\lim_{x\to\infty}\frac{x\left(a+\frac{b}{x}\right)}{x\left(\sqrt{1+\frac{a}{x}+\frac{b}{x^2}}+1\right)}$$

Taking x common from both numerator and denominator)

$$\frac{a}{(1+1)}=\frac{a}{2}$$

It is clear that it depends only on a.

332. If $f(a) = a^2$; $\phi(a) = b^2$ and $f'(a) = 3\phi'(a)$, then $\lim_{x\to a}\frac{\sqrt{f(x)}-a}{\sqrt{\phi(x)}-b}$

(a) $\frac{b^2}{a^2}$ (b) $\frac{b}{a}$ (c) $\frac{2b}{a}$ (d) $\frac{3b}{a}$

AP EAMCET-05.10.2021, Shift-II

Ans. (d) : We have,

$$\lim_{x\to a}\frac{\sqrt{f(x)}-a}{\sqrt{\phi(x)}-b}\qquad\left[\frac{0}{0}\text{form}\right]$$

On using L-Hospital rule, we get –

$$\lim_{x\to a}\frac{\frac{1}{2\sqrt{f(x)}}f'(x)}{\frac{1}{2\sqrt{\phi(x)}}\cdot\phi'(x)}$$

$$=\frac{\frac{f'(a)}{\sqrt{f(a)}}}{\frac{\phi'(a)}{\sqrt{\phi(a)}}}\qquad\left(\begin{array}{l}\because \text{Given, } f(a)=a^2\\ \phi(a)=b^2 \text{ and}\\ f'(a)=3\phi'(a)\end{array}\right)$$

$$=\frac{\frac{3\phi'(a)}{a}}{\frac{\phi'(a)}{b}}=\frac{3b}{a}$$

333. $\lim_{x\to 0}\left(\frac{4^x-1}{2^x-1}-\frac{\sqrt{4+3x}-2}{x}\right)=$

(a) 0 (b) $\frac{5}{4}$ (c) log 2 – 3 (d) $\frac{\log 4}{\log 2}-3$

AP EAMCET-24.04.2018, Shift-II

Ans. (b) : Given, $\lim_{x\to 0}\left(\frac{4^x-1}{2^x-1}-\frac{\sqrt{4+3x}-2}{x}\right)$

$$=\lim_{x\to 0}\left[\frac{\left(2^x\right)^2}{2^x-1}-\frac{\sqrt{4+3x}-2}{x}\times\frac{\sqrt{4+3x}+2}{\sqrt{4+3x}+2}\right]$$

$$=\lim_{x\to 0}\left[\frac{\left(2^x-1\right)\left(2^x+1\right)}{2^x-1}-\frac{4+3x-4}{x\left(\sqrt{4+3x}+2\right)}\right]$$

$$=\lim_{x\to 0}\left[\left(2^x+1\right)-\frac{3x}{x\left(\sqrt{4+3x}+2\right)}\right]$$

$$=\lim_{x\to 0}\left[\left(2^x+1\right)-\frac{3}{\left(\sqrt{4+3x}+2\right)}\right]$$

$$=\left(2^0+1\right)-\frac{3}{\sqrt{4}+2}$$

$$=2-\frac{3}{4}$$

$$=\frac{5}{4}$$

334. $\lim_{x\to\infty}\frac{\int_0^{2x}xe^{x^2}dx}{e^{4x^2}}$ equals

(a) 0 (b) ∞ (c) 2 (d) $\frac{1}{2}$

BITSAT-2015, 2013

Ans. (d) : Given,

$$\lim_{x\to\infty}\frac{\int_0^{2x}xe^{x^2}dx}{e^{4x^2}}$$

$$\lim_{x\to\infty}\frac{2xe^{4x^2}2}{e^{4x^2}8x}=\frac{1}{2}$$

335. **The value of the $\lim_{x\to\infty} x^{1/x}$ is equal to**

(a) 0 (b) 1 (c) e (d) e^{-1}

Jamia Millia Islamia-2010

Ans. (b) : We have, $\lim_{x\to\infty} x^{1/x}$

Let, $\lim_{x\to\infty} x^{1/x} = y$

Taking log on both sides,

$$\log y = \lim_{x\to\infty} \frac{\log x}{x}$$

It is $\frac{\infty}{\infty}$ form

Now, applying L' Hospital rule,

$$\log y = \lim_{x\to 0} \frac{1/x}{1} = 0$$

$y = e^o = 1$

336. **If $f(x) = \begin{cases} \frac{|x-4|}{x-4}, & x \neq 4 \\ 0, & x = 4 \end{cases}$ then $\lim_{x\to 4} f(x)$ is equal to**

(a) 1 (b) –1 (c) 0 (d) does not exist

Jamia Millia Islamia-2010

Ans. (d) : Given, $f(x) = \begin{cases} \frac{|x-4|}{x-4} & , x \neq 4 \\ 0 & x = 4 \end{cases}$

For $\lim_{x\to 4} f(x) = \lim_{x\to 4} \frac{|x-4|}{x-4}$

Taking right hand limit,

$$\lim_{x\to 4^+} \frac{x-4}{x-4} = 1 \quad \text{....(i)}$$

and taking left hand limit,

$$\lim_{x\to 4^-} \frac{4-x}{x-4} = -1 \quad \text{....(ii)}$$

From (i) & (ii),

RHL $\neq$ LHL

$1 \neq -1$

So, limit does not exist.

337. **The value of $\lim_{x\to 0} \frac{\int_0^{x^2} \cos^2 t\, dt}{x\sin x}$ is**

(a) 2 (b) 1 (c) 0 (d) None of these

Manipal UGET-2012

Ans. (b) : We have, $\lim_{x\to 0} \frac{\int_0^{x^2} \cos^2 t\, dt}{x \sin x}$

It is $\frac{0}{0}$ form

Using L'-Hospital's rule, we get

$$= \lim_{x\to 0} \frac{\cos^2(x^2).2x - 0}{x\cos x + \sin x} = \lim_{x\to 0} \frac{2\cos^2(x^2)}{\cos x + \frac{\sin x}{x}} = \frac{2}{2} = 1$$

338. **The value of $\lim_{x\to 2} \frac{(x-2)}{(x^3 - x^2 - x - 2)}$ is equal to**

(a) $\frac{3}{5}$ (b) $\frac{1}{5}$ (c) $\frac{2}{7}$ (d) $\frac{1}{7}$

J&K CET-2016

Ans. (d) : We have, $\lim_{x\to 2} \left(\frac{x-2}{x^3 - x^2 - x - 2} \right)$

$$\lim_{x\to 2} \left(\frac{1}{(x-2)(x^2+x+1)} \right)$$

$$\lim_{x\to 2} \left(\frac{1}{x^2+x+1} \right) = \frac{1}{x^2+x+1}$$

Putting x = 2,

$$\frac{1}{(2)^2+2+1} = \frac{1}{7}$$

339. **If $f(x) = \frac{2}{x-3}, g(x) = \frac{x-3}{x+4}$ and $h(x) = -\frac{2(2x+1)}{x^2+x-12}$ then $\lim_{x\to 3}[f(x)+g(x)+h(x)]$ is**

(a) –2 (b) –1 (c) $-\frac{2}{7}$ (d) 0

J&K CET-2019

Ans. (c) : Given, $f(x) = \frac{2}{x-3}$

$$g(x) = \frac{x-3}{x+4}$$

$$h(x) = -\frac{2(2x+1)}{x^2+x-12}$$

Now,

$$\lim_{x\to 3}[f(x)+g(x)+h(x)]$$

$$\lim_{x\to 3}\left[\frac{2}{x-3} + \frac{x-3}{x+4} - \frac{2(2x+1)}{x^2+x-12}\right]$$

$$\lim_{x\to\infty}\left[\frac{2(x+4)+(x-3)^2 - 2(2x+1)}{(x-3)(x+4)}\right]$$

$$\lim_{x\to\infty}\left[\frac{x^2-8x+15}{(x-3)(x+4)}\right]$$

$$\lim_{x\to\infty}\left[\frac{(x-3)(x-5)}{(x-3)(x+4)}\right] = \lim_{x\to\infty}\left[\frac{x-5}{x+4}\right]$$

Putting, x = 3

$$\frac{3-5}{3+4} = \frac{-2}{7}$$

340. **The value of $\lim_{x\to 0}\left\{\frac{\sin x - x}{x^3}\right\}$ equals to**

(a) 1/3 (b) –1/3 (c) 1/6 (d) –1/6

J&K CET-2019

Ans. (d) : We have, $\lim_{x\to 0}\left\{\frac{\sin x - x}{x^3}\right\}$

$$\lim_{x\to 0}\left\{\frac{\cos x - 1}{3x^2}\right\}$$

It is $\frac{0}{0}$ form

By using L Hospital's rule,

$$\lim_{x\to 0}\left\{\frac{-\sin x}{6x}\right\} = -\frac{1}{6}\left\{\frac{\sin x}{x}\right\} = -\frac{1}{6}$$

341. In n > 0, then the value of $\lim\limits_{x\to\infty}\dfrac{\log x}{x^n}$ is

(a) 0 (b) 1 (c) $\dfrac{1}{n}$ (d) $\dfrac{1}{n!}$

J&K CET-2019

Ans. (a) : We have, $\lim\limits_{x\to\infty}\dfrac{\log x}{x^n}$

Using L' Hospital's rule,

$$\lim_{x\to\infty}\frac{\log x}{x^n}=\lim_{x\to\infty}\frac{1}{n.x^{n-1}}=\lim_{n\to\infty}\frac{1}{nx^n}=0$$

