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FORMULAE & TRICKS

QUANTITATIVE ABILITY

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CAT, XAT, MAT, All MBA Entrance & Government Job Examinations

Shweta Arora

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SYLLABUS COVERED

Formulae & Tricks (Quatitative Ability)

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- Table of Contents 3 - 4
- About the Author 5 - 5
- Preface 6 - 6
- Tips to Crack Quantitative Ability in the First Attempt 7 - 7
- Deciphering Quantitative Ability & Its Importance 8 - 8

Chapter 1: Number System **1 - 19**
- Number Properties
- Arithmetic Operations on Even and Odd Numbers
- Vbodmas Rule - Order of Arithmetic Operations
- Some Important Algebraic Formulae Used in Number System
- To Find Square Root
- Surds & Indices
- Properties of Prime & Composite Numbers
- Remainders
- Wilson Theorem
- Fermat's Theorem
- Cyclicity Theorem
- Factors & Multiples
- HCF/GCD and LCM
- Unit Digit/Last Digit
- Factorial and Its Application

Chapter 2: Arithmetic **20 - 51**
- Basics of Percentage
- Profit,Loss and Discount
- Simple Interest and Compund Interest
- Ratio and Proportion & Partnership
- Partnership
- Average
- Mixture and Allegation
- Time, Speed and Distance
- Time and work
- Pipes and Cisterns
- Clocks and Calendars

Chapter 3: Geometry & Mensuration **52 - 104**
- Lines, Angles & Triangles
- Similarity of Triangles
- Congruency of Triangles
- Quadrilaterals
- Polygons
- Mensuration
- Rectangular Parallelepiped
- Co-ordinate Geometry
- Height and Distances

Table of Contents

Chapter 4: Algebra 105 - 122

- Important Formulae
- Equations & Polynomials
- Inequalities & Absolute Value
- Functions & Graphs
- Logarithm & Exponents
- Progression & Series
- Binominal Theorem

Chapter 5: Modern Mathematics 123 - 136

- Permutations & Combinations
- Probability
- Set Theory
- Venn Diagrams

Chapter 6: Tricks for Mental Calculations 137 - 144

- Multiplication
- Cross Multiplication for 2-Digit
- Cross Multiplication for 3-Digit
- Square of a Number
- Cube of a 2-Digit Number
- Prime Factors

Chapter 7: Memorisation Aids 145 - 152

- Tables till 20
- Square till 30
- Cube till 20
- Common square roots
- Common cube roots
- Prime numbers till 100
- Conversion Factors
- Trigonometric Values
- Important Pythagorean Triplets
- Important Mathematical Constants
- Divisibility Rules
- Logarithm Table
- Fractions to Decimal
- Fractions to Percentage Table
- Conversion Factors

About the Author

As an alumna of IIM Ahmedabad with two years of corporate experience post-MBA and seven years dedicated to guiding MBA aspirants, I can well understand the aspirations and challenges faced by students on their journey to B-schools.

When I was preparing for MBA exams, there was no such term called a "formula book". We, as aspirants, used to revise the entire syllabus by going through each question again and again - which consumed a lot of time.

Based on my experience of mentoring 1.6 million followers across platforms, many of them being MBA aspirants who are now studying at top B-schools, I have understood the importance of creating a formula book that can help you revise the syllabus and key concepts in no time.

As a CAT 99 percentile scorer and having cleared 12 IIM interviews on my first attempt, my mission is to empower MBA aspirants to unlock their full potential and provide them with an essential preparation resource that will boost their chances of making it to their dream B-school.

How to Use This Book:

- **Approach** each chapter systematically, reinforcing concepts and formulae as you progress through your primary study material.
- **Allocate** dedicated time each weekend to review the formulae covered during the week, ensuring comprehensive retention.
- **Revisit** all completed chapters monthly to solidify your understanding and enhance your confidence in tackling exam questions.
- **Utilize** the provided space to note down any additional formulae or insights you come across during your preparation journey.

As you embark on your journey towards top B-schools, use this book systematically to reinforce concepts, enhance retention, and solidify your understanding. Let this book be your trusted companion as you strive towards achieving your dreams.

Best Wishes,
Shweta Arora

Preface

Welcome to **"Formulae and Tricks for Quantitative Ability"!** This book is designed to be the comprehensive list of Formulae & Tricks for mastering the quantitative aptitude section, an integral part of various MBA entrance and competitive exams.

