6 Years 126 Sets

IIT/JEE MAIN MATHEMATICS

Chapterwise, Topicwise Typewise & Sub Type

Solved Papers

Revision Notes & Formulas

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CHAPTER WISE ANALYSIS CHART

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IIT JEE Mains Years wise Trend Analysis Chart

Years	No. of Papers	No of Questions
2019 (January)	8	8 × 30 = 240
2019 (April)	8	8 × 30 = 240
2020 (January)	6	6 × 30 = 180
2020 (September)	10	10 × 30 = 300
2021 (February)	6	6 × 30 = 180
2021 (March)	6	6 × 30 = 180
2021 (July)	8	8 × 30 = 240
2021 (August)	8	8 × 30 = 240
2022 (June)	12	12 × 30 = 360
2022 (July)	10	10 × 30 = 300
2023 (January)	12	12 × 30 = 360
2023 (April)	12	12 × 30 = 360
2024 (January)	10	10 × 30 = 300
2024 (April)	10	10 × 30 = 300
Total	126	3780

Syllabus

- **UNIT 1: SETS, RELATIONS, AND FUNCTIONS:**
  Sets and their representation: Union, intersection, and complement of sets and their algebraic properties; Power set; Relation, Type of relations, equivalence relations, functions; one-one, into and onto functions, the composition of functions
- **UNIT 2: COMPLEX NUMBERS AND QUADRATIC EQUATIONS:**
  Complex numbers as ordered pairs of reals, Representation of complex numbers in the form a + ib and their representation in a plane, Argand diagram, algebra of complex number, modulus, and argument (or amplitude) of a complex number, Quadratic equations in real and complex number system and their solutions Relations between roots and co-efficient, nature of roots, the formation of quadratic equations with given roots.
- **UNIT3: MATRICES AND DETERMINANTS:**
  Matrices, algebra of matrices, type of matrices, determinants, and matrices of order two and three, evaluation of determinants, area of triangles using determinants, Adjoint, and evaluation of inverse of a square matrix using determinants and, Test of consistency and solution of simultaneous linear equations in two or three variables using matrices.
- **UNIT 4: PERMUTATIONS AND COMBINATIONS:**
  The fundamental principle of counting, permutation as an arrangement and combination as selection, Meaning of P (n,r) and C (n,r), simple applications.
- **UNIT 5: BINOMIAL THEOREM AND ITS SIMPLE APPLICATIONS:**
  Binomial theorem for a positive integral index, general term and middle term, and simple applications.
- **UNIT 6: SEQUENCE AND SERIES:**
  Arithmetic and Geometric progressions, insertion of arithmetic, geometric means between two given numbers, Relation between A.M and G.M.
- **UNIT 7: LIMIT, CONTINUITY, AND DIFFERENTIABILITY:**
  Real–valued functions, algebra of functions, polynomials, rational, trigonometric, logarithmic, and exponential functions, inverse function. Graphs of simple functions. Limits, continuity, and differentiability. Differentiation of the sum, difference, product, and quotient of two functions. Differentiation of trigonometric, inverse trigonometric, logarithmic, exponential, composite, and

implicit functions; derivatives of order up to two, Applications of derivatives: Rate of change of quantities, monotonic-Increasing and decreasing functions, Maxima and minima of functions of one variable.

- **UNIT 8: INTEGRAL CALCULAS:**

Integral as an anti-derivative, Fundamental integral involving algebraic, trigonometric, exponential, and logarithmic functions. Integrations by substitution, by parts, and by partial functions. Integration using trigonometric identities.

Evaluation of simple integrals of the type

$\int \frac{dx}{x^2+a^2}$, $\int \frac{dx}{\sqrt{x^2 \pm a^2}}$, $\int \frac{dx}{a^2-x^2}$, $\int \frac{dx}{\sqrt{a^2-x^2}}$, $\int \frac{dx}{ax^2-bx+c}$, $\int \frac{dx}{\sqrt{ax^2-bx+c}}$, $\int \frac{(px+q)dx}{ax^2+bx+c}$,

$\int \frac{(px+q)dx}{\sqrt{ax^2+bx+c}}$, $\int \sqrt{a^2 \pm x^2}\, dx$, $\int \sqrt{x^2-a^2}\, dx$

The fundamental theorem of calculus, properties of definite integrals. Evaluation of definite integrals, determining areas of the regions bounded by simple curves in standard form.

- **UNIT 9: DIFFRENTIAL EQUATIONS**

Ordinary differential equations, their order, and degree, the solution of differential equation by the method of separation of variables, solution of a homogeneous and linear differential equation of the type

$$\frac{dy}{dx}+p(x)y=q(x)$$

- **UNIT 10: CO-ORDINATE GEOMETRY**

Cartesian system of rectangular coordinates in a plane, distance formula, sections formula, locus, and its equation, the slope of a line, parallel and perpendicular lines, intercepts of a line on the co-ordinate axis.

  - **Straight line**

Various forms of equations of a line, intersection of lines, angles between two lines, conditions for concurrence of three lines, the distance of a point form a line, co-ordinate of the centroid, orthocentre, and circumcentre of a triangle,

  - **Circle, conic sections**

A standard form of equations of a circle, the general form of the equation of a circle, its radius and central, equation of a circle when the endpoints of a diameter are given, points of intersection of a line and a circle with the centre at the origin and sections of conics, equations of conic sections (parabola, ellipse, and hyperbola) in standard forms,

- **UNIT 11: THREE DIMENSIONAL GEOMETRY**

Coordinates of a point in space, the distance between two points, section formula, directions ratios, and direction cosines, and the angle between two intersecting lines. Skew lines, the shortest distance between them, and its equation. Equations of a line

- **UNIT 12: VECTOR ALGEBRA**

Vectors and scalars, the addition of vectors, components of a vector in two dimensions and three-dimensional space, scalar and vector products,

- **UNIT 13: STATISTICS AND PROBABILITY**

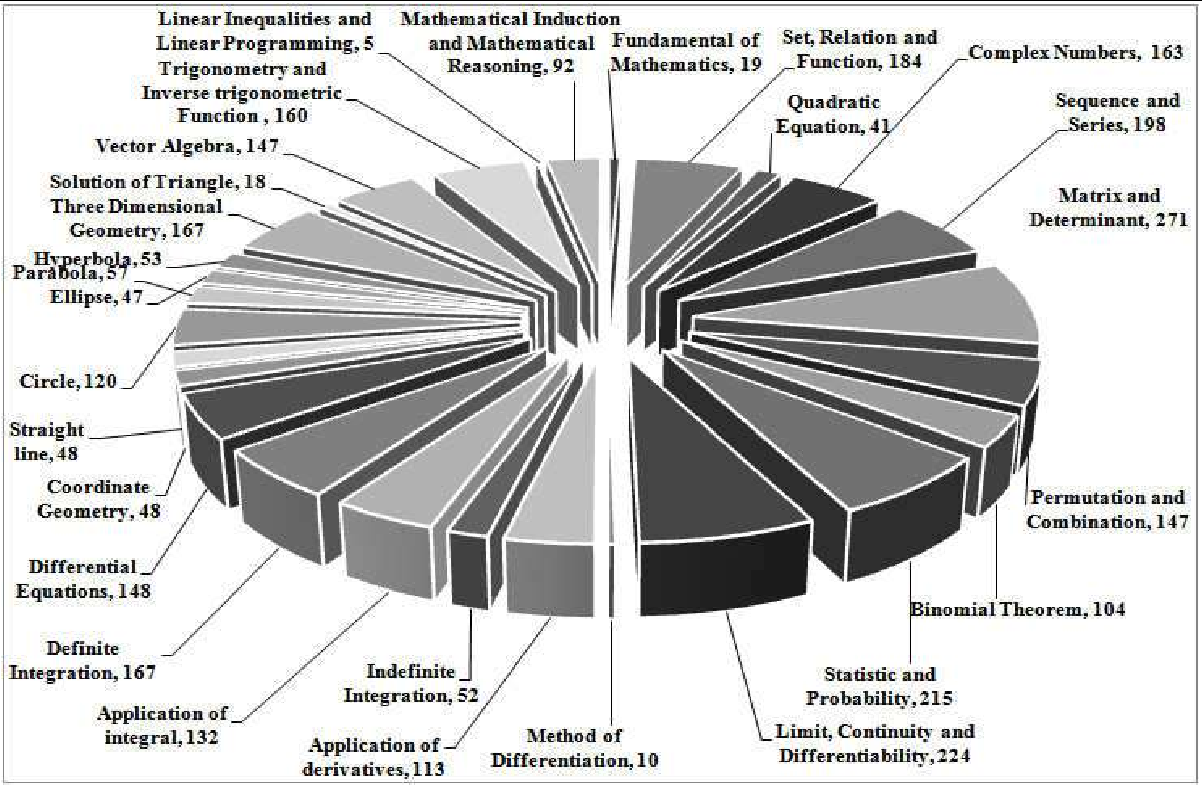
Measures of discretion; calculation of mean, median, mode of grouped and ungrouped data calculation of standard deviation, variance, and mean deviation for grouped and ungrouped data.

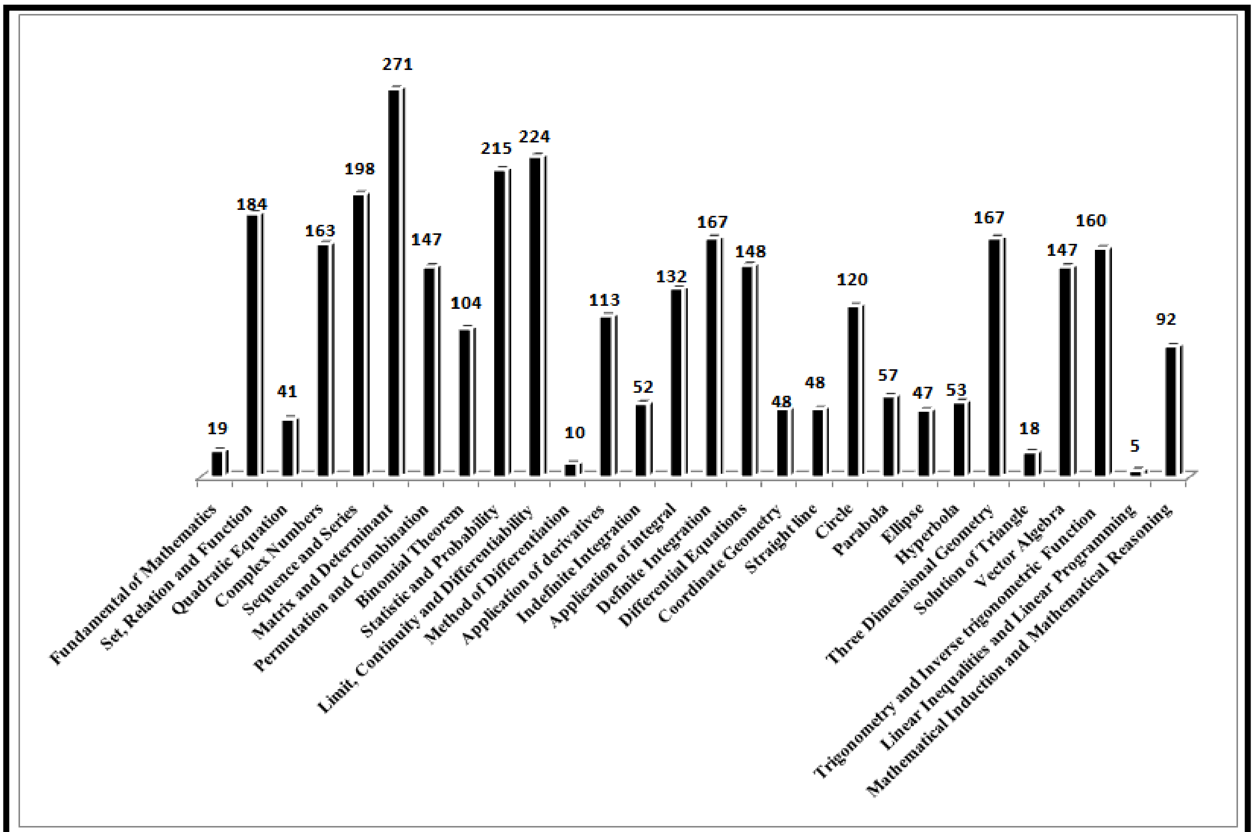
Probability: Probability of an event, addition and multiplication theorems of probability, Baye's theorem, probability distribution of a random variate,

- **UNIT 14: TRIGONOMETRY**

Trigonometrical identities and trigonometrical functions, inverse trigonometrical functions, and their properties.

Trend Analysis of IIT Math Questions through Pie Chart and Bar Graph





01.

Fundamental of Mathematics

Formula

■ **Intervals:**

Intervals are basically subsets of R and are commonly used in solving inequalities or in finding domains. If there are two numbers a, b ∈ R such that a < b, we can define four types of intervals as follows:

Symbols Used

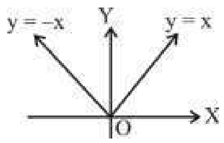
- Open interval : (a, b) = {x : a < x < b} i.e. end points are not included. ( ) or] [
- Closed interval : [a, b] = {x : a ≤ x ≤ b} i.e. end points are also included. [ ]
  This is possible only when both a and b are finite.
- Open - closed interval : (a, b] = {x : a < x ≤ b}
  ( ] or]]
  (iv) Closed - open interval : [a, b) = x : a ≤ x < b}
  [ ) or [[

□ **The infinite intervals are defined as follows:**

- (a, ∞) = {x : x > a}
- [a, ∞) = {x : x ≥ a}
- (– ∞, b) = {x : x < b}
- [∞, b) = {x : x ≤ b}
- (– ∞, ∞) = {x : x ∈ R}

□ **Modulus Function**

$$y = |x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$



"It is the numerical value of x".
"It is symmetric about y-axis" where domain ∈R and range ∈ [0, ∞].

□ **Properties of Modulus:**

For any a, b ∈ R

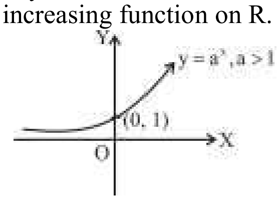
- $|a| \geq 0$
- $|a| = |-a|$,
- $|a| \geq a$
- $|a| \geq -a$
- $|ab| = |a|\,|b|$
- $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$
- $|a + b| \leq |a| + |b|$
- $|a - b| \geq ||a| - |b||$

□ **Exponential Function**

Here, $f(x) = a^x$, a > 0, a ≠ 1, and x ∈ R, where domain ∈ R, Range ∈ (0, ∞).

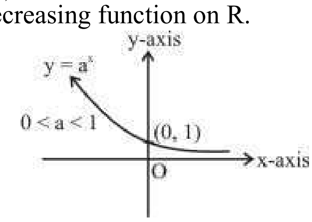
❍ **Case I. a > 1**

Here, $f(x) = y = a^x$ increase with the increase in x, i.e., f(x) is increasing function on R.



❍ **Case II. 0 < a < 1**

Here, $f(x) = a^x$ decrease with the increase in x, i.e., f(x) is decreasing function on R.



"In general, exponential function increases or decreases as (a > 1) or (0 < a < 1) respectively".

□ **Logarithmic Function**

The function $f(x) = \log_a x$; (x, a > 0) and a ≠ 1 is a logarithmic function.

Thus, the domain of logarithmic function is all real positive numbers and their range is the set R of all real numbers.

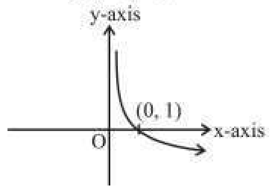
We have seen that $y = a^x$ is strictly increasing when a > 1 and strictly decreasing when 0 < a < 1.

Thus, the function is invertible. The inverse of this function is denoted by $\log_a x$, we write

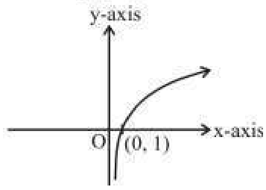
$y = a^x \Rightarrow x = \log_a y$;

where x∈R and y∈(0, ∞) writing $y = \log_a x$ in place of $x = \log_a y$, we have the graph of $y = \log_a x$.

If 0 < 0 < 1



If a > 1



Thus, logarithmic function is also known as inverse of exponential function.

■ **Properties of logarithmic function**

➢ $\log_e(ab) = \log_e a + \log_e b$ {a, b > 0}

➢ $\log_e\left(\frac{a}{b}\right) = \log_e a - \log_e b$ {a, b > 0}

➢ $\log_e a^m = m \log_e a$ {a > 0 and m ∈ R}

➢ $\log_a a = 1$ {a > 0 and a ≠ 1}

➢ $\log_{b^m} a = \frac{1}{m}\log_b a$ {a, b > 0, b ≠ 1 and m}

➢ $\log_b a = \frac{1}{\log_a b}$ {a, b>0 and a, b ≠ 1}

➢ $\log_b a = \frac{\log_m a}{\log_m b}$ {a, b > 0 ≠ {1} and m > 0}

➢ $a^{\log_a m} = m$ {a, m > 0 and a ≠ 1}

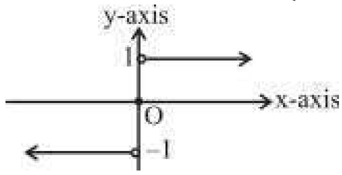
➢ $a^{\log_c b} = b^{\log_c a}$ {a, b, c > 0 and c ≠ 1}

➢ If $\log_m x > \log_m y \Rightarrow \begin{cases} x > y, \text{ if } m > 1 \\ x < y, \text{ if } 0 < m < 1 \end{cases}$ {m, x, y, > 0 and m ≠ 1}

■ **Signum function; y = Sgn(x)**

It is defined by;

$$y = \text{Sgn}(x) = \begin{cases} \frac{|x|}{x} \text{ or } \frac{x}{|x|}; & x \neq 0 \\ 0; & x = 0 \end{cases} = \begin{cases} +1, \text{ if } x > 0 \\ -1, \text{ if } x < 0 \\ 0, \text{ if } x = 0 \end{cases}$$



Here, Domain of f(x) ∈R. and

Range of f(x) ∈ {–1, 0, 1}.

■ **Greatest integer function**

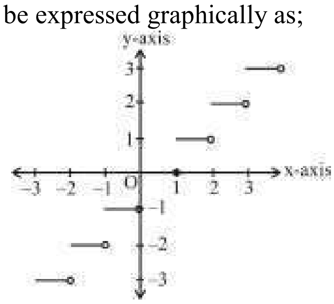
[x] indicates the integral part of x which is nearest and smaller integer to x. It is also known as floor of x.

Thus, [2.3] = 2, [0.23] = 0, [2] = 2, [–8.0725] = –9, …. In general;

n ≤ x < n + 1 (n ∈ Integer) ⇒ [x] = n.

Here, f(x) = [x] could be expressed graphically as;

x	[x]
$0 \le x < 1$	0
$1 \le x < 2$	1
$2 \le x < 3$	2



Domain of function- f(x) ∈ (–∞, ∞).

Range of function f(x) ∈ I.

■ **Properties of greatest integer function**

● [x] = x holds, if x is integer.

● [x + I] = [x] + I, if I is integer.

● [x + y] ≥ [x] + [y].

● If [ϕ (x)] ≥ I, then ϕ (x) ≥ I.

● If [ϕ (x)] ≤ I, then ϕ (x) < I + 1.

● [–x] = –[x], if x ∈ integer.

● [–x] = –[x] –1, if x ∉ integer.

"It is also known as stepwise function/floor of x."

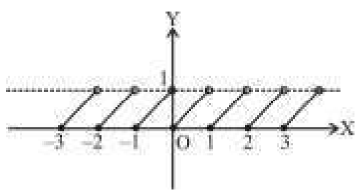
■ **Fractional part of function**

Here, {.} denotes the fractional part of x. Thus, in y = {x}

x = [x] + {x} = I + f ; where I = [x] and f = {x}

∴ **y = {x} = x –[x], where 0 ≤ {x} < I ; shown as:**

x	{x}
$0 \le x < 1$	x
$1 \le x < 2$	$x-1$
$2 \le x < 3$	$x-2$
$-1 \le x < 0$	$x+1$
$-2 \le x < -1$	$x+2$



■ **Properties of fractional part of x**

● {x} = x ; if 0 ≤ x <1

● {x} = 0 ; if x∈ integer.

● {–x} = 1 –{x} ; if x ∈ integer.

Type I: Greatest Integer function and G.C.D

1. The remainder, when $19^{200} + 23^{200}$ is divided by 49, is ________.

JEE Mains 01/02/2023 Shift-I

Ans. (29) : $(19)^{200} + (23)^{200} \div 49$

$= (23)^{200} + (19)^{200}$

$= (21+2)^{200} + (21-2)^{200}$

if n is even then expression

$(x + y)^n + (x - y)^n$

$= 2\left[{}^nC_0x^ny^0 + {}^nC_2x^{n-2}y^2 + ... + {}^nC_nx^0y^n\right]$

$(21+2)^{200} + (21-2)^{200} =$

$\left[{}^{200}C_0 21^{200}2^0 + {}^{200}C_2 21^{199}2^2 + ... + {}^{200}C_{200}21^0 2^{200}\right]$

$= m(49) + 2 \times 1 \times 2^{200}$

$\Rightarrow 2(2)^{200} = (2)^{201}$

$(2^3)^{67} = (7+1)^{63}$

$= \left[{}^{67}C_0 7^{67}1^0 + {}^{67}C_2 7^{65}1^2 + ... + {}^{67}C_{67}7^0 1^{67}\right]$

$= m(49) + (67 \times 7) + 1$

$= \frac{67 \times 7 + 1}{49}$

$= \frac{469 + 1}{49}$

$= \frac{470}{49} = \frac{490 - 20}{49}$

$= \frac{490}{49} - \frac{20}{49}$

Remainder = 49 – 20

= 29

2. The remainder, when 7^{103} is divided by 17, is ____.

JEE Mains 13/04/2023 Shift-II

Ans. (12) : $7^{103} = 7.7^{102}$

$= 7\,(7^2)^{51}$

$= 7\,(51-2)^{51} \rightarrow$ remainder $= 7\,(-2)^{51}$

$-7\left(2^3\right)(16)^{12} = -56(17-1)^{12} \rightarrow \text{Remainder } = -56(-1)^{12}$

Remainder = –56 + 17 k

= – 56 +68

= 12

3. If gcd (m, n) = 1 and $1^2 - 2^2 + 3^2 - 4^2 + ... + (2021)^2 - (2022)^2 + (2023)^2 = 1012\, m^2n$ then $m^2 - n^2$ is equal to:

(a) 180 (b) 220

(c) 200 (d) 240

JEE Mains 06/04/2023 Shift-II

Ans. (d) : Given,

$1^2 - 2^2 + 3^2 - 4^2 + ... + (2021)^2 - (2022)^2 + (2023)^2 = 1012\, m^2n$

$= (1 - 2)(1 + 2) + (3 - 4)(3 + 4) + ... + (2021 - 2022)(2021 + 2022) + (2023)^2 = (1012)\, m^2n$

$\Rightarrow (-1)\,[1 + 2 + 3 + 4 + ... + 2022] + (2023)^2 = (1012)\, m^2n$

$\Rightarrow \quad 1012m^2n = \frac{2023(2024)}{2} = 2023\times1012$

$1012m^2n = 2023\times1012$

$\Rightarrow \quad m^2n = 2023$

$\Rightarrow \quad m^2n = (17)^2 \times 7$

$\therefore \quad m = 17, n = 7$

Hence, $m^2 - n^2 = (17)^2 - 7^2 = 289 - 49 = 240$

4. The largest natural number n such that 3^n divides 66! is ____.

JEE Mains 08/04/2023 Shift-I

Ans. (31) :

$$\left[\frac{66}{3}\right]+\left[\frac{66}{9}\right]+\left[\frac{66}{27}\right]$$

$22 + 7 + 2 = 31$

Type II: Fractional Part of a Number

5. Fractional part of the number is $\frac{4^{2022}}{15}$ equal to

(a) $\frac{4}{15}$ (b) $\frac{8}{15}$

(c) $\frac{1}{15}$ (d) $\frac{14}{15}$

JEE Mains 13/04/2023 Shift-I

Ans. (c) : Sol.

$$\left\{\frac{4^{2022}}{15}\right\}=\left\{\frac{2^{4044}}{15}\right\}=\left\{\frac{(1+15)^{1011}}{15}\right\}=\frac{1}{15}$$

$$\left[\because (1+x)^n = 1+nx+\frac{n\times(n-1)}{2!}x^2+.......\right]$$

6. If the fractional part of the number $\frac{2^{403}}{15}$ is $\frac{k}{15}$ then k is equal to

(a) 14 (b) 6

(c) 4 (d) 8

JEE Main 09.01.2019, Shift-I

Ans. (d) : Given, $\frac{2^{403}}{15} = 2^3\times\frac{2^{400}}{15}$

$= 8\times\frac{16^{100}}{15} = \frac{8}{15}(1+15)^{100}$

Now, using binomial theorem,

$\frac{8}{15}(1+15n)$

$\frac{8}{15}+\frac{8}{15}\times15n \quad [n\in N]$

$\frac{8}{15}+8n$

Therefore comparing fractional part, we get –

$\frac{8}{15}=\frac{k}{15}$

$k = 8$

Type III: Divisibility and remainder theorem

7. Among the statements:

(S_1) : $2023^{2022} - 1999^{2022}$ is divisible by 8

(S_2) : $13(13)^n - 11n - 13$ is divisible by 144 for infinitely many $n \in \mathbb{N}$

(a) only (S_2) is correct

(b) only (S_1) is correct

(c) both (S_1) and (S_2) are incorrect

(d) both (S_1) and (S_2) are correct

JEE Mains 06/04/2023 Shift-II

Ans. (d) :

$\because \; x^n - y^n = (x - y)\,[x^{n-1} + x^{n-2}y + x^{n-3}y^2 + ... + y^{n-1}]$

$x^n - y^n$ is divisible by $x - y$

Stat 1 → $(2023)^{2022} - (1999)^{2022}$

$(2023) - (1999) = 24.k$

$\therefore$ $(2023)^{2022} - (1999)^{2022}$ is divisible by 8

Stat 2 →

$$\left(13\times(1+12)^n\right)=13\Big[\overbrace{{}^nC_0(1)^n(12)^0}^{1}+\overbrace{{}^nC_1(1)^{n-1}(12)^1}^{12n}+......$$

$${}^nC_n(12)^n\Big]-12n-13$$

$= 13(12n) - 12n + 13\left[{}^nC_2(12)^2 + ...{}^nC_n(12)^n\right]$

$= 156n - 12n + 13\left[{}^nC_2(12)^2 + ...{}^nC_n(12)^n\right]$

$= 144n - 144m$

If ($n = 144m$, $m\in N$) then it is divisible by 144 for infinite values of n.

8. $25^{190} - 19^{190} - 8^{190} + 2^{190}$ is divisible by

(a) 34 but not by 14

(b) 14 but not by 34

(c) Both 14 and 34

(d) Neither 14 not 34

JEE Mains 08/04/2023 Shift-II

Ans. (a) : Sol.

