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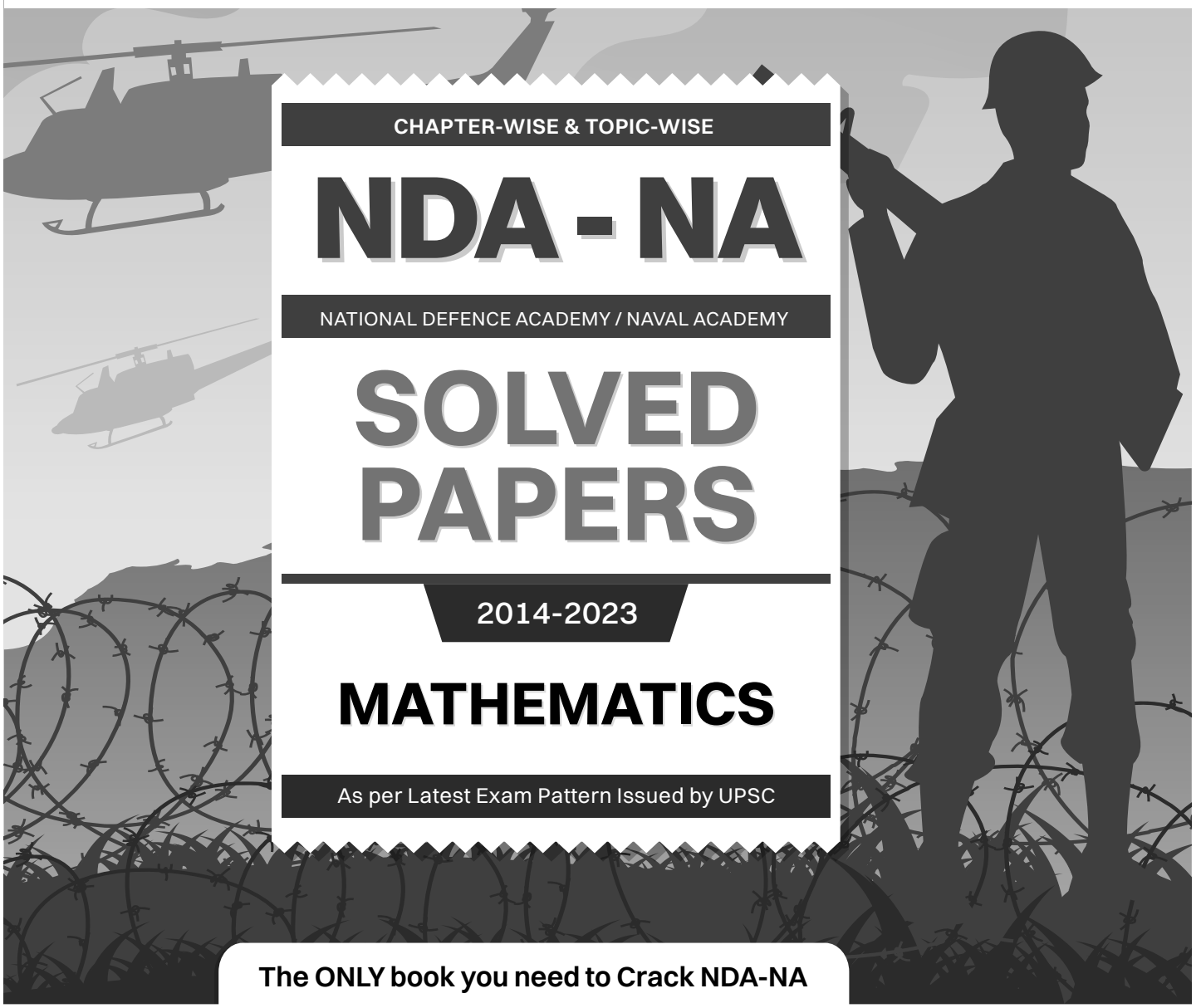

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NDA/NA

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0562-2857671

contact@oswaalbooks.com

www.OswaalBooks.com

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PREFACE

Welcome to the world of National Defence Academy (NDA), one of the most prestigious military academies in the world. Aspiring to join the NDA and serve your country is a noble and challenging endeavour, and cracking the NDA entrance examination is the first step towards achieving that dream.

This book, "NDA/NA Chapter-wise & Topic-wise Solved Papers - Mathematics," is designed to help you in your preparation for the NDA entrance examination. It is a Comprehensive Question Bank with Conceptual Revision Notes & detailed solutions are provided in a step-by-step manner, making it easier for you to understand the concepts and techniques required to solve the questions accurately and efficiently.

Some benefits of studying from Oswaal NDA-NA Solved papers are:

- **100% updated** with Fully Solved April & September 2023 Papers.
- **Concept Clarity** with Concept based Revision notes & Mind Maps
- **Extensive Practice** with 1200+ Questions and Two Sample Question Papers.
- **Crisp Revision** with Concept Based Revision notes, Mind Maps & Mnemonics.
- **Expert Tips** helps you get expert knowledge master & crack NDA/NA in first attempt.
- **Exam insights** with 5 Year-wise (2019-2023) Trend Analysis, empowering students to be 100% exam ready.

This book has been developed with the highest editorial standards, keeping in mind the rigor and meticulousness required of an exam resource catering to NDA/NA. The features of the book make it a must-have for anyone preparing for NDA/NA 2023-24. We hope it will help students to supplement their NDA/NA preparation strategy and secure a high rank.

We wish the readers great success ahead!

All the Best!

Tips to Crack NDA in the First Attempt

The NDA Exam is conducted by the Union Public Service Commission or UPSC for candidates who wants to join Army, Navy & Airforce Wing of National Defence Academy/Naval Academy, NDA is recognised as one of the reputed National level Examinations in India. Cracking the NDA/NA Exam in the very first attempt, given the difficulty level, can be a laborious task but is quite attainable if done diligently as well as smartly. Here are some tips that you must follow by heart to crack the exam in the very first attempt:

1

Think Right

Calming yourself and thinking positive is the first and the best course of action that one is required to take. Think and believe that the exam goal is achievable if worked upon smartly.

2

Start studying from the beginning

All the aspirants are aware of how vast, comprehensive and detailed the syllabus of the NDA exam is. To crack the exam in the first attempt you have to start preparing for the exam from the beginning of your 12th class. It is only then that you will be able to complete the entire syllabus. Following this approach will also allow you plenty of time to revise.

3

Respect the syllabus and arrange the materials accordingly

While preparing for the NDA exam nothing can be labelled as less important. Questions can come from the most unexpected topics too. Laying down your whole syllabus in front of you will help you to decide on the study material you require.

4

Get the right tools and study material

Gathering and preparing from the appropriate study material is something you cannot be ignorant towards. You can refer to Oswaal Books NDA/NA Year-Wise 11 Solved Papers along with Question Banks to enhance your preparation. Both the reference books are on the lines of the current syllabus and can be entrusted upon before the examination.

5

Schedule total time for each subject

Creating a schedule which gives due time to all the subjects is a must. Giving proper time to all the subjects daily will help you cover the syllabus on time, giving you enough time for revision.

6

Understand the concepts

No one can crack the NDA/NA exam just by mugging up all the concepts and topics. The syllabus of the exam is in-depth such that you need to understand every concept.

7

Practice a lot of Sample Papers

Year-wise Solved Papers will not only help you in understanding the examination pattern, but they will also help you in figuring out the questions that come up every year and this might give you an edge over other students. You can refer to Oswaal NDA/NA Question Bank, as they include all the typologies of Questions asked in the Examination, Previous Years Papers with solutions, Mind Maps, etc. Referring to various sample papers might also help you in comprehending the areas which require more work.

8

Revise whenever you get time

Make sure you revise as much as possible. The revision will help you in keeping the concepts fresh in your mind.

9

Analysing your performance

While you are solving papers, make sure you keep a track of time i.e. how much time does it take to solve one section or one question? Make a report of the sections and type of questions which take minimum and maximum time.

CONTENTS

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Syllabus

PAPER-I
MATHEMATICS

(Code No. 01) (Maximum Marks - 300)

1. **ALGEBRA:** Concept of set, operations on sets, Venn diagrams. De Morgan laws, Cartesian product, relation, equivalence relation. Representation of real numbers on a line. Complex numbers—basic properties, modulus, argument, cube roots of 19 unity. Binary system of numbers. Conversion of a number in decimal system to binary system and vice-versa. Arithmetic, Geometric and Harmonic progressions. Quadratic equations with real coefficients. Solution of linear inequations of two variables by graphs. Permutation and Combination. Binomial theorem and its applications. Logarithms and their applications.
2. **MATRICES AND DETERMINANTS:** Types of matrices, operations on matrices. Determinant of a matrix, basic properties of determinants. Adjoint and inverse of a square matrix, Applications-Solution of a system of linear equations in two or three unknowns by Cramer's rule and by Matrix Method.
3. **TRIGONOMETRY:** Angles and their measures in degrees and in radians. Trigonometrical ratios. Trigonometric identities Sum and difference formulae. Multiple and Sub-multiple angles. Inverse trigonometric functions. Applications-Height and distance, properties of triangles.
4. **ANALYTICAL GEOMETRY OF TWO AND THREE DIMENSIONS:** Rectangular Cartesian Coordinate system. Distance formula. Equation of a line in various forms. Angle between two lines. Distance of a point from a line. Equation of a circle in standard and in general form. Standard forms of parabola, ellipse and hyperbola. Eccentricity and axis of a conic. Point in a three dimensional space, distance between two points. Direction Cosines and direction ratios. Equation two points. Direction Cosines and direction ratios. Equation of a plane and a line in various forms. Angle between two lines and angle between two planes. Equation of a sphere.
5. **DIFFERENTIAL CALCULUS:** Concept of a real valued function–domain, range and graph of a function. Composite functions, one to one, onto and inverse functions. Notion of limit, Standard limits—examples. Continuity of functions—examples, algebraic operations on continuous functions. Derivative of function at a point, geometrical and physical interpretation of a derivative—applications. Derivatives of sum, product and quotient of functions, derivative of a function with respect to another function, derivative of a composite function. Second order derivatives. Increasing and decreasing functions. Application of derivatives in problems of maxima and minima.
6. **INTEGRAL CALCULUS AND DIFFERENTIAL EQUATIONS:** 20 Integration as inverse of differentiation, integration by substitution and by parts, standard integrals involving algebraic expressions, trigonometric, exponential and hyperbolic functions. Evaluation of definite integrals—determination of areas of plane regions bounded by curves—applications. Definition of order and degree of a differential equation, formation of a differential equation by examples. General and particular solution of a differential equations, solution of first order and first degree differential equations of various types—examples. Application in problems of growth and decay.
7. **VECTOR ALGEBRA:** Vectors in two and three dimensions, magnitude and direction of a vector. Unit and null vectors, addition of vectors, scalar multiplication of a vector, scalar product or dot product of two vectors. Vector product or cross product of two vectors. Applications—work done by a force and moment of a force and in geometrical problems.
8. **STATISTICS AND PROBABILITY:** Statistics: Classification of data, Frequency distribution, cumulative frequency distribution—examples. Graphical representation—Histogram, Pie Chart, frequency polygon— examples. Measures of Central tendency—Mean, median and mode. Variance and standard deviation—determination and comparison. Correlation and regression. Probability : Random experiment, outcomes and associated sample space, events, mutually exclusive and exhaustive events, impossible and certain events. Union and Intersection of events. Complementary, elementary and composite events. Definition of probability—classical and statistical—examples. Elementary theorems on probability—simple problems. Conditional probability, Bayes' theorem—simple problems. Random variable as function on a sample space. Binomial distribution, examples of random experiments giving rise to Binominal distribution.

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Scheme of Examination

1. The subjects of the written examination, the time allowed and the maximum marks allotted to each subject will be as follows:—

Subject	Code	Duration	Maximum Marks
Mathematics	01	2½ Hours	300
General Ability Test	02	2½ Hours	600
	Total		900
SSB Test/Interview:			900

2. THE PAPERS IN ALL THE SUBJECTS WILL CONSIST OF OBJECTIVE TYPE QUESTIONS ONLY. THE QUESTION PAPERS (TEST BOOKLETS) OF MATHEMATICS AND PART "B" OF GENERAL ABILITY TEST WILL BE SET BILINGUALLY IN HINDI AS WELL AS ENGLISH.

3. In the question papers, wherever necessary, questions involving the metric system of Weights and Measures only will be set.

4. Candidates must write the papers in their own hand. In no circumstances will they be allowed the help of a scribe to write answers for them.

5. The Commission have discretion to fix qualifying marks in any or all the subjects at the examination.

6. The candidates are not permitted to use calculator or Mathematical or logarithmic table for answering objective type papers (Test Booklets). They should not therefore, bring the same inside the Examination Hall.

Height and Weight Standards

For Female Candidates joining NDA (Army):

Age (yrs)	Minimum weight for all ages	Age: 17 to 20 yrs	Age: 20 + 01 day - 30 yrs	Age : 30 + 01 Day - 40 yrs	Age: Above 40 yrs
Height (cm)	Weight (kg)	Weight (kg)	Weight (kg)	Weight (kg)	Weight (kg)
140	35.3	43.1	45.1	47.0	49.0
141	35.8	43.7	45.7	47.7	49.7
142	36.3	44.4	46.4	48.4	50.4
143	36.8	45.0	47.0	49.1	51.1
144	37.3	45.6	47.7	49.8	51.8
145	37.8	46.3	48.4	50.5	52.6
146	38.4	46.9	49.0	51.2	53.3
147	38.9	47.5	49.7	51.9	54.0
148	39.4	48.2	50.4	52.6	54.8
149	40.0	48.8	51.1	53.3	55.5
150	40.5	49.5	51.8	54.0	56.3
151	41.0	50.2	52.4	54.7	57.0
152	41.6	50.8	53.1	55.4	57.8
153	42.1	51.5	53.8	56.2	58.5
154	42.7	52.2	54.5	56.9	59.3
155	43.2	52.9	55.3	57.7	60.1
156	43.8	53.5	56.0	58.4	60.8
157	44.4	54.2	56.7	59.2	61.6
158	44.9	54.9	57.4	59.9	62.4
159	45.5	55.6	58.1	60.7	63.2
160	46.1	56.3	58.9	61.4	64.0
161	46.7	57.0	59.6	62.2	64.8
162	47.2	57.7	60.4	63.0	65.6
163	47.8	58.5	61.1	63.8	66.4

For Male Candidates joining NDA (Army):

Height requirement varies as per the stream of entry. Weight should be proportionate to height as per the chart given below:-

Age (yrs)	Minimum weight for all ages	Age: 17 to 20 yrs	Age: 20 + 01 day - 30 yrs	Age : 30 + 01 Day - 40 yrs	Age: Above 40 yrs
Height (cm)	Weight (kg)	Weight (kg)	Weight (kg)	Weight (kg)	Weight (kg)
140	35.3	43.1	45.1	47.0	49.0
141	35.8	43.7	45.7	47.7	49.7
142	36.3	44.4	46.4	48.4	50.4
143	36.8	45.0	47.0	49.1	51.1
144	37.3	45.6	47.7	49.8	51.8
145	37.8	46.3	48.4	50.5	52.6
146	38.4	46.9	49.0	51.2	53.3
147	38.9	47.5	49.7	51.9	54.0
148	39.4	48.2	50.4	52.6	54.8
149	40.0	48.8	51.1	53.3	55.5
150	40.5	49.5	51.8	54.0	56.3
151	41.0	50.2	52.4	54.7	57.0
152	41.6	50.8	53.1	55.4	57.8
153	**42.1**	**51.5**	**53.8**	**56.2**	**58.5**
154	**42.7**	**52.2**	**54.5**	**56.9**	**59.3**
155	**43.2**	**52.9**	**55.3**	**57.7**	**60.1**
156	**43.8**	**53.5**	**56.0**	**58.4**	**60.8**
157	**44.4**	**54.2**	**56.7**	**59.2**	**61.6**
158	**44.9**	**54.9**	**57.4**	**59.9**	**62.4**
159	**45.5**	**55.6**	**58.1**	**60.7**	**63.2**
160	**46.1**	**56.3**	**58.9**	**61.4**	**64.0**
161	**46.7**	**57.0**	**59.6**	**62.2**	**64.8**
162	**47.2**	**57.7**	**60.4**	**63.0**	**65.6**
163	**47.8**	**58.5**	**61.1**	**63.8**	**66.4**
164	**48.4**	**59.2**	**61.9**	**64.6**	**67.2**
165	**49.0**	**59.9**	**62.6**	**65.3**	**68.1**
166	**49.6**	**60.6**	**63.4**	**66.1**	**68.9**
167	**50.2**	**61.4**	**64.1**	**66.9**	**69.7**
168	**50.8**	**62.1**	**64.9**	**67.7**	**70.6**
169	**51.4**	**62.8**	**65.7**	**68.5**	**71.4**
170	**52.0**	**63.6**	**66.5**	**69.4**	**72.3**
171	**52.6**	**64.3**	**67.3**	**70.2**	**73.1**
172	**53.3**	**65.1**	**68.0**	**71.0**	**74.0**
173	**53.9**	**65.8**	**68.8**	**71.8**	**74.8**
174	**54.5**	**66.6**	**69.6**	**72.7**	**75.7**

...CONTD.

Age (yrs)	Minimum weight for all ages	Age: 17 to 20 yrs	Age: 20 + 01 day - 30 yrs	Age : 30 + 01 Day - 40 yrs	Age: Above 40 yrs
Height (cm)	Weight (kg)	Weight (kg)	Weight (kg)	Weight (kg)	Weight (kg)
175	55.1	67.4	70.4	73.5	76.6
176	55.8	68.1	71.2	74.3	77.4
177	56.4	68.9	72.1	75.2	78.3
178	57.0	69.7	72.9	76.0	79.2
179	57.7	70.5	73.7	76.9	80.1
180	58.3	71.3	74.5	77.8	81.0
181	59.0	72.1	75.4	78.6	81.9
182	59.6	72.9	76.2	79.5	82.8
183	60.3	73.7	77.0	80.4	83.7
184	60.9	74.5	77.9	81.3	84.6
185	61.6	75.3	78.7	82.1	85.6
186	62.3	76.1	79.6	83.0	86.5
187	62.9	76.9	80.4	83.9	87.4
188	63.6	77.8	81.3	84.8	88.4
189	64.3	78.6	82.2	85.7	89.3
190	65.0	79.4	83.0	86.6	90.3
191	65.7	80.3	83.9	87.6	91.2
192	66.4	81.1	84.8	88.5	92.2
193	67.0	81.9	85.7	89.4	93.1
194	67.7	82.8	86.6	90.3	94.1
195	68.4	83.7	87.5	91.3	95.1
196	69.1	84.5	88.4	92.2	96.0
197	69.9	85.4	89.3	93.1	97.0
198	70.6	86.2	90.2	94.1	98.0
199	71.3	87.1	91.1	95.0	99.0
200	72.0	88.0	92.0	96.0	100.0
201	72.7	88.9	92.9	97.0	101.0
202	73.4	89.8	93.8	97.9	102.0
203	74.2	90.7	94.8	98.9	103.0
204	74.9	91.6	95.7	99.9	104.0
205	75.6	92.5	96.7	100.9	105.1
206	76.4	93.4	97.6	101.8	106.1
207	77.1	94.3	98.6	102.8	107.1
208	77.9	95.2	99.5	103.8	108.2
209	78.6	96.1	100.5	104.8	109.2
210	79.4	97.0	101.4	105.8	110.3

NDA vs CDS: Know All the Similarities & Differences

The National Defence Academy (NDA) and the Combined Defence Services (CDS) Exams are gateways to the Indian Armed Forces. Though both the exams are conducted by the Union Public Service Commission, i.e. UPSC, there are many similarities and differences in the recruitment, training, salary, perks and promotion opportunities, etc.