Quantitative Ability is a crucial component of most entrance exams, including but not limited to CAT, XAT, MAT, and various entrance & government job exams. It evaluates your mathematical and analytical skills, covering topics such as Number System, Arithmetic, Algebra, Geometry, Modern Mathematics and Tricks for Mental Calculation.

This book aims to equip the aspirants with the essential formulas, shortcuts, and strategies required to tackle quantitative problems efficiently. Whether you are a student preparing for competitive exams or a professional aiming to enhance the quantitative skills.

Key features of this book include:

1. **Comprehensive Coverage:** We have meticulously compiled all the important formulas, concepts, and tricks required to solve quantitative problems across various topics.
2. **Simplified Explanation:** Complex concepts are explained in a simplified manner, ensuring clarity and ease of understanding.
3. **Abundant Examples:** Each concept is accompanied by some examples to illustrate its application, helping the aspirants grasp the underlying principles effectively.
4. **Time-Saving Techniques:** To emphasize time-saving strategies and shortcuts, enabling aspirants to solve problems swiftly during the exam.
5. **Tips for Exam Success:** In addition to providing formulae and tricks, it provides valuable tips and strategies to enhance the exam performance and boost the confidence.

We believe that with dedicated practice and a thorough understanding of the concepts presented in this book, aspirants can excel in the quantitative aptitude section of any exam. So, embark on this learning journey with us, and let's sharpen the quantitative abilities together!

With Best wishes
Team Oswaal

Tips to Crack Quantitative Ability in the First Attempt

Quantitative Ability is a crucial section in many competitive exams in India, such as bank exams, CAT, XAT, MAT, UPSC, SSC, and other MBA entrance exams. Aspirants is required essential knowledge and tactics of Quantitative Ability for enhancing analytical and problem-solving skills. Cracking the Quantitative Ability section of a competitive exam in the first attempt requires hard work, dedication, and a strategic approach. Here are some tips that can help aspirants to achieve success in the first attempt:

1. Think Right

Calming yourself and thinking positive is the first and the best course of action that one is required to take. Think and believe that the exam goal is achievable if worked upon smartly.

2. Start studying from the beginning

All the aspirants are aware of how vast, comprehensive and detailed the syllabus of the Quantitative section is. To crack the exam in the first attempt you have to start preparing for the exam from the beginning. It is only then that you will be able to complete the entire syllabus. Following this approach will also allow you plenty of time to revise.

3. Understand the syllabus & Pattern and arrange the materials accordingly

While preparing for the Quantitative Ability nothing can be labelled as less important. Questions can come from the most unexpected topics too. Laying down the whole syllabus in front of you will help to decide on the study material to require.

4. Get the right tools and study material

Gathering and preparing from the appropriate study material is something you cannot be ignorant towards. You can refer to Oswaal 'Formulae & Tricks of Quantitative Ability' to enhance your preparation.

5. Understand the concepts

No one can crack the Quantitative Ability section just by mugging up all the concepts and topics. The syllabus of the exam is in-depth such that you need to understand every concept.

6. Lots of Practice of Different type Questions

Oswaal 'Formulae & Tricks of Quantitative Ability' will not only help you in understanding the concepts, but they will also help you in figuring out the different concepts that come up every exam and this might give you an edge over other students. Referring to various Topic-wise Questions might also help you in comprehending the areas which require more work.

7. Analysing your performance

While you are solving questions, make sure you keep a track of time i.e. how much time does it take to solve one section or one question? Make a report of the sections and type of questions which take minimum and maximum time.

Deciphering Quantitative Ability & Its Importance

Quantitative Ability refers to the capability to understand and solve mathematical problems quickly and accurately. It involves the application of basic Number System, Arithmetic, Algebra, Geometry, Modern Mathematics and other mathematical concepts to solve problems related to Quantitative Analysis, Data Interpretation, and Logical Reasoning.

Some common topics included in Quantitative Ability assessments are:

1. **Number system:** This is the system of writing or representing numbers using certain rules & patterns.
2. **Arithmetic:** This involves solving problems related to addition, subtraction, multiplication, division, percentages, ratio, and proportion etc.
3. **Algebra:** This includes solving problems related to linear and quadratic equations, inequalities, algebraic problems, and simultaneous equations.
4. **Geometry:** This includes problems related to lines, angles, triangles, circles, any polygons.
5. **Modern Mathematics:** It Includes to analyse data, identity patterns, and develops creative solutions to complex problems.