$x^n - y^n = (x - y)\,(x^{n-1} + x^{n-2}y + x^{n-3}y^2 + ... + y^{n-1})$

$(25^{190} - 19^{190}) - (8^{190} - 2^{190})$

$(25-19)\,k_1 - (8-2)k_2$

$6k_1 - 6k_2$

$6(k_1 - k_2)$

div by 2 & 3 both

$(25^{190} - 8^{190}) - (19^{190} - 2^{190})$

$(25 - 8)\,a - (19 - 2)b$

$17a - 17b = 17(a - b)$ div by 17

$(25^{190} + 2^{190}) - (19^{190} + 8^{190})$

$\left((25^2)^{95}+(2^2)^{95}\right)-\left((19)^2)^{95}-(8^2)^{95}\right)$

$(628 + 4)\,(x) - (361 + 64)\,(y)$

$629\,x - 425\,y$

$629\,x - 425\,y$

If div by 2 & 17 both ⇒ div by 34

If div by 2 but not div by 7

So, div by 34 but not by 14

9. Let the number $(22)^{2022} + (2022)^{22}$ leave the remainder α when divided by 3 and β when divided by 7. Then $(\alpha^2 + \beta^2)$ is equal to

(a) 13 (b) 20
(c) 10 (d) 5

JEE Mains 10/04/2023 Shift-II

Ans. (d) : Given,

$(22)^{2022} + (2022)^{22}$

Divided by 3

$(21 + 1)^{2022} + (2022)^{22}$

$= 3k + 1$

$(\alpha = 1)$

Divided by 7

$(21 + 1)^{2022} + (2023 - 1)^{22}$

$7k + 1 + 1 \quad (\beta = 2)$

$7k + 2$

So $\alpha^2 + \beta^2 \Rightarrow 5$

10. The total number of four digit numbers such that each of the first three digits is divisible by the last digit, is equal to_____.

JEE Main-29.06.2022, Shift-II

Ans. (1086) : : Let a, b, c, d is four digit number so the first three digits a, b, c divisible by d.

If the d = 1, 2, 3, 4

	No. of such numbers
d = 1	$9 \times 10 \times 10 = 900$
d = 2	$4 \times 5 \times 5 = 100$
d =3	$3 \times 4 \times 4 = 48$
d = 4	$2 \times 3 \times 3 = 18$
d = 5	$1 \times 2 \times 2 = 4$
d = 6	$4 \times 4 = 16$

So, total 4 digit numbers = 900 + 100 + 48 + 18 + 4 + 16 = 1086

11. The number of 3 digit numbers, that are divisible by either 3 or 4 but not divisible by 48, is

(a) 400 (b) 472
(c) 432 (d) 507

JEE Main-29.01.2023, Shift-II

Ans. (c) : Total number of three digit = 900

Divisible by 3 = 300 $\left(\because \frac{900}{3} = 300\right)$

No. divisible by 12 = 75

No. divisible by 4 = $\frac{900}{4}$ = 225

Number divisible by either 3 or 4

= 300 + 225 – 75 = 450

We have to remove divisible by 48 = 18

Required number of numbers = 450 – 18 = 432

12. Number of 4-digit numbers that are less than or equal to 2800 and either divisible by 3 or by 11, is equal to :

JEE Main-31.01.2023, Shift-I

Ans. (710) : Lower four digit number = 1000

Higher four digit number = 2799

Which is divisible by = 3

$T_n = a + (n - 1) d$

$2799 = 1002 + (n - 1) 3$

$(n - 1) 3 = 1797$

$3n - 3 = 1797$

$3n = 1800$

$n = 600$

Divisible by 11,

$1 \text{ to } 2799 \rightarrow \left[\frac{2799}{11}\right] = 254$

$1 \text{ to } 999 \rightarrow \left[\frac{999}{11}\right] = 90$

So, the numbers = 254 – 90 = 164

Divisible by 33

1 to 2799 = 84

1 to 999 = 30

1000 to 2799 = 54

$\therefore \quad n(3) + n(11) - n(33)$

$= 600 + 164 - 54 = 710$

13. Let the number $(22)^{2022} + (2022)^{22}$ leave the remainder α when divided by 3 and β when divided by 7. Then $(\alpha^2 + \beta^2)$ is equal to

(a) 10 (b) 5
(c) 20 (d) 13

JEE Main-10.04.2023, Shift-II

Ans. (b) : $(22)^{2022} + (2022)^{22}$

For α

Divided by 3

$(21 + 1)^{2022} + (2022)^{22} = 3k + 1$

$\alpha = 1$

And for β divided by 7

$(21 + 1)^{2022} + (2023 - 1)^{22} = 7k + 1 + 1 = 7k + 2$

$\beta = 2$

Hence, α = 1 and β = 2

Therefore, $\alpha^2 + \beta^2 = 1^2 + 2^2 = 1 + 4 = 5$

14. A natural number has prime factorisation given by $n = 2^x 3^y 5^z$ where y and z are such that $y + z = 5$ and $y^{-1} + z^{-1} = \frac{5}{6}$, y > z. Then, the number of odd divisors of n, including 1, is

(a) 11 (b) 6
(c) 6x (d) 12

JEE Main 26.02.2021, Shift,- I

Ans. (d) : Given,

$n = 2^x 3^y 5^z$

$y + z = 5$

$\frac{1}{y} + \frac{1}{z} = \frac{5}{6}$

$\Rightarrow \quad \frac{z + y}{yz} = \frac{5}{6}$

$\Rightarrow \quad \frac{5}{yz} = \frac{5}{6}$

$\Rightarrow \quad yz = 6$

$\because (y-z)^2 = (y+z)^2 - 4yz$

$\therefore \quad (y-z)^2 = 25 - 4 \times 6$

$\Rightarrow \quad (y-z)^2 = 25 - 24$

$\Rightarrow \quad (y-z) = \pm 1$

Also, $\quad y + z = 5 \quad ....(i)$

and, $\quad y - z = \pm 1 \quad ....(ii)$

From equation (i) & (ii) we get

$y = 3$ or 2

$z = 2$ or 3

$n = 2^x . 3^y . 3^z$

$\Rightarrow \quad n = 2^0 . 3^2 . 5^3$

Total odd divisors = (3 + 1) (2 + 1) = 12

15. If $(2021)^{3762}$ is divided by 17, then the remainder is ....

JEE Main 17.03.2021, Shift - I

Ans. (4) : Given,

$(2021)^{3762} = (2023 - 2)^{3762}$

$= (-2 + 2023)^{3762}$

$\sum_{r=0}^{3762} {}^{3762}C_r (-2)^{3762} (2023)^r$

Therefore,

${}^{3762}C_0 (-2)^{3762} (2023)^{0 + 17\lambda}$

Here, $\lambda \in 1$

$= 17\lambda + 2^2 (2^4)^{940} = 17\lambda + 4 (16)^{940}$

$= 17\lambda + 4 (17 - 1)^{940}$

Now,

$= 17\lambda + 4(17\mu + 1) = 17\lambda + 4$

Hence, the remainder of $(2021)^{3762}$ is 4.

16. $3 \times 7^{22} + 2 \times 10^{22} - 44$ when divided by 18 leaves the remainder

JEE Main 27.08.2021, Shift - II

Ans. (15) : Given,

$3 \times 7^{22} + 2 \times 10^{22} - 44$

$\Rightarrow \quad 3 \times (6 + 1)^{22} + 2 \times (9 + 1)^{22} - 44$

Now, binomial expansion

$\Rightarrow 3[{}^{22}C_0 + {}^{22}C_1 (6) + {}^{22}C_2 (6)^2 + ... + {}^{22}C_{22}(6)^{22}] + 2[{}^{22}C_0 + {}^{22}C_1 (9) + ...+ {}^{22}C_{22} (9)^{22}] - 44$

$\Rightarrow 3.{}^{22}C_0 + 18 [{}^{22}C_1 + {}^{22}C_2 (6) + ...+ {}^{22}C_{22}(6)^{21}] + 2.{}^{22}C_0 + 18 [{}^{22}C_1 + {}^{22}C_2 (9) + ...+ {}^{22}C_{22} (9)^{21}] - 44$

$\Rightarrow \quad 18A + 3 + 18B + 2 - 44$

$= 18 (A + B) - 39$

$= 18 C - 3 \times 18 + 15 \quad \because C = A + B$

$= 18 (C - 3) + 15 \quad \because C - 3 = \lambda$

$= 18\lambda + 15$

Hence, the remainder is 15.

17. The remainder when $(2021)^{2023}$ is divided by 7 is :

(a) 1 (b) 2
(c) 5 (d) 6

JEE Main-26.06.2022, Shift-I

Ans. (c): Given, $(2021)^{2023}$

$= (7 \times 288 + 5)^{2023}$

Here, 7×288 goes to 0 because 288 is a multiple of 7.

So, $\quad 5^{2023}$

$= (7 - 2)^{2023}$

$= (-2)^{2023}$

$= -1 \times 2^1 (2^3)^{674}$

$= -1 \times 2 (7 + 1)^{674}$

$= -2(1 + 7)^{674}$

$= -2 + 7$

$= 5.$

18. The remainder when 3^{2022} is divided by 5 is

(a) 1 (b) 2
(c) 3 (d) 4

JEE Main-24.06.2022, Shift-I

Ans. (d) : Given, 3^{2022}

$= (3^2)^{1011}$

$= (9)^{1011}$

$= (10 - 1)^{1011}$

$= {}^{1011}C_0 . 10^{1011} - {}^{1011}C_1 . 10^{1010} + ...+ {}^{1011}C_{1010} 10^1 - {}^{1011}C_{1011}$

$= 10k - 1$, where k = integer

$= 10k - 1 - 4 + 4$

$= 10 k - 5 + 4$

$= 5 (2 k - 1) + 4$

So, when it is divided by 5, remainder will be '4'

19. If $(20)^{19} + 2(21) (20)^{18} + 3(21)^2 (20)^{17} + ...+ 20(21)^{19} = k(20)^{19}$, then k is equal to ____:

JEE Mains 06/04/2023 Shift-II

Ans. (400) :

If $(20)^{19} + 2(21) (20)^{18} + 3(21)^2 (20)^{17} + ...+ (20)(21)^{19} = k(20)^{19}$ then k

$$20^{19}\left(1 + 2.\left(\frac{21}{20}\right) + 3\left(\frac{21}{20}\right)^2 + ... + 20\left(\frac{21}{20}\right)^{19}\right) = k(20)^{19}$$

$$k = 1 + 2\left(\frac{21}{20}\right) + 3\left(\frac{21}{20}\right)^2 + ... + 20\left(\frac{21}{20}\right)^{19} \quad ....(i)$$

$$k\left(\frac{21}{20}\right) = \left(\frac{21}{20}\right) + 2\left(\frac{21}{20}\right)^2 + ... + 19\left(\frac{21}{20}\right)^{19} + 20\left(\frac{21}{20}\right)^{20} \quad ...(ii)$$

On subtracting equation (ii) from (i), we get –

$$k\left(\frac{-1}{20}\right) = 1 + \frac{21}{20} + \left(\frac{21}{20}\right)^2 + ... + \left(\frac{21}{20}\right)^{19} - 20\left(\frac{21}{20}\right)^{20}$$

$$k\left(\frac{-1}{20}\right) = \frac{1\left(\left(\frac{21}{20}\right)^{20} - 1\right)}{\left(\frac{21}{20} - 1\right)} - 20\left(\frac{21}{20}\right)^{20}$$

$$k\left(\frac{-1}{20}\right) = 20\left(\frac{21}{20}\right)^{20} - 20 - 20\left(\frac{21}{20}\right)^{20}$$

$$k\left(\frac{-1}{20}\right) = -20$$

$-k = -20 \times 20$

$k = 400$

02.

Set, Relation and Function

Formula

- **Laws of Algebra of sets (Properties of sets):**
- **Commutative law:** $(A \cup B) = B \cup A$; $A \cap B = B \cap A$
- **Associative law:** $(A \cup B) \cup C = A \cup (B \cup C)$; $(A \cap B) \cap C = A \cap (B \cap C)$
- **Distributive law:**
  $A \cap (B \cap C) = (A \cup B) \cap (A \cup C)$; $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
- **De-morgan law:** $(A \cup B)' = A' \cap B'$; $(A \cap B)' = A' \cup B'$
- **Identity law:** $A \cap U = A$; $A \cup \phi = A$
- **Complement law:** $A \cup A' = U$, $A \cap A' = \phi$, $(A')' = A$
- **Idempotent law:** $A \cap A = A$, $A \cup A = A$

❖ **Some Important results on number of elements in sets:**

If A, B C are finite sets and U be the finite universal set then

- $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
- $n(A - B) = n(A) - n(A \cap B)$
- $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$
- Number of elements in exactly two of the sets A, B, C $= n(A \cap B) + n(B \cap C) + n(C \cap A) - 3n(A \cap B \cap C)$
- Number of elements in exactly one of the sets A, B, C $= n(A) + n(B) + n(C) - 2n(A \cap B) - 2n(B \cap C) - 2n(A \cap C) + 3n(A \cap B \cap C)$

■ **Types of relations:**

In this section we intend to define various types of relations on a given set A.

- **Void relation:** Let A be a set. Then $\phi \subseteq A \times A$ and so it is a relation on A. This relation is called the void or empty relation on A.
- **Universal relation:** Let A be a set. Then $A \times A \subseteq A \times A$ and so it is a relation on A. This relation is called the universal relation on A.
- **Identity relation:** Let A be a set. Then the relation $I_A = \{(a, a): a \in A\}$ on A is called the identity relation on A. In other words, a relation I_A. on A is called the identity relation if every element of A is related to itself only.
- **Reflexive relation:** A relation R on a set A is said to be reflexive if every element of A is related to itself. Thus, R on a set A is not reflexive if there exists an element $a \in A$ such that $(a, a) \notin R$.

☛ **Note:** Every identity relation is reflexive but every reflexive relation in not identity.

- **Symmetric relation:** A relation R on a set A is said to be a symmetric relation
  iff $(a, b) \in R \Rightarrow (b, a) \in R$ for all $a, b \in A$. i.e. $a R b \Rightarrow b R a$ for all $a, b \in A$.
- **Transitive relation:** Let A be any set. A relation R on A is said to be a transitive relation
  iff $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R$ for all $a, b, c \in A$
  i.e. $a R b$ and $b R c \Rightarrow a R c$ for all $a, b, c \in A$
- **Equivalence relation:** A relation R on a set A is said to be an equivalence relation on A iff
  - ➲ it is reflexive i.e. $(a, a) \in R$ for all $a \in A$
  - ➲ it is symmetric i.e. $(a, b) \in R \Rightarrow (b, a) \in R$ for all $a, b \in A$
  - ➲ it is transitive i.e. $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R$ for all a, b and $c \in A$

Type I — Set, Operation on Set and Venn Diagram

1. The number of elements in the set
$S \ \{(x,y,z): x,y,z \ \ Z, x \ \ 2y \ \ 3z \ \ 42, x,y,z \ \ 0\}$
equals________ .

JEE Mains 01/02/2024 Shift-I

Ans.(169) : We have,

$x + 2y + 3z = 42, \quad x, y, z \geq 0$

$\Rightarrow \quad x + 2y = 42 - 3z$

There are following cases-

1) $z = 0$ $\quad x + 2y = 42 \rightarrow 22$ case
2) $z = 1$ $\quad x + 2y = 39 \rightarrow 20$ case
3) $z = 2$ $\quad x + 2y = 36 \rightarrow 19$ case
4) $z = 3$ $\quad x + 2y = 33 \rightarrow 17$ case
5) $z = 4$ $\quad x + 2y = 30 \rightarrow 16$ case
6) $z = 5$ $\quad x + 2y = 27 \rightarrow 14$ case
7) $z = 6$ $\quad x + 2y = 24 \rightarrow 13$ case
8) $z = 7$ $\quad x + 2y = 21 \rightarrow 11$ case
9) $z = 8$ $\quad x + 2y = 18 \rightarrow 10$ case
10) $z = 9x + 2y = 15 \rightarrow 8$ case
11) $z = 10$ $\quad x + 2y = 12 \rightarrow 7$ case
12) $z = 11$ $\quad x + 2y = 9 \rightarrow 5$ case
13) $z = 12$ $\quad x + 2y = 6 \rightarrow 4$ case
14) $z = 13$ $\quad x + 2y = 3 \rightarrow 2$ case
15) $z = 14$ $\quad x + 2y = 0 \rightarrow 1$ case

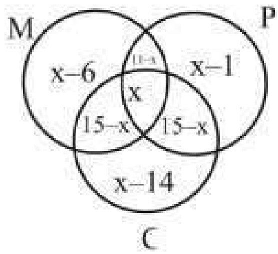
Therefore the number of elements in the set = 169.

2. **A group of 40 students appeared in an examination of 3 subjects – Mathematics, Physics and Chemistry. It was found that all students passed in atleast one of the subjects, 20 students passed in Mathematics, 25 students passed in Physics, 16 students passed in Chemistry, atmost 11 students passed in both Mathematics and Physics, atmost 15 students passed in both Physics and Chemistry, atmost 15 students passed in both Mathematics and Chemistry. The maximum number of students passed in all the three subjects is _______.**

JEE Mains 30/01/2024 Shift-I

Ans.(10) : According to question,

n(M) = 20, n(P) =25, n(C) = 16



11 x 0 15–x ≥ 0

x 11 x ≤15

x = number of student pass in all 3 subjects.

Max(x) = 11 it is not possible

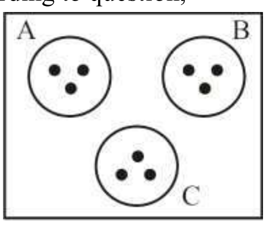
Max (x) = 10

3. **Let the set S = {2, 4, 8, 16, ...,512} be partitioned into 3 sets A, B, C with equal number of elements such that A ∪ B ∪ C = S and A ∩ B = B ∩ C = A ∩ C = ϕ. The maximum number of such possible partitions of S is equal to:**

(a) 1680 (b) 1520
(c) 1710 (d) 1640

JEE Mains 05/04/2024 Shift-II

Ans. (a) : According to question,



$S=\{2,2^2,2^3.........2^9\}$

n (S) = 9

Maximum number of possible partition of S

$= {}^9C_3 \times {}^6C_3 \times {}^3C_3$

$= \frac{9!}{3!6!} \times \frac{6!}{3!3!} \times 1$

$= \frac{9\times8\times7}{3\times2} \times \frac{6\times5\times4}{3\times2} \times 1$

= 84×20

=1680

4. **Let S be the set of positive integral values of a for which $\frac{ax^2+2(a+1)x+9a+4}{x^2-8x+32} < 0, \forall x \in \mathbb{R}$. Then, the number of elements in S is:**

(a) ∞ (b) 3
(c) 0 (d) 1

JEE Mains 31/01/2024 Shift-I

Ans. (c) : We have,

$\frac{ax^2+2(a+1)x+9a+4}{x^2-8x+32} < 0$

D = 64 – 4 × 32 < 0

& a = 1 > 0

$\therefore x^2-8x+32 > 0 \forall x \in R$

$ax^2+2(a+1)x+9a+4 < 0 \forall x \in R$

Only possible when,

a < 0 & D < 0

But we need positive integral value of a.

So,

$|S| = 0$

5. **Let A and B be two finite sets with m and n elements respectively. The total number of subsets of the set A is 56 more than the total number of subsets of B. Then the distance of the point P (m,n) from the point Q (–2,–3) is.**

(a) 10 (b) 6
(c) 4 (d) 8

JEE Mains 27/01/2024 Shift-II

Ans. (a) : A and B be two finite sets with m and n elements.

Given, $2^m = 2^n+56$

$2^m-2^n = 56$

$2^n(2^{m-n}-1) = 56 = 2^3\times7$

n = 3 $2^{m-n} = 8$

m = 6

∴ P(m, n) = P(6, 3) and Q(–2, –3)

Distance between P and Q are-

$PQ = \sqrt{64+36} = 10$

6. **If S = {a ∈ R : |2a – 1| = 3[a] + 2 {a}}, where [t] denotes the greatest integer less than or equal to t and {t} represent the fractional part of t, then $72\sum_{a\in S} a$ is equal to ________.**

JEE Mains 05/04/2024 Shift-I

Ans. (18) : Given,

S = {a ∈ R : |2a – 1| = 3[a] + 2 {a}}

|2a – 1| = 3[a] + 2{a}

|2a – 1| = [a] + 2a

Case-1: $a > \frac{1}{2}$

2a – 1 = [a] + 2a

[a] = –1 ∴ a ∈ [–1, 0) Reject

Case -2 $a < \frac{1}{2}$

$-2a + 1 = [a] + 2a$

$-2a + 1 = 0 + 2a$

$4a = 1$

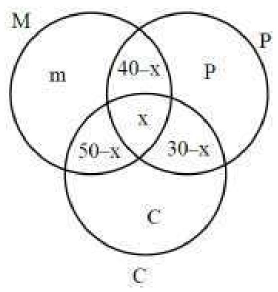
Hence, $a = \frac{1}{4}$

Therefore, $72\sum_{a\in S} a = 72 \times \frac{1}{4} = 18$

7. In a survey of 220 students of a higher secondary school, it was found that at least 125 and at most 130 students studied Mathematics; at least 85 and at most 95 studied Physics; at least 75 and at most 90 studied Chemistry; 30 studied both Physics and Chemistry; 50 studied both Chemistry and Mathematics; 40 studied both Mathematics and Physics and 10 studied none of these subjects. Let m and n respectively be the least and the most number of students who studied all the three subjects. Then m + n is equal to _____

JEE Mains 04/04/2024 Shift-I

Ans. 45



$n(M\cup P\cup C) = 220 - 10 = 210$

$n(M) \in [125, 130]$

$n(P) \in [85, 95]$

$n(C) \in [75, 90]$

$125 \le m + 90 - x \le 130$ ...(i)

$85 \le P + 70 - x \le 95$ ...(ii)

$75 \le C + 80 - x \le 90$ ...(iii)

Also, $m + P + C + 120 - 2x = 210$

$15 \le x \le 45$ & $30 - x \ge 0$

$15 \le x \le 30$

$30 + 15 = 45$

8. The number of elements in the set $\{n \in Z : |n^2 - 10n + 19| < 6\}$ is ______.

JEE Mains 10/04/2023 Shift-I

Ans. (6) : We have,

$-6 < n^2 - 10n + 19 < 6$

$\Rightarrow n^2 - 10n + 25 > 0$

$\Rightarrow (n-5)^2 > 0$

$\Rightarrow n \in Z - \{5\}$ ... (i)

and $n^2 - 10n + 13 < 0$

$\Rightarrow 5 - 2\sqrt{3} < n < 5 + 2\sqrt{3}$

$\Rightarrow 1.6 < n < 8.4$

$\Rightarrow n = \{2, 3, 4, 5, 6, 7, 8\}$ ... (ii)

From (i) $\cap$ (ii)

$N = \{2, 3, 4, 6, 7, 8\}$

So, Number of elements in the set = 6

9. The number of elements in the set $\{n \in N : 10 \le n \le 100$ and $3^n - 3$ is a multiple of 7$\}$ is _____

JEE Mains 15/04/2023 Shift-I

Ans. (15) :

$n \in [10, 100]$

$3^n - 3$ is multiple of 7

$3^n = 7\lambda + 3$

$n = 1, 7, 13, 20, \ldots..97$

Number of possible values of n = 15

10. Let $A = \{x \in \mathbb{R} : [x + 3] + [x + 4] \le 3\}$,

$B = \left\{x \in \mathbb{R} : 3^x\left(\sum_{r=1}^{\infty}\frac{3}{10^r}\right)^{x-3} < 3^{-3x}\right\}$, where [t] denotes greatest integer function. Then,

(a) $A \subset B, A \ne B$

(b) $A \cap B = \phi$

(c) $A = B$

(d) $B \subset C, A \ne B$

JEE Mains 06/04/2023 Shift-I

Ans. (c) :

$A = \{x \in \mathbb{R} : [x + 3] + [x + 4] \le 3\}$,

$[x] + 3 + [x] + 4 \le 3$

$2[x] + 7 \le 3$

$2[x] \le -4$

$[x] \le -1$

$A \Rightarrow x \in (-\infty, -1)$

$$B = \left\{x \in R : 3^x\left(\sum_{r=1}^{\infty}\frac{3}{10^r}\right)^{x-3} < 3^{-3x}\right\}$$

$$3^x\left(\sum_{r=1}^{\infty}\frac{3}{10^r}\right)^{x-3} < 3^{-3x} \quad ...(i)$$

$$\sum_{r=1}^{\infty}\frac{3}{10^r} = \frac{3}{10} + \frac{3}{10^2} + \frac{3}{10^3} + ... + \frac{3}{10^{\infty}}$$

$$= \frac{3}{10}\left(1 + \frac{1}{10} + \frac{1}{10^2} + ...\right)$$

$$= \frac{3}{10}\left(\frac{1}{1 - \frac{1}{10}}\right) = \frac{3}{10} \times \frac{10}{9} = \frac{1}{3}$$

From equation (i)

$$3^x\left(\sum_{r=1}^{\infty}\frac{3}{10^r}\right)^{x-3} < 3^{-3x}$$

$$3^x\left(\frac{1}{3}\right)^{x-3} < 3^{-3x}$$

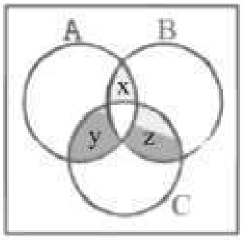
$(3)^{x-x+3} < 3^{-3x}$
$3^3 < 3^{-3x}$
$3 < -3x$
$x < -1$
$B \Rightarrow x < -1$
Hence, A = B

11. An organization awarded 48 medals in event 'A', 25 in event 'B' and 18 in event 'C'. If these medals went to total 60 men and only five men got medals in all the three events, then how many received medals in exactly two of three events?

(a) 15 (b) 9
(c) 21 (d) 10

JEE Mains 11/04/2023 Shift-I

Ans. (c) : Given,
n (A) = 48
n (B) = 25
n (C) = 18
$n(A\cup B\cup C) = 60$ [Total]
$n(A\cap B\cap C) = 5$



$n(A \cup B\cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$
$60 = 48 + 25 + 18 - (x+5) - (z+5) - (y+5) + 5$
$x + y + z = 21$

12. Let S= $\{1,2,3,5,7,10,11\}$. The number of non-empty subsets of S that have the sum of all elements a multiple of 3, is _____.

JEE Mains 25/01/2023 Shift-I

Ans. (43) : Elements of the type 3k = 3
Elements of the type 3k + 1 = 1, 7, 9
Element of the type 3k + 2 = 2, 5, 11
Subsets containing one element $S_1 = 1$
Subsets containing two elements
$S_2 = {}^3C_1 \times {}^3C_1 = 9$
Subsets containing three elements
$S_3 = {}^3C_1 \times {}^3C_1 + 1 + 1 = 11$
Subsets containing four elements
$S_4 = {}^3C_3 + {}^3C_3 + {}^3C_2 \times {}^3C_2 = 11$
Subsets containing five elements
$S_5 = {}^3C_2 \times {}^3C_2 \times 1 = 9$
Subsets containing six elements $S_6 = 1$
Subsets containing seven elements $S_7 = 1$
$\Rightarrow$ sum = 43

13. Let A = {1, 2, 3, 4, 5, 6, 7} and B = {3, 6, 7, 9}. Then the number of elements in the set $\{C \subseteq A : C \cap B \neq \phi\}$ is _______.