For those who are planning to join Indian Army, Navy or Air Force, it is essential to know the differences and similarities in NDA and CDS. The similarities are given below:

Parameter	NDA	CDS
Age	16.5-19.5 Years	19-25 Years
Eligibility	Men only	Men & Women
Educational Qualification	10+2	Degree
Scheme of Examination	Written + SSB	Written + SSB
Frequency of the Exam	Twice/Year	Twice/Year
Duration of Training	4-4.5 Years 3 Yrs. at NDA and 1 Yr. at IMA (For Army cadets) 3 Yrs. at NDA and 1 Yr. at Naval Academy (For Naval cadets)/ 3 Yrs. at NDA and 1 & 1/2 Yrs. at AFA Hyderabad (For AF cadets)	18 months for IMA Cadets 37-40 months for Navy Officers 74 months for Air Force Officers
Training Centres	National Defence Academy, Khadakwasla, Pune Indian Military Academy, Dehradun Indian Naval Academy, Ezhimala Indian Air Force Academy, Hyderabad	Indian Military Academy (IMA), Dehradun for Army Cadets Indian Naval Academy, Ezhimala for Navy Cadets Indian Air Force Academy, Hyderabad for Air Force Officers Officers Training Academy (OTA), Chennai
Degrees awarded	Army Cadets - B.Sc./B.Sc. (Computer)/BA /B.Tech. degree Naval Cadets - B.Tech. degree Air Force Cadets - B.Tech. degree	Army Cadets in IMA - PG Diploma in 'Military and Defence Management OTA Chennai – Post Graduate Diploma in Defence Management and Strategic Studies
Rank assigned after training	Lieutenant	Lieutenant
Stipend during training	Rs. 21,000/- p.m. (fixed)	Rs. 21,000/- p.m. (fixed)

Promotional Avenues

Rank	Min. Commissioned Service for Promotion	
	NDA Officer	CDS Officer
Lieutenant	On Commission	On Commission
Captain	02 Years	02 Years
Major	06 years	06 years
Lieutenant Colonel	13 years	13 years
Colonel(Selection)	15 years	15 years
Colonel (Time Scale)	26 years	26 years
Brigadier	On Selection	23 years
Major General	On Selection	25 years
Lieutenant General	On Selection	28 years
General	On Selection	No restrictions

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Trend Analysis (2023-2019)

Units No.	Chapter Name		Number of Question(s) in						
		2023	2022	2022	2021		2020	2019	
		I	I	II	I	II	I	I	II
1.	Algebra	23	30	29	25	33	20	27	30
2.	Matrices & Determinants	11	11	9	11	10	8	8	5
3.	Trigonometery	17	16	17	19	7	24	22	16
4.	Analytical Geometry of Two and Three Dimensions	15	14	11	15	15	15	16	10
5.	Differential Calculus	15	10	14	11	17	15	11	26
6.	Integral Calculus and Differential Equations	14	14	17	14	15	13	11	8
7.	Vector Algebra	5	5	5	6	5	5	5	5
8.	Statistics and Probability	20	20	18	19	18	20	20	18
9.	Mathematical Induction	–	–	–	–	–	–	–	–
10.	Speed, Distance & Time	–	–	–	–	–	–	–	–
11.	Applied Mathematics	–	–	–	–	–	–	–	2
	Total	120	120	120	120	120	120	120	120

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MATHEMATICS

I

QUESTION PAPER 2023

Time : 2 : 30 Hours **Total Marks : 300**

Instructions :

1. *This Test Booklet contains **120** items (questions). Each item comprises four responses (answers). You will select the response which you want to mark on the Answer Sheet. In case you feel that there is more than one correct response, mark the response which you consider the best. In any case, choose **ONLY ONE** response for each item.*
2. *You have to mark all your responses **ONLY** on the separate Answer Sheet provided. See directions in the Answer Sheet.*
3. ***All** items carry equal marks.*
4. ***Penalty for wrong answers :***
   THERE WILL BE PENALTY FOR WRONG ANSWERS MARKED BY A CANDIDATE IN THE OBJECTIVE TYPE QUESTION PAPERS.
   (i) *There are four alternatives for the answer to every question. For each question for which a wrong answer has been given by the candidate, **one-third** of the marks assigned to that question will be deducted as penalty.*
   (ii) *If a candidate gives more than one answer, it will be treated as a **wrong answer** even if one of the given answers happens to be correct and there will be same penalty as above to that question.*
   (iii) *If a question is left blank, i.e., no answer is given by the candidate, there will be **no penalty** for that question.*

1. If ω is a non-real cube root of 1, then what is the value of $\left|\frac{1-\omega}{\omega+\omega^2}\right|$?

(a) $\sqrt{3}$ **(b)** $\sqrt{2}$

(c) 1 **(d)** $\frac{4}{\sqrt{3}}$

2. What is the number of 6-digit numbers that can be formed only by using 0, 1, 2, 3, 4 and 5 (each once); and divisible by 6 ?

(a) 96 **(b)** 120

(c) 192 **(d)** 312

3. What is the binary number equivalent to decimal number 1011 ?

(a) 1011 **(b)** 111011

(c) 11111001 **(d)** 111110011

4. Let A be a matrix of order 3×3 and $|A| = 4$. If $|2\,\text{adj}(3A)| = 2^{\alpha}3^{\beta}$ then what is the value of $(\alpha + \beta)$?

(a) 12 **(b)** 13

(c) 17 **(d)** 24

5. If α and β are the distinct roots of equation $x^2 - x + 1 = 0$, then what is the value of $\left|\frac{\alpha^{100}+\beta^{100}}{\alpha^{100}-\beta^{100}}\right|$?

(a) $\sqrt{3}$ **(b)** $\sqrt{2}$

(c) 1 **(d)** $\frac{1}{\sqrt{3}}$

6. Let A and B be symmetric matrices of same order, then which one of the following is correct regarding (AB – BA)?

1. Its diagonal entries are equal but nonzero
2. The sum of its non-diagonal entries is zero

Select the correct answer using the code given below:

(a) 1 only **(b)** 2 only

(c) Both 1 and 2 **(d)** Neither 1 nor 2

7. Consider the following statements in respect of square matrices A, B, C each of same order n :

1. $AB = AC \Rightarrow B = C$ if A is non-singular

2. If $BX = CX$ for every column matrix X having n rows then $B = C$

Which of the statements given above is/are correct ?

(a) 1 only **(b)** 2 only

(c) Both 1 and 2 **(d)** Neither 1 nor 2

8. The system of linear equations $x + 2y + z = 4$, $2x + 4y + 2z = 8$ and $3x + 6y + 3z = 10$ has

(a) a unique solution

(b) infinite many solutions

(c) no solution

(d) exactly three solutions

9. Let AX = B be a system of 3 linear equations with 3-unknowns. Let X_1 and X_2 be its two distinct solutions. If the combination $aX_1 + bX_2$ is a solution of AX = B; where a, b are real numbers, then which one of the following is correct ?

(a) $a = b$ **(b)** $a + b = 1$
(c) $a + b = 0$ **(d)** $a - b = 1$

10. What is the sum of the roots of the equation $\begin{vmatrix} 0 & x-a & x-b \\ 0 & 0 & x-c \\ x+b & x+c & 1 \end{vmatrix} = 0$?

(a) $a + b + c$ **(b)** $a - b + c$
(c) $a + b - c$ **(d)** $a - b - c$

11. If $2 - i\sqrt{3}$ where $i = \sqrt{-1}$ is a root of the equation $x^2 + ax + b = 0$, then what is the value of $(a + b)$?

(a) -11 **(b)** -3
(c) 0 **(d)** 3

12. If $z = \frac{1+i\sqrt{3}}{1-i\sqrt{3}}$ where $i = \sqrt{-1}$, then what is the argument of z ?

(a) $\frac{\pi}{3}$ **(b)** $\frac{2\pi}{3}$
(c) $\frac{4\pi}{3}$ **(d)** $\frac{5\pi}{6}$

13. If a, b, c are in AP, then what is $\begin{vmatrix} x+1 & x+2 & x+3 \\ x+2 & x+3 & x+4 \\ x+a & x+b & x+3 \end{vmatrix}$ equal to ?

(a) -1 **(b)** 0
(c) 1 **(d)** 2

14. $\log_x a$, a^x and $\log_b x$ are in GP, then what is x equal to ?

(a) $\log_a(\log_b a)$ **(b)** $\log_b(\log_a b)$
(c) $\frac{\log_a(\log_b a)}{2}$ **(d)** $\frac{\log_b(\log_a b)}{2}$

15. If $2^{\frac{1}{c}}$, $2^{\frac{b}{ac}}$, $2^{\frac{1}{a}}$ are in GP, then which one of the following is correct?

(a) a, b, c are in AP **(b)** a, b, c are in GP
(c) a, b, c are in HP **(d)** ab, be, ca are in AP

16. The first and the second terms of an AP are $\frac{5}{2}$ and $\frac{23}{12}$ respectively. If n^{th} term is the largest negative term, what is the value of n ?

(a) 5
(b) 6
(c) 7
(d) n cannot be determined

17. For how many integral values of k, the equation $x^2 - 4x + k = 0$, where k is an integer has real roots and both of them lie in the interval (0, 5) ?

(a) 3 **(b)** 4
(c) 5 **(d)** 6

18. In an AP, the first term is x and the sum of the first n terms is zero. What is the sum of next m terms ?

(a) $\frac{mx(m+n)}{n-1}$ **(b)** $\frac{mx(m+n)}{1-n}$
(c) $\frac{nx(m+n)}{m-1}$ **(d)** $\frac{nx(m+n)}{1-m}$

19. Consider the following statements :
1. $(25)! + 1$ is divisible by 26
2. $(6)! + 1$ is divisible by 7
Which of the above statements is/are correct ?

(a) 1 only **(b)** 2 only
(c) Both 1 and 2 **(d)** Neither 1 nor 2

20. If z is a complex number such that $\frac{z-1}{z+1}$ is purely imaginary, then what is $|z|$ equal to ?

(a) $\frac{1}{2}$ **(b)** $\frac{2}{3}$
(c) 1 **(d)** 2

21. How many real numbers satisfy the equation $|x - 4| + |x - 7| = 15$?

(a) Only one **(b)** Only two
(c) Only three **(d)** Infinitely many

22. A mapping $f : A \to B$ defined as $f(x) = \frac{2x+3}{3x+5}, x \in A$. If f is to be onto, then what are A and B equal to ?

(a) $A = R \setminus \{-\frac{5}{3}\}$ and $B = R \setminus \{-\frac{2}{3}\}$

(b) $A = R$ and $B = R \setminus \{-\frac{5}{3}\}$

(c) $A = R \setminus \{-\frac{3}{2}\}$ and $B = R \setminus \{0\}$

(d) $A = R \setminus \{-\frac{5}{3}\}$ and $B = R \setminus \{\frac{2}{3}\}$

23. α and β are distinct real roots of the quadratic equation $x^2 + ax + b = 0$. Which of the following statements is/are sufficient to find α ?

1. $\alpha + \beta = 0, \alpha^2 + \beta^2 = 2$
2. $\alpha\beta^2 = -1, a = 0$

Select the correct answer using the code given below:

(a) 1 only **(b)** 2 only
(c) Both 1 and 2 **(d)** Neither 1 nor 2

24. If the sixth term in the binomial expansion of $\left(x^{-\frac{8}{3}} + x^2 log_{10} x\right)^8$ is 5600, then what is the value of x ?

(a) 6 **(b)** 8
(c) 9 **(d)** 10

25. How many terms are there in the expansion of $(3x - y)^4(x + 3y)^4$?

(a) 9 **(b)** 12
(c) 15 **(d)** 17

26. p, q, r and s are in AP such that $p + s = 8$ and $qr = 15$. What is the difference between largest and smallest numbers ?

(a) 6 **(b)** 5
(c) 4 **(d)** 3

27. Consider the following statements for a fixed natural number n :

1. $C(n, r)$ is greatest if $n = 2r$
2. $C(n, r)$ is greatest if $n = 2r - 1$ and $n = 2r + 1$

Which of the statements given above is/are correct ?

(a) 1 only **(b)** 2 only
(c) Both 1 and 2 **(d)** Neither 1 nor 2

28. m parallel lines cut n parallel lines giving rise to 60 parallelograms. What is the value of $(m + n)$?

(a) 6 **(b)** 7
(c) 8 **(d)** 9

29. Let x be the number of permutations of the word 'PERMUTATIONS' and y be the number of permutations of the word 'COMBINATIONS'. Which one of the following is correct ?

(a) $x = y$ **(b)** $y = 2x$
(c) $x = 4y$ **(d)** $y = 4x$

30. 5-digit numbers are formed using the digits 0, 1, 2, 4, 5 without repetition. What is the percentage of numbers which are greater than 50,000 ?

(a) 20% **(b)** 25%
(c) $\frac{100}{3}\%$ **(d)** $\frac{110}{3}\%$

*Consider the following for the next **two (02)** items that follow :*

Let $\sin\beta$ be the GM of $\sin\alpha$ and $\cos\alpha$; $\tan\gamma$ be the AM of $\sin\alpha$ and $\cos\alpha$.

31. What is $\cos 2\beta$ equal to ?

(a) $(\cos\alpha - \sin\alpha)^2$ **(b)** $(\cos\alpha + \sin\alpha)^2$
(c) $(\cos\alpha - \sin\alpha)^3$ **(d)** $\frac{(\cos\alpha - \sin\alpha)^2}{2}$

32. What is the value of $\sec 2\gamma$?

(a) $\frac{3 - \sin 2\alpha}{5 + 2\sin 2\alpha}$ **(b)** $\frac{5 + \sin 2\alpha}{3 - \sin 2\alpha}$
(c) $\frac{3 - 2\sin 2\alpha}{4 + \sin 2\alpha}$ **(d)** $\frac{3 - \sin 2\alpha}{4 + 3\sin 2\alpha}$

*Consider the following for the next **two (02)** items that follow :*

A flagstaff 20 m long standing on a pillar 10 m high subtends an angle $\tan^{-1}(0.5)$ at a point P on the ground. Let θ be the angle subtended by the pillar at this point P

33. If x is the distance of P from bottom of the pillar, then consider the following statements:

1. x can take two values which are in the ratio 1 : 3
2. x can be equal to height of the flagstaff

Which of the statements given above is/are correct ?

(a) 1 only **(b)** 2 only
(c) Both 1 and 2 **(d)** Neither 1 nor 2

34. What is a possible value of $\tan\theta$?

(a) $\frac{3}{4}$ **(b)** $\frac{2}{3}$
(c) $\frac{1}{3}$ **(d)** $\frac{1}{4}$

*Consider the following for the next **two (02)** items that follow :*

The perimeter of a triangle ABC is 6 times the AM of sine of angles of the triangle. Further $BC = \sqrt{3}$ and $CA = 1$.

35. What is the perimeter of the triangle ?

(a) $\sqrt{3} + 1$ **(b)** $\sqrt{3} + 2$
(c) $\sqrt{3} + 3$ **(d)** $2\sqrt{3} + 1$

36. Consider the following statements :

1. ABC is right angled triangle
2. The angles of the triangle are in AP

Which of the statements given above is/are correct ?

(a) 1 only **(b)** 2 only
(c) Both 1 and 2 **(d)** Neither 1 nor 2

*Consider the following for the next **two (02)** items that follow :*

Let $x = \dfrac{\sin^2 A + \sin A + 1}{\sin A}$ where $0 < A \le \dfrac{\pi}{2}$

37. What is the minimum value of x ?
(a) 1 **(b)** 2
(c) 3 **(d)** 4

38. At what value of A does x attain the minimum value ?
(a) $\dfrac{\pi}{6}$ **(b)** $\dfrac{\pi}{4}$
(c) $\dfrac{\pi}{3}$ **(d)** $\dfrac{\pi}{2}$

*Consider the following for the next **two (02)** items that follow :*

In the triangle ABC, $a^2+b^2 + c^2 = ac + \sqrt{3}\, bc$

39. What is the nature of the triangle ?
(a) Equilateral
(b) Isosceles
(c) Right angled triangle
(d) Scalene but not right angled

40. If $c = 8$, what is the area of the triangle ?
(a) $4\sqrt{3}$ **(b)** $6\sqrt{3}$
(c) $8\sqrt{3}$ **(d)** $12\sqrt{3}$

*Consider the following for the next **two (02)** items that follow :*

Consider the function
$f(x) = |x-2| + |3-x| + |4-x|$, where $x \in R$.

41. At what value of x does the function attain minimum value ?
(a) 2 **(b)** 3
(c) 4 **(d)** 0

42. What is the minimum value of the function ?
(a) 2 **(b)** 3
(c) 4 **(d)** 0

*Consider the following for the next **two (02)** items that follow :*

Consider the sum
$S = 0! + 1! + 2! + 3! + 4! + \ldots + 100!$

43. If the sum S is divided by 8, what is the remainder ?
(a) 0
(b) 1
(c) 2
(d) Cannot be determined

44. If the sum S is divided by 60, what is the remainder ?
(a) 1 **(b)** 3
(c) 17 **(d)** 34

*Consider the following for the next **two (02)** items that follow :*

In a triangle PQR, P is the largest angle and $\cos P = \dfrac{1}{3}$. Further the in-circle of the triangle touches the sides PQ, QR and RP at N, L and M respectively such that the lengths PN, QL and RM are n, $n + 2$, $n + 4$ respectively where n is an integer.