To improve the quantitative ability skills, aspirants can practice solving mathematical problems regularly, familiarize with different formulae and concepts, and improve the mental calculation speed.

Quantitative Ability, also known as Mathematical Aptitude, is a vital component of most competitive exams in India. It tests a aspirant's ability to solve numerical problems accurately and quickly. Here are some of the competitive exams where Quantitative Aptitude plays a significant role:

MBA Entrance Exams: Quantitative Ability is a crucial section in MBA entrance exams like CAT, XAT, MAT, etc. The section includes questions on topics like basic Number System, Arithmetic, Algebra, Geometry, Modern Mathematics.

SSC Exams: Quantitative Ability is also an important section of various Staff Selection Commission (SSC) exams, including SSC CGL, SSC CHSL, SSC CAPFs etc. The section includes questions on topics like Time and Distance, Profit and Loss, Percentage, Ratio and Proportion, etc.

Bank Exams: Quantitative Ability is an essential component of bank exams like IBPS PO, IBPS Clerk, SBI PO, SBI Clerk, etc. The section includes questions on topics like Number System, Arithmetic, Algebra, Geometry, Modern Mathematics.

Railway Exams: Quantitative Aptitude is a crucial section in railway exams like RRB NTPC, RRB JE, etc. The section includes questions on topics like Number System, Simplification, Arithmetic, Geometry, etc.

UPSC Civil Services (Prelims) Exam: Quantitative Aptitude is a part of the CSAT (Civil Services Aptitude Test) paper in the UPSC Civil Services Exam. The section includes questions on topics like Number System, Simplification, and Arithmetic.

In conclusion, Quantitative Ability is a critical component of various competitive exams in India, and it is essential to have a good understanding of mathematical concepts to perform well in these exams.

CHAPTER 1

NUMBER SYSTEM

FORMULAE AND TRICKS

NUMBER PROPERTIES

COMPLEX NUMBERS: The number that can be written in the form of a + bi, where "a" and "b" are the real number and "i" is an imaginary number, is known as complex numbers, i.e., 2 + 3i.

IMAGINARY NUMBERS: The imaginary numbers are the complex numbers that can be written in the form of the product of a real number and the imaginary unit, i.e., 3i, –7i.

REAL NUMBERS: All positive and negative integers, fractions and decimal numbers without imaginary numbers are called real numbers. It is represented by the symbol "R".

i.e., 1, 2, $\frac{1}{2}$, $\frac{1}{5}$ etc.

RATIONAL NUMBERS: Any number that can be written as a ratio of one number over another number is called rational number. This means that any number that can be written in the form of $\frac{p}{q}$ where q cannot be 0, i.e., 12, 0, $\frac{-3}{4}$, 0.7, etc.

IRRATIONAL NUMBERS: The number that cannot be expressed as the ratio of one over another, i.e., $\frac{p}{q}$ is known as irrational numbers, i.e., π, $\sqrt{2}$, e, $\sqrt{5}$, etc.

INTEGERS: Numbers that have no fractional part, such as –1, 0, 1, 2 and 3 are called integers. Integers include the counting numbers (1, 2, 3, ...), their negative counterparts (–1, –2, –3, ...) and 0.

WHOLE NUMBERS: Whole numbers, also called non-negative integers, do not include any fractional or decimal parts, i.e., 0, 1, 2, 3, 4, ……etc.

NATURAL NUMBERS: Natural numbers are known as counting numbers that contain the positive integers from 1 to ∞.

PRIME NUMBERS: A positive integer with exactly two factors, 1 and itself, is known as prime number. The number 1 does not qualify as prime because it has only one factor, not two. The number 2 is the smallest prime number and it is also the only even prime number.

The numbers 2, 3, 5, 7, 11, 13, etc. are prime numbers.

COMPOSITE NUMBERS: A composite number is a number that has more than two factors. For example, 8 is a composite number, as the number 8 is divisible by 1, 2, 4 and 8. Other examples of composite numbers are 6, 8, 9, 10 and so on.

EVEN INTEGERS: Any integer divisible by 2 or which can be written in the form of 2K is even. 0 is also comes into the category of even.

i.e., even integers = 2, 4, 8, 24, 36, etc.

ODD INTEGERS: Any integer which is not divisible by 2 or which can be written in the form of 2K + 1. So, odd integers = 3, 5, 7, 13, 15, etc.

