YOUTH COMPETITION TIMES

VOLUME III

Co-ordinate Geometry Chapterwise Solved Papers

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In the event of any dispute, the judicial area will be Prayagraj.

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Syllabus for JEE (Main) - 2024

Syllabus for JEE Main Paper-1 (B.E./B.Tech.)

MATHEMATICS

UNIT 1: SETS, RELATIONS, AND FUNCTIONS: Sets and their representation: Union, intersection, and complement of sets and their algebraic properties; Power set; Relation, Type of relations, equivalence relations, functions; one-one, into and onto functions, the composition of functions.

UNIT 2: COMPLEX NUMBERS AND QUADRATIC EQUATIONS: Complex numbers as ordered pairs of reals, Representation of complex numbers in the form $a + ib$ and their representation in a plane, Argand diagram, algebra of complex number, modulus, and argument (or amplitude) of a complex number, Quadratic equations in real and complex number system and their solutions Relations between roots and co-efficient, nature of roots, the formation of quadratic equation with given roots.

UNIT 3: MATRICES AND DETERMINATS: Matrices, algebra of matrices, type of matrices, determinants, and matrices of order two and three, evaluation of determinants, area of triangles using determinants, Adjoint, and evaluation of inverse of a square matrix using determinants and, Test of consistency and solution of simultaneous linear equations in two or three variables using matrices.

UNIT 4: PERMUTATIONS AND COMBINATIONS: The fundamental principle of counting, permutation as an arrangement and combination as section, Meaning of $P(n, r)$ and $C(n, r)$, simple applications.

UNIT 5: BINOMIAL THEOREM AND ITS SIMPLE APPLICATIONS: Binomial theorem for a positive integral index, general term and middle term, and simple applications.

UNIT 6: SEQUENCE AND SERIES: Arithmetic and Geometric progressions, insertion of arithmetic, geometric means between two given numbers, Relation between A.M and G.M.

UNIT 7: LIMIT, CONTINUITY, AND DIFFERENTIABILITY: Real-valued functions, algebra of functions, polynomials, rational, trigonometric, logarithmic, and exponential functions, inverse function. Graphs of simple functions. Limits, continuity, and differentiability. Differentiation of the sum, difference, product, and quotient of two functions. Differentiation of trigonometric, inverse trigonometric, logarithmic, exponential, composite, and implicit functions; derivatives of order up to two, Applications of derivatives: Rate of change of quantities, monotonic-Increasing and decreasing functions, Maxima and minima of functions of one variable.

UNIT 8: INTEGRAL CALCULAS: Integral as an anti-derivative, Fundamental integral involving algebraic, trigonometric, exponential, and logarithmic functions. Integrations by substitution, by parts, and by partial functions. Integrations by substitution, by parts, and by partial functions. Integration using trigonometric identities.

Evaluation of simple integrals of the type

$$\int \frac{dx}{x^2+a^2}, \int \frac{dx}{\sqrt{x^2 \pm a^2}}, \int \frac{dx}{a^2-x^2}, \int \frac{dx}{\sqrt{a^2-x^2}}, \int \frac{dx}{ax^2+bx+c}, \int \frac{dx}{\sqrt{ax^2+bx+c}}, \int \frac{(px+q)dx}{ax^2+bx+c}, \int \frac{(px+q)dx}{\sqrt{ax^2+bx+c}},$$

$$\int \sqrt{a^2 \pm x^2}\,dx, \int \sqrt{x^2-a^2}\,dx$$

The fundamental theorem of calculus, properties of definite integrals. Evaluation of definite integrals, determining areas of the regions bounded by simple curves in standard form.

UNIT 9 : DIFFERENTIAL EQUATION : Ordinary differential equations, their order, and degree, the solution of differential equation by the method of separation of variables, solution of a homegeneous and linear differential equation of the type

$$\frac{dy}{dx}+p(x)y=q(x)$$

UNIT 10 : CO-ORDINATE GEOMETRY : Cartesian system of rectangular coordinates in a plane, distance formula, sections formula, locus, and its equation, the slope of a line, parallel and perpendicular lines, intercepts of a line on the co-ordinate axis.

Straight line : Various forms of equations of a line, intersection of lines, angles between two lines, conditions for concurrence of three lines, the distance of a point form a line, co-ordinate of the centroid orthocentre, and circumcentre of a triangle.

Circle, conic sections : A standard form of equations of a circle, the general form of the equation of a circle, its radius and central, equation of a circle when the endpoints of a diameter are given, points of intersection of a line and a circle with the centre at the origin and sections of conics, equations of conic sections (parabola, ellipse, and hyperbola) in standard forms.

UNIT 11 : THREE DIMENSIONAL GEOMETRY : Coordinates of a point in space, the distance between two points, section formula, directions ratios, and direction consines, and the angle between two intersecting lines. Skew lines, the shortest distance between them, and its equation. Equations of a line

UNIT 12: VECTOR ALGEBRA: Vectors and scalars, the addition of vectors, components of a vector in two dimensions and three-dimensional space, scalar and vector products.

UNIT 13: STATISTICS AND PROBABILITY: Measures of discretion; calculation of mean, median, mode of grouped and ungrouped data calculation of standard deviation, variance, and mean deviation for grouped and ungrouped data.

Probability: Probability of an event, addition and multiplication theorems of probability, Baye's theorem, probability distribution of a random variate.

UNIT 14: TRIGONOMETRY : Trigonometrical identities and trigonometrical funtions, inverse trigonometrical functions, and their properties.

All India Engineering Entrance Examination & JEE-Main Previous Years Papers Analysis Chart

Sl No	Exam	Proposed Year		Total Question
Joint Entrance Examination (JEE) Main				
1.	NTA JEE Main (April Session)	April 2024	24 Paper	720
2.	NTA JEE Main (January Session)	January 2024	20 Paper	600
3.	NTA JEE Main	15.04.2023	Shift-I	30
4.	NTA JEE Main	13.04.2023	Shift-I	30
5.	NTA JEE Main	13.04.2023	Shift-II	30
6.	NTA JEE Main	12.04.2023	Shift-I	30
7.	NTA JEE Main	11.04.2023	Shift-I	30
8.	NTA JEE Main	11.04.2023	Shift-II	30
9.	NTA JEE Main	10.04.2023	Shift-I	30
10.	NTA JEE Main	10.04.2023	Shift-II	30
11.	NTA JEE Main	08.04.2023	Shift-I	30
12.	NTA JEE Main	08.04.2023	Shift-II	30
13.	NTA JEE Main	06.04.2023	Shift-I	30
14.	NTA JEE Main	06.04.2023	Shift-II	30
15.	NTA JEE Main	01.02.2023	Shift-I	30
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18.	NTA JEE Main	24.01.2023	Shift-II	30
19.	NTA JEE Main	25.01.2023	Shift-I	30
20.	NTA JEE Main	25.01.2023	Shift-II	30
21.	NTA JEE Main	29.01.2023	Shift-I	30
22.	NTA JEE Main	29.01.2023	Shift-II	30
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30.	NTA JEE Main	28.07.2022	Shift-II	30
31.	NTA JEE Main	27.07.2022	Shift-I	30
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34.	NTA JEE Main	26.07.2022	Shift-II	30
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40.	NTA JEE Main	28.06.2022	Shift-II	30
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42.	NTA JEE Main	27.06.2022	Shift-II	30
43.	NTA JEE Main	26.06.2022	Shift-I	30
44.	NTA JEE Main	26.06.2022	Shift-II	30
45.	NTA JEE Main	25.06.2022	Shift-I	30
46.	NTA JEE Main	25.06.2022	Shift-II	30
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57.	NTA JEE Main	27.07.2021	Shift-I	30
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74.	NTA JEE Main	25.02.2021	Shift-II	30
75.	NTA JEE Main	24.02.2021	Shift-I	30
76.	NTA JEE Main	24.02.2021	Shift-II	30
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78.	NTA JEE Main	06.09.2020	Shift-II	30
79.	NTA JEE Main	05.09.2020	Shift-I	30
80.	NTA JEE Main	05.09.2020	Shift-II	30
81.	NTA JEE Main	04.09.2020	Shift-I	25
82.	NTA JEE Main	04.09.2020	Shift-II	25
83.	NTA JEE Main	03.09.2020	Shift-I	30
84.	NTA JEE Main	03.09.2020	Shift-II	30
85.	NTA JEE Main	02.09.2020	Shift-I	25
86.	NTA JEE Main	02.09.2020	Shift-II	25
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88.	NTA JEE Main	09.01.2020	Shift-II	30
89.	NTA JEE Main	08.01.2020	Shift-I	30
90.	NTA JEE Main	08.01.2020	Shift-II	30
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92.	NTA JEE Main	07.01.2020	Shift-II	30
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108.	NTA JEE Main	09.01.2019	Shift-II	30
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114.	JEE Main	08.04.2017		30
115.	JEE Main	02.04.2017		30
116.	JEE Main	2016		30
117.	JEE Main	2015		30
118.	JEE Main	2014		30
119.	JEE Main	2013		30
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121.	AIEEE	2011		30
122.	AIEEE	2010		30
123.	AIEEE	2009		30
124.	AIEEE	2008		30
	AIEEE	2007		30
125.	AIEEE	2006		30
126.	AIEEE	2005		30
127.	AIEEE	2004		30
128.	AIEEE	2003		30
129.	AIEEE	2002		30
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130.	ASSAM-CEE	2023		40
131.	ASSAM-CEE	2022		40
132.	ASSAM-CEE	2021		40
133.	ASSAM-CEE	2020		40
134.	ASSAM-CEE	2019		40
135.	ASSAM-CEE	2018		40
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137.	A.P. EAPCET	15.05.2023	Shift-II	80
138.	A.P. EAPCET	16.05.2023	Shift-I	80
139.	A.P. EAPCET	16.05.2023	Shift-II	80
140.	A.P. EAPCET	17.05.2023	Shift-I	80
141.	A.P. EAPCET	17.05.2023	Shift-II	80
142.	A.P. EAPCET	18.05.2023	Shift-I	80
143.	A.P. EAPCET	18.05.2023	Shift-II	80
144.	A.P. EAPCET	19.05.2023	Shift-I	80
145.	A.P. EAMCET	04.07.2022	Shift-I	80
146.	A.P. EAMCET	04.07.2022	Shift-II	80
147.	A.P. EAMCET	05.07.2022	Shift-I	80
148.	A.P. EAMCET	05.07.2022	Shift-II	80
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151.	A.P. EAMCET	07.07.2022	Shift-I	80
152.	A.P. EAMCET	07.07.2022	Shift-II	80
153.	A.P. EAMCET	08.07.2022	Shift-I	80
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157.	A.P. EAMCET	23.08.2021	Shift-II	80
158.	A.P. EAMCET	19.08.2021	Shift-II	80
159.	A.P. EAMCET	20.08.2021	Shift-I	80
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161.	A.P. EAMCET	19.08.2021	Shift-I	80
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200.	A.P. EAMCET	2008		80
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206.	A.P. EAMCET	2002		80
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208.	A.P. EAMCET	2000		80
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210.	A.P. EAMCET	1998		80

211.	A.P. EAMCET	1997		80
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229.	AMU	2011		50
230.	AMU	2010		70
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232.	AMU	2008		70
233.	AMU	2007		70
234.	AMU	2006		70
235.	AMU	2005		70
236.	AMU	2004		70
237.	AMU	2003		70
238.	AMU	2002		100
239.	AMU	2001		100
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240.	BCECE	2018		50
241.	BCECE	2017		50
242.	BCECE	2016		50
243.	BCECE	2015		50
244.	BCECE	2014		50
245.	BCECE	2013		50
246.	BCECE	2012		50
247.	BCECE	2011		50
248.	BCECE	2010		50
249.	BCECE	2009		50
250.	BCECE	2008		50
251.	BCECE	2007		50
252.	BCECE	2006		50
253.	BCECE	2005		50
254.	BCECE	2004		50
255.	BCECE	2003		50
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256.	BITSAT	2023		40
257.	BITSAT	2022		40
258.	BITSAT	2021		40
259.	BITSAT	2019		40
260.	BITSAT	2018		40
261.	BITSAT	2017		40
262.	BITSAT	2016		40

263.	BITSAT	2015		40
264.	BITSAT	2014		40
265.	BITSAT	2013		40
266.	BITSAT	2012		40
267.	BITSAT	2011		40
268.	BITSAT	2010		40
269.	BITSAT	2009		40
270.	BITSAT	2008		40
271.	BITSAT	2007		40
272.	BITSAT	2006		40
273.	BITSAT	2005		40
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275.	Chhattisgarh-PET	2022		100
276.	Chhattisgarh-PET	2021		100
277.	Chhattisgarh-PET	2020		100
278.	Chhattisgarh-PET	2019		100
279.	Chhattisgarh-PET	2018		100
280.	Chhattisgarh-PET	2017		100
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285.	Chhattisgarh-PET	2012		100
286.	Chhattisgarh-PET	2011		100
287.	Chhattisgarh-PET	2010		100
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290.	Chhattisgarh-PET	2007		100
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297.	COMEDK-JEE	2020		60
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304.	COMEDK-JEE	2013		60
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330.	J & K-CET	2013		75
331.	J & K-CET	2012		75
332.	J & K-CET	2011		75
333.	J & K-CET	2010		75
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335.	J & K-CET	2008		75
336.	J & K-CET	2007		75
337.	J & K-CET	2006		75
338.	J & K-CET	2005		75
339.	J & K-CET	2004		75
340.	J & K-CET	2003		75
Jharkhand (JCECE)				
341.	JCECE	2019		50
342.	JCECE	2018		50
343.	JCECE	2017		50
344.	JCECE	2016		50
345.	JCECE	2015		50
346.	JCECE	2014		50
347.	JCECE	2013		50
348.	JCECE	2012		50
349.	JCECE	2011		50
350.	JCECE	2010		50
351.	JCECE	2009		50
352.	JCECE	2008		50
353.	JCECE	2007		50
354.	JCECE	2006		50
355.	JCECE	2005		50
356.	JCECE	2004		50
357.	JCECE	2003		50
358.	JCECE	2002		50
359.	JCECE	2001		50
Jamia Millia Islamia				
360.	Jamia Millia Islamia	2015		60
361.	Jamia Millia Islamia	2014		60
362.	Jamia Millia Islamia	2013		60
363.	Jamia Millia Islamia	2012		60
364.	Jamia Millia Islamia	2011		60
365.	Jamia Millia Islamia	2010		60

366.	Jamia Millia Islamia	2009		60
367.	Jamia Millia Islamia	2008		60
368.	Jamia Millia Islamia	2007		60
369.	Jamia Millia Islamia	2006		60
370.	Jamia Millia Islamia	2005		60
371.	Jamia Millia Islamia	2004		60
Kerala-KEAM				
372.	Kerala KEAM	2023		60
373.	Kerala KEAM	2022		60
374.	Kerala KEAM	2021		60
375.	Kerala KEAM	2020		60
376.	Kerala KEAM	2019		60
377.	Kerala KEAM	2018		60
378.	Kerala KEAM	2017		60
379.	Kerala KEAM	2016		60
380.	Kerala KEAM	2015		60
381.	Kerala KEAM	2014		60
382.	Kerala KEAM	2013		60
383.	Kerala KEAM	2012		60
384.	Kerala KEAM	2011		60
385.	Kerala KEAM	2010		60
386.	Kerala KEAM	2009		60
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388.	Kerala KEAM	2007		60
389.	Kerala KEAM	2006		60
390.	Kerala KEAM	2005		60
391.	Kerala KEAM	2004		60
Karnataka-CET (KCET)				
392.	Karnataka-CET	2023		60
393.	Karnataka-CET	2022		60
394.	Karnataka-CET	2021		60
395.	Karnataka-CET	2020		60
396.	Karnataka-CET	2019		60
397.	Karnataka-CET	2018		60
398.	Karnataka-CET	2017		60
399.	Karnataka-CET	2016		60
400.	Karnataka-CET	2015		60
401.	Karnataka-CET	2014		60
402.	Karnataka-CET	2013		60
403.	Karnataka-CET	2012		60
404.	Karnataka-CET	2011		60
405.	Karnataka-CET	2010		60
406.	Karnataka-CET	2009		60
407.	Karnataka-CET	2008		60
408.	Karnataka-CET	2007		60
409.	Karnataka-CET	2006		60
410.	Karnataka-CET	2005		60
411.	Karnataka-CET	2004		60
412.	Karnataka-CET	2003		60
413.	Karnataka-CET	2002		60
414.	Karnataka-CET	2001		60
415.	Karnataka-CET	2000		60
Kishore Vaigyanik Protsahan Yojana (KVPY)				
416.	KVPY-SB-SX	2023		15
417.	KVPY-SB-SX	2022		15

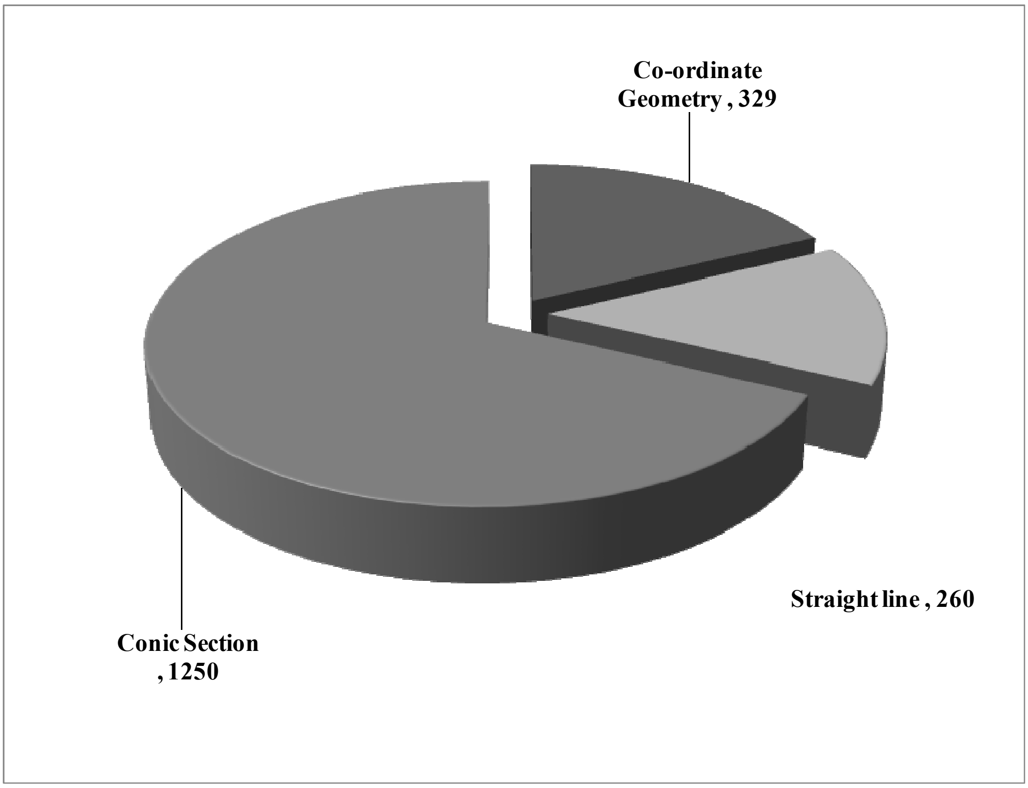
418.	KVPY-SB-SX	2021		15
419.	KVPY-SA	2021		15
420.	KVPY-SA	2020		15
421.	KVPY-SB-SX	2018		15
422.	KVPY-SA	2017		15
423.	KVPY-SB-SX	2016		15
424.	KVPY-SB-SX	2015		15
425.	KVPY-SA	2014		15
426.	KVPY-SB-SX	2013		15
427.	KVPY-SA	2012		15
428.	KVPY-SA	2009		15
429.	KVPY-SB-SX	2009		15
Madhya Pradesh Pre Engineering Test (MPPET)				
430.	MPPET	2013		50
431.	MPPET	2012		50
432.	MPPET	2009		50
433.	MPPET	2008		50
Manipal-UGET				
434.	Manipal	2023		50
435.	Manipal	2022		50
436.	Manipal	2021		50
437.	Manipal	2020		50
438.	Manipal	2019		50
439.	Manipal	2018		50
440.	Manipal	2017		50
441.	Manipal	2016		50
442.	Manipal	2015		50
443.	Manipal	2014		50
444.	Manipal	2013		50
445.	Manipal	2012		50
446.	Manipal	2011		50
447.	Manipal	2010		50
448.	Manipal	2009		50
449.	Manipal	2008		50
(Maharashtra) MHT-CET				
450.	MHT-CET	2022	All Shifts	500
451.	MHT-CET	2021	All Shifts	500
452.	MHT-CET	13.10.2020	Shift-I	100
453.	MHT-CET	13.10.2020	Shift-II	100
454.	MHT-CET	14.10.2020	Shift-I	100
455.	MHT-CET	14.10.2020	Shift-II	100
456.	MHT-CET	15.10.2020	Shift-I	100
457.	MHT-CET	15.10.2020	Shift-II	100
458.	MHT-CET	16.10.2020	Shift-I	100
459.	MHT-CET	16.10.2020	Shift-II	100
460.	MHT-CET	19.10.2020	Shift-I	100
461.	MHT-CET	19.10.2020	Shift-II	100
462.	MHT-CET	20.10.2020	Shift-I	100
463.	MHT-CET	20.10.2020	Shift-II	100
464.	MHT-CET	02.05.2019	Shift-I	100
465.	MHT-CET	02.05.2019	Shift-II	100

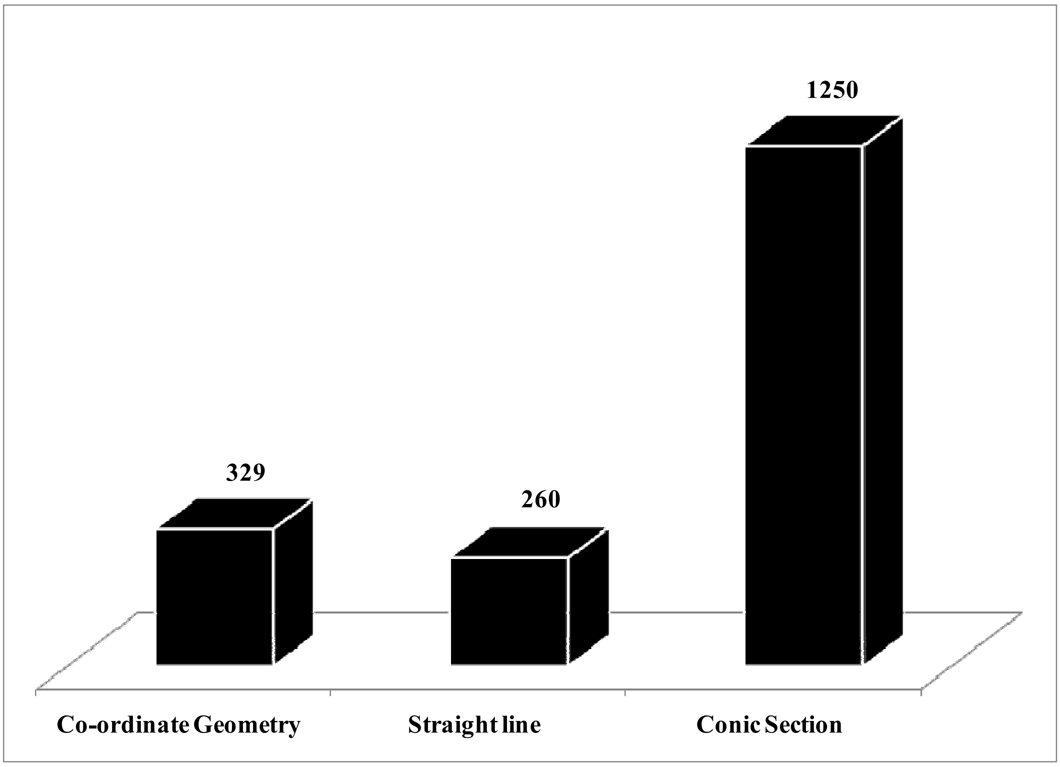
466.	MHT-CET	03.05.2019		100
467.	MHT-CET	2018		100
468.	MHT-CET	2017		100
469.	MHT-CET	2016		100
470.	MHT-CET	2015		100
471.	MHT-CET	2014		100
472.	MHT-CET	2013		100
473.	MHT-CET	2012		100
474.	MHT-CET	2011		100
475.	MHT-CET	2010		100
476.	MHT-CET	2009		100
477.	MHT-CET	2008		100
478.	MHT-CET	2007		100
479.	MHT-CET	2006		100
480.	MHT-CET	2005		100
481.	MHT-CET	2004		100
Rajasthan PET				
482.	Rajasthan PET	2012		40
483.	Rajasthan PET	2011		40
484.	Rajasthan PET	2010		40
485.	Rajasthan PET	2009		40
486.	Rajasthan PET	2008		40
487.	Rajasthan PET	2007		40
488.	Rajasthan PET	2006		40
489.	Rajasthan PET	2005		40
490.	Rajasthan PET	2004		40
491.	Rajasthan PET	2003		40
492.	Rajasthan PET	2002		40
493.	Rajasthan PET	2001		40
SCRA				
494.	SCRA	2015		60
495.	SCRA	2014		60
496.	SCRA	2013		60
497.	SCRA	2012		60
498.	SCRA	2010		60
499.	SCRA	2009		60
SRM-JEEE				
500.	SRM-JEEE	2022		40
501.	SRM-JEEE	2021		40
502.	SRM-JEEE	2020		40
503.	SRM-JEEE	2019		40
504.	SRM-JEEE	2018		40
505.	SRM-JEEE	2016		40
506.	SRM-JEEE	2015		40
507.	SRM-JEEE	2014		40
508.	SRM-JEEE	2013		40
509.	SRM-JEEE	2012		40
510.	SRM-JEEE	2011		40
511.	SRM-JEEE	2010		40
512.	SRM-JEEE	2009		40
513.	SRM-JEEE	2008		40