45. What is the value of n ?
(a) 4 **(b)** 6
(c) 8 **(d)** 10

46. What is the length of the smallest side ?
(a) 12 **(b)** 14
(c) 16 **(d)** 18

*Consider the following for the next **two (02)** items that follow :*

Given that
$\sin x + \cos x + \tan x + \cot x + \sec x + \operatorname{cosec} x = 7$

47. The given equation can be reduced to
(a) $\sin^2 2x - 44 \sin 2x + 36 = 0$
(b) $\sin^2 2x + 44 \sin 2x - 36 = 0$
(c) $\sin^2 2x - 22 \sin 2x + 18 = 0$
(d) $\sin^2 2x + 22 \sin 2x - 18 = 0$

48. If $\sin 2x = a - b\sqrt{c}$, where a and b are natural numbers and c is prime number, then what is the value of $a - b + 2c$?
(a) 0 **(b)** 14
(c) 21 **(d)** 28

*Consider the following for the next **two (02)** items that follow :*

A quadratic equation is given by

$(3 + 2\sqrt{2})x^2 - (4 + 2\sqrt{3})x + (8 + 4\sqrt{3}) = 0$

49. What is the HM of the roots of the equation ?
(a) 2 **(b)** 4
(c) $2\sqrt{2}$ **(d)** $2\sqrt{3}$

50. What is the GM of the roots of the equation ?
(a) $\sqrt{2}\left(\sqrt{6} - \sqrt{3} + \sqrt{2} - 1\right)$
(b) $\sqrt{2}\left(\sqrt{6} + \sqrt{3} - \sqrt{2} - 1\right)$
(c) $\left(\sqrt{6} - \sqrt{3} + \sqrt{2} - 1\right)$
(d) $\left(\sqrt{6} + \sqrt{3} + \sqrt{2} - 1\right)$

*Consider the following for the next **two (02)** items that follow :*

Let $\Delta(a, b, c, \alpha) = \begin{vmatrix} a & b & a\alpha + b \\ b & c & b\alpha + c \\ a\alpha + b & b\alpha + c & 0 \end{vmatrix}$

51. If $\Delta(a, b, c, \alpha) = 0$ for every $\alpha > 0$, then which one of the following is correct ?
(a) a, b, c are in AP **(b)** a, b, c are in GP
(c) $a, 2b, c$ are in AP **(d)** $a, 2b, c$ are in GP

52. If $\Delta(7, 4, 2, \alpha) = 0$, then α is a root of which one of the following equations ?
(a) $7x^2 + 4x + 2 = 0$ **(b)** $7x^2 - 4x + 2 = 0$
(c) $7x^2 + 8x + 2 = 0$ **(d)** $7x^2 - 8x + 2 = 0$

*Consider the following for the next **two (02)** items that follow :*

Given that $m(\theta) = \cot^2\theta + n^2\tan^2\theta + 2n$, where n is a fixed positive real number.

53. What is the least value of $m(\theta)$?
(a) n **(b)** $2n$
(c) $3n$ **(d)** $4n$

54. Under what condition does m attain the least value ?
(a) $n = \tan^2\theta$ **(b)** $n = \cot^2\theta$
(c) $n = \sin^2\theta$ **(d)** $n = \cos^2\theta$

*Consider the following for the next **two (02)** items that follow :*

A quadrilateral is formed by the lines $x = 0$, $y = 0$, $x + y = 1$ and $6x + y = 3$.

55. What is the equation of diagonal through origin ?
(a) $3x + y = 0$ **(b)** $2x + 3y = 0$
(c) $3x - 2y = 0$ **(d)** $3x + 2y = 0$

56. What is the equation of other diagonal ?
(a) $x + 2y - 1 = 0$ **(b)** $x - 2y - 1 = 0$
(c) $2x + y + 1 = 0$ **(d)** $2x + y - 1 = 0$

*Consider the following for the next **two (02)** items that follow :*

P(x, y) is any point on the ellipse $x^2 + 4y^2 = 1$. Let *E, F be the foci of the ellipse.*

57. What is PE + PF equal to ?
(a) 1 **(b)** 2
(c) 3 **(d)** 4

58. Consider the following points :
1. $\left(\frac{\sqrt{3}}{2}, 0\right)$
2. $\left(\frac{\sqrt{3}}{2}, \frac{1}{4}\right)$
3. $\left(\frac{\sqrt{3}}{2}, -\frac{1}{4}\right)$

Which of the above points lie on latus rectum of ellipse ?
(a) 1 and 2 only **(b)** 2 and 3 only
(c) 1 and 3 only **(d)** 1, 2 and 3

*Consider the following for the next **two (02)** items that follow :*

The line $y = x$ partitions the circle $(x - a)^2 + y^2 = a^2$ in two segments.

59. What is the area of minor segment ?
(a) $\frac{(\pi - 2)a^2}{4}$ **(b)** $\frac{(\pi - 1)a^2}{4}$
(c) $\frac{(\pi - 2)a^2}{2}$ **(d)** $\frac{(\pi - 1)a^2}{2}$

60. What is the area of major segment ?
(a) $\frac{(3\pi - 2)a^2}{4}$ **(b)** $\frac{(3\pi + 2)a^2}{4}$
(c) $\frac{(3\pi - 2)a^2}{2}$ **(d)** $\frac{(3\pi + 2)a^2}{2}$

*Consider the following for the next **two (02)** items that follow :*

Let A(1, –1, 2) and B(2, 1, –1) be the end points of the diameter of the sphere $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz - 1 = 0$.

61. What is $u + v + w$ equal to ?
(a) – 2 **(b)** – 1
(c) 1 **(d)** 2

62. If P(x, y, z) is any point on the sphere, then what is $PA^2 + PB^2$ equal to ?
(a) 15 **(b)** 14
(c) 13 **(d)** 6·5

*Consider the following for the next **two (02)** items that follow :*

Consider two lines whose direction ratios are (2, – 1, 2) and $(k, 3, 5)$. They are inclined at an angle $\frac{\pi}{4}$.

63. What is the value of k ?
(a) 4 **(b)** 2
(c) 1 **(d)** – 1

64. What are the direction ratios of a line which is perpendicular to both the lines ?
(a) (1, 2, 10) **(b)** (– 1, – 2, 10)
(c) (11, 12, – 10) **(d)** (11, 2, – 10)

*Consider the following for the next **two (02)** items that follow :*

Let $\vec{a} = 3\hat{i} + 3\hat{j} + 3\hat{k}$ and $\vec{c} = \hat{j} - \hat{k}$. Let $\vec{b}$ be such that $\vec{a} \cdot \vec{b} = 27$ and $\vec{a} \times \vec{b} = 9\vec{c}$

65. What is $\vec{b}$ equal to ?

(a) $3\hat{i} + 4\hat{j} + 2\hat{k}$ **(b)** $5\hat{i} + 2\hat{j} + 2\hat{k}$
(c) $5\hat{i} - 2\hat{j} + 6\hat{k}$ **(d)** $3\hat{i} + 3\hat{j} + 4\hat{k}$

66. What is the angle between $(\vec{a} + \vec{b})$ and $\vec{c}$

(a) $\frac{\pi}{2}$ **(b)** $\frac{\pi}{3}$
(c) $\frac{\pi}{4}$ **(d)** $\frac{\pi}{6}$

*Consider the following for the next **two (02)** items that follow :*

Let a vector $\vec{a} = 4\hat{i} - 8\hat{j} + \hat{k}$ make angles α, β, γ with the positive directions of x, y, z axes respectively.

67. What is $\cos\alpha$ equal to ?

(a) $\frac{1}{3}$ **(b)** $\frac{4}{9}$
(c) $\frac{5}{9}$ **(d)** $\frac{2}{3}$

68. What is $\cos2\beta + \cos2\gamma$ equal to ?

(a) $-\frac{32}{81}$ **(b)** $-\frac{16}{81}$
(c) $\frac{16}{81}$ **(d)** $\frac{32}{81}$

*Consider the following for the next **two (02)** items that follow :*

The position vectors of two points A and B are $\hat{\imath} - \hat{\jmath}$ and $\hat{\jmath} + \hat{k}$ respectively.

69. Consider the following points :
1. $(-1, -3, 1)$
2. $(-1, 3, 2)$
3. $(-2, 5, 3)$

Which of the above points lie on the line joining A and B ?
(a) 1 and 2 only **(b)** 2 and 3 only
(c) 1 and 3 only **(d)** 1, 2 and 3

70. What is the magnitude of $\overrightarrow{AB}$?
(a) 2 **(b)** 3
(c) $\sqrt{6}$ **(d)** $\sqrt{3}$

*Consider the following for the next **three (03)** items that follow :*

Let $f(x) = Pe^x + Qe^{2x} + Re^{3x}$, where P, Q, R are real numbers. Further $f(0) = 6$, $f'(ln\,3) = 282$ and $\int_0^{ln2} f(x)dx = 11$

71. What is the value of Q ?
(a) 1 **(b)** 2
(c) 3 **(d)** 4

72. What is the value of R ?
(a) 1 **(b)** 2
(c) 3 **(d)** 4

73. What is $f'(0)$ equal to ?
(a) 18 **(b)** 16
(c) 15 **(d)** 14

*Consider the following for the next **two (02)** items that follow :*

Suppose E is the differential equation representing family of curves $y^2 = 2cx + 2c\sqrt{c}$ where c is a positive parameter.

74. What is the order of the differential equation ?
(a) 1 **(b)** 2
(c) 3 **(d)** 4

75. What is the degree of the differential equation ?
(a) 2
(b) 3
(c) 4
(d) Degree does not exist

*Consider the following for the next **three (03)** items that follow :*

Let $f(x) = \begin{vmatrix} \cos x & x & 1 \\ 2\sin x & x^2 & 2x \\ \tan x & x & 1 \end{vmatrix}$

76. What is $f(0)$ equal to ?
(a) -1 **(b)** 0
(c) 1 **(d)** 2

77. What is $\lim_{x\to0} \frac{f(x)}{x}$ equal to ?

(a) -1 **(b)** 0
(c) 1 **(d)** **2**

78. What is $\lim_{x\to0} \frac{f(x)}{x^2}$ equal to ?

(a) -1 **(b)** 0
(c) 1 **(d)** **2**

*Consider the following for the next **two (02)** items that follow :*

Let $f(x) = \sin[\pi^2]x + \cos[-\pi^2]x$ where [.] is a greatest integer function

79. What is $f\left(\frac{\pi}{2}\right)$ equal to ?

(a) -1 **(b)** 0
(c) 1 **(d)** **2**

80. What is $f\left(\frac{\pi}{4}\right)$ equal to ?

(a) $-\frac{1}{\sqrt{2}}$ **(b)** -1

(c) 1 **(d)** $\frac{1}{\sqrt{2}}$

*Consider the following for the next **three (03)** items that follow :*

Let $I_1 = \int_0^{\pi} \frac{x}{1+\cos^2 x} dx$ and $I_2 = \int_0^{\pi} \frac{x}{1+\sin^2 x} dx$

81. What is the value of $\frac{I_1 + I_2}{I_1 - I_2}$?

(a) 1 **(b)** π
(c) $\pi^2/2$ **(d)** $\frac{\pi+1}{\pi-1}$

82. What is the value of $8I_1^2$?

(a) π **(b)** π^2
(c) π^3 **(d)** π^4

83. What is the value of I_2 ?

(a) $\frac{\pi}{\sqrt{2}}$ **(b)** $\frac{\pi^2}{2\sqrt{2}}$
(c) $\frac{3\pi}{2\sqrt{2}}$ **(d)** $\frac{\pi}{4\sqrt{2}}$

*Consider the following for the next **two (02)** items that follow :*

Let $l = \int_a^b \frac{|x|}{x} dx,\ a < b$

84. What is l equal to when $a < 0 < b$?

(a) $a + b$ **(b)** $a - b$

(c) $b - a$ **(d)** $\frac{(a+b)}{2}$

85. What is l equal to when $a < b < 0$?

(a) $a + b$ **(b)** $a - b$

(c) $b - a$ **(d)** $\frac{(a+b)}{2}$

*Consider the following for the next **three (03)** items that follow :*

Let $f(x) = |lnx|,\ x \neq 1$

86. What is the derivative of $f(x)$ at $x = 0{\cdot}5$?

(a) -2 **(b)** -1
(c) 1 **(d)** 2

87. What is the derivative of $fof(x)$ at $x = 2$?

(a) $-\frac{1}{2}$ **(b)** -1

(c) $\frac{1}{2}$ **(d)** 2

88. What is the derivative of $fof(x)$, where $1 < x < 2$?

(a) $\frac{1}{lnx}$ **(b)** $\frac{1}{xlnx}$

(c) $-\frac{1}{lnx}$ **(d)** $-\frac{1}{xlnx}$

*Consider the following for the next **two (02)** items that follow :*

Let $f(x) = \begin{cases} x+6, & x \leq 1 \\ px+q, & 1 < x < 2 \\ 5x, & x \geq 2 \end{cases}$

and $f(x)$ is continuous

89. What is the value of p ?

(a) 2 **(b)** 3
(c) 4 **(d)** 5

90. What is the value of q ?

(a) 2 **(b)** 3
(c) 4 **(d)** 5

91. Consider the following statements :

1. $f(x) = lnx$ is increasing in $(0, \infty)$
2. $g(x) = e^x + e^x$ is decreasing in $(0, \infty)$

Which of the statements given above is/are correct ?

(a) 1 only **(b)** 2 only
(c) Both 1 and 2 **(d)** Neither 1 nor 2

92. What is the derivative of $\sin^2 x$ with respect to $\cos^2 x$?

(a) -1 **(b)** 1
(c) $\sin 2x$ **(d)** $\cos 2x$

93. For what value of m with $m < 0$, is the area bounded by the lines $y = x$, $y = mx$ and $x = 2$ equal to 3 ?

(a) $-\frac{1}{2}$ **(b)** -1

(c) $-\frac{3}{2}$ **(d)** -2

94. What is the derivative of $\text{cosec}(x°)$?

(a) $-\text{cosec}(x°)\cot(x°)$

(b) $-\frac{\pi}{180}\text{cosec}(x°)\cot(x°)$

(c) $\frac{\pi}{180}\text{cosec}(x°)\cot(x°)$

(d) $-\frac{\pi}{180}\text{cosec}(x)\cot(x)$

95. A solution of the differential equation $\left(\frac{dy}{dx}\right)^2 - x\frac{dy}{dx} = 0$ is

(a) $y = x^2/2 + c$ **(b)** $y = 2x + 4$

(c) $y = x^2 + 1$ **(d)** $y = \frac{(x^2 - x)}{2}$

96. If $f(x) = x^2 + 2$ and $g(x) = 2x - 3$, then what is $(fg)(1)$ equal to ?

(a) 3 **(b)** 1

(c) –2 **(d)** –3

97. What is the range of the function $f(x) = x + |x|$ if the domain is the set of real numbers ?

(a) $(0, \infty)$ **(b)** $[0, \infty)$

(c) $(-\infty, \infty)$ **(d)** $[1, \infty)$

98. If $f(x) = x(4x^2 - 3)$, then what is $f(\sin\theta)$ equal to ?

(a) $-\sin3\theta$ **(b)** $-\cos3\theta$

(c) $\sin3\theta$ **(d)** $-\sin4\theta$

99. What is $\lim_{x\to5}\frac{5-x}{|x-5|}$ equal to ?

(a) – 1 **(b)** 0

(c) 1 **(d)** Limit does not exist

100. What is $\lim_{x\to1}\frac{x^9-1}{x^3-1}$ equal to ?

(a) – 1 **(b)** – 3

(c) 3 **(d)** Limit does not exist

101. The mean and variance of five obser-vations are 14 and 13.2 respectively. Three of the five observations are 11, 16 and 20. What are the other two observations ?

(a) 8 and 15 **(b)** 9 and 14

(c) 10 and 13 **(d)** 11 and 12

102. Let A and B be two independent events such that P(A') = 0.7, P(B') = k, P(A ∪ B) = 0.8. What is the value of k ?

(a) $\frac{5}{7}$ **(b)** $\frac{4}{7}$

(c) $\frac{2}{7}$ **(d)** $\frac{1}{7}$

103. A biased coin with the probability of getting head equal to $\frac{1}{4}$ is tossed five times. What is the probability of getting tail in all the first four tosses followed by head ?

(a) $\frac{81}{512}$ **(b)** $\frac{81}{1024}$

(c) $\frac{81}{256}$ **(d)** $\frac{27}{1024}$

104. A coin is biased so that heads comes up thrice as likely as tails. In four independent tosses of the coin, what is probability of getting exactly three heads ?

(a) $\frac{81}{256}$ **(b)** $\frac{27}{64}$

(c) $\frac{27}{256}$ **(d)** $\frac{9}{256}$

105. Let X and Y be two random variables such that X + Y = 100. If X follows Binomial distribution with parameters $n = 100$ and $p = \frac{4}{5}$, what is the variance of Y?

(a) 1 **(b)** $\frac{1}{2}$

(c) **16** **(d)** $\frac{1}{16}$

106. If two lines of regression are $x + 4y + 1 = 0$ and $4x + 9y + 7 = 0$, then what is the value of x when $y = -3$?

(a) –13 **(b)** –5

(c) 5 **(d)** 7

107. The central angles p, q, r and s (in degrees) of four sectors in a Pie Chart satisfy the relation $9p = 3q = 2r = 6s$. What is the value of $4p - q$?

(a) 12 **(b)** 24

(c) 30 **(d)** 36

108. The observations 4, 1, 4, 3, 6, 2, 1, 3, 4, 5, 1, 6 are outputs of 12 dices thrown simultaneously. If m and M are means of lowest 8 observations and highest 4 observations respectively, then what is $(2m + M)$ equal to ?

