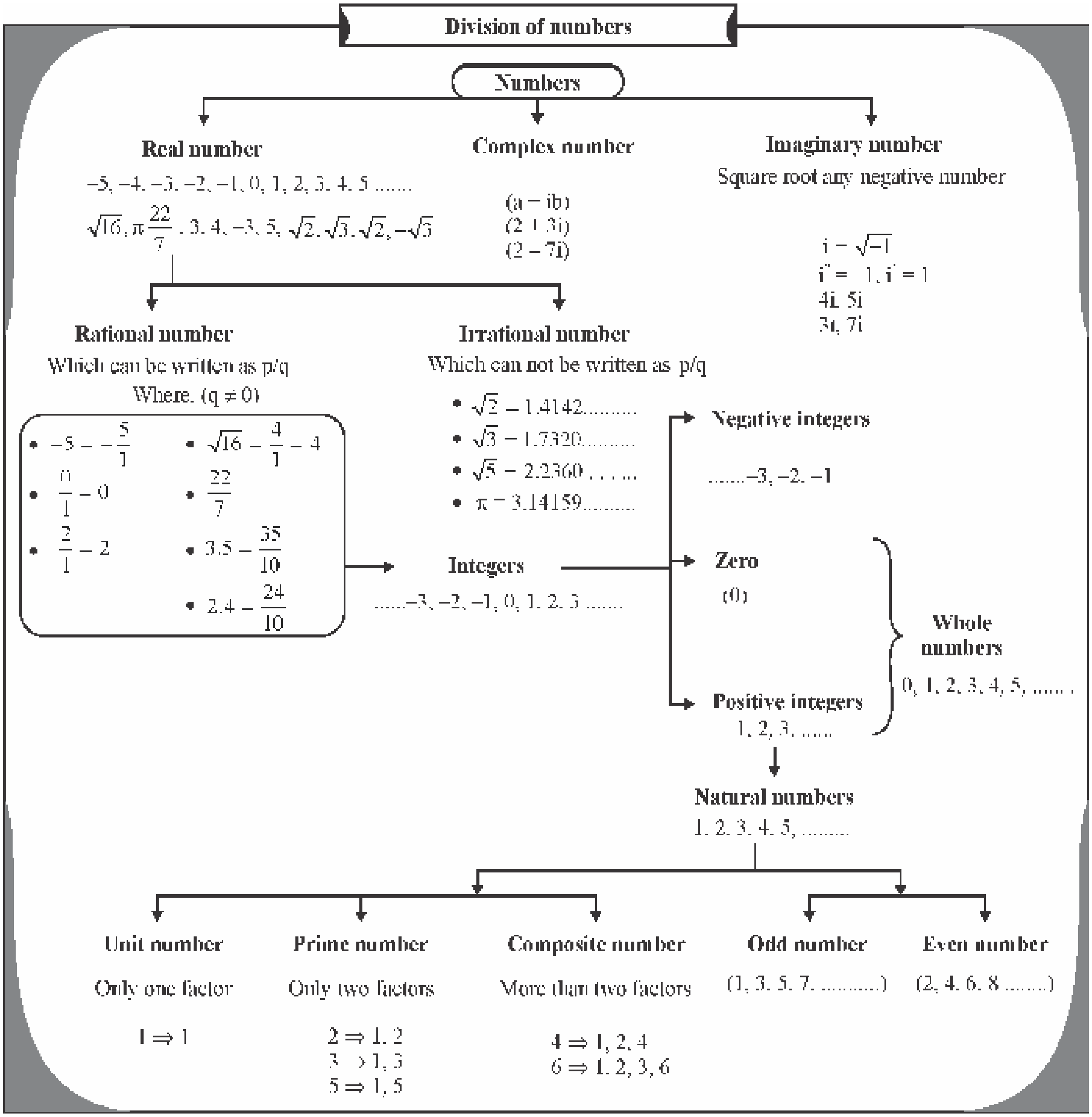
01 Number system

(Rational and irrational numbers & BODMAS rule)