JEE Main-26.07.2022, Shift-II

Ans. (112) : Given,
A = {1, 2, 3, 4, 5, 6, 7} and B = {3, 6, 7, 9}.
$\because$ The number of subset = 2^n
Then, number of subset A = 2^7
= 128
$C \cap B = \phi$ when set C contains the elements
C = {1, 2, 4, 5}
$S = \{C \subseteq A ; C\cap B \neq \phi\}$
$= \text{Total} - (C \cap B = \phi)$
$= 128 - 2^4 = 128 - 16 = 112$

14. The number of elements in the set $\{n \in Z : |n^2 - 10n + 19| < 6\}$' is ________.

JEE Main-10.04.2023, Shift-I

Ans. (6) : Given,
$n\in Z: |n^2 - 10n + 19| < 6$
$\Rightarrow \left|(n-5)^2 - 6\right| < 6$
$\Rightarrow -6 < (n-5)^2 - 6 < 6$
$0 < (n-5)^2 < 12$
$\Rightarrow (n-5)^2 = 1,4,9$
$\Rightarrow n - 5 = \pm1, \pm2, \pm3$
So, the number of elements in the set is 6.

15. A survey shows that 63% of the Indians like tea whereas 76% like coffee. If x% of the Indians like both tea and coffee, then

(a) x = 39 (b) x = 63
(c) $39 \le x \le 63$ (d) none of these

JEE Main 04.09.2020 Shift-I

Ans. (c) : Given, number of the Indians like tea –
n(T) = 63
Number of the Indians like coffee
n(C) = 76
And number of the Indians like both tea and coffee
$n(T \cap C) = x$
Then, $n(T \cup C) = n(T) + n(C) - n(T \cap C)$
$100 = 63 + 76 - x$
$x = 139 - 100$
$x = 39$
Also, $n(T \cap C) \le n(T)$
$x \le 63$
So, $39 \le x \le 63$

16. Let A = {n ∈ N : H.C.F. (n, 45) = 1} and Let B = {2k : k ∈{1, 2,....,100}}. Then the sum of all the elements of A ∩ B is_____.

JEE Main-26.06.2022, Shift-I

Ans. (5264) : Given, A = {n∈N : H. C. F. (n, 45) = 1}
And, B = {2k : k∈ {1, 2, 3 …….100}
Since, $45 = 3^2 \times 5$
Then, A must be a set that does not consist of either 3 multiples or 5 multiples.

$\Rightarrow$ A = {1, 2, 4, 7, 8, 11, 13 ……}
And, B = {2, 4, 6, …….200}
So, $A \cap B$ = {1, 2, 4, 7, 8, 11, 13, 14, ……} $\cap$ {2, 4, 6, 8,……..,200}
$\Rightarrow$ $A \cap B$ = {2, 4, 8, 14, ……,200}
Since, find the sum of the element in $A \cap B$.
Then,
$\Rightarrow$ [2 + 4 + 8 + 14 + ………+ 200]
$\Rightarrow$ 2 [1 + 2 + 4 + 7+ ……….+ 100]
$\Rightarrow$ 2 [sum of the natural number up to 100 – sum of multiples (3, 5)]

$$\Rightarrow 2\left[\frac{100\times101}{2}-\frac{3(33\times34)}{2}-\frac{5\times20\times21}{2}+\frac{15\times6\times7}{2}\right]$$

$\Rightarrow$ 2[5050 – 3 (561) – 5(210) + 15 × 21]
$\Rightarrow$ 2 [5050 – 1683 – 1050 + 315]
$\Rightarrow$ 2 × 2632 = 5264.

Type II Cartesian Product of Sets

17. Let the set $C = \{(x,y) \mid x^2 - 2^y = 2023, x, y \in N\}$. Then $\sum_{(x,y)\in C}(x+y)$ is equal to _____.

JEE Mains 29/01/2024 Shift-II

Ans. : (46) We have-
$x^2 \quad 2^y \quad 2023$
Let, x = 45 and y = 1,which is satisfying the given equation.
$45^2 = 2025$
$45^2 - 2^1 = 2023$
$\Rightarrow x = 45, y = 1$
So,
$\sum_{(x,y)\ C}(x \quad y) \quad 46.$

18. Let [α] denote the greatest integer ≤ α. Then $[\sqrt{1}]+[\sqrt{2}]+[\sqrt{3}]+...[\sqrt{120}]$ is equal to _____.

JEE Mains 13/04/2023 Shift-II

Ans. (825) : $S = [\sqrt{1}]+[\sqrt{2}]+[\sqrt{3}]+....+[\sqrt{120}]$

$[\sqrt{1}] = [1] = 1$

$[\sqrt{2}] = [1.414] = 1$

$[\sqrt{3}] = [1.732] = 1$

$[\sqrt{1}] \to [\sqrt{3}] = 1\times3$

$[\sqrt{4}] \to [\sqrt{8}] = 2\times5$

$[\sqrt{9}] \to [\sqrt{15}] = 3\times7$

$[\sqrt{100}] \to [\sqrt{120}] = 10\times21$

$S = 1\times3+2\times5+3\times7+...+10\times21$

$= \sum_{r=1}^{10} r(2r+1)$

$= 2\sum_{r=1}^{10} r^2 + \sum_{r=1}^{10} r$

$= \frac{2\times10\times11\times21}{6} + \frac{10\times11}{2}$

= 770 + 55
= 825

19. Let the number of elements in sets A and B be five and two respectively. Then the number of subsets of A × B each having at least 3 and at most 6 elements is:
(a) 752 (b) 772
(c) 782 (d) 792

JEE Mains 08/04/2023 Shift-I

Ans. (d) : Given,
Number of elements in set A, n (A)= 5
For set n(B) = 2
n(A × B) = 10
No of ways of selection of r things out of n things $= {}^{n}C_{r}$
$= {}^{10}C_3 + {}^{10}C_4 + {}^{10}C_5 + {}^{10}C_6 = 792$

20. Let the number of elements in sets A and B five and two respectively. Then the number of subsets of A × B each having at least 3 and at most 6 element is:
(a) 792 (b) 752 (c) 782 (d) 772

JEE Main-08.04.2023, Shift-I

Ans. (a) : Number of element in set A = 5
And no. of element in set B = 2
The no. of element in ordered pair A × B = 2 × 5 = 10
n (A + B) = 10
Then, The number of subsets of A × B each having at least 3 and at most 6 elements is-
$= {}^{10}C_3 + {}^{10}C_4 + {}^{10}C_5 + {}^{10}C_6$
We know that,
$${}^{n}c_{r} = \frac{n!}{r!(n-r)!}$$
$$= \frac{10!}{3!\times7!}+\frac{10!}{4!\times6!}+\frac{10!}{5!\times5!}+\frac{10!}{6!\times4!}$$
$$= \frac{10\times9\times8\times7!}{3\times2\times1\times7!}+\frac{10\times9\times8\times7\times6!}{4\times3\times2\times1\times6!}$$
$$+\frac{10\times9\times8\times7\times6\times5!}{5\times4\times3\times2\times1\times5!}+\frac{10\times9\times8\times7\times6!}{4\times3\times2\times1\times6!}$$
=120 + 210 + 252 + 210 = 792

21. Let A = {2, 3, 4} and B = {8, 9, 12}. Then the number of elements in the relation R = {((a_1, b_1), (a_2, b_2)) ∈ (A × B, A × B) : a_1 divides b_2 and a_2 divides b_1} is :
(a) 36 (b) 12 (c) 18 (d) 24

JEE Main-10.04.2023, Shift-II

Ans. (a) : Let A = { 2, 3, 4}
And, B = {8, 9, 12}

A	B
2	8
3	9
4	12

a_1 divides b_2 and a_2 divides b_1 each element has 2 choice
3 × 2 = 6 and 3 × 2 = 6
Now total number of elements = 6 × 6 = 36 .

Type III — Types of Relation and its Counting

22. The number of symmetric relations defined on the set {1, 2, 3, 4} which are not reflexive is ______.

JEE Mains 30/01/2024 Shift-II

Ans. : (960) We know that,
Total number of relation which reflexive and symmetric both = $2^{\frac{n^2-n}{2}}$
Total number of relation which symmetric = $2^{\frac{n^2+n}{2}}$
Number of relation which are not reflexive
$= 2^{\frac{n^2+n}{2}} - 2^{\frac{n^2-n}{2}}$
$\therefore$ n = 4
$= 2^{\frac{16+4}{2}} - 2^{\frac{16-4}{2}}$
$= 2^{10} - 2^{6}$
$= 2^{6}(16-1)$
= 64 ×15 = 960

23. Let A = {1,2,3,….,20}. Let R_1 and R_2 be the two relation on A such that
R_1 {(a,b) : b is divisible by a}
R_2 {(a,b) : a is an integral multiple of b}.
Then, number of elements in $R_1 - R_2$ is equal to_______.

JEE Mains 01/02/2024 Shift-I

Ans. (46) : We have,
A = {1, 2, 3,….20}
R_1 = {(a, b) : b is divisible by a}
R_1 = {(1,1), (1,2)…(1, 20), (2,2), (2,4)…(2,20) (3,3), (3,6) ….(3, 18), (4,4), (4,8)….(4,20) (5,5), (5,10) (5,15), (5,20), (6,6), (6,12), (6,18) (7,7), (7,14), (8,8), (8,16), (9,9), (9,18), (10,10) (10, 20), (11, 11), (12, 12)……. (20, 20)}
$n(R_1) = 20+10+6+5+4+3+2+2+2+2+1+....+1$
$n(R_1) = 66$
$\because n(R_1 - R_2) = n(R_1) - n(R_1 \cap R_2)$
And n ($R_1 \cap R_2$) = {(1,1), (2,2) (3,3) …. (20,20)}=20
n($R_1 - R_2$) = 66 – 20 = 46

24. Consider the relations R_1 and R_2 defined as aR_1b a^2 b^2 1 for all a, b ∈ R and $(a,b)R_2(c,d)$ a d b c for all $(a,b),(c,d)$ N N. Then
(a) R_1 and R_2 both are equivalence relations
(b) Only R_1 is an equivalence relation
(c) Only R_2 is an equivalence relation
(d) Neither R_1 nor R_2 is an equivalence relation

JEE Mains 01/02/2024 Shift-II

Ans. (c) :
$a R_1 b \Leftrightarrow a^2 + b^2 = 1$ a, b∈R
For Reflexive-
$a R_1 b \Leftrightarrow a^2 + a^2 = 1$
Which is not true ∀ a ∈ R.
Hence R_1 is not reflexive.
Therefore, R_1 is not equivalence relation.
(a, b) R_2 (c, d) ⇒ a + d = b + c
For reflexive:-
(a, b) R_2 (a, b) = a + b = b + a
It's true ∀ (a , b) ∈ N × N
Hence, R_2 is reflexive.
For symmetric-
(a, b), (c, d) ∈ N × N
(a, b) R_2 (c, d) = a + d = b + c
(c, d) R_2 (a, b) = c + d = d + a
∴ a + b = b + c
(a, b) R_2 (c, d) ⇒ (c, d) R_2 (a, b) ∀ (a, b), (c, d) ∈N × N
Hence R_2 is symmetric.
For transitive:-
(a, b), (c, d), (e, f) ∈ N × N
(a, b) R_2 (c, d) ⇒ a + d = b + c
(c, d) R_2 (e, f) ⇒ c + f = d + e
∴ a + b + c + f = b + d + c + e
a + f = b + c
(a, b) R_2 (e, f)
Hence, R_2 is transitive.
Therefore, R_2 is equivalence relation.

25. Let S = {1,2,3,…,10}. Suppose M is the set of all the subsets of S, then the relation R {(A,B) : A B ;A,B M} is:
(a) reflexive only
(b) symmetric and reflexive only
(c) symmetric and transitive only
(d) symmetric only

JEE Mains 27/01/2024 Shift-I

Ans. (d) : Let S = {1, 2, 3, ... 10}
R = {(A, B) : A ∩B ≠ ϕ; A, B ∈M}
For reflexive-
m is subset of 'S'
So, ϕ ∈ m
for ϕ ∩ ϕ = ϕ
but relation is A ∩ B ≠ ϕ
So it is not reflexive.

For symmetric,
ARB = A ∩ B ≠ ϕ
= BRA = A ∩ B ≠ ϕ
So it is symmetric
For transitive
if A = {(1, 2) (2, 3)}
B = {(2, 3) (3, 4)}
C = {(3, 4) (5, 6)}
ARB and BRC but A does not relate to C so it not transitive.

26. Let the relations R_1 and R_2 on the set X = {1, 2, 3, ..., 20} be given by R_1 = {(x, y) : 2x – 3y = 2} and R_2 = {(x, y) : –5x + 4y = 0 }. If M and N be the minimum number of elements required to be added in R_1 and R_2, respectively, in order to make the relations symmetric, then M + N equals
(a) 8 (b) 16 (c) 12 (d) 10

JEE Mains 06/04/2024 Shift-I

Ans. (d) : x = {1, 2, 3, ...20}
R_1 = {(x, y) : 2x – 3y = 2}
R_2 = {(x, y) : –5x + 4y = 0}
R_1 ={(4, 2), (7, 4), (10, 6), (13, 8), (16, 10), (19, 12)}
For symmetry
={(2, 4), (4, 7), (6,10), (8, 13), (10, 16), (12, 19)}
R_2 ={(4, 5), (8, 10), (12, 15), (16, 20)}
For symmetry
R_2 ={(5,4), (10, 8), (15,12), (20, 16)}
in R_1 6 element needed i. e. M = 6
in R_2 4 element needed i. e. N = 4
So, the value of M + N = 6 + 4 = 10 element

27. Let A = {2, 3, 6, 8, 9, 11} and B = {1, 4, 5, 10, 15} Let R be a relation on A × B define by (a, b) R (c, d) if and only if 3ad – 7bc is an even integer. Then the relation R is
(a) reflexive but not symmetric.
(b) transitive but not symmetric.
(c) reflexive and symmetric but not transitive.
(d) an equivalence relation.

JEE Mains 08/04/2024 Shift-II

Ans. (c) : A = {2, 3, 6, 8, 9, 11)
B = {1, 4, 5, 10, 15}
R is defined as (a, b) R (c, d) such that 3ad – 7bc is an even integer.
Reflexive : (a, b) R (a, b)
⇒ 3ab – 7ba = –4ab always even so it is reflexive.
Symmetric : If 3ad – 7bc = Even
Case- I : odd no. odd no.
Case-II : even no. even no.
(c, d) R (a, b) ⇒ 3bc – 3ab
Case-I : odd no. odd no.
Case-II : even no. even no.
so it has symmetric relation on R
Transitive :
(3, 1) R (6, 4)
⇒ 12 – 6 = 6, which is an even integer, satisfying the above relation
(6, 4) R (3, 1)
⇒ 6 – 12 = –6, which is an even integer, satisfying the above relation
but (3, 4) R (3, 1) does not satisfy relation
so it is not transitive.

28. If R is the smallest equivalence relation on the set {1, 2, 3, 4} such that $\{(1, 2),(1, 3)\} \subset$ R, then the number of elements in R is _____
(a) 15 (b) 8
(c) 12 (d) 10

JEE Mains 29/01/2024 Shift-II

Ans. (d) : Given,
set {1, 2, 3, 4}
Smallest equivalence relation ={(1, 1), (2, 2), (3, 3), (4, 4), (3, 1),(2, 1), (2, 3),(3,2), (1, 2), (1, 3)}
Thus, no. of elements = 10

29. Let a relation R on N×N be defined as: (x_1, y_1) $R(x_2, y_2)$ if and only if $x_1 \le x_2$ or $y_1 \le y_2$
Consider the two statements:
(I) R is reflexive but not symmetric.
(II) R is transitive
Then which one of the following is true?
(a) Only (II) is correct
(b) Only (I) is correct
(c) Both (I) and (II) are correct
(d) Neither (I) nor (II) is correct

JEE Mains 04/04/2024 Shift-II

Ans. (b) : All $((x_1, y_1), (x_1, y_1))$ are is R where $x_1, y_1 \in N$ ∴ R is reflexive
((1, 1), (2, 3)) ∈ R but ((2, 3), (1, 1)) ∉ R
∴ R is not symmetric
((2,4), (3,3))∈R and ((3,3), (1,3))∈R but ((2,4), (1,3))∉R
∴ R is not transitive

30. Let A = {1,2,3…….100}. Let R be a relation on A defined by (x,y)∈R if and only if 2x = 3y. Let R_1 be a symmetric relation on A such that R R_1 and the number of elements in R_1 is n. Then, the minimum value of n is_______.

JEE Mains 31/01/2024 Shift-II

Ans. :(66) Given,
A {1,2,...100}
And R= {(x, y): 2x= 3y}
⇒ R {(3,2),(6,4),(9,6)....(99,96)}
⇒ n(R) 33
∵ R R_1 and R_1 be a symmetric relation on A i.e
R_1 contains (y,x) such that 2y = 3x
i.e, R_1 {(3,2),(6,4),(9,6).....(99,66), (2,3),(4,6),(6,9)......(66,99)}
⇒minimum number of elements in R_1 = 66

31. Let A = {1, 2, 3, 4, 5}. Let R be a relation on A defined by xRy if and only if $4x \le 5y$. Let m be the number of element in R and n be the minimum number of elements from A × A that are required to be added to R to make it a symmetric relation. Then m + n is equal to:

(a) 24 (b) 23
(c) 25 (d) 26

JEE Mains 06/04/2024 Shift-II

Ans. (c) : Given: $4x \le 5y$
if x = 1
So, 4 < 5y i.e (1, 1), (1, 2), (1, 3), (1, 4) ,(1, 5)
x = 2, 8 < 5y i.e (2, 2), (2, 3),(2,4), (2, 5)
x = 3, 12 < 5y i.e. (3, 3), (3, 4), (3, 5)
x = 4, 16 < 5y i.e (4, 4), (4, 5)
x = 5, 20 < 5y i.e (5, 4), (5, 5)
Then
R = {(1,1) , (1, 2), (1, 3), (1, 4) ,(1, 5), (2, 2), (2, 3),(2,4) (2, 5), (3, 3), (3, 4), (3, 5), (4, 4), (4, 5), (5, 4), (5, 5)}
i.e. 16 elements.
i.e. m = 16
Now to make R a symmetric relation add
{(2, 1) (3, 2)(4, 3)(3, 1)(4, 2)(5, 3)(4, 1)(5,2)(5,1)}
i.e. n = 9
So m + n = 25

32. Let A = {1, 2, 3, 4} and R = {(1, 2)(2, 3)(1, 4)} be a relation on A. Let S be the equivalence relation on A such the R ⊂ S and the number of elements in S is n. Then, the minimum value of n is ________.

JEE Mains 31/01/2024 Shift-I

Ans. (16) : Given,
A = {1, 2, 3, 4}
R = {(1, 2)(2, 3)(1, 4)}
S is equivalence relation, relation must be reflexive, symmetric & transitive.
For Reflexive,
{(1, 1),(2, 2),(3, 3),(4 ,4)}
For Symmetric,
{(2, 1),(4, 1),(3, 2)}
For transitive,
{(1, 3),(3, 1),(4, 2),(2, 4)}
S = {(1, 1)(2, 2)(3, 3)(4, 4)(1, 2)(2, 1)(2, 3)(3, 2)(1, 4)(4, 1)(1, 3)(3, 1)(2, 4)(4, 2)(4, 3)(3, 4)}
All elements are included,
∴ The number of elements are 16

33. Let R be a relation on Z × Z defined by (a, b) R (c, d) if and only if ad–bc is divisible by 5. Then R is :

(a) Reflexive and transitive but not symmetric
(b) Reflexive, symmetric and transitive
(c) Reflexive and symmetric but not transitive
(d) Reflexive but neither symmetric not transitive

JEE Mains 29/01/2024 Shift-I

Ans. (c): For reflexive:
(a, b) R (a, b)
⇒ ab – ab = 0 is divisible by 5
So, (a, b) R (a,b) ∀ ab ∈ Z
∴ R is reflexive relation.
For symmetric:
(a, b) R (b, c)
If $ac - b^2$ is divisible by 5
Then, $-(b^2 - ac)$ is also divisible by 5.
⇒ (b, c) R (a, b) ∀ a, b, c, d ∈ Z
∴ R is symmetric relation on R.
For transitive:
If (a, b) R (c, d)
⇒ ad – bc divisible by 5 and (c, d) R (e, f)
⇒ cf – de divisible by 5
$ad - bc = 5k_1$ ∵ $k_1, k_2 \in Z$
$cf - de = 5k_2$
∴ $afd - bcf = 5k_1 f$
$bcf - bde = 5k_2 b$
$afd - bde = 5(k_1 f + k_2 b)$
$d(af - be) = 5(k_1 f + k_2 b)$
af – be is not divisible by 5 for every a. b, c, d, e, f ∈ Z.
∴ R is not transitive.
Thus R is reflexive and symmetric but not transitive.
Hence, option (c) is correct.

34. Let A = {2, 3, 6, 7} and B = {4, 5, 6, 8}. Let R be a relation defined on A × B by (a_1, b_1) R (a_2, b_2) such that $a_1 + a_2 = b_1 + b_2$. Then the number of element in R is _________.

JEE Mains 09/04/2024 Shift-I

Ans. (25) : A = {2, 3, 6, 7}
B = {4, 5, 6, 8}
(a_1, b_1) R (a_2, b_2)
$a_1 + a_2 = b_1 + b_2$

1.	(2, 4) R (6, 4)	2. (2, 4) R (7, 5)	
3.	(2, 5) R (7, 4)	4. (3, 4) R (6, 5)	
5.	(3, 5) R (6, 4)	6. (3, 5) R (7, 5)	
7.	(3, 6) R (7, 4)	8. (3, 4) R (7, 6)	× 2
9.	(6, 5) R (7, 8)	10. (6, 8) R (7, 5)	
11.	(7, 8) R (7, 6)	12. (6, 8) R (6, 4)	
13.	(6, 6) R (6, 6)		

Total 24 + 1 = 25

35. Let R be a relation of ℝ, given by $R = \{(a,b) : 3a - 3b + \sqrt{7} \text{ is an irrational number}\}$. Then R is

(a) reflexive and transitive but not symmetric.
(b) an equivalence relation
(c) reflexive but neither symmetric nor transitive
(d) reflexive and symmetric but not transitive

JEE Mains 01/02/2023 Shift-I

Ans. (c) :
$R = \{(a,b) : 3a - 3b + \sqrt{7} \text{ is an irrational number}\}$
Reflexive - let (a, a)∈R
⇒ $3a - 3a + \sqrt{7} = \sqrt{7}$

(a, a) : $\sqrt{7} \in R$ is an irrational number and it is Reflexive over R.

for symmetric-

Let $\left(\frac{\sqrt{7}}{3}, 0\right) \in R$

$\Rightarrow \quad 3\times\frac{\sqrt{7}}{3} - 3\times 0 + \sqrt{7} = 2\sqrt{7} \in Q^c$,i.e. $2\sqrt{7}$ is an irrational no.

but for $\left(0, \frac{\sqrt{7}}{3}\right)$

$3(0) - 3\times\frac{\sqrt{7}}{3} + \sqrt{7} = 0 \notin Q^C$,i.e. not an irrational no.

$\Rightarrow \left(0, \frac{\sqrt{7}}{3}\right) \notin R$

$\therefore$ R is not symmetric.

For transitive -

Let $(0, 3) \in R$ and $\left(3, \frac{\sqrt{7}}{3}\right) \in R$

but $\left(0, \frac{\sqrt{7}}{3}\right) \notin R$

So, R is not transitive .

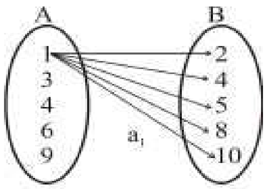
36. Let A = {1, 3, 4, 6, 9} and B = {2, 4, 5, 8, 10}. Let R be a relations defined on A×B such that R = {((a_1, b_1), (a_2,b_2)): $a_1 \le b_2$ and $b_1 \le a_2$}. Then the number of elements in the set R is

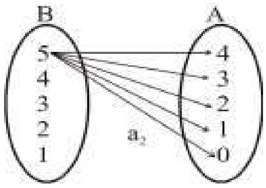
(a) 52 (b) 160

(c) 26 (d) 180

JEE Mains 11/04/2023 Shift-II

Ans. (b) :





Total element = 5 ×5 = 25

Total Subset = 2^{25}

= 5(4+3+2+1+0) = 5 ×10 = 50

= 4× 10 = 40

= 4 ×10 = 40

= 2× 10 = 20

= 1×10 = 10

Total = 160

37. Let A = $\{-4,-3,-2,0,1,3,4\}$ and R = $\{(a,b) \in A\times A : b = |a| \text{ or } b^2 = a+1\}$ be a relation on A. Then the minimum number of elements, that must be added to relation R so that it becomes reflexive and symmetric, is ______.

JEE Mains 13/04/2023 Shift-II

Ans. (7) : A = $\{-4,-3,-2,0,1,3,4\}$ and R = $\{(a,b) \in A\times A : b = |a| \text{ or } b^2 = a+1\}$

$R = [(-4,4),(-3,3),(3,-2),(0,1),(0,0),(1,1),(4,4),(3,3)]$

For reflexive, add $\Rightarrow (-2,-2),(-4,-4),(-3,-3)$

For symmetric, add $\Rightarrow (4,-4),(3,-3),(-2,3),(1,0)$

Total = 3 + 4 = 7

38. Let A = {1, 2, 3, 4} and R be a relation on the set A × A defined by R = {((a, b, (c, d):2a + 3b = 4c + 5d}. Then the number of elements in R is ______

JEE Mains 15/04/2023 Shift-I

Ans. (6) : Given,

A = {1, 2, 3, 4}

R = {(a, b), (c, d)}

2a + 3b = 4c + 5d = α (let)

2a = {2, 4, 6, 8} 4c = {4, 8, 12, 16}

3b = {3, 6, 9, 12} 5d = {5, 10, 15, 20}

$$2a+3b = \begin{Bmatrix} 5,8,11,14 \\ 7,10,13,16 \\ 9,12,15,18 \\ 11,14,17,20 \end{Bmatrix} = 4c+5d = \begin{Bmatrix} 9,14,19,24 \\ 13,18.... \\ 17,22.... \\ 21,26.... \end{Bmatrix}$$

Possible value of α = 9, 13, 14, 14, 17, 18

Pairs of {(a, b), (c, d)} = 6

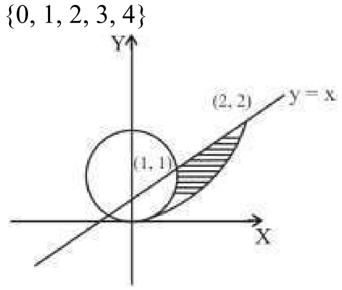
39. Let A = {1, 2, 3, 4, ….., 10} and B = {0, 1, 2, 3, 4}. The number of elements in the relation R = {(a, b) ∈ A × A : 2 (a – b)^2 + 3 (a – b) ∈ B} is ___.