(a) 10 **(b)** 12

(c) 17 **(d)** 21

109. A bivariate data set contains only two points (–1, 1) and (3, 2). What will be the line of regression of y on x ?

(a) $x - 4y + 5 = 0$ **(b)** $3x + 2y - 1 = 0$

(c) $x + 4y + 1 = 0$ **(d)** $5x - 4y + 1 = 0$

110. A die is thrown 10 times and obtained the following outputs:
1, 2, 1, 1, 2, 1, 4, 6, 5, 4
What will be the mode of data so obtained ?
(a) 6 **(b)** 4
(c) 2 **(d)** 1

111. Consider the following frequency distribution:

x	1	2	3	5
f	4	6	9	7

What is the value of median of the distribution?
(a) 1 **(b)** 2
(c) 3 **(d)** 3-5

112. For data –1, 1, 4, 3, 8, 12, 17, 19, 9, 11; if M is the median of first 5 observations and N is the median of last five observations, then what is the value of 4M – N ?
(a) 7 **(b)** 4
(c) 1 **(d)** 0

113. Let P, Q, R represent mean, median and mode. If for some distribution $5P = 4Q = \frac{R}{2}$, then what is $\frac{P+Q}{2P+0.7R}$ equal to ?
(a) $\frac{1}{12}$ **(b)** $\frac{1}{7}$
(c) $\frac{2}{9}$ **(d)** $\frac{1}{4}$

114. If G is the geometric mean of numbers 1, 2, 2^2, 2^3,. . ., 2^{n-1}, then what is the value of $1 + 2\log_2 G$?
(a) 1 **(b)** 4
(c) n - 1 **(d)** n

115. If H is the harmonic mean of numbers 1, 2, 2^2, 2^3, . . . , 2^{n-1}, then what is n/H equal to ?
(a) $2-\frac{1}{2^{n+1}}$ **(b)** $2-\frac{1}{2^{n-1}}$
(c) $2+\frac{1}{2^{n-1}}$ **(d)** $2-\frac{1}{2^n}$

116. Let P be the median, Q be the mean and R be the mode of observations x_1, x_2, x_3, ... x_n. Let $S=\sum_{i=1}^{n}(2x_i-a)^2$ S takes minimum value, when a is equal to
(a) P **(b)** $\frac{Q}{2}$
(c) 2Q **(d)** R

117. One bag contains 3 white and 2 black balls, another bag contains 2 white and 3 black balls. Two balls are drawn from the first bag and put it into the second bag and then a ball is drawn from the second bag. What is the probability that it is white ?
(a) $\frac{6}{7}$ **(b)** $\frac{33}{70}$
(c) $\frac{3}{10}$ **(d)** $\frac{1}{70}$

118. Three dice are thrown. What is the probability that each face shows only multiples of 3 ?
(a) $\frac{1}{9}$ **(b)** $\frac{1}{18}$
(c) $\frac{1}{27}$ **(d)** $\frac{1}{3}$

119. What is the probability that the month of December has 5 Sundays ?
(a) **1** **(b)** $\frac{1}{4}$
(c) $\frac{3}{7}$ **(d)** $\frac{2}{7}$

120. A natural number n is chosen from the first 50 natural numbers. What is the probability that $n+\frac{50}{n}<50$?
(a) $\frac{23}{25}$ **(b)** $\frac{47}{50}$
(c) $\frac{24}{25}$ **(d)** $\frac{49}{50}$

ANSWER KEY

Q No	Answer Key	Topic	Chapter
1	**a**	Cube root of unity	Complex Numbers
2	**d**	Number of ways	Permutations and Combinations
3	**Bonus**	Binary operation	Sets
4	**b**	Adjoint of a matrix	Matrices
5	**d**	Cube root of unity	Complex Numbers
6	**b**	Properties of matrices	Matrices
7	**d**	Properties of determinants	Determinants
8	**b**	System of equations	Determinants
9	**b**	Properties of determinants	Determinants
10	**b**	Expansion of determinant	Determinants
11	**d**	Roots of Equations	Complex Numbers
12	**b**	Argument	Complex Numbers
13	**b**	Expansion of determinant	Determinants
14	**c**	Geometric Progression	Sequence and Series
15	**a**	Geometric Progression	Sequence and Series
16	**b**	Arithmetic Progression	Sequence and Series
17	**a**	Nature of roots	Quadratic Equations
18	**b**	Suum of n terms	Sequence and Series
19	**b**	Factorial	Permutations and Combinations
20	**c**	Modulus	Complex Numbers
21	**b**	Roots of Equations	Equations
22	**d**	Onto Functions	Relations and Functions
23	**a**	Roots of Equations	Quadratic Equations
24	**d**	N^{th} term	Binomial Theorem
25	**c**	Binomial Expansion	Binomial Theorem
26	**a**	Arithmetic Progression	Sequence and Series
27	**c**	Combinations	Permutations and Combinations
28	**d**	Combinations	Permutations and Combinations
29	**c**	Number of permutations	Permutations and Combinations
30	**b**	Number of ways	Permutations and Combinations
31	**a**	Trigonometric Identities	Trigonometry
32	**b**	Trigonometric Identities	Trigonometry
33	**a**	Height and Distance	Trigonometry
34	**c**	Height and Distance	Trigonometry
35	**c**	Triangle	Trigonometry
36	**c**	Triangle	Trigonometry
37	**c**	Arithmetic and Geometric Progression	Trigonometry
38	**d**	Minimum Value	Trigonometry
39	**c**	Triangle property	Trigonometry

Q No	Answer Key	Topic	Chapter
40	**c**	Area of triangle	Trigonometry
41	**b**	Extreme values	Continuity and Differentiability
42	**a**	Extreme values	Continuity and Differentiability
43	**c**	Factorial	Permutations and Combinations
44	**d**	Factorial	Permutations and Combinations
45	**c**	Triangle	Trigonometry
46	**a**	Triangle	Trigonometry
47	**a**	Trignometric Relation	Trigonometry
48	**d**	Trignometric Relation	Trigonometry
49	**b**	Harmonic Mean	Sequence and Series
50	**a**	Geometric Mean	Sequence and Series
51	**b**	Expansion of determinant	Determinants
52	**c**	Properties of determinants	Determinants
53	**d**	Trigonometric expressions	Trigonometry
54	**b**	Trigonometric expressions	Trigonometry
55	**c**	Equation of a line	Straight lines
56	**d**	Equation of a line	Straight lines
57	**b**	Ellipse	Conic Section
58	**d**	Ellipse	Conic Section
59	**a**	Circle	Conic Section
60	**b**	Circle	Conic Section
61	**a**	Sphere	3D Geometry
62	**b**	Sphere	3D Geometry
63	**a**	Direction ratios	Three Diimensional Geomtery
64	**d**	Direction ratios	Three Diimensional Geomtery
65	**b**	Product of two vectors	Vector Algebra
66	**a**	Product of two vectors	Vector Algebra
67	**b**	Direction cosines	3D Geometry
68	**a**	Direction cosines	3D Geometry
69	**b**	Line	3D Geometry
70	**c**	Line	3D Geometry
71	**b**	Definite integral	Calculus
72	**c**	Definite integral	Calculus
73	**d**	Differentiation	Calculus
74	**a**	Order and degree	Differential equations
75	**b**	Order and degree	Differential equations
76	**b**	Evaluation of limits	Limits
77	**b**	Evaluation of limits	Limits
78	**a**	Evaluation of limits	Limits
79	**b**	Trigonometric functions	Trigonometry
80	**d**	Trigonometric Functions	Trigonometry
81	**Bonus**	Definite Integral	Calculus

Q No	Answer Key	Topic	Chapter
82	**d**	Definite Integral	Calculus
83	**b**	Definite Integral	Calculus
84	**a**	Definite Integral	Calculus
85	**c**	Definite Integral	Calculus
86	**a**	Differentiation	Calculus
87	**c**	Differentiation	Calculus
88	**d**	Differentiation	Calculus
89	**b**	Continuity	Calculus
90	**c**	Continuity	Calculus
91	**a**	Increasing-decreasing functions	Calculus
92	**a**	Differentiation	Calculus
93	**a**	General Equation of a line	Straight Lines
94	**b**	Differentiation	Calculus
95	**a**	Variable separable	Differential Equations
96	**d**	Operations on functions	Functions
97	**b**	Range	Functions
98	**a**	Value of a function	Functions
99	**d**	Evaluation of limits	Limits
100	**c**	Evaluation of limits	Limits
101	**c**	Mean and variance	Statistics
102	**c**	Independent events	Probability
103	**b**	Independent events	Probability
104	**b**	Independent events	Probability
105	**c**	Binomial distribution	Probability
106	**c**	Regression	Statistics
107	**d**	Angles	Trigonometry
108	**a**	Mean	Statistics
109	**a**	Regression	Statistics
110	**a**	Mode	Statistics
111	**c**	Median	Statistics
112	**d**	Median	Statistics
113	**d**	Mean, median, mode	Statistics
114	**d**	Geometric mean	Sequence and Series
115	**b**	Harmonic mean	Sequence and Series
116	**c**	Derivative	Continuity and Differentiability
117	**b**	Total Probability	Probability
118	**c**	Probability	Probability
119	**c**	Probability	Probability
120	**b**	Probability	Probability

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MATHEMATICS

I

SOLVED PAPER
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ANSWERS WITH EXPLANATION

1. Option (a) is correct.

Explanations: We have,

$$\left|\frac{1-\omega}{\omega+\omega^2}\right| = \left|\frac{1-\omega}{-1}\right| = |-1+\omega|$$

$$= \left|-1+\left(\frac{-1+1\sqrt{3}}{2}\right)\right|$$

$$= \sqrt{\left(\frac{-3}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2} = \sqrt{3}$$

2. Option (d) is correct.

Explanations: For number to be divisible by 6, the number should be divisible by 2 and 3 both.

Now, number is divisible by 2 if units place digit is 0, 2, or 4:

Also, sum of all digits = 0 + 1 + 2 +3 + 4 + 5 = 15

Case I: If units digit is 0; then no. of ways = 5! = 120

Case II: If units digit is either 2 or 4, then no. of ways = 2 × 4! × 4 = 192

So, total number of 6 digit number formed

= 120 + 192 = 312

3. Option (Bonus) is correct.

Explanations: To covert 1011 decimal number, we have,

Divisible by 2	Quotient	Remainder	Binary Bit
1011 ÷ 2	505	1	1
505 ÷ 2	252	1	1
252 ÷ 2	126	0	0
126 ÷ 2	63	0	0
63 ÷ 2	31	1	1
31 ÷ 2	15	1	1
15 ÷ 2	7	1	1
7 ÷ 2	3	1	1
3 ÷ 2	1	1	1
1 ÷ 2	0	1	1

1011 = (1111110011)

4. Option (b) is correct.

Explanations: $|2 \text{ adj } (3A)| = 2^3 |\text{adj } (3A)|$ (i)

Now, $|3A| = 3^3 |A| = 3^3.4 = 3^3.2^2$

$|\text{adj } (3A)| = |3A|^{3-1} = |3A|^2 = |3^3.2^2|^2 = 3^6.2^4$

from (i), we have

$|2 \text{ adj } (3A)| = 2^3.2^4.3^6 = 2^7.3^6 = 2^\alpha.3^\beta$

$\Rightarrow \quad \alpha = 7$ and $\beta = 6$

$\therefore \alpha + \beta = 7 + 6 = 13$

5. Option (d) is correct.

Explanations: We have,

$x^2 - x + 1$

$$x = \frac{1 \pm \sqrt{3}i}{2} \Rightarrow x = -\omega \text{ or } -\omega^2$$

So, $\alpha = -\omega$ and $\beta = -\omega^2$

$$\left|\frac{\alpha^{1\omega}+\beta^{1\omega}}{\alpha^{1\omega}-\beta^{1\omega}}\right| = \left|\frac{\omega^{1\omega}+\omega^{2\omega}}{\omega^{1\omega}-\omega^{2\omega}}\right|$$

$$= \left|\frac{1+\omega^{1\omega}}{1-\omega^{1\omega}}\right| = \left|\frac{1+\omega}{1-\omega}\right|$$

$$\left|\frac{\pm^{100}+\beta^{100}}{\pm^{100}-\beta^{100}}\right| = \left|\frac{(-\omega)^{100}+\left(-\omega^2\right)^{100}}{(-\omega)^{100}-\left(-\omega^2\right)^{100}}\right|$$

$$= \left|\frac{\omega^{100}+\left(1+\omega^{100}\right)}{\omega^{100}+\left(1-\omega^{100}\right)}\right| = \left|\frac{1+\omega^{100}}{1-\omega^{100}}\right| = \left|\frac{1+\omega^{3\times 33}\omega}{1-\omega^{3\times 33}\omega}\right|$$

$$= \left|\frac{1+\omega}{1-\omega}\right| = \left|\frac{1+\left(\frac{-1-\sqrt{3}i}{2}\right)}{1+\left(\frac{-1+\sqrt{3}i}{2}\right)}\right|$$

$$= \left|\frac{1+\sqrt{3}i}{3+\sqrt{3}i}\right| = \frac{\sqrt{1+3}}{\sqrt{9+3}} = \frac{1}{\sqrt{3}}$$

6. Option (b) is correct.

Explanations: When A and B be symmetric matrices then (AB – BA) is skew symmetric.

7. Option (d) is correct.

Explanations:

$$\because \begin{bmatrix}3 & 5\\ 7 & 3\end{bmatrix}\begin{bmatrix}K\\ 2K\end{bmatrix}=\begin{bmatrix}7 & 3\\ 3 & 5\end{bmatrix}\begin{bmatrix}K\\ 2K\end{bmatrix}$$

$$\begin{bmatrix}13K\\ 13K\end{bmatrix}=\begin{bmatrix}13K\\ 13K\end{bmatrix}$$

$$\begin{bmatrix}3 & 5\\ 7 & 3\end{bmatrix}\neq\begin{bmatrix}7 & 3\\ 3 & 5\end{bmatrix}$$

So, both statements are wrong.

8. Option (b) is correct.

Explanations: We have,

$x + 2y + z = 4$

$2x + 4y + 2z = 8$

$\Rightarrow 2(x + 2y + z) = 8$

$\Rightarrow x + 2y + z = 4$

and $3x + 6y + 3z = 10$

$\Rightarrow 3(x + 2y + z) = 10$

$$\Rightarrow x + 2y + z = \frac{10}{3}$$

So, the linear equations have infinity many solutions.

9. Option (b) is correct.

Explanations: We know that if X_1 and X_2 are solution of system of equations $AX = B$, $B = 0$ then $aX_1 + bX_2$ is also solution iff $a + b = 1$

10. Option (b) is correct.

Explanations: $\begin{vmatrix}0 & x-a & x-b\\ 0 & 0 & x-c\\ x+b & x+c & 1\end{vmatrix}=0$

$\Rightarrow 0 - (x - a)(0 - (x - c)(x + b)) + (x - b)(0 - 0) = 0$

$\Rightarrow (x - a)\,(x + b)\,(x - c) = 0$

$\Rightarrow x = a, x = -b$ or $x = c$

Sum of roots $= a - b + c$

11. Option (d) is correct.

Explanations: $2 - i\sqrt{3}$ is a root of $x^2 + ax + b$.

So, $2 + i\sqrt{3}$ is also the root of $x^2 + ax + b$.

Sum of roots = 4

$-a = 4 \Rightarrow a = -4$

Product of roots = 4 + 3 = 7

$\Rightarrow b = 7$

So, $a + b = -4 + 7 = 3$

12. Option (b) is correct.

Explanations: We have,

$$z=\frac{1+i\sqrt{3}}{1-i\sqrt{3}}\times\frac{1+i\sqrt{3}}{1+i\sqrt{3}}=\frac{1-3+2\sqrt{3}\,i}{1+3}$$

$$=\frac{-2+2\sqrt{3i}}{4}=\frac{-1}{2}+\frac{\sqrt{3}}{2}i$$

$$\text{Now, } \tan\theta=\left(\frac{\frac{\sqrt{3}}{2}}{\frac{-1}{2}}\right)=-\sqrt{3}$$

$$\Rightarrow \theta=\tan^{-1}\left(-\sqrt{3}\right)=\pi-\frac{\pi}{3}=\frac{2\pi}{3}$$

13. Option (b) is correct.

Explanations: We have,

$2b = a + c$ (i) (a, b, c in AP)

$$\text{Let } \Delta=\begin{vmatrix}x+1 & x+2 & x+3\\ x+2 & x+3 & x+4\\ x+a & x+b & x+c\end{vmatrix}$$

$$=\begin{vmatrix}x+1 & 1 & 2\\ x+2 & 1 & 2\\ x+a & b-a & c-a\end{vmatrix}=\begin{vmatrix}x+1 & 1 & 2\\ 1 & 0 & 0\\ x+a & b-a & c-a\end{vmatrix}$$

$(x + 1)(0 - 0) - 1(c - a - 0) + 2(b - a - 0)$

$= a - c + 2b - 2a$

$= -a - c + a + c$ [Using (i)]

$= 0$

14. Option (c) is correct.

Explanations: Since, $\log_x a$, a^x, $\log_b x$ are in G.P

$$\therefore \left(a^x\right)^2=\left(\log_x a\right)\left(\log_b x\right)$$

$$\Rightarrow a^{2x}=\frac{\log a}{\log x}\frac{\log x}{\log b}=\log_b a$$

Taking log both sides, we get

$2x \log_a = \log (\log_b^a)$

$$x=\frac{1}{2}\log_a(\log_b^a)$$

15. Option (a) is correct.

Explanations: $2^{1/c}$, $2^{b/ac}$, $2^{1/a}$ are in G.P

$2^{2b/ac} = 2^{1/c}.2^{1/a} = 2^{2b/ac} = 2^{1/c+1/a}$

$$=\frac{2b}{ac}=\frac{1}{c}+\frac{1}{a}=2b=a+c$$

Hence, a, b, c are in A.P

16. Option (b) is correct.

Explanations: We have,

$$a_n=0=\frac{5}{2}+(n-1)\left(\frac{-7}{12}\right)$$

$$\Rightarrow n-1=\frac{30}{7}\Rightarrow n=\frac{37}{7}$$

So, largest negative term will be for integer $n = 6$

17. Option (a) is correct.

Explanations: We have,

$f(x) = x^2 - 4x + x$ has real roots

$D > 0 = (4)^2 - 4k, 1 > 0 = 16 - 4k > 0$

$k < 4$ (i)

Now, roots of above equation are lying in the internal (0, 5).

$f(0) > 0 = k > 0$ (ii)

and $f(5) > 0 = 25 - 20 + k > 0 = k > -5$ (iii)

from (i), (ii), and (iii) we have,

$k = (0, 4)$

Possible integral values of l are 1, 2 and 3 i.e. 3 is number.

18. Option (b) is correct.

Explanations: We have

$a = x, S_n = 0$

$$\Rightarrow \frac{n}{2}\left[2a+(n-1)d\right]=0 \Rightarrow 2x+(n-1)d=0$$

$$\Rightarrow d=\left(\frac{-2x}{n-1}\right)$$

$$=\frac{m+n}{2}\left[2x+(m+n-1)d\right]-0$$

$$=\frac{m+n}{2}\left[2x+md-2x\right]$$

$$=\left(\frac{m+n}{2}\right)m\left(\frac{-2x}{n-1}\right)$$

$$=\frac{mx(m+n)}{1-n}$$

19. Option (b) is correct.

Explanations:

(1) as $5! = 120$
and $5! + 1 = 121$ has 1 at unit place.
so, $25! + 1$ also has 1 at units place.
$25! + 1$ is not divisible by 26.

(2) $6! = 720$
$6! + 1 = 721$, which is divisible by 7.
So, only (2) is true.

20. Option (c) is correct.

Explanations: Let $x = x + iy$

then $\frac{z-1}{z+1}=\frac{x+iy-1}{x+iy+1}$

$$=\frac{(x-1)+iy}{(x+1)+iy}\times\frac{(x+1)-iy}{(x+1)-iy}$$

$$=\frac{x^2+x+ixy-x-1+iy+ixy+iy-i^2y^2}{(x+1)^2-i^2y^2}$$

$$=\frac{x^2+y^2-1+2iy}{x^2+1+2x+y^2} \qquad (\because i^2=-1)$$

If $\frac{z-1}{z+1}$ is purely imaginary number, then

$$\text{Re}\left(\frac{z-1}{z+1}\right)=0$$

$\Rightarrow x^2 + y^2 = 0$

$\Rightarrow x^2 + y^2 = 1 \Rightarrow |z|^2 = 1$ or $|z| = 1$

Thus the value of $|z| = 1$

21. Option (b) is correct.

Explanations: We have, $|x - 4| + |x - 7| = 15$

There are two cases arise.

Case I: When $x < 4$

$-x + 4 - x + 7 = 15 \Rightarrow n = -2$

Case II: When $x \geq 7$

So, only 2 Solution possible.

22. Option (d) is correct.

Explanations: $f(x)$ is onto

$$3x + 5 = 0 \Rightarrow x = -\frac{5}{3}$$

So, A $= \{x = R-(-5/3)\}$

Let, $y = 2x + 3/3x + 5 \Rightarrow 3xy + 5y = 2x + 3$

$$= x = 3 - \frac{5y}{3y} - 2$$

$$3y - 2 = 0 = y = \frac{2}{3}$$

$B = \{y = R - (2 - 3)\}$

23. Option (a) is correct.

Explanations: We have,

$\alpha + \beta = 0$ (i)

$\alpha^2 + \beta^2 = 2$

$(\alpha + \beta)^2 - 2\alpha\beta = 2$

$2\alpha\beta = -2$ [Using (i)]

$\alpha\beta = -1$

Now, $(\alpha - \beta)^2 = \alpha^2 + \beta^2 - 2\alpha\beta = 2 - 2(-1) = 4$

$\alpha - \beta = +2$ (ii)

Solving (i) and (ii), we get

$\alpha = 1$ and $\alpha + \beta = +1$

So, only (1) is sufficient to find x.