342. $\lim\limits_{x\to0}\dfrac{\sin x^2}{1-\cos x}$ is

(a) 1/2 (b) 0 (c) 1 (d) 2

J&K CET-2015

Ans. (d) : We have, $\lim\limits_{x\to0}\dfrac{\sin x^2}{1-\cos x}$

$$\lim_{x\to0}\frac{\sin x^2}{1-\cos x}=\lim_{x\to0}\frac{\cos x^2 2x}{\sin x}$$

By using L' Hospital's rule,

$$\frac{\lim\limits_{x\to0}2\cos x^2}{\lim\limits_{x\to0}\left(\dfrac{\sin x}{x}\right)}=2$$

343. $\lim\limits_{n\to\infty}\dfrac{0+2+4+6+...+2n}{1+3+5+7+...+(2n-1)}$

(a) is equal to 0 (b) is equal to 1
(c) is equal to 2 (d) does not exist

J&K CET-2015

Ans. (b) : We have, $\lim\limits_{x\to\infty}\dfrac{0+2+4+6+.....+2n}{1+3+5+7+....(2n-1)}$

$$\lim_{x\to\infty}\frac{\frac{n}{2}\left[(2\times2)+(n-1)2\right]}{\frac{n}{2}\left[(2\times1)+(n-1)2\right]}=\lim_{x\to\infty}\frac{[4+2n-2]}{[2+2n-2]}$$

$$\lim_{n\to\infty}\frac{[2+2n]}{2n}=\lim_{n\to\infty}\frac{1+n}{n}=\lim_{n\to\infty}\frac{1}{n}+1=1$$

344. If a > 0, b > 0 then $\lim\limits_{n\to\infty}\left(\dfrac{a+b^{1/n}-1}{a}\right)^n=$

(a) a^b (b) b^a (c) $b^{1/a}$ (d) $a^{1/b}$

AP EAMCET-06.07.2022, Shift-I

Ans. (c) : Given, a > 0, b > 0

$$\lim_{n\to\infty}\left(\frac{a+b^{1/n}-1}{a}\right)^n$$

$$\lim_{n\to\infty}\left(\frac{a+b^{1/n}-1}{a}\right)^n \quad \text{(indeterminate from } 1^\infty)$$

$$e^{\lim\limits_{n\to\infty}\left[\frac{b^{1/n}-1}{a}\right]\times n}=e^{\frac{1}{a}\lim\limits_{n\to\infty}\left[\frac{b^{1/n}-1}{1/n}\right]=e^{\frac{1}{a}\log b}}=e^{\log b^{\frac{1}{a}}}=b^{1/a}$$

345. $\lim\limits_{n\to\pi/6}\dfrac{3\sin x-\sqrt3\cos x}{6x-\pi}$

(a) $\dfrac{-1}{\sqrt3}$ (b) $\dfrac{1}{\sqrt3}$ (c) $\dfrac{1}{\sqrt2}$ (d) $\dfrac{-1}{\sqrt2}$

AP EAMCET-06.07.2022, Shift-I

Ans. (b) : We have, $\lim\limits_{x\to\frac{\pi}{6}}\dfrac{3\sin x-\sqrt3\cos x}{6x-\pi}$

It is $\dfrac{0}{0}$ from

Using L-Hospital rule, $\lim\limits_{x\to\frac{\pi}{6}}\dfrac{3\cos x+\sqrt3\sin x}{6}$

$$\frac{3\cos\frac{\pi}{6}+\sqrt3\sin\frac{\pi}{6}}{6}=\frac{3\times\frac{\sqrt3}{2}+\sqrt3\times\frac{1}{2}}{6}=\frac{4\sqrt3}{12}=\frac{1}{\sqrt3}$$

346. If $\lim\limits_{n\to\infty}x_n$ exists and is finite,

$x_1=2,\ x_{n+1}=\dfrac{a+bx_n}{b+cx_n}\ \forall n\in N$ and c > b > a > 0

then $\lim\limits_{n\to\infty}x_n=$

(a) $\sqrt{ab/c}$ (b) $\sqrt{a/c}$ (c) $\sqrt{c/b}$ (d) $\sqrt{a/b}$

AP EAMCET-08.07.2022, Shift-I

Ans. (b) : We have, $x_{n+1}=\dfrac{a+bx_n}{b+cx_n}$

$$\lim_{x\to\infty}x_{n+1}=\frac{a+b\lim\limits_{n\to\infty}x_n}{b+c\lim\limits_{n\to\infty}x_n}$$

Let, $y=\lim\limits_{n\to\infty}x_{n+1}=\lim\limits_{n\to\infty}x_n$

$$y=\frac{a+by}{b+cy}$$

$by+cy^2=a+by$

$cy^2=a$

$$y^2=\frac{a}{c}\Rightarrow y=\sqrt{\frac{a}{c}}$$

$$\lim_{n\to\infty}x_n=\sqrt{\frac{a}{c}}$$

347. $\lim\limits_{n\to\infty}\left(\dfrac{n}{n^2+1^2}+\dfrac{n}{n^2+2^2}+\dfrac{n}{n^2+3^2}+.....+\dfrac{1}{5n}\right)$ is equal to

(a) $\tan^{-1}(3)$ (b) $\tan^{-1}(2)$ (c) $\pi/4$ (d) $\pi/2$

JEE Main 12.01.2019, Shift - II

Ans. (b) : We have,

$$\lim_{n\to\infty}\left(\frac{n}{n^2+1^2}+\frac{n}{n^2+2^2}+\frac{n}{n^2+3^2}+....+\frac{1}{5n}\right)$$

$$\lim_{n\to\infty}\left(\frac{n}{n^2+1^2}+\frac{n}{n^2+2^2}+....+\frac{n}{n^2+(2n)^2}\right)$$

$$\lim_{n\to\infty}\sum_{r=1}^{2n}\frac{n}{n^2+r^2}$$

$$\int_0^2\frac{1}{1+x^2}dx=\left|\tan^{-1}x\right|_0^2=\tan^{-1}2$$

348. $\lim\limits_{n\to\infty}\left(\dfrac{(n+1)(n+2)....3n}{n^{2n}}\right)^{\frac{1}{n}}$ is equal to

(a) $\dfrac{18}{e^4}$ (b) $\dfrac{27}{e^2}$ (c) $\dfrac{9}{e^2}$ (d) $3\log3-2$

JEE Mains-2016

Ans. (b) : We have,

$$\lim_{n\to\infty}\left(\frac{(n+1)(n+2)......(n+2n)}{n^{2n}}\right)^{1/n}$$

Let, $\ln y=\lim_{x\to\infty}\frac{1}{n}\left[\ln\left(\frac{n+1}{n}\right)+\ln\left(\frac{n+2}{n}\right)+.....+\ln\left(\frac{n+2n}{n}\right)\right]$

$=\lim_{n\to\infty}\frac{1}{n}\left[\ln\left(1+\frac{1}{n}\right)+\ln\left(1+\frac{2}{n}\right)+.....+\ln\left(1+\frac{2n}{n}\right)\right]$

$=\lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{2n}\ln\left(1+\frac{r}{n}\right)$

$=\int_0^2\ln(1+x)dx=(x+1)\ln(x+1)-(x+1)\Big|_0^2$

$= 3(\ln 3) - 3 + 1 = 3(\ln 3) - 2 = \ln 27 - \ln e^2$

$\ln y=\ln\left(\frac{27}{e^2}\right) \Rightarrow y=\frac{27}{e^2}$

349. $\lim_{n\to\infty}\left[\frac{1}{n}+\frac{n}{(n+1)^2}+\frac{n}{(n+2)^2}+....+\frac{n}{(2n-1)^2}\right]$ **is equal to**

(a) 1 (b) $\frac{1}{2}$ (c) $\frac{1}{3}$ (d) $\frac{1}{4}$

JEE Main 25.02.2021, Shift - II

Ans. (b) : We have,

$$\lim_{n\to\infty}\left[\frac{1}{n}+\frac{n}{(n+1)^2}+\frac{n}{(n+2)^2}+....+\frac{n}{(2n-1)^2}\right]$$

We know that,

$$\lim_{n\to\infty}\sum_{r=0}^{n-1}\frac{n}{(n+r)^2}$$

$$\int_0^1\frac{dx}{(x+1)^2}=\left[\frac{-1}{(x+1)}\right]_0^1=\frac{1}{2}$$

350. Let f : (0, 2) → R be defined as f(x) = $\log_2\left[1+\tan\left(\frac{\pi x}{4}\right)\right]$ **Then,**

$\lim_{n\to\infty}\frac{2}{n}\left[f\left(\frac{1}{n}\right)+f\left(\frac{2}{n}\right)+......f(1)\right]$ **is equal to ..... .**

JEE Main 16.03.2021, Shift-I

Ans. (1) : Given, $f(0, 2)\to R$

$$f(x)=\log_2\left[1+\tan\left(\frac{\pi x}{4}\right)\right]$$

For, $\lim_{n\to 0}\frac{2}{n}\left(f\left(\frac{1}{n}\right)+f\left(\frac{2}{n}\right)+....+f(1)\right)$

$$E=2\lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{n}f\left(\frac{r}{n}\right)$$

$$E=\frac{2}{\ln 2}\int_0^1\ln\left(1+\tan\frac{\pi x}{4}\right)dx \qquad ....(i)$$

Now, replacing $x\to 1-x$

$$E=\frac{2}{\ln 2}\int_0^1\ln\left(1+\tan\frac{\pi}{4}(1-x)\right)dx$$

$$E=\frac{2}{\ln 2}\int_0^1\ln\left(1+\tan\left(\frac{\pi}{4}-\frac{\pi}{4}x\right)\right)dx$$

$$E=\frac{2}{\ln 2}\int_0^1\ln\left(1+\frac{1-\tan\frac{\pi}{4}x}{1+\tan\frac{\pi}{4}x}\right)dx$$

$$E=\frac{2}{\ln 2}=\int_0^1\ln\left(\frac{2}{1+\tan\frac{\pi x}{4}}\right)dx$$

$$E=\frac{2}{\ln 2}\int_0^1\left(\ln 2-\ln\left(1+\frac{\tan\pi x}{4}\right)\right)dx \qquad ....(iii)$$