JEE Mains 06/04/2023 Shift-I

Ans. (18) : Given,

A = {1, 2, 3, 4,............10}

and B = {0, 1, 2, 3, 4}



R = {(a, b) ∈ A × A : 2 (a – b)^2 + 3(a – b) ∈ B}

Now 2 (a – b)^2 + 3 (a – b) = (a – b) (2 (a – b) + 3)

⇒ a = b or a – b = –2∈ B

When a = b ⇒ 10 order pairs

Number of order pair, a – b = – 2 ⇒ 8 order pairs

Number of total elements = 18

40. The number of the relations, on the set {1, 2, 3} containing (1, 2) and (2, 3) which are reflexive and transitive but not symmetric, is______.

JEE Mains 12/04/2023 Shift-I

Ans. (3) :

A = {1, 2, 3}

For Reflexive (1, 1) (2, 2) (3, 3) $\in$R

For transitive : (1, 2) and (2, 3) $\in$R $\Rightarrow$ (1, 3) $\in$R

Not symmetric : (2, 1) and (3, 2) $\notin$ R

R_1 = {(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3)}

R_2 = {(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3), (2, 1)}

R_3 = {(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3), (2, 1)}

41. Let A = {0, 3, 4, 6, 7, 8, 9, 10} and R be the relation defined on A such that R = {x, y) $\in$ A × A : x – y is odd positive integer or x – y = 2}. The minimum number of elements that must be added to the relation R, so that it is a symmetric relation, is equal to ____.

JEE Mains 08/04/2023 Shift-I

Ans. (19) : Given,

A = {0, 3, 4, 6, 7, 8, 9, 10}

R = {x–y = odd positive integer x –y = 2}

Here, 3,7,9, are odd number i.e. 3_{C_1}

0,4,6,8,10 are even number so 5_{C_1}

minimum order pair to be added must be = 15 + 4 = 19

R ={(6, 4),(8, 6),(9, 7),(10, 8),(3, 0), (7, 0), (9, 0), (4, 3), (6, 3), (8, 3), (10, 3), (7, 4), (9, 4), (7, 6), (9, 6), (8, 7), (10, 7), (9, 8), (10, 9)}

Hence 19 element should be add in R for making its.

42. Let A = {1, 2, 3, 4, 5, 6, 7}. Then the relation R = {(x, y) $\in$A × A : x + y = 7} is

(a) Symmetric but neither reflexive nor transitive
(b) Transitive but neither symmetric nor reflexive
(c) An equivalence relation
(d) Reflexive but neither symmetric nor transitive

JEE Mains 08/04/2023 Shift-II

Ans. (a) : Sol.

A = {1, 2, 3, 4, 5, 6, 7}

R = {(x, y) $\in$ A×A : x + y = 7}

y = 7 – x

R = {(1, 6) (2, 5) (3, 4) (4, 3) (5, 2) (6, 1)}

Symmetric but neither reflexive nor transitive

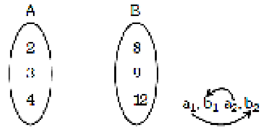
R = {(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)}

43. Let A = {2, 3, 4} and B = {8, 9, 12}. Then the number of elements in the relation R = {((a_1, b_1), (a_2, b_2))$\in$ (A × B, A × B) : a_1 divides b_2 and a_2 divides b_1} is

(a) 18 (b) 24
(c) 12 (d) 36

JEE Mains 10/04/2023 Shift-II

Ans. (d) : Given,



For a_1 divides b_2, each elements has 2 choices

$\Rightarrow$ 3 × 2 = 6

Also, for a_2 divides b_1, each elements has 2 choices

$\Rightarrow$ 3 × 2 = 6

$\therefore$ Number of elements in R = 6 × 6 = 36

44. Among the relations

$$S = \{(a,b) : a,b \in R - \{0\}, 2 + \frac{a}{b} > 0\}$$

$$\text{and } T = \{(a,b) : a,b \in R,\ a^2 - b^2 \in Z\},$$

(a) Both S and T are symmetric
(b) S is transitive but T is not
(c) Neither S nor T is transitive
(d) T is symmetric but S is not

JEE Mains 31/01/2023 Shift-II

Ans. (d) : From 2nd relation T = $a^2 - b^2$ = – I a, b $\in$R

Then, (b, a) on relation T

$\Rightarrow b^2 - a^2$ = – I b, a $\in$R

$\therefore$ T is symmetric

Now, from equation first

$$S = \left\{(a,b) : a,b \in R - \{0\}, 2 + \frac{a}{b} > 0\right\}$$

$$2 + \frac{a}{b} > 0$$

$$\Rightarrow \frac{a}{b} > -2,$$

$$\Rightarrow \frac{b}{a} < \frac{-1}{2}$$

If (b, a) one relation S then

$2 + \frac{b}{a}$ not necessarily positive

$\therefore$ S is not symmetric

45. Let R be a relation on N×N defined by (a, b) R (c, d) if and only if ad (b–c) = bc (a–d). Then R is

(a) transitive but neither reflexive nor symmetric
(b) symmetric but neither reflexive nor transitive
(c) symmetric and transitive but not reflexive
(d) reflexive and symmetric but not transitive

JEE Mains 31/01/2023 Shift-I

Ans. (b) : (a, b) R (c, d) $\Rightarrow$ ad (b – c) = bc (a–d)

Symmetric :

(c, d) R (a, b) $\Rightarrow$ cb (d–a) = da (c– b)

$\Rightarrow$ $bc = (a-b) = ad(b-c)$

Reflexive :

(a, b) R (a, b) $\Rightarrow$ ab (b – a) $\neq$ ba (a – b) $\Rightarrow$ Not reflexive

Transitive : (2, 3) R (3, 2) and (3, 2) R (5, 30) but ((2, 3), (5, 30) $\notin$ R $\Rightarrow$ Not transitive

46. **The minimum number of elements that must be added to the relation R = {(a, b), (b, c)} on the set {a, b, c} so that it becomes symmetric and transitive is:**

(a) 7 (b) 3
(c) 5 (d) 4

JEE Mains 30/01/2023 Shift-I

Ans. (a) : Given relation R = {(a, b), (b, c)}
For symmetric (a, b), (b, c) ∈R
⇒ (b, a), (c, b) ∈R
For transitive, (a, b), (b, c) ∈R
(a, c) ∈R
Now, For symmetric– ∵ (a, c) ∈R ⇒ (c, a) ∈R
And, For transitive– ∵ (a, b), (b, a) ∈R
⇒ (a, a) ∈R
And, (b, c), (c, b) ∈R
⇒ (b, b) & (c, c) ∈R
Therefore, elements to be added
{(b, a), (c, b) (a, c) (c, a), (a, a) (b, b), (c, c)}
∴ Number of elements to be added = 7

47. **Let R be a relation defined on ℕ as a R b if 2s + 3b is a multiple of 5, a, b ∈ N. Then R is**

(a) transitive but not symmetric
(b) an equivalence relation
(c) not reflexive
(d) symmetric but not transitive

JEE Mains 29/01/2023 Shift-II

Ans. (b) : For (a,a) ⇒ 2a+3b
= 2a+3a = 5a which is divisible by 5
So, (a,a) ∈ R , a∈ N reflexive
Let (a,b) ∈ R ⇒ 2a + 3b = $5k_1$
and 5a + 5b = $5k_2$
then,
5a + 5b –2a– 3b = 5 (k_2–k_1)
2b + 3a : 5k
(b,a) ∈ R is symmetric
Let (a,b) and (b, c) both ∈ R
2a +3b = $5k_1$
2b + 3c = $5k_2$
then , 2a + 3b + 3c = 5(k_1+ k_2)
2a + 3c = 5k – 5b
(a,c) ∈ R for transitive
So, it is equivalence relation-

48. **Let S= $\{1,2,3,5,7,10,11\}$. The number of non-empty subsets of S that have the sum of all elements a multiple of 3, is _____.**

JEE Mains 25/01/2023 Shift-I

Ans. (43) : Elements of the type 3k = 3
Elements of the type 3k + 1 = 1, 7, 9
Element of the type 3k + 2 = 2, 5, 11
Subsets containing one element S_1 = 1
Subsets containing two elements
$S_2 = {}^3C_1 \times {}^3C_1 = 9$
Subsets containing three elements
$S_3 = {}^3C_1 \times {}^3C_1 + 1 + 1 = 11$
Subsets containing four elements
$S_4 = {}^3C_3 + {}^3C_3 + {}^3C_2 \times {}^3C_2 = 11$
Subsets containing five elements
$S_5 = {}^3C_2 \times {}^3C_2 \times 1 = 9$
Subsets containing six elements S_6 = 1
Subsets containing seven elements S_7 = 1
⇒ sum = 43

49. **The minimum number of elements that must be added to the relation R = {(a, b), (b, c), (b, d)} on the set {a, b, c, d} so that it is an equivalence relation, is _______.**

JEE Mains 24/01/2023 Shift-II

Ans. (13) : Set = {a, b, c, d}
R = {(a, b), (b, c), (b, d)}
To make the given relation R as an equivalence relation-
Reflexive → (a, a), (b, b), (c, c), (d, d)
Symmetric → (a, b) ∈R
⇒ (b, a) ∈R
$(a,b)\ (b,c)\ (b,d)$
$(b,a)\ (c,b)\ (d,b)$
Transitive→ (a, b) and (b, c) ∈ R
(a, c), (a, d), (c, d), (d, c), (d, a), (c, a)
n = 4
set (A) = n^2
set (A) = 4^2
set A = 16
So, 13 elements more to be added to make an equivalence relation.

50. **The relation R = {(a, b): gcd (a, b) = 1, 2a ≠ b, a, b ∈ Z } is :**

(a) reflexive but not symmetric
(b) neither symmetric nor transitive
(c) symmetric but not transitive
(d) transitive but not reflexive

JEE Mains 24/01/2023 Shift-I

Ans. (b) : Given that
R = {(a,b) : gcd (a,b) = 1, 2a ≠ b, a b ∈ z }
• For reflexive relation :
(a,a) ⇒ gcd (a,a) = 1
Which is true for every a ∈ z.
⇒ For symmetric relation:
Taking a = 2 , b = 1 ⇒ gcd (2,1) = 1
Also, 2a = 4 ≠b
Now, when a = 1, b = 2⇒ gcd (1,2) = 1
Also, now 2a = 2 = b
Hence, a = 2b
⇒ R is not symmetric.
• For transitive relation:
Let a = 14, b = 19 , c = 21
gcd (a,b) = 1
gcd (b,c) = 1
gcd (a,c) = 7
Hence, R is not transitive.
Therefore, R is neither Symmetric nor transitive.
Thus, option (b) is correct answer.

51. Let P(S) denote the power set S = {1, 2, 3, ..., 10} . Define the relations R_1 and R_2 on P(S) as AR_1B if $(A \cap B^c) \cup (B \cap A^c) = \phi$ and AR_2B if $A \cup B^c = A^c \cup B, \forall A, B \in P(S)$. Then:

(a) Only R_1 is an equivalence relation
(b) Both R_1 and R_2 are not equivalence relations
(c) both R_1 and R_2 are equivalence relations
(d) only R_2 is an equivalence relation

JEE Mains 01/02/2023 Shift-II

Ans. (c) : Given,

$S = \{1,2,3.........10\}, n = 10$

Total number of element in P(S) = 2^{10}

AR_1B is defined as : $(A \cap B^c) \cup (B \cap A^c) = \phi$

$\Rightarrow A \cap B^c = \phi$ and $B \cap A^c = \phi$

$\Rightarrow A = B$.

Thus AR_1B is an equivalence relation.

and AR_2B is defined as $A \cup B^c = B \cup A^c \forall A, B \in P(S)$

$\Rightarrow A = B$.

Thus AR_2B is an equivalence relation.

So, both of them have an equivalence relation on S.

52. Let R be a relation from the set {1, 2, 3..........,60} to itself such that R = {(a, b) : b = pq, where p, q ≥ 3 are prime numbers}. Then the number of elements in R is :

(a) 600 (b) 660
(c) 540 (d) 720

JEE Main-29.07.2022, Shift-I

Ans. (b) : Given set,

A = {1, 2, 3, 4 ….60}

And, function R = {(a, b) : b = pq}

$1 \le pq \ge 60$

Number of possible values of a = 60 for b = pq

If p = 3, q = 3, 5, 7, 11, 13, 17, 19
If p = 5, q = 5, 7, 11
If p = 7, q = 7

a = 60 b = 11

a.b = 60 × 11

So, the number of elements in R is = 660.

53. Let R_1 and R_2 be relations on the set {1, 2, …, 50} such that
$R_1 = \{(p, p^n)$: p is a prime and n ≥ 0 is an integer} and $R_2 = \{(p, p^n)$: p is a prime and n = 0 or 1}. Then, the number of elements in $R_1 - R_2$ is ______.

JEE Main-28.06.2022, Shift-I

Ans. (8) : Here, $\{p, p^n\} \in \{1, 2, ….50\}$

Possible choice of P are –

2, 3, 5, 7, 11, 13, 17, 23, 29, 31, 37, 41, 43 and 47. we can calculate no. of elements in R_1 as $(2, 2^0), (2, 2^1)$ .. $(2, 2^5)$

$(3, 3^0), …,(3, 3^3)$
$(5, 5^0), …,(5, 5^2)$
$(7, 7^0), …,(7, 7^2)$
$(11, 11^0), …,(11, 11^1)$

Every number of P^n should lie in the given set {1, 2, 3……50}

And rest for all other two elements each

$n(R_1) = 6 + 4 + 3 + 3 + (2 \times 10) = 36$

Similarly for R_2

$(2, 2^0), (2, 2^1)$
$(47, 47^0), (47, 47^1)$

$\therefore$ $n(R_2) = 2 \times 14 = 28$

$\therefore$ $n(R_1) - n(R_2) = 36 - 28 = 8$

54. Let P(S) denote the power set of S = {1, 2, 3…..,10}. Define the relation R_1 and R_2 on P(S) as AR_1B if $(A \cap B^C) \cup (B \cap A^C) = \phi$ and AR_2B if $A \cup B^C = B \cup A^C, \forall$ A,B ∈ P(S). Then

(a) both R_1 and R_2 are not equivalence relations
(b) only R_2 is an equivalence relation
(c) only R_1 is an equivalence relation
(d) both R_1 and R_2 are equivalence relations

JEE Main-01.02.2023, Shift-II

Ans. (d) : P (S) = power set S

S = {1, 2, 3, ….10}

Given, $AR_1B \Rightarrow (A \cap B^c) \cup (B \cap A^c) = \phi$

$\Rightarrow A \cap B^c = \phi$ and $(B \cap A^c) = \phi$

$\Rightarrow A = B$

$\therefore$ AR_1B is an equivalence relation.

$AR_2B \Rightarrow A \cup B^c = B \cup A^c$

$\Rightarrow AB$

$\therefore$ AR_2B is an equivalence relation.

Hence, R_1 and R_2 are equivalence relation.

55. Let r be a relation from R (set of real numbers) to R defined by r = {(a, b) | a, b ∈ R and $a - b + \sqrt{3}$ is an irrational number}. The relation r is

(a) an equivalence relation (b) reflexive only
(c) symmetric only (d) transitive only

AMU-2009
JEE Main – 01.02.2023 Shift-1

Ans. (a) : Given,

r = {a, b | a, b ∈ R}

And , r ⇒ $a - b + \sqrt{3}$ is an irrational number.

For reflexive relation –

Then, $aRa = a - a + \sqrt{3}$

$\Rightarrow$ $aRa = \sqrt{3}$

And, $bRb = b - b + \sqrt{3} \Rightarrow$ $bRb = \sqrt{3}$

Therefore r is reflexive.

For symmetric relation –

Let, a, b ∈ R

$a - b + \sqrt{3} = b - a + \sqrt{3}$ is an irrational number

b,a ∈ R

Therefore r is symmetric.

For transitive relation –

Let (a, b) ∈ R and (b, c) ∈ R

$a - b + \sqrt{3} = b - c + \sqrt{3}$ is an irrational number

Now, $a - c + 2\sqrt{3}$ is an also irrational number

$\therefore$ (a, c) ∈ R

Thus r is transitive relation

Hence, r is an equivalence relation.

56. The minimum number of elements that must be added to the relation R = {(a, b), (b, c)} on the set {a, b, c} so that it becomes symmetric and transitive is :

(a) 3 (b) 7 (c) 4 (d) 5

JEE Main-30.01.2023, Shift-I

Ans. (b) : Given relation,
R = { (a, b) , (b, c)} on the set {a, b, c}
Now, required elements in sets for symmetrices and transitive are –
R = { (a, a), (b, b), (c, c), (b, a), (c, b), (a, c), (c, a) }
R = { (a, b), (b, c)}
Then, total number is 9.
So, minimum 7 elements must be added to becomes symmetric and transitive.

57. The minimum number of elements that must be added to the relation R = {(a, b), (b, c), (b, d)} on the set {a, b, c, d} so that it is an equivalence relation, is.

JEE Main-24.01.2023, Shift-II

Ans. (13) : Given that, R = { (a, b), (b, c), (b, d)}
On the set {a, b, c, d} to become equivalence.
For symmetric
(b, a) (c, a) (c, d), (d, c) (a, d) (d, a) (a, c)
For reflexive
(a, a) (b, b) (c, c) , (d, d)
For transitive
(c, b) (d, b)
Total number of element to be added = 7 + 4 + 2 = 13

58. Let R be a relation defined an N as a R b is 2a + 3b is a multiple of 5, a, b ∈ N. Then R is

(a) transitive but not symmetric
(b) an equivalence relation
(c) symmetric but no transitive
(d) not reflexive

JEE Main-29.01.2023, Shift-II

Ans. (b) : Given Relation, R = {(2a + 3b)} multiple of 5, a, b ∈ N
Let (a, b) ∈R
f (a, b) = 2a + 3b
For reflexive –
f (a, a) = 2a + 3a = 5a
i.e. it is divisible by 5.
⇒ (a, a) ∈R
For symmetric –
f (a, b) = 2a + 3b = 5α
f (b, a) = 2b + 3a

$$= 2b + \left(\frac{5\alpha - 3b}{2}\right) \times 3$$

$$= \frac{15\alpha}{2} - \frac{5}{2}b = \frac{5}{2}(3\alpha - b)$$

$$= \frac{5}{2}(2a + 2b - 2\alpha) = 5\,(a + b - \alpha)$$

f (b, a) is divisible by 5 ⇒ (b, a) ⇒ R
For transitive –
f (a, b) = 2a + 3b is divisible by 5
⇒ 2a + 3b = 5α
f (b, c) = 2b + 3c, is divisible by 5
2b + 3c = 5β
2a + 5b + 3c = 5(α + β)
2a + 3c = 5(α + β – b)
⇒ aRc
So, 2a + 3c is divisible by 5
⇒ (a, c) ∈R
Which is transitive.
Hence, R is equivalence relation.

59. Let R be a relation on N×N defined by (a, b) R (c, d) if and only if ad (b–c) = bc (a–d). Then R is

(a) transitive but neither reflexive nor symmetric
(b) symmetric but neither reflexive nor transitive
(c) symmetric and transitive but not reflexive
(d) reflexive and symmetric but not transitive

JEE Main-31.01.2023, Shift-I

Ans. (b) : Let R be relation defined by (a, b) R (c, d) ⇔ ad (b – c) = bc (a – d)
For reflexive –
(a, b) R (a, b) ⇒ ab (b – a) = ba (a – b)
∴ It is not reflexive.
For symmetric ⇒ (a, b) R (c, d) = ad (b – c) = bc (a – d) and
(c, d) R (a, b) = cb (d – a) = da (c – b)
It is true
Which is symmetric.
For transitive –
(a, b) R (c, d) = ad (b – c) = bc (a – d)
(c, d) R (e, f) = cf (d – e) = de (c – f)
So,
adcf (b – c) (d – e) = bcde (c – d) (c – f)
af (b – c) (d – e) = be (a – d) (c – f)
It is not transitive .

60. Among the relations

$S = \{(a, b) : a, b \in R - \{0\}, 2 + \frac{a}{b} > 0\}$ and $T = \{(a, b) : a, b \in R, a^2 - b^2 \in Z\}$.

(a) S is transitive but T is not transitive
(b) both S and T are symmetric
(c) neither S nor T is transitive
(d) T is symmetric but S is not symmetric

JEE Main-31.01.2023, Shift-II

Ans. (d) : Given relations
$S = \{ (a, b) : a, b \in R - \{0\} , 2 + \frac{a}{b} > 0 \}$
And, $T = \{ (a, b) : a, b \in R, a^2 - b^2 \in Z \}$.
Now, $T = a^2 - b^2 \in Z$
Then (b, a) on Relation R
$b^2 - a^2 \in Z$
Hence T is symmetric.
For,

$$S = \left\{(a, b) : a, b \in R - \{0\}, 2 + \frac{a}{b} > 0\right\}$$

$$2 + \frac{a}{b} > 0 \Rightarrow \frac{a}{b} > -2 \Rightarrow \frac{b}{a} < \frac{-1}{2}$$

If (b, a) ∈ S then,

$2 + \frac{b}{a}$ not necessarily positive.

So, S is not symmetric.

61. Let A = {1, 3, 4, 6, 9) and B = {2, 4, 5, 8, 10}. Let R be a relation defined on A × B such that R = {((a$_1$, b$_1$), (a$_2$, b$_2$)) : $a_1 \le b_2$ and $b_1 \le a_2$}. Then the number of elements in the set R is

(a) 26 (b) 160 (c) 180 (d) 52

JEE Main-11.04.2023, Shift-II

Ans. (b) : Given set,

A = {1, 3, 4, 6, 9)

and B = {2, 4, 5, 8, 10}

R = A × B ⇒ {(a_1 b_1) (a_2 b_2) : $a_1 \le b_2$ $b_1 \le a_2$}

Let,

$a_1 = 1$ then b_2 has 5 choices

$a_1 = 4$ then b_2 has 4 choices

$a_1 = 6$ then b_2 has 2 choices

$a_1 = 9$ then b_2 has 1 choices

Now,

$b_1 = 2$ then a_2 has 4 choices

$b_1 = 4$ then a_2 has 3 choices

$b_1 = 5$ then a_2 has 2

$b_1 = 8$ a_2 has 1 choices

So, total number of element

R = 160

62. Let A = {1, 2, 3, 4, 5, 6, 7}. Then the relation R = {(x, y) ∈ A × A : x + y = 7} is

(a) transitive but neither symmetric nor reflexive
(b) reflexive but neither symmetric nor transitive
(c) an equivalence relation
(d) symmetric but neither reflexive nor transitive

JEE Main-08.04.2023, Shift-II

Ans. (d) : A = {1, 2, 3, 4, 5, 6, 7}. defined on the set

R = {(x, y) ∈ A × A : x + y = 7}

R = {(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)}

For symmetric:- xRy = yRx

(1, 6) ∈ R, (6, 1) ∈ R and (5, 2) ∈ R, (2, 5) ∈ R

So R is symmetric

For Reflexive:- xRx

(1, 1) ∉R (2, 2) ∉ R (3, 3) ∉ R and (5, 5) ∉ R

So, R is not reflexive

For transitive

(1, 6) ∈ R and (6, 1) ∈ R but (1, 1) ∉ R and (2, 5) ∈ R (5, 2) ∈ R but (2, 2) ∉ R so R is not transitive.

63. Let A = {0, 3, 4, 6, 7, 8, 9, 10} and R be the relation defined on A such that R = {(x, y) ∈A × A : x –y is odd positive integer or x – y = 2}. The minimum number of elements that must be added to the relation R, so that it is a symmetric relation, equal to_____

JEE Main-08.04.2023, Shift-I

Ans. (19) : Given,

Set A = {0, 3, 4, 6, 7, 8, 9, 10}

Relation R defined in A.

R = [{x, y}∈ A × A : x – y is odd positive integer or x – y = 2]

R = {(6, 4), (8, 6), (9, 7), (10, 8), (3, 0), (7, 0), (9, 0), (4, 3), (6, 3), (8, 3), (10, 3), (7, 9), (9, 4), (7, 6), (9, 6), (8, 7), (10, 7), (9, 8), (10, 9)}

Hence, 19 element should be add in R for making it symmetric.

64. Let R$_1$ and R$_2$ be two relations defined as follows

$R_1 = \{(a, b) \in R^2 : a^2 + b^2 \in Q\}$

$R_2 = \{(a, b) \in R^2 : a^2 + b^2 \notin Q\}$

where Q is the set of all rational numbers. Then

(a) R_1 and R_2 are both transitive
(b) Neither R_1 nor R_2 is transitive
(c) R_1 is transitive but R_2 is not transitive
(d) R_2 is transitive but R_1 is not transitive

JEE Main 03.09. 2020 Shift-II

Ans. (b) : Let R_1 and R_2 be two relations

$R_1 = \{(a, b) \in R^2 : a^2 + b^2 \in Q\}$

$R_2 = \{(a, b) \in R^2 : a^2 + b^2 \notin Q\}$

For R_1 –

Consider,

$a = 1 + \sqrt{2}$, $b = 1-\sqrt{2}$ and $c = 8^{1/4}$

(a, b) ∈ R_1 because,

$a^2 + b^2 = \left(1+\sqrt{2}\right)^2 + \left(1-\sqrt{2}\right)^2$

$= 1 + 2 + 2\sqrt{2} + 1 + 2 - 2\sqrt{2} = 6 \in Q$

And (b, c) ∈ R_1 because,

$b^2 + c^2 = \left(1-\sqrt{2}\right)^2 + \left(8^{\frac{1}{4}}\right)^2 = 1+2-2\sqrt{2}+2\sqrt{2} = 3 \in Q$

But (a, c) ∉ R_1 because,

$a^2 + c^2 = \left(1+\sqrt{2}\right)^2 + \left(8^{1/4}\right)^2 = 1+2+2\sqrt{2}+2\sqrt{2}$

$= 3+4\sqrt{2} \notin Q$

Hence, R_1 is not transitive.

Now, For R_2 -

Consider, $a = 1 + \sqrt{3}$, $b = \sqrt{3}$, $c = 1-\sqrt{3}$

(a, b) ∈ R_2 because,

$a^2 + b^2 = \left(1+\sqrt{3}\right)^2 + \left(\sqrt{3}\right)^2$

$= 1 + 3 + 2\sqrt{3} + 3 = 7 + 2\sqrt{3} \notin Q$

(b, c) ∈ R_2 because,

$b^2 + c^2 = \left(\sqrt{3}\right)^2 + \left(1-\sqrt{3}\right)^2$

$= 3 + 1 + 3 = 2\sqrt{3} = 7 - 2\sqrt{3} \notin Q$

But (a, c) ∉ R_2 because,

$a^2 + c^2 = \left(1+\sqrt{3}\right)^2 + \left(1-\sqrt{3}\right)^2$

$= 1 + 3 + 2\sqrt{3} + 1 + 3 - 2\sqrt{3} = 8 \in Q$

So, R_2 is not transitive.

Hence, neither R_1 nor R_2 is transitive.