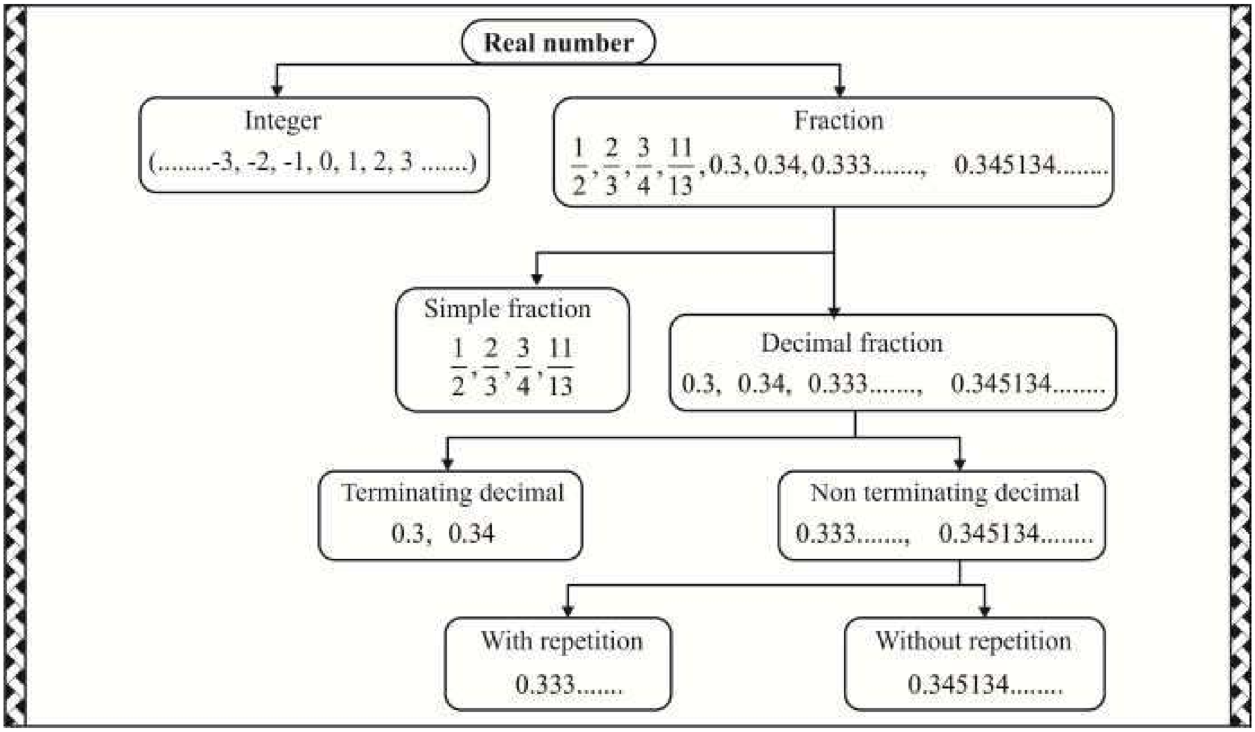


Co-prime/Relatively prime number

☞ A pair of numbers which H.C.F. (Highest common factor) is 1, is called co-prime number. **Ex.** (2, 3), (3, 4), (3, 5), (6, 7), (8, 11).

Twin-prime number

☞ A pair of prime numbers in which the difference is two is called twin prime number. Ex. (3, 5), (5, 7), (11, 13)



☞ Decimals with repetition can be expressed as rational numbers.

The test of prime number

- Let a is any give number and n is the smallest number.

  where, $n^2 \geq a$

  Now divide the given number by 'n' and smaller than each prime number. If 'a' is not completely divisible by any of these numbers, then 'a' will be a prime number otherwise not.

Ex. Test of 241–:

$241 \Rightarrow 16^2 \geq 241$

Prime number less than 16

= 2, 3, 5, 7, 11, 13

∵ 241 is not divisible by any prime number less than 16)

∴ 241 is a prime number.

Ex. Test of 437–:

$437 \Rightarrow 21^2 \geq 437$

Prime number less than 21

= 2, 3, 5, 7, 11, 13, 17, 19,

∵ 437 is completly divisible by 19

∴ 437 is a composite number.

Number of prime numbers

Prime numbers between 1-10	**4**
Prime numbers between 1-50	**15**
Prime numbers between 1-100	**25**
Prime numbers between 1-200	**46**
Prime numbers between 1-1000	**168**

☞ First prime number = 2

☞ Each prime number can be written as (6k ± 1) form. But every (6k ± 1) from may not be necessarily prime number.