Adding equation (i) & (ii), we get -

E = 1

351. If f : R → R is given by f(x) = x + 1, then the value of

$\lim_{n\to\infty}\frac{1}{n}\left[f(0)+f\left(\frac{5}{n}\right)+f\left(\frac{10}{n}\right)+....+f\left(\frac{5(n-1)}{n}\right)\right]$

is

(a) 3/2 (b) 5/2 (c) 1/2 (d) 7/2

JEE Main 20.07.2021, Shift-II

Ans. (d) : We have,

$$\lim_{n\to\infty}\frac{1}{n}\left[f(0)+f\left(\frac{5}{n}\right)+f\left(\frac{10}{n}\right)+....+f\left(\frac{5(n-1)}{n}\right)\right]$$

$I=\sum_{r=0}^{n-1}f\left(\frac{5r}{n}\right)\frac{1}{n}$, $I=\int_0^1 f(5x)dx$

$I=\int_0^1(5x+1)dx$, $I=\left[\frac{5x^2}{2}+x\right]_0^1$, $I=\frac{5}{2}+1=\frac{7}{2}$

352. If $U_n=\left(1+\frac{1}{n^2}\right)\left(1+\frac{2^2}{n^2}\right)^2......\left(1+\frac{n^2}{n^2}\right)^n$ **then** $\lim_{n\to\infty}(U_n)^{\frac{-4}{n^2}}$ **is equal to**

(a) $e^2/16$ (b) 4/e (c) $16/e^2$ (d) $4/e^2$

JEE Main 27.08.2021, Shift-I

Ans. (a) : Given, $U_n=\left(1+\frac{1}{n^2}\right)\left(1+\frac{2^2}{n^2}\right).....\left(1+\frac{n^2}{n^2}\right)^n$

$$U_n=\sum_{r=1}^{n}\left(1+\frac{r^2}{n^2}\right)^r$$

$$L=\lim_{n\to\infty}(U_n)^{-4/n^2}$$

$$\log L=\lim_{n\to\infty}\frac{-4}{n}\sum_{r=1}^{n}\cdot\frac{r}{n}\log\left(1+\frac{r^2}{n^2}\right)$$

$$-4\int_0^1 x\log(1+x^2)dx$$

Put, $1 + x^2 = t$

Now,

$2xdx = dt$

$= -2\int_1^2\log(t)dt=-2[t\log t-t]_1^2$

$=\log L=-2(2\log 2-1)$

$L=e^{-2(2\log 2-1)}$

$=e^{-2\left(\log\frac{4}{e}\right)}\Rightarrow\left(\frac{e}{4}\right)^2=\frac{e^2}{16}$

353. $\lim_{x\to 0}\frac{x\cot(4x)}{\sin^2 x\cot^2(2x)}$ is equal to

(a) 0 (b) 1 (c) 4 (d) 2

JEE Main 11.01.2019, Shift-II

Ans. (b) : We have, $\lim_{x\to 0}\frac{x\cot(4x)}{\sin^2 x\cot^2(2x)}$

$$\lim_{x\to 0}\frac{x\tan^2 2x}{\tan 4x\sin^2 x}\Rightarrow \lim_{x\to 0}\frac{\frac{x\tan^2 2x}{4x^2}4x^2}{\frac{\tan 4x}{4x}4x\cdot\frac{\sin^2 x}{x^2}x^2}=1$$

354. $\lim_{x\to\frac{\pi}{4}}\frac{\cot^3 x-\tan x}{\cos\left(x+\frac{\pi}{4}\right)}$ is

(a) $4\sqrt{2}$ (b) 4 (c) 8 (d) $8\sqrt{2}$

JEE Main 12.01.2019, Shift-I

Ans. (c) : We have, $\lim_{x\to\frac{\pi}{4}}\frac{\cot^3 x-\tan x}{\cos\left(x+\frac{\pi}{4}\right)}$

$$\lim_{x\to\frac{\pi}{4}}\left(\frac{1-\tan^4 x}{\tan^3 x\cos\left(x+\frac{\pi}{4}\right)}\right)$$

$$\lim_{x\to\frac{\pi}{4}}\frac{(1+\tan^2 x)(1-\tan^2 x)}{1.\frac{1}{\sqrt{2}}(\cos x-\sin x)}$$

$$2\sqrt{2}\lim_{x\to\frac{\pi}{4}}\frac{(\cos^2 x-\sin^2 x)}{\cos^2 x(\cos x-\sin x)}$$

$$4\sqrt{2}\lim_{x\to\frac{\pi}{4}}(\cos x+\sin x)$$

$$4\sqrt{2}.\sqrt{2}=8$$

355. $\lim_{x\to 1^-}\frac{\sqrt{\pi}-\sqrt{2\sin^{-1}x}}{\sqrt{1-x}}$ is equal to

(a) $\sqrt{\frac{\pi}{2}}$ (b) $\sqrt{\frac{2}{\pi}}$ (c) $\sqrt{\pi}$ (d) $\frac{1}{\sqrt{2\pi}}$

JEE Main 12.01.2019, Shift-II

Ans. (b) : We have, $\lim_{x\to 1^{-1}}\frac{\sqrt{\pi}-\sqrt{2\sin^{-1}x}}{\sqrt{1-x}}$

By using L-Hospital rule,

$$\frac{0-\sqrt{2}\cdot\frac{1}{2\sqrt{\sin^{-1}x}}\times\frac{1}{\sqrt{1-x^2}}}{\frac{1}{2\sqrt{1-x}}(-1)}\Rightarrow\sqrt{2}\cdot\frac{1}{\sqrt{\frac{\pi}{2}}}-\frac{1}{\sqrt{1+1}}=\sqrt{\frac{2}{\pi}}$$

356. $\lim_{x\to\pi/2}\frac{\cot x-\cos x}{(\pi-2x)^3}$ equals

(a) $\frac{1}{24}$ (b) $\frac{1}{16}$ (c) $\frac{1}{8}$ (d) $\frac{1}{4}$

JEE Main-2017

Ans. (b) : We have, $\lim_{x\to\frac{\pi}{2}}\frac{\cot x-\cos x}{(\pi-2x)^3}$

$$\lim_{x\to\frac{\pi}{2}}\frac{1}{8}\cdot\frac{\cos x(1-\sin x)}{\sin x\left(\frac{\pi}{2}-x\right)^3}$$

$$\lim_{h\to 0}\frac{1}{8}\cdot\frac{\cos\left(\frac{\pi}{2}-h\right)\left[1-\sin\left(\frac{\pi}{2}-h\right)\right]}{\sin\left(\frac{\pi}{2}-h\right)\left(\frac{\pi}{2}-\frac{\pi}{2}+h\right)^3}$$

$$\frac{1}{8}\lim_{h\to 0}\frac{\sin h(1-\cos h)}{\cos h.h^3}$$

$$\frac{1}{4}\lim_{h\to 0}\frac{\sin h.\sin^2\left(\frac{h}{2}\right)}{h^3\cos h}$$

$$\frac{1}{4}\lim_{h\to 0}\left(\frac{\sin h}{h}\right)\left(\frac{\sin\frac{h}{2}}{\frac{h}{2}}\right)^2\cdot\frac{1}{\cos h}\frac{1}{4}$$

$$\frac{1}{4}\times\frac{1}{4}=\frac{1}{16}$$

357. Let $p=\lim_{x\to 0^+}\left(1+\tan^2\sqrt{x}\right)^{1/2x}$, then log p is equal to

(a) 2 (b) 1 (c) $\frac{1}{2}$ (d) $\frac{1}{4}$

JEE Main-2016

Ans. (c) : We have, $P=\lim_{n\to 0^+}\left(1+\tan^2\sqrt{x}\right)^{\frac{1}{2x}}$

It is 1^∞ form.

$$=e^{\lim_{x\to 0^+}\frac{\tan^2\sqrt{x}}{2x}}=e^{1/2}$$

$$\log p=\log e^{1/2}=\frac{1}{2}$$

358. $\lim_{x\to 0}\frac{(1-\cos 2x)(3+\cos x)}{x\tan 4x}$, is equal to

(a) $-\frac{1}{4}$ (b) $\frac{1}{2}$ (c) 1 (d) 2

JEE Main-2013

Ans. (d) : Given, $\lim_{x\to 0}\frac{(1-\cos 2x)(3+\cos x)}{x\tan 4x}$

$$\Rightarrow\frac{(1-1+2\sin^2 x)(3+\cos x)}{(x)\times(4x)}\times\frac{4x}{\tan 4x}$$

$$\Rightarrow\frac{2\sin^2 x}{4x^2}\times(3+\cos x)\times 1\Rightarrow\frac{1}{2}\left(\frac{\sin x}{x}\right)^2\times(3+\cos x)$$

$$\Rightarrow\frac{1}{2}\times 4\times 1=2$$

359. $\lim_{n\to\infty}\tan\left\{\sum_{r=1}^{n}\tan^{-1}\left(\frac{1}{1+r+r^2}\right)\right\}$ is equal to

JEE Main 24.02.2021, Shift-I

Ans. (1) : We have given, $\lim_{n\to\infty}\tan\left\{\sum_{r=1}^{N}\tan^{-1}\left(\frac{1}{1+r+r^2}\right)\right\}$

$$\tan\left\{\lim_{n\to\infty}\sum_{r=1}^{n}\tan^{-1}(r+1)-\tan^{-1}r\right\}$$

$$\tan\left\{\lim_{n\to\infty}\left(\tan^{-1}(n+1)-\frac{\pi}{4}\right)\right\}\Rightarrow\tan\times\left(\frac{\pi}{2}-\frac{\pi}{4}\right)$$

$$\tan\frac{\pi}{4}=1$$

360. The value of

$$\lim_{h\to 0}2\left\{\frac{\sqrt{3}\sin\left(\frac{\pi}{6}+h\right)-\cos\left(\frac{\pi}{6}+h\right)}{\sqrt{3}h(\sqrt{3}\cosh-\sinh)}\right\}$$ **is**