24. Option (d) is correct.

Explanations: We have,

$T_{5+1} = 5600$

$8_{C5}(x^{-8/3})^{8-5} = (x^2\log_{10}x)^5 = 5600$

$56.x^{-8}.x^{10}(\log_{10}x)^5 = 5600$

$x^2(\log_{10}x)^5 = (10)^2.(\log_{10}10)^5$

So, $x = 10$

25. Option (c) is correct.

Explanations: We have,

$$(3x-y)^4(x+3y)^4 = [(3x-y)(x+3y)]^4$$
$$= (3x^2 + 9xy - xy - 3y^2)^4$$
$$= (3x^2 + 8xy - 3y^2)^4$$

Here, $r = 3$ and $n = 4$

Required number of terms $= {}^{n+r-1}C_{r-1}$

$= {}^{4+3-1}C_{3-1}$
$= {}^{6}C_{2} = 15$

26. Option (a) is correct.
Explanations: Let $P = 1 - 3d, q = a - d, r = a + d$
Then,
$P + S = 8$
$a - 3d + a + 3d = 8 \Rightarrow a = 4$
Also,
$qr = 15$
$= a^2 - d^2 = 15$
$= d^2 = 16 - 15$
$d = +1$
If $d = +1$ and $a = 4$, then
Largest number = 7 and smallest number = 1
Required difference = 7 – 1 = 6

27. Option (c) is correct.
Explanations:
Both statements are true.

28. Option (d) is correct.
Explanations:
Selection of 2 parallel lines from m lines $= {}^{m}C_{2}$
Selection of 2 parallel lines from n lines $= {}^{n}C_{2}$
No. of parallelograms formed $= {}^{m}C_{2}.{}^{n}C_{2}$
$= 60 = {}^{m}C_{2}.{}^{n}C_{2}$
$= {}^{5}C_{2} \times {}^{4}C_{2} = {}^{m}C_{2}.{}^{n}C_{2}$
$\therefore\ m = 5$ and $n = 4$
So, $m + n = 5 + 4 = 9$

29. Option (c) is correct.
Explanations: No. of permutations of the word PERMUTATIONS = 12!/2! (T accurs twice)
No. of permutations of the word COMBINATIONS =12!/2! 2! 2!
(As 0, I, M occurs twice)
y = 12!/2!. 1/4
$= 4y = x$

30. Option (b) is correct.
Explanations: Total 5 digit numbers that can be formed using 0, 1, 2, 4 and 5 without repetition $= 4 \times 4! = 96$
No. of 5 digit numbers greater that 50000 $= 1 \times 4! = 24$
(Ten thousand should be filled by 5 only)
Required percentage = 24/96 × 100 = 25%

(31-32.)
We have
$\sin^2\beta = \sin\alpha \cos\alpha$ (i)
and $2\tan\gamma = \sin\alpha + \cos\alpha$ (ii)

31. Option (a) is correct.
Explanations:
Now, $\cos2\beta = 1 - 2\sin^2\beta$
$= 1 - 2\sin\alpha\cos\alpha$
$= (\sin\alpha - \cos\alpha)^2$

32. Option (b) is correct.
Explanations:

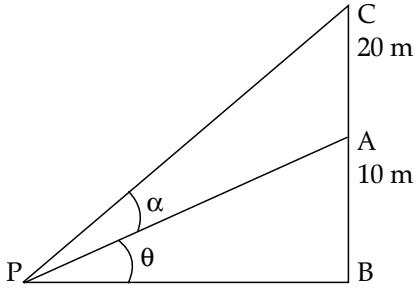
$$\cos2\gamma = \frac{1-\tan^2\gamma}{1+\tan^2\gamma}$$

$$\Rightarrow \sec2\gamma = \frac{1+\tan^2\gamma}{1-\tan^2\gamma} = \frac{1+\left(\frac{\sin\alpha+\cos\alpha}{2}\right)^2}{1-\left(\frac{\sin\alpha+\cos\alpha}{2}\right)^2}$$

$$= \frac{5+2\sin\alpha\cos\alpha}{3-2\sin\alpha\cos\alpha}$$

$$\Rightarrow \sec2\gamma = \frac{5+\sin2\alpha}{3-\sin2\alpha}$$

(33-34).
From the given question, figure should be as follows:



Let AB be the pillar and α be the angle formed by flagstaff.

33. Option (a) is correct.
Explanations: It is given that,

$$\tan\theta = \frac{AB}{AP} = \frac{10}{x}$$

$$\tan(\theta+\alpha) = \frac{30}{x}$$

$$\Rightarrow \frac{\tan\theta+\tan\alpha}{1-\tan\theta\tan\alpha} = \frac{30}{x}$$

$$\Rightarrow \frac{\frac{10}{x}+\frac{1}{2}}{1-\left(\frac{10}{x}\right)\left(\frac{1}{2}\right)} = \frac{30}{x}$$

$$\Rightarrow \frac{20+x}{2x-10} = \frac{30}{x}$$

$$\Rightarrow 20x + x^2 - 60x + 300 = 0$$

$$\Rightarrow x^2 - 40x + 300 = 0$$

$$\Rightarrow (x-30)(x-10) = 0$$

$$\Rightarrow x = 30 \text{ or } x = 10$$

Ratio of two values of $x = 1 : 3$
And $x \neq 20$ m
So, only (1) is correct.

34. Option (c) is correct.

Explanations:

Now, $\tan\theta = \frac{10}{30}$ or $\tan\theta = \frac{10}{10}$

$\tan\theta = \frac{1}{3}$ or 1

(35-36).

Let A, B, C be the angle of ΔABC

Now,

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} = k$$

$\sin A = ak$, $\sin B = bk$ and $\sin c = ck$

It is given that,

$$a+b+c = 6\times\left(\frac{\sin A+\sin B+\sin C}{3}\right)$$

$$\Rightarrow 2k = 1 \Rightarrow k = \frac{1}{2}$$

$$\text{So, } \frac{\sin A}{a} = k \Rightarrow \sin A = BC\frac{1}{2} = \frac{\sqrt{3}}{2} \Rightarrow A = \frac{\pi}{3}$$

$$\frac{\sin B}{b} = k \Rightarrow \sin B = AC\frac{1}{2} = \frac{1}{2} \Rightarrow B = \frac{\pi}{6}$$

$$C = \pi - \left(\frac{\pi}{3}+\frac{\pi}{6}\right) = \frac{\pi}{2}$$

35. Option (c) is correct.

Explanations: Perimeter of triangle

$= \sqrt{3} + 1 + 2 = 3 + \sqrt{3}$

36. Option (c) is correct.

Explanations: $C = \pi/3 = 1/2\,(\pi/2 + \pi/6)$

$C = 1/2\,(A+B)$

A, C, B are in A.P

Both (1) and (2) are true.

37. Option (c) is correct.

Explanations: We have,

$$x = \frac{\sin^2 A+\sin A+1}{\sin A}$$

$$= \sin A + 1 + \frac{1}{\sin A}$$

$$\text{Now, } \sin A + \frac{1}{\sin A} \geq 2 \qquad (\because AM > GM)$$

$$\Rightarrow \sin A + \frac{1}{\sin A} + 1 \geq 3$$

$$\Rightarrow x \geq 3$$

Minimum value of $x = 3$

38. Option (d) is correct.

Explanations: Now, $x = 3$

$\sin^2 A + \sin A + 1 = 3 \sin A$

$\sin A - 2 \sin A + 1 = 0$

$(\sin A - 1)^2 = 0$

$\sin A = 1 = A = \pi/2$

39. Option (c) is correct.

Explanations: We know that

$a^2 = b^2 + c^2 - 2bc \cos A$

$b^2 = c^2 + a^2 - 2ca \cos B$

$c^2 = a^2 + b^2 - 2ab \cos C$

Adding above equations, we get

$a^2 + b^2 + c^2 = 2a^2 + 2b^2 + 2c^2$

$= 2bc\ c\cos A = 2ca\ c\cos B - 2ab\ c\cos C$

$a^2 + b^2 + c^2 = 2ab\ c\cos C + 2bc\ c\cos A + 2ac\ c\cos B$

Now, it is given that,

$a^2 + b^2 + c^2 = ac + \sqrt{3}\,bc$

$2ab\ c\cos C + 2bc\ c\cos A + 2ac\ c\cos B = ac + \sqrt{3}\,bc$

On comparing, we get

ABC is right angled triangle.

40. Option (c) is correct.

Explanations:

Now, area of ABC = 1/2 × AC × BC

$= 1/2 \times 4\sqrt{3} \times 4$

$\left(\frac{1}{2} = \frac{BC}{8} = BC = 4 \text{ and } \frac{\sqrt{3}}{2} = \frac{AC}{8} = AC = 4\sqrt{3}\right)$

$= 8\sqrt{3}$

(41-42).

We have,

$f(x) = |x-2| + |3-x| + |4-x|$

$$f(x) = |x-2|+|3-x|+|4-x|$$

$$\Rightarrow f(x) = \begin{cases} -x+2+3-x+4-x, x \in (-\infty, 2) \\ x+2+3-x+4-x, x \in [2,3) \\ x+2+3-x+4-x, x \in [3,4) \\ x+2+3-x+4-x, x \geq 4 \end{cases}$$

$$\Rightarrow f(x) = \begin{cases} 9-3x, x < 2 \\ 5-x, x \in [2,3) \\ x-1, x \in [3,4) \\ 3x-g, x \geq 4 \end{cases}$$

$$\Rightarrow f^1(x) = \begin{cases} -3, x < 2 \\ -1, x \in (2,3) \\ 1, x \in (3,4) \\ 3, x \geq 4 \end{cases}$$

41. Option (b) is correct.

Explanations: Since sign changes from negative to positive a $x = 3$

$f(x)$ is minimum at $x = 3$

42. Option (a) is correct.

Explanations: Minimum value of $f(x) = f(3)$

$= |3-2| + |3-3| + |4-3|$

$= 1 + 0 + 1 = 2$

43. Option (c) is correct.

Explanations: Given, $s = 0! + 1! + 2! + ... + 100!$
From 41 onwards every terms has 4×2, which is divisible by 8.
Remaining sum $= 0! + 1! + 2! + 3!$
$= 1 + 1 + 2 + 6 = 10$
Now, remainder when 10 is divisible by 8 is 2 so, required remainder = 2

44. Option (d) is correct.

Explanations: Similarly from 5! onwards every terms has 10, which is divisible by 60
Remainder $= 0! + 1! + 2! + 3! + 4!$
$= 1 + 1 + 2 + 6 + 24 = 34$

45. Option (c) is correct.

Explanations: We have
PN = PM (Tangents from an external point)
PN = PM = n
Similarly, QL = QN = $n + 2$
and, RM = RL = $n + 4$
So, sides of triangle are,
PQ = $2n + 2$, QR = $2n + 6$, PR = $2n + 4$
Now, cos P = 1/3

$$\Rightarrow \frac{(PQ)^2 + (PR)^2 - (QR)^2}{2PQ \cdot PR} = \frac{1}{3}$$

$$\Rightarrow \frac{(2n+2)^2 + (2n+4)^2 - (2n+6)^2}{2\cdot(2n+2)^2(2n+4)} = \frac{1}{3}$$

$$\Rightarrow \frac{4\left((n+1)^2 + (n+2)^2 - (n+3)^2\right)}{4(n+1)(2n+4)} = \frac{1}{3}$$

$$\Rightarrow \frac{n^2+1+2n+n^2+4+4n-n^2-n-6n}{2n^2+6n+4} = \frac{1}{3}$$

$n = 8$, or $n = -2$

46. Option (a) is correct.

Explanations: Length of smallest side
$= 2n + 2 = 18$

47. Option (a) is correct.

Explanations: We have,

$$\sin x + \cos x + \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} + \frac{1}{\cos x} + \frac{1}{\sin x} = 7$$

$$\sin x + \cos x + \frac{1}{\sin x.\cos x} + \frac{\sin x + \cos x}{\sin x.\cos x} = 7$$

$$\Rightarrow (\sin x + \cos x)\left(1 + \frac{2}{\sin x}\right) = 7 - \frac{2}{\sin 2x}$$

Squaring both sides, we get,

$$\Rightarrow (1 + \sin 2x)\left(1 + \frac{2}{\sin x}\right)^2 = 7 - \frac{2}{\sin 2x}$$

$\Rightarrow \sin^2 2x - 44 \sin 2x + 36 = 0$

48. Option (d) is correct.

Explanations: $\sin^2 2x - 44 \sin 2x + 36 = 0$
$a = 22$, $b = 8$ and $c = 7$
So, $a - b + 2c = 22 - 8 + 14 = 28$

49. Option (b) is correct.

Explanations: Let a and b are the roots of the given equation

$$\therefore \alpha + \beta = \frac{4 + 2\sqrt{3}}{3 + 2\sqrt{2}} \text{ and } \alpha\beta = \frac{8 + 4\sqrt{3}}{3 + 2\sqrt{2}}$$

$$\text{HM of } \alpha \text{ and } \beta = \frac{2\alpha\beta}{\alpha + \beta}$$

$$= \frac{2\cdot\left(8 + 4\sqrt{3}\right)}{4 + 2\sqrt{3}} \times \frac{4 - 2\sqrt{3}}{4 - 2\sqrt{3}}$$

$$= \frac{2\left(32 - 16\sqrt{3} + 16\sqrt{3} - 24\right)}{16 - 12} = \frac{16}{4} = 4$$

50. Option (a) is correct.

Explanations: GM of α and $\beta = \sqrt{\alpha\beta}$

$$= \sqrt{\frac{8 + 4\sqrt{3}}{3 + 2\sqrt{2}}} = \sqrt{\frac{2\left(4 + 2\sqrt{3}\right)}{3 + 2\sqrt{2}}}$$

$$= \sqrt{2}\left(\sqrt{\frac{\left(\sqrt{3}+1\right)^2}{\left(\sqrt{2}+1\right)^2}}\right.$$

$$= \sqrt{2}\left(\frac{\sqrt{3}+1}{\sqrt{2}+1} \times \frac{\sqrt{2}-1}{\sqrt{2}-1}\right)$$

$$= \sqrt{2}\left(\sqrt{6} - \sqrt{3} + \sqrt{2} - 1\right)$$

51. Option (b) is correct.

Explanations:

$$\therefore \begin{vmatrix} a & b & a\alpha + b \\ b & c & b\alpha + c \\ a\alpha + b & b\alpha + c & 0 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 0 & b & a\alpha + b \\ 0 & c & b\alpha + c \\ a\alpha^2 + 2b\alpha + c & b\alpha + c & 0 \end{vmatrix} = 0$$

$= 0 - 0 + (a\alpha^2 + 2b\alpha + c)(b^2\alpha + bc - ac\alpha - bc)$
$= 0$
$= b^2\alpha + ac\alpha = 0$ or $b^2 - ac = 0 = b^2 = ac$
So, a, b, c are in G.P

52. Option (c) is correct.

Explanations: $(7, 4, 2, \alpha) = 0$
$7\alpha^2 + 8\alpha + 2 = 0$
So, α is root of the equation, $7x^2 + 8x + 2 = 0$

53. Option (d) is correct.

Explanations: We have,
$m(0) = \cot\theta^2 + n^2\tan^2\theta + 2n$

$= (\cot\theta + n\tan\theta)^2$

$m(\theta) > 0$

Now, $\frac{\cot\theta + n\tan\theta}{2} \geq \sqrt{n}$

$= (\cot\theta + n\tan\theta)^2 > 4n$

$\therefore$ Minimum value of $m(\theta) = 4n$

54. Option (b) is correct.

Explanations: $(\cot\theta + n\tan\theta)^2 - 4x = 0$

$\Rightarrow (\cot\theta - n\tan\theta)^2 - 4x = 0$

$\Rightarrow \cot\theta = n\tan\theta$

$\Rightarrow x = \cot^2\theta$

55. Option (c) is correct.

Explanations: Equation of line of the quadrilateral is, $x = 0$, $y = 0$, $x + y = 1$ and $6x + y = 3$

Point of intersection of these lines are

$$A\left(\frac{1}{2},0\right), B(0,0), C(0,1), D\left(\frac{2}{5},\frac{3}{5}\right)$$

So, equation of diagonal passes through B is ,

$$BD = y - 0 = \frac{3/5}{2/5}(x-0)$$

$2y = 3x \Rightarrow 3x - 2y = 0$

56. Option (d) is correct.

Explanations: Equation of diagonal AC is

$$y - 0 = \frac{1-0}{0-1/2}\left(x - \frac{1}{2}\right)$$

$$\Rightarrow y = -2\left(x - \frac{1}{2}\right) \Rightarrow y = -2x + 1 \Rightarrow 2x + y - 1 = 0$$

57. Option (b) is correct.

Explanations: The given ellipse is,

$$\frac{x^2}{1} + \frac{y^2}{\left(\frac{1}{2}\right)^2} = 1$$

As we know, sum of distances of any point P from two is,

PE + PE = $2a = 2$

58. Option (d) is correct.

Explanations: Equation of latus return of ellipse is $x = 2\sqrt{3}/2$

So, points 1, 2 and 3 will be on it.

59. Option (a) is correct.

Explanations: Given equation of circle is

$$(x-a)^2 + y^2 = a^2$$

Now, $y = x$ intersect it 2 parts

Point of intersection of line and circle is, (0, 0) and (a, a)

So, required area $= \int_0^a \sqrt{a^2 - (x-a)^2}\,dx - \int_0^a x\,dx$

$$= \left(\frac{x-a}{2}\sqrt{2ax - x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x-a}{a}\right) - \left(\frac{x^2}{2}\right)\right)_0^a$$

$$= \frac{a^2}{2}\sin^{-1}(0) - \frac{a^2}{2}\sin^{-1}(-1) - \frac{a^2}{2}$$

$$= \frac{a^2}{2}\times 0 + \frac{a^2}{2}\times\frac{\pi}{2} - \frac{a^2}{2}$$

$$= \frac{a^2}{4}(\pi - 2)$$

60. Option (b) is correct.

Explanations: Area of major segment

$= \pi r^2$ – Area of minor segment

$$= \pi a^2 - \frac{\pi a^2}{4} - \frac{a^2}{2}$$

$$= \frac{3\pi a^2}{4} + \frac{a^2}{2} = \frac{a^2}{4}(3\pi + 2)$$

61. Option (a) is correct.

Explanations: End points of diameter are A(1, –1, 2) and B(2, 1, –1)

Centre of sphere $= \left(\frac{3}{2}, 0, \frac{1}{2}\right)$

and radius $= \sqrt{\left(\frac{3}{2}-1\right)^2 + (0+1)^2 + \left(\frac{1}{2}-2\right)^2}$

$$= \sqrt{\frac{1}{4} + 1 + \frac{9}{4}} = \sqrt{\frac{14}{4}} = \sqrt{\frac{7}{2}}$$

Equation of space is

$$\Rightarrow x^2 + y^2 + z^2 + \frac{9}{4} + \frac{1}{4} - 3x - z = \frac{7}{2}$$

$\Rightarrow x^2 + y^2 + z^2 + 9/4 + 1/4 - 3x - z = 7/2$

$\Rightarrow x^2 + y^2 + z^2 - 3x - z - 1 = 0$

So, from given equation of sphere we have

$2a = -3$, $2v = 0$ and $2w = -1$

$\Rightarrow a = -3/2$, $v = 0$, $w = -1/2$

So, $u + v + w = 4/2 = -2$

62. Option (b) is correct.

Explanations:

$PA^2 + PB^2 = AB^2$

$= (2-1)^2 + (1+1)^2 + (-1-2)^2$

$= 1 + 4 + 9 = 14$

63. Option (a) is correct.

Explanations: dr1 (2, –1, 2) and $(k, 3, 5)$ indicated at $\pi/4$

$$\therefore \cos\frac{\pi}{4} = \frac{2k - 3 + 10}{\sqrt{4+1+4}\sqrt{k^2+9+25}}$$

$$\Rightarrow \frac{1}{\sqrt{2}} = \frac{2k+7}{3\sqrt{k^2+34}}$$

$\Rightarrow$ $9(k^2 + 34) = 2(2k + 7)^2$
$\Rightarrow$ $9k^2 + 306 = 2(4k^2 + 49 + 28k)$
$\Rightarrow$ $9k^2 + 306 - 8k^2 - 98 - 56k = 0$
$\Rightarrow$ $k^2 - 56k + 209 = 0$
$\Rightarrow$ $k^2 - 52k - 4k + 208 = 0$
$\Rightarrow$ $(k - 52)(k - 4) = 0 \Rightarrow k = 52$ or $k = 4$

64. Option (d) is correct.

Explanations: Let the drs of line perpendicular to given lines be (a, b, c)
Then, $2a - b + 2c = 0$
and $4a + 3b + 5c = 0$

$$\Rightarrow \frac{a}{-5-6} = \frac{b}{8-10} = \frac{c}{6-4}$$
$$\Rightarrow \frac{a}{-11} = \frac{b}{-2} = \frac{c}{10}$$

So, (11, 2, –10) as the required drs.