Arithmetic Operations on Even and Odd Numbers

- Even + Even = Even
- Odd + Odd = Even
- Even + Odd = Odd
- Even – Even = Even
- Odd – Odd = Even
- Even – Odd = Odd
- Odd – Even = Odd
- Even × Any natural number = Even
- Odd × Odd = Odd
- Even/Even = Even or Odd (if divisible)
- Even/Odd = Even (if divisible)
- Odd/Odd = Odd (if divisible)
- Odd/Even = Never divisible
- Odd + Odd + Odd + Odd + ... Odd number of times = Odd number
- Odd + Odd + Odd + Odd + ... Even number of times = Even number
- $(\text{Even})^{\text{odd}}$ = Even
- $(\text{Odd})^{\text{even}}$ = Odd

Vbodmas Rule:- (Order of Arithmetic Operations)

- V – Vinculum
- B – Brackets
- O – Of
- D – Division
- M – Multiplication
- A – Addition
- S – Subtraction

Example: $[2 + \{3 + 4 \times (2 + 3)\} - 10]$

Solution: $[2 + \{3 + 4 \times 5\} - 10] = [2 + \{3 + 20\} - 10] = [2 + 23 - 10] = 15$

Some Important Algebraic Formulae Used In Number System

- $(a + b)^2 = a^2 + 2ab + b^2$
- $(a - b)^2 = a^2 - 2ab + b^2$
- $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$
- $(a + b)^3 = a^3 + b^3 + 3ab\,(a + b)$
- $(a - b)^3 = a^3 - b^3 - 3ab\,(a + b)$
- $a^2 - b^2 = (a + b)(a - b)$
- $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
- $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
- $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$
- If, $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$

To Find Square Root

A number n is called the square root of a number $n^2 = (n \times n)$

The symbol for square root is '$\sqrt{\ }$'.

Example:

Square root of 0.4356 is $\sqrt{0.4356} = 0.66$

Square root of 1.1236 is $\sqrt{1.1236} = 1.06$

Square root of 5.3361 is $\sqrt{5.3361} = 2.31$

Generally there are two methods for finding the square root:

(i) Prime Factorisation method

(ii) Division method

(i) ***Factorisation Method:***

In this method first we find out the prime factors and then we pair them as given below.

EXAMPLE: Find the square root of 92.16.

SOLUTION: Factors of 92.16 $= \dfrac{2^{10} \times 3^2}{10^2}$

$$\sqrt{92.16} = \frac{2^5 \times 3}{10}$$

$$\sqrt{92.16} = 9.6$$

2	92.16
2	46.08
2	23.04
2	11.52
2	5.76
2	2.88
2	1.44
2	0.72
2	0.36
2	0.18
3	0.09
3	0.03
	0.01

(ii) ***Division Method:***

In this method first of all we make pairs from the right side towards left and then solve as given below.

EXAMPLE: Find the square root of 10404.

SOLUTION:

	102
1	1 04 04
1	1
202	0 04 04
02	4 04
	× ×

$\therefore \sqrt{10404} = 102$

SURDS & INDICES

- $a \times a \times a \times a \times \ldots\ldots\ldots\ldots\ldots \times n$ times $= a^n$.
- $a^{-1} = \dfrac{1}{a}$ (a can't be 0)
- $a^{-n} = \dfrac{1}{a^n}$
- $a^n = \dfrac{1}{a^{-n}}$
- $(a)^0 = 1$
- $a^p \times a^q \times a^r = a^{p+q+r}$ (a can't be 0)
- $\dfrac{a^m}{a^n} = a^{m-n}$ [when $m > n$]
- $\dfrac{a^m}{a^n} = \dfrac{1}{a^{n-m}}$ [when $n > m$]
- $\dfrac{a^m}{a^n} = 1$ [when $m = n$]
- $\left(\dfrac{a}{b}\right)^m = \left(\dfrac{b}{a}\right)^{-m}$
- $\left(a^m\right)^n = \left(a^n\right)^m = a^{m\times n} = a^{n\times m}$
- $(abc)^n = a^n \times b^n \times c^n$
- $\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$ [b cannot be 0]