514.	SRM-JEEE	2007		40
Telangana EAMCET				
515.	TS-EAMCET	12.05.2023	Shift-I	80
516.	TS-EAMCET	12.05.2023	Shift-II	80
517.	TS-EAMCET	13.05.2023	Shift-I	80
518.	TS-EAMCET	13.05.2023	Shift-II	80
519.	TS-EAMCET	14.05.2023	Shift-I	80
520.	TS-EAMCET	14.05.2023	Shift-II	80
521.	TS-EAMCET	18.07.2022	Shift-I	80
522.	TS-EAMCET	18.07.2022	Shift-II	80
523.	TS-EAMCET	19.07.2022	Shift-I	80
524.	TS-EAMCET	19.07.2022	Shift-II	80
525.	TS-EAMCET	20.07.2022	Shift-I	80
526.	TS-EAMCET	20.07.2022	Shift-II	80
527.	TS-EAMCET	06.08.2021	Shift-I	80
528.	TS-EAMCET	06.08.2021	Shift-II	80
529.	TS-EAMCET	05.08.2021	Shift-I	80
530.	TS-EAMCET	05.08.2021	Shift-II	80
531.	TS-EAMCET	04.08.2021	Shift-I	80
532.	TS-EAMCET	04.08.2021	Shift-II	80
533.	TS-EAMCET	09.09.2020	Shift-I	80
534.	TS-EAMCET	09.09.2020	Shift-II	80
535.	TS-EAMCET	10.09.2020	Shift-I	80
536.	TS-EAMCET	10.09.2020	Shift-II	80
537.	TS-EAMCET	11.09.2020	Shift-I	80
538.	TS-EAMCET	11.09.2020	Shift-II	80
539.	TS-EAMCET	14.09.2020	Shift-I	80
540.	TS-EAMCET	14.09.2020	Shift-II	80
541.	TS-EAMCET	03.05.2019	Shift-I	80
542.	TS-EAMCET	03.05.2019	Shift-II	80
543.	TS-EAMCET	04.05.2019	Shift-I	80
544.	TS-EAMCET	04.05.2019	Shift-II	80
545.	TS-EAMCET	06.05.2019	Shift-I	80
546.	TS-EAMCET	05.05.2018	Shift-I	80
547.	TS-EAMCET	05.05.2018	Shift-II	80
548.	TS-EAMCET	02.05.2018	Shift-I	80
549.	TS-EAMCET	04.05.2018	Shift-II	80
550.	TS-EAMCET	07.05.2018	Shift-I	80
551.	TS-EAMCET	24.04.2017	Shift-I	80
552.	TS-EAMCET	2016		80
553.	TS-EAMCET	2015		80
554.	TS-EAMCET	2014		80
Tripura JEE				
555.	Tripura JEE	2023		50
556.	Tripura JEE	2022		50
557.	Tripura JEE	2021		50
558.	Tripura JEE	2019		50
(Uttar Pradesh) UPTU/UPSEE				
559.	UPTU/UPSEE	2020		50
560.	UPTU/UPSEE	2019		50
561.	UPTU/UPSEE	2018		50

562.	UPTU/UPSEE	2017		50
563.	UPTU/UPSEE	2016		50
564.	UPTU/UPSEE	2015		50
565.	UPTU/UPSEE	2014		50
566.	UPTU/UPSEE	2013		50
567.	UPTU/UPSEE	2012		50
568.	UPTU/UPSEE	2011		50
569.	UPTU/UPSEE	2010		50
570.	UPTU/UPSEE	2009		50
571.	UPTU/UPSEE	2008		50
572.	UPTU/UPSEE	2007		50
573.	UPTU/UPSEE	2006		50
574.	UPTU/UPSEE	2005		50
575.	UPTU/UPSEE	2004		50
VITEEE				
576.	VITEEE	2023		40
577.	VITEEE	2022		40
578.	VITEEE	2021		40
579.	VITEEE	2020		40
580.	VITEEE	2019		40
581.	VITEEE	2018		40
582.	VITEEE	2017		40
583.	VITEEE	2016		40
584.	VITEEE	2015		40
585.	VITEEE	2014		40
586.	VITEEE	2013		40
587.	VITEEE	2012		40
588.	VITEEE	2011		40
589.	VITEEE	2010		40
590.	VITEEE	2009		40
591.	VITEEE	2008		40
592.	VITEEE	2007		40
593.	VITEEE	2006		40
WEST BENGAL				
594.	West Bengal	2023		30
595.	West Bengal	2022		30
596.	West Bengal	2021		30
597.	West Bengal	2020		30
598.	West Bengal	2019		30
599.	West Bengal	2018		30
600.	West Bengal	2017		30
601.	West Bengal	2016		30
602.	West Bengal	2015		30
603.	West Bengal	2014		30
604.	West Bengal	2013		30
605.	West Bengal	2012		30
606.	West Bengal	2011		30
607.	West Bengal	2010		30
608.	West Bengal	2009		30
609.	West Bengal	2008		30
	Total			**36020**

Trend Analysis of previous year paper of IIT JEE Mathematics through Bar graph and Pie chart.





01. Co-Ordinate Geometry

A. Co-Ordinate system

1. Transformation of Axis

1. The vertices of a triangle are A(–1, 3), B(–2, 2) and C(3, –1). A new triangle is formed by shifting the sides of the triangle by one unit inwards. Then the equation of the side of the new triangle nearest to origin is:

(a) $x - y - \left(2+\sqrt{2}\right) = 0$

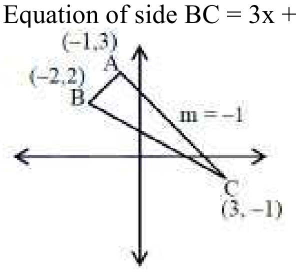
(b) $-x + y - \left(2-\sqrt{2}\right) = 0$

(c) $x + y - \left(2-\sqrt{2}\right) = 0$

(d) $x + y + \left(2-\sqrt{2}\right) = 0$

JEE MAIN-04.04.2024, Shift-I

Ans. (c) : Equation of side AB = y – 2 = 1(x+2)
$x - y + 4 = 0$
Equation of side BC = $3x + 5y - 4 = 0$



equation of AC $\rightarrow x + y = 2$
ABC be the new triangle formed
equation of line parallel to AC $x + y = d$

$$\left|\frac{d-2}{\sqrt{2}}\right| = 1$$

$d = 2-\sqrt{2}$
eq^n of new required line

$$\boxed{x + y = 2-\sqrt{2}}$$

2. A variable line L passes through the point (3, 5) and intersects the positive coordinate axes at the points A and B. The minimum area of the triangle OAB, where O is the origin, is :

(a) 30 (b) 25
(c) 40 (d) 35

JEE MAIN-09.04.2024, Shift-I

Ans. (a) : Equation of line L in intercepted form-

$$\frac{x}{a} + \frac{y}{b} = 1$$

Line is passing through (3, 5)-

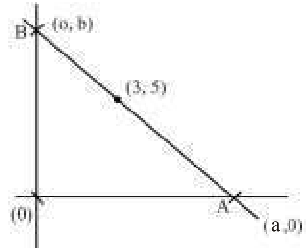
$$\frac{3}{a} + \frac{5}{b} = 1$$

$$\frac{\frac{3}{a} + \frac{5}{6}}{2} \geq \sqrt{\frac{15}{ab}}$$

$$\frac{15}{2ab} \geq \sqrt{\frac{15}{ab}}$$

$$\frac{1}{4} \geq \frac{15}{ab}$$

$ab \geq 60$



$$A_{min} = \frac{1}{2}ab$$

$$= \frac{1}{2} \times 60$$

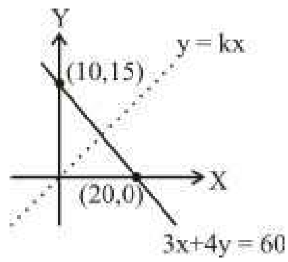
$= 30$

3. A triangle is formed by X-axis, Y-axis and the line 3x + 4y = 60. Then the number of points P(a,b) which lie strictly inside the triangle, where a is an integer and b is a multiple of a , is

JEE MAIN-25.01.2023, Shift-II

Ans. (31) : As b is multiple of a
The required point lie on the
Line $y = kx$ ($k \in z$)
$\therefore 3x + 4kx = 60$

$$x = \frac{60}{3+4k}$$



if

K=1	,	8 internal Points
K=2	,	5 internal Points
K=3	,	3 internal Points
K=4	,	3 internal Points
K=5	,	2 internal Points
K=6	,	2 internal Points
K=7	,	1 internal Points
K=8	,	1 internal Points
:		:
K=14,		1 internal Points

$\therefore$ Total = 31 Points

4. **If P_1 and P_2 be the length of perpendiculars from the origin upon the straight lines $x\sec\theta + y\,\mathrm{cosec}\theta = a$ and $x\cos\theta - y\sin\theta = a\cos 2\theta$ respectively, then the value of $4P_1^{\,2} + P_2^{\,2}$.**

(a) a^2 (b) $2a^2$ (c) $a^2/2$ (d) $3a^2$

BITSAT-2014

Ans. (a): We have P_1 = length of perpendicular from (0, 0) on $x\ \sec\theta + y\ \mathrm{cosec}\theta = a$

i.e. $P_1 = \left|\dfrac{a}{\sqrt{\sec^2\theta + \mathrm{cosec}^2\theta}}\right| = |a\sin\theta\cos\theta|$

$$= \left|\frac{a}{2}\sin 2\theta\right|$$

$\therefore \quad 2P_1 = |a\sin 2\theta|$

P_2 = Length of the perpendicular from (0, 0) on $x\cos\theta - y\sin\theta = a\cos 2\theta$

$P_2 = \left|\dfrac{a\cos 2\theta}{\sqrt{\cos^2\theta + \sin^2\theta}}\right| = |a\cos 2\theta|$

Now, $4P_1^{\,2} + P_2^{\,2} = a^2\sin^2 2\theta + a^2\cos^2 2\theta$

$4P_1^{\,2} + P_2^{\,2} = a^2\left(\sin^2 2\theta + \cos^2 2\theta\right)$

$\therefore \quad 4P_1^2 + P_2^2 = a^2$

5. **The angle between the lines whose intercepts on the axis are a, –b and b, –a respectively, is**

(a) $\tan^{-1}\dfrac{a^2 - b^2}{ab}$ (b) $\tan^{-1}\dfrac{b^2 - a^2}{2}$

(c) $\tan^{-1}\dfrac{b^2 - a^2}{2ab}$ (d) None of these

BITSAT-2017

Ans. (c) : From the given data

Equation of lines are $\dfrac{x}{a} - \dfrac{y}{b} = 1$ and $\dfrac{x}{b} - \dfrac{y}{a} = 1$

$m_1 = \dfrac{b}{a}$ and $m_2 = \dfrac{a}{b}$

$\because \tan\theta = \dfrac{m_1 - m_2}{1 + m_1 m_2}$

$$\therefore\ \theta = \tan^{-1}\frac{\frac{b}{a} - \frac{a}{b}}{1 + \frac{b}{a}\cdot\frac{a}{b}} = \frac{\frac{b^2 - a^2}{ab}}{2}$$

$$\theta = \tan^{-1}\frac{b^2 - a^2}{2ab}$$

6. **A ray of light coming from the point (1, 2) is reflected at a point A on the x-axis and then passes through the point (5, 3). The co-ordinates of the point A is**

(a) $\left(\dfrac{13}{5}, 0\right)$ (b) $\left(\dfrac{5}{13}, 0\right)$

(c) (–7, 0) (d) None of these

BITSAT-2016

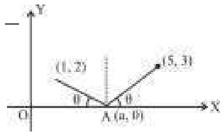
Ans. (a) : Let the co-ordinates of A be (a, 0). Then the slope of the reflected ray is

$\dfrac{3 - 0}{5 - a} = \tan\theta$ (say) ....(i)

Then the slope of the incident ray

$\dfrac{2 - 0}{1 - a} = \tan(\pi - \theta)$ .....(ii)

From equation (i) and (ii), we get –



$\tan\theta + \tan(\pi - \theta) = 0$

$$\frac{3 - 0}{5 - a} + \frac{2 - 0}{1 - a} = 0$$

$$\frac{3}{5 - a} + \frac{2}{1 - a} = 0$$

$3 - 3a + 10 - 2a = 0$

$13 - 5a = 0$

$a = \dfrac{13}{5}$

Thus, the co-ordinates of A are $\left(\dfrac{13}{5}, 0\right)$.

7. **The equation of the line with gradient –3/2, which is concurrent with the lines $4x + 3y - 7 = 0$ and $8x + 5y - 1 = 0$, is**

(a) $3x + 2y - 2 = 0$ (b) $3x + 2y - 63 = 0$

(c) $2y - 3x - 2 = 0$ (d) $2y - 3x - 63 = 0$

BITSAT-2016

Ans. (a) : The equation of a line concurrent with the lines

$4x + 3y - 7 = 0$ and $8x + 5y - 1 = 0$ is

$(4x + 3y - 7) + \lambda(8x + 5y - 1) = 0$

$(4 + 8\lambda)x + (3 + 5\lambda)y - 7 - \lambda = 0$ …..(i)

The gradient of this line is $-\dfrac{3}{2}$, therefore

$-\dfrac{8\lambda + 4}{5\lambda + 3} = \dfrac{3}{2}$

$\lambda = 1$

So, from equation (i), the required line is

$12x + 8y - 8 = 0$

$3x + 2y - 2 = 0$

8. **Equation of line passing through the point (1, 2) and perpendicular to the line y = 3x – 1 is**

(a) x – 3y = 0 (b) x + 3y = 0

(c) x + 3y –7 = 0 (d) x + 3y + 7 = 0

Karnataka CET-2017

Ans. (c) : Given,

The equation of line y = 3x – 1 .....(i)

We know,

General equation of line is y = mx + c

Now, Comparing equation (i) with general equation

Slope (m) = 3

If two lines are perpendicular,

Then, $m_1 m_2 = -1$

therefore, slope of line perpendicular to y = 3x – 1

$3 \times m_2 = -1$

$m_2 = -1/3$

Now, equation of line $y = -\frac{1}{3}x + c$

$3y + x = 3c$ .....(ii)

The line $3y + x = 3c$ passes through the point (1, 2)

$\therefore$ $3 \times 2 + 1 = 3c$

$c = 7/3$

Now, Equation of line becomes

$x + 3y = 3c$

$x + 3y = 3\times\frac{7}{3}$

$x + 3y - 7 = 0$

9. If the straight lines 2x + 3y – 3 = 0 and x + ky + 7 = 0 are perpendicular, then the value of k is

(a) 2/3 (b) 3/2 (c) –2/3 (d) –3/2

Karnataka CET-2016

Ans. (c) : Given,

Equation of straight lines are

$2x + 3y - 3 = 0$ .....(i)

$x + ky + 7 = 0$ ..... (ii)

Equation (i) and (ii) can be written as shown below

$y = -\frac{2}{3}x + 1$

$\therefore$ Slope $(m_1) = -\frac{2}{3}$

And, $y = -\frac{1}{k}x - \frac{7}{k}$

Slope $(m_2) = -\frac{1}{k}$

We know that both the straight line are perpendicular to each other.

$\therefore$ $m_1m_2 = -1$

$-\frac{2}{3}\times\left(-\frac{1}{k}\right) = -1, \quad k = -\frac{2}{3}$

10. A straight line passes through the points (5, 0) and (0, 3). The length of perpendicular from the point (4, 4) on the line is

(a) $\frac{15}{\sqrt{34}}$ (b) $\frac{\sqrt{17}}{2}$ (c) $\frac{17}{2}$ (d) $\sqrt{\frac{17}{2}}$

Karnataka CET-2014

Ans. (d) : We know,

The equation Straight line passes through point (5, 0) and (0, 3)

$(y - y_1) = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

$(y - 0) = \frac{3 - 0}{0 - 5}(x - 5)$

$y = -\frac{3}{5}(x - 5)$

$5y = -3x + 15$

$3x + 5y - 15 = 0$

Therefore, the distance of this line from the point (4, 4) is

$d = \frac{|3\times4+5\times4-15|}{\sqrt{3^2+5^2}} = \frac{17}{\sqrt{34}}$

$d = \frac{\sqrt{17}\times\sqrt{17}}{\sqrt{17}\times\sqrt{2}} = \sqrt{\frac{17}{2}}$

11. Find the transformed equation of the straight line xy – x – y + 1 = 0, when the origin is shifted to the point (1, 1) after translation of axis.

(a) xy = 5 (b) xy = 2

(c) xy = 0 (d) xy = 8

COMEDK-2017

Ans. (c) : Let the coordinates of a point p changes from (x,y)to (x',y')in new coordinates axis where origin has the coordinates h=1, k=1

Then, x=x'+1, y=y'+1.

Substituting these values in the given equation of straight line

$(x'+1)(y'+1)-(x'+1)-(y'+1)+1=0$

$x'y'+x'+y'+1-x'-1-y'-1+1=0$

$x'y'=0$

Therefore, the equation of straight line in the new system is xy = 0.

12. If the axis are shifted to the point (1, –2) without solution, then the equation $2x^2 + y^2 - 4x + 4y = 0$ becomes

(a) $2X^2 + 3Y^2 = 6$ (b) $2X^2 + Y^2 = 6$

(c) $X^2 + 2Y^2 = 6$ (d) None of these

VITEEE-2011

Ans. (b) : Given the point (1,–2)

The equation is $2x^2 + y^2 - 4x + 4y = 0$

Now, substituting x = X + 1 and y = Y – 2 in given equation, we get

$2(X+1)^2 + (Y-2)^2 - 4(X+1) + 4(Y-2) = 0$

$2X^2 + Y^2 = 6$

13 If the axis are rotated through an angle of 30° in the clockwise direction, the point $(4, 2\sqrt{3})$ in the new system is

(a) (2,3) (b) $(2,\sqrt{3})$ (c) $(\sqrt{3},2)$ (d) $(\sqrt{3},5)$

JCECE-2015

Ans. (d) : Given,

The coordinate of (x, y) is $(4, 2\sqrt{3})$

$x = 4, \ y = 2\sqrt{3}$

$\theta = -30°$ [Clock wise]

We know, $X = x\cos\theta + y\sin\theta$ and,

$Y = x\sin\theta + y\cos\theta$

$\therefore$ $X = 4\cos30° - 2\sqrt{3}\sin30°$

$= 4\times\frac{\sqrt{3}}{2} - 2\sqrt{3}\times\frac{1}{2} = \sqrt{3}$

$Y = 4\sin30° + 2\sqrt{3}\cos30°$

$= 4\times\frac{1}{2} + 2\sqrt{3}\times\frac{\sqrt{3}}{2} = 2 + 3 = 5$

The given Point in the new system $(\sqrt{3}, 5)$

14. **If the axis are rotated through an angle 45° in the positive direction without changing the origin, then the co-ordinates of the point $(\sqrt{2}, 4)$ in the old system are**

(a) $(1-2\sqrt{2}, 1+2\sqrt{2})$ (b) $(1+2\sqrt{2}, 1-2\sqrt{2})$
(c) $(2\sqrt{2}, \sqrt{2})$ (d) $(\sqrt{2}, 2)$

AP EAMCET-2002

Ans. (a) : Given,
The axis are rotated through an angle = 45°
The co-ordinate of the point = $(\sqrt{2}, 4)$
We know that,
$x' = x \cos\theta - y \sin\theta$
$y' = x \sin\theta + y \cos\theta$
Now,
$x' = x \cos 45° - y \sin 45°$
$x' = \frac{x}{\sqrt{2}} - \frac{y}{\sqrt{2}} = \frac{\sqrt{2}}{\sqrt{2}} - \frac{4}{\sqrt{2}} = 1 - 2\sqrt{2}$
$y' = x \sin\theta + y \cos\theta$
$y' = \sqrt{2} \sin 45° + 4 \cos 45°$
$y' = \frac{\sqrt{2}}{\sqrt{2}} + \frac{4}{\sqrt{2}}$
$y' = 1 + 2\sqrt{2}$
$\therefore$ The co-ordinate is $(x', y') = (1-2\sqrt{2}, 1+2\sqrt{2})$

15. **If the axis are transformed to the point (–1, 1) then the equation $3x^2 + y^2 + 2x + 4y + 15 = 0$ would transform to_______**

(a) $3x^2 + 2y^2 - 4x + 6y + 23 = 0$
(b) $3x^2 + y^2 - 4x + 6y + 21 = 0$
(c) $3x^2 + y^2 + 4x - 6y - 21 = 0$
(d) $3x^2 + y^2 + 4x + 6y + 21 = 0$

APEAPCET-20.08.2021, Shift-I

Ans. (b): Given,
The axis transformed to the point (–1, 1)
Initial equation $3x^2 + y^2 + 2x + 4y + 15 = 0$
We know that,
The co-ordinate of transformed point
$x = x + h$
$y = y + k$
$\therefore$ $x = x - 1$
$y = y + 1$
Now, The equation transform to
$3(x-1)^2 + (y+1)^2 + 2(x-1) + 4(y+1) + 15 = 0$
$3(x^2 + 1 - 2x) + y^2 + 1 + 2y + 2x - 2 + 4y + 4 + 15 = 0$
$3x^2 + 3 - 6x + y^2 + 1 + 2y + 2x - 2 + 4y + 4 + 15 = 0$
$3x^2 - 4x + y^2 + 6y + 21 = 0$
$3x^2 + y^2 - 4x + 6y + 21 = 0$

16. **When the axes are rotated through an angle 45°. the new coordinates of a point P are (1,–1). The coordinates of P in the original system are______**

(a) $(\sqrt{2}, \sqrt{2})$ (b) $(\sqrt{2}, 0)$ (c) $(0, \sqrt{2})$ (d) $(-\sqrt{2}, 0)$

AP EAPCET-25.08.2021, Shift-II

Ans. (b) : Given,
The angle of rotation of axis = 45°
The new co-ordinates of a point P = (1, –1)
We know that,
$x = x \cos\theta - y \sin\theta$
$y = x \sin\theta + y \cos\theta$
Now, $x = 1 \times \cos 45° - (-1)\sin 45° = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \sqrt{2}$
$y = 1 \times \sin 45° + (-1) \cos 45° = \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} = 0$
$\therefore$ Original co-ordinate = $(\sqrt{2}, 0)$

17. **Find the transformed equation of $x \cos\theta + y \sin\theta = p$, when the axes are rotated through an angle θ.**

(a) $x = p$ (b) $y = p$
(c) $x + y = p$ (d) $x - y = p$

AP EAMCET-17.09.2020, Shift-I

Ans. (a) : Given,
The transformed equation is,
$x \cos\theta + y \sin\theta = p$ ....(i)
The axis of rotation = θ
We know that,
$x = x' \cos\theta - y' \sin\theta$
$y = x' \sin\theta + y' \cos\theta$
Now, in equation (i) –
$x \cos\theta + y \sin\theta = p$
$(x' \cos\theta - y' \sin\theta)\cos\theta + (x' \sin\theta + y' \cos\theta)\sin\theta = p$
$x' \cos^2\theta - y' \sin\theta\cos\theta + x' \sin^2\theta + y' \sin\theta\cos\theta = p$
$x'[\cos^2\theta + \sin^2\theta] = p$
$x' = p$
$x = p$

18. **When the origin is shifted to (2, 3) the transformed equation $x^2 + 3xy - 2y^2 + 17x - 7y - 11 = 0$, then the original equation of curve is ....**

(a) $x^2 - 2y^2 - 3xy + 4x - y + 20 = 0$
(b) $x^2 - 2y^2 + 3xy + 4x - y - 20 = 0$
(c) $x^2 - 2y^2 - 3xy - 4x - y + 20 = 0$
(d) $x^2 - 2y^2 - 3xy + 4x - y - 20 = 0$

AP EAMCET-18.09.2020, Shift-II

Ans. (b) : Given,
The coordinate of shifted point is (2, 3)
The equation is $x^2+3xy - 2y^2 + 17x - 7y - 11 = 0$ ...(i)
To get the original equation of curve, replace (x, y) by (x – 2, y – 3)
Now, Putting the value of x and y in equation (i)
$(x-2)^2+3(x-2)(y-3) - 2(y-3)^2 + 17(x-2) - 7(y-3) - 11 = 0$
$x^2 + 4 - 4x + 3(xy - 3x - 2y + 6) - 2(y^2 + 9 - 6y) + 17x - 34 - 7y + 21 - 11 = 0$
$x^2 + 4 - 4x + 3xy - 9x - 6y + 18 - 2y^2 - 18 + 12y + 17x - 34 - 7y + 21 - 11 = 0$
$x^2 + 3xy - 2y^2 + 4x - y - 20 = 0$
$x^2 - 2y^2 + 3xy + 4x - y - 20 = 0$

19. The polar equation of the line perpendicular to the line sinθ– cosθ = $\frac{1}{r}$ and passing through the point $\left(2, \frac{\pi}{6}\right)$ is

(a) $\sin\theta + \cos\theta = \frac{\sqrt{3}+1}{r}$

(b) $\sin\theta - \cos\theta = \frac{\sqrt{3}+1}{r}$

(c) $\sin\theta + \cos\theta = \frac{\sqrt{3}-1}{r}$

(d) $\cos\theta - \sin\theta = \frac{\sqrt{3}}{r}$

AP EAMCET-2011

Ans. (a) : Given,

The polar equation is, $\sin\theta - \cos\theta = \frac{1}{r}$

$\therefore$ r sinθ – r cosθ = 1 ....(i)

The perpendicular equation passing through the point $(2, \pi/6)$

We know that,

The conversion of polar coordinate into cartesian coordinate

$x = r\cos\theta$

$y = r\sin\theta$

$\therefore$ From equation (i)

$y - x = 1$ ....(ii)

The slope of equation (ii) is

$m_1 = 1$

The slope of perpendicular equation $m_2 = -1$

Now, The equation of line which passes through point $(2, \pi/6)$

$(y - y_1) = m_2(x - x_1)$

$(r\sin\theta - 2\sin\pi/6) = -1(r\cos\theta - 2\cos\pi/6)$

$\left(r\sin\theta - 2\times\frac{1}{2}\right) = -1\left(r\cos\theta - 2\times\frac{\sqrt{3}}{2}\right)$

$r\sin\theta - 1 = -r\cos\theta + \sqrt{3}$

$r(\sin\theta + \cos\theta) = 1 + \sqrt{3}$

$\sin\theta + \cos\theta = \frac{1+\sqrt{3}}{r}$

20. The origin is translated to (1, 2). The point (7, 5) in the old system undergoes the following transformations successively.