Ex. $(6 \times 2 - 1) = 13$ Prime number

$25 = (6 \times 4 + 1)$ Composite number

Divisibility Rules

Divisibility of 2, 4, 8 and 16

- **Divisibility of 2 –:** If the digit at unit place of a number is either '0' or divisible by 2, then the number is divisible by 2.

  Ex. 8570, 7242, 9376

- **Divisibility of 4 –:** If the last two digits (ten's place, units place) of a number is either '00' or divisible by 4, then the number is divisible by 4.

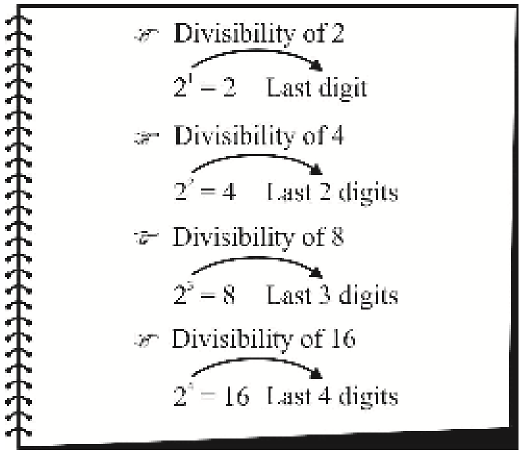
  Ex. 8700, 6924, 6376

- **Divisibility of 8 –:**If the last three digits (Hundred's place, tenth place, units place) of a number is either '000' or divisible by 8, then the number is divisible by 8.

  Ex. 63000, 9248, 7464

- **Divisibility of 16 –:** If the last three digits (Thousand's place, hundred's place, then's place, units place) of a number is either '0000' or divisible by 16, then the number is divisible by 16.

  Ex. 630000, 948464



Divisibility of 3 and 9

- **Divisibility of 3 –:** If the sum of its all digits of a number is divisible by 3, then the number is divisible by 3.

  Ex. 78141

  $\Rightarrow \frac{7+8+1+4+1}{3} = \frac{21}{3} = 7 \text{ (divisible)}$

  Hence, the number 78141 will be divisible by 3

  Ex. 246753

  $\Rightarrow \frac{2+4+6+7+5+3}{3} = \frac{27}{3} = 9 \text{ (divisible)}$

  Hence, the number 246753 will be divisible by 3

- **Divisibility of 9 –:** If the sum of its all digits of a number is divisible by 9, then the number is divisible by 9)

Ex. 764352

$\Rightarrow \frac{7+6+4+3+5+2}{9} \quad \frac{27}{9} = 3 \text{ (divisible)}$

Hence, the number 764352 will be divisible by 9

Ex. 432432

$\Rightarrow \frac{4 \quad 3 \quad 2 \quad 4 \quad 3 \quad 2}{9} \quad \frac{18}{9} \quad 2 \text{ (divisible)}$

Hence, the number 432432 will be divisible by 9

☞ In divisibility of 3 and 9, we can use 'digital sum' in place of sum.

Digital sum –: It is just a position of remainder when it is divided by 9. That is, the sum of the digits should be 9. If it is more than 9 then add the digits together.

Ex. 10 Digtital sum 1 0 1

11 Digtital sum 1 1 2

84 Digtital sum 8 4 12 1 2 3

786 Digtital sum 7 8 6 21 2 1 3

☞ Cut all digits whose sum is 9

☞ Digital sum of a perfect square number 0 or 9, 1, 4, 7

☞ To calculate digital sum in fraction number, then always make digital sum 1 in denominator.

Denominator	Multiply	Digital sum
4	4 × 7 = 28	1
7	7 × 4 = 28	1
5	5 × 2 = 10	1
2	2 × 5 = 10	1
8	8 × 8 = 64	1

Note– If the denominator of a number is 3, 6 or 9 then 1 can not be made for the digital sum.

Divisibility of 5, 10, 25 and 100

- **Divisibility of 5 –:** If the digit at unit place of a number is either 0 or 5 then the number is divisible by 5.

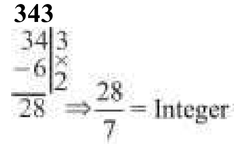
  Ex. 24520, 28735
- **Divisibility of 10) –:** If the digit at unit place of a number is 0 then the number is divisible by 10.

  Ex. 570120, 4567890
- **Divisibility of 25 –:** If the last two digits (ten's, unit's place) of a number either 25, 50, 75 or 00, then the number is divisible by 25.