(a) $\frac{4}{3}$ (b) $\frac{2}{\sqrt{3}}$ (c) $\frac{3}{4}$ (d) $\frac{2}{3}$

JEE Main 26.02.2021, Shift-I

Ans. (a) : Given,

$$\lim_{h\to 0}2\left\{\frac{\sqrt{3}\sin\left(\frac{\pi}{6}+h\right)-\cos\left(\frac{\pi}{6}+h\right)}{\sqrt{3}h\left(\sqrt{3}\cosh-\sinh\right)}\right\}$$

$$\lim_{h\to 0}2\frac{\left[\frac{\sqrt{3}}{2}\cosh+\frac{3}{2}\sinh-\frac{\sqrt{3}}{2}\cosh+\frac{\sinh}{2}\right]}{h\left(3\cosh-\sqrt{3}\sinh\right)}$$

$$\lim_{h\to 0}2\times\left(\frac{2\sinh}{h}\right)\times\frac{1}{\left(3\cosh-\sqrt{3}\sinh\right)}$$

$$\lim_{h\to 0}\frac{4}{3\cosh-\sqrt{3}\sinh}\Rightarrow\frac{4}{3\cos 0^\circ-\sqrt{3}\sin 0^\circ}$$

$$\frac{4}{3-\sqrt{3}\times 0}=\frac{4}{3}$$

361. $\lim_{x\to 0}\left(\tan\left(\frac{\pi}{4}+x\right)\right)^{1/x}$ **is equal to**

(a) e (b) 2 (c) 1 (d) e^2

JEE Main 02.09.2020, Shift-II
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Ans. (d) : Given, $\lim_{x\to 0}\left(\tan\left(\frac{\pi}{4}+x\right)\right)^{1/x}$

$$A=\lim_{x\to 0}\left[\tan\left(\frac{\pi}{4}+x\right)\right]^{1/x}$$

$$=e^{\lim_{x\to 0}\frac{1}{x}\left[\tan\left(\frac{\pi}{4}+x\right)-1\right]}$$

$$A=e^{\lim_{x\to 0}\left[\frac{\tan\left(\frac{\pi}{4}+x\right)-1}{x}\right]}$$

$$A=e^{\lim_{x\to 0}\frac{\sec^2(\pi/4+x)}{1}}\Rightarrow A=e^{\sec^2\left(\frac{\pi}{4}+0\right)}$$

$$A=e^{\left(\sqrt{2}\right)^2}\Rightarrow A=e^2$$

362. If $\lim_{x\to 0}\left\{\frac{1}{x^8}\left(1-\cos\frac{x^2}{2}-\cos\frac{x^2}{4}+\cos\frac{x^2}{2}\cos\frac{x^2}{4}\right)\right\}=$ 2^{-k}**, then the value of k is**

JEE Main 03.09.2020 Shift -I

Ans. (8) : Given,

$$\lim_{x\to 0}\left\{\frac{1}{x^8}\left(1-\cos\frac{x^2}{2}-\cos\frac{x^2}{4}+\cos\frac{x^2}{2}-\cos\frac{x^2}{4}\right)\right\}$$

$$\lim_{x\to 0}\frac{1}{x^8}\left(1-\cos\frac{x^2}{2}\right)\left(1-\cos\frac{x^2}{4}\right)=2^{-k}$$

$$\lim_{x\to 0}\frac{2\sin^2\left(\frac{x^2}{4}\right).2\sin^2\frac{x^2}{8}}{x^8}=2^{-k}$$

$$\lim_{x\to 0}4\left\{\left[\frac{\sin\frac{x^2}{4}}{\frac{x^2}{4}}\times\frac{\sin\frac{x^2}{8}}{\frac{x^2}{8}}\right]^2\frac{\frac{x^8}{(32)^2}}{x^8}\right\}=2^{-k}$$

$$\Rightarrow\frac{4}{2^{10}}=2^{-k}\Rightarrow 2^{-k}=2^{2-10}\Rightarrow 2^{-k}=2^{-8}$$

So, $k=8$

363. If α is the positive root of the equation, p(x) = $x^2-x-2=0$**, then** $\lim_{x\to\alpha^+}\frac{\sqrt{1-\cos(p(x))}}{x+\alpha-4}$ **is equal to**

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{2}$ (c) $\frac{3}{\sqrt{2}}$ (d) $\frac{3}{2}$

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Ans. (c) : Given, $P(x)=x^2-x-2=0$

$$\lim_{x\to\alpha}+\frac{\sqrt{1-\cos(p(x))}}{x+\alpha-4}$$

$x^2-x-2=0\Rightarrow x^2-2x+x-2=0$

$x=2\ \&-1$

$\therefore$ roots are $(2\ \&-1)$

$$\lim_{x\to 2^+}\frac{\sqrt{1-\cos\left(x^2-x-2\right)}}{x+2-4}\Rightarrow\lim_{x\to 2^+}\frac{\sqrt{2\sin^2\frac{\left(x^2-x-2\right)}{2}}}{(x-2)}$$

$$\lim_{x\to 2}=\frac{\sqrt{2}\sin\left(\frac{(x-2)(x+1)}{2}\right)}{\frac{(x-2)(x+1)}{2}\times\frac{2}{x+1}}\Rightarrow\lim_{x\to 2^+}-\sqrt{2}\times\frac{x+1}{2}$$

$$\left(\because\lim_{x\to 2^+}\frac{\sin\left(\frac{(x-2)(x+1)}{2}\right)}{\frac{(x-2)(x+1)}{2}}=1\right)$$

$$\lim_{x\to 2^+}\frac{x+1}{\sqrt{2}}=\frac{2+1}{\sqrt{2}}=\frac{3}{\sqrt{2}}$$

364. $\lim\limits_{x\to0}\dfrac{\sin^2x}{\sqrt{2}-\sqrt{1+\cos x}}$ equals

(a) $4\sqrt{2}$ (b) $\sqrt{2}$ (c) $2\sqrt{2}$ (d) 4

JEE Main 08.04.2019, Shift-I

Ans. (a) : Given, $\lim\limits_{x\to0}\dfrac{\sin^2 x}{\sqrt{2}-\sqrt{1+\cos x}}$

$$\Rightarrow \lim_{x\to0}\frac{\left(\frac{\sin^2x}{x^2}\right)\left(\sqrt{2}+\sqrt{1+\cos x}\right)}{\left(\frac{1-\cos x}{x^2}\right)}$$

$$=\frac{(1)^2.\left(2\sqrt{2}\right)}{1/2} = 2\sqrt{2}\times2 = 4\sqrt{2}$$

365. $\lim\limits_{x\to0}\dfrac{x+2\sin x}{\sqrt{x^2+2\sin x+1}-\sqrt{\sin^2x-x+1}}$

(a) 6 (b) 2 (c) 3 (d) 1

JEE Main 12.04.2019, Shift-II

Ans. (b) : Given,

$$\lim_{x\to0}\frac{x+2\sin x}{\sqrt{x^2+2\sin x+1}-\sqrt{\sin^2 x-x+1}}$$

$$\lim_{x\to0}\frac{(x+2\sin x)\times\left(\sqrt{x^2+2\sin x+1}+\sqrt{\sin^2 x-x+1}\right)}{\left(\sqrt{x^2+2\sin x+1}-\sqrt{\sin^2 x-x+1}\right)\left(\sqrt{x^2+2\sin x+1}+\sqrt{\sin^2 x-x+1}\right)}$$

$$\lim_{x\to0}\frac{(x+2\sin x)\left(\sqrt{x^2+2\sin x+1}+\sqrt{\sin^2 x-x+1}\right)}{x^2+2\sin x+1-\sin^2x+x-1}$$

$$\lim_{x\to0}\frac{x+2\sin x}{x^2+2\sin x-\sin^2x+x}(1+1)$$

$$\lim_{x\to0}\frac{(x+2\sin x)\times2}{x^2+2\sin x-\sin^2x+x}$$

$$\lim_{x\to0}\frac{2(1+2\cos x)}{2x+2\cos x-2\sin x\cos x+1}$$

(According to L'H rule)

$$=\frac{2(1+2\cos0)}{0+2\cos0-0+1}=\frac{2(1+2)}{2+1}=\frac{3\times2}{3}=2$$

366. **For each $x\in R$, let [x] be the greatest integer less than or equal to x.**

Then, $\lim\limits_{x\to0^-}\dfrac{x([x]+|x|)\sin[x]}{|x|}$ is equal to

(a) 0 (b) sin 1 (c) –sin 1 (d) 1

JEE Main 09.01.2019, Shift-II

Ans. (c) : Given, $\lim\limits_{x\to0^-}\dfrac{x(|x|)+|x|\sin(x)}{|x|}$

We have,

As $x\to0$, $[x]=-1$ and $|x|=-x$

So, $\lim\limits_{x\to0^-}\dfrac{x(-1-x)\sin(-1)}{-x}$

$$=\lim_{x\to0^-}-\frac{(1+x)\sin1}{1}=\frac{-(1+0)\sin1}{1}=-\sin 1$$

367. **If the value of $\lim\limits_{x\to0}(2-\cos x\sqrt{\cos 2x})^{\left(\frac{x+2}{x^2}\right)}$ is equal to e^a, then a is equal to…**

JEE Main 20.07.2021, Shift-I

Ans. (3) : Given, $\lim\limits_{x\to0}\left(2-\cos x\sqrt{\cos2x}\right)^{\frac{x+2}{x^2}}$

$$\lim_{x\to0}\left(1+1-\cos x\sqrt{\cos 2x}\right)^{\frac{x+2}{x^2}}$$

$$e^{\lim\limits_{x\to0}\left(1-\cos x\sqrt{\cos2x}\right)\left(\frac{x+2}{x^2}\right)}$$