I. Moves to the new point under the given translation of origin.

II. Translated through 2 units along the negative direction of the new X-axis.

III. Rotated through an angle $\frac{\pi}{4}$ about the origin of new system in the clockwise direction. The final position of the point (7, 5) is

(a) $\left(\frac{9}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right)$ (b) $\left(\frac{7}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

(c) $\left(\frac{7}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right)$ (d) $\left(\frac{5}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right)$

AP EAMCET-2013

Ans. (c) : Given,

The old coordinate of point P = (7, 5)

The coordinate shifted by (1, 2)

We know that,

The new coordinate after translation

P' = (7 – 1, 5 – 2)

P' = (6, 3)

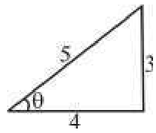
Now, P' translated 2 units in negative direction

P' = (6 – 2, 3)

P'= (4, 3)

Now, Convert it to the polar coordinate

$r = \sqrt{4^2 + 3^2} = 5$



$\because$ $x = r\cos\theta$

$y = r\sin\theta$

$\therefore$ P' = (r cosθ, r sinθ)

$P' = \left[r\cos(\theta - \pi/4), r\sin(\theta - \pi/4)\right]$

$$P' = \begin{bmatrix} 5\left(\cos\theta\cos\frac{\pi}{4} + \sin\theta\sin\frac{\pi}{4}\right), \\ 5\left(\sin\theta\cos\frac{\pi}{4} - \cos\theta\sin\frac{\pi}{4}\right) \end{bmatrix}$$

$$P' = \left[5\left(\frac{4}{5}\times\frac{1}{2} + \frac{3}{5}\times\frac{1}{\sqrt{2}}\right), 5\left(\frac{3}{5}\times\frac{1}{\sqrt{2}} - \frac{4}{5}\times\frac{1}{\sqrt{2}}\right)\right]$$

$$P' = \left(\frac{7}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right)$$

$$P' = \left(\frac{7}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right)$$

21. If the point P(1,3) undergoes the following transformations successively.

(i) Reflection with respect to line y = x

(ii) Translation through 3 units along the positive direction of the X-axis.

(iii) Rotation through and angles of $\frac{\pi}{6}$ about the origin in the clockwise direction.

Then the final position of the point P is

(a) $\left(\frac{6\sqrt{3}+1}{2}, \frac{\sqrt{3}-6}{2}\right)$ (b) $\left(\frac{\sqrt{7}}{2}, \frac{5}{\sqrt{2}}\right)$

(c) $\left(\frac{6+\sqrt{3}}{2}, \frac{1-6\sqrt{3}}{2}\right)$ (d) $\left(\frac{6+\sqrt{3}-1}{2}, \frac{6+\sqrt{3}}{2}\right)$

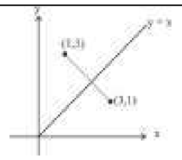
AP EAMCET-2014

Ans. (a) : Given,

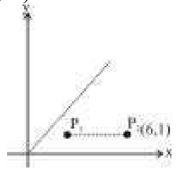
The coordinate of the point P(1, 3)

We know that,

The point reflection, new position $P_1 = (3, 1)$



Now,
The translation of point (3,1) by 3 unit along the positive direction of the x-axis
$P_2 = (3 + 3, 1)$
$P_2 = (6, 1)$



Now, The point P_2 rotate by $\pi/6$ then the new position.

$\therefore \quad r = \sqrt{6^2+1^2} = \sqrt{37}$

$x = r\cos\theta$

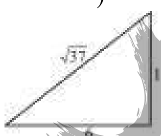
$x = \sqrt{37}\cos(\theta - \pi/6)$

$y = r\sin\theta$

$y = \sqrt{37}\sin(\theta - \pi/6)$

Now, $x = \sqrt{37}(\cos\theta\cos\pi/6 + \sin\theta\sin\pi/6)$

Here, $\tan\theta = 1/6$



$\cos\theta = \dfrac{6}{\sqrt{37}} = \dfrac{6}{r}$

$\sin\theta = \dfrac{1}{\sqrt{37}} = \dfrac{1}{r}$

$\therefore \quad x = \left(r.\dfrac{6}{r}.\dfrac{\sqrt{3}}{2} + r.\dfrac{1}{r}.\dfrac{1}{2}\right)$

$x = 3\sqrt{3} + \dfrac{1}{2} \Rightarrow x = \dfrac{6\sqrt{3}+1}{2}$

$y = \sqrt{37}(\sin\theta\cos\pi/6 - \cos\theta\sin\pi/6)$

$y = \left(r.\dfrac{1}{r}.\dfrac{\sqrt{3}}{2} - r.\dfrac{6}{r}.\dfrac{1}{2}\right)$,

$y = \dfrac{\sqrt{3}-6}{2}$

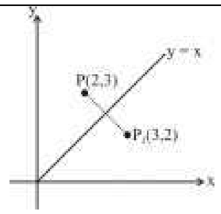
$\therefore \quad$ coordinate $\left(\dfrac{6\sqrt{3}+1}{2}, \dfrac{\sqrt{3}-6}{2}\right)$

22. The point (2, 3) is first reflected in the straight line y = x and then translated through a distance of 2 units along the positive direction X-axis. The coordinates of the transformed point are

(a) (5, 4) (b) (2, 3)
(c) (5, 2) (d) (4, 5)

AP EAMCET-2015

Ans. (c) : Given,
The coordinate of the point is (2,3)
We know that,
The coordinate reflect in the straight line y = x



$\therefore$ The coordinate of reflected point is $P_2(3,2)$
Now, The point P_2 translated in the positive direction x-axis by 2 units
$\therefore$ The New coordinate after translation is

$P_3 = (3 + 2, 2) \Rightarrow P_3 = (5, 2)$

23. If the axis are rotated through an angle 45°, the coordinates of the point $(2\sqrt{2}, -3\sqrt{2})$ in the new system are______

(a) $(3\sqrt{3}, -5)$ (b) $(-1, -5)$
(c) $(5\sqrt{3}, -7)$ (d) $(7, -\sqrt{3})$

AP EAMCET-19.08.2021, Shift-I

Ans. (b): Given,
Coordinate (x, y) = $(2\sqrt{2}, -3\sqrt{2})$
Let new coordinates be (x', y'). Then,

$x' = x\cos\theta + y\cos\theta$

And $\quad y' = -x\,x\text{in}\theta + y\cos\theta$

Here, $\theta = 45°$

$\therefore \quad x' = 2\sqrt{2}\ \cos 45° + (-3\sqrt{2})\ \sin 45°$

$= 2\sqrt{2}\times\dfrac{1}{\sqrt{2}} - 3\sqrt{2}\times\dfrac{1}{\sqrt{2}}$

$= 2 - 3 = -1$

And, $\quad y' = -2\sqrt{2}\ \sin 45° + (-3\sqrt{2})\ \cos 45°$

$= -2\sqrt{2}\times\dfrac{1}{\sqrt{2}} - 3\sqrt{2}\times\dfrac{1}{\sqrt{2}} = -2 - 3 = -5$

New, coordinate is (– 1, – 5).

24. Without changing the direction of the axis, the origin is transferred to the point (2, 3). Then the equation $x^2 + y^2 - 4x - 6y + 9 = 0$ changes to

(a) $x^2 + y^2 + 4 = 0$
(b) $x^2 + y^2 = 4$
(c) $x^2 + y^2 - 8x - 12y + 48 = 0$
(d) $x^2 + y^2 = 9$

WB JEE-2018

Ans. (b) : Given,
The equation is $x^2 + y^2 - 4x - 6y + 9 = 0$
The origin is transferred to the point (2, 3)
We know that,

$x' = x + 2$

$y' = y + 3$

Now, $(x + 2)^2 + (y + 3)^2 - 4(x + 2) - 6(y + 3) + 9 = 0$

$x^2 + 4 + 4x + y^2 + 9 + 6y - 4x - 8 - 6y - 18 + 9 = 0$

$x^2 + y^2 - 4 = 0$

25. **If the coordinates of a point P changes to (2,–6) when the coordinate axis are rotated through an angle of 135^0, then the coordinates of P in the original system are**

(a) (–2, 6) (b) (–6, 2)

(c) $\left(2\sqrt{2}, 4\sqrt{2}\right)$ (d) $\left(\sqrt{2}, -\sqrt{2}\right)$

AP EAMCET-22.04.2018, Shift-I

Ans. (c) : Given,

The coordinated of change point P(2, –6)

Angle of rotation (θ) = 135^o

We know that,

$x = x\cos\theta - y\sin\theta$

$y = y\cos\theta + x\sin\theta$

So, $x = 2\cos 135^o - (-6)\sin 135$

$$x = 2\left(-\frac{1}{\sqrt{2}}\right) + 6\left(\frac{1}{\sqrt{2}}\right) = \frac{4}{\sqrt{2}} = 2\sqrt{2}$$

And, $y = Y\cos\theta + X\sin\theta$

$$y = (-6)\left(-\frac{1}{\sqrt{2}}\right) - 2\left(\frac{1}{\sqrt{2}}\right) \Rightarrow y = \frac{6}{\sqrt{2}} + \frac{2}{\sqrt{2}}$$

$$= \frac{8}{\sqrt{2}} = 4\sqrt{2}$$

∴ The original coordinate $\left(2\sqrt{2}, 4\sqrt{2}\right)$

26. **The transformed equation of $3x^2 - 6xy + 8y^2 = 8$ when the axes are rotated about the origin through an angle $\frac{\pi}{4}$ in the positive direction, is**

(a) $5x^2 + 10xy + 17y^2 + 16 = 0$

(b) $5x^2 + 10xy + 17y^2 - 16 = 0$

(c) $5x^2 - 10xy + 17y^2 - 16 = 0$

(d) $5x^2 - 10xy + 17y^2 + 16 = 0$

AP EAMCET-23.04.2018, Shift-II

Ans. (b) : Given,

The transformation equation $3x^2 - 6xy + 8y^2 = 8$

Angle of rotation about the origin = $\frac{\pi}{4}$

We know that,

$x = X\cos\theta - Y\sin\theta$

$y = X\sin\theta + Y\cos\theta$

∴ $x = X\cos\pi/4 - Y\sin\pi/4$

$$x = \frac{X - Y}{\sqrt{2}}$$

Similarly

$$y = X\sin\frac{\pi}{4} + Y\cos\frac{\pi}{4}$$

$$y = \frac{X}{\sqrt{2}} + \frac{Y}{\sqrt{2}} = \frac{X + Y}{\sqrt{2}}$$

Now,

$$3\left(\frac{X - Y}{\sqrt{2}}\right)^2 - 6\left(\frac{X - Y}{\sqrt{2}}\right)\left(\frac{X + Y}{\sqrt{2}}\right) + 8\left(\frac{X + Y}{\sqrt{2}}\right)^2 = 8$$

Now, putting X = x and Y = y

$$\frac{3}{2}(x - y)^2 - 3\left(x^2 - y^2\right) + \frac{8}{2}(x + y)^2 = 8$$

$$3(x-y)^2 - 6(x^2 - y^2) + 8(x + y)^2 = 16$$

$$3(x^2 + y^2 - 2xy) - 6x^2 + 6y^2 + 8(x^2 + y^2 + 2xy) - 16 = 0$$

$$3x^2 + 3y^2 - 6xy - 6x^2 + 6y^2 + 8x^2 + 8y^2 + 16xy - 16 = 0$$

$$5x^2 + 17y^2 + 10xy - 16 = 0$$

27. **A light ray emerging from a point source at A(2, 3) is reflected on the y-axis at point 'B' and passes through point C(5, 10), then the coordinates of 'B' are**

(a) (5, 0) (b) (0, 5) (c) (0, 2) (d) (2, 0)

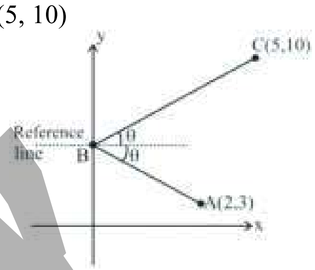
AP EAMCET-17.09.2020, Shift-II

Ans. (b) : Given,

The coordinate of source point is A(2, 3)

The coordinate of point after reflected from

Point B is C (5, 10)



Now,

Let the coordinate of point B is (0, y)

We know the angle of incident is equal to angle of reflection.

∴ Slope of line AB from reference line =

Slope of line BC from reference line

$$\therefore \frac{3 - y}{2 - 0} = \frac{-10 - y}{5 - 0}$$

$15 - 5y = -20 + 2y$

$7y = 35$

$y = 5$

∴ The coordinate of point B is (0, 5)

28. **If the point P changes to (4, –3) when the axes are rotated through an angle of 135° then the coordinates of the point P, with respect to the original system is _______**

(a) $\left(\frac{1}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right)$ (b) $\left(\frac{1}{\sqrt{2}}, \frac{-7}{\sqrt{2}}\right)$

(c) $\left(\frac{-1}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right)$ (d) $\left(\frac{-1}{\sqrt{2}}, \frac{-7}{\sqrt{2}}\right)$

AP EAMCET-05.10.2021, Shift-II
AP EAMCET-22.09.2020, Shift-II

Ans. (c) : Given,

The coordinate of point P (x, y)

The coordinate of change P(4, –3)

The angle of rotated through is 135^o

We know that,

$x = X\cos\theta - Y\sin\theta$

$y = X\sin\theta + Y\cos\theta$

∴ $x = 4(\cos 135^o) - (-3)\sin 135^o$

$$x = 4 \times \left(-\frac{1}{\sqrt{2}}\right) + 3 \times \frac{1}{\sqrt{2}}$$

$$x = \frac{-4 + 3}{\sqrt{2}}$$

$x = -\frac{1}{\sqrt{2}}$

Similarly,

$y = x \sin 135^o + y \cos 135$

$= 4 \times \frac{1}{\sqrt{2}} + (-3)\times\left(-\frac{1}{\sqrt{2}}\right) = \frac{7}{\sqrt{2}}$

$\therefore$ The coordinate of point P = $\left(\frac{-1}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right)$

29. The angle by which axes are to be rotated without changing the origin so that the transformed equation of $x^2 + 4xy - y^2 = 0$ in new coordinates (X, Y) does not contain XY term is.

(a) $\frac{1}{2}\tan^{-1}(2)$ (b) $\tan^{-1}(2)$

(c) $\frac{\pi}{8}$ (d) $\frac{\pi}{4}$

TS EAMCET-05.08.2021, Shift-II

Ans.. (a): Given,

Equation $x^2 + 4xy - y^2 = 0$

Replace $x \to x \cos\theta - y \sin\theta$

$y \to x \sin\theta + y \cos\theta$

Then,

$(x \cos\theta - y \sin\theta)^2 + 4 (x \cos\theta - y \sin\theta)(x \sin\theta + y \cos\theta) - (x \sin\theta + y\cos\theta)^2 = 0$

$x^2 \cos^2\theta + y^2 \sin^2\theta - 2xy \sin\theta \cos\theta + 4x^2 \cos\theta \sin\theta + 4xy \cos^2\theta - 4yx \sin^2\theta - 4y^2 \cos\theta \sin\theta - x^2 \sin^2\theta - y^2\cos^2\theta - 2xy \sin\theta \cos\theta = 0$

Eliminate the coefficient of xy, then

$\Rightarrow -2 \sin\theta \cos\theta - 2 \sin\theta \cos\theta + 4 \cos^2\theta - 4 \sin^2\theta = 0$

$\Rightarrow -4 \sin\theta \cos\theta + 4 (\cos^2\theta - \sin^2\theta) = 0$

$\Rightarrow -2 \sin2\theta + 4 \cos2\theta = 0$

$\Rightarrow 4 \cos2\theta = 2\sin2\theta$

$\Rightarrow 1 = \frac{1}{2}\tan 2\theta \Rightarrow \tan2\theta = 2 \Rightarrow 2\theta = \tan^{-1}2$

$\Rightarrow \theta = \frac{1}{2}\tan^{-1}(2)$

30. When the coordinate axis are rotated through an angle θ in anti clockwise direction, if the transformed equation of $x^2 + y^2 + 2xy + 2x + 6y + 1 = 0$ is $(2+\sqrt{3})X^2 + 2XY + (2-\sqrt{3})Y^2 + aX + bY + 2 = 0$, then 3a – b =

(a) 10 (b) $2(1+2\sqrt{3})$

(c) 20 (d) $2(3+\sqrt{3})$

TS EAMCET-11.09.2020, Shift-II

Ans. (c) : If coordinate axis are rotated through an angle θ in anti-clockwise direction, then,

$x = X \cos \theta - Y \sin \theta$

and $y = X \sin \theta + Y \cos \theta$,

On substituting the values of x and y in equation

$x^2 + y^2 + 2xy + 2x + 6y + 1 = 0$, we get

$(X\cos\theta - Y\sin\theta)^2 + (X\sin\theta + Y\cos\theta)^2$

$+2(X\cos\theta - Y\sin\theta)(X\sin\theta + Y\cos\theta)$

$+2(X\cos\theta - Y\sin\theta) + 6(X\sin\theta + Y\cos\theta) + 1 = 0$

$\Rightarrow (\cos^2\theta + \sin^2\theta + 2\sin\theta\cos\theta)X^2$

$+(-2\sin\theta\cos\theta + 2\sin\theta\cos\theta + 2\cos^2\theta - 2\sin^2\theta)XY$

$+(\sin^2\theta + \cos^2\theta - 2\sin\theta\cos\theta)Y^2$

$+(2\cos\theta + 6\sin\theta)X + (6\cos\theta - 2\sin\theta)Y + 1 = 0$

$\Rightarrow (1+\sin2\theta) + 2\cos2\theta XY + (1-\sin2\theta)Y^2$

$+(2\cos\theta + 6\sin\theta)X + (6\cos\theta - 2\sin\theta)Y + 1 = 0$

On comparing with transformed given equation

$(2+\sqrt{3})X^2 + 2XY + (2-\sqrt{3})Y^2 + aX + bY + 2 = 0$

we get

$\sin 2\theta = \frac{\sqrt{3}}{2}, \cos 2\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6}$

$\therefore$ $a = 4(\cos\theta + 3\sin\theta) = 2\sqrt{3} + 6$

and $b = 4(3\cos\theta - \sin\theta) = 6\sqrt{3} - 2$

$\therefore 3a - b = 6\sqrt{3} + 18 - 6\sqrt{3} + 2 = 20$

31. Let C be a curve $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ in a cartesian plane. By rotating the coordinate axis through an angle $\frac{\pi}{4}$ in the positive direction, if the transformed equation of C is $Y^2 + XY - X = 0$, then $(h^2 - ab) - 2gf =$

(a) 0 (b) 2 (c) 1 (d) –1

TS EAMCET-11.09.2020, Shift-I

Ans. (a) : Equation of given curve C is

$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ ...(i)

Now, on rotating the axis through an angle $\frac{\pi}{4}$ in the positive direction, so we should replace x by

$\left(X\cos\frac{\pi}{4} - Y\sin\frac{\pi}{4}\right) = \left(\frac{X}{\sqrt{2}} - \frac{Y}{\sqrt{2}}\right)$ and

y by $\left(X\sin\frac{\pi}{4} + Y\cos\frac{\pi}{4}\right) = \left(\frac{X}{\sqrt{2}} + \frac{Y}{\sqrt{2}}\right)$, then

transformed equation of C is

$\Rightarrow \frac{a}{2}(X-Y)^2 + h(X^2 - Y^2) + \frac{b}{2}(X+Y)^2$

$+\frac{2g}{\sqrt{2}}(X-Y) + \frac{2f}{\sqrt{2}}(X+Y) + c = 0$

$\Rightarrow \left(\frac{a}{2} + h + \frac{b}{2}\right)X^2 + (b-a)XY + \left(\frac{a}{2} - h + \frac{b}{2}\right)Y^2$

$+\sqrt{2}(g+f)X + \sqrt{2}(f-g)Y + c = 0$

On comparing with $Y^2 + XY - X = 0$, we get

$a + b + 2h = 0,\ b - a = 1,\ a + b - 2h = 2$

$\sqrt{2}(g+f) = -1, f - g = 0$ and $c = 0$

$\therefore$ $f = g = \frac{-1}{2\sqrt{2}}$ and $a + b = 1$

$\therefore$ $b = 1,\ a = 0,$ so $h = -\frac{1}{2}$

So, the value of $(h^2 - ab) - 2gf = \left(\frac{1}{4} - 0\right) - 2\frac{1}{8} = \frac{1}{4} - \frac{1}{4} = 0$

32. If $a\alpha^2 + b\beta^2 + c\alpha\beta + d = 0$ is the transformed equation of $4x^2 + \sqrt{3}xy + 5y^2 - 4 = 0$ obtained by using $\alpha = \frac{\sqrt{3}}{2}x + \frac{y}{2}$ and $\beta = -\frac{x}{2} + \frac{\sqrt{3}}{2}y$, then $c(a + b + d) =$

(a) 0 (b) $13\sqrt{3}$ (c) $5\sqrt{3}$ (d) 6

TS EAMCET 14.09.2020, Shift-II

Ans. (c) : $a\alpha^2 + b\beta^2 + c\alpha\beta + d = 0$ is the transformed equation of $4x^2 + \sqrt{3}xy + 5y^2 - 4 = 0$

where, $\alpha = \frac{\sqrt{3}}{2}x + \frac{y}{2}$ ...(i)

$\beta = \frac{-x}{2} + \frac{\sqrt{3}y}{2}$ ...(ii)

From Eqs. (i) and (ii), we get

$$x = \frac{\sqrt{3}}{2}\alpha + \frac{\beta}{2} \Rightarrow y = \frac{\alpha}{2} - \frac{\sqrt{3}}{2}\beta$$

Putting the value of x and y in

$4x^2 + \sqrt{3}xy + 5y^2 - 4 = 0$, we get

$$4\left(\frac{\sqrt{3}}{2}\alpha + \frac{\beta}{2}\right)^2 + \sqrt{3}\left(\frac{\sqrt{3}}{a}\alpha + \frac{\beta}{2}\right)\left(\frac{\alpha}{2} - \frac{\sqrt{3}}{2}\beta\right) + 5\left(\frac{\alpha}{2} - \frac{\sqrt{3}}{2}\beta\right)^2 - 4 = 0$$