65. Option (b) is correct.

Explanations: Let $\vec{b} = ai = bj = ck$
Then, $\vec{a} \cdot \vec{b} = 27$
$\Rightarrow 3a + 3b + 3c = 27 \Rightarrow a + b + c = 9$ (i)
Also,

$$\Rightarrow \begin{vmatrix} i & j & k \\ 3 & 3 & 3 \\ a & b & c \end{vmatrix} = 9(j - k)$$

$\Rightarrow i(3c - 3b) - j(3c - 3a) + k(3b - 3a) = 9(j - k)$
$\Rightarrow 3c - 3b = 0, 3c - 3A = 9, 3b - 3a = -9$
$\Rightarrow c = b, a - c = 3, a - b = 3 \Rightarrow c = a - 3, b = a - 3$
From (i), we have
$a + a - 3 + a - 3 = 9$
$\Rightarrow 3a = 15 \Rightarrow a = 5$
$b = 5 - 3 = 2 = c$
So, $b = 5i + 2j + 2k$

66. Option (a) is correct.

Explanations: Now, $a + b = 8i + 5j + 5k$
$c = j - k$
$(a+b).c = |a + b||c| \cos\theta$,
where θ is the required angle

$$\Rightarrow 0 = \sqrt{8^2 + 5^2 + 5^2}\sqrt{1+1}\cos\theta$$
$$\Rightarrow \cos\theta = 0 \Rightarrow \theta = \frac{\pi}{2}$$

67. Option (b) is correct.

Explanations: We have, $a = 4i - 8j + k$

$$\therefore \cos\alpha = \frac{4}{\sqrt{4^2 + 8^2 + 1^2}} = \frac{4}{9}$$

68. Option (a) is correct.

Explanations: Also, $\cos\beta = 8/9$ and $\cos\gamma = 1/9$
Now, $\cos2\beta + \cos2\gamma = 2\cos^2\beta - 1 + 2\cos^2\gamma - 1$

$$= 2\left(\frac{64}{81} + \frac{1}{81}\right) - 2 = \frac{-32}{81}$$

69. Option (b) is correct.

Explanations: We have, A = (1, –1, 0) and B = (0, 1, 1)
Equation of line AB is,

$$\frac{x-1}{0-1} = \frac{y+1}{1+1} = \frac{z-0}{1-0}$$
$$\frac{1-x}{1} = \frac{y+1}{2} = \frac{z}{1}$$

Now, only (2) and (3) satisfy this equation.

70. Option (c) is correct.

Explanations:
We have, $A = \hat{i} + \hat{j} + 0.\hat{k}$ and $B = oi + j + k$
$AB = (0 - 1)i + (1+1)i + (1 - 0) = i + 2j + k$

$$\left|\overline{AB}\right| = \sqrt{1^2 + 2^2 + 1} = \sqrt{6}$$

(71-73).

We have,
$f(x) = Pe^x + Qe^{2x} + Re^{3x}$
It is given that, $f(0) = 6$
$P + Q + R = 6$ (i)

$$\int_0^{\ln 2} f(x)\,dx = 11$$
$$\Rightarrow \left(Pe^x + \frac{Qe^{2x}}{2} + \frac{Re^{3x}}{3}\right)_0^{\ln 2} = 11$$

71. Option (b) is correct.

72. Option (c) is correct.

73. Option (d) is correct.

$f(0) = P + 2Q + 3R = 1 + 4 + 9 = 14$

(74-76).

We have,
$y^2 = 2c\,x + 2c$
$= y^4 + 4y^2(y)^2x^2 - 4y^3(y)x - 4y^3.(y)^3 = 0$

74. Option (a) is correct.

75. Option (b) is correct.

76. Option (b) is correct.

Explanations: We have,

$$f(x) = \begin{vmatrix} \cos x & x & 1 \\ 2\sin x & x^2 & 2x \\ \tan x & x & 1 \end{vmatrix}$$
$$\therefore f(0) = \begin{vmatrix} \cos 0 & 0 & 1 \\ 2\sin 0 & 0 & 0 \\ \tan 0 & 0 & 1 \end{vmatrix} = 0$$

77. Option (b) is correct.

Explanations:

$$\lim_{x\to 0}\frac{f(x)}{x} = \lim_{x\to 0}\begin{vmatrix} \cos x & x & 1 \\ 2\sin x/x & x & 2 \\ \tan x & x & 1 \end{vmatrix} = 0$$

78. Option (a) is correct.
Explanations:

$$\lim_{x\to 0}\frac{f(x)}{x^2}=\lim_{x\to 0}\begin{vmatrix}\cos x & 1 & 1\\ 2\sin x/x & 1 & 2\\ \tan x & 1 & 1\end{vmatrix}$$

$$=\begin{vmatrix}1 & 1 & 1\\ 2 & 1 & 2\\ 0 & 1 & 1\end{vmatrix}=-1$$

79. Option (b) is correct.
Explanations: We have,
$f(x) = \sin[\pi^2]x + \cos[-\pi^2]x$
$= \sin 9x + \cos(-10x)$
$= \sin 9x + \cos 10x$
$= 1 + (-1) = 0$

80. Option (d) is correct.
Explanations:

$$f\left(\frac{\pi}{4}\right)=\sin\frac{9\pi}{4}+\cos\frac{5\pi}{2}$$

$$=\sin\left(2\pi+\frac{\pi}{4}\right)+0=\sin\frac{\pi}{4}=\frac{1}{\sqrt{2}}$$

81. Option (Bonus) is correct.

Explanations: Since $I_1 = I_2 = \dfrac{\pi^2}{2\sqrt{2}}$

So, 81 and 83 is bonus.

82. Option (d) is correct.

Explanations: $8I^2 = 8\left[\dfrac{\pi^2}{2\sqrt{2}}\right]^2 = \dfrac{8\pi^4}{8} = \pi^4$

83. Option (b) is correct.

84. Option (a) is correct.
Explanations: Now, $a < 0 < b$
$l = b - (-a) = a + b$

85. Option (c) is correct.
Explanations: Now, $a < b < 0$
$l = -(-b) + (-a) = b - a$

86. Option (a) is correct.
Explanations: $f'(0.5) = -1/0.5 = -2$

87. Option (c) is correct.
Explanations: $F'(2) = 1/2$

88. Option (d) is correct.
Explanations:

$$\begin{cases} ln(-ln\,x) & ,x<0.1\\ -ln(-ln\,x) & ,x\in(0.1,1)\\ -ln(-ln\,x) & ,x\in(1,2)\end{cases}$$

$$\therefore\ \frac{d}{dx}\left(fof(x)=\frac{-1}{\ln x}\cdot\frac{1}{x}=\frac{-1}{x\ln x}\right.$$

89. Option (b) is correct.
Explanations: $f(x)$ is continuous
$7 = P + q$ (i)
$10 = 2P + q$ (ii)
Solving (i) and (ii) we get
$P = 3$

90. Option (c) is correct.
Explanations: Also, $p = 3, q = 4$

91. Option (a) is correct.
Explanations: Only (1) is true

92. Option (a) is correct.
Explanations:

$$\Rightarrow\frac{du}{dx}=2\sin x\cos x=\sin 2x$$

$$\text{and}\ \ \frac{dv}{dx}=2\cos x(-\sin x)=-\sin 2x$$

$$\therefore\ \frac{du}{dv}=\frac{du}{dx}\Big/\frac{dv}{dx}=\frac{\sin 2x}{-\sin 2x}=-1$$

93. Option (a) is correct
Let the equation of line segments of ABC are given A = (2, 2), B(2, 2 m) and C(0, 0)
Since area of ABC = 3
$= |1/2(0 + 2(2m - 0) + 2(0 - 2)| = 3$
$= 4m - 4 = \pm 6$
$m = 5/2$ or $m = -1/2$
$\because\ m < 0$
$\therefore\ m = -1/2$

94. Option (b) is correct.
Explanations:

$$x^\circ=\frac{\pi x}{180}\ \text{radians}$$

$$\therefore\ \frac{d}{dx}(\text{cosec}\,x^\circ)=\frac{d}{dx}\left(\text{cosec}\,\frac{\pi x}{180}\right)$$

$$=\frac{-\pi}{180}\text{cosec}\,x^\circ\cdot\cot x^\circ$$

95. Option (a) is correct.
Explanations:

$$\left(\frac{dy}{dx}\right)^2-x\frac{dy}{dx}=0$$

$$\Rightarrow\frac{dy}{dx}\left(\frac{dy}{dx}-x\right)=0$$

$$\Rightarrow\frac{dy}{dx}=0\ or\ \frac{dy}{dx}=x$$

$$\Rightarrow y=\text{C(constant) or } y=\frac{x^2}{2}+c$$

96. Option (d) is correct.
Explanations: $f(x) = x^2 + 2$, $g(x) = 2x - 3$
$f(1) = 1 + 2 = 3$ and $g(1) = -1$
$(fg)(1) = 3$

97. Option (b) is correct.
Explanations: We have

$$\Rightarrow f(x)=\begin{cases}0 & ,x<0\\ 2x & ,x\geq 0\end{cases}$$

Range of $f(x) = [0, \infty)$

98. Option (a) is correct.
Explanations:

$f(x) = x(4x^2 - 3)$

$f(\sin\theta) = \sin\theta(4\sin^2\theta - 3)$

$= 4\sin 3\theta - 3\sin\theta = -\sin 3\theta$

99. Option (d) is correct.
Explanations:

$$\lim_{x\to 5}\frac{5-x}{|x-5|}$$

LHL = 1 and RHL = –1
So, limit at $x = 5$ does not exist.

100. Option (c) is correct.
Explanations: We have

$$\lim_{x\to 1}\frac{x^9-1}{x^3-1}=\lim x-1\left(\frac{x^9-1}{x-1}\times\frac{x-1}{x^3-1}\right)$$

$$=\frac{9\cdot(1)^8}{3\cdot(1)^2}=3$$

101. Option (c) is correct.
Explanations: Let the other two observations be x and y.

$$\therefore \text{ Mean}=\frac{11+16+20+x+y}{5}$$

$= 14 \times 5 = 47 + x + y$

$= x + y = 23$ (i)

Now, variance $= \frac{1}{5}[(11-14)^2 + (16-14)^2 + (20-14)^2 + (x-14) + (y-14)^2]$

$= 13.2 \times 5 = 9 + 4 + 36 + (x-14)^2 + (y-14)^2$

$= x^2 + y^2 - 28(x + y) + 2x.196 + 49 = 66$

$= x^2 + y^2 - 28(23) = -375$

$= x^2 + y^2 = 269$ (ii)

$x = 13, y = 10$

102. Option (c) is correct.
Explanations: Since A and B are independent events

$0.8 = (1 - 0.7) + (1 - k) - (1 - 0.7)(1 - k)$

$0.8 = 0.3 + 1 - k - 0.3(1 - k)$

$0.5 = 1 - k - 0.3 + 0.3k$

$0.1k = 0.2 = k = 2/7$

103. Option (b) is correct

Explanations: P (getting head) $= \frac{1}{4}$

P(getting tail) $= 1-\frac{1}{4}=\frac{3}{4}$

Now, required probability

$$\left(\frac{3}{4}\right)^4=\frac{81}{1024}$$

104. Option (b) is correct
Explanations: We have

$p(\text{H}) = \frac{3}{4}$ and $p(\text{T}) = \frac{1}{4}$

Required probability $= 4 \times (3/4)^3\, 1/4 = 27/64$

105. Option (c) is correct.

Explanations: We have, $n = 100$, P $= \frac{4}{5}$, $q = \frac{1}{5}$

For random variable x

$\text{V}(x) = npq = 80 \times \frac{1}{5} = 16$

Now, $y = 100 - x$
and, var $(y) = v(100 - x) = 0 + (-1)^2 v(x) = 16$

106. Option (c) is correct.
Explanations: We have

$x + 4y + 1 = 0$ (i)

$4x + 9y + 7 = 0$ (ii)

From (i), $bxy = -4$
From (ii), $byx = -1$
$r^2 = 4$ which is not possible as $0 < r^2 < 1$
$4x + 9y + 7 = 0$ is a line of regression x on y.
$4x = -(9y + 7)$
$x = -(9y + 7)/4 = -(-27 + 7)/4 = 5$

107. Option (d) is correct.
Explanations: Sum of all angles = 360°
$= p + q + r + s = 360°$

$$=\frac{k}{9}+\frac{k}{3}+\frac{k}{2}+\frac{k}{6}=360^{\circ}$$

So, $4p - q = 144 - 108 = 36°$

108. Option (a) is correct
Explanations:

m = mean of 8 lowest observations

$= 1 + 1 + 1 + 2 + 3 + 3 + 4 + \frac{4}{8}$

$= m = \frac{19}{8}$

Now, M = Mean of highest 4 observation

$$\text{M} = 6 + 6 + 5 + \frac{4}{4} = \frac{21}{4}$$

$$\therefore \quad 2m + \text{M} = \frac{19}{4}+\frac{21}{4}=\frac{40}{4}=10$$

109. Option (a) is correct
Explanations: We have,

$x = -1 + 3 = 2, y = 1 + 2 = 3$
$xy = -1 + 6 = 5, x^2 = 1 + 9 = 10$
Now, $x = \frac{2}{2} = 1$ and $y = \frac{3}{2}$
So, line of regression y on x is,
$y - y = b_{yx}(x - x)$
$y - \frac{3}{2} = \frac{1}{4}(x - 1) = x - 4y + 5 = 0$

110. Option (a) is correct
Explanations: Mode = 1

111. Option (c) is correct
Explanations:

x	f	cf
1	4	4
2	6	10
3	9	19
5	7	26
	26	

Now, N/2 = 13
Median = 3 as cf just greater than lies for $x = 3$.

112. Option (d) is correct
Explanations: Arranging observations of first five and last five observations, we get
Set-I: –1, 1, 3, 4, 8
Set-II: 9, 11, 12, 17, 19
M = Median of set I = 3
N = Median of set II = 12
$4M - N = 4 \times 3 - 12 = 12 - 12 = 0$

113. Option (d) is correct
Explanations: Let $P = \frac{k}{5}$, $Q = \frac{k}{4}$ and $R = 2k$

$$P + Q/2P + 0.7R = \frac{\left(\frac{9k}{20}\right)}{\left(\frac{9k}{5}\right)} = \frac{1}{4}$$

114. Option (d) is correct
Explanations: G.M. of $1, 2, 2^2 ..... 2^{n-1}$
$= \sqrt[n]{1.2.2^2(.....)2^3....2^{n-1}}$
$= \sqrt[n]{2^{1+2+3+...+n-1}}$
$= \sqrt[n]{2^{(n-1)n/2}}$
$= G = 2^{(n-1)/2}$
Now, $1 + 2\log_2 G = 1 + 2(n - 1/2)$
$= 1 + n - 1 = n$

115. Option (b) is correct
Explanations: $HM = \frac{n}{1+\frac{1}{2}=\frac{1}{2^2}+....\frac{1}{2^{n-1}}}$

$$\Rightarrow \frac{n}{H} = 1+\frac{1}{2} = \frac{1}{2^2}+....\frac{1}{2^{n-1}}$$
$$= \frac{1\left(1-\frac{1}{2^n}\right)}{1-\frac{1}{2}} = 2\left(1-\frac{1}{2^n}\right)$$
$$\Rightarrow \frac{n}{H} - 2 - \frac{1}{2^{n-1}}$$

116. Option (c) is correct
Explanations: Let $P = (2x_i - a)^2$
On differentiating, we get
$\frac{dp}{da} = 8x_i - 4a$
$\frac{dp}{da} = 0$
$8x_i = 4a$
$a = 2x_i$
$a = \frac{2x_i}{n} = 2Q$

117. Option (Bonus) is correct
Explanations:
Let E1 : Two balls are white
E2 : Two balls are black
E3 : One ball is white & other is black
A: A white ball is from by B
Probability
$$= \frac{{}^3C_2}{{}^5C_2}+\frac{4}{7}+\frac{{}^2C_2}{{}^5C_2}+\frac{2}{7}+\frac{{}^3C_1\times{}^2C_1}{{}^5C_2}\times\frac{3}{7} = \frac{33}{70}$$

118. Option (c) is correct
Explanations: Total number of ways
$= 6 \times 6 \times 6$
Out of these, multiple of 3 shows $2 \times 2 \times 2$ times
Required probability $= \frac{8}{216} = \frac{1}{27}$

119. Option (c) is correct
Explanations: There are 31 days in december
i.e., $31 = \frac{28}{7} + 3$
Now, on these 3 days, one can be sunday
Required probability $= \frac{3}{7}$

120. Option (b) is correct
Explanations: $n + \frac{50}{n} < 50$
So, n can be 2, 3, ..., 48
Favourable cases = 47
Total cases = 50
Required probability $= \frac{47}{50}$