We know that,

$$\therefore \quad \cos x=1-\frac{x^2}{21}+\frac{x^4}{4!}$$

And $\cos2x = 1-\dfrac{(2x)^2}{2!}+\dfrac{(2x)^4}{4!}$

$$=\left(1-\frac{x^2}{2}+\frac{x^4}{2^4}......\right)\left(1-2x^2+\frac{2}{3}x^4.....\right)^{1/2}$$

We have to extract till the coefficient of x^2 as denominator is x^2

So, $\left(1-\dfrac{x^2}{2}\right)\left(1-2x^2\right)^{1/2}=\left(1-\dfrac{x^2}{2}\right)\left(1-x^2\right)$

$$\left(1-\frac{x^2}{2}-x^2+\frac{x^4}{2}\right)=\left(1-\frac{3}{2}x^2\right)$$

So, $e^{\lim\limits_{x\to0}\left(1-\cos x\sqrt{\cos2x}\right)\left(\frac{x+2}{x^2}\right)}$

$$=e^{\lim\limits_{x\to0}\left[1-1+\left(\frac{3x^2}{2}\right)\right]\left(\frac{x+2}{x^2}\right)}=e^{\lim\limits_{x\to0}\left(\frac{3x^2}{2}\right)\left(\frac{x+2}{x^2}\right)}=e^3$$

According to information

$e^3=e^a$

So, $a=3$

368. **If $\lim\limits_{x\to0}\dfrac{\sin^{-1}x-\tan^{-1}x}{3x^3}$ is equal to L, then the value of (6L + 1) is**

(a) $\dfrac{1}{6}$ (b) $\dfrac{1}{2}$ (c) 6 (d) 2

JEE Main 18.03.2021, Shift-I

Ans. (d) : Given, $\lim\limits_{x\to0}\dfrac{\sin^{-1}x-\tan^{-1}x}{3x^3}$

Applying 'L' hospital value,

According to $\lim\limits_{x\to c}\dfrac{f(x)}{g(x)}=\lim\limits_{x\to c}\dfrac{f'(x)}{g'(x)}$

$$\lim_{x\to0}\frac{\sin^{-1}x-\tan^{-1}x}{3x^3}=\lim_{x\to0}\frac{\frac{d}{dx}\sin^{-1}x-\frac{d}{dx}\tan^{-1}x}{\frac{d}{dx}(3x^3)}$$

$$=\lim_{x\to0}\frac{\frac{1}{\sqrt{1-x^2}}-\frac{1}{1+x^2}}{9x^2}=\lim_{x\to0}\frac{1+x^2-\sqrt{1-x^2}}{9x^2\times\sqrt{1-x^2}\times(1+x^2)}$$

$$=\lim_{x\to0}\frac{1+x^2-\sqrt{1-x^2}}{9x^2}$$

Again applying 'L' hospital rule

$$\lim_{x\to 0}\frac{1+x^2-\sqrt{1-x^2}}{9x^2}=\lim_{x\to 0}\frac{0+2x-\frac{1}{2\sqrt{1-x^2}}(-2x)}{18x}$$

$$\lim_{x\to 0}=\frac{2x+\frac{x}{\sqrt{1-x^2}}}{18x}=\frac{2x+\frac{x}{\sqrt{1-0}}}{18}=\frac{3}{18}=\frac{1}{6}$$

We have,

$$\lim_{x\to 0}\frac{\sin^{-1}x-\tan^{-1}x}{3x^3}=L$$

So, $\quad (6L+1)=6\times\frac{1}{6}+1=2$

369. The value of $\lim\limits_{x\to 0^+}\frac{\cos^{-1}(x-[x]^2).\sin^{-1}(x-[x]^2)}{x-x^3}$ where [x] denotes the greatest integer ≤ x is

(a) π (b) 0 (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{2}$

JEE Main 17.03.2021, Shift-I

Ans. (d) : Given, $\lim\limits_{x\to 0}+\frac{\cos^{-1}\left(x-[x]^2\right).\sin^{-1}\left(x-[x]^2\right)}{x-x^3}$

Let, $x = m$

$$=\lim_{m\to 0}+\frac{\cos^{-1}(m-0).\sin^{-1}(m-0)}{m-m^3}=\lim_{m\to 0}+\frac{\cos^{-1}m.\sin^{-1}m}{m(1-m)(1+m)}$$

$$=\lim_{m\to 0}\left(\frac{\sin^{-1}m}{m}\right)\left[\frac{\cos^{-1}m}{(1-m)(1+m)}\right]=1.\frac{\pi}{2}=\frac{\pi}{2}$$

370. The value of $\lim\limits_{\theta\to 0}\frac{\tan(\pi\cos^2\theta)}{\sin(2\pi\sin^2\theta)}$ is equal to

(a) $-\frac{1}{2}$ (b) $-\frac{1}{4}$ (c) 0 (d) $\frac{1}{4}$

JEE Main 17.03.2021, Shift-II

Ans. (a) : Given,

$$\lim_{x\to 0}\frac{\tan\left(\pi\cos^2\theta\right)}{\sin\left(2\pi\sin^2\theta\right)}$$

$$=\lim_{x\to 0}\left[\frac{\tan\left[\pi\left(1-\sin^2\theta\right)\right]}{\sin\left(2\pi\sin^2\theta\right)}\right]$$

$$=\lim_{x\to 0}\frac{\frac{\tan\left(\pi-\pi\sin^2\theta\right)}{\pi\sin^2\theta}\times\pi\sin^2\theta}{\frac{\sin\left(2\pi\sin^2\theta\right)}{\left(2\pi\sin^2\theta\right)}\times 2\pi\sin^2\theta}=-\frac{1}{2}$$

371. $\lim\limits_{x\to 2}\frac{3^x+3^{3-x}-12}{3^{-x/2}-3^{1-x}}$ is equal to……

JEE Main 07.01.2020, Shift-I

Ans. (36) : Given,

$$\lim_{x\to 2}\frac{3^x+3^{3-x}-12}{3^{-x/2}-3^{1-x}}=\lim_{x\to 2}\frac{3^{2x}-12\cdot 3^x+27}{3^{x/2}-3}$$

$$\lim_{x\to 2}\frac{\left(3^x-9\right)\left(3^x-3\right)}{\left(3^{x/2}-3\right)}$$

$$\lim_{x\to 2}\frac{\left(3^{x/2}-3\right)\left(3^{x/2}+3\right)\left(3^x-3\right)}{\left(3^{x/2}-3\right)}$$

$$\lim_{x\to 2}\left(3^{x/2}+3\right)\left(3^x-3\right)$$

$$=\left(3^{\frac{2}{2}}+3\right)\left(3^2-3\right)=6\times 6=36$$

372. $\lim\limits_{x\to 0}\left(\frac{3x^2+2}{7x^2+2}\right)^{1/x^2}$ is equal to

(a) e^2 (b) e (c) $\frac{1}{e^2}$ (d) $\frac{1}{e}$

JEE Main 08.01.2020, Shift-I

Ans. (c) : Given, $\lim\limits_{x\to 0}\left(\frac{3x^2+2}{7x^2+2}\right)^{1/x^2}$

$$L=e^{\lim\limits_{x\to 0}\frac{1}{x^2}\left(\frac{3x^3+2}{7x^2+2}-1\right)}$$

$\because$ If $\lim\limits_{x\to a}(f(x))^{g(x)}$ have in determinant form 1^∞

Then,

$$\lim_{x\to a}f(x)^{g(x)}=e^{\lim\limits_{x\to a}f(x)(f(x)-1)}$$

$$L=e^{\lim\limits_{x\to 0}\frac{1}{x^2}\left(\frac{3x^2+2-7x^2-2}{7x^2+2}\right)}=e^{\lim\limits_{x\to 0}\frac{1}{x^2}\left(\frac{-4x^2}{7x^2+2}\right)}=e^{\lim\limits_{x\to 0}\left(\frac{-4}{7x^2+2}\right)}$$

$\therefore \quad L=e^{\frac{-4}{0+2}}\Rightarrow L=e^{-2}$

or $\quad L=\frac{1}{e^2}$

373. If $\lim\limits_{x\to 1}\frac{x^4-1}{x-1}=\lim\limits_{x\to k}\frac{x^3-k^3}{x^2-k^2}$, then k is

(a) $\frac{4}{3}$ (b) $\frac{3}{8}$ (c) $\frac{3}{2}$ (d) $\frac{8}{3}$

JEE Main 10.04.2019, Shift-I

Ans. (d) : Given,

$$\lim_{x\to 1}\frac{x^4-1}{x-1}=\lim_{x\to k}\frac{x^3-k^3}{x^2-k^2}$$

$$\lim_{x\to 1}\frac{x^4-1}{x-1}=\lim_{x\to 1}\frac{(x-1)\left(x^3+x^2+x+1\right)}{(x-1)}$$

$$\lim_{x\to 1}\frac{x^3+x^2+x+1}{1}=1+1+1+1$$

$$=4 \qquad ....(i)$$

$$\lim_{x\to k}\frac{x^3-k^3}{x^2-k^2}=\lim_{x\to k}\frac{\left(x^2+kx+k^2\right)(x-k)}{(x+k)(x-k)}$$

$$=\lim_{x\to k}\frac{x^2+kx+k^2}{x+k}=\frac{k^2+k\cdot k+k^2}{k+k}=\frac{3k^2}{2k} \qquad ....(ii)$$