65. Let A = {2, 3, 4, 5, ... 30} and '≃' be an equivalence relation on A×A, defined by (a, b) ≃ (c, d), if and only if ad = bc. Then, the number of ordered pairs, which satisfy this equivalence relation with ordered pair (4, 3) is equal to

(a) 5 (b) 6
(c) 8 (d) 7

JEE Main 16.03.2021 Shift-II

Ans. (d) : Given,
Set A = {2, 3, 4, 5, ... 30} where A×A is defined by
(a, b) ≃ (c, d). Hence, (a, b) ≃ (c, d) implies that it reflexive, symmetric and transitive conditions.
Given, (a, b) ≃ (c, d)
ad = bc
Now ordered pair (4, 3)
(4, 3) ≃ (c, d)
4d = 3c
$$\frac{4}{3}=\frac{c}{d}$$
(c, d) ∈ {2, 3, 4, 5,….. 30}
$$\frac{c}{d}=\frac{4}{3}$$
(c, d) = (4, 3) (8, 6) (12, 9) (16, 12) (20, 15) (24, 18) (28, 21)
Hence, n. of order pair = 7.

66. Let R = {(P, Q)|, P and Q are at the same distance from the origin} be a relation, then the equivalence class of (1, –1) is the set
(a) $S = \{(x, y)|x^2 + y^2 = 4\}$
(b) $S = \{(x, y)|x^2 + y^2 = 1\}$
(c) $S = \{(x, y)|x^2 + y^2 = \sqrt{2}\}$
(d) $S = \{(x, y)|x^2 + y^2 = 2\}$

JEE Main 26.02.2021 Shift-I

Ans. (d) : Equivalence class of (1, –1) is a circle with centre.
Radius of circle at (1, –1) from origin
$$r=\sqrt{(1-0)^2+(-1+0)^2}=\sqrt{2}$$
Equation of circle
$$x^2 + y^2 = r^2$$
$$x^2 + y^2 = \left(\sqrt{2}\right)^2$$
$$x^2 + y^2 = 2$$
Which is symmetric, reflexive and transitive.
So relation
$S = \{(x, y)| x^2 + y^2 = 2\}$
is equivalence relation.

67. Which of the following is not correct for relation R on the set of real numbers?
(a) (x, y)∈R ⇔ 0 < |x|– |y| ≤ 1 is neither transitive nor symmetric.
(b) (x, y)∈ R ⇔ 0 <|x–y|≤ 1 is symmetric and transitive.
(c) (x, y)∈ R ⇔|x| –| y|≤ 1 is reflexive but not symmetric
(d) (x, y) ∈ R ⇔ |x–y| ≤ 1is reflexive and symmetric.

JEE Main 31.08.2021 Shift-I

Ans. (b) : (x, y)∈ R ⇒ 0 <|x–y|≤ 1
(1, 2)∈ R ⇒ 0 <|1–2|≤ 1
⇒ 0 <|–1|≤ 1
(2, 3) ∈ R ⇒ 0 <|2–3|≤ 1
⇒ 0 <|1–1|≤ 1
But (1, 3) ∈ R ⇒ 0 <|1–3|≤ 1
⇒ 0 <|–2|≤ 1
Hence, it is not transitive.

68. Define a relation R over a class of n × n real matrices A and B as "ARB, if there exists a non -singular matrix P such that $PAP^{-1} = B$". Then which of the following is true?
(a) R is symmetric, transitive but not reflexive.
(b) R is reflexive, symmetric but not transitive.
(c) R is an equivalence relation.
(d) R is reflexive, transitive but not symmetric.

JEE Main 18.03.2021, Shift-II

Ans. (c) : A and B are matrices of n × n order and ARB if there exists a non-singular matrix P(det (P) ≠ 0)
Such that $PAP^{-1} = B$
For reflexive –
ARA ⇒ $PAP^{-1} = A$ ….(i) must be true
For P = I, Equation (i) is true so 'R' is reflexive
For symmetric –
ARB ⇔ $PAP^{-1} = B$ …..(i) is true
For BRA if $PBP^{-1} = A$ ……(ii) must be true
∵ $PAP^{-1} = B$
$P^{-1}PAP^{-1} = P^{-1}B$
$IAP^{-1}P = P^{-1}BP$
$A = P^{-1}BP$ …..(iii)
From equation (ii) and (iii) $PBP^{-1} = P^{-1}BP$ can be true some $P = P^{-1}$
⇒ $P^2 = I$ (∵ det (P) ≠ 0)
So, R is symmetric.
For transitive –
ARB ⇔ $PAP^{-1} = B$ ….is true
BRC ⇔ $PBP^{-1} = C$ is true
Now, $P\,PAP^{-1}P^{-1} = C$
$P^2A\,(P^2)^{-1} = C$
⇒ ARC
So, 'R' is transitive relation
⇒ Hence, R is equivalence.

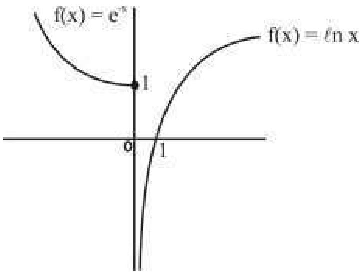
Type IV — Properties of Function and its Graph

69. Let f : R   R and g : R   R be defined
as f(x) $\begin{matrix}\log_e x, & x \quad 0\\ e^{\;x}, & x \quad 0\end{matrix}$ and g(x) $\begin{matrix}x, & x \quad 0\\ e^x, & x \quad 0\end{matrix}$.
Then, gof: R   R is :
(a) one-one but not onto
(b) neither one-one nor onto
(c) onto but not one-one
(d) both one-one and onto

JEE Mains 01/02/2024 Shift-I

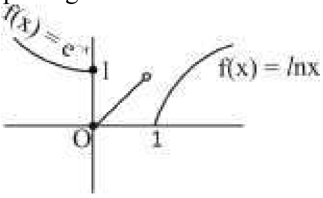
Ans. (b): $g(f(x)) \begin{matrix} f(x), & f(x) \quad 0 \\ e^{f(x)}, & f(x) \quad 0 \end{matrix}$

$$g(f(x)) = \begin{cases} \ell n x, & x \geq 1 \\ e^{\ell n x}, & 0 < x < 1 \\ e^{-x}, & x \leq 0 \end{cases}$$



$$g(f(x)) = \begin{cases} e^{-x}, & x \leq 0 \\ x, & 0 < x < 1 \\ \ell n x, & x \geq 1 \end{cases}$$

Now graph of gof is



$\because$ Range $= [0, \infty) \neq R \Rightarrow$ onto function does not exists.
Also if horizontal line intersect the graph two or more than two points then graph is many one and into
$\Rightarrow$ gof is neither one-one nor onto.

70. Consider the function f: R → R defined by $f(x) = \dfrac{2x}{\sqrt{1+9x^2}}$. If composition of $f,\underbrace{(f\,o\,f\,o\,f\,o....o\,f)}_{10\,times}(x) = \dfrac{2^{10}x}{1+9\alpha x^2}$, then the value of $\sqrt{3\alpha+1}$ is equal to....

JEE Mains 04/04/2024 Shift-II

Ans. :(1024)

$$f(f(x)) = \frac{2f(x)}{\sqrt{1+9f^2(x)}} \frac{\frac{2(2x)}{\sqrt{1+9x^2}}}{\sqrt{1+9\left(\frac{2x}{\sqrt{1+9x^2}}\right)^2}}$$

$$= \frac{4x}{\sqrt{1+9x^2+9.2^2x^2}}$$

$$= \frac{2^2x}{\sqrt{1+9x^2+9.2^2x^2}}$$

$$f(f(f(x))) = \frac{2^3x/\sqrt{1+9x^2}}{\sqrt{1+9(1+2^2)\frac{2^2x^2}{1+9x^2}}} = \frac{2^3x}{\sqrt{1+9x^2(1+2^2+2^4)}}$$

$\therefore$ By observation
$\alpha = 1 + 2^2 + 2^4 + ... + 2^{18}$

formula $\dfrac{a(r^n-1)}{r-1}$

$a = 1, n = 10, r = 2^2$

$$\alpha = 1\left(\frac{(2^2)^{10}-1}{2^2-1}\right) = \frac{2^{20}-1}{3}$$

$3\alpha + 1 = 2^{20} \rightarrow 3\alpha + 1 = (2^{10})^2 \rightarrow \sqrt{3\alpha+1} = 2^{10} = \boxed{1024}$

71. If $f(x) = \dfrac{4x+3}{6x-4}, x \neq \dfrac{2}{3}$ and (fof) (x) = g(x), where $g: \mathbb{R}-\left\{\dfrac{2}{3}\right\} \rightarrow \mathbb{R}-\left\{\dfrac{2}{3}\right\}$, then (gogog) (4) is equal to

(a) 4 (b) –4

(c) $-\dfrac{19}{20}$ (d) $\dfrac{19}{20}$

JEE Mains 31/01/2024 Shift-I

Ans. (a) : Given that,

$$f(x) = \frac{4x+3}{6x-4}$$

$$g(x) = f\{f(x)\} = \frac{4f(x)+3}{6f(x)-4} = \frac{4\left(\frac{4x+3}{6x-4}\right)+3}{6\left(\frac{4x+3}{6x-4}\right)-4} = x$$

$\Rightarrow$ g(x) = x

Now,

$$g\{g\{g(x)\}\} = x$$

$$g(g(g(4))) = 4$$

72. Let f(x) = $2^x - x^2$, x ∈ R. If m and n are respectively, the number of points at which the curves y = f(x) and y = f' (x) intersect the x-axis, then the value of m + n is _____.

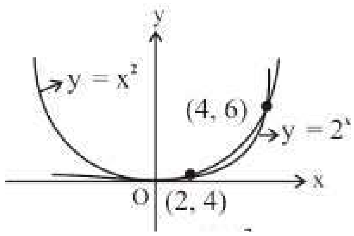
JEE Mains 29/01/2024 Shift-I

Ans. (5) : $y = 2^x - x^2$ meet the x – axis

y = 0

$\Rightarrow$ $2^x - x^2 = 0$

$2^x = x^2$



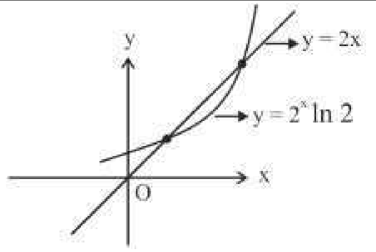
Number of point of intersection = 3

m = 3

y = f'(x)

$y = 2^x \ln 2 - 2x$ meet the x-axis at y = 0

$2^x \ln 2 = 2x$



Number of point of intersection = 2

n = 2

so, m + n = 5

73. If a function f satisfies f (m + n) = f(m) + f(n) for all m, n ∈ N and f(1) = 1, then the largest natural number λ such that $\sum_{k=1}^{2022} f(\lambda+k) \le (2022)^2$ is equal to ______.

JEE Mains 09/04/2024 Shift-I

Ans. (1010) : f (m + n) = f(m) + f(n)

⇒ f(x) = kx

⇒ f(1) = 1

⇒ k = 1

f(x) = x

Now $\sum_{k=1}^{2022} f(\lambda+k) \le (2022)^2$

⇒ $\sum_{k=1}^{2022} (\lambda+k) \le (2022)^2$

⇒ $2022\lambda + \frac{2022 \times 2023}{2} \le (2022)^2$

⇒ $\lambda \le 2022 - \frac{2023}{2}$

⇒ $\lambda \le 1010.5$

∴ Largest natural number λ is 1010.

74. Let f(x) = $x^5 + 2x^3 + 3x + 1$, x ∈ R and g (x) be a function such that g(f(x)) = x for all x∈ R. Then $\frac{g(7)}{g'(7)}$ is equal to:

(a) 7 (b) 42 (c) 1 (d) 14

JEE Mains 05/04/2024 Shift-I

Ans. (d) : Given, $f(x) = x^5 + 2x^3 + 3x + 1$

$f'(x) = 5x^4 + 6x^2 + 3$

$f'(1) = 5 + 6 + 3 = 14$

g'(f(x))f '(x) = 1

For f(x) = 7

⇒ $x^5 + 2x^3 + 3x + 1 = 7$

⇒ $x(x^4 + 2x^2 + 3) - 6 = 0$

⇒ x = 1

∴ $g'(7)f'(1) = 1 \Rightarrow g'(7) = \frac{1}{f'(1)} = \frac{1}{14}$

Now, x = 1, f(x) = 7 ⇒ g(7) = 1

$\frac{g(7)}{g'(7)} = \frac{1}{1/14} = 14$

75. Let f(x) = $x^5 + 2e^{x/4}$ for all x ∈ R. Consider a function g(x) such that (g of) (x) = x for all x ∈ R. Then the value of 8g'(2) is:

(a) 16 (b) 4

(c) 8 (d) 2

JEE Mains 04/04/2024 Shift-I

Ans. (a) : Given,

gof(x) = x

g(f (x)) = x

g'(f (x)) f '(x) = 1

$g'(f(x)) = \frac{1}{f'(x)}$ --------- (i)

$f'(x) = x^5 + 2e^{x/4}$

$f'(x) = 5x^4 + 2e^{x/4} \times \frac{1}{4}$

$g'(f(x)) = \frac{1}{5x^4 + \frac{e^{x/4}}{2}}$

So, g'(2) means

f(x) = 2

So,

$f(x) = x^5 + 2e^{x/4}$

$2 = x^5 + 2e^{x/4}$

This eq^n is satisfied

When x = 0

$g'(2) = \frac{1}{0 + \frac{1}{2}}$

Where x = 0

g'(2) = 2

So, 8g' (2) = 8+2 = 16

76. If $f(x) = \frac{(\tan 1^\circ)x + \log_e(123)}{x\log_e(1234) - (\tan 1^\circ)}, x > 0$, then the least value of $f(f(x)) + f\left(\left(\frac{4}{x}\right)\right)$ is:

(a) 2 (b) 4

(c) 8 (d) 0

JEE Mains 10/04/2023 Shift-I

Ans. (b) : Given,

$f(x) = \frac{(\tan 1)x + \log 123}{x\log 1234 - \tan 1}$

Let A = tan1, B = log123, C = log1234

$f(x) = \frac{Ax + B}{xC - A}$

$f(f(x)) = \frac{A\left(\frac{Ax+B}{Cx-A}\right) + B}{C\left(\frac{Ax+B}{Cx-A}\right) - A}$

$= \frac{A^2x + AB + xBC - AB}{ACx + BC - ACx + A^2}$

$$= \frac{x(A^2+BC)}{(A^2+BC)} = x$$

$$f(f(x)) = x$$

$$f\left(f\left(\frac{4}{x}\right)\right) = \frac{4}{x}$$

$$f(f(x)) + f\left(f\left(\frac{4}{x}\right)\right)$$

$\because$ AM ≥ GM

$$x + \frac{4}{x} \geq 4$$

77. For x ∈R, Two real valued functions f(x) and g(x) are such that, g(x) = $\sqrt{x}+1$ and fog(x) = x + 3 –$\sqrt{x}$. Then f(0) is equal to

(a) 5 (b) 0 (c) –3 (d) 1

JEE Mains 13/04/2023 Shift-I

Ans. (a) : Sol.

$g(x) = \sqrt{x}+1$

$fog(x) = x + 3 - \sqrt{x}$

$f(g(x)) = (\sqrt{x}+1)^2 - 3(\sqrt{x}+1) + 5$

$= g^2(x) - 3g(x) + 5$

Replacing g(x) by x,

$\Rightarrow f(x) = x^2 - 3x + 5$

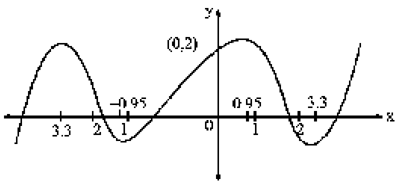
$\therefore f(0) = 5$

But, if we consider the domain of the composite function fog(x) then in that case f(0) will be not defined as g(x) cannot be equal to zero.

78. The number of points, where the cure y = x^5 – $20x^3$ + 50x + 2 crosses the x-axis it ___:

JEE Mains 06/04/2023 Shift-II

Ans. (5) :



$y = x^5 - 20x^3 + 50x + 2$

$y' = 5x^4 - 60x^2 + 50$

$y' = 5(x^4 - 12x^2 + 10) = 0$

$x^4 - 12x^2 + 10 = 0$

$(x^2-6)^2 + 10 - (6)^2 = 0$

$(x^2-6)^2 + 10 - (6)^2 = 0$

$(x^2-6)^2 = 26$

$x^2 - 6 = \pm\sqrt{26}$

$x^2 = 6 \pm \sqrt{26}$

$x = \pm\sqrt{6\pm\sqrt{26}}$

The number of points where the curve cuts the x-axis = 5.

79. Consider a function f : IN → IR, satisfying f(1) + 2f(2) + 3f(3) + ... + xf(x) = x(x + 1)f(x); x ≥ 2 with f(1) = 1.

Then $\frac{1}{f(2022)} + \frac{1}{f(2028)}$ is equal to

(a) 8100 (b) 8200

(c) 8000 (d) 8400

JEE Mains 29/01/2023 Shift-II

Ans. (a) : Given that,

f : N → R such that f(1) = 1

Now, f(1) + 2f(2) + 3f(3) + ... +xf(x) = x(x + 1) f(x), x ≥ 2

Here, f(1) + 2f(2) = 2(2 + 1) f(2)

$\Rightarrow$ f(1) + 2f(2) = 6 f(2)

$\Rightarrow$ f(1) = 4f(2)

$\Rightarrow f(2) = \frac{f(1)}{4}$

$\Rightarrow f(2) = \frac{1}{4}$, $\{\because f(1) = 1\}$

And f(1) + 2f(2) + 3f(3) = 3(3 + 1) f(3)

$\Rightarrow 1 + 2\left(\frac{1}{4}\right) + 3f(3) = 12\,f(3)$

$\Rightarrow 9f(3) = \frac{3}{2}$

$\Rightarrow f(3) = \frac{1}{6}$

Similarly, f(1) + 2f(2) + 3f(3) + 4f(4) = 4(4 + 1)f(4)

$\Rightarrow 16f(4) = 1 + 2\times\frac{1}{4} + 3\times\frac{1}{6} == 1 + \frac{1}{2} + \frac{1}{2} = 2$

$\Rightarrow f(4) = \frac{1}{8}$

Now, In general, $f(x) = \frac{1}{2x}$, if x = x then

or $f(n) = \frac{1}{2n} \Rightarrow 2n = \frac{1}{f(n)}$

Here, $\frac{1}{f(2022)} = 2\times2022$ and $\frac{1}{f(2028)} = 2\times2028$

$\frac{1}{f(2022)} = 4044$ and $\frac{1}{f(2028)} = 4056$

now, $\frac{1}{f(2022)} + \frac{1}{f(2028)} = 4044 + 4056 = 8100$

80. Let f(x) be a function such that f(x + y) = f(x).f(y) for all x, y ∈ N. If f(1) = 3 and $\sum_{k=1}^{n} f(k) = 3279$, then the value of n is

(a) 8 (b) 7

(c) 9 (d) 6

JEE Mains 24/01/2023 Shift-II

Ans. (b) : f(x + y) = f(x) . f(y)

$f(2) = f(1).\ f(1) = 3^2$

$f(3) = f(2)\ f(1) = 3^3$

$f(4) = 3^4$

$f(n) = 3^n$

$$\sum_{k=1}^{n} f(k) = 3279$$

f (1) +f (2) + f(3) +f(4) + ... + f(n) = 3279

$\Rightarrow 3 + 3^2 + 3^3 + ... + 3^n = 3279$

$$\frac{3(3^n - 1)}{(3-1)} = 3279$$

$(3^n - 1) = 2(1093)$

$3^n = 2186 + 1$

$3^n = 2187$

$3^n = 3^7$

$\boxed{n = 7}$

81. If $f(x) = \frac{2^{2x}}{2^{2x}+2}, x \in R$, then $f\left(\frac{1}{2023}\right) + f\left(\frac{2}{2023}\right)$ + ... + $f\left(\frac{2022}{2023}\right)$ is equal to

(a) 2010 (b) 2011

(c) 1011 (d) 1010

JEE Mains 24/01/2023 Shift-II

Ans. (c) : Given,

$$f(x) = \frac{4^x}{4^x+2}$$

$$f(1-x) = \frac{4^{1-x}}{4^{1-x}+2}$$

$$\Rightarrow f(x) + f(1-x) = \frac{4^x}{4^x+2} + \frac{4^{1-x}}{4^{1-x}+2}$$

$$= \frac{4^x}{4^x+2} + \frac{4}{4+2.4^x}$$

$$= \frac{4^x}{4^x+2} + \frac{2}{2+4^x} = \frac{4^x+2}{4^x+2} = 1$$

$$\Rightarrow f\left(\frac{1}{2023}\right) + f\left(\frac{2}{2023}\right) + f\left(\frac{3}{2023}\right) + ... + f\left(\frac{2022}{2023}\right)$$

$$\therefore \quad f\left(\frac{1}{2023}\right) + f\left(\frac{2022}{2023}\right) = 1$$

$$f\left(\frac{2}{2023}\right) + f\left(\frac{2021}{2023}\right) = 1$$

$$\vdots$$

$$f\left(\frac{1011}{2023}\right) + f\left(\frac{1012}{2023}\right) = 1$$

$\Rightarrow$ 1 + 1 + 1 + ... (1011 times) = 1011

82. Let f : R – {0, 1} → R be a function such that $f(x) + f\left(\frac{1}{1-x}\right) = 1 + x$. Then f (2) is equal to

(a) $\frac{7}{3}$ (b) $\frac{7}{4}$ (c) $\frac{9}{2}$ (d) $\frac{9}{4}$

JEE Mains 01/02/2023 Shift-II

Ans. (d) : f : R – {0, 1} → R

$$f(x) + f\left(\frac{1}{1-x}\right) = 1 + x$$

Put x = 2,

f(2) + f(–1) = 3 ...(i)

Put x = –1,

$f(-1) + f\left(\frac{1}{2}\right) = 0$ ...(ii)

Put x = 1/2

$f\left(\frac{1}{2}\right) + f(2) = \frac{3}{2}$ ...(iii)

Subtracting equation (i) and (ii), we get -

$f(2) + f(-1) - f(-1) - f(-1) - f\left(\frac{1}{2}\right) = 3$

$f(2) - f\left(\frac{1}{2}\right) = 3$ ...(iv)

On adding equation (iii) and (iv), we get-

$$f(2) - f\left(\frac{1}{2}\right) + f(2) + f\left(\frac{1}{2}\right) = 3 + \frac{3}{2}$$

$$2f(2) = \frac{9}{2}$$

$$f(2) = \frac{9}{4}$$

83. If f(x) and g(x) are two polynomials such that the polynomial $p(x) = f(x^3) + xg(x^3)$ is divisible by $x^2 + x + 1$, then p(1) is equal to ....

JEE Main 18.03.2021, Shift - II

Ans. (0) : Given, polynomial,

$p(x) = f(x^3) + x\ g(x^3)$

Putting the value of x = 1, we get-

p(1) = f(1) + g(1) .....(i)

According to question, p(x) is divisible by $x^2 + x + 1$,

$p(x) = Q(x)\ (x^2 + x + 1)$

$\therefore$ $p(\omega) = 0 = p(\omega^2)$ where ω, ω^2

Non-real cube roots of units,

Now, $p(x) = f(x^3) + x\ g(x^3)$

$p(\omega) = f(\omega^3) + \omega\ g(\omega^3) = 0$

$f(1) + \omega\ g(1) = 2$ ……(ii)

$p(\omega^2) = f(\omega^6) + \omega^2\ g(\omega^6) = 0$

$f(1) + \omega^2\ g(1) = 0$ …. (iii)

On adding equation (ii) and (iii) we get-

$2f(1) + (\omega + \omega^2)y(1) = 0$

2f(1) = g(1) ….(iv)

On subtracting equation (ii) and (iii), we get–

$(\omega - \omega^2)\ g(1) = 0$

From equation (iv), g(1) = 0 = f(1)

Putting the value in equation (i) we get-

p(1) = f(1) + g(1)

p(1) = 0 + 0 = 0

84. Consider a function f: N → R, satisfying f(1) + 2f (2) + 3f(3)+......+ xf(x) = x(x + 1) f(x) : x ≥ 2 with f(1) = 1. Then $\frac{1}{f(2022)}+\frac{1}{f(2028)}$ is equal to

(a) 8400 (b) 8100
(c) 8200 (d) 8000

JEE Main-29.01.2023, Shift-II

Ans. (b) : Given, a function f : N → R, satisfying –
f(1) + 2f (2) + 3f(3) + ……+ xf(x) = x(x + 1)f (x)
Replace x by x + 1, we get –
x(x + 1) f(x) + (x + 1)f (x + 1) = (x + 1) (x + 2)f(x + 1)

$$\frac{x}{f(x+1)}+\frac{1}{f(x)}=\frac{(x+2)}{f(x)}$$

xf(x) = (x + 1)f(x + 1) = $\frac{1}{2}$, x ≥ 2

$$f(2)=\frac{1}{4},\ f(3)=\frac{1}{6}$$

Now, $f(2022)=\frac{1}{4044}$

$$f(2028)=\frac{1}{4056}$$

So, $\frac{1}{f(2022)}+\frac{1}{f(2028)}=4044+4056$
= 8100.

85. Let f : R → R be a function defined $f(x)=\frac{2e^{2x}}{e^{2x}+e}$. Then $f\left(\frac{1}{100}\right)+f\left(\frac{2}{100}\right)+f\left(\frac{3}{100}\right)+......+f\left(\frac{99}{100}\right)$ is equal to______.

JEE Main-27.06.2022, Shift-I

Ans. (99) : Given, a function f : R → R is defined on –

$$f(x)=\frac{2e^{2x}}{e^{2x}+e} \quad .....(i)$$

Replace (x) by (1 – x), we get –

$$f(1-x)=\frac{2e^{2(1-x)}}{e^{2(1-x)}+e}$$

$$f(1-x)=\frac{2e^{2-2x}}{e^{2-2x}+e} \quad ....(ii)$$

On adding equation (i) and equation (ii), we get –

$$f(x)+f(1-x)=\frac{2e^{2x}}{e^{2x}+e}+\frac{2e^{2-2x}}{e^{2-2x}+e}$$

$$=\frac{2e^{2x}}{e^{2x}+e}+\frac{2e^{2}\times e^{-2x}}{e^{2}\times e^{-2x}+e}$$

$$=2\left[\frac{e^{2x}}{e^{2x}+e}+\frac{e^{2}}{e^{2}+e^{2x+1}}\right]$$

$$=2\left[\frac{e^{2x-1}}{e^{2x-1}+1}+\frac{1}{1+e^{2x-1}}\right]$$

= 2

So, $f\left(\frac{1}{100}\right)+f\left(\frac{2}{100}\right)+f\left(\frac{3}{100}\right)+.....+f\left(\frac{99}{100}\right)$

$$=\left\{f\left(\frac{1}{100}\right)+f\left(\frac{99}{100}\right)\right\}+\left\{f\left(\frac{2}{100}\right)+f\left(\frac{98}{100}\right)\right\}$$

$$+......+\left\{f\left(\frac{49}{100}\right)+f\left(\frac{51}{100}\right)\right\}+f\left(\frac{1}{2}\right)$$

$$=\{2+2+2+.......49\text{times}\}+\frac{2e}{e+e}$$

= 98 + 1
= 99.