$\Rightarrow$ $5\alpha^2 + 4\beta^2 + \sqrt{3}\alpha\beta - 4 = 0$

Here, $a = 5,\ b = 4,\ c = \sqrt{3}, d = -4$

$\therefore$ $c(a + b + d) = \sqrt{3}(5 + 4 - 4) = 5\sqrt{3}$

33. If a variable line is moving such that the intercepts made by it on the coordinate axes are reciprocal to each other, then the points P(x, y) on such lines satisfy

(a) $x + y > 4$ (b) $4xy > 1$

(c) $4xy < 1$ (d) $x + y = 4$

TS EAMCET-14.09.2020, Shift-I

Ans. (c) : Equation of line is moving such that intercept mode by it on the coordinate axis and reciprocal to each other is

$$ax + \frac{y}{a} = 1 \Rightarrow a^2x - a + y = 0$$

$a \in R$

$\therefore$ $(1)^2 - 4xy > 0 \Rightarrow 4xy < 1$

34. When the origin is shifted to the point $\left(\frac{3}{2}, \frac{3}{2}\right)$ by the translation of coordinate axes, then the transformed equation of $32x^2 + 8xy + 32y^2 - 108x - 108y + 99 = 0$ is

(a) $72X^2 + 56Y^2 - 63 = 0$

(b) $X^2 - 14XY - 7Y^2 - 2 = 0$

(c) $32X^2 - 16XY + 32Y^2 - 225 = 0$

(d) $32X^2 + 8XY + 32Y^2 - 63 = 0$

TS EAMCET-10.09.2020, Shift-I

Ans. (d) : Substituting $x = X + \frac{3}{2},\ y = Y + \frac{3}{2}$ in the equation $32x^2 + 8xy + 32y^2 - 108x - 108y + 99 = 0$, we get

$$32\left(X + \frac{3}{2}\right)^2 + 8\left(X + \frac{3}{2}\right)\left(Y + \frac{3}{2}\right) + 32\left(Y + \frac{3}{2}\right)^2 - 108\left(X + \frac{3}{2}\right) - 108\left(Y + \frac{3}{2}\right) + 99 = 0$$

$$\Rightarrow 32\left[X^2 + 3X + \frac{9}{4}\right] + 8\left(\frac{2X + 3}{2}\right)\left(\frac{2Y + 3}{2}\right) + 32\left[Y^2 + 3Y + \frac{9}{4}\right] - 108X - 162 - 108Y - 162 + 99 = 0$$

$$\Rightarrow 32X^2 + 96X + 72 + 2(4XY + 6X + 6Y + 9) + 32Y^2 + 96Y + 72 - 108X - 162 - 108Y - 162 + 99 = 0$$

$$\Rightarrow 32X^2 + 32Y^2 + 8XY - 63 = 0$$

35. The transformed equation of the curve $2x^2 + y^2 - 3x + 5y - 8 = 0$ when the origin is translated to the point $(-1, 2)$ is

(a) $2x^2 + y^2 - 7x + 9y + 11 = 0$

(b) $2x^2 + y^2 + 7x + 9y + 11 = 0$

(c) $2x^2 + y^2 - x + y + 11 = 0$

(d) $2x^2 + y^2 + 7x - 9y + 11 = 0$

TS EAMCET-04.08.2021, Shift-I

Ans. (a): Given equation of curve

$2x^2 + y^2 - 3x + 5y - 8 = 0$

Put origin is translated to the point (– 1, 2)

We know that the coordinate of transformed point

$x = X + h \Rightarrow y = Y + K$

$\therefore$ $x = X - 1 \Rightarrow y = Y + 2$

$\Rightarrow 2(x - 1)^2 + (y + 2)^2 - 3(x - 1) + 5(y + 2) - 8 = 0$

$\Rightarrow 2(x^2 + 1 - 2x) + y^2 + 4 + 4y - 3x + 3 + 5y + 10 - 8 = 0$

$\Rightarrow 2x^2 + 2 - 4x + y^2 + 4 + 4y - 3x + 3 + 5y + 2 = 0$

$\Rightarrow 2x^2 + y^2 - 7x + 9y + 11 = 0$

$\Rightarrow 2x^2 + y^2 - 7x + 9y + 11 = 0$

$\Rightarrow 2x^2 + y^2 - 7x + 9y + 11 = 0$

2. Distance and Sections Formula (Internal and External division)

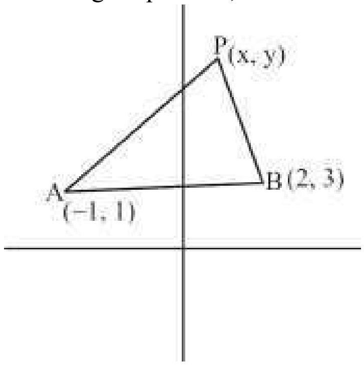
36. Let A (–1, 1) and B (2, 3) be two points and P be a variable point above the line AB such that the area of ΔPAB is 10. If the locus of P is ax + by = 15, then 5a + 2b is:

(a) $-\frac{12}{5}$ (b) $-\frac{6}{5}$

(c) 4 (d) 6

JEE MAIN-05.04.2024, Shift-II

Ans. (a) : According to question,



Given,

Locus of P is ax + by = 15

Let the point P be (h, k)

Now,

Area of ΔPAB

$=\frac{1}{2}\left|\left[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right]\right|$

$\Rightarrow \frac{1}{2}\left|-1(3-k)+2(k-1)+h(1-3)\right|=10$

$\Rightarrow \left|-3+k+2k-2+h-3h\right|=20$

$\Rightarrow \left|3k-2h-5\right|=20$

Therefore,

3k – 2h – 5 = 20

–2h + 3k = 25

$-\frac{6}{5}h+\frac{9}{5}k=15$

On comparing with ax + by = 15

$a=-\frac{6}{5}, b=\frac{9}{5}$

Hence, 5a + 2b

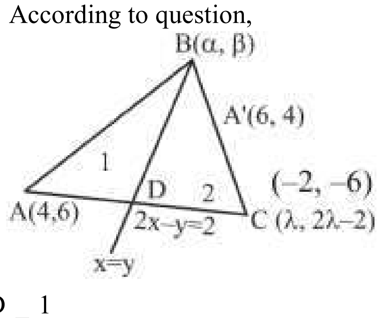
$=5\times\left(-\frac{6}{5}\right)+2\times\frac{9}{5}=-\frac{12}{5}$

37. In a ΔABC, suppose y = x is the equation of the bisector of the angle B and the equation of the side AC is 2x – y = 2. If 2AB = BC and points A and B are respectively (4, 6) and (α, β), then α + 2β is equal to

(a) 42 (b) 45
(c) 39 (d) 48

JEE MAIN-29.01.2024, Shift-I

Ans. (a) : According to question,



$\frac{AB}{BC}=\frac{AD}{DC}=\frac{1}{2}$

$D\left(\frac{\lambda+8}{3},\frac{2\lambda-2+12}{3}\right)$ lies on y = x

$\frac{\lambda+8}{3}=\frac{2\lambda+10}{3}$

3λ + 24 = 6λ + 30

3λ = – 6

⇒ λ = –2

Therefore, C (λ, 2λ –2) ≡ C (–2, –6)

B, A', C are line on the same line

$\therefore \frac{4-\alpha}{6-\alpha}=\frac{-6-4}{-2-6}=\frac{10}{8}=\frac{5}{4}$

16 – 4α = 30 – 5α

α = 30 – 16

α = 14 = β

Now,

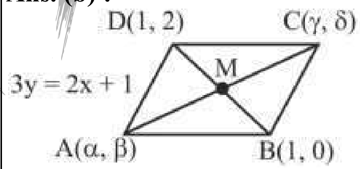
α + 2β = 42

38. Let α, β, γ, δ ∈ ℤ and let A(α, β), B(1, 0), C(γ, δ) and D(1, 2) be the vertices of a parallelogram ABCD. If AB = $\sqrt{10}$ and the points A and C lie on the line 3y = 2x + 1, then 2(α + β + γ + δ) is equal to

(a) 12 (b) 8
(c) 10 (d) 5

JEE MAIN-31.01.2024, Shift-I

Ans. (b) :



∵ M is mid point of both diagonal

$\left(\frac{\alpha+\gamma}{2},\frac{\beta+\delta}{2}\right)=(1,\ 1)$

α + γ = 2 ... (i)

β + δ = 2 ... (ii)

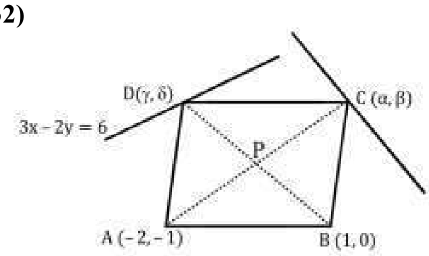
α + β + γ + δ = 4

2(α + β + γ + δ) = 8

39. Let A(–2,–1), B(1,0), C(α,β) and D (γ,δ) be the vertices of a parallelogram ABCD. If the point C lies on 2x – y = 5 and the point D lies on 3x–2y = 6, the value of | | is equal to_______.

JEE MAIN-31.01.2024, Shift-II

Ans. : (32)



Let, diagonal are intersect at point P

So, P is mid point of diagonals AC and BD.

$$\left(\frac{\alpha-2}{2},\frac{\beta-1}{2}\right)=\left(\frac{\gamma+1}{2},\frac{\delta}{2}\right)$$

$\alpha - 2 = \gamma + 1, \beta - 1 = \delta$

$\alpha - \gamma = 3$ .....(i)

$\beta - \delta = 1$ ....(ii)

And, (γ, δ) lies on $3x - 2y = 6$

$\Rightarrow$ $3\gamma - 2\delta = 6$ .....(iii)

And, $(\alpha, \beta,)$ lies on $2x - y = 5$

$2\alpha - \beta = 5$ ....(iv)

Now, (ii) + (iv) –

$2\alpha - \delta = 6$ .....(v)

And, (i) × 3 + (iii) –

$3\alpha - 2\delta = 15$ ......(vi)

Now,

(v) × 2 + (vi) –

$4\alpha - 2\delta = 12$

$\underline{3\alpha - 2\delta = 15}$

$\alpha = -3$

So, from (i) –

$\alpha - \gamma = 3$

$\gamma = -6$

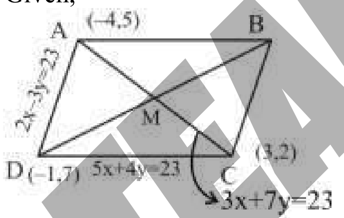
Similarly, $\beta = -11, \delta = -12$

Now, $|\alpha + \beta + \gamma + \delta| = |-3 - 11 - 6 - 12| = 32$

40. Let the equations of two adjacent sides of a parallelogram ABCD be 2x – 3y = –23 and 5x + 4y = 23. If the equation of its one diagonal AC is 3x + 7y = 23 and the distance of A from the other diagonal is d, then 50 d² is equal to ____.

JEE MAIN-10.04.2023, Shift-II

Ans. (529) : Given,



Equation of two adjacent side-

$2x - 3y = -23$ ……(i)

and, $5x + 4y = 23$ ……(ii)

Equation of diagonal AC

$3x + 7y = 23$ ……(iii)

On Solving (i) and (ii)–

$23x = -23$

$x = -1$

& $y = 7$

Then coordinate of point D = (–1, 7)

From equation (ii) and (iii)–

$x = 3$

$y = 2$

Then coordinate of point C = (3, 2)

From equation (i) and (iii)

$x = -4$

$y = 5$

Then, A = (–4, 5)

Co-ordinate of mid point $M=\left(-\frac{1}{2},\frac{7}{2}\right)$

Equation of diagonal BD is

$$y-\frac{7}{2}=\frac{\frac{7}{2}-7}{-\frac{1}{2}+1}\left(x+\frac{1}{2}\right)$$

$\Rightarrow 7x + y = 0$

Distance of A from diagonal BD

$$d=\frac{23}{\sqrt{50}}$$

$\Rightarrow 50d^2 = (23)^2$

$\Rightarrow 50d^2 = 529$

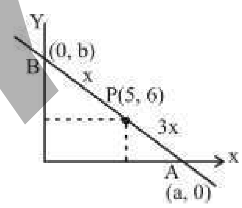
41. The lines cut X and Y axis at the points A and B respectively. The point $(5,6)$ divides the line segment AB internally in the ratio 3 : 1, then equation of line is

(a) $2x-5y=-20$ (b) $2x+y=16$

(c) $2x-y=4$ (d) $2x+5y=40$

MHT CET-2020

Ans. (d) :



Let the coordinates of A = (a, 0)

The coordinate of B = (0, b)

$$\therefore \quad 5=\frac{3x\times 0+x\times a}{3x+x}$$

$$5=\frac{a}{4}\Rightarrow a=20$$

Similarly,

$$6=\frac{3x\times b+x\times 0}{3x+x}$$

$3b = 24$

$b = 8$

$\therefore$ Equation of line

$$\frac{x}{20}+\frac{y}{8}=1$$

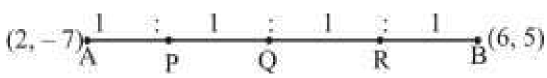
$2x + 5y = 40$

42. The line joining A(2, –7) and B(6, 5) is divided into 4 equal parts by the points P, Q and R such that AQ = RP = QB. The midpoint of PR is

(a) (–8, 1) (b) (4, 12) (c) (8, –2) (d) (4, –1)

Karnataka CET-2010

Ans.(d): According to the question,



Given, AQ = RP = QB

So, midpoint of P and R is same as midpoint of A and B because AQ = QB

$$\therefore \text{ Coordinate of Q}=\left(\frac{6+2}{2},\frac{5-7}{2}\right)$$

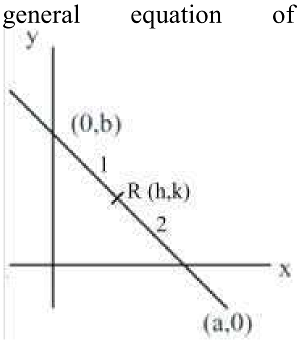
$= (4, -1)$

43. Point R (h, k) divides a line segment between the axis in the ratio 1:2. Find equation of the line.

(a) 2kx + hy = 3hk (b) 2kx + hy = 2hk
(c) 2kx – hy = 3hk (d) None of the above

BCECE-2013

Ans. (a): We know,
The general equation of straight line



$$\frac{x}{a}+\frac{y}{b}=1 \quad .....(i)$$

$\therefore$ $x_1 = a, y_1 = 0$
$x_2 = 0, y_2 = b$

$$h = \frac{mx_2+nx_1}{m+n}$$

$$h = \frac{1\times 0+2\times a}{3} = \frac{2a}{3} \Rightarrow a = \frac{3h}{2}$$

Similarly,

$$k = \frac{my_2+ny_1}{m+n}$$

$$k = \frac{1\times b+2\times 0}{3} \Rightarrow b = 3k$$

Putting value of a, b in equation (i)

$$\frac{2x}{3h}+\frac{y}{3k}=1$$

2kx + hy = 3kh

44. The ratio in which yz-plane divide it he line joining the points A (3, 1, – 5) and B (1, 4, – 6) is

(a) –3 : 1 (b) 3 : 1 (c)–1 : 3 (d) 1 : 3

CG PET- 2013

Ans. (a) : Given,
The coordinate of end points of line are
A(3, 1, –5) and B(1, 4, –6)
Let the yz plane divide the line in k : 1 ratio.
Let the coordinate of points c is (x, y, z)
We know that,
In yz – Plane, x = 0

$$\therefore (x, y, z) = \left(\frac{1\times k+3}{k+1},\frac{4k+1}{k+1},\frac{-6k-5}{k+1}\right)$$

$$(0,y,z)=\left(\frac{k+3}{k+1},\frac{4k+1}{k+1},\frac{-6k-5}{k+1}\right)$$

$$\therefore \quad \frac{k+3}{k+1}=0$$

k + 3 = 0
k = –3
$\therefore$ The ratio is –3 : 1

45. If algebraic sum of distances of a variable line from points (2, 0), (0, 2) and (–2, –2) is zero, then the line passes through the fixed point

(a) (–1, –1) (b) (1, 1) (c) (2, 2) (d) (0, 0)

AMU-2011

Ans. (d) : The variable line be ax + by + c = 0
So, the sum of the perpendicular from the points (2, 0), (0, 2) and (–2, –2)

$$\left|\frac{2\times a+b\times 0+c}{\sqrt{a^2+b^2}}\right|+\left|\frac{a\times 0+b\times 2+c}{\sqrt{a^2+b^2}}\right|+\left|\frac{a\times(-2)+b\times(-2)+c}{\sqrt{a^2+b^2}}\right|=0$$

$$\pm\left(\frac{2a+c}{\sqrt{a^2+b^2}}\right)\pm\left(\frac{2b+c}{\sqrt{a^2+b^2}}\right)\pm\left(\frac{-2a-2b+c}{\sqrt{a^2+b^2}}\right)=0$$

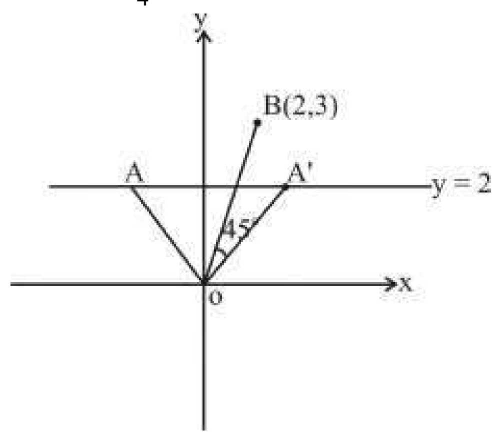
2a + c + 2b + c –2a –2b + c = 0
0a + 0b + 3c = 0
This is linear relation between a, b and c by comparing ax + by + c = 0
So, the co-ordinate of fixed point are (0,0)

46. The distance between the two points A and A' which lie on y = 2 such that both the line segments AB and A' B (where B is the point (2, 3)) subtend angle $\frac{\pi}{4}$ at the origin, is equal to :

(a) 10 (b) $\frac{48}{5}$ (c) $\frac{52}{5}$ (d) 3

JEE Main-29.06.2022, Shift-I

Ans. (c) : Given,
The equation on of line is y = 2
Subtend angle = $\frac{\pi}{4}$ at origin from point A and A'



Now, The coordinate of point A and A' is $(x_1, 2)$ and $(x_2, 2)$ respectively and slope of line OB is m_1 and OA' is m_2

$$m_1 = \frac{2}{x_1} \text{ and } m_2 = \frac{3}{2}$$

The angle between two lines having slope m_1 and m_2 is θ

$$\tan\theta = \left|\frac{m_1-m_2}{1+m_1m_2}\right|$$

$$\tan \pi/4 = \left|\frac{2/x_1-3/2}{1+(2/x_1)(3/2)}\right|$$

$$1 = \left|\frac{2/x_1 - 3/2}{1 + 3/x_1}\right|$$

$$\pm 1 = \frac{4 - 3x_1}{2(x_1 + 3)}$$

$4 - 3x_1 = \pm(2x_1 + 6)$

$4 - 3x_1 = 2x_1 + 6$ …..(i)

and $4 - 3x_1 = -2x_1 - 6$ …..(ii)

From equation (i)

or $x_1 = 10$

From equation (ii)

$$x_1 = \frac{-2}{5}$$

x is positive for A and negative for A'

$x_1 = 10$ and $x_2 = \frac{-2}{5}$

$\therefore$ A = (10, 2) and A' = $\left(\frac{-2}{5}, 2\right)$

Now, The distance AA' = $\sqrt{\left(10 + \frac{2}{5}\right)^2 + (2-2)^2}$

$$AA' = \frac{52}{5} \text{units}$$

47. The distance between the parallel lines $9x^2 - 6xy + y^2 + 18x - 6y + 8 = 0$ is

(a) $\frac{1}{\sqrt{10}}$ (b) $\frac{2}{\sqrt{10}}$ (c) $\frac{4}{\sqrt{10}}$ (d) $\sqrt{10}$

EAMCET-1994

Ans. (b) : Given,

The equation of pair of straight lines

$9x^2 - 6xy + y^2 - 18x - 6y + 8 = 0$ ....(i)

We know that,

The general equation of pair of straight lines

$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ ....(ii)

Now comparing the coefficients in equation (i) and (ii)

a = 9, h = –3, b = 1

g = –9, f = –3, c = 8

We know that,

Distance between two parallel lines

$$d = 2\sqrt{\frac{g^2 - ac}{a(a+b)}}$$

So, $$d = 2\sqrt{\frac{9^2 - 9(8)}{a(a+1)}}$$

$$= 2\sqrt{\frac{81 - 72}{9 \times 10}} = 2\sqrt{\frac{9}{9 \times 10}}$$

$\therefore$ $$d = \frac{2}{\sqrt{10}}$$

48. The distance between parallel lines given by the equation $x^2 + 2\sqrt{2}\,xy + 2y^2 + 4x - 8 + 4\sqrt{2}\,y = 0$, is

(a) 4 (b) $2\sqrt{2}$ (c) $4\sqrt{2}$ (d)8

EAMCET-1995

Ans. (a) : Given,

The equation of pair of straight lines

$x^2 + 2\sqrt{2}xy + 2y^2 + 4x - 8 + 4\sqrt{2}\,y = 0$ ......(i)

We know that,

The general equation of pair of straight lines

$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ ....(ii)

Now, comparing the coefficients from equation (i) and (ii)

a = 1, h = $\sqrt{2}$, b = 2, g = 2, f = $2\sqrt{2}$, c = –8

$\therefore$ The distance between two parallel lines

$$d = 2\sqrt{\frac{g^2 - ac}{a(a+b)}}$$

$$d = 2\sqrt{\frac{4 - 1 \times (-8)}{1(3)}}$$

$$d = 2\sqrt{\frac{12}{3}}$$

$$d = 2\sqrt{4}$$

d = 4

49. the ratio in which the point P, whose abscissa is 3, divides the join of A (6, 5) and B(-1, 4) is equal to .....

(a) 3 : 4 (b) 4 : 3 (c)3 : 2 (d)2 : 3

AP EAMCET-18.09.2020, Shift-II

Ans. (a) : Given,

The coordinate of end point are A(6, 5) and B(–1, 4)

Let Point (x, y) divides the line in the ratio of m : n

$$(x, y) = \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$$

Where (x_1, y_1) and (x_2, y_2) are the end point of lines.

Let the ratio in which P divides line be r : 1

$\therefore$ Coordinates of P is $\left(\frac{-r+6}{r+1}, \frac{4r+5}{r+1}\right)$

$$\frac{-r+6}{r+1} = 3 \Rightarrow -r + 6 = 3r + 3$$

4r = 3

$$r = \frac{3}{4}$$

50. If A(2, –1) and B(6, 5) are two points the ratio in which the foot of the perpendicular from (4, 1) to AB divides it, is

(a) 8 : 15 (b) 5 : 8 (c)–5 : 8 (d) –8 : 5

AP EAMCET-2007

Ans. (b) : Given,

The coordinate of end points of line are (2, –1) and (6, 5)

We know that, $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

$$y - (-1) = \frac{5 - (-1)}{6 - 2}(x - 2)$$

$$y + 1 = \frac{6}{4}(x - 2)$$

$$y + 1 = \frac{3}{2}(x - 2)$$

$3x - 2y - 8 = 0$ .....(i)

The foot of the perpendicular from (4, 1) to equation (i)

$$\frac{h-4}{3}=\frac{k-1}{-2}=-\frac{(12-2-8)}{9+4}$$

$\therefore \quad h = \frac{46}{13}$

Ratio = $x_1 - x : x - x_2$

$$= \left(2-\frac{46}{13}\right):\left(\frac{46}{13}-6\right)$$

$= 5 : 8$

51. The perpendicular distance from the point $(1, \pi)$ to the line joining (1, 0) and $\left(1, \frac{\pi}{2}\right)$, (in polar coordinates) is

(a) 2 (b) $\sqrt{3}$ (c) 1 (d) $\sqrt{2}$

AP EAMCET-2013

Ans. (d) : Given,
The coordinate of point from which perpendicular drawn on the line is $(1, \pi)$
We know that,
The polar coordinate representation is (r, θ)
$\therefore r = 1, \theta = \pi$
$x = r\cos\theta, y = r\sin\theta$
The coordinate (1, 0), $(1, \pi)$ and $(1, \pi/2)$ should
$(1, \pi) = (\cos\pi, \sin\pi)$
$= (-1, 0)$ .....(i)
Similarly, $(1, \pi/2) = (\cos\pi/2, \sin\pi/2)$
$= (0, 1)$ .....(ii)
and $(1, 0) = (\cos 0, \sin 0)$
$= (1, 0)$ .....(iii)
Now, The equation of line joining (0, 1) and (1,0)

$$\frac{y}{x-1}=\frac{y-1}{x}$$

$\therefore \quad xy = xy - (x + y) + 1$
$x + y = 1$
Then, the perpendicular distance form point (–1, 0) to x + y = 1 is

$$d = \left|\frac{-1+0-1}{\sqrt{2}}\right|$$

$$\therefore \quad d = \left|\frac{-2}{\sqrt{2}}\right|$$

$d = \sqrt{2}$

52. The ratio in which the YZ-plane divides the line joining (2, 4, 5) and (3, 5, –4) is_____

(a) 2 : 3 internally (b) 3 : 2 internally
(c) 3 : 2 externally (d) 2 : 3 externally

AP EAMCET-19.08.2021, Shift-I

Ans. (d): Let the yz – plane divide the line segment joining the points (2, 4, 5) and (3, – 5, 4) in m :1
We know that on yz – plane the co-ordinate of x is 0

$$\frac{m\times 3+1\times 2}{m+1}=0$$

$3m + 2 = 0$

$m = -\frac{2}{3}$

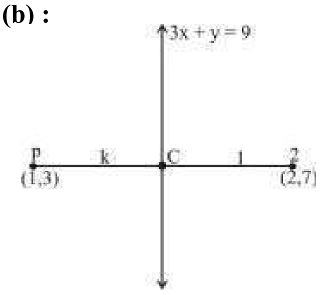
Hence, yz- plane divide the line segment joining the points (2, 4, 5) and (3, – 5, 4) in 2 : 3 externally,

53. The straight line 3x + y = 9 divides the line segment joining the points (1, 3) and (2, 7) in the ratio

(a) 3 : 4 externally (b) 3 : 4 internally
(c) 4 : 5 internally (d) 5 : 6 externally

WB JEE-2010

Ans. (b) :



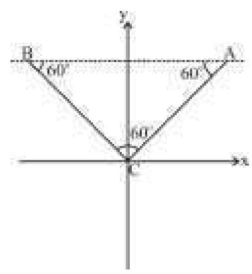
Then, coordinates of C = $\left(\frac{2k+1}{k+1}, \frac{7k+3}{k+1}\right)$
But C, lies on 3x + y = 9
Therefore, $3\left(\frac{2k+1}{k+1}\right)+\frac{7k+3}{k+1}-9=0$
$(6k + 3) + (7k + 3) - 9k - 9 = 0$
$4k - 3 = 0$
$k = 3/4$
So, the required ratio is 3 : 4 internally.