- $a^{m^n} = a^{m \times m \times m \times \ldots\ldots\ldots \times n \text{ times}}$
- $a^{m^n} \neq \left(a^m\right)^n \neq \left(a^n\right)^m$.
- If $a^m = a^n$ then $m = n$ ·
- If $a^m = b^m$ then $a = b$.
- $(-1)^n = 1$ [when n is even]
- $(-1)^n = -1$ [when n is odd]
- $\sqrt[n]{a} = (a)^{\frac{1}{n}}$
- $\sqrt[n]{ab} = (ab)^{\frac{1}{n}} = a^{\frac{1}{n}} \times b^{\frac{1}{n}} = \sqrt[n]{a} \times \sqrt[n]{b}$
- $\left(\sqrt[n]{a}\right)^m = (a)^{\frac{m}{n}}$
- $\sqrt[n]{\frac{a}{b}} = \left(\frac{a}{b}\right)^{\frac{1}{n}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \left[\frac{b}{a}\right]^{-n}$
- $\left[\sqrt[z]{\left(\sqrt[y]{\left(\sqrt[x]{a}\right)^p}\right)^q}\right]^r = (a)^{\frac{pqr}{xyz}}$

Properties of Prime & Composite Numbers

- A prime number has only 2 distinct factors, 1 and itself.
- 2 is the smallest as well as the only even prime number.
- There are a total of 25 prime numbers up to 100 and there are 46 prime numbers up to 200. Prime numbers up to 100 are; 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89 and 97.
- There are infinite prime numbers.
- 1 is neither prime nor composite.
- Two consecutive prime numbers are called twin primes. There are infinite number of such twin primes, e.g., 11, 13 or 5, 7, etc.
- The only triplet of prime numbers is 3, 5 and 7.
- All prime numbers greater than 3 can be written in the form of 6K ± 1 but the reverse of this may or may not be true.
- A composite number has more than two distinct factors or at least three distinct factors.

Checking Whether a Number *N* is Prime or Not

- **Step 1:** A prime number must leave remainder of +1 or –1 when divided by 6. So, to become a prime number, the number must satisfy this condition. If a number doesn't satisfy this condition, then it's not prime and if it satisfies this condition, then move to step 2.
- **Step 2:** Find the approximate square root of N.
- **Step 3:** Check the divisibility of N with all the prime numbers from 2 to $\sqrt{N}$. If N is divided by any one of these, then it is a composite number, else it's a prime number.

EXAMPLE: If the product of three consecutive positive integers is 15,600 then the sum of the squares of these integers is **(CAT 2017)**

(1) 1,777 **(2)** 1,785 **(3)** 1,875 **(4)** 1,877

SOLUTION: **Option (4) is correct.**

Given, that $(x-1)(x)(x+1) = 15,600$.

$\Rightarrow x^3 - x = 15,600$. The nearest cube to 15,600 is 15,625 = 25^3

Hence, making a calculated approximation, we can say that x = 25

Smart Approach:
15600 = 156 × 100 = 25 × 4 × 12 × 13
= 24 × 25 × 26
24, 25, 26 and sum of their squares
= 1,877

Hence, the three numbers are 24, 25, 26 and sum of their squares = 1,877.

EXAMPLE: If the sum of squares of two numbers is 97, then which one of the following cannot be their product? **(CAT 2018)**

(1) −32 **(2)** 16 **(3)** 48 **(4)** 64

SOLUTION: **Option (4) is correct.**

Let a and b are the two numbers

Now, A.M.>G.M.

$\frac{(a + b)}{2}$ > square root a.b

$a^2 + b^2 + 2a.b > 4\ a.b$

$a^2 + b^2 > 2a.b$

$97 > 2a.b$

$a.b < 48.5$

So, 64 is not possible product of a and b

EXAMPLE: How many pairs (m, n) of positive integers satisfy the equation $m^2 + 105 = n^2$? **(CAT 2019)**

SOLUTION: **Option (4) is correct.**

Given $m^2 + 105 = n^2$

Or $n^2 - m^2 = 105$

$(n-m) \times (n+m) = 105$

105 can be written as

$105 \times 1 = 21 \times 5 = 15 \times 7 = 35 \times 3$

So, only four cases are possible to get values of n, m as positive. Thus the number of solutions = 4

EXAMPLE: The mean of all 4-digit even natural numbers of the form '*aabb*', where $a > 0$, is **(CAT 2020)**

(1) 4864 **(2)** 5050 **(3)** 5544 **(4)** 4466

SOLUTION: **Option (3) is correct.**

Possible numbers in form of *aabb*

1100	2200	....	9900
1122	2222	....	9922
1144	2244	....	9944
1166	2266	....	9966
1188	2288	....	9988

For each column:

As all numbers have same common difference.