86. Let f : N → R be a function such that f (x + y) = 2f (x) f (y) for natural numbers x and y. If f(1) = 2, then the value of α for which $\sum_{k=1}^{10} f(\alpha+k)=\frac{512}{3}(2^{20}-1)$ holds, is

(a) 2 (b) 3 (c) 4 (d) 6

JEE Main-25.06.2022, Shift-I

Ans. (c) : Given,
f : N → R, f (x + y) = 2 f (x) f (y) ….(i)
f (1) = 2

$$\sum_{k=1}^{10} f(\alpha+k)=2f(\alpha)\sum_{k=1}^{10} f(k)$$

= 2f (α){f(1) + f (2) + …. + f(10)} ….(ii)

Form equation (i),
$f(2) = 2 f^2(1) = 2^3$
$f(3) = 2 f(2) f(1) = 2^5$
……………………..
……………………..
……………………..
$f(10) = 2^9 f^{10}(1) = 2^{19}$
∴ $f(\alpha) = 2^{2\alpha-1}$; α ∈ N

Form equation (ii)

$$\sum_{k=1}^{10} f(\alpha+k)=2\left(2^{2\alpha-1}\right)\left(2+2^3+2^5+....+2^{19}\right)$$

$$\frac{512}{3}\left(2^{20}-1\right)=2^{2\alpha}\left(2.\frac{\left(2^{20}-1\right)}{3}\right)$$

$$\frac{512}{3}\left(2^{20}-1\right)=\frac{2^{2\alpha+1}}{3}\left(2^{20}-1\right)$$

Comparing both side, we get–
$2^{2\alpha+1}=512$
$2^{2\alpha+1}=2^9$
2α + 1 = 9
2α = 8
Hence, α = 4

87. Let f(x) = ax² + bx + c be such that f(1) = 3, f(–2) = λ and f(3) = 4. If f(0) + f(1) + f(–2) + f(3) = 14 then λ is equal to

(a) –4 (b) $\frac{13}{2}$ (c) $\frac{23}{2}$ (d) 4

JEE Main-28.07.2022, Shift-II

Ans. (d): Given, $f(x) = ax^2 + bx + c$

Then, $f(1) = a + b + c = 3$ ….. (i)

$f(-2) = 4a - 2b + c = \lambda$ …… (ii)

$f(3) = 9a + 3b + c = 4$ ……. (iii)

$\because$ $f(0) + f(1) + f(-2) + f(3) = 14$

$\therefore$ $c + 3 + \lambda + 4 = 14$

$c + \lambda = 7$

$\lambda = 7 - c$

Solving (i) and (ii):-

$$\begin{array}{l} 2a+2b+2c=6 \\ \underline{4a-2b+c=\lambda} \\ 6a+3c=6+\lambda \end{array}$$

From (ii) and (iii):-

$$\begin{array}{l} 12a-6b+3c=3\lambda \\ \underline{18a+6b+2c=8} \\ 30a+5c=3\lambda+8 \end{array}$$

Now, we have–

$6a + 3c = 6 + \lambda$ …..(iv)

$30a + 5c = 3\lambda + 8$…..(v)

Solving (iv) and (v), we get –

$$\begin{array}{l} 30a+15c=30+5\lambda \\ \underline{30a+5c=8+3\lambda} \\ 10c=22+2\lambda \end{array}$$

$\therefore$ $c=\frac{22}{10}+\frac{\lambda}{5}$

Then, $\lambda = 7-\frac{22}{10}-\frac{\lambda}{5}$

Or $\frac{6}{5}\lambda=\frac{70-22}{10}=\frac{48}{10}$

So, $\lambda = \frac{48}{10}\times\frac{5}{6}=\frac{8}{2}=4$

88. Let A = {1, 2, 3, ... 10} and f : A → A be defined as f(x) = $\begin{cases} x+1, \text{ if } x \text{ is odd} \\ x, \quad \text{If } x \text{ is even} \end{cases}$ Then, the number of possible functions g : A → A, such that gof = f is

(a) 10^5 (b) $^{10}C_5$

(c) 5^5 (d) 5 !

JEE Main 26.02. 2021, Shift -II

Ans. (a) : Given,

set A = {1, 2, 3 ....10}

$\therefore$ g : A → A such that

g(f(k)) = f (k)

If k is even then g (k) = k .....(i)

If k is odd then g (k + 1) = k + 1 ......(ii)

From equation (i) and (ii)

g (k) = k, if k is even

If k is odd then g(k) can take any value is set A

So, the no. of g(k) = 10^5

89. Let a, b, c ∈ R. If $f(x) = ax^2 + bx + c$ is such that a + b + c = 3 and f (x + y) = f(x) + f(y) + xy, ∀ x, y ∈R, then ` $\sum_{n=1}^{10} f(n)$ is equal to

(a) 330 (b) 165

(c) 190 (d) 255

JEE Main 2017

Ans. (a) : Given,

$f(x) = ax^2 + bx + c$

a, b, c ∈ R

f(1) = a + b + c

f(2) = f(1 + 1) = f(1) + f(1) + 1 = 2f(1) + 1

f(3) = f (2 + 1) = f(2) + f(1) + 2

= 2f(1) + 1 + f(1) + 2

f (3) = 3f (1) + 3

f(4) = f(3 + 1) = f(3) + f(1) + 3.1

3f(1) + 3 + f(1) + 3

4f(1) + 6

f(5) = f(4 + 1) = f(4) + f(1) + 4.

= 4f(1) + 6 + f(1) + 4

= 5 f(1) + 10

Now, $\sum_{n=1}^{10} f(n) = f(1)+f(2)+f(3)+f(4)...f(10)$

= f(1) + 2f(1) + 1+ 3f(1) + 3 + 4 f(1) + 6 + 5f(1) + 10 + 6f(1) + 15

= f(1) [1 + 2 + 3 + 4 + 5 + ...10] + (1 + 3 + 6 + 10 + 15 + 21 + 28 + 36 + 49)

$= f(1)\times\frac{10\times11}{2}+165$

= 3 × 55 + 165

= 165 + 165

= 330

90. Let f : R → R be a function which satisfies f(x + y) = f(x) + f(y), ∀ x, y ∈R. If f (1) = 2 and g(n) = $\sum_{k=1}^{(n-1)} f(k), n \in N$, then the value of n, for which g(n) = 20 is

(a) 5 (b) 20

(c) 4 (d) 9

JEE Main 2.09. 2020, Shift -II

Ans. (a) : Given,

f (x + y) = f(x) + f(y) and f(1) = 2

f(2) = f(1 + 1) = f(1) + f(1) = 2f(1)

f (3) = f(2 + 1) = f(2) + f(1) = 3f(1)

f(4) = f(3 + 1) = f(3) + f(1) = 4 f(1)

f(n) = nf(1) = 2n

Then,

$$g(n)=\sum_{k=1}^{n-1} f(n)$$

$$g(n)=\sum_{k=1}^{n-1} 2n$$

$$g(n) = 2\sum_{k=1}^{n-1} n$$

$$20 = 2\frac{n(n-1)}{2}$$

n (n – 1) = 20

n (n – 1) = 5 × 4 = 20

n = 5

91. For a suitable chosen real constant a, let a function f : R – {a} → R be defined by f(x) = $\frac{a-x}{a+x}$. Further suppose that for any real number x ≠ –a and f(x) ≠ –a, (fof)(x) = x. Then, $f\left(-\frac{1}{2}\right)$ is equal to

(a) $\frac{1}{3}$ (b) $-\frac{1}{3}$

(c) –3 (d) 3

JEE Main 06.09. 2020 Shift-II

Ans. (d) : We have,

$f(x) = \frac{a-x}{a+x}\left[x \in R - \{-a\}\right]$

fof (x) = x

f [f (x)] = x

$\frac{a-f(x)}{a+f(x)} = x$

$\frac{a-\left(\frac{a-x}{a+x}\right)}{a+\left(\frac{a-x}{a+x}\right)} = x$

$\frac{(a^2-a)+x(a+1)}{(a^2+a)+x(a-1)} = x$

$(a^2 – a) + x (a + 1) = (a^2 + a) x + x^2 (a – 1)$

$x^2(a – 1) + x (a^2 + a – a – 1) – a^2 + a = 0$

$x^2 (a – 1) + x (a^2 – 1) – (a^2 – a) = 0$

$x^2 + x (a + 1) – a = 0$

a = 1

$f(x) = \frac{1-x}{1+x}$

$f\left(-\frac{1}{2}\right) = \frac{1-\left(\frac{-1}{2}\right)}{1+\left(\frac{-1}{2}\right)}$

$f\left(-\frac{1}{2}\right) = \frac{\frac{3}{2}}{\frac{1}{2}}$

$f\left(\frac{-1}{2}\right) = 3$

92. Suppose that a function f : R → R satisfies f(x + y) = f(x) f(y) for all x, y ∈R and f(1) = 3. If $\sum_{i=1}^{n} f(i) = 363$, then n is equal to

JEE Main 06.09. 2020 Shift-II

Ans. (5) : Given function f : R → R

Satisfies f(x + y) = f(x) f(y)

And $\sum_{i=1}^{n} f(i) = 363$

f (1) = 3

$f(2) = f(1 + 1) = f(1) f(1) = [f(1)]^2 = 3^2$

$f(3) = f(2 + 1) = f(2) f(1) = [f(1)]^3 = 3^3$

$f(4) = f(3 + 1) = f(3) \cdot f(1) = [f(1)]^4 = 3^4$

$f(n) = [f(1)]^n$

$\sum_{i=1}^{n} f(i) = f(1)+(f(2))+(f(3))......(f(n))^n$

$363 = 3 + 3^2 + 3^3 + ...3^n$

$363 = \frac{3(3^n-1)}{3-1}$

$3^n – 1 = \frac{363\times 2}{3}$

$3^n – 1 = 242$

$3^n = 243$

$3^n = 3^5$

n = 5

93. If a + α = 1, b + β = 2 and af(x) + αf$\left(\frac{1}{x}\right)$ = bx + $\frac{\beta}{x}$, x ≠0, then the value of expression $\frac{f(x)+f\left(\frac{1}{x}\right)}{x+\frac{1}{x}}$ is ....

JEE Main 24.02. 2021 Shift-II

Ans. (2) : Given,

$af(x) + \alpha f\left(\frac{1}{x}\right) = b(x) + \frac{\beta}{x} \cdot x \neq 0$ ….(i)

a + α = 1 and b + β = 2

Replace, x by $\frac{1}{x}$ then

$af\left(\frac{1}{x}\right)+\alpha f(x) = b\frac{1}{x}+\beta x$ ….(ii)

Now adding (i) and (ii) we get -

$af(x)+\alpha f\left(\frac{1}{x}\right)+af\left(\frac{1}{x}\right)+\alpha f(x) = bx + \frac{\beta}{x}+\frac{b}{x}+\beta x$

$a + \alpha)\, f(x) + (a + \alpha)\, f\left(\frac{1}{x}\right) = (b + \beta)\, x + (b + \beta)\left(\frac{1}{x}\right)$

$1.\, f(x)+1\cdot f\left(\frac{1}{x}\right) = 2x+\frac{2}{x}$

$f\left(x+f\left(\frac{1}{x}\right)\right) = 2\left(x+\frac{1}{x}\right)$

$\frac{f(x)+f\left(\frac{1}{x}\right)}{x+\frac{1}{x}} = 2$

94. If $f(x)=\frac{2^{2x}}{2^{2x}+2}, x \in R$ then

$$f\left(\frac{1}{2023}\right)+f\left(\frac{2}{2023}\right)+f\left(\frac{3}{2023}\right)+......+f\left(\frac{2022}{2023}\right)$$

is equal to

(a) 1011 (b) 2010
(c) 1012 (d) 2011

JEE Main-24.01.2023, Shift-II

Ans. (a) : Given,

$$f(x)=\frac{2^{2x}}{2^{2x}+2}$$

Put, $x \to 1-\mathbf{x}$ then we get-

$$f(1-x)=\frac{2^{2(1-x)}}{2^{2(1-x)}+2}$$

$$\Rightarrow \frac{4^{1-x}}{4^{1-x}+2}$$

Then, adding we get-

$$f(x)+f(1-x)=\frac{4^x}{4^x+2}+\frac{4^{1-x}}{4^{1-x}+2}$$

$$\Rightarrow \frac{4^x}{4^x+2}+\frac{\frac{4}{4^x}}{\frac{4}{4^x}+2}$$

$$\Rightarrow \frac{4^x}{4^x+2}+\frac{2}{4^x+2}$$

$$\Rightarrow \frac{4^x+2}{4^x+2}=1$$

Now,

$$f\left(\frac{1}{2023}\right)+f\left(\frac{2}{2023}\right)+f\left(\frac{3}{2023}\right)+.....+f\left(\frac{2022}{2023}\right)$$

=(1 + 1 + 1 + 1 +……+1, 1011 times)
=1011

95. Let $x=\left(8\sqrt{3}+13\right)^{13}$ and $y=\left(7\sqrt{2}+9\right)^{9}$. If [t] denotes the greatest integer ≤ t, then

(a) [x] is even but [y] is odd
(b) [x] + [y] is even
(c) [x] and [y] are both odd
(d) [x] is odd but [y] is even

JEE Main-30.01.2023, Shift-II

Ans.(b): Given,

$$x=\left(8\sqrt{3}+13\right)^{13} \text{ and } y=\left(7\sqrt{2}+9\right)^{9}$$

$$x=\left(8\sqrt{3}+13\right)^{13}C_0\left(8\sqrt{3}\right)^{13}+{}^{13}C_1\left(8\sqrt{3}\right)^{12}(13)^1+.......$$

$$x'=\left(8\sqrt{3}-13\right)^{13}={}^{13}C_0\left(8\sqrt{3}\right)^{13}-{}^{13}C_1\left(8\sqrt{3}\right)^{12}(13)^1+.......$$

$$x-x'=2\left[{}^{13}C_1.\left(8\sqrt{3}\right)^{12}(13)^1+{}^{13}C_3\left(8\sqrt{3}\right)^{10}.(13)^3......\right]$$

Therefore, x – x' is even integer, hence [x] is even.
Now,

$$y=\left(7\sqrt{2}+9\right)^{9}={}^{9}C_0\left(7\sqrt{2}\right)^{9}+{}^{9}C_1\left(7\sqrt{2}\right)^{8}(9)^1+{}^{9}C_2\left(7\sqrt{2}\right)^{7}9^2.....$$

$$y'=\left(7\sqrt{2}-9\right)^{9}={}^{9}C_0\left(7\sqrt{2}\right)^{9}-{}^{9}C_1\left(7\sqrt{2}\right)^{8}9^1+{}^{9}C_2\left(7\sqrt{2}\right)^{7}(9)^2+...$$

$$y-y'=2\left[{}^{9}C_1\left(7\sqrt{2}\right)^{8}(9)^1+{}^{9}C_3\left(7\sqrt{2}\right)^{6}.(9)^3+...\right]$$

y – y′ = even integer, hence [y] is even.

96. If $f(x)=\log_e\left(\frac{1-x}{1+x}\right)$, |x| < 1, then $f\left(\frac{2x}{1+x^2}\right)$ is equal to

(a) $2f(x)$ (b) $2f(x^2)$
(c) $(f(x))^2$ (d) $-2f(x)$

JEE Main 08.04. 2019 Shift-I

Ans. (a) : Given that,

$$f(x)=\log\left(\frac{1-x}{1+x}\right)$$

So, $$f\left(\frac{2x}{1+x^2}\right)=\log\left(\frac{1-\frac{2x}{1+x^2}}{1+\frac{2x}{1+x^2}}\right)$$

$$=\log\left(\frac{1+x^2-2x}{1+x^2+2x}\right)$$

$$=\log\left[\frac{(1+x)^2}{(1-x)^2}\right]$$

$$=2\log\left[\frac{(1+x)}{(1-x)}\right]$$

$$=2f(x)$$

97. A function f(x) is given by

$f(x)=\frac{5^x}{5^x+5}$, then the sum of the series

$f\left(\frac{1}{20}\right)+f\left(\frac{2}{20}\right)+f\left(\frac{3}{20}\right)+......f\left(\frac{39}{20}\right)$ is equal to

(a) $\frac{29}{2}$ (b) $\frac{49}{2}$ (c) $\frac{39}{2}$ (d) $\frac{19}{2}$

JEE Main 25.02. 2021 Shift-II

Ans. (c) : Given that, $f(x)=\frac{5^x}{5^x+5}$

$$f(2-x)=\frac{5^{(2-x)}}{5^{(2-x)}+5}$$

$$=\frac{\frac{25}{5^x}}{\frac{25}{5^x}+5}$$

$$=\frac{\frac{25}{5^x}}{\frac{25+5\times5^x}{5^x}}$$

$$f(2-x)=\frac{5}{5+5^x}$$

Therefore,

$$f(x)+f(2-x)=\frac{5^x}{5^x+5}+\frac{5}{5+5^x}=1$$

Hence,

$$\left[f\left(\frac{1}{20}\right)+f\left(\frac{39}{20}\right)\right]+\left[f\left(\frac{19}{20}\right)+f.\left(\frac{21}{20}\right)\right]+\left[f\left(\frac{20}{20}\right)\right]$$

$= 1 + 1 + 1 + \ldots..19 \text{ times} + \frac{1}{2}$

$=19\times1+\frac{1}{2}=\frac{39}{2}$

98. The inverse function of $f(x)=\frac{8^{2x}-8^{-2x}}{8^{2x}+8^{-2x}}$, $x\in(-1, 1)$ is

(a) $\frac{1}{4}(\log_8 e)\log_e\left(\frac{1-x}{1+x}\right)$

(b) $\frac{1}{4}\log_e\left(\frac{1-x}{1+x}\right)$

(c) $\frac{1}{4}(\log_8 e)\log_e\left(\frac{1+x}{1-x}\right)$

(d) $\frac{1}{4}\log_e\left(\frac{1+x}{1-x}\right)$

JEE Main 08.01.2020 Shift -I

Ans. (c) : We have function,

$$f(x)=\frac{8^{2x}-8^{-2x}}{8^{2x}+8^{-2x}}$$

Let,

$f(x) = y$

$\Rightarrow \quad x = f^{-1}(y)$

Now,

$$y=\frac{8^{2x}-8^{-2x}}{8^{2x}+8^{-2x}}$$

Applying componendo and dividendo rule-

$$\left\{\begin{array}{l}\frac{a}{b}=\frac{c}{d}\\ \frac{a+b}{a-b}=\frac{c+d}{c-d}\end{array}\right\}$$

So,

$$\frac{y+1}{y-1}=\frac{(8^{2x}-8^{-2x})+(8^{2x}+8^{-2x})}{(8^{2x}-8^{-2x})-(8^{2x}+8^{-2x})}$$

$$\frac{y+1}{y-1}=\frac{2\times8^{2x}}{-2\times8^{-2x}}$$

$$\frac{y+1}{y-1}=\frac{-8^{2x}}{\frac{1}{8^{2x}}}=-8^{2x}\cdot8^{2x}$$

$$\frac{y+1}{1-y}=8^{4x}$$

On taking log both side with base 8, we get –

$$\log_8\left(\frac{y+1}{1-y}\right)=\log_8 8^{4x}$$

$$\log_8\left(\frac{y+1}{1-y}\right)=4x\log_8 8$$

$$4x=\log_8\left(\frac{y+1}{1-y}\right)$$

$$x=\frac{1}{4}\log_8\left(\frac{y+1}{1-y}\right)$$

$$f^{-1}(x)=\frac{1}{4}\log_8\left(\frac{1+x}{1-x}\right)$$

$$f^{-1}(x)=\frac{1}{4}(\log_8 e)\left(\log_e\frac{1+x}{1-x}\right)$$

99. Let $f : R - \{3\} \to R - \{1\}$ be defined by $f(x) = \frac{x-2}{x-3}$. Let $g: R \to R$ be given as $g(x) = 2x - 3$. Then, the sum of all the values of x for which $f^{-1}(x) + g^{-1}(x) = \frac{13}{2}$ is equal to

(a) 7 (b) 2
(c) 5 (d) 3

JEE Main 18.03.2021, Shift - II

Ans. (c) : Given that-

$$f(x)=\frac{x-2}{x-3}$$

And, $g(x) = 2x - 3$

Let, $f(x)=y=\frac{x-2}{x-3}$

$yx - 3y = x - 2$

$(y - 1)x = 3y - 2$

$$x=\frac{3y-2}{y-1}$$

$$\therefore \quad f^{-1}(x)=\frac{3x-2}{x-1}$$

Now,

$g(x) = y = 2x - 3$

$$x=\frac{y+3}{2}$$

$$\because \quad g^{-1}(x)=\frac{x+3}{2}$$

$$\therefore \quad f^{-1}(x)+g^{-1}(x)=\frac{13}{2}$$

$$\frac{3x-2}{x-1}+\frac{x+3}{2}=\frac{13}{2}$$

$$\frac{2(3x-2)+(x+3)(x-1)}{(x-1)\times2}=\frac{13}{2}$$

$$\frac{6x-4+x^2+3x-x-3}{2(x-1)}=\frac{13}{2}$$

$$\frac{x^2+8x-7}{2(x-1)}=\frac{13}{2}$$
$2x^2 + 16x - 14 = 13 \times 2 (x - 1)$
$2x^2 + 16x - 26x - 14 + 26 = 0$
$2x^2 - 10x + 12 = 0$
$x^2 - 5x + 6 = 0$
$x^2 - (2 + 3) x + 6 = 0$
$x^2 - 2x - 3x + 6 = 0$
$x (x - 2) - 3 (x - 2) = 0$
$(x - 2) (x - 3 = 0$
$x = 2, 3$
So, sum of all values of x = 2 + 3 = 5

100. For $x \in \left(0, \frac{3}{2}\right)$ let $f(x) = \sqrt{x}$ $g(x) = \tan x$ and $h(x) = \frac{1-x^2}{1+x^2}$. If $\phi(x)$ = ((hof)og) (x), then $\phi\left(\frac{\pi}{3}\right)$ is equal to

(a) $\tan\frac{\pi}{12}$ (b) $\tan\frac{11\pi}{12}$
(c) $\tan\frac{7\pi}{12}$ (d) $\tan\frac{5\pi}{12}$

JEE Main 12.04.2019 Shift -I

Ans. (b) : Given that,

$f(x) = \sqrt{x}$, $g(x) = \tan x$, $h(x) = \frac{1-x^2}{1+x^2}$

Now, $\phi(x)$ = (hof)og) (x) = h(f(g(x)))

$= h(f(\tan x)) = h\left(\sqrt{\tan x}\right)$

$$= \frac{1-\left(\sqrt{\tan x}\right)^2}{1+\left(\sqrt{\tan x}\right)^2} = \frac{1-\tan x}{1+\tan x}$$

Or, $$\phi(x) = \frac{\tan\frac{\pi}{4} - \tan x}{1+\tan x \tan\frac{\pi}{4}}$$

$\phi(x) = \tan\left(\frac{\pi}{4} - x\right)$

Hence, $\phi\left(\frac{\pi}{3}\right) = \tan\left(\frac{\pi}{4} - \frac{\pi}{3}\right) = \tan\left(-\frac{\pi}{12}\right) = -\tan\frac{\pi}{12}$

$\phi\left(\frac{\pi}{3}\right) = \tan\left(\pi - \frac{\pi}{12}\right)$

$\phi\left(\frac{\pi}{3}\right) = \tan\frac{11\pi}{12}$

101. If $g(x) = x^2 + x - 1$ and (gof) (x) = $4x^2 - 10x + 5$, then $f\left(\frac{5}{4}\right)$ is equal to

(a) $-\frac{1}{2}$ (b) $\frac{3}{2}$ (c) $-\frac{3}{2}$ (d) $\frac{1}{2}$

JEE Main 07.01.2020 Shift - I

Ans. (a) : We have-
we have-
$g(x) = x^2 + x - 1$
$(gof)(x) = 4x^2 - 10x + 5$
Now, $g(f(x)) = (f(x))^2 + f(x) - 1$
$g(f(x)) = 4x^2 - 10x + 5$

$g(f(5/4)) = 4\left(\frac{5}{4}\right)^2 - 10\left(\frac{5}{4}\right) + 5$

$g(f(5/4)) = \frac{25}{4} - \frac{50}{4} + 5$

$g(f(5/4)) = -\frac{5}{4}$

So, $g\left(f\left(\frac{5}{4}\right)\right) = \left[f\left(\frac{5}{4}\right)\right]^2 + f\left(\frac{5}{4}\right) - 1$

$-\frac{5}{4} = \left[f\left(\frac{5}{4}\right)\right]^2 + f\left(\frac{5}{4}\right) - 1$

$\left[f\left(\frac{5}{4}\right)\right]^2 + f\left(\frac{5}{4}\right) + \frac{1}{4} = 0$

$\left[f\left(\frac{5}{4}\right) + \frac{1}{2}\right]^2 = 0$

Hence, $f\left(\frac{5}{4}\right) = -\frac{1}{2}$

102. Let $f : R - \left\{\frac{\alpha}{6}\right\} \rightarrow R$ be defined by $f(x) = \frac{5x+3}{6x-\alpha}$, Then, the value of α for which (fof) (x) = x for all $x \in R - \left\{\frac{\alpha}{6}\right\}$ is

(a) No such α exists (b) 5
(c) 8 (d) 6

JEE Main 20.07.2021 Shift-II

Ans. (b) : Function defined by-

$f : R\left\{\frac{\alpha}{6}\right\} \rightarrow R$, $f(x) = \frac{5x+3}{6x-\alpha}$

Now, (fof) (x) = x

$f(f(x)) = \frac{5f(x)+3}{6f(x)-\alpha}$

$$x = \frac{5\left(\frac{5x+3}{6x-\alpha}\right)+3}{6\left(\frac{5x+3}{6x-\alpha}\right)-\alpha}$$

$$x = \frac{\frac{25x+15}{6x-\alpha}+3}{\frac{30x+18}{6x-\alpha}-\alpha}$$

So, $\frac{25x+15}{6x-\alpha} + 3 = x\left[\frac{30x+18}{6x-\alpha} - \alpha\right]$

$$\frac{25x+15+18x-3\alpha}{6x-\alpha}=x\left[\frac{30x+18-6\alpha x+\alpha^2}{6x-\alpha}\right]$$

$\Rightarrow \quad 43x - 3\alpha + 15 = 30x^2 + 18x - 6\alpha x^2 + \alpha^2 x$

$30x^2 + 18x - 6\alpha x^2 + \alpha^2 x - 43x + 3\alpha - 15 = 0$

$(30 - 6\alpha)x^2 + (\alpha^2 - 25)x + 3\alpha - 15 = 0$

$6(5 - \alpha)x^2 + (\alpha + 5)(\alpha - 5)x + 3(\alpha - 5) = 0$

$-6(\alpha - 5)x^2 + (\alpha + 5)(\alpha - 5)x + 3(\alpha - 5) = 0$

$(\alpha - 5)[-6x^2 + (\alpha + 5)x + 3] = 0$

So, $(\alpha - 5) = 0$

$\alpha = 5$

Type V — Types of Function and Number of Functions

103. The function f:N – {1} → N; defined by f(n) = the highest prime factor of n, is:

(a) neither one-one nor onto
(b) one-one only
(c) both one-one and onto
(d) onto only

JEE Mains 27/01/2024 Shift-I

Ans. (a) :

$f : N - \{1\} \to N$

f (n) = the highest prime factor of n, $n \in N$-

$f(2) = 2$	$f(2) = f(4) (2 \neq 4)$
$f(3) = 3$ $\Rightarrow$	Since, at n = 2 and n = 4 have a common image in the co- domain set.
$f(4) = 2$	So, it is not a one - one function.