  Ex. 8725, 68750, 931275, 8600
- **Divisibility of 100 –:** If the last two digits (ten's, unit's place) of a number 00, then the number is divisible by 100.

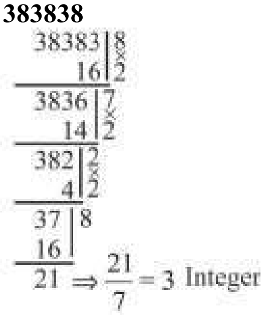
  Ex. 689200
- **Divisibility of 7 –:** If the number obtained by subtracting twice the unit digit from the remaining number excluding the unit digit, is divisible by 7, then that number will be divisible by 7. Repeat this process again and again for larger numbers.

  Ex. 343



  Hence, 343 is divisible by 7

  Ex. 383838



  Hence, 383838 is divisible by 7
- **Divisibility of 11 –:** If the difference of the sum of the digits in even position and the sum of the digits in odd position is zero or multiple of 11.

  Ex. $\bar{3}\underline{5}\bar{2}\underline{1}\bar{4}\underline{3}$

Sum of even position = 4 + 2 + 3 = 9

Sum of odd position = 3 + 1 + 5 = 9

$\Rightarrow |9-9| = 0$

Hence, the number 352143 is divisible by 11

Ex. $7\bar{1}9\bar{4}0$

Sum of even position = 4 + 1 = 5

Sum of odd position = 0 + 9 + 7 = 16

$$\Rightarrow \frac{|5-16|}{11} = 1 \text{ (Integer)}$$

Hence, the number 71940 is divisible by 11

Divisibility of 7, 11, 13

- Make pairs of three digits from the right side of a numbers. Find the difference between sum of pairs at even places and sum of pairs at odd places–
- ☞ If the difference is 0, then the number will be divisible by 7, 11 and 13.
- ☞ If the difference is divisible by any of 7, 11 and 13, then the number will also be divisible by that.

Ex. 786786

786 786 = |786 – 786| ⇒ 0

Hence, the number is divisible by 7, 11 and 13.

Ex. 1001

001 001 = |001 – 001| ⇒ 0

Hence, the number is divisible by 7, 11 and 13.

Ex. 786730

786 730 = |786 – 730|

⇒ 56 (Divisible by 7)

Hence, the number is divisible by 7

Ex. 5786

005 786 = |005 – 786|

⇒ 781 (Divisible by 11)

Hence, the number is divisible by 11

Ex. 91689

091 689 = |091 – 689|

⇒ 598 (Divisible by 13)

Hence, the number is divisible by 13

Ex. 786709

786 709 = |786 – 709|

⇒ 77 (Divisible by 7 and 11)

Hence, the number is divisible by 7 and 13.

- When a number is divisible by another number, It is also divisible by the factor of the number.

Ex. 48 is divisible by 12

Then, 48 is also divisible by factor (1, 2, 3, 4, 6, 12) of 12.

- When a number is divisible by two or more co-prime numbers, It is also divisible by their products.

Ex. 12 is divisible by 2 and 3.

∵ (2, 3) → Co-prime number

∴ 12, 12 is divisible by (2 × 3).

- When a number is a factor of two given number It is also a factor of their sum and difference.

Ex. ∵ 6 is factor of 30 and 6 is factor of 18.

Then, 6 is factor of {(30 + 18) = 48} and {(30 – 18) = 12}

- When a number is a factor of another number, It is also a factor of any multiple of that number.

Ex. ∵ 4 is factor of 12

Then, 4 is also factor of multiple (12, 24, 36, ...) of 12.

- ☞ If a number is formed by repeating a digit six times, it will be divisible by 3, 7, 11, 13, 37.

Ex. (111111), (222222), (333333)

- ☞ If a number is formed by repeating 2 digit 3 times, it will be divisible by 3, 7, 13, 37.

Ex. 383838, 171717, 595959

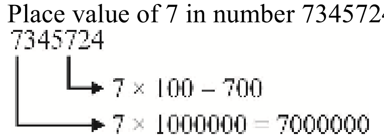
- ☞ If a number repeats the same digit 3, 6, 9, 12 (multiple of 3), then that number will be divisible by 3 and 37.