So average would be 3rd number of every column like 1st column average = 1144

2nd column average = 2244

For 3rd row: The numbers 1144, 2244, 3344 are having same common difference.

So average of these numbers = 5544

That will be average of all such numbers.

> Smart Approach:
> Number = 1000a +100a + 10b + b
> = 1100a + 11b
> The smallest number is 1100
> (a = 1) and (b = 0).
> The largest number is 9988
> (a = 9) and (b = 8).
> $$\text{mean} = \frac{\{1100 + 9988\}}{2} = 5544$$

DIVISIBILITY TEST: Divisibility for 2, 4, 8, 16, ... 2^n, etc.

Number	Exponential Form in Power of 2	Number of Digits to be Checked	Divisibility Rule	Example
For 2	2^1	1	Check last 1 digit is divisible by 2 or not.	238: The last digit of number is 8 which is divisible by 2. Hence, 238 will be divisible by 2.
For 4	2^2	2	Check last 2 digits are divisible by 4 or not.	532: The last 2 digits of the number are 32 which is divisible by 4. Hence, 532 will be divisible by 4.
For 8	2^3	3	Check last 3 digits are divisible by 8 or not.	2184: The last 3 digits of the number are 184 which is divisible by 8. Hence, 2184 will be divisible by 8.

For 16	2^4	4	Check last 4 digits are divisible by 16 or not.	213216: The last 4 digits of the number are 3216 which is divisible by 16. Hence, 213216 will be divisible by 16.
For a number X which can be written in 2^n	2^n	n	Check last n digits are divisible by the number X or not.	

Divisibility of 3, 9, …………. 3^n, etc.

Number	Divisibility Rule	Example
For 3	Sum of the digits must be divisible by 3.	1383: The sum of the digits is $1 + 3 + 8 + 3 = 15$, which is divisible by 3. Hence, the number will be divisible by 3.
For 9	Sum of the digits must be divisible by 9	13833: The sum of the digits is $1 + 3 + 8 + 3 + 3 = 18$, which is divisible by 9. Hence, the number will be divisible by 9.
For a number X which can be written in 3^n	Sum of the digits must be divisible by X.	

Divisibility of 5, 25, …………. 5^n, etc.

Number	Exponential Form in Power of 2	Number of Digits to be Checked	Divisibility Rule	Example
For 5	5^1	1	Last 1 digit must be divisible by 5 or the last 1 digit must be 0.	235: The last digit of the number is 5 which is divisible by 5. Hence, 235 will be divisible by 5.
For 25	5^2	2	Last 2 digits must be divisible by 25 or the last 2 digits must be 0.	1675: The last 2 digits of the number are 75 which is divisible by 25. Hence, 1675 will be divisible by 25.
For a number X which can be written in 5^n	5^n	n	Last n digits must be divisible by X or the last n digit must be 0.	

Divisibility of 11 - The difference between the sum of the digits in the even places and the sum of the digits in the odd places should be either 0 or a multiple of 11. In such a case, the number is said to be divisible by 11.

EXAMPLE: Is 1419 divisible by 11?

SOLUTION: Sum of odd place digits $= 1 + 1 = 2$ and sum of even place digits $= 4 + 9 = 13$

So, the required difference $= 13 – 2 = 11$

Hence, the given number 1419 is divisible by 11.

Divisibility of composite numbers like; 6, 12, 88, 65, etc.

Number	Factorisation (In Co-Primes)	Divisibility Rule
For 6	3×2	The number must be divisible by both (prime numbers) 3 and 2 simultaneously.
For 12	4×3 2×6 (not co-prime)	The number must be divisible by both (prime numbers) 3 and 2 simultaneously.
For 88	11×8 22×4 (not co-prime)	The number must be divisible by both (prime numbers) 11 and 2 simultaneously.
For 65	13×5	The number must be divisible by both (prime numbers) 13 and 5 simultaneously.
For a number X which can be written in the product of co-prime	$A \times B$	The number must be divisible by both (prime numbers) A and B simultaneously.

Note

If Any no is written 6 times like (111111), 222222, 666666 it will exactly divide by 7,11,13,37.

Any no written like 3737, 2525, 2323 will be divisible by 101.