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MATHEMATICS

II

QUESTION PAPER 2023

Time : 2 : 30 Hours **Total Marks: : 300**

Instructions

1. *This Test Booklet contains **120** items (questions). Each item is printed in **English**. Each item comprises four responses (answers's). You will select the response which you want to mark on the Answer Sheet. In case you feel that there is more than one correct response, mark the response which you consider the best. In any case, choose **ONLY ONE** response for each item.*
2. *You have to mark all your responses **ONLY** on the separate Answer Sheet provided. See directions in the Answer Sheet.*
3. ***All** items carry equal marks.*
4. *Before you proceed to mark in the Answer Sheet the response to the various items in the Test Booklet, you have to fill in some particulars in the Answer Sheet as per instructions.*
5. ***Penalty for wrong answers :***
   THERE WILL BE PENALTY FOR WRONG ANSWERS MARKED BY A CANDIDATE IN THE OBJECTIVE TYPE QUESTION PAPERS.
   (i) *There are four alternatives for the answer to every question. For each question for which a wrong answer has been given by the candidate, **one-third** of the marks assigned to that question will be deducted as penalty.*
   (ii) *If a candidate gives more than one answer, it will be treated as a **wrong answer** even if one of the given answers happens to be correct and there will be same penalty as above to that question.*
   (iii) *If a question is left blank, i.e., no answer is given by the candidate, there will be **no penalty** for that question.*

1. If $z\bar{z} = |z + \bar{z}|$, where $z = x + iy$, i = $\sqrt{-1}$, then the locus of z is a pair of :
   (a) straight lines
   (b) rectangular hyperbolas
   (c) parabolas
   (d) circles
2. If 1 ! + 3! + 5! + 7! + ,… + 199! is divided by 24, what is the remainder?
   (a) 3 (b) 6
   (c) 7 (d) 9
3. What is the value of $\sqrt{12+5i} + \sqrt{12-5i}$, where $i = \sqrt{-1}$?
   (a) 24 (b) 25
   (c) $5\sqrt{2}$ (d) $5\left(\sqrt{2}-1\right)$
4. If A = $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, then what is the value of det (I + AA'), where I is the 3 × 3 identity matrix?
   (a) 15 (b) 6
   (c) 0 (d) –1
5. If A, B and C are square matrices of order 3 and det(BC) = 2 det(A), then what is the value of det($2A^{-1}$ BC)?
   (a) 16 (b) 8
   (c) 4 (d) 2
6. If the n^{th} term of a sequence is $\frac{2n+5}{7}$, then what is the sum of its first 140 terms?
   (a) 2840 (b) 2780
   (c) 2920 (d) 5700
7. Let A be a skew-symmetric matrix of order 3. What is the value of det($4A^4$) – det ($3A^3$) + det($2A^2$) – det(A)+ det(-I) where I is the identity matrix of order 3 ?
   (a) –1 (b) 0
   (c) 1 (d) 2
8. If A = $\begin{bmatrix} 0 & 3 & 4 \\ -3 & 0 & 5 \\ -4 & -5 & 0 \end{bmatrix}$, then which one of the following statements is correct?
   (a) A^2 is symmetric matrix with det(A^2) = 0.
   (b) A^2 is symmetric matrix with det(A^2) ≠ 0.
   (c) A^2 is skew-symmetric matrix with det(A^2) = 0.
   (d) A^2 is skew-symmetric matrix with det(A^2) ≠ 0.
9. If A = $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}$, then which of the following statements are correct?
   1. A^n will always be singular for any positive integer n.
   2. A^n will always be a diagonal matrix for any positive integer n.

3. A^n will always be a symmetric matrix for any positive integer n.

Select the correct answer using the code given below:

(a) 1 and 2 only (b) 2 and 3 only
(c) 1 and 3 only (d) 1, 2 and 3

10. If $(a + b)$, $2b$, $(b + c)$ are in HP, then which one of the following is correct?
(a) a, b and c are in AP
(b) a – b, b – c and c – a are in AP
(c) a, b and c are in GP
(d) a – b, b – c and c – a are in GP

11. Let $t_1, t_2, t_3 \ldots$ be in GP. What is $(t_1 t_3 \ldots t_{21})^{\frac{1}{11}}$ equal to?

(a) t_{10} (b) t_{10}^2
(c) t_{11} (d) t_{11}^2

12. Which one of the following is a square root of $-\sqrt{-1}$?

(a) 1 + i (b) $\frac{1-i}{\sqrt{2}}$
(c) $\frac{1+i}{\sqrt{2}}$ (d) $\frac{1}{\sqrt{2}}i$

13. What is the maximum number of points of intersection of 10 circles?
(a) 45 (b) 60
(c) 90 (d) 120

14. A set S contains (2n + 1) elements. There are 4096 subsets of S which contain at most n elements. What is n equal to?
(a) 5 (b) 6
(c) 7 (d) 8

15. If $\begin{vmatrix} x^2+3x & x-1 & x+3 \\ x+1 & -2x & x-4 \\ x-3 & x+4 & 3x \end{vmatrix} = ax^4 + bx^3 + cx^2 + dx + e$, then what is the value of e?
(a) –1 (b) 0
(c) 1 (d) 2

16. If all elements of a third order determinant are equal to 1 or –1, then the value of the determinant is :
(a) 0 only
(b) an even number but not necessarily 0
(c) an odd number
(d) 0, 1 or –1

17. If $A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 3 & 0 \\ 1 & 0 & 1 \end{bmatrix}$, then what is the value of $\det|\text{adj}(\text{adj}A)|$?
(a) 5 (b) 25
(c) 125 (d) 625

18. If $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, then what is the $23A^3 - 19A^2 - 4A$ equal to?
(a) Null matrix of order 3
(b) Identity matrix of order 3
(c) $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$
(d) $\begin{bmatrix} 7 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 7 \end{bmatrix}$

19. The value of the determinant of a matrix A of order 3 is 3. If C is the matrix of cofactors of the matrix A, then what is the value of determinant of C^2?
(a) 3 (b) 9
(c) 81 (d) 729

20. If $A_k = \begin{bmatrix} k-1 & k \\ k-2 & k+1 \end{bmatrix}$, then what is $\det(A_1) + \det(A_2) + \det(A_3) + \ldots + \det(A_{100})$ equal to?
(a) 100 (b) 1000
(c) 9900 (d) 10000

21. The Cartesian product A × A has 16 elements among which are (0, 2) and (1, 3). Which of the following statements is/are correct?
1. It is possible to determine set A.
2. A × A contains the element (3, 2).

Select the correct answer using the code given below:

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

22. Let A = {1, 2, 3, …, 20}. Define a relation R from A to A by R = $\{(x, y) : 4x - 3y = 1\}$, where $x, y \in A$. Which of the following statements is/are correct?
1. The domain of R is {1, 4, 7, 10, 13, 16}.
2. The range of R is {1, 5, 9, 13, 17}.
3. The range of R is equal to codomain of R.

Select the correct answer using the code given below:

(a) 1 only (b) 2 only
(c) 1 and 2 (d) 2 and 3

23. Consider the following statements:
1. The relation f defined by $f(x) = \begin{cases} x^3, & 0 \le x \le 2 \\ 4x, & 2 \le x \le 8 \end{cases}$ is a function.
2. The relation g defined by $g(x) = \begin{cases} x^3, & 0 \le x \le 4 \\ 3x, & 4 \le x \le 8 \end{cases}$ is a function.

Which of the statements given above is/are correct?

(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

24. Consider the following statements:
    1. $A = (A \cup B) \cup (A - B)$
    2. $A \cup (B - A) = (A \cup B)$
    3. $B = (A \cup B) - (A - B)$

    Which of the statements given above are correct?
    (a) 1 and 2 only (b) 2 and 3 only
    (c) 1 and 3 only (d) 1, 2 and 3

25. A function satisfies $f(x-y) = \frac{f(x)}{f(y)}$, where $f(y) \neq 0$.

    If $f(1) = 0.5$, then what is $f(2) + f(3) + f(4) + f(5) + f(6)$ equal to?
    (a) $\frac{15}{32}$ (b) $\frac{17}{32}$
    (c) $\frac{29}{64}$ (d) $\frac{31}{64}$

26. What is $2\cot\left(\frac{1}{2}\cos^{-1}\frac{\sqrt{5}}{3}\right)$ equal to?
    (a) –1 (b) 1
    (c) $3+\sqrt{5}$ (d) $3-\sqrt{5}$

27. If $\sec^{-1} p - \operatorname{cosec}^{-1} q = 0$, where $p > 0$, $q > 0$; then what is the value of $p^{-2} + q^{-2}$?
    (a) 1 (b) 2
    (c) $\frac{1}{2}$ (d) $\frac{1}{2\sqrt{2}}$

28. What is $1+\sin^2\left[\cos^{-1}\left(\frac{3}{\sqrt{17}}\right)\right]$ equal to?
    (a) $\frac{25}{17}$ (b) $\frac{8}{17}$
    (c) $\frac{9}{17}$ (d) $\frac{47}{17}$

29. If $\tan(\pi\cos\theta) = \cot(\pi\sin\theta)$, $0 < \theta < \frac{\pi}{2}$; then what is the value of $8\sin^2\left(\theta+\frac{\pi}{4}\right)$?
    (a) 16 (b) 2
    (c) 1 (d) $\frac{1}{2}$

30. If $\tan\alpha = \frac{1}{7}$, $\sin\beta = \frac{1}{\sqrt{10}}$; $0 < \alpha, \beta < \frac{\pi}{2}$, then what is the value of $\cos(\alpha + 2\beta)$?
    (a) $-\frac{1}{2}$ (b) $-\frac{1}{\sqrt{2}}$
    (c) $\frac{1}{\sqrt{2}}$ (d) $\frac{1}{2}$

Consider the following for the next two (02) items that follow:
Consider the equation $(1-x)^4 + (5-x)^4 = 82$.

31. What is the number of real roots of the equation?
    (a) 0 (b) 2
    (c) 4 (d) 8
32. What is the sum of all the roots of the equation?
    (a) 24 (b) 12
    (c) 10 (d) 6

Consider the following for the next three (03) items that follow:
Consider equation –I : $z^3 + 2z^2 + 2z + 1 = 0$ and equation – II : $z^{1985} + z^{100} + 1 = 0$.

33. What are the roots of equation–I?
    (a) $1, \omega, \omega^2$ (b) $-1, \omega, \omega^2$
    (c) $1, -\omega, \omega^2$ (d) $-1, -\omega, -\omega^2$
34. Which one of the following is a root of equation-II?
    (a) –1 (b) $-\omega$
    (c) $-\omega^2$ (d) ω
35. What is the number of common roots of equation-I and equation-II?
    (a) 0 (b) 1
    (c) 2 (d) 3

Consider the following for the next two (02) items that follow:
A quadratic equation is given by $(a+b)x^2 - (a+b+c)x + k = 0$, where a, b, c are real.

36. If $k = \frac{c}{2}, (c \neq 0)$, then the roots of the equation are:
    (a) Real and equal (b) Real and unequal
    (c) Real iff a > c (d) Complex but not real
37. If k = c, then the roots of the equation are:
    (a) $\frac{a+c}{a+b}$ and $\frac{b}{a+b}$
    (b) $\frac{a+c}{a+b}$ and $-\frac{b}{a+b}$
    (c) 1 and $\frac{c}{a+b}$
    (d) –1 and $-\frac{c}{a+b}$

Consider the following for the next three (03) items that follow:
Let $(1+x)^n = 1 + T_1x + T_2x^2 + T_3x^3 + \ldots + T_nx^n$.

38. What is $T_1 + 2T_2 + 3T_3 + \ldots + nT_n$ equal to?
    (a) 0 (b) 1
    (c) 2^n (d) $n2^{n-1}$
39. What is $1 - T_1 + 2T_2 - 3T_3 + \ldots + (-1)^n nT_n$ equal to?
    (a) 0 (b) -2^{n-1}
    (c) $n2^{n-1}$ (d) 1
40. What is $T_1 + T_2 + T_3 + \ldots + T_n$ equal to?
    (a) 2^n (b) $2^n - 1$
    (c) 2^{n-1} (d) $2^n + 1$

Consider the following for the next two (02) items that follow:

Let $f(x) = x^2 - 1$ and gof(x) = $x - \sqrt{x} + 1$.

41. Which one of the following is a possible expression for $g(x)$?

(a) $\sqrt{x+1} - \sqrt[4]{x+1}$ (b) $\sqrt{x+1} - \sqrt[4]{x+1} + 1$

(c) $\sqrt{x+1} + \sqrt[4]{x+1}$ (d) $x + 1 - \sqrt{x+1} + 1$

42. What is g(15) equal to?

(a) 1 (b) 2

(c) 3 (d) 4

Consider the following for the next two (02) items that follow:

Let a function f be defined on R – [0] and $2f(x) + f\left(\frac{1}{x}\right) = x + 3$.

43. What is $f(0.5)$ equal to?

(a) $\frac{1}{2}$ (b) $\frac{2}{3}$

(c) 1 (d) 2

44. If f is differentiable, then what is f′(0.5) equal to?

(a) $\frac{1}{4}$ (b) $\frac{2}{3}$

(c) 2 (d) 4

Consider the following for the next (02) items that follow:

A function is defined by

$$f(x) = \begin{vmatrix} x+1 & 2 & 3 \\ 2 & x+4 & 6 \\ 3 & 6 & x+9 \end{vmatrix}$$

45. The function is decreasing on:

(a) $\left[-\frac{28}{3}, 0\right]$ (b) $\left[0, \frac{28}{3}\right]$

(c) $\left[0, \frac{50}{3}\right]$ (d) $\left[0, \frac{56}{3}\right]$

46. The function attains local minimum value at:

(a) $x = -\frac{28}{3}$ (b) $x = -1$

(c) $x = 0$ (d) $x = \frac{28}{3}$

Consider the following for the next (02) items that follow:

Given that $4x^2 + y^2 = 9$.

47. What is the maximum value of y?

(a) $\frac{3}{2}$ (b) 3

(c) 4 (d) 6

48. What is the maximum value of xy?

(a) $\frac{9}{4}$ (b) $\frac{3}{2}$

(c) $\frac{4}{9}$ (d) $\frac{2}{3}$

Consider the following for the next (02) items that follow:

A function is defined by $f(x) = \pi + \sin^2 x$.

49. What is the range of the function?

(a) [0, 1] (b) $[\pi, \pi + 1]$

(c) $[\pi - 1, \pi + 1]$ (d) $[\pi - 1, \pi - 1]$

50. What is the period of the function?

(a) 2π (b) π

(c) $\frac{\pi}{2}$ (d) The function is non-periodic

Consider the following for the next (02) items that follow:

A parabola passes through (1, 2) and satisfies the differential equation $\frac{dy}{dx} = \frac{2y}{x}, x > 0, y > 0$.

51. What is the directrix of the parabola?

(a) $y = -\frac{1}{8}$ (b) $y = \frac{1}{8}$

(c) $x = -\frac{1}{8}$ (d) $x = \frac{1}{8}$

52. What is the length of latus rectum of the parabola?

(a) 1 (b) $\frac{1}{2}$

(c) $\frac{1}{4}$ (d) $\frac{1}{8}$

Consider the following for the next (02) items that follow:

Let $f(x) = \frac{a^{x-1} + b^{x-1}}{2}$ and $g(x) = x - 1$.

53. What is $\lim_{x \to 1} \frac{f(x) - 1}{g(x)}$ equal to?

(a) $\frac{\ln(ab)}{4}$ (b) $\frac{\ln(ab)}{2}$

(c) ln (ab) (d) 2ln (ab)

54. What is $\lim_{x \to 1} f(x)^{\frac{1}{g(x)}}$ equal to?

(a) $\sqrt{ab}$ (b) ab

(c) 2ab (d) $\frac{\sqrt{ab}}{2}$

Consider the following for the next (02) items that follow:

Let $f(x) = \sqrt{2 - x} + \sqrt{2 + x}$.

55. What is the domain of the function?

(a) (–2, 2) (b) [–2, 2]

(c) R –(–2, 2) (d) R – [–2, 2]

56. What is the greatest value of the function?

(a) $\sqrt{3}$ (b) $\sqrt{6}$

(c) $\sqrt{8}$ (d) 4

Consider the following for the next (02) items that follow:

Let $f(x) = |x|$ and $g(x) = [x] - 1$, where [.] is the greatest integer function.

Let $h(x) = \dfrac{f(g(x))}{g(f(x))}$.

57. What is $\lim\limits_{x\to 0^+} h(x)$ equal to?
(a) –2 (b) –1
(c) 0 (d) 1

58. What is $\lim\limits_{x\to 0-} h(x)$ equal to?
(a) –2 (b) –1
(c) 0 (d) 2

Consider the following for the next (02) items that follow:

Let f(x) = $\begin{cases} \dfrac{x-3}{|x-3|}+a; & x<3 \\ a-b; & x=3 \\ \dfrac{x-3}{|x-3|}+b; & x>3 \end{cases}$ and

$f(x)$ be continuous at $x = 3$.

59. What is the value of a?
(a) –1 (b) 1
(c) 2 (d) 3

60. What is the value of b?
(a) –1 (b) 1
(c) 2 (d) 3

Consider the following for the next (02) items that follow:

Let I = $\int\limits_{-2\pi}^{2\pi} \dfrac{\sin^4 x+\cos^4 x}{1+3^x}dx$

61. What is $\int\limits_0^{\pi}(\sin^4 x+\cos^4 x)dx$ equal to?
(a) $\dfrac{3\pi}{8}$ (b) $\dfrac{3\pi}{4}$
(c) $\dfrac{3\pi}{2}$ (d) 3π

62. What is I equal to?
(a) 0 (b) $\dfrac{3\pi}{4}$
(c) $\dfrac{3\pi}{2}$ (d) 3π

Consider the following for the next (02) items that follow:

Let $f(x) = \begin{cases} ax(x+1)+b, & x<1 \\ x-1, & 1\le x\le 2 \end{cases}$

63. If the function $f(x)$ is differentiable at $x = 1$, then what is the value of $(a + b)$?
(a) $-\dfrac{1}{3}$ (b) –1
(c) 0 (d) 1

64. What is $\lim\limits_{x\to 0} f(x)$ equal to?
(a) $-\dfrac{1}{3}$ (b) $-\dfrac{2}{3}$
(c) 0 (d) 1

65. If $f(x) = |\ln|x||$ where $0 < x < 1$, then what is $f'(0.5)$ equal to?
(a) –2 (b) –1
(c) 0 (d) 2

66. If $f'(x) = \cos(\ln x)$ and $y = f\left(\dfrac{2x-3}{x}\right)$, then what is $\dfrac{dy}{dx}$ equal to?
(a) $\cos\left(\ln\left(\dfrac{2x-3}{x}\right)\right)$ (b) $-\dfrac{3}{x^2}\sin\left(\ln\left(\dfrac{2x-3}{x}\right)\right)$
(c) $\dfrac{3}{x^2}\cos\left(\ln\left(\dfrac{2x-3}{x}\right)\right)$ (d) $-\dfrac{3}{x^2}\cos\left(\ln\left(\dfrac{2x-3}{x}\right)\right)$

67. What is $\int\limits_0^{8\pi}|\sin x|\,dx$ equal to?
(a) 2 (b) 4
(c) 8 (d) 16

68. What is the area between the curve $f(x) = x|x|$ and x-axis for $x \in [-1, 1]$?
(a) $\dfrac{2}{3}$ (b) $\dfrac{1}{2}$
(c) $\dfrac{1}{4}$ (d) $\dfrac{1}{3}$

69. What are the order and the degree respectively of the differential equation $x^2\left(\dfrac{d^3y}{dx^3}\right)^2+\left(\dfrac{dy}{dx}\right)^4+\sin x=0$?
(a) 3, 4 (b) 1, 4
(c) 2, 2 (d) 3, 2

70. What is the differential equation of all parabolas of the type $y^2 = 4a(x – b)$?
(a) $\dfrac{d^2y}{dx^2}+\left(\dfrac{dy}{dx}\right)^2=0$ (b) $\dfrac{d^2y}{dx^2}+x^2\left(\dfrac{dy}{dx}\right)^2=0$
(c) $y^2\dfrac{d^2y}{dx^2}+\left(\dfrac{dy}{dx}\right)^2=0$ (d) $y\dfrac{d^2y}{dx^2}+\left(\dfrac{dy}{dx}\right)^2=0$

Consider the following for the next two (02) items that follow:

Let $a_1, a_2, a_3 \ldots$ be in AP such that $a_1 + a_5 + a_{10} + a_{15} + a_{20} + a_{25} + a_{30} + a_{34} = 300$.