54. The equations $y = \pm\sqrt{3}x$, y = 1 are the sides of

(a) An equilateral triangle
(b) A right angled triangle
(c) An isosceles triangle
(d) An obtuse angled triangle

WB JEE-2010

Ans. (a) :



Given,
$y = \pm\sqrt{3}x$, $\quad y = 1$
$y = \sqrt{3}x$ ....(i)
$y = -\sqrt{3}x$ ....(ii)
$y = 1$ ....(iii)
From, equation (i) & (ii), we get–
$x = 0, y = 0$
So, $c = (0, 0)$
Again, equation (ii) & (iii), we get–

$x = -\frac{1}{\sqrt{3}}$, $y = 1$

So, $B = \left(-\frac{1}{\sqrt{3}}, 1\right)$

Similarly, $A = \left(\frac{1}{\sqrt{3}}, 1\right)$

From distance formula,
$BC = AB = AC = 2/\sqrt{3}$
So, it is an equilateral triangle.

55. Given that the points P (3, 2,– 4), Q (5, 4, – 6) and R (9, 8, –10) are collinear, the ratio in which Q divides PR externally is

(a) 1:2 (b) 2:1 (c) 1:1 (d) 2:2

AMU-2016

Ans. (a) : Given,
The point P (3, 2, –4)
and Q (5, 4, –6) and R (9, 8, –10) divides PR internally.
Let Q divided PR in the ratio k : 1

$$Q=\left(\frac{9k+3}{k+1},\frac{8k+2}{k+1},\frac{-10k-4}{k+1}\right)$$

$$(5, 4, -6)=\left(\frac{9k+3}{k+1},\frac{8k+2}{k+1},\frac{-10k-4}{k+1}\right)$$

$$\frac{9k+3}{k+1}=5 \Rightarrow 9k+3=5k+5$$

$$k = 1/2$$

Thus, point Q divides PR in the ratio 1: 2

56. The harmonic conjugate of (2, 3, 4) with respect to the points (3, –2, 2) and (6, –17, –4) is

(a) (11, – 16, 2) (b) $\left(\frac{1}{2},\frac{1}{3},\frac{1}{4}\right)$

(c) (0, 0, 0) (d) $\left(\frac{18}{5},\frac{-5}{1},\frac{4}{5}\right)$

AP EAMCET-22.09.2020, Shift-II

Ans. (d) : Given,
P(2, 3, 4), A(3, –2, 2) & B(6, –17, –4)
Let P divides AB in the ratio = k : 1

then, $2=\frac{6k+3}{k+1}$

$2k + 2 = 6k + 3 \Rightarrow k = -1/4$

harmonic conjugate Q divided to the ratio = –k : 1
= 1/4 : 1

Hence, coordinates of Q,

$$\left(\frac{1/4\times6+1\times3}{1/4+1},\frac{1/4\times(-17)+1\times-2}{1/4+1},\frac{1/4-4+1\times2}{1/4+1}\right)$$

$$\left(\frac{18}{5},\frac{-25}{5},\frac{4}{5}\right), \quad \left(\frac{18}{5},\frac{-5}{1},\frac{4}{5}\right)$$

57. If (a, 8) is a point on the join of (2, 5) and (4, –1) then

(a) $a=\frac{8}{3}$ (b) $a=\frac{3}{8}$ (c) a = 1 (d) a = – 1

AP EAMCET-18.09.2020, Shift-I

Ans. (c) : Given,
The coordinate of points are (2, 5) and (4, –1)
We know that,
The equation of straight lines is

$$y-y_1=\frac{y_2-y_1}{x_2-x_1}(x-x_1)$$

$$\therefore \quad y-5=\frac{-1-5}{4-2}(x-2)$$

$$y-5=\frac{-6}{2}(x-2)$$

$$y - 5 = -3(x - 2) \quad ......(i)$$

Now, point (a, 8) will satisfy the equation (i)

$\therefore$ $8 - 5 = -3(a - 2)$

$3 = -3(a - 2)$

$a - 2 = -1$

$a = 1$

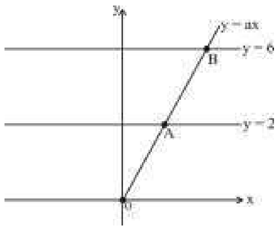
58. If the length of the intercept made on the line y = ax by the lines y = 2 and y = 6 is less than 5

(a) $a \in (-\infty,\infty)$ (b) $a\in\left(\frac{-4}{3},\frac{4}{3}\right)$

(c) $a\in\left(\frac{-3}{3},\frac{4}{3}\right)$ (d) $a<\frac{-4}{3}$ or $a>\frac{4}{3}$

AP EAMCET-21.09.2020, Shift-II

Ans. (d) : Given,
The equation of line is y = ax
We know that,
The coordinate of point
A is $\left(\frac{2}{a},2\right)$
Similarly, the coordinate
of point B is $\left(\frac{6}{a},6\right)$



Now, the distance between AB is

$$\sqrt{\left(\frac{6}{a}-\frac{2}{a}\right)^2+(6-2)^2}<5$$

$$\left(\frac{6}{a}-\frac{2}{a}\right)^2+4^2<25$$

$$\left(\frac{6}{a}-\frac{2}{a}\right)^2<9, \quad \left(\frac{4}{a}\right)^2<9.$$

$$\frac{16}{a^2}<9$$

$$a^2>\frac{16}{9}$$

$$\therefore \quad a>\pm\sqrt{\frac{16}{9}}$$

$$a>\pm\frac{4}{3}$$

$\therefore$ a > 4/3 or a < – 4/3

59. A straight line through the origin O meets the parallel lines $4x+2y=9$ and $2x+y+6=0$ at P and Q respectively. The point O divides the segment PQ in the ratio

(a) 1:2 (b) 3:4 (c) 2:1 (d) 4:3

WB JEE-2020

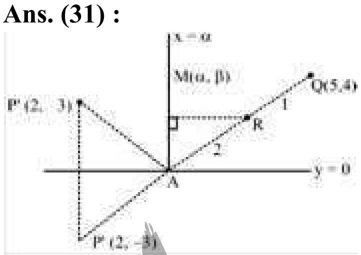
Ans. (b) : Given,
The equation of parallel lines are 4x + 2y = 9 ....(i)
and 2x + y + 6 = 0 .....(ii)
We know that,
The equation of line passing through origin is
y = mx .....(iii)
Now, from equation (i) and (iii)

$4x + 2y = 9$
$2x + y = 9/2$
$2x + mx = 9/2$
$x(2 + m) = \frac{9}{2}$
$x = \frac{9}{2(2+m)}$
Now, substitute the value of x in y of equation (i)
$y = \frac{9m}{2(2+m)}$
The coordinate of point of intersection on line
$4x + 2y = 9$ is $P\left(\frac{9}{2(2+m)}, \frac{9m}{2(2+m)}\right)$
Similarly, the coordinate of point of intersection on the line $2x + y + 6 = 0$ is $Q\left(\frac{-6}{2+m}, \frac{-6m}{2+m}\right)$
Now, The section formula for the ratio
$\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$ [Where m and n be the ratios]
Let the ratio is $\lambda : 1$
$\therefore \left(\frac{mx_2+nx_1}{m+n}\right) = 0$
$$\frac{\frac{-6\lambda}{2+m}+\frac{9(1)}{2m+4}}{\lambda+1} = 0$$
$$\frac{-6\lambda}{2+m}+\frac{9}{2(2+m)} = 0$$
$-12\lambda = -9$
$\lambda = 3/4$
$\therefore$ Ratio is $\frac{3}{4}:1 = 3:4$

60. The ratio in which the line joining points A (–1, –1) and B (2, 1) divides the line joining C (3, 4) D (1, 1)
(a) 7: 5 Internally (b) 7: 5 Externally
(c) 7: 11 Internally (d) 7: 11 Externally
AP EAMCET-23.08.2021, Shift-I

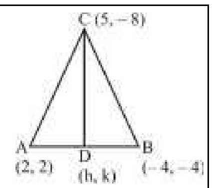
Ans. (b): Given, A (– 1, –1) , B (2, 1), C (3, 4) , D(1, 2)
Let the ratio be $\lambda : 1$
equation of line through AB
$y + 1 = \frac{2}{3}(x+1)$
$3y + 3 = 2x + 2$
$2x - 3y - 1 = 0$
Let, E be the point where line intersect
$E = \left(\frac{\lambda+3}{\lambda+1}, \frac{2\lambda+4}{\lambda+1}\right)$
$$2\left(\frac{\lambda+3}{\lambda+1}\right) - 3\left(\frac{2\lambda+4}{\lambda+1}\right) - 1 = 0$$
$2\lambda + 6 - 6\lambda - 12 - \lambda - 1 = 0$
$-5\lambda - 7 = 0$
$\lambda = \frac{-7}{5}$
7: 5 Externally

61. A ray of light passing through the point P(2, 3) reflects on the x-axis at point A and the reflected ray passes through the point Q(5, 4). Let R be the point that divides the line segment AQ internally into the ratio 2 : 1. Let the co-ordinates of the foot of the perpendicular M from R on the bisector of the angle PAQ be (α, β) Then, the value of 7α + 3β is equal to _______.
JEE Main-28.06.2022, Shift-I

Ans. (31) :


By observation we see that A (α, 0)
And β = y-coordinate of R
$= \frac{2\times4+1\times0}{2+1} = \frac{8}{3}$ …(i)
Now P' is image of P in y = 0 which will be P'(2, –3)
$\therefore$ Equation of P'Q is $(y+3) = \frac{4+3}{5-2}(x-2)$
i.e. $3y + 9 = 7x - 14$
$A \equiv \left(\frac{23}{7}, 0\right)$ by solving with y = 0
$\therefore \alpha = \frac{23}{7}$ …(ii)
By equation (i) and (ii)
$7\alpha + 3\beta = 23 + 8 = 31$

62. The vertices of ΔABC are A (2, 2), B (–4, –4) and C (5, – 8). Find the length of a median of a triangle, which is passing through the point C.
(a) $\sqrt{65}$ (b) $\sqrt{117}$ (c) $\sqrt{85}$ (d) $\sqrt{116}$
GUJCET-2011

Ans. (c) :


D is a mid-point of AB
$h = \frac{-4+2}{2} = -1, \; k = \frac{-4+2}{2} = -1$
(h, k) = (–1, –1)
Distance from (5, – 8) and (–1, –1) is
$D = \sqrt{(5+1)^2+(-8+1)^2} = \sqrt{36+49} = \sqrt{85}$

63. A straight line passing through origin O intersects the lines 10x – 8y – 10 = 0 and $\frac{x}{4}-\frac{y}{5}+1=0$ at right angles and at the points P and Q respectively. Then the ratio in which O divides the line segment PQ is
(a) 1 : 2 (b) 1 : 4 (c) 1 : 1 (d) 3 : 4
TS EAMCET-05.08.2021, Shift-II

Ans.. (b): Let,

Equation of required line

$y = mx+c$

Given, it is normal to $10x - 8y = 10$

and $5x - 4y + 20 = 0$

So, $m_1m_2 = -1$

$$m\times\left(\frac{5}{4}\right) = -1$$

$$m = -\frac{4}{5}$$

P will be the point of intersection of

$5x - 4y = 5$

$4x + 5y = 0$

$$P : \left(\frac{25}{41}, \frac{-20}{41}\right)$$

Q will be the point of intersection of

$5x - 4y = -20$

$4x + 5y = 0$

$$Q : \left(\frac{-100}{41}, \frac{80}{41}\right)$$

Let O divides PQ in $\lambda : 1$

$$(0,0) = \left(\frac{-100\lambda+25}{41(\lambda+1)}, \frac{80\lambda-20}{41(\lambda+1)}\right)$$

$$\lambda = \frac{1}{4}$$

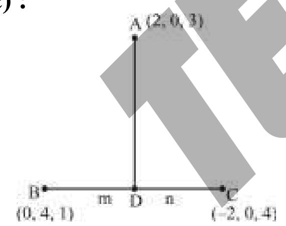
$\therefore \quad \lambda : 1 = 1 : 4$

64. Let D be the foot of the perpendicular drawn from the point A (2, 0, 3) to the line joining the points B (0, 4, 1) and C (–2, 0, 4). Then, the ratio in which D divides BC is

(a) 3 : 2 (b) $2\sqrt{6} : \sqrt{17}$ (c) 18: 11 (d) 16: 9

TS EAMCET-19.07.2022, Shift-I

Ans. (c) :



Equation of line BC is-

$$\frac{x-0}{-2} = \frac{y-4}{-4} = \frac{z-1}{3} = K$$

$\therefore$ Any point on the line BC i.e. D is = $(-2K, -4K + 4, 3K + 1)$

Direction ratio's of line AD is-

$(-2K - 2, -4K + 4 - 0, 3K + 1 - 3)$

i.e. $(-2K - 2), (-4K + 4)$ and $(3K - 2)$

Direction ratio's of line BC = $(-2, -4, 3)$

Since line AD and BC are perpendicular to each other

$-2(-2K - 2) - 4(-4K + 4) + 3(3K - 2) = 0$

$(4K + 16K + 9K) + (4 - 16 - 6) = 0$

$$29K = 18 = K = \frac{18}{29}$$

$\therefore$ Co-ordinates of D = $\left(\frac{-36}{29}, \frac{44}{29}, \frac{83}{29}\right)$

Now,

$$\frac{-2m}{m+n} = \frac{-36}{29} \quad .....(i)$$

$$\frac{4n}{m+n} = \frac{44}{29} \quad .....(ii)$$

$$\frac{4m+n}{m+n} = \frac{83}{29} \quad ....(iii)$$

Solving the equation (i) we get

$-58m = -36m - 36n$

$22m - 36n = 0$

$$\frac{m}{n} = \frac{18}{11}$$

65. Let the line L drawn perpendicular to the lines $2x - 3y + 4 = 0$ and $6x - 9y + 7 = 0$ meet them at A and B, respectively. If P (1, 1) is a point on L, then the ratio in which P divides AB is

(a) 9 : 4 internally (b) 9 : 4 externally

(c) 4 : 9 internally (d) 4 : 9 externally

TS EAMCET-19.07.2022, Shift-I

Ans. (b) :

Since two lines are parallel to each other.

$$\therefore \quad PA = \frac{2-3+4}{\sqrt{13}} = \frac{3}{\sqrt{13}}$$

$$PB = \frac{6-9+7}{\sqrt{36+81}} = \frac{4}{3\sqrt{13}}$$

Now, $$\frac{AP}{BP} = \frac{3}{\sqrt{13}} \times \frac{3\sqrt{13}}{4} = \frac{9}{4}$$

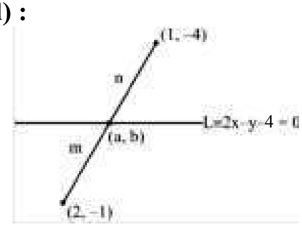
or AP : BP :: 9 : 4 (externally)

66. If the line $2x - y - 4 = 0$ divides the line segment joining the points (2, – 1) and (1, – 4) at the point (a, b) in the ratio m : n, then $4\left(\mathbf{a} - \mathbf{b}\left(\frac{\mathbf{m}}{\mathbf{n}}\right)^2\right) =$

(a) –5 (b) 14 (c) 11 (d) 10

TS EAMCET-19.07.2022, Shift-I

Ans. (d) :



Equation of line through (2, –1) and (1, –4) is-

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$\Rightarrow \quad y - (-1) = \frac{-4+1}{1-2}(x-2)$$

$y + 1 = 3(x - 2)$

or $3x - y = 7$ .... (i)

$2x - y - 4 = 0$ .....(ii)

Solving equation (i) and (ii), we get the point of intersection (a, b) i.e.

$a = 3, b = 2$

Also, $a = \dfrac{m+2n}{m+n}$ and $b = \dfrac{-4m-n}{m+n}$

or $a = \dfrac{\frac{m}{n}+2}{\frac{m}{m}+1}$ and $b = \dfrac{\frac{-4m}{n}-1}{\frac{m}{n}+1}$

$\Rightarrow \quad \dfrac{a}{b} = \dfrac{\frac{m}{n}+2}{\frac{-4m}{n}-1} = \dfrac{3}{2}$

Which gives-

$\dfrac{m}{n} = -\dfrac{1}{2}$

Thus,

$$4\left[a-b\left(\frac{m}{n}\right)^2\right] = 4\left[3-2\left(-\frac{1}{2}\right)^2\right] = 4\left(3-\frac{1}{2}\right) = 4\times\frac{5}{2}$$

$$4\left[a-b\left(\frac{m}{n}\right)^2\right] = 10$$

67. The quadrilateral formed by the points A(1, 2, 5), B(–1, 6, 1), C(3, 4, –3) and D(5, 0, 1) is a

(a) Parallelogram (b) Rectangle
(c) Square (d) Rhombus

TS EAMCET-14.09.2020, Shift-I

Ans. (c) : Given,
Vertices of quadrilateral

$A(1, 2, 5), B(-1, 6, 1), C(3, 4, -3), D(5, 0, 1)$

$\therefore \quad AB = \sqrt{4+16+16} = 6$

$BC = \sqrt{16+4+16} = 6$

$CD = \sqrt{4+16+16} = 6$

$AD = \sqrt{16+4+16} = 6$

$AC = \sqrt{4+4+64} = \sqrt{72}$

$BD = \sqrt{36+36+0} = \sqrt{72}$

Here, $AB = BC = CD = AD$ and $AC = BD$

$\therefore$ ABCD forms a square.

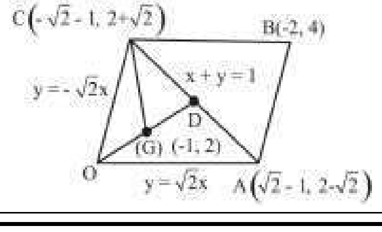
68. Let OABC be a parallelogram. The equation of one diagonal AC is $x+y-1=0$ and the combined equation of the sides OA, OC is $2x^2 - y^2 = 0$. If G is centroid of the triangle OAC, then BG =

(a) $2\sqrt{5}$ (b) $\dfrac{4}{3}\sqrt{5}$ (c) $\dfrac{2}{3}\sqrt{15}$ (d) $\dfrac{4}{9}\sqrt{5}$

TS EAMCET-14.09.2020, Shift-I

Ans. (b) : Equation of diagonal of AC of a parallelogram OABC is $x + y - 1 = 0$

Equation of OA and OC is $2x^2 - y^2 = 0$



$\because \quad y = \pm\sqrt{2}x$

Coordinate of $A\left(\sqrt{2}-1, 2-\sqrt{2}\right)$

$C = \left(-\sqrt{2}-1, 2+\sqrt{2}\right)$

D is mid-point of AC

$$D = \left(\frac{\sqrt{2}-1-\sqrt{2}-1}{2}, \frac{2-\sqrt{2}+2+\sqrt{2}}{2}\right) = (-1, 2)$$

D is also mid-point of OB

$\because B(-2, 4)$

Centroid

$$G = \left(\frac{\sqrt{2}-1-\sqrt{2}-1+0}{3}, \frac{2-\sqrt{2}+2+\sqrt{2}+0}{3}\right)$$

$$G = \left(\frac{-2}{3}, \frac{4}{3}\right)$$

$$BG = \sqrt{\left(\frac{-2}{3}+2\right)^2 + \left(\frac{4}{3}-4\right)^2} = \sqrt{\frac{16+64}{9}} = \frac{4\sqrt{5}}{3}$$

69. If the points A(–1, 0, 7), B(3, 2, t), C (5, k, –2) are collinear, then the ratio in which the point P (t, k – 2t, t + k) divides the line segment BC is

(a) –2 : 3 (b) –1 : 2 (c) 4 : 3 (d) 1 : 1

TS EAMCET-10.09.2020, Shift-I

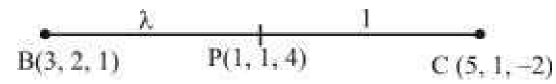
Ans. (b) : Given,
A(–1, 0, 7), B(3, 2, t) and C(5, k, –2) are collinear.

$\therefore \quad \dfrac{3+1}{5-3} = \dfrac{2-0}{k-2} = \dfrac{t-7}{-2-t}$

$\Rightarrow \quad 2 = \dfrac{2}{k-2} = \dfrac{t-7}{-2-t} \Rightarrow k = 3, t = 1$

$\therefore \quad B = (3, 2, 1), C = (5, 3, -2)$ and $P(1, 1, 4)$

Let P divides BC in the ratio λ : 1



$\dfrac{5\lambda+3}{\lambda+1} = 1$

$\Rightarrow 5\lambda + 3 = \lambda + 1 \Rightarrow 4\lambda = -2 \Rightarrow \lambda = -\dfrac{1}{2}$

$\therefore$ Required ratio = –1 : 2

70. The point on the line $4x - y - 2 = 0$ which is equidistant from the points (–5, 6)and (3, 2) is

(a) (2, 6) (b) (4, 14) (c) (1, 2) (d) (3, 10)

TS EAMCET-04.08.2021, Shift-I

Ans. (b): Let, the point on line,

$4x - y - 2 = 0$ be $P(x, y)$

Let, $A = (-5, 6)$ and $B = (3, 2)$

$4x - y - 2 = 0$ ....(i)

Point, P is equivalent from point A and B

$\therefore AP = PB$

By distance formula,

$$(x+5)^2+(y-6)^2=(x-3)^2+(y-2)^2$$

$$x^2+25+10x+y^2+36-12y=x^2+9-6x+y^2+4-4y$$

$$16x-8y+48=0$$

$$4x-2y+12=0 \qquad \text{.............(ii)}$$

subtracting eq^n. (ii) – eq^n.(i) we get -

$y=14$

$x=4$

So, point on the line is $(4,14)$

71. The distance of the point (3, 5) from 2x + 3y –14 = 0 measured parallel to x – 2y = 1 is

(a) $\frac{7}{\sqrt{5}}$ (b) $\frac{7}{\sqrt{13}}$ (c) $\sqrt{5}$ (d) $\sqrt{13}$

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Ans. (c) : Let the equation of the line parallel to x–2y = 1 is $x-2y+\lambda=0$

Since, it passes through (3, 5)

$$3-10+\lambda=0 \Rightarrow \lambda=7$$

$\therefore$ The line is $x-2y+7=0$

The point of intersection of $x-2y+7=0$ and $2x+3y-14=0$ is (1, 4)

$\therefore$ The distance between (3, 5) and (1, 4)

$$=\sqrt{(3-1)^2+(5-4)^2}=\sqrt{4+1}=\sqrt{5}$$

3. Co-ordinates of different of centers of triangles (centroid, circumcenter, orthocenter & incenter etc.)

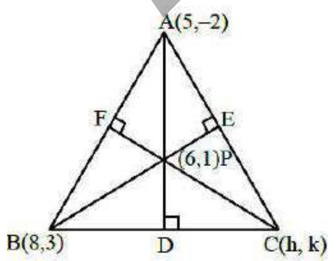
72. If P(6, 1) be the orthocenter of the triangle whose vertices are A (5, –2), B (8, 3) and C (h, k), then the point C lies on the circle.