Comparing by equation (i) and (ii), we get –

$$\frac{3k^2}{2k}=4\Rightarrow\frac{3}{2}k=4$$

$$k=\frac{8}{3}$$

374. Let f(x) = 5 – |x – 2| and g(x) = |x + 1|, x ∈ R. If f(x) attains maximum value at α and g(x) attains minimum value of β, then

$$\lim_{x\to-\alpha\beta}\frac{(x-1)(x^2-5x+6)}{x^2-6x+8}$$ **is equal to**

(a) 1/2 (b) –3/2 (c) –1/2 (d) 3/2

JEE Main 12.04.2019, Shift-II

Ans. (a) : Given, $f(x) = 5-|x-2|$

and $g(x) = |x+1|$, $x \in R$

$\because$ maximum of g(x) occurred at x = 2,

So, $\alpha = 2$

Minimum of g(x) occurred at x = – 1

So, $\beta = -1$

Hence, $\alpha\beta = 2x - 1 = -2$

$$\lim_{x\to-\alpha\beta}\frac{(x-1)(x^2-5x+6)}{x^2-6x+8}$$

Putting $\alpha\beta = -2$

$$\therefore \lim_{x\to2}\frac{(x-1)(x-3)(x-2)}{(x-4)(x-2)} \Rightarrow \lim_{x\to2}\frac{(x-1)(x-3)}{(x-4)}$$

So, $=\frac{(2-1)(2-3)}{(2-4)} = \frac{1\times(-1)}{-2} = \frac{1}{2}$

375. If $\lim_{x\to0}\frac{\alpha\times e^x-\beta\log_e(1+x)+\gamma x^2e^{-x}}{x\sin^2x}$ = 10, α, β γ ∈ R, then the value of α + β + γ is …

JEE Main 20.07.2021, Shift-II

Ans. (3) : Given,

$$\lim_{x\to0}\frac{\alpha\times e^x-\beta\log_e(1+x)+\gamma x^2e^{-x}}{x\sin^2x} = 10$$

$$=\lim_{x\to0}\frac{\alpha\times\left(1+\frac{x}{1!}+\frac{x^2}{2!}+....\right)-\beta\left(x-\frac{x^2}{2}+\frac{x^3}{3}.....\right)+\gamma x^2\left(1-\frac{x}{1!}+\frac{x^2}{2!}....\right)}{x\sin^2x}$$

$$=\lim_{x\to0}\frac{x(\alpha-\beta)+x^2\left(\alpha+\frac{\beta}{2}+\gamma\right)+x^3\left(\frac{\alpha}{2}-\frac{\beta}{3}-\gamma\right)....}{x\sin^2x}$$

$\because$ Degree of denominator = 3

$\therefore$ For limit to exist,

$\alpha - \beta = 0$ ....(i)

and $\alpha + \frac{\beta}{2} + \gamma = 0$ ....(ii)

For terms greater than degree 3,

as $x \to 0$

$$\frac{\alpha}{2}-\frac{\beta}{3}-\gamma=10$$

From equation (i) $\beta = \alpha$

From equation (ii) $\gamma = \left(\frac{\alpha}{2}+\alpha\right) = \frac{-3\alpha}{2}$

On putting there in equation (iii)

$$\frac{\alpha}{2}-\frac{\alpha}{3}+\frac{3\alpha}{2}=10$$

$$\frac{3\alpha-2\alpha+9\alpha}{6}=10$$

Now, $\alpha = 6$

$\alpha = \beta$

$\therefore$ $\beta = 6$

and $\gamma = -\frac{3\alpha}{2} = -3\times\frac{6}{2} = -9$

So, $\alpha + \beta + \gamma = 6 + 6 - 9 = 3$

376. The value of $\lim_{x\to0}\frac{[r]+[2r]+...+[nr]}{n^2}$, where r is non-zero real number and [r] denotes the greatest integer less than or equal to r, is equal to

(a) $\frac{r}{2}$ (b) r (c) 2r (d) 0

JEE Main 17.03.2021, Shift-II

Ans. (a) : Given, $\lim_{x\to0}\frac{|r|+|2r|+.....+|nr|}{n^2}$

$r \le [r] < r + 1$

$2r \le [2r] < 2r + 1$

$3r \le [3r] < 3r + 1$

⋮ ⋮ ⋮

$nr \le [nr] < nr + 1$

Adding,

$\Rightarrow$ (r + 2r + 3r + 4r + ...+nr) ≤ [r] + [2r] + [3r] + [4r] + ...[nr] < (r + 1) + [2r + 1] + [3r + 1] + [4r + 1] + ... + [nr + 1]

$\Rightarrow$ r (1 + 2 + 3 + 4 +....+n ≤ [r] + [2r] + [3r] +....[nr] < (r + 2r + 3r + ...+ nr) + (1 + 1+ 1 + ...+1)

$$\Rightarrow r.\frac{n(n+1)}{2} \le [r]+[2r]+[3r]+....+[nr] < \frac{r.[n(n+1)]}{2}+n$$

$$\Rightarrow \frac{r\frac{n(n+1)}{2}}{n^2} \le \frac{[r]+[2r]+[3r]+....+[nr]}{n^2}$$

$$< \frac{r\left(\frac{n(n+1)}{2}\right)+n}{n^2}$$

Now, $\lim_{n\to\infty}\frac{n(n+1)\cdot r+n}{2\cdot n^2} = \lim_{n\to\infty}\frac{n(n+1)\cdot r+2n}{2n^2}$

$$=\lim_{n\to\infty}\frac{n^2\left\{\left(1+\frac{1}{n}\right)r+\frac{2}{n}\right\}}{2n^2}$$

$$=\frac{(1+0)r+0}{2}=\frac{r}{2} \qquad ....(ii)$$

From equation (i) and (ii)

$\lim_{n\to\infty}\frac{[r]+[2r]+[3r]+.....+[nr]}{n^2}$ (By sand witch theorem)

$=\frac{r}{2}$

377. If $\lim_{x\to0}\frac{ae^x-b\cos x+ce^{-x}}{x\sin x}$ = 2, then a + b + c is equal to…

JEE Main 16.03.2021, Shift-I

Ans. (4) : Given,

$$\lim_{x\to0}\frac{ae^x-b\cos x+ce^{-x}}{x\sin x}$$

$$\lim_{x\to0}\frac{ae^x-b\cos x+ce^{-x}}{x\sin x}=2$$

$$\lim_{x\to 0}\frac{a\left(1+x+\frac{x^2}{2!}+....+\right)-b\left(1-\frac{x^2}{2!}+\frac{x^4}{4!}....+\right)+c\left(1-x+\frac{x^2}{2!}...+\right)}{x\left(x-\frac{x^3}{3!}+....+\right)}$$

$$\Rightarrow \lim_{x\to 0}\frac{(a-b+c)+(a-c)x+\left(\frac{a}{2}+\frac{b}{2}+\frac{c}{2}\right)x^2+....+}{x^2-\frac{x^4}{6}+....}$$

In numerator, all the coefficients of x^p, where $p < 2$ has to be zero, then only limit will exist.

So, $a - b + c = 0 \Rightarrow a - c = 0$

$\Rightarrow$ $a = c, b = 2a$

Salving limit, $\frac{a+b+c}{2}=2$

$a + b + c = 4 \Rightarrow a + 2a + a = 4$

$4a = 4 \Rightarrow a = 1$

So, $b = 2 \times 1 = 2$

$c = 1$

Hence, $a + b + c = 1 + 2 + 1 = 4$

378. If $\lim\limits_{x\to 0}\frac{ax-(e^{4x}-1)}{ax(e^{4x}-1)}$ exists and is equal to b, then the value of a – 2b is…

JEE Main 25.02.2021, Shift-II

Ans. (5) : Given, $\lim\limits_{x\to 0}\frac{ax-\left(e^{4x}-1\right)}{ax\left(e^{4x}-1\right)}$

We know that, $\left(e^x=1+x+\frac{x^2}{2!}+....\right)$

$$\lim_{x\to 0}\frac{ax-\left[\left(1+4x+\frac{(4x)^2}{2!}+....\right)-1\right]}{ax\left[\left(1+4x+\frac{(4x)^2}{2!}.....\right)-1\right]}$$

$$\lim_{x\to 0}\frac{(a-4)x-\frac{(4x)^2}{2!}+...}{ax\left(4x+\frac{(4x)^2}{2!}+....\right)}\Rightarrow \lim_{x\to 0}\frac{(a-4)-8x-\frac{(4x)^3}{3!\,x}}{a\left(4x+\frac{(4x)^2}{2!}+....\right)}$$

Limit exists if

$a - 4 = 0 \Rightarrow a = 4$

$$\lim_{x\to 0}\frac{-8x\left(1+\frac{4}{3}x^2+.....\right)}{4\times 4x\left(1+\frac{4x}{2!}+.....\right)}$$

Putting a = 4

Then, $\frac{-8}{16}=\frac{-1}{2}$

$b=\frac{-1}{2}$

So, $a - 2b = 4 - 2 \times -\frac{1}{2} = 5$

379. Let [t] denote the greatest integer ≤ t. if for some $\lambda \in R - \{0, 1\}$, $\lim\limits_{x\to 0}\left|\frac{1-x+|x|}{\lambda-x+[x]}\right| = L$, then L is equal to

(a) 1 (b) 2 (c) $\frac{1}{2}$ (d) 0

JEE Main 03.09.2020, Shift-I

Ans. (b) : Given, $|t|\le t$

$\lambda \in R - <0\,,\,\}$

$$\lim_{x\to 0}\left|\frac{1-x+|x|}{\lambda-x+[x]}\right|=L$$

$$L=\lim_{x\to 0^-}\left|\frac{1-x-x}{\lambda-1}\right|=\lim_{x\to 0^+}\left|\frac{1-x+x}{\lambda-0}\right|$$

$$L=\frac{1}{|\lambda-1|}=\frac{1}{\lambda}$$

$$|\lambda-1|=|\lambda|=\frac{1}{2}$$

$\therefore$ $L = 2$

380. If $\lim\limits_{x\to\infty}\left(\sqrt{x^2-x+1}-ax\right)=b$, then the ordered pair (a, b) is

(a) $\left(1,\frac{1}{2}\right)$ (b) $\left(1,-\frac{1}{2}\right)$ (c) $\left(-1,\frac{1}{2}\right)$ (d) $\left(-1,-\frac{1}{2}\right)$