Example: A six-digit number is divisible by 33. If 54 is added to the number, then the new number formed will also be divisible by: **(SSC CGL 2023 Tier-I)**

(1) 1.7 **(2)** 2.2 **(3)** 3.5 **(4)** 4.3

Solution: Option (4) is correct.

Given: A six-digit number is divisible by 33

Formula used: Dividend = divisor × quotient + remainder

Calculation: Dividend = divisor × quotient + remainder

$\Rightarrow 33 \times q + 0 = 33q$

If 54 is added to the dividend then,

New number = $33q + 54$

$\Rightarrow 3 \times (11q + 18)$

So, we can clearly say that the new number is divisible by 3.

Shortcut: Just look for a number which is dividing 33 and 54 both.

3 is such number.

Some Important Expressions

- $a^n + b^n$ is always divisible by $a + b$ when n is ODD.
- $a^n - b^n$ is always divisible by $a + b$ when n is EVEN.
- $a^n - b^n$ is always divisible by $a - b$.

EXAMPLE: The number of integers x such that $0.25 \le 2^x \le 200$ and $2^x + 2$ is perfectly divisible by either 3 or 4, is **[CAT 2018]**

SOLUTION: **The correct answer is [5].**

x	2^x	$2^x + 2$	Divisible by 3 or 4	Falls in the given range
0	1	3	Yes	Yes
1	2	4	Yes	Yes
2	4	6	Yes	Yes
3	8	10	No	No
4	16	18	Yes	Yes
5	32	34	No	Yes
6	64	66	Yes	Yes
7	128	130	No	No

From the table required integers are $(0, 1, 2, 4 \,\&\, 6)$.

EXAMPLE: If N and x are positive integers such that $N^N = 2^{160}$ and $N^2 + 2^N$ is an integral multiple of 2^x, then the largest possible x is **[CAT 2018]**

SOLUTION: **The correct answer is [10].**

$N^N = \left(2^5\right)^{32}$

$\Rightarrow N^N = 32^{32}$

$\therefore N = 32$

$\Rightarrow 32^2 + 2^{32} = 2^{10} + 2^{32} = 2^{10}\left(1 + 2^{22}\right)$

Hence, largest possible value of x is 10.

REMAINDERS

- Dividend = Divisor × Quotient + Remainder.
- When a number 'n' is the divisor, the remainder 'r' can take any integral value from 0 to $n - 1$. The remainder can also be considered to be $(r - n)$.

REMAINDERS ARE MULTIPLICATIVE

Remainder when the product of numbers is divided by a number is the same as the product of remainders when the numbers are individually divided by the number.

$$\text{Remainder } \frac{(a \times b)}{c} = \text{Rem}\left(\frac{a}{c}\right) \times \text{Rem}\left(\frac{b}{c}\right)$$

EXAMPLE: What will be the remainder when $172 \times 173 \times 174$ is divided by 19?

SOLUTION: Rem $\frac{(172 \times 173 \times 174)}{19}$ = Rem $\left(\frac{172}{19}\right)$ × Rem $\left(\frac{173}{19}\right)$ × Rem $\left(\frac{174}{19}\right)$

$= (1 \times 2 \times 3) = 6$

Final remainder = 6

EXAMPLE: What is the remainder when **(UPSC CSAT 2023)**

$85 \times 87 \times 89 \times 91 \times 95 \times 96$ is divided by 100?

(1) 0 **(2)** 1 **(3)** 2 **(4)** 4

SOLUTION: Option (1) is Correct.

If you will divide $85 \times 87 \times 89 \times 91 \times 95 \times 96$ by 100, it will be divided completely.

$100 = 5 \times 5 \times 4$

So, divide 85 by 5, it gets divided fully. Now divided 95 by 5, and it is divided fully.

Now divide 96 by 4, and it too gets divided fully.

So, the full product $(85 \times 87 \times 89 \times 91 \times 95 \times 96)$ is fully divisible by 100 (which is $5 \times 5 \times 4$).

So, the remainder will be zero.

EXAMPLE: What is the larget power of 10 that divides the product $29 \times 28 \times 27 \times \dots 2 \times 1$? **(CDS 2023-I)**

(1) 4 **(2)** 5 **(3)** 6 **(4)** 7

SOLUTION: Option (3) is correct.

We need to count the number of trailing zeros in that product.

It is obtained by multiplying a factor of 10, which is equivalent to multiplying by 2 and 5.