Ex. (111), (222222), (333333333), (444444444444)

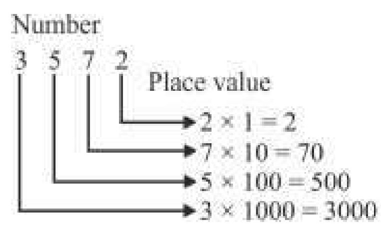
Place value and face value

Place value –: The place value of a digit describes its place in a given number.

Ex. Place value of 7 in number 7345724–



Ex.



Ex. Write 'Eleven thousand eleven hundred eleven' in digits–

```
  11000
   1100
+    11
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  12111
```
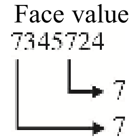
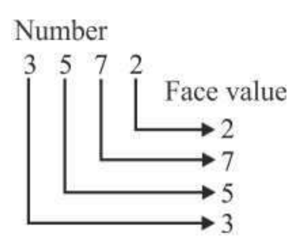

Face value –: Face value is the value of the digit itself in a number. It does not depend upon its position in the number.

Ex. Face value of 7 in number 7345724–



Ex.



- ☞ The face value as well as place value of zero is always zero.

Place value of a decimal number

78623.4679

$7 \times 10000 = 70000$
$8 \times 1000 = 8000$
$6 \times 100 = 600$
$2 \times 10 = 20$
$3 \times 1 = 3$

$9 \times \frac{1}{10000} = 0.0009$
$7 \times \frac{1}{1000} = 0.007$
$6 \times \frac{1}{100} = 0.06$
$4 \times \frac{1}{10} = 0.4$

Division operation in numbers

divisor **(d)** | Dividend **(D)** | quotient **(q)**

remainder **(r)**

D = dq + r Where, $0 \le r < d$

Ex. Find the number in which dividing by 15 gives quotient 14 and remainder 13?

Solve– D = dq + r
D = 15 × 14 + 13
D = 223

Ex. By dividing a number by 11 and 5 successively, the remainder remains 2 and 3 respectively, what will be the remainder if the number is divided by 55?

Solve– ∵ 11 × 5 = 55
11 and 5 are factors of 55
∴ D = 11 × 3 + 2
D = 35

Ex. When two different number are divided by a divisor, the remainder becomes 547 and 349 respectively when the sum of both numbers is divided by the same divisor, the remainder is 211, find the divisor.

Solve–
First quotient = q_1
Second quotient = q_2
Common divisor = d

∴ First number = $dq_1 + 547$
Second number = $dq_2 + 349$

then, $\frac{dq_1 \quad 547 \quad dq_2 \quad 349}{d}$ Remainder 211

∴ d = 547 + 349 – 211
d = 685

Ex. When a number is divided by 441, the remainder is 40. If the same number is divided by 21, the remainder will be?

Solve– ∵ 21 is the factor of 441
∴ $\frac{40}{21}$ Remainder **19**
Hence, the remainder will be 19.

Ex. When a number is divided by 231, the remainder is 45. If the same number is divided by 17, the remainder will be?

Solve–
∵ 17 is not the factor of 231
∴ The remainder can not be determined

Unit digit

- **The last digit of a number is called the unit digit.**

4364357 → Unit digit

- 763 + 542 ⇒ 1305 → Unit digit
- 765 + 849 ⇒ 1614 → Unit digit
- 763 – 542 ⇒ 221 → Unit digit
- 765 – 347 ⇒ 418 → Unit digit
- 765 – 947 ⇒ –182 → Unit digit
- 765 – 943 ⇒ –178 → Unit digit

- In subtraction problems, while finding the unit digit, the smaller number is subtracted from the larger number.
- The last digit of the answer obtained will be unit digit. The answer obtained can be positive or negative, but not the unit digit.

Finding the unit digit when number is raised to the power

- **When the unit digit of a number is 0, 1, 5 and 6 and it has any power, then its unit digit will be the same digit.**

- (1530)[illegible] → Unit digit
- (761)[illegible] → Unit digit
- (765)[illegible] → Unit digit
- (786)[illegible] → Unit digit

- **When the unit digit of a number is 2, 3, 4, 7, 8, and 9 and it has any power, then find the unit digit–**
- Digit last two digits of power by 4 and find out remainder