(a) $x^2+y^2-65=0$ (b) $x^2+y^2-74=0$

(c) $x^2+y^2-61=0$ (d) $x^2+y^2-52=0$

JEE MAIN-06.04.2024, Shift-II

Ans. (a) :



Slope of line = $\frac{y_2-y_1}{x_2-x_1}$

So, Slope of AD = 3

Slope of BC (m_2)

$$m_1m_2=-1$$

$$m_2=-\frac{1}{3}$$

Equation of line ,$(y-y_1)=m(x-x_1)$

Equation of BC is 3y + x – 17 = 0 ... (i)

Similarly for slope of BE = 1

Slope of AC = –1

Equation of AC is x + y – 3 = 0 ... (ii)

Now solving equation (i) and (ii) we get,

x = – 4 and y = 7

So, point C is (–4, 7)

Point C (–4, 7) satisfy the equation of circle $x^2+y^2-65=0$

Hence, point C lies on the circle $x^2+y^2-65=0$

73. Let a rectangle ABCD of sides 2 and 4 be inscribed in another rectangle PQRS such that the vertices of the rectangle ABCD lie on the sides of the rectangle PQRS. Let a and b be the sides of the rectangle PQRS when its area is maximum. Then $(a+b)^2$ is equal to:

(a) 72

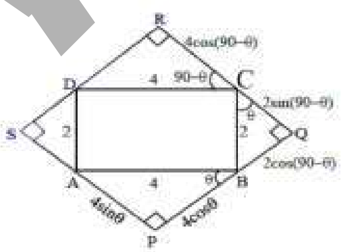
(b) 60

(c) 80

(d) 64

JEE MAIN-05.04.2024, Shift-I

Ans. (a) : According to question,



From figure PB= 4 cos θ

BQ = 2 sin θ

So, PQ = PB+ BQ = 4 cos θ + 2 sinθ

And CR = 4 sinθ, CQ = 2 cos θ

So, QR = CR + CQ = 4 sinθ + 2 cos θ

Area of rectangle PQRS=

$$(4\cos\theta+2\sin\theta)(2\cos\theta+4\sin\theta)$$

$$=8\cos^2\theta+16\sin\theta\cos\theta+4\sin\theta\cos\theta+8\sin^2\theta$$

= 8 + 20 sinθcosθ

= 8 + 10 sin2θ

For maximum sin2θ = 1, θ = 45°

Maximum Area = 8 + 10 = 18

Hence, the value of

$$(a+b)^2=(4\cos\theta+2\sin\theta+2\cos\theta+4\sin\theta)^2$$

$$(a+b)^2=(6\sin\theta+6\cos\theta)^2$$

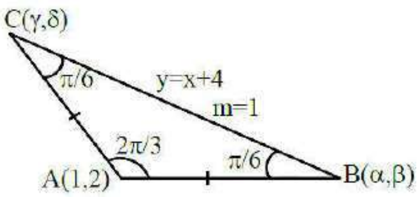
$$(a+b)^2=36(\sin\theta+\cos\theta)^2$$

$$(a+b)^2=36(\sqrt{2})^2$$

$$(a+b)^2=72$$

74. Consider a triangle ABC having the vertices A(1, 2), B(α, β) and C(γ, δ) and angles $\angle BAC=\frac{2\pi}{3}$. If the points B and C lie on the line y = x + 4, then $\alpha^2+\gamma^2$ is equal to....

Ans :(14)



Equation of line passes through point A(1, 2) which makes angle $\frac{\pi}{6}$ from y = x + 4 is

$$y-2=\frac{1\pm\tan\frac{\pi}{6}}{1\mp\tan\frac{\pi}{6}}(x-1)$$

$$y-2=\frac{\sqrt{3}\pm1}{\sqrt{3}\mp1}(x-1)$$

$\oplus$ $\qquad\qquad$ $\ominus$

$y-2=(2+\sqrt{3})(x-1)$ $\qquad$ $y-2=(2-\sqrt{3})(x-1)$

solve with y = x + 4 $\qquad$ solve with y = x + 4

$x+2=(2+\sqrt{3})x-2-\sqrt{3}$ $\qquad$ $x+2=(2-\sqrt{3})x-2+\sqrt{3}$

$\alpha = x=\frac{4+\sqrt{3}}{1+\sqrt{3}}$ $\qquad$ $\beta=x=\frac{4-\sqrt{3}}{1-\sqrt{3}}$

$$\alpha^2+\gamma^2=\left(\frac{4+\sqrt{3}}{1+\sqrt{3}}\right)^2+\left(\frac{4-\sqrt{3}}{1-\sqrt{3}}\right)^2$$

$$=\frac{(4+\sqrt{3})^2}{(1+\sqrt{3})^2}+\frac{(4-\sqrt{3})^2}{(1-\sqrt{3})^2}$$

$$=\frac{16+3+8\sqrt{3}}{1+3+2\sqrt{3}}+\frac{16+3-8\sqrt{3}}{1+3-2\sqrt{3}}$$

$$=\frac{19+8\sqrt{3}}{4+2\sqrt{3}}+\frac{19-8\sqrt{3}}{4-2\sqrt{3}}$$

$$=\frac{76-38\sqrt{3}+32\sqrt{2}-48+76+38\sqrt{3}-32\sqrt{3}-48}{16-12}$$

$$\alpha^2+\gamma^2=\frac{152-96}{4}=\frac{56}{4}=14$$

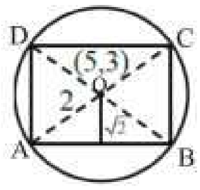
75. A square is inscribed in the circle $x^2+y^2-10x-6y+30=0$. One side of this square is parallel to y = x + 3. If (x_i, y_i) are the vertices of the square, then $\sum\left(x_i^2+y_i^2\right)$ is equal to:

(a) 148 $\qquad$ (b) 156

(c) 160 $\qquad$ (d) 152

JEE MAIN-04.04.2024, Shift-I

Ans. (d) :



$x^2+y^2-10x-6y+30=0$

Radius = 2

Line of equation y = x + 3

Slope of BC = –1

Slope of OB = $\frac{m-1}{1+m}$

m = 0

Parametric form of equation-

$$\frac{x-x_1}{\cos\theta}=\frac{y-y_1}{\sin\theta}=\pm r$$

$$\frac{x-5}{\cos 0}=\frac{y-3}{\sin 0}=\pm 2$$

x = 5 +2 $\qquad$ y = 3 ± 0

x = 5 ± 2

y = 3 ± 0

pts (5, 5), (3, 3), (7, 3), (5, 1)

$\sum\left(x_i^2+y_1^2\right)=25+25+9+9+49+9+25+1$

= 152

Option (d)

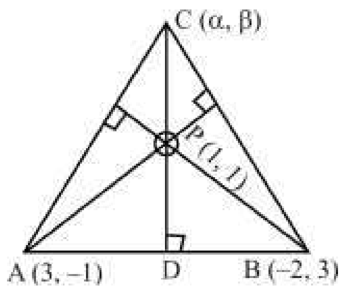
76. Two vertices of a triangle ABC are A (3, –1) and B (–2, 3), and its orthocentre is P (1, 1). If the coordinates of the point C are (α, β) and the centre of the circle circumscribing the triangle PAB is (h, k), then the value of (α + β) + 2 (h + k) equals :

(a) 51 $\qquad$ (b) 81

(c) 5 $\qquad$ (d) 15

JEE MAIN-09.04.2024, Shift-II

Ans. (c) :



$M_{AB}=\frac{4}{-5}\Rightarrow M_{DP}=\frac{5}{4}$

Equation of PC is $y-1=\frac{5}{4}(x-1)$ ....(1)

$M_{AP}=\frac{2}{-2}=-1\Rightarrow M_{BC}=+1$

Equation of BC -

y – 3 = (x + 2) $\qquad$ .... (2)

On solving (1) and (2),

α = 21

⇒ β = y = x + 5 = 26

α + β = 47

Equation of ⊥bisector of AP

y – 0 = (x – 2) .... (3)

Equation of ⊥bisector of AB

$$y - 1 = \frac{5}{4}\left(x - \frac{1}{2}\right) \text{ .... (4)}$$

On solving (3) & (4)

$$(x - 3)\, 4 = 5x - \frac{5}{2}$$

$$x = \frac{-19}{2} = h$$

$$y = \frac{-23}{2} = k$$

$\Rightarrow$ $2(h + k) = -42$

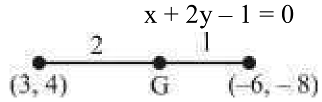
So, $\alpha+\beta + 2(h+k) = 47 - 42 = 5$

77. If the orthocentre of the triangle formed by the lines $2x + 3y - 1 = 0$, $x + 2y - 1 = 0$ and $ax + by - 1 = 0$, is the centroid of another triangle, whose circumecentre and orthocentre respectively are (3, 4) and (–6, –8), then the value of $|a - b|$ is ______.

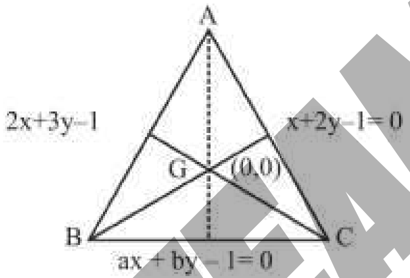
JEE MAIN-08.04.2024, Shift-I

Ans. (16) : Given, Lines $2x + 3y - 1 = 0$ ....(i)

$x + 2y - 1 = 0$ ...(ii)



$$G\left(\frac{-6+6}{3}, \frac{-8+8}{3}\right) = G(0, 0)$$



We know that, equation of line $y = mx + c$

By equation (i) $y = \frac{1}{3} - \frac{2}{3}x$

$$m_1 = -\frac{2}{3}$$

We know that, $m_1 \times m_2 = -1$

So, $m_2 = \frac{3}{2}$

$$y = \frac{3}{2}x \quad \text{...(iii)}$$

Put the value of y in equation (ii)

$$x + 3x - 1 = 0$$

$$x = \frac{1}{4}, \quad y = \frac{3}{8}$$

$$C\left(\frac{1}{4}, \frac{3}{8}\right)$$

By equation (ii)

$$y = \frac{1}{2} - \frac{x}{2}$$

$$m_1 = -\frac{1}{2}$$

So, $m_2 = 2$

$$y = 2x \quad \text{...(iv)}$$

Put the value of y in equation (i)

$$2x + 6x - 1 = 0$$

$$x = \frac{1}{8} \quad y = \frac{1}{4}$$

$$B\left(\frac{1}{8}, \frac{1}{4}\right)$$

Given, Equation

$$ax + by - 1 = 0$$

From point C, $\frac{a}{4} + \frac{3b}{8} - 1 = 0$ ...(v)

Point B, $\frac{a}{8} + \frac{b}{4} - 1 = 0$ ... (vi)

Solving Equation (v) and (vi)

$$\left(\frac{3}{16} - \frac{1}{4}\right)b = \frac{1}{2} - 1$$

$$\left(\frac{3-4}{16}\right)b = \frac{-1}{2}$$

$b = 8$

$a = -8$

$$|a - b| = |-8 - 8|$$

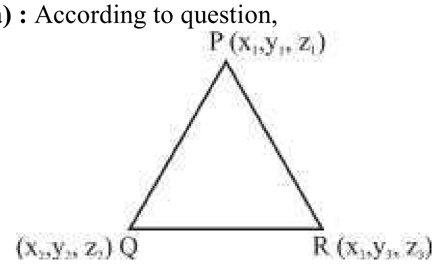
$$= 16$$

78. Let P and Q be the points on the line $\frac{x \quad 3}{8} \quad \frac{y \quad 4}{2} \quad \frac{z \quad 1}{2}$ which are at a distance of 6 units from the point R(1, 2, 3). If the centroid of the triangle PQR is (, ,), then $^2 \quad ^2 \quad ^2$ is:

(a) 18 (b) 24
(c) 26 (d) 36

JEE MAIN-01.02.2024, Shift-II

Ans. (a) : According to question,



Centriod at Δ PQR

$$= \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}, \frac{z_1, + z_2 + z_3}{3}\right)$$

$$\frac{x+3}{8} = \frac{y-4}{2} = \frac{z+1}{2} = k$$

Point on the line P (8k – 3, 2k + 4, 2k – 1) and R(1, 2, 3)

$$\therefore \text{ PR} = 6 = \sqrt{(8k-4)^2 + (2k+2)^2 + (2k-4)^2}$$

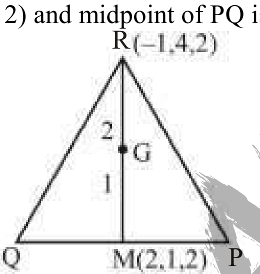
$9 = (4k - 2)^2 + (k + 1)^2 + (k - 2)^2$
$9 = 18k^2 - 18k + 9$
$18k(k - 1) = 0$
$k = 0$ & $k = 1$
Therefore, P (–3, 4, –1) Q (5, 6, 1) and R (1, 2, 3)
Centriod of Δ PQ R = $\left(\frac{-3+5+1}{3}, \frac{4+6+2}{3}, \frac{-1+1+3}{3}\right)$
$= (1, 4, 1)$
$\therefore$ $(1, 4, 1) = (\alpha, \beta, \gamma)$
$\alpha = 1,\ \beta = 4, \gamma = 1$
Hence, $\alpha^2 + \beta^2 + \gamma^2 = (1)^2 + (4)^2 + (1)^2$
$= 1 + 16 + 1$
$= 18$

79. Let PQR be a triangle with R (–1, 4, 2). Suppose M (2, 1, 2) is the midpoint of PQ. The distance of the centroid of ΔPQR from the point of intersection of the lines $\frac{x-2}{0} = \frac{y}{2} = \frac{z+3}{-1}$ and $\frac{x-1}{1} = \frac{y+3}{-3} = \frac{z+1}{1}$ is

(a) $\sqrt{99}$ (b) 9
(c) $\sqrt{69}$ (d) 69

JEE MAIN-29.01.2024, Shift-I

Ans. (c) : Given,
R(–1, 4, 2) and midpoint of PQ is M (2, 1, 2).



Centroid of Δ PQR =
$G\left(\frac{4-1}{3}, \frac{2+4}{3}, \frac{4+2}{3}\right) \equiv G(1, 2, 2)$

$\frac{x-2}{0} = \frac{y}{2} = \frac{z+3}{-1} = \lambda \Rightarrow (2, 2\lambda, -\lambda - 3)$ ... (L_1)

$\frac{x-1}{1} = \frac{y+3}{-3} = \frac{z+1}{1} = \mu \Rightarrow (\mu+1, -3\mu-3, \mu-1)$ ...(L_2)

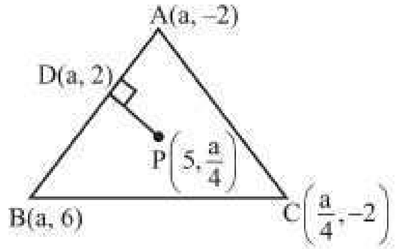
$\mu + 1 = 2 \quad \Rightarrow \mu = 1$
$2\lambda = -3\mu - 3$
$2\lambda = -6$
$\lambda = -3$
Point of intersection P(2, –6, 0)
PG = $\sqrt{1+64+4} = \sqrt{69}$

80. Let $\left(5, \frac{a}{4}\right)$ be the circumcenter of a triangle with vertices A (a, –2), B (a, 6) and $C\left(\frac{a}{4}, -2\right)$. Let α denote the circumradius, β denote the area and γ denote the perimeter of the triangle. Then α + β + γ is

(a) 30 (b) 53 (c) 62 (d) 60

JEE MAIN-29.01.2024, Shift-I

Ans. (b) : D is mid point of AB
AB is perpendicular to PD
$2 = \frac{a}{4} \quad \Rightarrow a = 8$



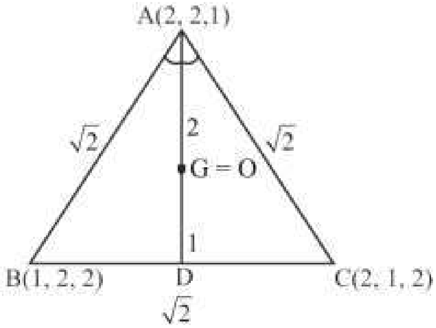
A(8, –2), B(8, 6), C(2, –2), P(5, 2)
α = AP = $\sqrt{9+16} = 5$ unit
AB = 8, BC = 10, AC = 6
so, this is a right angle triangle
$\therefore$ Area = $\beta = \frac{1}{2}(8)(6) = 24$ unit
Perimeter = γ = 24 unit
Hence, $\alpha + \beta + \gamma = 53$

81. Let the position vectors of the vertices A, B and C of a triangle be $2\hat{i}+2\hat{j}+\hat{k}$, $\hat{i}+2\hat{j}+2\hat{k}$ and $2\hat{i}+\hat{j}+2\hat{k}$ respectively. Let l_1, l_2 and l_3 be the lengths of perpendiculars drawn from the ortho center of the triangle on the sides AB, BC and CA respectively, then $l_1^2 + l_2^2 + l_3^2$ equals.

(a) $\frac{1}{5}$ (b) $\frac{1}{2}$
(c) $\frac{1}{4}$ (d) $\frac{1}{3}$

JEE MAIN-27.01.2024, Shift-II

Ans. (b) : Given, $2\hat{i}+2\hat{j}+\hat{k}$, $\hat{i}+2\hat{j}+2\hat{k}$ and $2\hat{i}+\hat{j}+2\hat{k}$



$AB = \sqrt{(2-1)^2 + (2-2)^2 + (1-2)^2} = \sqrt{2}$

$BC = \sqrt{(1-2)^2 + (2-1)^2 + (2-2)^2} = \sqrt{2}$

$CA = \sqrt{(2-2)^2 + (1-2)^2 + (2-1)^2} = \sqrt{2}$

$\Rightarrow$ AB = BC = CA
Its an equilateral Δ
Here, D is midpoint of BC
$D\left(\frac{3}{2}, \frac{3}{2}, 2\right)$

$\therefore$ Orthocenter and centroid will be same

And G = $\left(\frac{5}{3},\frac{5}{3},\frac{5}{3}\right)$

$l_1 = \frac{1}{\sqrt{6}}$, $l_2 = \frac{1}{\sqrt{6}}$,$l_3 \frac{1}{\sqrt{6}}$

$\therefore l_1^2 + l_2^2 + l_3^2 = \frac{3}{6} = \frac{1}{2}$

82. Let a, b, c be the lengths of three sides of a triangle satisfying the condition $(a^2 \quad b^2)x^2 \quad 2b(a \quad c)x \quad (b^2 \quad c^2) \quad 0$. If the set of all possible values of x is the interval $(\; , \;)$, $12(\;^2 \quad \;^2)$ is equal to_______.

JEE MAIN-31.01.2024, Shift-II

Ans. :(36) Given that –

$(a^2 + b^2)x^2 - 2b(a + c)x + b^2 + c^2 = 0$

$a^2x^2 - 2bax + b^2 + b^2x^2 - 2bcx + c^2 = 0$

$(ax - b)^2 + (bx - c)^2 = 0$

$ax - b = 0$ and $(bx - c) = 0$

Now,

Case (i) $a + b > c$

$a + ax > bx$ (put b = ax, c = bx)

$a + ax > ax^2$ (put b = ax)

$x^2 - x - 1 < 0$

$\frac{1-\sqrt{5}}{2} < x < \frac{1+\sqrt{5}}{2}$

Case (ii) $b + c > a$

$ax + bx > a$ (put b = ax, c = bx)

$ax + ax^2 > a$ (put b = ax)

$x^2 + x - 1 > 0$

$x < \frac{-1-\sqrt{5}}{2}$ or $x > \frac{-1+\sqrt{5}}{2}$

Case (iii) $c + a > b$

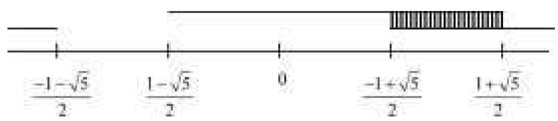
$ax^2 + a > ax$

$x^2 - x + 1 > 0$

Always true $x \in R$

Combine (i), (ii) and (iii) –

$x \in \left(\frac{\sqrt{5}-1}{2},\frac{\sqrt{5}+1}{2}\right)$



$\frac{1 \quad \sqrt{5}}{2}$ $\quad$ $\frac{1 \quad \sqrt{5}}{2}$

Now $12 \quad \frac{1 \quad \sqrt{5}}{2}^2 \quad \frac{1 \quad \sqrt{5}}{2}^2$

36

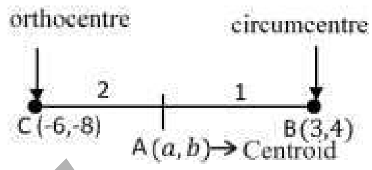
83. Let A(a,b),B(3,4) and C(–6,–8) respectively denote the centroid, circumcenter and orthocenter of a triangle. Then, the distance of the point P(2a + 3, 7b + 5) from the line 2x + 3y – 4 = 0 measured parallel to the line x – 2y –1 = 0 is

(a) $\frac{\sqrt{5}}{17}$ (b) $\frac{15\sqrt{5}}{7}$

(c) $\frac{17\sqrt{5}}{7}$ (d) $\frac{17\sqrt{5}}{6}$

JEE MAIN-31.01.2024, Shift-II

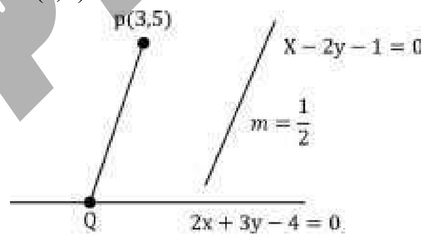
Ans. (c) :



$a \quad \frac{2 \quad 3 \quad 6}{3} \quad 0$

$b \quad \frac{2 \quad 4 \quad 8}{3} \quad 0$

So point P(3,5)



Equation of PQ

$y - 5 = \frac{1}{2}(x - 3)$

$2y \quad 10 \quad x \quad 3$

$2y \quad x \quad 7 \quad 0$

Intersection point

$2(2y \quad x \quad 7 \quad 0)$

$\frac{3y \quad 2x \quad 4 \quad 0}{7y \quad 18}$

$y \quad \frac{18}{7}$

Put, this value in $2y - x - 7 = 0$

$2 \quad \frac{18}{7} \quad x \quad 7 \quad 0$

$\frac{36 \quad 49}{7} \quad x$

$x \quad \frac{13}{7}$

Point Q $\frac{13}{7},\frac{18}{7}$

Now, P(3,5) and $Q\left(\frac{-13}{7},\frac{18}{7}\right)$

So, distance between P and Q are

$$PQ \quad \sqrt{\left(\frac{13}{7} \quad 3\right)^2 \quad \left(\frac{18}{7} \quad 5\right)^2}$$

$$PQ \quad \sqrt{\left(\frac{34}{7}\right)^2 \quad \left(\frac{17}{7}\right)^2}$$

$$PQ \quad \frac{\sqrt{1156 \quad 289}}{7} \quad \frac{17\sqrt{5}}{7}$$

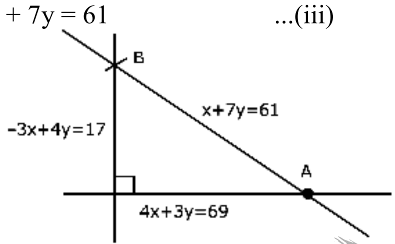
84. Let C(α, β) be the circumcenter of the triangle formed by the lines
4x + 3y = 69
4y – 3x = 17
x + 7y = 61
Then $(\alpha - \beta)^2 + \alpha + \beta$ is equal to

(a) 18 (b) 15
(c) 16 (d) 17

JEE MAIN-08.04.2023, Shift-I

Ans. (d) : Given,

4x + 3y = 69 ...(i)
4y – 3x = 17 ...(ii)
x + 7y = 61 ...(iii)



In equation (iii) multiply by 4 then subtracted to the equation (i)

$$\begin{array}{r} 4x + 28y = 244 \\ 4x + 3y = 69 \\ - \quad - \quad - \\ \hline 25y = 175 \end{array}$$

y = 7, x = 12
A(12, 7)

For point B, in equation (iii) ×3 then adding equation (ii)

–3x + 4y = 17
3x + 21y = 183
25y = 200
y = 8, x = 5
B(5, 8)

$\therefore$ Circum center $= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

$$\left(\frac{12+5}{2}, \frac{7+8}{2}\right) = \left(\frac{17}{2}, \frac{15}{2}\right)$$

$$\alpha = \frac{17}{2},\ \beta = \frac{15}{2}$$

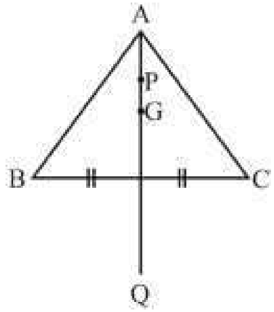
So, the value of $(\alpha - \beta)^2 + \alpha + \beta$ = 1 + 16 = 17

85. If the points P and Q are respectively the circumecenter and the orthocenter of a ΔABC, the $\overrightarrow{PA} + \overrightarrow{PB} + \overrightarrow{PC}$ is equal to

(a) $2\overrightarrow{QP}$ (b) $\overrightarrow{PQ}$
(c) $2\overrightarrow{PQ}$ (d) $\overrightarrow{PQ}$

JEE MAIN-10.04.2023, Shift-II

Ans. (d) :



We know that centroid divides the distance between orthocenter and circumcentre in 2:1.