$\Rightarrow$ For y = 4 in the co- domain set, there is not any such natural number in the domain set as per the given function, also 4 is not a prime number , so, it has not any pre - image in the domain set.

So, it is also not an onto function.

$\Rightarrow$ Neither one – one nor onto

104. The function $f(x) = \dfrac{x^2 + 2x - 15}{x^2 - 4x + 9}$, x∈R is

(a) Both one-one and onto
(b) Onto but not one-one
(c) Neither one-one nor onto
(d) One-one but not onto.

JEE Mains 06/04/2024 Shift-I

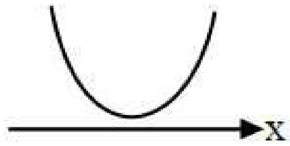
Ans. (c) : Given, $f(x) = \dfrac{x^2 + 2x - 15}{x^2 - 4x + 9}$, x∈R

$$f(x) = \frac{(x+5)(x-3)}{x^2 - 4x + 9}$$

Let $g(x) = x^2 - 4x + 9$

$D < 0$

$g(x) > 0$ for $x \in R$



$\therefore \begin{bmatrix} f(-5) = 0 \\ f(3) = 0 \end{bmatrix}$

So, f (x) is many-one.

again,

$yx^2 - 4xy + 9y = x^2 + 2x - 15$

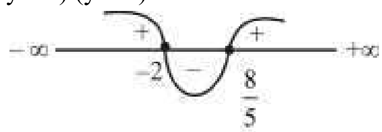
$x^2(y - 1) - 2x(2y + 1) + (9y + 15) = 0$

for $\forall x \in R \Rightarrow D \geq 0$

$D = 4(2y + 1)^2 - 4(y - 1)(9y + 15) \geq 0$

$5y^2 + 2y - 16 \leq 0$

$(5y - 8)(y + 2) \leq 0$



$y \in \left[-2, \frac{8}{5}\right]$ range

Note: If function is defined from f: R → R then only correct answer is option (c)

105. Let A = {1, 2, 3, ..., 7} and let P(A) denote the power set of A. If the number of functions f : A → P (A) such that a ∈f (a), ∀ a∈A is m^n, m and n∈N and m is least, then m + n is equal to ________.

JEE Mains 30/01/2024 Shift-I

Ans. (44): We know that,

Let A is a set

$n(A) = m$

$n(P(A)) = 2^m$

So,

$f : A \to P(A),\ a \in f(a)$

i.e., a will contain with subset which contain element a.

A = {1, 2, 3, 4, 5, 6, 7,}

Total option for 1 = 2^6

Similarly every other element have option = 2^6

Total option = $(2^6)^7 = 2^{42} = m^n$

Then, m + n = 2 + 42 = 44

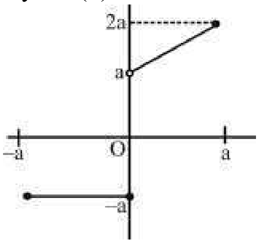
106. Let $f(x) = \begin{cases} -a, \text{ if } -a \leq x \leq 0 \\ x + a, \text{ if } 0 < x £ a \end{cases}$ where a > 0 and g(x) = (f|x|) – |f(x)|/2.

Then the function g : [–a, a] → [–a, a] is

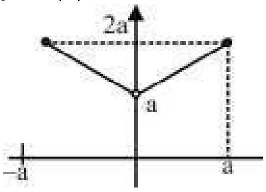
(a) neither one-one nor onto.
(b) both one-one and onto.
(c) one-one.
(d) onto

JEE Mains 08/04/2024 Shift-II

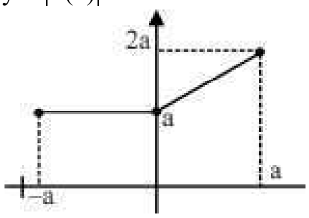
Ans. (a) : y = f (x)



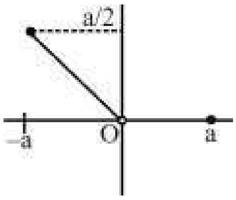
y = f |x|



y = |f (x)|



$$g(x) = \frac{f(|x|) - |f(x)|}{2}$$



107. If the function $f : (-\infty, -1) \rightarrow (a, b]$ defined by $f(x) \quad e^{x^3 \ 3x \ 1}$ is one-one and onto, the distance of the point $P(2b \quad 4, a \quad 2)$ from the line $x \quad e^{\ 3}y \quad 4$ is :

(a) $\sqrt{1 \quad e^6}$ (b) $2\sqrt{1 \quad e^6}$

(c) $4\sqrt{1 \quad e^6}$ (d) $3\sqrt{1 \quad e^6}$

JEE Mains 31/01/2024 Shift-II

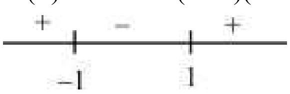
Ans. (b) : Given,

$f : (\quad , \ 1) \quad (a, b]$

$f(x) \quad e^{x^3 \ 3x \ 1}$

$f\ (x) \quad e^{x^3 \ 3x \ 1}(3x^2 \quad 3)$

$f\ (x) \quad e^{x^3 \ 3x \ 1}3(x \quad 1)(x \quad 1)$



$(\quad , \ 1] \qquad F^n$

So x

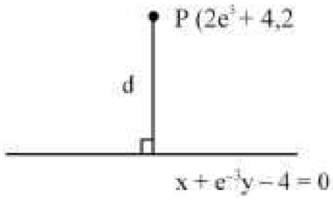
$\Rightarrow f(x) = e^{-\infty} = 0 = a$

x 1

$\Rightarrow f(-1) = e^{-1+3+1} = e^3 = b$

$(a, b] = (0, e^3]$

$\therefore \ P(2e^3 \quad 4, 2)$



$$d \quad \left| \frac{2e^3 \quad 4 \quad e^{\ 3}(2) \quad 4}{\sqrt{1 \quad (e^{\ 3})^2}} \right|$$

$$d \quad \frac{2(e^3 \quad e^{\ 3})}{\sqrt{\frac{e^6 \quad 1}{e^6}}}$$

$$v \quad \frac{\frac{2}{e^3}(1 \quad e^6)}{\frac{\sqrt{1 \quad e^6}}{e^3}}$$

$$\boxed{d \quad 2\sqrt{1 \quad e^6}}$$

108. Let *f*, g : R → R be defined as: *f*(x) = |x – 1| and $g(x) = \begin{cases} e^x, & x \geq 0 \\ x+1, & x \leq 0 \end{cases}$. Then the function *f*(g(x)) is

(a) Neither one-one nor onto

(b) One-one but not onto

(c) Both one-one and onto

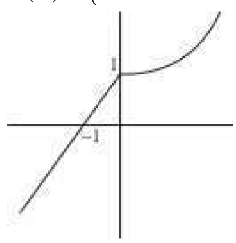
(d) Onto but not one-one.

JEE Mains 05/04/2024 Shift-II

Ans. (a) : Given,

f (x) = |x–1|

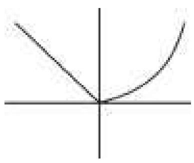
$$f(x) = \begin{cases} 1-x & x<1 \\ x-1 & x>1 \end{cases}$$



Now, f(g(x)) = |g(x) –1|

$$fog = \begin{cases} |e^x - 1| & x \geq 0 \\ |x+1-1| & x \leq 0 \end{cases}$$

$$fog = \begin{cases} e^x - 1 & x \geq 0 \\ -x & x \leq 0 \end{cases}$$



Hence, neither one- one nor onto function.

109. Let A = {(x, y) : 2x + 3y = 23, x, y ∈ N} and B = {x : (x, y)∈A}. Then the number of one-one functions from A to B is equal to ________.

JEE Mains 09/04/2024 Shift-II

Ans. (24) : 2x + 3y = 23

x = 1 y = 7
x = 4 y = 5
x = 7 y = 3
x = 10 y = 1

So, A = $\{(1,7),(4,5),(7,3),(10,1)\}$

B = $\{1,4,7,10\}$

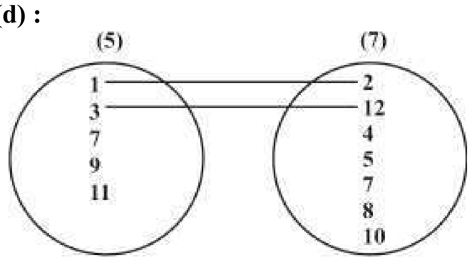
The number of one-one functions from A to B is equal to 4!= 24

110. Let A={1,3,7,9,11} and B ={2,4,5,7,8,10,12}. Then the total number of one-one maps f:A→B, such that f(1) + f(3) = 14, is:

(a) 180 (b) 120
(c) 480 (d) 240

JEE Mains 05/04/2024 Shift-I

Ans. (d) :



A={1,3,7,9,11}
B ={2,4,5,7,8,10,12}
f(1) + f(3) = 14

Case – I f(1) = 2, f(3) = 12
f (1) = 12, f(3) = 2

Total one-one function = 2 × 5 × 4 × 3 = 120

Case – II f(1) = 4, f(3) = 10
f(1) = 10, f(3) = 4

Total number of one-one function = 2× 5 × 4 × 3 =120

Hence, the total number of one-one function in case - I and II = 120 + 120 = 240

111. Let A = {1, 2, 3, 4, 5} and B = { 1, 2, 3, 4, 5, 6}. Then the number of functions f:A→B satisfying f(1) + f(2) = f(4)–1 is equal to ....

JEE Mains 11/04/2023 Shift-II

Ans. (360) : f(1) +f(2) + 1 = f(4) ≤ 6
f(1) + f(2) ≤ 5

Case (i) f(1) = 1 ⇒ f(2) = 1, 2, 3, 4 ⇒ 4 mappings
Case (ii) f(1) = 2 ⇒ f(2) = 1, 2, 3 ⇒ 3 mappings
Case (iii) f(1) = 3 ⇒ f(2) = 1 , 2 ⇒ 2 mappings
Case (iv) f(1)4 ⇒ f(2) = 1 ⇒ 1 mapping
f(5) & f(6) both have 6 mappings each
Number of functions = (4 + 3 + 2 +1) × 6 × 6 = 360

112. Let R = {a, b, c, d, e} and S = {1, 2, 3, 4}. Total number of onto functions f : R → S such that f(a) ≠ 1, is equal to ____.

JEE Mains 08/04/2023 Shift-II

Ans. (180): Sol.

Total onto function

$\frac{\lfloor 5}{\lfloor 3 \lfloor 2} \times \lfloor 4 = 240$

Now when f(a) = 1

$\lfloor 4 + \frac{\lfloor 4}{\lfloor 2 \lfloor 2} \times \lfloor 3 = 24 + 36 = 60$

So required f^n = 240 – 60 = 180

Or

If f(a) ≠ 1

For a (remaining 4 to all 4 + remaining 4 to all 3)

$= {}^3C_1\left(4! + \frac{4!}{2!2!} \times 3!\right)$

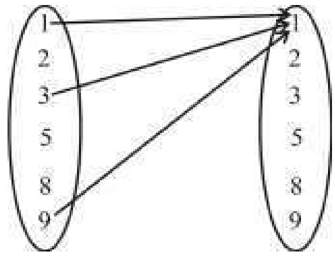
$= 3(24 + 36) = 180$

113. Let A = {1, 2, 3, 5, 8, 9}. Then the number of possible functions f : A → A such that f(m.n) = f(m). f(n) for every m, n ∈ A with m.n ∈ A is equal to _______.

JEE Mains 30/01/2023 Shift-II

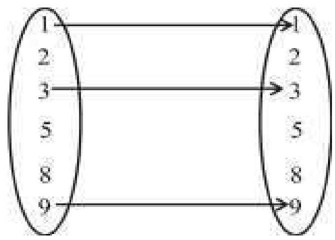
Ans. (432) : Total number of function = 6^6

f (m.n) = f (m) f (n)
m= n= 1
f (1) = f (1). f (1)
f (1) = 1
m = 3, n= 3
f (9) = f (3). f (3)
f(3) = 1
f (9) = 1
or,
f (3) = 3
f (9) = 9
Similarly,



So, f (2) , f (5) and f (8)

1×6× **1**× 6× 1×6 = 216



1×6× **1**× 6× 1×6 = 216

Total possible function = 216 + 216 = 432

114. Let S = {1, 2, 3, 4, 5, 6}. Then the number of one-one functions f : S → P(S), where P(S) denote the power set of S, such that f(n) ⊂ f(m) where n < m is _______.

JEE Mains 30/01/2023 Shift-I

Ans. (3240) : Given,
S = {1, 2, 3, 4, 5, 6}
⇒ n(S) = 6
P(S) = {ϕ, {1}, … {6}, {1, 2}, … {5, 6}, … {1, 2, 3, 4, 5, 6}}

Case–1
f(6) = S i.e. 1 option.
f(5) = any 5 elements subset A of S i.e. 6 options.
f(4) = any 4 element subset B of A i.e. 5 options.
f(3) = any 3 element subset C of B i.e. 4 options.
f(2) = any 2 element subset D of C i.e. 3 options.
f(1) = any 1 element subset E of D or empty subset i.e 3 options.
Total function = 1080.

Case-2
f(6) = any 5 element subset A of S i.e. 6 options.
f(5) = any 4 elements subset B of A i.e. 5 options.
f(4) = any 3 element subset C of B i.e. 4 options.
f(3) = any 2 element subset D of C i.e. 3 options.
f(2) = any 1 element subset E of D i.e. 2 options.
f(1) = empty subset i.e. 1 option.
Total functions = 720.

Case-3
f(6) = S
f(5) = any 4 element subset A of S i.e. 15 options.
f(4) = any 3 elements subset B of A i.e. 4 options.
f(3) = any 2 element subset C of D i.e. 3 options.
f(2) = any 1 element subset D of C i.e. 2 options.
f(1) = empty subset i.e. 1 option.
Total functions = 360.

Case-4
f(6) = S
f(5) = any 5 element A of S i.e. 6 options.
f(4) = any 3 elements subset B of A i.e. 10 options.
f(3) = any 2 element subset C of B i.e. 3 options.
f(2) = any 1 element subset D of C i.e. 2 options.
f(1) = empty subset i.e. 1 option.
Total functions = 360.

Case-5
f(6) = S
f(5) = any 5 element A of S i.e. 6 options.
f(4) = any 4 elements subset B of A i.e. 5 options.
f(3) = any 2 element subset C of B i.e. 6 options.
f(2) = any 2 element subset D of C i.e. 2 options.
f(1) = empty subset i.e. 1 option.
Total functions = 360.

Case-6
f(6) = S
f(5) = any 5 element A of S i.e. 6 options.
f(4) = any 4 elements subset B of A i.e. 5 options.
f(3) = any 3 element subset C of B i.e. 4 options.
f(2) = any 2 element subset D of C i.e. 3 options.
f(1) = empty subset i.e. 1 option.
Total functions = 360.
∴ Number of such functions = 3240

115. Let f: R → R be a function such that $f(x) = \frac{x^2+2x+1}{x^2+1}$. Then

(a) f(x) is one-one in (1, ∞) but not in (–∞, ∞)
(b) f(x) is one-one in (–∞, ∞)
(c) f(x) is many-one in (–∞, –1)
(d) f(x) is many-one in (1, ∞)

JEE Mains 29/01/2023 Shift-I

Ans. (a) : Given,

$$f(x) = \frac{x^2+2x+1}{x^2+1}, \text{ where } f : R \to R$$

$$f(x) = \frac{(x^2+1)+2x}{x^2+1} = \frac{x^2+1}{x^2+1} + \frac{2x}{x^2+1}$$

$$= 1 + \frac{2x}{x^2+1}$$

Differentiate

$$f'(x) = \frac{0+(x^2+1)2-2x(2x)}{(x^2+1)^2}$$

$$f'(x) = \frac{2(x^2+1)-4x^2}{(x^2+1)^2} = \frac{2x^2+2-4x^2}{(x^2+1)^2}$$

$$f'(x) = \frac{2-2x^2}{(x^2+1)^2} = \frac{2(1-x)(1+x)}{(x^2+1)^2}$$

Hence function f(x) is one-one are in [1, ∞] but not in (∞, ∞)

116. The number of functions f: {1,2,3,4} → {a∈:Z| a| ≤ 8} satisfying $f(n) + \frac{1}{n} f(n+1) = 1$, ∀ n∈{1,2,3} is

(a) 2
(b) 3
(c) 4
(d) 1

JEE Mains 25/01/2023 Shift-II

Ans. (a) : Given,

$$f(n) + \frac{1}{n}.f(n+1) = 1$$

n. f (n) +f (n + 1) = n

If, n = 1
f (1) +f (2) = 1 ....(i)

If, n = 2
2f (2) + f (3) = 2 ...(i)

If, n = 3
3.f(3) + f (4) = 3 ... (iii)

From equation (i), we get-
2 f (1) + 2f (2) = 2 ... (iv)

On subtracting equation (iv) from (ii), we get-

$f(3) - 2f(1) = 0$

$f(3) = 2f(1)$ ... (v)

In equation (iii), we get

$3.(2f(1)) + f(4) = 3$

$6f(1) + f(4) = 3$

$f(4) = 3 - 6f(1)$

Now, $-8 \le f(4) \le 8$

$-8 \le 3 - 6f(1) \le 8$

$\frac{-5}{6} \le f(1) \le \frac{11}{6}$

$\therefore$ f(1) = 0, 1

Case-I $f(1) = 0$, $f(2) = 1$

$f(3) = 0$, $\Rightarrow f(4) = 3$

Case-II $f(1) = 1$, $f(2) = 0$

$f(3) = 2$, $\Rightarrow f(4) = -1$

The number of possible function is 2.

117. Let $f(x) = 2x^n + \lambda$, $\lambda \in R$, m∈N, and $f(4) = 133$, $f(5) = 255$. Then the sum of all the positive integer divisors of $(f(3) - f(2))$ is

(a) 61
(b) 58
(c) 59
(d) 60

JEE Mains 25/01/2023 Shift-II

Ans. (d) : Given function,

$f(x) = 2x^n + \lambda, \lambda \in R$

$f(4) = 133$

$f(5) = 255$

$133 = 2.4^n + \lambda$ ……(i)

$255 = 2.5^n + \lambda$ ……(ii)

On subtracting equation (i) from (ii), we get–

$122 = 2(5^n - 4^n)$

$61 = 5^n - 4^n$

here, n = 3

From equation (i), we get–

$133 = 2.4^3 + \lambda$

$= 2.64 + \lambda$

$133 = 128 + \lambda$

$\Rightarrow \lambda = 5$

$f(x) = 2\times 3 + 5$

$\Rightarrow f(3) = 2.3^3 + 5 = 2.27 + 5 = 54 + 5 = 59$

$f(2) = 2.2^3 + 5 = 2.8 + 5 = 21$

$f(3) - f(2) = 59 - 21 = 38$

$= 2 \times 19$

Sum of all the positive integers

divisors = 2 + 19 + 38 + 1

= 60

118. For some a.b.c. $\in N$, let $f(x) = ax - 3$ **and** $g(x) = x^b + c, x \in R$. **If** $(fog)^{-1}(x) = \left(\frac{x-7}{2}\right)^{1/3}$,

then (fog) (ac)+(gof) (b) is equal to ______.

JEE Mains 25/01/2023 Shift-I

Ans. (2039) : Let fog (x) = h (x)

$\Rightarrow h^{-1}(x) = \left(\frac{x-7}{2}\right)^{\frac{1}{3}}$

$\Rightarrow h(x) = fog(x) = 2x^3 + 7$

$fog(x) = a(x^b + c) - 3$

$\Rightarrow$ a = 2, b = 3, c = 5

$\Rightarrow$ fog(ac) = fog(10) = 2007

$g(f(x) = (2x - 3)^3 + 5$

$\Rightarrow$ gof(b) = gof(3) = 32

$\Rightarrow$ sum = 2039

119. The total number of functions, f : {1, 2, 3, 4} – {1, 2, 3, 4, 5, 6} such that f(1) + f(2) = f(3), is equal to:

(a) 60 (b) 90
(c) 108 (d) 126

JEE Main-25.07.2022, Shift-I

Ans. (b) : Given,

A = {1, 2, 3, 4}

B = {1, 2, 3, 4, 5, 6}

Here f(3) can be 2, 3, 4, 5, 6

Then, f(3) = 2, (f(1), f(2)) → (1, 1) → 6 cases

f(3) = 3, (f(1), f(2)) → (1, 2), (2, 1)

→ 2 × 6 = 12 cases

f(3) = 4, (f(1), f(2)) → (1, 3), (3, 1), (2, 2)

→3

6 = 18 cases

f(3) = 5, (f(1), f(2)) → (1, 4), (4, 1), (2, 3), (3, 2)

→ 4 × 6 = 24 cases

f(3) = 6, (f(1)) , f(2)) → (1, 5), (5, 1), (2, 4), (4, 2), (3, 3)

→ 5 × 6 = 30 cases

Total number of cases = 6 + 12 + 18 + 24 + 30 = 90

120. The number of functions f, from the set A = {x ∈ N : $x^2 - 10x + 9 \le 0$} to the set B = {n^2 : n ∈ N} such that $f(x) \le (x-3)^2 + 1$, for every x ∈ A, is _______.

JEE Main-27.07.2022, Shift-II

Ans. (1440) : Given,

$(x^2 - 10x + 9) \le 0$

$(x-1)(x-9) \le 0$

$x \in [1, 9]$

A = {1, 2, 3, 4, 5, 6, 7, 8, 9}

Now,

$f(x) \le (x-2)^2 + 1$

$x = 1 : f(1) \le 5 \Rightarrow 1^2, 2^2$

$x = 2 : f(2) \le 2 \Rightarrow 1^2$

$x = 3 : f(3) \le 1 \Rightarrow 1^2$

$x = 4 : f(4) \le 2 \Rightarrow 1^2$

$x = 5 : f(5) \le 5 \Rightarrow 1^2, 2^2$

$x = 6 : f(6) \le 10 \Rightarrow 1^2, 2^2, 3^2$

$x = 7 : f(7) \le 17 \Rightarrow 1^2, 2^2, 3^2, 4^2$

$x = 8 : f(8) \le 26 \Rightarrow 1^2, 2^2, 3^2, 4^2, 5^2$

$x = 9 : f(9) \le 37 \Rightarrow 1^2, 2^2, 3^2, 4^2, 5^2, 6^2$

Total number of function = 2(6!) = 2(720) = 1440

121. The number of functions f: {1, 2, 3, 4} → {a ∈ Z: |a| ≤ 8} satisfying f(n) $+\frac{1}{n}f(n+1)$ = 1, ∀ n ∈ {1, 2, 3} is

(a) 2 (b) 1
(c) 4 (d) 3

JEE Main-25.01.2023, Shift-II

Ans. (a) : Given,

$f(n) + \frac{1}{n}f(n+1) = 1$

n.f(n) + f (n + 1) = 1

When n = 1

f(1) + f(2) = 1 …(i)

When n = 2

2f(2) + f(3) = 2 …(ii)

When n = 3

3f(3) + f(4) = 3 …(iii)

Now, multiple by 2 in equation (i), we get –

2f (1) + 2f (2) = 2 …(iv)

On subtracting equation (iv) from (ii), we get –

f(3) – 2f (1) = 0

f(3) = 2f(1) …(iv)

Now, putting the value in equation (iii), we get–

3[2f (1)] + f(4) = 3

6f(1) + f(4) = 3

f(4) = 3 – 6f (1)

Therefore, – 8 ≤ f (4) ≤ 8

– 8 ≤ 3 – 6 f (1) ≤ 8

– 11 ≤ – 6f (1) ≤ 5

$-\frac{5}{6} \le f(1) \le \frac{11}{6}$

f (1) = 0, 1

Case – I : f (1) = 0, f(2) = 1

f(3) = 0, f(4) = 3

Case – II: f(1) = 1, f(2) = 0

f(3) = 2, f(4) = –3

There can be 2 function such that like this.

122. Let A = {1, 2, 3, 4, 5} and B = {1, 2, 3, 4, 5, 6}. Then the number of function f : A → B satisfying f(1) + f(2) = f(4) – 1 is equal to ______

JEE Main-11.04.2023, Shift-II

Ans. (360) : Given,

A = { 1, 2, 3, 4, 5}

And, B = {1, 2, 3, 4, 5, 6}

Now,

f(1) + f(2) + 1 = f (4) ≤ 6

f (1) + f (2) ≤ 5

C – I

f (1) = 1 ⇒ f(2) = 1, 2, 3, 4 ⇒ 4 mappings

C–II

f (1) = 2 ⇒ f (2) = 1, 2, 3, ⇒ 3 mappings

C. II

f (1) = 3 ⇒ f (2) = 1, 2 ⇒ 2 mapping

C.IV

f(1) = 4 ⇒ f(2) = 1 ⇒ 1 mapping

And f (5) and f(6) both have 6 and 6 mapping.

Hence, the number of function = (4 + 3 + 2 + 1) × 6 × 6

= 10 × 36

= 360

123. The number of functions f from {1, 2, 3, ... 20} onto {1, 2, 3, ... 20}such that f(k) is a multiple of 3, whenever k is a multiple of 4, is

(a) (15)! × 6! (b) $5^6 \times 15$
(c) 5! × 6! (d) $6^5 \times (15)!$

JEE Main 11.01.2019 Shift - II

Ans. (a) : Let, the multiple of 3 is f (k).

f(k) = (3, 6, 9, 12, 15, 18)

for k = 4, 8, 12, 16, 20

For these k we have 6.5.4.3.2 ways = 6!

For other numbers we have 15! ways.

So total = 15! 6!.

124. Let f(x) = $2x^n + \lambda$, λ ∈ R, n ∈ N, and f(4) = 133, f(5) = 255. Then the sum of all the positive integer divisors of {(f(3) – f(2)} is

(a) 59 (b) 60
(c) 61 (d) 58

JEE Main-25.01.2023, Shift-II

Ans. (b) : Given,

$f(x) = 2x^n + \lambda$, λ ∈ R and n ∈ N

f(4) = 133,

f(5) = 255.

$f(4) = 133 = 2 \times (4)^n + \lambda$ ….(i)

$f(5) = 255 = 2 \times (5)^n + \lambda$ ….(ii)

Now, subtracting the equation-

$2\{(5)^n - (4)^n\} = 255 - 133$

$(5)^n - (4)^n = \frac{122}{2}$

$(5)^n - (4)^n = 61$

$(5)^n - (4)^n = (5)^3 - (4)^3$

n = 3

From equation (i) –

$2 \times (4)^3 + \lambda = 133$

λ = 133 – 128

λ = 5

Now, f(3) – f(2)

$=\{2(3)^3 + \lambda) - (2(2)^3 + \lambda\}$

$=2(3^3 - 2^3) = 2(27 - 8) = 38$

The number of divisor is 1, 2, 19, 38

Sum of divisor 1 + 2 + 19 + 38 = 60

125. Let A = {x : x ∈ R ; x is not a positive integer} Define f : A → R as f (x) = $\frac{2x}{x-1}$, then f is

(a) injective but not surjective
(b) surjective but not injective
(c) bijective
(d) neither injective nor surjective

Jee Mains- 09.01.2019, shift-II

Ans. (a) : Given,

$$f(x)=\frac{2x}{x-1}$$

$$f'(x)=\frac{(x-1)2-2x(1)}{(x-1)^2}$$

$$f'(x)=\frac{2x-2-2x}{(x-1)^2}$$

$$f'(x)=\frac{-2}{(x-1)^2},\ \forall\ x\in A.$$

We see that f is decreasing in its domain
So, f is one-one (injective)
Let, y = f(x)

$$y=\frac{2x}{x-1}$$

$$xy-y=2x$$
$$xy-2x=y$$
$$x(y-2)=y$$
$$x=\frac{y}{y-2}$$

Consider y = 3, then $x=\frac{3}{3-2}=3>0$

Since, x is not a positive integer.
So, f is not onto (Surjective).