$\frac{\text{Last two digits of power}}{4}$

Remainder ⇒ 1, 2, 3, 0

Remainder	Power
1	1
2	2
3	3
0	4

☞ $[172]^{4325}$

$\frac{25}{4} \xrightarrow{\text{Remainder}} 1 \xrightarrow{\text{Power}} 1$

$2^1 \Rightarrow 2$ ↳ Unit digit

☞ $[978]^{4798}$

$\frac{98}{4} \xrightarrow{\text{Remainder}} 2 \xrightarrow{\text{Power}} 2$

$8^2 \Rightarrow 64$ ↳ Unit digit

☞ $[567]^{8759}$

$\frac{59}{4} \xrightarrow{\text{Remainder}} 3 \xrightarrow{\text{Power}} 3$

$7^3 \Rightarrow 343$ ↳ Unit digit

☞ $[6543]^{9972}$

$\frac{72}{4} \xrightarrow{\text{Remainder}} 0 \xrightarrow{\text{Power}} 4$

$3^4 \Rightarrow 81$ ↳ Unit digit

When the number is in the form of N!

☞ **When the power is in the form of n!–**

$1! = 1$
$2! = 2 \times 1$
$3! = 3 \times 2 \times 1$
$4! = 4 \times 3 \times 2 \times 1$
$5! = 5 \times 4 \times 3 \times 2 \times 1$
$5! = 5 \times 4!$
$n! = n(n-1)!$

$\frac{n!}{4} \xrightarrow{\text{Remainder}} 0 \xrightarrow{\text{Power}} 4$

Where, $n! \geq 4$

Ex. $992^{786!}$

$\because 786! > 4! \xrightarrow{\text{Remainder}} 0 \xrightarrow{\text{Power}} 4$

$\therefore 2^4 = 16$ ↳ Unit digit

☞ **When the number is in the form of multiplication of n!–**

Number	0!	1!	2!	3!	4!
Unit digit	1	1	2	6	4

- 5! and greater than 5! give unit digit 0.

Unit digit of multiplication by 5

☞ $5 \times \text{Odd number} \xrightarrow{\text{Unit digit}} 5$

Ex. $5 \times 1 = 5 \xrightarrow{\text{Unit digit}} 5$

Ex. $5 \times 3 = 15 \xrightarrow{\text{Unit digit}} 5$

☞ $5 \times \text{Even number} \xrightarrow{\text{Unit digit}} 0$

Ex. $5 \times 2 = 10 \xrightarrow{\text{Unit digit}} 0$

Ex. $5 \times 4 = 20 \xrightarrow{\text{Unit digit}} 0$

☞ $5 \times \text{Odd number} \times \text{Even number} \xrightarrow{\text{Unit digit}} 0$

Ex. $5 \times 1 \times 2 = 10 \xrightarrow{\text{Unit digit}} 0$

Ex. $5 \times 3 \times 4 = 60 \xrightarrow{\text{Unit digit}} 0$

- The unit digit of a perfect square number can be 0, 1, 4, 5, 6 or 9 but if the unit digit of a number is 0, 1, 4, 5, 6 or 9 then it is not necessary that it is a perfect square number.

Zero Place
Number of trailing zeroes

- A zero is formed by a pair of 5 and 2, i.e. by multiplying 5 and 2, we get zero
- In any question, as many pairs of five and two are formed, The same zero is formed. Therefore, to solve the question the powers of 5 and 2 are seen and whose power is less, the same zero is created.

☞ $5 \times 2 = 10$

$5^1 \times 2^1 \xrightarrow{\text{No. of pair}} 1 \xrightarrow{\text{No. of zero}} 1$

☞ $25 \times 4 = 100$

$5^2 \times 2^2 \xrightarrow{\text{No. of pair}} 2 \xrightarrow{\text{No. of zero}} 2$

☞ $125 \times 4 = 500$

$5^3 \times 2^2 \xrightarrow{\text{No. of pair}} 2 \xrightarrow{\text{No. of zero}} 2$

(Which power less)

☞ $25 \times 8 = 200$

$5^2 \times 2^3 \xrightarrow{\text{No. of pair}} 2 \xrightarrow{\text{No. of zero}} 2$

(Which power less)

☞ $125 \times 8 = 1000$

$5^3 \times 2^3 \xrightarrow{\text{No. of pair}} 3 \xrightarrow{\text{No. of zero}} 3$

Ex. Multiplying 25 × 16 × 2 × 5 will be how many zeros on the right side.