$\because\ \overrightarrow{GQ} = 2\overrightarrow{PG}$

Let D is the midpoint of length BC.
So, By internal division formula

$$\overline{O} = \frac{mx_2 + nx_1}{m+n}$$

$$\overrightarrow{PD} = \frac{(1)\overrightarrow{PB} + (1)\overrightarrow{PC}}{1+1}$$

$$2\overrightarrow{PD} = \overrightarrow{PB} + \overrightarrow{PC}$$

Both side adding $\overrightarrow{PA}$

$$\overrightarrow{PA} + \overrightarrow{PB} + \overrightarrow{PC} = \overrightarrow{PA} + 2\overrightarrow{PD} \quad \ldots(i)$$

$\because$ We know that (G) divides the points A and midpoint of opposite side BC that is D in the ratio of 2:1

$$\because\ \overrightarrow{PG} = \frac{(1)\overrightarrow{PA} + 2\overrightarrow{PD}}{2+1}$$

$$3\overrightarrow{PG} = \overrightarrow{PA} + 2\overrightarrow{PD} \quad \ldots(ii)$$

$\because$ Equation (i) and (ii) has same R.H.S

$\because$ Their L.H.S should same

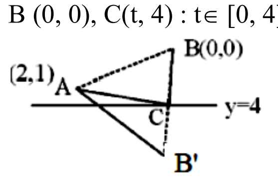
$$\overrightarrow{PA} + \overrightarrow{PB} + \overrightarrow{PC} = 3\overrightarrow{PG}$$
$$\overrightarrow{PA} + \overrightarrow{PB} + \overrightarrow{PC} = 2\overrightarrow{PG} + \overrightarrow{PG}$$
$$\overrightarrow{PA} + \overrightarrow{PB} + \overrightarrow{PC} = \overrightarrow{GQ} + \overrightarrow{PG}$$
$$\overrightarrow{PA} + \overrightarrow{PB} + \overrightarrow{PC} = \overrightarrow{PQ}$$

86. Consider the triangles with vertices A(2, 1), B(0, 0) and C(t, 4), t ∈ [0, 4]. If the maximum and the minimum perimeters of such triangles are obtained at t = α and t = β respectively, then 6α + 21β is equal to ____

JEE MAIN-15.04.2023, Shift-I

Ans. (48) : Given vertices of triangle-

A(2, 1), B (0, 0), C(t, 4) : t∈ [0, 4]



$B_1(0, 8)$ ≡ image of B w.r.t. y = 4

For AC + BC + AB to be minimum

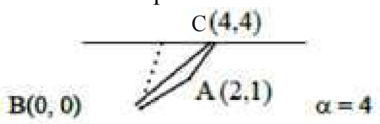
$$m_{AB'} = \frac{-7}{2}$$

Line $AB_1 = 7x + 2y = 16$

$$C\left(\frac{8}{7}, 4\right)$$

$$\beta = \frac{8}{7}$$

For maximum perimeter



$$AB = \sqrt{5} : BC = 4\sqrt{2}, AC = \sqrt{13}$$

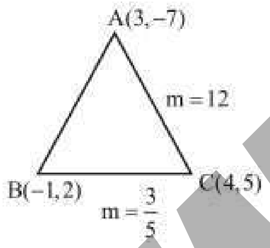
Hence, $6\alpha + 21\beta = 24 + 24 = 48$

87. If (α, β) is the orthocenter of the triangle ABC with vertices A(3, –7), B(–1, 2) and C(4, 5), then 9α – 6β + 60 is equal to

(a) 30 (b) 35
(c) 40 (d) 25

JEE MAIN-15.04.2023, Shift-I

Ans. (d) :



Altitude of BC : $y + 7 = \frac{-5}{3}(x - 3)$

$3y + 21 = -5x + 15$
$5x + 3y + 6 = 0$

Altitude of AC : $y - 2 = \frac{-1}{12}(x + 1)$

$12y - 24 = -x - 1$
$x + 12y = 23$

$$\alpha = \frac{-47}{19},\ \beta = \frac{121}{57}$$

Hence, $9\alpha - 6\beta + 60 = 9\times\left(-\frac{47}{19}\right) - 6\times\left(\frac{121}{57}\right) - 60 = 25$

88. Let P $\left(\frac{2\sqrt{3}}{\sqrt{7}}, \frac{6}{\sqrt{7}}\right)$, Q, R, and S be four points on the ellipse $9x^2 + 4y^2 = 36$. Let PQ and RS be mutually perpendicular and pass through the origin. If $\frac{1}{(PQ)^2} + \frac{1}{(RS)^2} = \frac{p}{q}$, Where p and q are coprime, then p + q is equal to

(a) 137 (b) 143
(c) 157 (d) 147

JEE MAIN-12.04.2023, Shift-I

Ans. (c) : Given,

$$9x^2 + 4y^2 = 36$$

$$\frac{x^2}{4} + \frac{y^2}{9} = 1$$

PQ passes through origin and ellipse is symmetrical in both axes

$$\Rightarrow Q\left(\frac{-2\sqrt{3}}{\sqrt{7}}, -\frac{\sqrt{6}}{\sqrt{7}}\right)$$

$$PQ^2 = \left(\frac{4\sqrt{3}}{\sqrt{7}}\right)^2 + \left(\frac{2\times 6}{\sqrt{7}}\right) = \frac{48+144}{7} = \frac{192}{7}$$

Slope of PQ is $m_1 = \frac{6}{2\sqrt{3}} = \sqrt{3}$

Equation of RS is $y = -\frac{1}{\sqrt{3}}x$

$$x = \pm\frac{6\sqrt{3}}{\sqrt{31}},\quad y = \mp\frac{6}{\sqrt{31}}$$

$$(RS)^2 = \left(\frac{12\sqrt{3}}{\sqrt{31}}\right)^2 + \left(\frac{12}{\sqrt{31}}\right)^2 = \frac{576}{31}$$

Hence, $\frac{1}{(PQ)^2} + \frac{1}{(RS)^2} = \frac{7}{192} + \frac{31}{576}$

$$= \frac{21+31}{576} = \frac{13}{144}$$

Then, $\frac{p}{q} = \frac{13}{144}$

Hence, $p + q = 13 + 144 = 157$

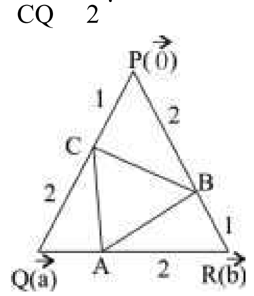
89. Let PQR be a triangle. The points A, B and C are on the sides QR, RP and PQ respectively such that $\frac{QA}{AR} = \frac{RB}{BP} = \frac{PC}{CQ} = \frac{1}{2}$. Then $\frac{\text{Area}(\Delta PQR)}{\text{Area}(\Delta ABC)}$ is equal to

(a) 4 (b) 2
(c) $\frac{5}{2}$ (d) 3

JEE MAIN-24.01.2023, Shift-I

Ans. (d) : Given triangle is PQR.

and $\frac{QA}{AR} = \frac{RB}{BP} = \frac{PC}{CQ} = \frac{1}{2}$.



Let position vector of P,Q and R be $\vec{0}$ $\vec{a}$ and $\vec{b}$

Now, Position vector of A = $\dfrac{1(\vec{b})+2(\vec{a})}{1+2} = \dfrac{2\vec{a}+\vec{b}}{3}$

Position vector of B $= \dfrac{2\vec{b}+\vec{0}(1)}{2+1} = \dfrac{2\vec{b}}{3}$

And, Position vector of C = s

So, $\overrightarrow{CA} = \vec{A}-\vec{C} = \dfrac{2\vec{a}+\vec{b}}{3} - \dfrac{\vec{a}}{3}$

$\overrightarrow{CA} = \dfrac{\vec{a}+\vec{b}}{3}$

$\overrightarrow{AB} = \vec{B}-\vec{A} = \dfrac{2\vec{b}}{3} - \left(\dfrac{2\vec{a}+\vec{b}}{3}\right)$

$\overrightarrow{AB} = \dfrac{\vec{b}-2\vec{a}}{3}$

Now,

Area of $\Delta PQR = = \dfrac{1}{2}\left|\overrightarrow{PQ}\times\overrightarrow{PR}\right| = \dfrac{1}{2}\left|\vec{a}\times\vec{b}\right|$

And Area of ΔABC =

$\dfrac{1}{2}\left|\overrightarrow{CA}\times\overrightarrow{AB}\right| = \dfrac{1}{2}\left|\left(\dfrac{\vec{a}+\vec{b}}{3}\right)\times\left(\dfrac{\vec{b}-2\vec{a}}{3}\right)\right| = \dfrac{1}{2}\left|\dfrac{\vec{a}\times\vec{b}}{3}\right|$

Now,

$$\frac{\text{Area of }\Delta PQR}{\text{Area of }\Delta PQR} = \frac{\frac{1}{2}\left|\vec{a}\times\vec{b}\right|}{\frac{1}{2}\left|\frac{\vec{a}\times\vec{b}}{3}\right|} = 3$$

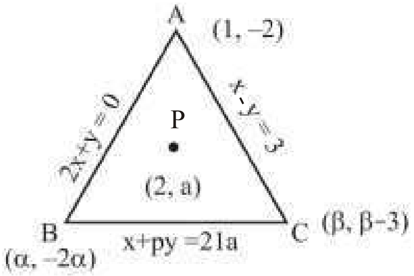
Hence, option (d) is correct.

90. The equations of the sides AB, BC and CA of a triangle ABC are : $2x + y = 0$, $x + py = 21a$, $(a \neq 0)$ and $x - y = 3$ respectively. Let P(2, a) be the centroid of ΔABC. Then $(BC)^2$ is equal to

JEE MAIN-24.01.2023, Shift-II

Ans. (122) :

$$2x+y=0$$
$$x-y=3$$
$$3x=3$$
$$\boxed{x=1}$$
$$\boxed{y=-2}$$



Centroid = $\dfrac{x_1+x_2+x_3}{3}, \left(\dfrac{y_1+y_2+y_3}{3}\right)$

$2 = \dfrac{1+\alpha+\beta}{3}, a = \dfrac{-2-2\alpha+\beta-3}{3}$

$\alpha+\beta = 5, \quad -2\alpha+\beta-5 = 3a$

$\boxed{-2\alpha+\beta = 5+3a}$

$$\alpha+\beta=5$$
$$-2\alpha \pm \beta = \_5 + \_3a$$
$$3\alpha = -3a$$
$$\boxed{\alpha=-a}$$

$\Rightarrow$ $\alpha + \beta = 5$

$-a + \beta = 5$

$\boxed{\beta = 5+a}$

$(\alpha, -2\alpha)$ lies on $x + py = 21a$

$\alpha + p(-2\alpha) = 21a$

$\alpha - 2pa = 21a$

$2pa - 22a = 0$

$2a(p - 11) = 0$

$a \neq 0$ or $p - 11 = 0$

$\boxed{p=11}$

Now equation of BC,

$x + 11y = 21a$

$\beta + 11(\beta - 3) = 21a$

$\beta + 11\beta - 33 = 21a$

$5 + a + 11(5 + a) - 33 = 21a$

$5 + a + 55 + 11a - 33 = 21a$

$27 = 9a$

$\boxed{a=3}$

$\alpha = -a$

$\alpha = -3$

$\beta = 5 + a$

$\beta = 5 + 3$

$\boxed{\beta=8}$

Coordinate of point B = $(\alpha, -2\alpha) = (-3, 6)$

Coordinate of point C= $(\beta, \beta - 3) = (8, 5)$

$BC^2 = (8 + 3)^2 + (5 - 6)^2$

$BC^2 = (11)^2 + (-1)^2$

$BC^2 = 121 + 1$

$\boxed{BC^2 = 122}$

91. A triangle is formed by the tangents at the point (2, 2) on the curves $y^2 = 2x$ and $x^2 + y^2 = 4x$, and the line $x + y + 2 = 0$. If r is the radius f its circumcircle, then r^2 is equal to ______.

JEE MAIN-29.01.2023, Shift-II

Ans. (10) : Tangent at $y^2 = 2x$

$T : 2y = = 2\left(\dfrac{x+2}{2}\right)$

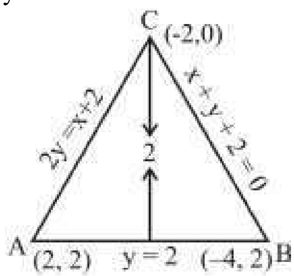
$2y = x + 2$

Tangent $x^2+y^2 = 4x$

$2x + 2y = \dfrac{4\times(x+2)}{2}$

$2x+2y = 2x+4$

y = 2



$AB = c = \sqrt{6^2 + 0} = 6$

$BC = a = \sqrt{4+4} = \sqrt{8}$

$CA = b = \sqrt{16+4} = \sqrt{20}$

$\Delta = \frac{1}{2} \times AB \times 2 = 6$

$r = \frac{abc}{4\Delta} = \frac{6 \times \sqrt{8} \times \sqrt{20}}{4 \times 6}$

$r = \frac{2\sqrt{2} \times 2\sqrt{5}}{4}$

$r = \sqrt{10}$

$\boxed{r^2 = 10}$

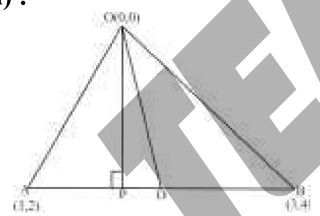
92. O(0, 0), A(1, 2), B(3, 4) are the vertices of ΔOAB. The joint equation of the altitude and median drawn from O is

(a) $x^2 + 7xy - y^2 = 0$ (b) $x^2 + 7xy + y^2 = 0$

(c) $3x^2 - xy - 2y^2 = 0$ (d) $3x^2 + xy - 2y^2 = 0$

MHT CET-2017

Ans. (d) :



D is midpoint of $AB \Rightarrow D \equiv (2,3)$

Hence equation of OD,

$$\frac{y-0}{3-0} = \frac{x-0}{2-0}$$

$$2y = 3x \Rightarrow 3x - 2y = 0 \quad \text{….(i)}$$

Slope of $AB = \frac{4-2}{3-1} = 1 \Rightarrow$ slope of OD is –1.

$\therefore$ Equation of OP,

$$(y-0) = -(x-0)$$

$$x + y = 0 \quad \text{…. (ii)}$$

Required joint equation is

$$(3x - 2y)(x + y) = 0$$

$$3x^2 + xy - 2y^2 = 0$$

93. If a, c, b are in G.P., then the area of the triangle formed by the lines ax + by + c = 0 with the coordinates axes is equal to

(a) 1 (b) 2 (c) $\frac{1}{2}$ (d) Nones of these

BITSAT-2020

Ans. (c) : Areas of the triangle

$= \frac{1}{2}$(x intercept) × (y intercept)

$= \frac{1}{2}\left(-\frac{c}{a}\right)\left(-\frac{c}{b}\right) = \frac{1}{2}\frac{c^2}{ab} = \frac{1}{2}$ unit

$[\because a, c, b \text{ are in G.P.} \Rightarrow c^2 = ab]$

94. The points $A(-a, -b), B(0, 0), C(a, b)$ and $D(a^2, ab)$ are

(a) Vertices of a rectangle

(b) Vertices of a parallelogram

(c) Vertices of a square

(d) Collinear

MHT CET-2020

Ans. (d) :

Distance between the points $A(-a,-b)$ and B(0,0)

$= \sqrt{(0+a)^2 + (0+b)^2} = \sqrt{a^2+b^2}$

Distance between the points $B(0,0)$ and C(a,b)

$\sqrt{(a-0)^2 + (b-0)^2} = \sqrt{a^2+b^2}$

Distance between the points $C(a,b)$ and $D(a^2,ab)$

$= \sqrt{(a^2-a)^2 + (ab-b)^2} = \sqrt{[a(a-1)]^2 + [b(a-1)]^2}$

$= \sqrt{a^2(a-1)^2 + b^2(a-1)^2} = \sqrt{(a^2+b^2)(a-1)^2}$

$= (a-1)\sqrt{a^2+b^2}$

Similarly, distance between the points $A(-a,-b)$ and $D(a^2,ab) = \sqrt{(a^2+a)^2 + (ab+b)^2} = (a+1)\sqrt{a^2+b^2}$

$AB + BC + CD = \sqrt{a^2+b^2} + \sqrt{a^2+b^2} + (a-1)\sqrt{a^2+b^2}$

$= (a+1)\sqrt{a^2+b^2} = AD$

Hence, the point are collinear.

95. If P(2, 2), Q(–2, 4) and R(3, 4) are the vertices of ΔPQR then the equation of the median through vertex R is

(a) x + 3y – 9 = 0 (b) x + 3y + 9 = 0

(c) x – 3y – 9 = 0 (d) x – 3y + 9 = 0

MHT CET-2019

Ans. (d) : Given,

$P = (2,2) Q = (-2,4) R = (3,4)$

Let S be the midpoint of PQ.

$$S = \left(\frac{2+(-2)}{2}, \frac{2+4}{2}\right) \Rightarrow S = (0,3)$$

$\therefore$ Equation of median RS is

$$\frac{y-3}{3-4} = \frac{x-0}{0-3} \text{ i.e. } \frac{y-3}{-1} = \frac{x}{-3}$$

$$-3y + 9 = -x \text{ i.e. } x - 3y + 9 = 0$$

96. The minimum area of the triangle formed by the variable line 3 $\cos\theta.x + 4\sin\theta.y=12$ and the co-ordinate axes is

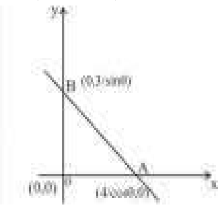
(a) 144 (b) 25/2 (c) 49/4 (d) 12

Karnataka CET-2013

Ans. (d) : The equation of line formed a triangle with x-axis and y-axis is

$$3\cos\theta.x + 4\sin\theta.y = 12$$

$$\frac{x}{(4/\cos\theta)} + \frac{y}{(3/\sin\theta)} = 1$$



Now, The area of triangle AOB will be

$$= \frac{1}{2}\times\text{base}\times\text{height} = \frac{1}{2}\times\frac{4}{\cos\theta}\times\frac{3}{\sin\theta} = \frac{12}{\sin 2\theta}$$

The minimum area of the triangle will be when the denominator will be maximum i.e. sin2θ will be maximum.

Area of $\Delta AOB = 12$

$\left[\begin{array}{l}\because \sin 2\theta \text{ maximum} \\ \text{will be } 1\end{array}\right]$

97. A = (cosθ, sinθ), B = (sinθ, – cosθ) are two points. The locus of the centroid of ΔOAB, where 'O' is the origin is

(a) $x^2+y^2=3$ (b) $9x^2+9y^2=2$

(c) $2x^2+2y^2=9$ (d) $3x^2+3y^2=2$

Karnataka CET-2013

Ans. (b) : Given,

Vertex of triangle AOB are

A = (cosθ, sinθ), B = (sinθ, –cosθ), O = (0, 0)

Let the co-ordinate of centroid of the triangle be (h, k)

So, $(h, k) = \left[\frac{\cos\theta+\sin\theta+0}{3}, \frac{\sin\theta-\cos\theta+0}{3}\right]$

$\cos\theta + \sin\theta = 3h$ ....(i)

$\sin\theta - \cos\theta = 3k$ ....(ii)

Now Square and add both equation (i) & (ii), we get-

$$(\cos\theta+\sin\theta)^2 + (\sin\theta-\cos\theta)^2 = 9h^2+9k^2$$

$$1+\sin 2\theta+1-\sin 2\theta = 9(h^2+k^2)$$

$$2 = 9(h^2+k^2)$$

Now, Interchange (h, k) with (x,y)

$\therefore$ $9x^2+9y^2=2$

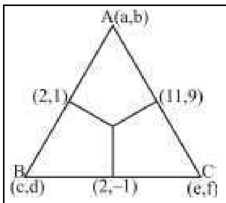
98. The points (11, 9), (2, 1) and (2, –1) are the midpoints of the sides of the triangle. Then the centroid is

(a) (5, 3) (b) (–5, –3) (c) (5, –3) (d) (3, 5)

Karnataka CET-2012

Ans. (a) : Given,

The co-ordinates of mid-points of triangle are as follows (11, 9), (2,1) and (2,–1)



Now,

Let the co-ordinates of vertices of triangle ABC are (a,b),(c,d) and (e,f)

Now, $\left(\frac{a+c}{2}, \frac{b+d}{2}\right) = (2,1)$

Similarly, $\left(\frac{a+e}{2}, \frac{b+f}{2}\right) = (11,9)$

$\left(\frac{c+e}{2}, \frac{d+f}{2}\right) = (2,-1)$

We know, The co-ordinates of the centroid are

$$\left(\frac{a+c+e}{3}, \frac{b+d+f}{3}\right)$$

$$= \left(\frac{11+2+2}{3}, \frac{9+1-1}{3}\right) = (5,3)$$

99. The mid points of the sides of triangle are (1, 5, –1), (0, 4, –2) and (2, 3, 4), then centroid of the triangle is

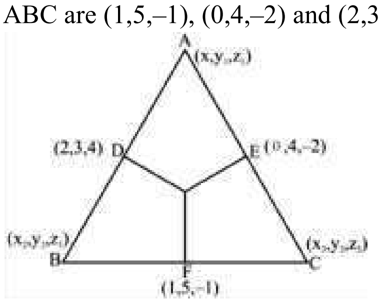
(a) (1, 4, 3) (b) $\left(1, 4, \frac{1}{3}\right)$

(c) (–1, 4, 3) (d) $\left(\frac{1}{3}, 2, 4\right)$

Karnataka CET-2021

Ans. (b) : Given,

The co-ordinates of mid points of a triangle ABC are (1,5,–1), (0,4,–2) and (2,3,4)



Let us assume the co-ordinate of A, B and C are (x_1, y_1, z_1), (x_2, y_2, z_2) and (x_3, y_3, z_3) respectively.

Now,

We know that,

$\therefore$ $\left[\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right] = (2,3,4)$

$\left[\frac{x_2+x_3}{2}, \frac{y_2+y_3}{2}, \frac{z_2+z_3}{2}\right] = (1,5,-1)$

$\left[\frac{x_1+x_3}{2}, \frac{y_1+y_3}{2}, \frac{z_1+z_3}{2}\right] = (0,4,-2)$

Now, $\frac{x_1+x_2}{2} + \frac{x_2+x_3}{2} + \frac{x_1+x_3}{2} = 1+2+0$

$\therefore \quad \frac{2(x_1+x_2+x_3)}{2}=3$

$x_1+x_2+x_3=3$

$x_3=3-(x_1+x_2)$

$=3-4 \qquad \left[\because \frac{x_1+x_2}{2}=2\right]$

$x_3=-1$

$\frac{x_2+x_3}{2}=1$

$x_2+x_3=2$

$x_2-1=2 \quad \Rightarrow \quad x_2=3$

$x_1=0-x_3 \quad \Rightarrow \quad x_1=1$

Similarly, $\frac{y_1+y_2}{2}=3, \frac{y_2+y_3}{2}=5, \frac{y_3+y_1}{2}=4$

$\therefore \quad 2(y_1+y_2+y_3)=24$

$y_1+y_2+y_3=12$

$y_3=6,\ y_2=4,\ y_1=2$

and, $\frac{z_1+z_2}{2}=4, \frac{z_2+z_3}{2}=-1, \frac{z_3+z_1}{2}=-2$

$z_1+z_2+z_3=1$

$\therefore \quad z_1=3, \quad z_2=5, \quad z_3=-7$

$\therefore \quad A=(1,2,3)$

$B=(3,4,5)$

$C=(-1,6,-7)$

Now,

Co-ordinate of centroid

$$G=\left[\frac{x_1+x_2+x_3}{3},\frac{y_1+y_2+y_3}{3},\frac{z_1+z_2+z_3}{3}\right]$$

$$=\left[\frac{1+3+(-1)}{3},\frac{2+4+6}{3},\frac{3+5-7}{3}\right]$$

$$=[1,4,1/3]$$

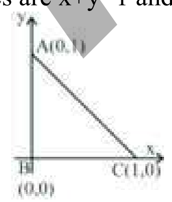
100. Orthocentre of the triangle formed by the lines x+y =1 and xy = 0 is

(a) (0,0) (b) (0,1) (c) (1,0) (d) (-1,1)

COMEDK-2011

CGPET-2010

Ans. (a) : Given lines are x+y=1 and xy=0



xy = 0 represents line x=0 and y=0.