JEE Main 27.08.2021, Shift-II

Ans. (b) : Given, $\lim\limits_{x\to\infty}\left(\sqrt{x^2-x+1}-ax\right)=b$

$$=\lim_{x\to\infty}\left(\sqrt{x^2-x+1}-ax\right)\frac{\left(\sqrt{x^2-x+1}+ax\right)}{\sqrt{x^2-x+1}+ax}=b$$

$$=\lim_{x\to\infty}\frac{x^2-x+1-a^2x^2}{\sqrt{x^2-x+1}+ax}=b$$

Limit exits, If

$a^2 = 1$

$a = \pm 1$

$$\lim_{x\to\infty}\frac{-x+1}{\sqrt{x^2-x+1}+ax}=b$$

$$\lim_{x\to\infty}\frac{-1+\frac{1}{x}}{\sqrt{1-\frac{1}{x}+\frac{1}{x^2}}+a}=b \quad =\frac{-1}{1+a}=b$$

But,

$a \ne -1 \Rightarrow a = 1$

$\therefore$ $b=\frac{-1}{2}$

So, $(a, b) = \left(1,-\frac{1}{2}\right)$

381. $\lim\limits_{x\to 2}\left(\sum\limits_{n=1}^{9}\frac{x}{n(n+1)x^2+2(2n+1)x+4}\right)$ is equal to

(a) $\frac{9}{44}$ (b) $\frac{5}{24}$ (c) $\frac{1}{5}$ (d) $\frac{7}{36}$

JEE Main 26.08.2021, Shift-II

Ans. (a) : Given,

$$L=\lim_{x\to 2}\left(\sum_{n=1}^{9}\frac{x}{n(n+1)x^2+2(2n+1)x+4}\right)$$

$$L=\sum_{n=1}^{9}\frac{2}{4(n^2+3n+2)}$$

$$=\frac{1}{2}\sum_{n=1}^{9}\left(\frac{1}{n+1}-\frac{1}{n+2}\right)=\frac{1}{2}\left(\frac{1}{2}-\frac{1}{11}\right)=\frac{1}{2}\times\frac{11-2}{22}=\frac{9}{44}$$

382. If f(x) = $\frac{[x]}{|x|}, x\neq 0$, where [.] denotes the greatest integer function, then f(1) is equal to

(a) –1 (b) ∞
(c) Non-existent (d) None of these

Jamia Millia Islamia-2013

Ans. (c) : $f(1+0)=\lim_{h\to 0}\frac{f(1+h)-f(1)}{h}$

$$=\lim_{h\to 0}\frac{\left(\frac{1+h}{1+h}\right)-\left(\frac{1}{1}\right)}{h}=\lim_{h\to 0}\frac{\left(\frac{1}{1+h}\right)-1}{h}=\lim_{h\to 0}\frac{(-h)}{h(1+h)}=-1$$

$$f(1-0)=\lim_{h\to 0}\frac{f(1-h)-f(1)}{-h}$$

$$=\lim_{h\to 0}\frac{\left(\frac{1-h}{1-h}\right)-\left(\frac{1}{1}\right)}{-h}=\lim_{h\to 0}\left(\frac{0-1}{-h}\right)=\infty$$

$\therefore$ $\gamma(1+0)\neq f(1-0)$

So, f(1) does not exist

383. If $\lim_{x\to 0}\frac{[(a-n)nx-\tan x]\sin nx}{x^2}=0, n\neq 0$ then the minimum possible positive value of a is

(a) 0 (b) –2 (c) 2 (d) 1

AP EAMCET-23.04.2019, Shift-I

Ans. (c) : Given, $\lim_{x\to 0}\frac{((a-n)nx-\tan x)\sin nx}{x^2}=0,\ n\neq 0$

$$\lim_{x\to 0}\left(\frac{(a-n)nx}{x}-\frac{\tan x}{x}\right)\frac{\sin(nx)n}{nx}=0$$

$$\left[\because \lim_{x\to 0}\frac{\sin nx}{nx}=1\right]$$

$$n(an-n^2-1)=0\Rightarrow a=n+\frac{1}{n},\ n\neq 0$$

$$\frac{n+\frac{1}{n}}{2}\geq\sqrt{n\frac{1}{n}} \qquad \text{(by AM} \geq \text{GM)}$$

$$\frac{a}{2}\geq 1\Rightarrow a\geq 2$$

Therefore, minimum possible positive value of a is 2.

384. Let [x] denote the greatest integer not exceeding x. If

$$l_1 \lim_{x\to 2^+}(x^2+[x]), l_2=\lim_{x\to 3^-}(2x-[x]) \text{ and } l_3=\lim_{x\to\frac{\pi}{2}}\left(\frac{\cos x}{x-\frac{\pi}{2}}\right)$$

then

(a) $l_2<l_3<l_1$ (b) $l_1<l_3<l_2$
(c) $l_1<l_2<l_3$ (d) $l_3<l_2<l_1$

AP EAMCET-23.04.2019, Shift-I

Ans. (d) : We have,

$$l_1=\lim_{x\to 2^+}(x^2+[x])$$

$$l_1=\lim_{x\to 2^+}(x^2+2) \qquad [\because x\to 2^+, [x]=2]$$

$l_1=4+2$

$l_1=6$

$$l_2=\lim_{x\to 3^-}(2x-[x])$$

$$l_2=\lim_{x\to 3^-}(2x-2) \qquad [\because x\to 3^-, [x]=2]$$

$l_2=2(3)-2\Rightarrow l_2=4$

$$l_3=\lim_{x\to\frac{\pi}{2}}\left(\frac{\cos x}{x-\frac{\pi}{2}}\right)$$

Put the value $x-\frac{\pi}{2}=y$ and as $x\to\frac{\pi}{2}$, then $y\to 0$

$$x=\frac{\pi}{2}+y=\lim_{y\to 0}\frac{\cos\left(\frac{\pi}{2}+y\right)}{y}=\lim_{y\to 0}-\frac{\sin\left(y+\frac{\pi}{2}\right)}{y}$$

$l_3=-1$

Therefore, $l_3<l_2<l_1$

385. $\lim_{n\to\infty}\frac{2^2+4^2+6^2+...+(2n)^2}{n^3}=$

(a) $\frac{2}{3}$ (b) $\frac{4}{3}$ (c) $\frac{3}{2}$ (d) $\frac{8}{7}$

AP EAMCET-21.09.2020, Shift-I

Ans. (b) : Given, $=\lim_{n\to\infty}\frac{2^2+4^2+6^2+......+(2n)^2}{n^3}$

$$=4\lim_{n\to\infty}\frac{1^2+2^2+3^2+......+n^2}{n^3}$$

$$=4\lim_{n\to\infty}\frac{n(n+1)(2n+1)}{6n^3}=4\lim_{n\to\infty}\frac{1\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)}{6}$$

$$=4\times\frac{2}{6}=\frac{4}{3}$$

386. $\lim_{x\to 0}\frac{a^x-1}{\sin(x)}=$

(a) log (a) (b) $\frac{1}{2}$log (a) (c) 0 (d) 1

AP EAMCET-21.09.2020, Shift-I

Ans. (a) : Given,

$$=\lim_{x\to 0}\frac{a^x-1}{\sin x}=\frac{\lim_{x\to 0}\frac{a^x-1}{x}}{\lim_{x\to 0}\frac{\sin x}{x}}=\frac{\log_e a}{1}=\log_e a$$

387. The value of $\lim_{x\to 1}(1-x)\tan\left(\frac{\pi x}{2}\right)=$

(a) $\frac{\pi}{2}$ (b) π (c) $\frac{2}{\pi}$ (d) 0

CG PET-2021

Ans. (a) : Given, $\lim_{x\to1}(1-x)\tan\left(\frac{\pi x}{2}\right)$

$$\lim_{x\to1}\frac{1-x}{\cot\left(\frac{\pi x}{2}\right)}$$

$$\lim_{x\to1}\frac{-1}{-\frac{\pi}{2}\operatorname{cosec}^2\left(\frac{\pi x}{2}\right)} \quad \text{(On applying L' Hospital rule)}$$

$$=\frac{\pi}{2}$$

388. $\lim_{x\to-2}\frac{\sin^{-1}(x+2)}{x^2+2x}$ **is equal to**

(a) 0 (b) ∞ (c) $-\frac{1}{2}$ (d) None of these

Manipal UGET-2019

Ans. (c) : Let, $y=\lim_{x\to-2}\frac{\sin^{-1}(x+2)}{x^2+2x}$

$$y=\lim_{x\to-2}\frac{\sin^{-1}(x+2)}{(x+2)}\times\frac{1}{x}$$

$$y=\lim_{x\to-2}(1)\times\frac{1}{x} \quad \left[\because \lim_{x\to\theta}\frac{\sin\theta}{\theta}=1\right]$$

$$y=-\frac{1}{2}$$

389. $\lim_{x\to0}\frac{1-\cos mx}{1-\cos nx}$ **is equal to**

(a) $\frac{m}{n}$ (b) $\frac{m^2}{n^2}$ (c) 0 (d) $\frac{n^2}{m^2}$

Manipal UGET-2017

Ans. (b) : Let, $\lim_{x\to0}\left(\frac{1-\cos mx}{1-\cos n x}\right)$

Using L' Hospital rule,

$$\lim_{x\to0}\frac{2\sin^2(mx/2)}{(mx/2)}.\frac{(nx/2)^2}{2\sin^2(n/2)}.\frac{m^2}{n^2}=\frac{m^2}{n^2}$$

390. $\lim_{n\to\infty}\left(\frac{1}{\sqrt{4n^2-1}}+\frac{1}{\sqrt{4n^2-2^2}}+.....+\frac{1}{\sqrt{3n^2}}\right)$ **is equal to**