126. The function f : R → R defined by f(x) = (x – 1) (x – 2)(x – 3) is

(a) one-one but not onto
(b) onto but not one-one
(c) both one-one and onto
(d) neither one-one nor onto

JEE Main-26.06.2022, Shift-II
JEE Main-27.07.2022, Shift-I

Ans. (b) : Given,

$$f(x)=(x-1)(x-2)(x-3)$$
$$f(1)=f(2)=f(3)=0$$

∴ f(x) is not one-one.
For each y ∈ R, there exists x ∈ R such that
f(x) = y.
∴ f is onto.
If a continuous function has more than one roots, then the function is always many-one.

127. Let a function $f:\mathbb{N}\rightarrow\mathbb{N}$ be defined by

$$\mathbf{f(n)=\begin{cases}2n, & n=2,4,6,8,.....\\ n-1, & n=3,7,11,15,.....\\ \frac{n+1}{2}, & n=1,5,9,13,.......\end{cases}}$$ **then, f is**

(a) one-one but not onto
(b) onto but not one-one
(c) neither one-one nor onto
(d) one-one and onto

JEE Main-28.06.2022, Shift-I

Ans. (d): Given,

$$f(n)=\begin{cases}2n, & n=2,4,6,8......\\ n-1, & n=3,7,11,15......\\ \frac{n+1}{2}, & n=1,5,9,13.....\end{cases}$$

If n = 2, 4, 6, 8, then 2n in multiple of 4.
If n = 3, 7, 11, 15 then (n–1) is not multiple of 4.
If n = 1, 5, 9, 13, then $\left(\frac{n+1}{2}\right)$ is the odd number.

Hence, Every numbers give exactly one value.
So, f is one – one and onto.

128. Let A = {1, 2, 3, 5, 8, 9}. Then the number of possible functions f : A→ A such that f (m.n) = f(m). f(n) for every m, n ∈ A with m. n ∈ A is equal to ______.

JEE Main-30.01.2023, Shift-II

Ans. (432) : Given,
A = {1, 2, 3, 5, 8, 9}
f (mn) = f (m). f(n)
∴ Put m = n = 1
f(1) = f(1) f (1)
f (1) = 1
Put m = n = 3
f (9) = f(3). f(3)
f (3) = 1 or 3
Total number of such function = 1 × 6 × 2 × 6 × 6 ×1
= 432

129. Let S = {1, 2, 3, 4, 5, 6}. Then the number of one-one functions f : S → P (S), where P(S) denote the power set of S, such that f(n) ⊂ f(m) where n < m is _______.

JEE Main-30.01.2023, Shift-I

Ans. (3240) : Given,
S = {1, 2, 3, 4, 5, 6}

Case – I

f(1) has only 1 element in {1}, {2}, {3}, {4}, {5}, {6}.
f(2) has 2 elements in which one is same as f(1) and so on.
Therefore,

$$^{6}C_1\cdot{}^{5}C_1\cdot{}^{4}C_1\cdot{}^{3}C_1\cdot{}^{2}C_1\cdot 1$$
$$=\frac{6!}{5!}\times\frac{5!}{4!}\times\frac{4!}{3!}\times\frac{3!}{2!}\times\frac{2!}{1!}\times 1$$
$$=6!$$
$$=6\times5\times4\times3\times2\times1$$
$$=720$$

Case – II

f(1) = ϕ

f(2)	f(3)	f(4)	f(5)	f(6)
1	2	3	4	5

: $^{6}C_1\cdot{}^{5}C_1\cdot{}^{4}C_1\cdot{}^{3}C_1\cdot{}^{2}C_1=720$

1	2	3	4	6

: $^{6}C_1\cdot{}^{5}C_1\cdot{}^{4}C_1\cdot{}^{3}C_1\cdot{}^{1}C_1=360$

1	2	4	5	6
1	2	3	5	6
1	3	4	5	6
2	3	4	5	6

$= 4 \times 360 = 1440$

Hence, the total $= 720 + 720 + 360 + 1440 = 3240$

130. The number bijective functions f : {1, 3, 5, 7 …..99} → {2, 4, 6, 8, ….100}, such that $f(3) \ge f(9) \ge f(15) \ge f(21) \ge \ldots \ge f(99)$ is _____

(a) ${}^{50}P_{17}$ (b) ${}^{50}P_{33}$

(c) $33! \times 17!$ (d) $\frac{50!}{2}$

JEE Main-25.07.2022, Shift-II

Ans. (b) : One to one functions define that each element of one set, say set (A) is mapped with a unique element of another set (B), solution of question,

As function is one-one and onto, out of 50 elements of domain set 17 elements are following restriction.

$f(3) \ge f(a) \ge f(15) \ldots\ldots \ge f(99)$

So number of ways $= {}^{50}C_{17}\ 33!$

$= {}^{50}P_{33}$

131. Let R = {a, b, c, d, e} and S = {1, 2, 3, 4}. Total number of onto function f : R → S such that f(a) ≠ 1, is equal to___ .

JEE Main-08.04.2023, Shift-II

Ans. (180) : Given,

R = {a, b, c, d, e} and S = {1, 2, 3, 4}

Now,

$$\text{Total number of onto function } = \frac{\lfloor 5}{\lfloor 3 \lfloor 2} \times \lfloor 4$$

$$= \frac{5 \times 4 \times 3! \times 4 \times 3 \times 2 \times 1}{3! \times 2 \times 1}$$

$$= 240$$

Now, when f(a) = 1

$$\lfloor 4 + \frac{\lfloor 4}{\lfloor 2 \lfloor 2} \times \lfloor 3 = 24 + 36 = 60$$

So, required $f^n = 240 - 60 = 180$

132. Let A = {x∈R : x is not a positive integer}. Define a function f : A→R as $f(x) = \frac{2x}{x-1}$, then f is

(a) Injective but not surjective.

(b) Not injective.

(c) Surjective but not injective.

(d) Neither injective nor surjective.

JEE Main 09.01.2019 Shift-II

Ans. (a) : Given,

$$f(x) = \frac{2x}{x-1}$$

On differentiating of given function with respect to x we get –

$$f'(x) = \frac{-1}{(x-1)^2} < 0 \forall x \in A$$

f is decreasing in its domain.

∴ f is injective

Let, y = f(x)

$$y = \frac{2x}{x-1}$$

$$x = \frac{y}{y-2}$$

If, $y = 3, \text{then } x = \frac{3}{3-2} = 1 > 0$

Since it is not positive integer

Hence, function is not surjective.

133. Let N be the set of natural numbers and two functions f and g be defined as f, g: N → N such that

$$\mathbf{f(n)} = \begin{cases} \frac{n+1}{2} & \text{if n is odd} \\ \frac{n}{2} & \text{if n is even} \end{cases}$$

and $g(n) = n - (-1)^n$. Then fog is

(a) one-one but not onto

(b) onto but not one-one

(c) both one-one and onto

(d) neither one-one nor onto

JEE Main 10.01.2019 Shift - II

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Ans. (b) : : For g(x), find the case where n is odd not even, fog is f(g(x)), Hence, prove that f(n) = fog(n).

Put n = 1, 2 even and odd in the expressing of fog(n), find if it's one-one.

Then check if fog(n) is onto by taking f(n) in cases of odd and even. Then prove that n is same as f(n).

One-one function.

$$f(n) = \begin{cases} \frac{n+1}{2} & \text{n is odd} \\ \frac{n}{2} & \text{n is even} \end{cases}$$

$$g(n) = n - (-1)^n \begin{cases} n+1, & \text{n is odd} \\ n-1, & \text{n is even} \end{cases}$$

$$f(g(x)) = \begin{cases} \frac{n}{2} & \text{n is even} \\ \frac{n+1}{2} & \text{n is odd} \end{cases}$$

∴ Onto but not one-one.

134. Let a function f : (0, ∞) → (0, ∞) be defined by $f(x) = \left|1 - \frac{1}{x}\right|$. Then, f is

(a) injective only.

(b) both injective as well as surjective.

(c) not injective but it is surjective.

(d) neither injective nor surjective.

JEE Main 11.01.2019 Shift - II

Ans. (c): Given,

$$f(x) = \left|1-\frac{1}{x}\right|$$

$$f(x) = \begin{cases} 1-\frac{1}{x} & x \in (1,\infty) \\ \frac{1}{x}-1 & x \in (0,1) \end{cases}$$

$$f'(x) = \begin{cases} \frac{1}{x^2} & x \in (1,\infty) \\ -\frac{1}{x^2} & x \in (0,1) \end{cases}$$

This shows f(x) is not injective.
Since range of the function is equal to codomain function is surjective.

135. Let x denote the total number of one-one functions from a set A with 3 elements to a set B with 5 elements and y denote the total number of one-one functions from the set A to the set A×B. Then,

(a) $2y = 91x$ (b) $2y = 273x$
(c) $y = 91x$ (d) $y = 273x$

JEE Main 25.02.2021 Shift -II

Ans. (c) : Given,
Total number of one-one function from a set A with 3 elements to a set B with 5 elements and y denoting total number of one one function.
As we know that, no of one-one function ${}^{q}C_p \times p!$

$$x = {}^{5}C_3 \times 3!$$
$$x = \frac{5!}{2!\,3!} \times 3!$$
$$x = \frac{5!}{2!}$$
$$x = 5 \times 4 \times 3 = 60$$

The number of one-one function

$$y = {}^{15}C_3 \times 3!$$
$$y = \frac{15!}{12!\,3!} \times 3!$$
$$y = \frac{15!}{12!}$$
$$y = 15 \times 14 \times 13 = 2730$$

Therefore,

$$\frac{x}{y} = \frac{60}{2730}$$
$$273x = 6y$$
$$2y = 91x$$

136. Let f : R →R be defined as f(x) = 2x –1 and g : R – {1} → R be defined as $g(x) = \frac{x-\frac{1}{2}}{x-1}$ Then, the composition function f(g(x)) is

(a) one-one but not onto
(b) onto but not one-one
(c) Neither one-one nor onto
(d) Both one-one and onto

JEE Main 24.02.2021, Shift-I

Ans. (a) : : We have,

$$f(x) = 2x - 1$$
$$g(x) = \frac{x-\frac{1}{2}}{x-1}$$

Now,

$$f(g(x)) = 2g(x) - 1$$
$$= 2\frac{\left(x-\frac{1}{2}\right)}{(x-1)} - 1$$
$$= \frac{2(2x-1)-2(x-1)}{2(x-1)}$$
$$= \frac{4x-2-2x+2}{2x-2}$$
$$= \frac{2x}{2x-2} = \frac{x}{x-1}$$

So, the range of f(g(x)) is R – {1}
Co domain is R
Hence, f(g(x)) is not onto as the range and co domain are not same.
We know that, if the function is one-one, then the function is always increasing or decreasing in its domain.

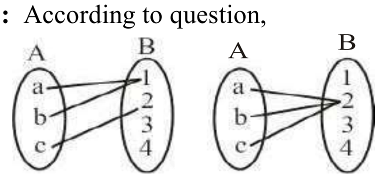
$$f(g(x)) = \frac{x}{x-1}$$
$$f'(g)(x) = \frac{(x-1)-x(1)}{(x-1)^2}$$
$$= \frac{-1}{(x-1)^2}$$

Therefore we can conclude that f'(g)(x) is always decreasing as there is a negative sign.
So, the function is one-one
Hence, f [g (x)] is one-one but not onto function.

137. Let A = {a, b, c} and B = {1, 2, 3, 4}. Then the number of elements in the set C = {f : A → B | 2∈ f(A) and f is not one - one} is

JEE Main 05.09.2020 Shift-II

Ans. (19) : According to question,



$$= \left[{}^{3}C_2 \times {}^{3}C_1 \times {}^{1}C_1 \times {}^{1}C_1\right] + \left[{}^{3}C_2 \times {}^{1}C_1 \times {}^{1}C_1 \times {}^{3}C_1\right]$$
$$= \left[\frac{3\times 2}{1\times 2} \times 3\right] + \left[\frac{3\times 2}{1\times 2} \times 3\right]$$
$$= 9 + 9 = 18$$

Therefore, the number elements in set C is –
$\therefore$ n(C) = 18 + 1 = 19

138. Let f, g : N → N, such that f (n + 1) = f(n) + f(1)∀ n∈N and g be any arbitrary function. Which of the following statements is not true?

(a) if fog is one-one, then g is one - one
(b) if f is onto, then f (n) = n, ∀n ∈N.
(c) f is one-one
(d) if g is onto, then fog is one-one

JEE Main 25.02.2021 Shift-I

Ans. (d) : Given,

f(n + 1) = f(n) + f(1)

f(n + 1) – f(n) = f(1)

Since, Above terms are in A.P. with common difference = f(1)

General term $T_n = f(1) + (n-1) f(1) = n f(1)$

f(n) = nf(1)

For fog to be one-one, g must be one-one.

For f to be onto, f(n) should take all the values of natural numbers.

f(n) = n

If g is many one, then fog is many one.

So, if g is onto then fog is one-one is incorrect.

139. Let A = {0, 1, 2, 3, 4, 5, 6, 7}. Then the number of bijective functions F : A →A such that f(1) + f(2) = 3 – f(3) is equal to ....

JEE Main 22.07.2021, Shift - II

Ans. (720) : Given,

f(1) + f(2) = 3 – f(3)

f(1) + f(2) + f(3) = 3

The only possibility is : 0 + 1 + 2 = 3

Elements, 1, 2, 3 in the domain can be mapped with 0, 1, 2 only.

So, number of bijective functions

= 3! × 5!

= 3 × 2 × 1 × 5 × 4 × 3 × 2 × 1

= 6 × 120 = 720

Type VI — Domain, Co-domain and Range of Function

140. If the domain of the function $f(x) = \log_e\left(\frac{2x+3}{4x^2+x-3}\right)+\cos^{-1}\left(\frac{2x-1}{x+2}\right)$ **is (α, β], then the value of 5β – 4α is equal to**

(a) 10 (b) 12
(c) 11 (d) 9

JEE Mains 30/01/2024 Shift-II

Ans. (b) : Given function,

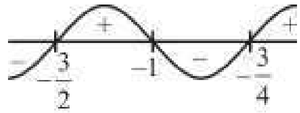
$f(x) = \log_e\left(\frac{2x+3}{4x^2+x-3}\right)+\cos^{-1}\left(\frac{2x-1}{x+2}\right)$

Now,

$$\frac{2x+3}{4x^2+x-3} > 0$$

$$\frac{2x+3}{(4x-3)(x+1)} > 0$$

$$x = -\frac{3}{2}, -1, \frac{3}{4}$$

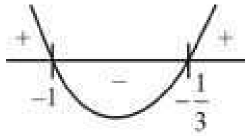


$x \in \left(-\frac{3}{2}, -1\right) \cup \left(\frac{3}{4}, \infty\right)$ .....(i)

and $-1 \le \frac{2x-1}{x+2} \le 1$

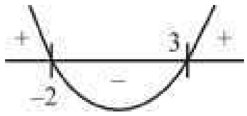
$\frac{2x-1}{x+2} \ge -1$ and $\frac{2x-1}{x+2} \le 1$

$\frac{3x+1}{x+2} \ge 0$ $\qquad \frac{2x-1-x-2}{x+2} \le 0$



$$\frac{x-3}{x+2} \le 0$$

$x \in (-\infty, -2) \cup [-\frac{1}{3}, \infty)$ ....(ii)



$x \in (-2, 3]$ .....(iii)

On intersection of (ii) & (iii), we get-

$x \in \left[-\frac{1}{3}, 3\right]$ .....(iv)

Again intersection of (i) & (iv), we get-

$x \in \left(\frac{3}{4}, 3\right]$

$\therefore \alpha = \frac{3}{4}$ and $\beta = 3$

Hence, 5β – 4α = 15 – 3 = 12

141. If the domain of the function

$$f(x) = \frac{\sqrt{x^2-25}}{(4-x^2)} + \log_{10}(x^2+2x-15)$$

is $(-\infty, \alpha) \cup [\beta, \infty)$**, then** $\alpha^2 + \beta^3$ **is equal to:**

(a) 140 (b) 175
(c) 125 (d) 150

JEE Mains 01/02/2024 Shift-II

Ans. (d) : Given function,

$$f(x) = \frac{\sqrt{x^2-25}}{(4-x^2)} + \log_{10}(x^2+2x-15)$$

Define the function-

x^2 25 0

$(x-5)(x+5) \ge 0$

x (, 5] [5,) ...(i)

and 4 x^2 0

x 2or 2 ...(ii)

$x^2 \quad 2x \quad 15 \quad 0$

$(x \quad 5)(x \quad 3) \quad 0$

$x \quad (\quad , 5) \quad (3, \quad)$ .....(iii)

Intersection of equation (i) ∩ (ii) ∩ (iii), we get-

$x \quad (\quad , 5) \quad [5, \quad)$

On comparing general term, we get-

$5, \quad 5$

Hence, $\alpha^2 + \beta^3 = 25 + 125 = 150$

142. If the domain of the function $f(x) = \cos^{-1}\left(\frac{2-|x|}{4}\right) + \{\log_e(3-x)\}^{-1}$ is $[-\alpha, \beta) - \{\gamma\}$, then $\alpha + \beta + \gamma$ is equal to :

(a) 12 (b) 9

(c) 11 (d) 8

JEE Mains 30/01/2024 Shift-I

Ans. (c) :

$$f(x) \quad \cos^{\ 1}\frac{2 \quad |x|}{4} \quad \frac{1}{\ln(3 \quad x)}$$

$-1 \le \frac{2-|x|}{4} \le 1$ $\qquad 3 - x \neq 0$

$\qquad x \neq 3$

$-4 \le 2 - |x| \le 4$ $\qquad 3 - x > 0$

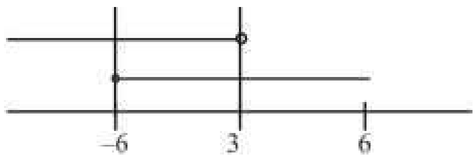
$-6 \le -|x| \le 2$ $\qquad x < 3$

$|x| \le 6$ $\qquad x \in (-\infty, 3)$

$x \in [-6, 6]$ $\qquad$ And, $\log_e(3-x) \neq 0$

$\qquad 3 - x \neq 1$

$\qquad x \neq 2$



∴ $x \in [-6, 3] - \{2\}$

Compare with $[-\alpha, \beta) - \{\gamma\}$

$\alpha = 6, \beta = 3, \gamma = 2$

Hence, $\alpha + \beta + \gamma = 6 + 3 + 2 = 11$

143. Let a, b, c be the lengths of three sides of a triangle satisfying the condition $(a^2 \quad b^2)x^2 \quad 2b(a \quad c)x \quad (b^2 \quad c^2) \quad 0$. If the set of all possible values of x is the interval $(\ , \ )$, $12(\ ^2 \quad ^2)$ is equal to_______.

JEE Mains 31/01/2024 Shift-II

Ans. :(36) Given that –

$(a^2 + b^2)x^2 - 2b(a + c)x + b^2 + c^2 = 0$

$a^2x^2 - 2bax + b^2 + b^2x^2 - 2bcx + c^2 = 0$

$(ax - b)^2 + (bx - c)^2 = 0$

$ax - b = 0$ and $(bx - c) = 0$

Now,

Case (i) $\qquad a + b > c$

$a + ax > bx$ (put b = ax, c = bx)

$a + ax > ax^2$ (put b = ax)

$x^2 - x - 1 < 0$

$\frac{1-\sqrt{5}}{2} < x < \frac{1+\sqrt{5}}{2}$

Case (ii) $\qquad b + c > a$

$ax + bx > a$ (put b = ax, c = bx)

$ax + ax^2 > a$ (put b = ax)

$x^2 + x - 1 > 0$

$x < \frac{-1-\sqrt{5}}{2}$ or $x > \frac{-1+\sqrt{5}}{2}$

Case (iii) $\qquad c + a > b$

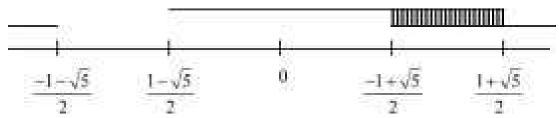
$ax^2 + a > ax$

$x^2 - x + 1 > 0$

Always true $x \in R$

Combine (i), (ii) and (iii) –

$x \in \left(\frac{\sqrt{5}-1}{2}, \frac{\sqrt{5}+1}{2}\right)$



$\frac{1 \quad \sqrt{5}}{2} \qquad \frac{1 \quad \sqrt{5}}{2}$

Now $\quad 12 \quad \left(\frac{1 \quad \sqrt{5}}{2}\right)^2 \quad \left(\frac{1 \quad \sqrt{5}}{2}\right)^2$

36

144. Let [t] be the greatest integer less than or equal to t. Let A be the set of all prime factors of 2310 and f : A → Z be the function $f(x) = \left[\log_2\left(x^2 + \left[\frac{x^3}{5}\right]\right)\right]$ The number of one-to-one functions from A to the range of f is :

(a) 20 (b) 120

(c) 25 (d) 24

JEE Mains 08/04/2024 Shift-I

Ans. (b) : f : A →B

n(A) = m, n (B) = n

number of one - one function is $^n p_m$ if n ≥ m

N = 2310 = 231 × 10

= 3 × 11 × 7 × 2 × 5

A = {2, 3, 5, 7, 11}

f:A→Z

$f(x) = \left[\log_2\left(x^2 + \left[\frac{x^3}{5}\right]\right)\right]$

$f(2) = [\log_2(5)] = 2$

$f(3) = [\log_2(14)] = 3$

$f(5) = [\log_2(25+25)] = 5$

$f(7) = [\log_2 (117)] = 6$
$f(11) = [\log_2 387] = 8$
Range of f : B = {2, 3, 5, 6, 8}

No. of one-one functions = $^5P_5 = \frac{5!}{0!} = 5! = 120$

145. Let $f(x) = \frac{1}{7 - \sin 5x}$ be a function defined on R. Then the range of the function f(x) is equal to:

(a) $\left[\frac{1}{8}, \frac{1}{5}\right]$ (b) $\left[\frac{1}{7}, \frac{1}{6}\right]$
(c) $\left[\frac{1}{7}, \frac{1}{5}\right]$ (d) $\left[\frac{1}{8}, \frac{1}{6}\right]$

JEE Mains 06/04/2024 Shift-II

Ans. (d) : Since, Range of sin x is [–1, 1] for all x.
$\Rightarrow \quad -1 \le \sin 5x \le 1$
We multiply by negative sign
So, $1 \ge \sin 5x \ge -1$
Now, $8 \ge 7 - \sin 5x \ge 6$
$\frac{1}{8} \ge \frac{1}{7 - \sin 5x} \ge \frac{1}{6}$
Therefore, the range of f(x) = $\left[\frac{1}{8}, \frac{1}{6}\right]$

146. Let f: $R - \left\{\frac{-1}{2}\right\} \to R$ and $g : R - \left\{\frac{-5}{2}\right\} \to R$ be defined as $f(x) = \frac{2x+3}{2x+1}$ and $g(x) = \frac{|x|+1}{2x+5}$ then the domain of the function fog is.

(a) $R - \left\{-\frac{5}{2}\right\}$ (b) R
(c) $R - \left\{-\frac{7}{4}\right\}$ (d) $R - \left\{-\frac{5}{2}, -\frac{7}{4}\right\}$

JEE Mains 27/01/2024 Shift-II

Ans. (a) : $f(x) = \frac{2x+3}{2x+1}, x \ne -\frac{1}{2}$

$g(x) = \frac{|x|+1}{2x+5}, x \ne -\frac{5}{2}$

Domain of f(g(x))

$x \ne -\frac{5}{2}$ and $\frac{|x|+1}{2x+5} \ne -\frac{1}{2}$

$g(x) \ne -\frac{1}{2}$

$\frac{|x|+1}{2x+5} \ne -\frac{1}{2}$

(i) $x \ge 0$

$\frac{x+1}{2x+5} = \frac{-1}{2}$

$2x+2 = -2x-5$

$4x = -7$

$x = \frac{-7}{4}$ (Rejected)

(ii) $x < 0$

$\frac{-x+1}{2x+5} = \frac{-1}{2}$

$-2x+2 = -2x - 5$

$2 = -5$ (not possible)

$\Rightarrow$ Domain of f(g(x)) = domain of g(x).

$\therefore$ Domain will be $R - \left\{-\frac{5}{2}\right\}$

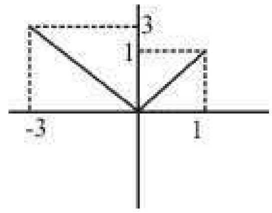
147. If $f(x) = \begin{cases} 2 + 2x, & -1 \le x < 0 \\ 1 - \frac{x}{3}, & 0 \le x \le 3 \end{cases}$; $g(x) = \begin{cases} -x, & -3 \le x \le 0 \\ x, & 0 < x \le 1 \end{cases}$, then range of (fog) (x) is :

(a) [0, 3) (b) [0, 1]
(c) [0, 1) (d) (0, 1]

JEE Mains 29/01/2024 Shift-I

Ans. (b) : Given,

$g(x) = \begin{cases} -x, & -3 \le x \le 0 \\ x, & 0 < x \le 1 \end{cases}$



Now,

$fog(x) = \begin{cases} 2 + 2g(x), & -1 \le g(x) < 0 \\ 1 - \frac{g(x)}{3}, & 0 \le g(x) \le 3 \end{cases}$

$= \begin{cases} 2 - 2x,; & -1 \le x \le 0 \\ 1 - \frac{x}{3}; & 0 < x \le 1 \end{cases} = \begin{cases} 1 + \frac{x}{3}; & -3 \le x \le 0 \\ 1 - \frac{x}{3}; & 0 < x \le 1 \end{cases}$

Hence, range of g(x) = [0, 1]

148. If the domain of the function $f(x) = \sin^{-1}\left(\frac{x-1}{2x+3}\right)$ is R – (α, β) then 12 αβ is equal to :

(a) 36 (b) 24
(c) 40 (d) 32

JEE Mains 09/04/2024 Shift-I

Ans. (d) : Given,

Domain of (x) = $\sin^{-1}\left(\frac{x-1}{2x+3}\right)$ is

$2x + 3 \ne 0$ and $x \ne \frac{-3}{2}$ and $\left|\frac{x-1}{2x+3}\right| \le 1$

$|x - 1| \le |2x + 3|$