Triangle formed by lines x + y=1, x=0

And y = 0 is ΔABC.

$\because \Delta ABC$ is right angled triangle at $\angle B$.

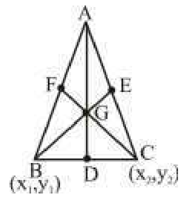
Orthocentre of ΔABC is at B(0, 0).

101. If a vertex of triangle is (3,3) and the mid points of two sides through this vertex are $\left(2,\frac{2}{3}\right)$ and $\left(4,\frac{3}{2}\right)$, then the centroid of the triangle is given by

(a) (1,3) (b) (3,0) (c) (3,4/9) (d) (0,3)

COMEDK-2012

Ans. (c) : : Let A=(3,3)



Given,

F= $\left(2,\frac{2}{3}\right)$ and E = $\left(4,\frac{3}{2}\right)$

Are mid-point of AB and AC

Let B=(x_1, y_1) and C=(x_2, y_2)

Using mid-point formula,

Co-ordinate of F= $\left(2,\frac{2}{3}\right)=\left(\frac{3+x_1}{2},\frac{3+y_1}{2}\right)$

$\Rightarrow (x_1,y_1)=\left(1,-\frac{5}{3}\right)$

Co–ordinate of E= $\left(4,\frac{3}{2}\right)=\left(\frac{3+x_2}{2},\frac{3+y_2}{2}\right)$

$\Rightarrow (x_2,y_2)=(5,0)$

Since, centroid of triangle is given by.

$$G(x,y)=\left(\frac{x_1+x_2+x_3}{3},\frac{y_1+y_2+y_3}{3}\right)$$

$$\therefore G(x,y)=\left(\frac{3+1+5}{3},\frac{3-\frac{5}{3}+0}{3}\right)=\left(3,\frac{4}{9}\right)$$

102. The x-coordinate of the incentre of the triangle that has the coordinates of mid points of its sides as (0, 1), (1, 1) and (1, 0) is

(a) $2-\sqrt{2}$ (b) $1+\sqrt{2}$ (c) $1-\sqrt{2}$ (d) $2+\sqrt{2}$

COMEDK-2013

Ans. (a) : The triangle whose midpoints are given to be (0,1), (1,0) and (1,1) happen to be a right angled triangle with vertices as shown.

1st solution: x-coordinate of in centre

$$=\frac{ax_1+bx_2+cx_3}{a+b+c}=\frac{2\times 2+2\sqrt{2}\times 0+2\times 0}{2+2+2\sqrt{2}}$$

$$=\frac{4}{4+2\sqrt{2}}=\frac{2}{2+\sqrt{2}}=2-\sqrt{2}$$

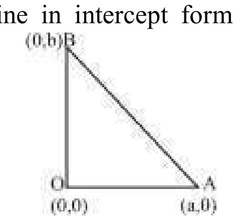
103. A straight line meets the coordinates axes at A and B, so that the centroid of the triangle OAB is (1,2) Then the equation of the line AB is

(a) x + y = 6 (b) 2x + y = 6

(c) x + 2y = 6 (d) 3x + y = 6

COMEDK-2015

Ans. (b) : Since, straight line meets the coordinate axes at A and B, so equation of line in intercept form is $\frac{x}{a}+\frac{y}{b}=1$.



$G\left(\frac{0+a+0}{3},\frac{0+0+b}{3}\right)=(1,2)$

$\Rightarrow \frac{a}{3}=1 \Rightarrow a=3, \frac{b}{3}=2 \Rightarrow b=6$

Hence, required equation of line is

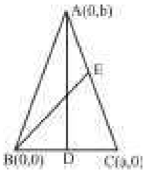
$\frac{x}{3}+\frac{y}{6}=1 \Rightarrow 2x+y=6$

104. The medians AD and BE of a triangle with vertices A(0, b), B(0, 0) and C(a, 0) are perpendicular to each other, if

(a) $a=\frac{b}{2}$ (b) $b=\frac{a}{2}$ (c) $ab = 1$ (d) $a=\pm\sqrt{2}b$

COMEDK-2015

Ans. (d) : We have, BE and AD are the medians. SO, E and D are the mid points of AC and BC respectively.



$\therefore$ Coordinates of E $=\left(\frac{a}{2},\frac{b}{2}\right)$

And coordinates of D $=\left(\frac{a}{2},0\right)$

Now, slope of median BE,

$$m_1=\frac{b}{a}$$

Also, slope of median AD,

$$m_2=\frac{-2b}{a}$$

Now, m_1 & m_2 are perpendicular if $m_1 m_2 = -1$

$\Rightarrow \frac{b}{a}\times\frac{-2b}{a}=-1\Rightarrow 2b^2=a^2\Rightarrow a=\pm\sqrt{2}b$

105. The sides of a triangle are x = 2, y + 1 = 0 and x + 2y = 4. Its circum centre is

(a) (4,0) (b) (2,–1) (c) (0,4) (d) (2,3)

COMEDK-2017

Ans. (a) : The vertices are (2, 1), (6, 1), (2,1) forming a right angled triangle. Circumcentre is the midpoint of the hypotenuse,

i.e., $\left(\frac{6+2}{2},\frac{1-1}{2}\right)=(4,0)$

106. A straight line through the point A (3, 4) is such that its intercept between the axes is bisected at A, its equation is

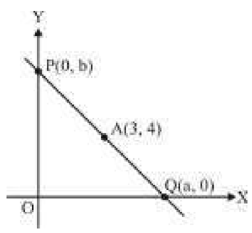
(a) $3x - 4y + 7 = 0$ (b) $4x + 3y = 24$
(c) $3x + 4y = 25$ (d) $x + y = 7$

VITEEE-2012

Ans. (b) : A is mid point of line PQ.

$\therefore 3=\frac{a+0}{2}\Rightarrow a=6$

and $4=\frac{0+b}{2}\Rightarrow b=8$



Thus, equation of line is

$\frac{x}{6}+\frac{y}{8}=1$

$\Rightarrow 4x + 3y = 24$

107. If a plane meets the coordinate axes at A, B and C such that the centroid of the triangle is (1, 2, 4), then the equation of the plane is

(a) $x+2y+4z=12$ (b) $4x+2y+z=12$
(c) $x+2y+4z=3$ (d) $4x+2y+z=3$

VITEEE-2014

Ans. (b) : Let the equation of the plane is

$$\frac{x}{\alpha}+\frac{y}{\beta}+\frac{z}{\gamma}=1$$

Then, A (α, 0, 0), B(0, β, 0) and C (0, 0, γ) are the points on the co-ordinate axes, The centroid of the triangle is (1, 2, 4).

$$\therefore\frac{\alpha}{3}=1\Rightarrow\alpha=3$$

$$\frac{\beta}{3}=2\Rightarrow\beta=6$$

and $\frac{\gamma}{3}=4\Rightarrow\gamma=12$

$\therefore$ The equation of the plane is

$$\frac{x}{3}+\frac{y}{6}+\frac{z}{12}=1\ \Rightarrow 4x+2y+z=12$$

108. The normals at three points P, Q and R of the parabola y^2 = 4ax meet at (h, k). The centroid of the Δ PQR lies on

(a) x = 0 (b) y = 0 (c) x = –a (d) y = a

VITEEE-2014

Ans. (b) : We know that, the sum of ordinates of feet of normals draw from a point the parabola, $y^2 = 4ax$ is always zero.

Now, as normals at three points P, Q and R of points P, Q and R of parabola $y^2 = 4ax$ meet at (h, k).

$\Rightarrow$ The normals from (h,k) to $y^2 = 4ax$ meet the parabola at P, Q and R.

$\Rightarrow$ y-coordinate y_1, y_2, y_3 of these points and R will be zero.

$\Rightarrow$ y-coordinate of the centroid of

ΔPQR i.e.,

$$\frac{y_1+y_2+y_3}{3}=\frac{0}{3}=0$$

$\therefore$ Centroid lies on y = 0

109. If the vertices of a triangle are A(0, 4, 1), B(2, 3, –1) and C(4, 5, 0), then the orthocentre of ΔABC, is

(a) (4, 5, 0) (b) (2, 3, –1) (c) (–2, 3, –1) (d) (2, 0, 2)

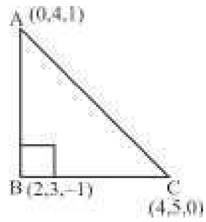
VITEEE-2014

Ans. (b) :

Vertices of Δ ABC are

A(0, 4, 1), B(2, 3, –1)

and C (4, 5, 0).



$AB=\sqrt{(2-0)^2+(3-4)^2+(-1-1)^2}$

$AB=\sqrt{4+1+4}$

$AB=\sqrt{9}$

AB = 3

$BC=\sqrt{(4-2)^2+(5-3)^2+(0+1)^2}$

$BC=\sqrt{4+4+1}$

$BC=\sqrt{9}$

BC = 3

$CA=\sqrt{(4-0)^2+(5-4)^2+(0-1)^2}$

$CA=\sqrt{16+1+1}$

$CA = 3\sqrt{2}$

$AB^2 + BC^2 = AC^2$

$\therefore$ ΔABC is a right angled triangle.

We know that, the orthocenter of a right angled triangle is the vertex containing the right angle.

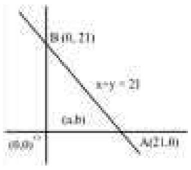
$\therefore$ Orthocenter is point B (2, 3, –1).

110. The number of integral points (integral point means both the coordinates should be integer exactly in the interior of the triangle with vertices (0, 0), (0, 21) and (21, 0) is

(a) 133 (b) 190 (c) 233 (d) 105

VITEEE-2013

Ans. (b) :



Let the vertices of triangle be A (21,0), B (0,21) and O (0,0)

Thus, any point in the interior of the triangle lies in first quadrant.

$\therefore \quad a > o \ \& \ b > o$

Point (a,b) lies on the same side of the AB where lies.

For (0,0)

$x + y \le 21$

therefore

$a + b - 21 < 0$

$\Rightarrow \quad a + b < 21$

For $a = 1$; $b < 21 - 1 \Rightarrow b < 20$

$b \in [1, 19]$ total 19 integral value.

For $a = 2$; $b < 21 - 2 \Rightarrow b < 19$

$b \in [1,18]$ total 18 integral values

Similarly

for $a = 19$; $b < 21 - 19 \Rightarrow b < 2$

$b = 1$ 1 integral values

Thus Number of integral points = 19+18+…..+1

$$= \frac{19(19+1)}{2} = \frac{19\times 20}{2} = 190$$

Thus there are total 190 integral points.

Which lies inside the triangle.

111. In an equilateral triangle, the inradius, circum-radius and one of the exradii are in the ratio

(a) 2 : 3 : 5 (b) 1 : 2 : 3

(c) 1 : 3 : 7 (d) 3 : 7 : 9

VITEEE-2011

Ans. (b) : We have, area of $\Delta = \frac{\sqrt{3}}{4}a^2, s = \frac{3a}{2}$

In radius $r = \frac{\Delta}{s} = \frac{a}{2\sqrt{3}}$

Circumradius $R = \frac{abc}{4\Delta} = \frac{a^3}{\sqrt{3}a^2} = \frac{a}{\sqrt{3}}$

and exradius $r_1 = \frac{\Delta}{s-a} = \frac{\sqrt{3}a^2/4}{a/2}$

$= \frac{\sqrt{3}}{2}a$

$\therefore$ Required ratio = r : R : r_1

$= \frac{a}{2\sqrt{3}} : \frac{a}{\sqrt{3}} : \frac{\sqrt{3}}{2}a = 1 : 2 : 3.$

112. Let O = (0, 0), A = (a, 11) and B = (b, 37) are the vertices of an equilateral triangle OAB, then a and b satisfy the relation

(a) $(a^2 + b^2) - 4ab = 138$

(b) $(a^2 + b^2) - ab = 124$

(c) $(a^2 + b^2) + 3ab = 130$

(d) $(a^2 + b^2) - 3ab = 138$

UPSEE-2017

Ans. (a) : Given,

co-ordinate of vertices of equilateral triangle

O = (0,0), A = (a, 11), B = (b, 37)

Let c is the middle point of side AB

$\therefore \quad C = \left(\frac{a+b}{2}, 24\right)$

$\therefore \quad OC = \sqrt{3}\,CA$

$(OC)^2 = \left(\sqrt{3}(CA)\right)^2$ [squaring both side]

$(OC)^2 = 3(CA)^2$

$$\left(\frac{a+b}{2} - 0\right)^2 + (24-0)^2 = 3\left[\left(a - \frac{a+b}{2}\right)^2 + (11-24)^2\right]$$

$$\frac{a^2 + b^2 + 2ab}{4} + 576 = 3\left[\left(\frac{a-b}{2}\right)^2 + (-13)^2\right]$$

$a^2 + b^2 - 4ab = 138$

113. The coordinates of the point on the line through the points A (3, 4, 1) and B (5, 1, 6) crosses XY-plane are

(a) $\left(\frac{13}{5}, \frac{23}{5}, 0\right)$ (b) $\left(\frac{3}{5}, \frac{2}{5}, 0\right)$

(c) (1, 1, 0) (d) $\left(-\frac{13}{5}, \frac{23}{5}, 0\right)$

JCECE-2018

Ans. (a) : Given,

The co-ordinate of the points A(3, 4, 1) and B(5,1,6)

Now,

The equation of line in space can be written as

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1} \quad \left[\begin{array}{l}\text{Where } (x_1, y_1, z_1) \text{ and} \\ (x_2, y_2, z_2) \text{ are the points}\end{array}\right]$$

$$\frac{x-3}{5-3} = \frac{y-4}{1-4} = \frac{z-1}{6-1}$$

$$\frac{x-3}{2} = \frac{y-4}{-3} = \frac{z-1}{5} = k$$

Now, In xy plane z = 0

$k = -\frac{1}{5}$

$\therefore \quad \frac{x-3}{2} = \frac{-1}{5}$ and $\frac{y-4}{-3} = -\frac{1}{5}$

$x - 3 = -\frac{2}{5}$ and $y - 4 = 3/5$

$x = -\frac{2}{5} + 3$ $\qquad$ $y = \frac{3}{5} + 4$

$x = \frac{13}{5}$ and $y = \frac{23}{5}$

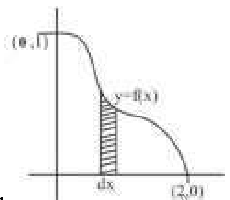
Hence, the required co-ordinate $\left(\frac{13}{5}, \frac{23}{5}, 0\right)$

114. If y = f (x) makes positive intercepts of 2 and 1 unit an x and y coordinate axes and encloses an area of $\frac{3}{4}$ square unit with the axes, then $\int_0^2 x f'(x) dx$ is is

(a) $\frac{3}{2}$ (b) 1 (c) $\frac{5}{4}$ (d) $-\frac{3}{4}$

BCECE-2018

Ans. (d) : Given,



$$\int_0^2 f(x)dx = 3/4$$

$$\therefore \int_0^2 x f'(x) dx$$

$$= x\int_0^2 f'(x)dx - \int_0^2 f(x)dx$$

$$= \left[x\int_0^2 f(x)dx\right]_0^2 - 3/4 = 2f(2) - 3/4$$

$$= 2 \times 0 - 3/4$$

$[y = f(x) \Rightarrow y = f(2)] = -3/4$

115. If A(2,3) and B(–2,1) are two vertices of a triangle and third vertex moves on the line 2x+3y=9, then the locus of the centroid of the triangle is

(a) 2x+3y=1 (b) 2x+y=3
(c) 2x–3y=1 (d) x–y=1

BCECE-2017

Ans. (a) : Given,
The co-ordinate of vertices of triangle are A(2,–3), B(–2,1) and C(x,y)
Now, Let the co-ordinate of centroid be (h,k)

$\therefore \quad h = \left(\frac{2-2+x}{3}\right) \Rightarrow x = 3h$

$k = \left(\frac{-3+1+y}{3}\right) \Rightarrow y = 3k + 2$

Where (x,y) are the point on the line 2x + 3y = 9

$\therefore \quad 2(3h) + 3(3k + 2) = 9$

$6h + 9k + 6 = 9$

$6h + 9k = 3$

$2h + 3k = 1$

Locus of the centroid of the triangle is 2x + 3y = 1

116. If the lines x + 3y – 9 = 0, 4x + by – 2 = 0 and 2x –y – 4 = 0 are concurrent, then b equal to

(a) –6 (b) 5 (c) 1 (d) –5

BCECE-2016

Ans. (d): Given,
lines x + 3y – 9 = 0, 4x + by – 2 = 0 and 2x – y – 4 = 0 are concurrent
$\therefore$ The determinant is zero for concurrent line –

i.e. $\begin{vmatrix} 1 & 3 & -9 \\ 4 & b & -2 \\ 2 & -1 & -4 \end{vmatrix} = 0$

$\Rightarrow 1(-4b - 2) - 3(-16 + 4) - 9(-4 - 2b) = 0$

$\Rightarrow -4b - 2 + 36 + 36 + 18b = 0$

$14b = -70$

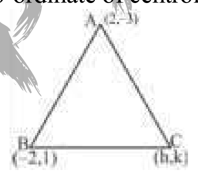
$b = -5$

117. Let A (2, – 3) and B (– 2, 1) be vertices of a ΔABC. If The centroid of this triangle moves on the line 2x + 3y = 1, then the locus of the vertex C is the line

(a) $2x + 3y = 9$ (b) $2x - 3y = 7$
(c) $3x + 2y = 5$ (d) $3x - 2y = 3$

BCECE-2014

Ans. (a) : Given,
The co-ordinate of vertices of triangle A(2,–3), B(–2,1) and C(h,k)
Now, co-ordinate of centroid lies on the line 2x + 3y = 1



$\Rightarrow \left(\frac{2-2+h}{3}, \frac{-3+1+k}{3}\right) \Rightarrow \left(\frac{h}{3}, \frac{-2+k}{3}\right)$

$\therefore \left(\frac{h}{3}, \frac{-2+k}{3}\right)$ satisfy the equation {2x + 3y = 1}

$\Rightarrow \quad 2\left(\frac{h}{3}\right) + 3\left(\frac{-2+k}{3}\right) = 1$

$\frac{2}{3}h + (-2 + k) = 1$

$\Rightarrow \quad 2h - 6 + 3k = 3$

$2h + 3k = 9$

Locus of the vertex C is the line 2x + 3y = 9

118. The equation of the plane meets the axes in A, and C such that centroid of the ΔABC is $\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$ is given by

(a) $x + y + z = 1$ (b) $x + y + z = 2$
(c) $\frac{x}{3} + \frac{y}{3} + \frac{z}{3} = 3$ (d) $x + y + z = \frac{1}{3}$

BCECE-2014

Ans. (a): Given,
The centroid of triangle ΔABC is = $\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$
the planes meets the axes in ABC
Now,
The centroid of the triangle ΔABC is $\left(\frac{a}{3}, \frac{b}{3}, \frac{c}{3}\right)$

$\Rightarrow \quad a = 1, b = 1 \;\&\; c = 1$

$\therefore$ The equation of plane meet the co-ordinate axes is

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$$

$$\frac{x}{1}+\frac{y}{1}+\frac{z}{1}=1$$

$$x+y+z=1$$

119. If $A(-1,3,2)$, $B(2,3,5)$ and $C(3,5,-2)$ are vertices of a ΔABC, then angles of are

(a) $\angle A=90°, \angle B=30°, \angle C=60°$

(b) $\angle A=\angle B=\angle C=60°$

(c) $\angle A=\angle B=45°, \angle C=90°$

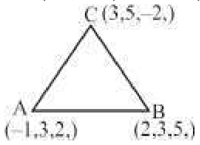
(d) None of the above

CG PET- 2014

Ans. (d) : Given,

The co-ordinate of vertices of triangle are

A(–1, 3, 2), B(2, 3, 5) and C(3,5, –2)



Now, $a = x_2 - x_1$, $b = y_2 - y_1$, $c = z_2 - z_1$

$\therefore$ AB = (3, 0, 3)

AC = (4, 2, –4)

BC = (–1, –2, + 7)

$$AB=\sqrt{(3)^2+0^2+(3)^2}=\sqrt{9+0+9}=3\sqrt{2}$$

$$BC=\sqrt{4^2+2^2+4^2}=\sqrt{16+4+16}=6$$

$$AC=\sqrt{1^2+2^2+7^2}=\sqrt{1+4+49}=3\sqrt{6}$$

$$\cos A=\frac{AB^2+AC^2-BC^2}{2\times AB\times AC}=\frac{18+54-36}{2\times3\sqrt{2}\times3\sqrt{6}}$$

$$=\frac{36}{18\times2\sqrt{3}}=\frac{1}{\sqrt{3}}$$

$$A=\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

$$\cos B=\frac{AB^2+BC^2-AC^2}{2AB\times BC}=\frac{18+36-54}{2\times3\sqrt{2}\times6}=0$$

$$B=90°$$

$$\cos C=\frac{BC^2+AC^2-AB^2}{2\times BC\times AC}=\frac{54+36-18}{2\times63\sqrt{6}}=\sqrt{\frac{2}{3}}$$

$$C=\cos^{-1}\left(\sqrt{\frac{2}{3}}\right)$$

Hence the triangle,

$$A=\cos^{-1}\left(\frac{1}{\sqrt{3}}\right), B=90°, C=\cos^{-1}\left(\frac{\sqrt{2}}{3}\right)$$

120. If G and G' are respectively centroid of ΔABC and $\Delta A'B'C'$. then AA' + BB' + CC' is equal to

(a) 2GG' (b) 3GG' (c) $\frac{2}{3}GG'$ (d) $\frac{1}{3}GG'$

CG PET- 2016

Ans. (b) : Let a.b.c be the position vectors of A,B and C respectively.

Then, the position vector of G is $\frac{a'+b+c'}{3}$

Let the position vectors of A'B' and C' be a'b'

Then, the position vector of G' is $\frac{a'+b+c'}{3}$

$$\therefore\ AA'+BB'+CC'=(a'-a)+(b'-b)+(c'-c)$$

$$\Rightarrow AA'+BB'+CC'=(a'+b'+c')-(a+b+c)$$

$$=\left(\frac{a'+b'+c'}{3}-\frac{a+b+c}{3}\right)=3GG'$$

121. The centroid of the triangle formed by the lines $x+y=1, 2x+3y=6$ and $4x-y=-4$ lies in the quadrant

(a) I (b) II (c) III (d) IV

CG PET- 2016

Ans. (b) : Given, x + y =1 …(i)

2x + 3y = 6 …(ii)

And 4x – y = – 4 …(iii)

On solving equation (i) and (ii), we get

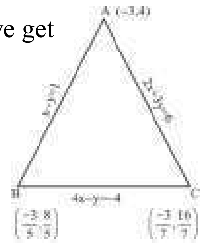
x = – 3 and y = 4

Now, solving equation (ii) and (iii), we get

$$x=-\frac{3}{7}\text{ and } y=\frac{16}{7}$$

And solving equation (i) and (iii), we get

$$x=-\frac{3}{5}\text{ and } y=\frac{8}{5}$$



$\therefore$ Centroid of ΔABC

$$=\left[\frac{x_1+x_2+x_3}{3},\frac{y_1+y_2+y_3}{3}\right]$$

$$=\left[\frac{(-3)+\left(-\frac{3}{5}\right)+\left(-\frac{3}{7}\right)}{3},\frac{4+\frac{8}{5}+\frac{16}{7}}{3}\right]$$

$$=\left[-\frac{141}{105},\frac{276}{105}\right]\cong(-x,y)$$

Hence, the centroid lies in II quadrant.

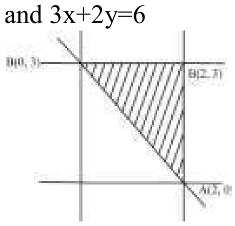
122. The orthocenter of the triangle formed by the line $x=2$, $y=3$ and $3x+2y=6$ at the point

(a) (2, 0) (b) (2, 3) (c) (0, 3) (d) None of these

CG PET- 2018

Ans. (b) : Given equation of lines are

x=2,y=3 and 3x+2y=6



Since, ΔABC is right angled at C. Now, orthocenter of ΔABC will the vertex at which right angle is forming.

$\therefore$ Orthocenter of ΔABC is (2,3).