(a) 0 (b) 1 (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{6}$

Manipal UGET-2017

Ans. (d) : Given, $\lim_{n\to\infty}\left[\frac{1}{\sqrt{4n^2-1}}+\frac{1}{\sqrt{4n^2-2^2}}+......\frac{1}{\sqrt{3n^2}}\right]$

$$\lim_{n\to\infty}\frac{1}{n}\left[\frac{n}{\sqrt{4n^2-1^2}}+\frac{n}{\sqrt{4n^2-2^2}}+......\frac{n}{\sqrt{4n^2-n^2}}\right]$$

$$\lim_{n\to\infty}\frac{1}{n}\left[\frac{1}{\sqrt{4-\left(\frac{1}{n}\right)^2}}+\frac{1}{\sqrt{4-\left(\frac{2}{n}\right)^2}}+......\frac{1}{\sqrt{4-\left(\frac{n}{n}\right)^2}}\right]$$

$$\lim_{n\to\infty}\frac{1}{n}\left[\sum_{r=1}^{n}\frac{1}{\sqrt{4-\left(\frac{r}{n}\right)^2}}\right]$$

$$\int_0^1\frac{1}{\sqrt{4-x^2}}dx=\left[\sin^{-1}\left(\frac{x}{2}\right)\right]_0^1=\frac{\pi}{6}$$

391. $\lim_{n\to\infty}\left[\frac{1}{n+1}+\frac{1}{n+2}+......+\frac{1}{6n}\right]$

(a) log 2 (b) log $(1+\sqrt{5})$ (c) log 6 (d) 0

Manipal UGET-2017

Ans. (c) : Given, $\lim_{n\to\infty}\left[\frac{1}{n+1}+\frac{1}{n+2}+...+\frac{1}{n+5n}\right]$

$$=\lim_{n\to\infty}\sum_{r=0}^{5n}\frac{1}{n+r}=\lim_{n\to\infty}\left[\frac{1}{n}\sum_{r=0}^{5n}\frac{n}{n+r}\right]$$

$$=\lim_{n\to\infty}\left[\frac{1}{n}\sum_{r=0}^{5n}\frac{1}{1+(r/n)}\right]$$

$$=\int_0^5\frac{1}{1+x}dx=\left[\log(1+x)\right]_0^5$$

$$=\log6-\log1=\log6$$

392. **The value of** $\lim_{x\to\frac{\pi}{6}}\frac{2\sin^2x+\sin x-1}{2\sin^2x-3\sin x+1}$ **is**

(a) 3 (b) −3 (c) 6 (d) 0

Manipal UGET-2016

Ans. (b) : We have,

$$\lim_{x\to\frac{\pi}{6}}\frac{2\sin^2x+\sin x-1}{2\sin^2x-3\sin x+1}$$

$$\lim_{x\to\frac{\pi}{6}}\frac{(2\sin x-1)(\sin x+1)}{(2\sin x-1)(\sin x-1)}$$

$$\lim_{x\to\frac{\pi}{6}}\frac{(\sin x+1)}{(\sin x-1)}=\frac{\left(\sin\frac{\pi}{6}+1\right)}{\left(\sin\frac{\pi}{6}-1\right)}=-3$$

393. $\lim_{n\to\infty}\left(\frac{1^2}{1-n^3}+\frac{2^2}{1-n^3}+.....+\frac{n^2}{1-n^3}\right)$ **is equal to**

(a) $\frac{1}{3}$ (b) $-\frac{1}{3}$ (c) $\frac{1}{6}$ (d) $-\frac{1}{6}$

Manipal UGET-2016

Ans. (b) : Let, $\lim_{n\to\infty}\frac{1}{1-n^3}\sum_{r=1}^{b}r^2$

$$=\lim_{n\to\infty}\frac{1}{n^3\left(\frac{1}{n^3}-1\right)}.\frac{n(n+1)(2n+1)}{6}$$

$$=\lim_{n\to\infty}\frac{n^3\left(1+\frac{1}{n}\right)(2/n)}{n^3(1/n^3)6}=-1/3$$

394. If $f(x) = \cot^{-1}[(3x - x^3)/(1-3x^2)]$ and $g(x) = \cos^{-1}[(1-x^2)/(1+x^2)]$, then

$$\lim_{x\to a}\frac{f(x)-f(a)}{g(x)-g(a)}\left[0<a<\frac{1}{2}\right] \text{ is}$$

(a) $-\frac{3}{2}$ (b) $\frac{1}{2}$ (c) $\frac{3}{2}$ (d) None of these

Rajasthan PET-2012

Ans. (a) : Let,

$$f(x)=\cot^{-1}\frac{3x-x^3}{1-3x^2}=\frac{\pi}{2}-3\tan^{-1}x$$

$$g(x)=\cos^{-1}\frac{1-x^2}{1+x^2}=2\tan^{-1}x$$

$$\lim_{x\to\infty}\frac{\frac{\pi}{2}-3\tan^{-1}x-\frac{\pi}{2}+3\tan^{-1}a}{2\tan^{-1}x-2\tan^{-1}a}$$

$$=\frac{-3}{2}\times\frac{\tan^{-1}x-\tan^{-1}a}{\tan^{-1}x-\tan^{-1}a}=\frac{-3}{2}$$

395. $\lim_{x\to\infty}\frac{x^n}{e^x}=0$ for

(a) n = 0 only (b) n is any whole number
(c) n = 2 only (d) no value of n

Rajasthan PET-2011

Ans. (b) : Given, $\lim_{x\to\infty}\frac{x^n}{e^x}=0$

Case I : n is a positive integer,

$$\lim_{x\to\infty}\frac{x^2}{e^x}=\lim_{x\to\infty}\frac{nx^{n-1}}{e^x}$$

$$\lim_{x\to\infty}\frac{n(n-1)x^{n-2}}{e^x}=......=\lim_{x\to\infty}\frac{n!}{e^x}$$

[By L' Hospital's rule repeatedly] = 0

Case II : n is a negative integer,

$$\lim_{x\to\infty}\frac{x^n}{e^n}=\lim_{x\to\infty}\frac{x^{-m}}{e^x}$$

[Put the value n = – m, where m is a positive repeatedly]

$$=\lim_{x\to\infty}\frac{1}{x^m e^x}=\frac{1}{\infty}=0$$

Hence, $\lim_{x\to\infty}\frac{x^n}{e^x}=0$ for all values of n.

396. $\lim_{x\to 2}\frac{2-\sqrt{2+x}}{2^{1/3}-(4-x)^{1/3}}$ is equal to

(a) $2.3^{-1/2}$ (b) $3.2^{-4/3}$
(c) $-3.2^{-4/3}$ (d) None of these

Ans. (c) : Given that,

$$\lim_{x\to 2}\frac{2-\sqrt{2+x}}{2^{1/3}-(4-x)^{1/3}} \qquad \left[\frac{0}{0}\text{ from}\right]$$

$$=\lim_{x\to 2}\frac{0-\frac{1}{2\sqrt{2+x}}}{0-\frac{1}{3}\frac{(-1)}{(4-x)^{2/3}}}$$

$$=\lim_{x\to 2}\frac{-3(4-x)^{2/3}}{2\sqrt{2+x}}$$

$$=\frac{-3}{2}\times\frac{(4-2)^{2/3}}{\sqrt{2+2}}=\frac{-3}{2}\times\frac{2^{2/3}}{2}$$

$$=-3\cdot 2^{\frac{2}{3}-2}=-3\cdot 2^{-4/3}$$

397. If $\lim_{x\to 0}\frac{\sin(\sin x)-\sin x}{ax^3+bx^5+c}=\frac{-1}{12}$, then

(a) $a=2, b\in R, c=0$ (b) $a=-2, b\in R, c=0$
(c) $a=1, b\in R, c=0$ (d) $a=-1, b\in R, c=0$

Manipal UGET-2020

Ans. (b) : Given that, $\lim_{x\to 0}\frac{\sin(\sin x)-\sin x}{ax^3+bx^5+c}=\frac{-1}{12}$

$$\frac{\sin(\sin 0)-\sin 0}{a(0)^3+b(0)^5+c}=\frac{-1}{12}$$

$$\frac{0}{c}=-\frac{1}{12}$$

c = 0

Now, $\lim_{x\to 0}\frac{\sin x(\sin x)-\sin x}{ax^3+bx^5+c}=-\frac{1}{12}\quad\left(\frac{0}{0}\text{form}\right)$

Applying L' Hospital's rule,

$$\lim_{x\to 0}\frac{\cos x(\sin x)\cos x-\cos x}{3ax^2+5bx^4+0}=-\frac{1}{12}$$

$$\lim_{x\to 0}\frac{\cos x(\cos(\sin x)-1)}{x^2(3a+5bx)}=-\frac{1}{12}$$

$$\lim_{x\to 0}\cos x\frac{2\sin^2\left(\frac{\sin x}{2}\right)}{x^2(3a+5bx)}=-\frac{1}{12}$$

$$\frac{\cos 0}{3a+5b(0)}\times\lim_{x\to 0}\frac{2\sin^2\left(\frac{\sin x}{2}\right)}{4\left(\frac{\sin x}{2}\right)^2\times\left(\frac{x}{\sin x}\right)^2}=\frac{-1}{12}$$

$$\frac{1}{3a+0}\times\frac{1}{2}\times\frac{1}{1}=-\frac{1}{12}\qquad\left(\because\lim_{\theta\to 0}\frac{\sin\theta}{\theta}=1\right)$$

$$\frac{1}{6a}=-\frac{1}{12}\Rightarrow a=-12\Rightarrow a=-2$$

Hence, $a=-2$, $b\in R$ and $c=0$

398. If $f(x)=\begin{cases}\frac{\sin[x]}{[x]} & \text{for}[x]\neq 0\\ 0 & \text{for}[x]=0\end{cases}$

Where, [x] denotes the greatest integer less than or equal to x, then $\lim_{x\to 0}f(x)$ is equal to

(a) 1 (b) –1 (c) 0 (d) does not exist

Manipal UGET-2020