71. What is $a_1 + a_5 – a_{10} – a_{15} – a_{20} – a_{25} + a_{30} + a_{34}$ equal to?
(a) 0 (b) 25
(c) 125 (d) 250

72. What is $\sum\limits_{n=1}^{34} a_n$ equal to?
(a) 900 (b) 1025
(c) 1200 (d) 1275

Consider the following for the next two (02) items that follow:

Let p = $\cos\left(\dfrac{\pi}{5}\right)\cos\left(\dfrac{2\pi}{5}\right)$ and q = $\cos\left(\dfrac{4\pi}{5}\right)\cos\left(\dfrac{8\pi}{5}\right)$.

73. What is the value of p + q?
(a) $-\frac{1}{2}$ (b) $-\frac{1}{4}$
(c) 0 (d) $\frac{1}{2}$

74. What is the value of pq?
(a) $-\frac{1}{16}$ (b) $-\frac{1}{4}$
(c) $\frac{1}{4}$ (d) $\frac{1}{16}$

Consider the following for the next two (02) items that follow:

Let $p=\frac{1}{3}-\frac{\tan 3x}{\tan x}$ and $q = 1 - 3\tan^2 x,\ 0 < x < \pi, x \neq \frac{\pi}{2}$.

75. What is pq equal to?
(a) 1 (b) 2
(c) $\frac{8}{3}$ (d) $-\frac{8}{3}$

76. For how many values of x does $\frac{1}{p}$ become zero?
(a) No value (b) Only one value
(c) Only two values (d) Only three values

Consider the following for the next two (02) items that follow:

Let $\sin x + \sin y = \sqrt{3}(\cos y - \cos x)$; $x + y = \frac{\pi}{2}$, $0 < x, y < \frac{\pi}{2}$.

77. What is a value of $\sin 3x + \sin 3y$?
(a) –1 (b) 0
(c) 1 (d) 3

78. What is the value of $\cos^3 x + \cos^3 y$?
(a) $\frac{3\sqrt{3}}{8}$ (b) $\frac{3\sqrt{6}}{8}$
(c) $\frac{3\sqrt{6}}{4}$ (d) 1

Consider the following for the next two (02) items that follow:

The angles A, B and C of a triangle ABC are in the ratio 3 : 5 : 4.

79. What is the value of $a+b+\sqrt{2}c$ equal to?
(a) $3a$ (b) $2b$
(c) $3b$ (d) $2c$

80. What is the ratio of $a^2 : b^2 : c^2$?
(a) $2:2+\sqrt{3}:3$ (b) $2:2-\sqrt{3}:2$
(c) $2:2+\sqrt{3}:2$ (d) $2:2-\sqrt{3}:3$

81. What is the equation of directrix of parabola $y^2 = 4bx$, where $b < 0$ and $b^2 + b - 2 = 0$?
(a) $x + 1 = 0$ (b) $x - 2 = 0$
(c) $x - 1 = 0$ (d) $x + 2 = 0$

82. The points $(-a, -b)$, $(0, 0)$, (a, b) and (a^2, ab) are:
(a) lying on the same circle
(b) vertices of a square
(c) vertices of a parallelogram that is not a square
(d) collinear

83. Given that $16p^2 + 49q^2 - 4r^2 - 56pq = 0$. Which one of the following is a point on a pair of straight lines $(px + qy + r)(px + qy - r) = 0$?
(a) $\left(2,\frac{7}{2}\right)$ (b) $\left(2,-\frac{7}{2}\right)$
(c) (4, –7) (d) (4, 7)

84. If $3x + y - 5 = 0$ is the equation of a chord of the circle $x^2 + y^2 - 25 = 0$, then what are the coordinates of the mid-point of the chord?
(a) $\left(\frac{3}{4},\frac{1}{4}\right)$ (b) $\left(\frac{3}{2},\frac{1}{2}\right)$
(c) $\left(\frac{3}{4},-\frac{1}{4}\right)$ (d) $\left(\frac{3}{2},-\frac{1}{2}\right)$

85. Consider the following in respect of the equation $\frac{x^2}{24-k}+\frac{y^2}{k-16} = 2$.
1. The equation represents an ellipse if $k = 19$.
2. The equation represents a hyperbola if $k = 12$.
3. The equation represents a circle if $k = 20$.

How many of the statements given above are correct?
(a) Only one (b) Only two
(c) All three (d) None

86. Consider the following statements in respect of hyperbola $\frac{x^2}{\cos^2\theta}-\frac{y^2}{\sin^2\theta} = 1$.
1. The two foci are independent of θ.
2. The eccentricity is sec θ.
3. The distance between the two foci is 2 units.

How many of the statements given above are correct?
(a) Only one (b) Only two
(c) All three (d) None

87. Consider the following in respect of the circle $4x^2 + 4y^2 - 4ax - 4ay + a^2 = 0$:
1. The circle touches both the axes.
2. The diameter of the circle is 2a.
3. The centre of the circle lies on the line $x + y = a$.

How many of the statements given above are correct?
(a) Only one (b) Only two.
(c) All three (d) None

88. For what values of k is the line $(k - 3)x - (5 - k^2)y + k^2 - 7k + 6 = 0$ parallel to the line $x + y = 1$?
(a) –1, 1 (b) –1, 2
(c) 1, –2 (d) 2, –2

89. The line $x + y = 4$ cuts the line joining P(–1, 1) and Q(5, 7) at R. What is PR : RQ equal to?
(a) 1 : 1 (b) 1 : 2
(c) 2 : 1 (d) 1 : 3

90. What is the sum of the intercepts of the line whose perpendicular distance from origin is 4 units and the angle which the normal makes with positive direction of x-axis is 15°?
(a) 8 (b) $4\sqrt{6}$
(c) $8\sqrt{6}$ (d) 16

91. What is the length of projection of the vector $\hat{i}+2\hat{j}+3\hat{k}$ on the vector $2\hat{i}+3\hat{j}-2\hat{k}$?
(a) $\frac{1}{\sqrt{17}}$ (b) $\frac{2}{\sqrt{17}}$
(c) $\frac{3}{\sqrt{17}}$ (d) $\frac{2}{\sqrt{14}}$

92. If $\left(\vec{a}\times\vec{b}\right)^2+\left(\vec{a}\cdot\vec{b}\right)^2 = 144$ and $\left|\vec{b}\right| = 4$, then what is the value of $\left|\vec{a}\right|$?
(a) 3 (b) 4
(c) 5 (d) 6

93. If θ is the angle between vector $\vec{a}$ and $\vec{b}$ such that $\vec{a}\cdot\vec{b}\geq 0$, then which one of the following is correct?
(a) $0\leq\theta\leq\pi$ (b) $\frac{\pi}{2}\leq\theta\leq\pi$
(c) $0\leq\theta\leq\frac{\pi}{2}$ (d) $0<\theta<\frac{\pi}{2}$

94. The vectors $60\hat{i}+3\hat{j}$, $40\hat{i}-8\hat{j}$ and $\beta\hat{i}-52\hat{j}$ are collinear if:
(a) β = 20 (b) β = 40
(c) β = –40 (d) β =26

95. Consider the following in respect of the vectors $\vec{a}=(0,1,1)$ and $\vec{b}=(1,0,1)$:
1. The number of unit vectors perpendicular to both $\vec{a}$ and $\vec{b}$ is only one.
2. The angle between the vectors is $\frac{\pi}{3}$.
Which of the statements given above is/are correct?
(a) 1 only (b) 2 only
(c) Both 1 and 2 (d) Neither 1 nor 2

96. If L is the line with direction ratios < 3, –2, 6 > and passing through (1, –1, 1), then what are the coordinates of the points on L whose distance from (1, –1, 1) is 2 units?
(a) $\left(-\frac{11}{7},\frac{13}{7},\frac{19}{7}\right)$ and $\left(\frac{1}{7},\frac{3}{7},\frac{5}{7}\right)$
(b) $\left(\frac{19}{7},-\frac{11}{7},\frac{13}{7}\right)$ and $\left(-\frac{1}{7},\frac{3}{7},-\frac{5}{7}\right)$
(c) $\left(\frac{13}{7},\frac{11}{7},\frac{19}{7}\right)$ and $\left(-\frac{1}{7},-\frac{3}{7},\frac{5}{7}\right)$
(d) $\left(\frac{13}{7},-\frac{11}{7},\frac{19}{7}\right)$ and $\left(\frac{1}{7},-\frac{3}{7},-\frac{5}{7}\right)$

97. Which one of the planes is parallel to the line $\frac{x-2}{3}=\frac{y-3}{4}=\frac{z-4}{5}$?
(a) $2x + 2y + z - 1 = 0$
(b) $2x - y - 2z + 5 = 0$
(c) $2x + 2y - 2z + 1 = 0$
(d) $x - 2y + z - 1 = 0$

98. What is the angle between the lines $2x = 3y = -z$ and $6x = -y = -4z$?
(a) 0° (b) 30°
(c) 60° (d) 90°

99. What is the equation of the sphere concentric with the sphere $x^2 + y^2 + z^2 - 2x - 6y - 8z - 5 = 0$ and which passes through the origin?
(a) $x^2+y^2+z^2-2x-8z=0$
(b) $x^2+y^2+z^2-2x-6y=0$
(c) $x^2+y^2+z^2-6y-8z=0$
(d) $x^2+y^2+z^2-2x-6y-8z=0$

100. A point P lies on the line joining A(1, 2, 3) and B(2, 10, 1). If z-coordinate of P is 7, what is the sum of other two coordinates?
(a) –15 (b) –13
(c) –11 (d) –9

101. The sum of deviations of a n numbers from 10 and 20 are p and q respectively. If $(p - q)^2 = 10000$, then what is the value of n?
(a) 10 (b) 20
(c) 50 (d) 100

102. If $\bar{X}=20$ is the mean of 10 observations $x_1, x_2, \ldots, x_{10}$; then what is the value of $\sum_{i=1}^{10}\left(\frac{3x_i-4}{5}\right)$?
(a) 0 (b) 12
(c) 112 (d) 1012

103. If the mean and the sum of squares of 10 observations are 40 and 16160 respectively, then what is the standard deviation?
(a) 16 (b) 6
(c) 5 (d) 4

104. Three dice are thrown. What is the probability of getting a sum which is a perfect square?
(a) $\frac{17}{108}$ (b) $\frac{5}{108}$
(c) $\frac{19}{108}$ (d) $\frac{23}{108}$

105. A, B, C and D are mutually exclusive and exhaustive events.
If 2P(A) = 3P(B)= 4P(C)= 5P(d), then what is 77P(A) equal to?
(a) 12 (b) 15
(c) 20 (d) 30

106. Two distinct natural numbers from 1 to 9 are picked at random. What is the probability that their product has 1 in its unit place?
(a) $\frac{1}{81}$ (b) $\frac{1}{72}$
(c) $\frac{1}{18}$ (d) $\frac{1}{36}$

107. Two dice are thrown. What is the probability that difference of numbers on them is 2 or 3?
(a) $\frac{7}{36}$ (b) $\frac{7}{18}$
(c) $\frac{5}{18}$ (d) $\frac{11}{36}$

108. What is the mean of the numbers 1,2,3, ..., 10 with frequencies ${}^9C_0, {}^9C_1, {}^9C_2, ..., {}^9C_9$ respectively?
(a) 1.1×2^8 (b) 1.2×7^4
(c) 5.5 (d) 0.55

109. The probability that a person recovers from a disease is 0·8. What is the probability that exactly 2 persons out of 5 will recover from the disease?
(a) 0.00512 (b) 0.02048
(c) 0.2048 (d) 0.0512

110. Suppose that there is a chance for a newly constructed building to collapse, whether the design is faulty or not. The chance that the design is faulty is 10%. The chance that the building collapses is 95% if the design is faulty, otherwise it is 45%. If it is seen that the building has collapsed, then what is the probability that it is due to faulty design?
(a) 0.10 (b) 0.19
(c) 0.45 (d) 0.95

111. If r is the coefficient of correlation between x and y, then what is the correlation coefficient between $(3x + 4)$ and $(-3y + 3)$?
(a) $-r$ (b) r
(c) $\sqrt{3}r$ (d) $-\sqrt{3}r$

112. A fair coin is tossed 6 times. What is the probability of getting a result in the 6th toss which is different from those obtained in the first five tosses?
(a) $\frac{7}{16}$ (b) $\frac{1}{16}$
(c) $\frac{1}{32}$ (d) $\frac{1}{64}$

113. If H is the Harmonic Mean of three numbers ${}^{10}C_4, {}^{10}C_5$, and ${}^{10}C_6$, then what is the value of $\frac{270}{H}$?
(a) 1 (b) $\frac{14}{17}$
(c) $\frac{17}{14}$ (d) $\frac{1}{31}$

114. In a class, there are n students including the students P and Q. What is the probability that P and Q sit together if seats are assigned randomly?
(a) $\frac{1}{n}$ (b) $\frac{2}{n}$
(c) $\frac{4}{n}$ (d) $\frac{1}{2n}$

115. In a Binomial distribution B(n, p), $n = 6$ and $9P(X = 4) = P(X = 2)$. What is p equal to?
(a) $\frac{1}{4}$ (b) $\frac{1}{2}$
(c) $\frac{3}{4}$ (d) $\frac{4}{5}$

Consider the following for the next five (05) items that follow:

Three boys P, Q, R and three girls S, T, U are to be arranged in a row for a group photograph.

116. What is the probability that all three boys sit together?
(a) $\frac{1}{5}$ (b) $\frac{1}{4}$
(c) $\frac{1}{3}$ (d) $\frac{1}{12}$

117. What is the probability that boys and girls sit alternatively?
(a) $\frac{4}{5}$ (b) $\frac{1}{10}$
(c) $\frac{5}{6}$ (d) $\frac{1}{7}$

118. What is the probability that no two girls sit together?
(a) $\frac{2}{5}$ (b) $\frac{3}{5}$
(c) $\frac{1}{18}$ (d) $\frac{1}{5}$

119. What is the probability that P and Q take the two end positions?
(a) $\frac{1}{15}$ (b) $\frac{7}{15}$
(c) $\frac{14}{15}$ (d) $\frac{11}{45}$

120. What is the probability that Q and U sit together?
(a) $\frac{2}{3}$ (b) $\frac{1}{4}$
(c) $\frac{5}{6}$ (d) $\frac{1}{3}$

Answer			
Q No	**Answer Key**	**Topic Name**	**Chapter Name**
1	(d)	Geometrical Representation	Complex Number
2	(c)	Factorial	Permutation and Combination
3	(c)	Square Roots	Complex Number
4	(a)	Values of Determinant	Determinants
5	(a)	Inverse of Matrices	Determinants
6	(c)	Special Series	Sequence and Series
7	(a)	Values of Determinant	Determinants
8	(a)	Values of Determinant	Determinants
9	(b)	Values of Determinant	Determinant
10	(c)	H.P.	Sequence and Series
11	(c)	G.P.	Sequence and Series
12	(b)	Values if i	Complex Number
13	(c)	Special Series	Sequence and Series
14	(b)	Relation of Determinants	Binomial Theorem
15	(b)	Values of Determinant	Determinants
16	(b)	Cofactor	Determinants
17	(d)	Adjoint	Determinants
18	(a)	Product of Matrices	Matrices
19	(c)	Adroit	Determinants
20	(d)	Values of Determinants	Determinants
21	(c)	Cartesian Product	Relations & Function
22	(b)	Range	Relations & Function
23	(a)	Function	Relations & Function
24	(c)	Complement of Set	Sets
25	(a)	Values of Function	Relations & Function
26	(c)	Identities	Trigonometry
27	(a)	Identities	Inverse Trigonometry
28	(a)	Identities	Inverse Trigonometry
29	(c)	Trigonometric Equations	Trigonometry
30	(c)	Identities	Trigonometry
31	(b)	Nature of Roots	Quadratic Equation
32	(b)	Sum of Roots	Quadratic Equation
33	(b)	Cube Roots of Unity	Complex Number
34	(d)	Cube Roots of Unity	Complex Number
35	(c)	Cube Roots of Unity	Complex Number
36	(b)	Nature of Roots	Quadratic Equations
37	(c)	Nature of Roots	Quadratic Equations
38	(d)	Relation of Coefficients	Binomial Theorem
39	(d)	Relation of Coefficients	Binomial Theorem

40	(b)	Relation of Coefficients	Binomial Theorem
41	(b)	Composite Function	Relation & Function
42	(c)	Values of Function	Relation & Function
43	(b)	Values of Function	Relation & Function
44	(c)	Values of Function	Differentiation
45	(a)	Increasing and Decreasing	Application of Derivatives
46	(c)	Maxima & Minima	Application of Derivatives
47	(b)	Maximum value of function	Application of Derivatives
48	(a)	Maximum and Minimum	Application of Derivatives
49	(b)	Range	Trigonometry
50	(b)	Period	Trigonometry
51	(a)	Solution of Different Equates	Differential Equations
52	(b)	Parabola	Conic section
53	(b)	Limit	Limit & Derivatives
54	(a)	Limit	Limit & Derivatives
55	(b)	Domain	Relation & Function
56	(c)	Greatest Value of Function	Application of Derivatives
57	(b)	Limit	Limit & Derivatives
58	(a)	Limit	Limit & Derivatives
59	(d)	Continuity	Continuity and Differentiability
60	(b)	Continuity	Continuity and Differentiability
61	(b)	Values of Definite Integral	Definite Integral
62	(d)	Properties of Definite Integral	Definite Integral
63	(a)	Differentiability	Continuity and Differentiability
64	(b)	Limit	Limit and Derivative
65	(a)	Value of Differentiation	Differentiation
66	(c)	Differentiation	Differentiation
67	(d)	Properties of Definite Integrals	Application of Integral
68	(a)	Area Bounded by a Curve	Definite Integral
69	(d)	Degree and Order	Differential Equation
70	(d)	Formation of Differential Equation	Differential Equation
71	(a)	A.P.	Sequence and Series
72	(d)	A.P.	Sequence and Series
73	(c)	Identities	Trigonometry
74	(a)	Identities	Trigonometry
75	(d)	Identities	Trigonometry
76	(c)	Identities	Trigonometry
77	(b)	Identities	Trigonometry
78	(b)	Values	Trigonometry
79	(c)	Properties of triangle	Trigonometry
80	(a)	Properties of triangle	Trigonometry