Since every even number contributes a factor of 2 and every multiple of 5 contributes a factor of 5, we need to determine the number of multiples of 5 among the numbers from 1 to 29.

The numbers 5, 10, 15, 20, and 25 have one factor of 5 each while 25 contributes an additional factor of 5. Hence, there are 6 factors of 5 in total.

Therefore, the largest power of 10 that divides the product of the numbers from 29 to 1 is 10^6.

EXAMPLE: What is the remainder when 65^{99} is divided by 11? **(CDS 2023-I)**

(1) 0 **(2)** 5 **(3)** 9 **(4)** 10

SOLUTION: Option (4) is Correct.

We have 65^{99}

Remainder of $\frac{65^{99}}{11}$

Remainder of $\frac{(-1)^{99}}{11}$

Remainder of $\frac{-1}{11} = 11 - 1 = 10$

$\therefore$ Required remainder = 10.

REMAINDERS ARE ADDITIVE: Remainder the sum of numbers when divided by another number is the same as the sum of remainders when the numbers are individually divided by the number.

$$\text{Remainder } \frac{(a+b)}{c} = \text{Rem}\left(\frac{a}{c}\right) + \text{Rem}\left(\frac{b}{c}\right)$$

EXAMPLE: What will be the remainder when $10^1 + 10^2 + 10^3$ is divided by 9?

SOLUTION: $\text{Rem}\left(\frac{10^1}{9}\right) + \text{Rem}\left(\frac{10^2}{9}\right) + \text{Rem}\left(\frac{10^3}{9}\right) = (1 + 1 + 1) = 3.$

So, final remainder = 3

NEGATIVE REMAINDER: Negative Remainder = Positive Remainder – Divisor

For example, remainder when 2^3 is divided by 9 is 8 but it can also be considered as $(8 - 9) = -1$. It is used to reduce the calculation only.

WILSON THEOREM

Remainder when $(n - 1)!$ is divided by n is always $n - 1$ where n must be a prime number.

IMPORTANT POINTS

- While calculating remainder, divisor can be used multiple times.
- While answering the question, only the positive remainder will be considered.
- Keep dividing the numerator until it becomes lesser than the divisor.

EXAMPLE: Find the remainder when 6! is divided by 7.

SOLUTION: $\text{Remainder}\left(\frac{720}{7}\right) = 6$

NOTE

Remainder when $(n - 1)!$ is divided by n is always 0 where n is not prime number. It is applicable only when the n is at least 5.

FERMAT'S THEOREM

If p is a prime number and N and p are coprime to each other than the remainder of $\frac{N^{p-1}}{p}$ is always 1.

EXAMPLE: Find the remainder of $\frac{6^4}{5}$.

SOLUTION: It can be written as Rem of $\left(\frac{6}{5}\times\frac{6}{5}\times\frac{6}{5}\times\frac{6}{5}\right) = 1$

CYCLICITY THEOREM

Remainder also gets repeats when the powers of the same number, incremented by one each time, are divided by the same divisor.

TO KNOW CYCLICITY

- Either it must start repeating or we get remainder as 1. For example, if remainder starts repeating after 4 powers than the cyclicity of the remainder will be 4. If we get remainder 1 at power 5 it means the cyclicity is 5.
- If we get –1 as remainder than the cyclicity of the remainder with 2 times the power at which we got –1 remainder. For example, if we get –1 as remainder at power 5 than the cyclicity of the remainder will be 10.

FACTORS & MULTIPLES

MULTIPLES: Multiples are integers formed by multiplying an integer by any other integer. For example, 9 is a multiple of 3 (3 × 3), as are 15 (5 × 3), 18 (6 × 3), etc. In addition, 5 is also a multiple of itself, i.e., 5 (1 × 5).

PRIME FACTORISATION: Prime factorisation is a way to express any number as a product of prime numbers. For example, the prime factorisation of 42 is 2 × 3 × 7. Prime factorisation is used in solving LCM, HCF, divisibility and many other calculations.

FACTORS: Positive integers that can divide a given number completely, is a factor of given number. Factors are equal to or smaller than the given number, i.e., factors of 15 = 1, 3, 5, 15.

Important Formulas & Tricks Related to Factors

- For any composite number N, which can be expressed as $N = a^p \times b^q \times c^r$. Here a, b and c are prime factors and p, q and r are positive integers. Then,
- Total number of prime factors = Sum of the exponents of all prime basis = $p + q + r$