Sol. 25 × 16 × 2 × 5

$\Rightarrow 5 \times 5 \times 2 \times 2 \times 2 \times 2 \times 5$

$\Rightarrow 5^3 \times 2^4$

$5^3 \times 2^4 \xrightarrow[\text{(Which power less)}]{\text{No. of pair}} 3 \xrightarrow{\text{No. of zero}} 3$

Ex. Multiplying 300 × 400 × 24 × 25 will be how many zeros on right side.

Sol. 300 × 400 × 24 × 25

$\Rightarrow 3 \times 4 \times 24 \times 25 \times 10000$

$\Rightarrow 3 \times 4 \times 2 \times 2 \times 2 \times 3 \times 5 \times 5 \times 10000$

$\Rightarrow 2^5 \times 5^2 \times 3^2 \times 10000$

$\underbrace{2^5 \times 5^2}_{(00)} \times 3^2 \times \underbrace{10000}_{(0000)}$

Number of zeroes = 6

Ex. Multiplying all natural numbers from 1 to 60, how many zeros will come to the right side.

Sol. 1 × 2 × 3 ×......... × 25 × ....× 50 × ...... × 60

$\frac{60}{5} = 12$

$\frac{12}{5} = 2 \qquad 12 + 2 = 14 \text{ (Zeroes)}$

☞ In the given question it is clear that on multiplying, the power of five is less than that of 2.

☞ Stop dividing when the quotient is less than 5.

Ex. Multiplying all natural number from 1 to 100, How many zeros will come to right side.

Sol. 1 × 2 × 3 ×..... × 25 × .....× 50 ×...... ×75 × ...... × 100

$\Rightarrow \frac{100}{5} = 20$

$\frac{100}{25} = 4 \qquad 20 + 4 = 24 \text{ (Zeroes)}$

Ex. Multiplying all natural numbers from 1 to 500, how many zeros will come to right side.

Sol. 1 × 2 × 3 ×.....× 25 × .....× 50 ×......× 100× ......×500

$\frac{500}{5} = 100$

$\frac{100}{5} = 20$

$\frac{20}{5} = 44 \qquad 100 + 20 + 4 = 124 \text{ (Zeroes)}$

Ex. Multiplying all natural numbers 1 to 1000, How many zeros will come to right side.

Ex. 1 × 2 × 3 ×.....× 25 × .....× 50 ×......× 100× ......×1000

$\frac{1000}{5} = 200$

$\frac{200}{5} = 40$

$\frac{40}{5} = 8$

$\frac{8}{5} = 1 \qquad 200 + 40 + 8 = 249 \text{ (Zeroes)}$

Ex. Multiplying all even numbers upto 80, How many zeros will come to right side.

Sol. 2 × 4 × 6 × .... × 80

$\frac{80}{10} = 8$

$\frac{8}{5} = 1 \qquad 8 + 1 = 9 \text{ (Zeroes)}$

☞ In multiplication of even number, first divide by 10, then by 5

Ex. Multiplying all the numbers 51 to 100, How many zeros will come to right side.

Sol. 51 × 52 × 53 100

$\Rightarrow$ [1 × 2 × 3 .... 100] – [1 × 2 × 3 .... 50]

$\Rightarrow \frac{100}{5} = 20 \qquad \frac{50}{5} = 10$

$\frac{20}{5} = 4 \qquad \frac{10}{5} = 2$

$\Rightarrow$ [20 + 4 = 24] [10 + 2 = 12]

$\Rightarrow$ [24] – [12] = 12 (Zeroes)

Ex. On solving 96! how many zeros will come to right side.

Sol. 96! = 96 × 95 × 94 × ....× 1

$\frac{96}{5} \quad 19$

$\frac{19}{5} \quad 3 \qquad 19 + 3 = 22 \text{ (Zeroes)}$

Ex. On solving 9860!, How many zeros will come to right side.

Sol. 9860! = 9860 × 9859 .... × 1

$\therefore \frac{9860}{5} = 1972$

$\frac{1972}{5} = 394$

$\frac{394}{5} = 78$

$\frac{78}{5} = 15$

$\frac{15}{5} = 3$

$\Rightarrow$ 1972 + 394 + 78 + 15 + 3 = 2462 (Zeroes)

Ex. Multiplying all the odd numbers 1 to 100, how many zeros will come to right side.

Sol. 1 × 3 × 5 × 7 × 9 × 11 99

''Number of zeroes is zero''

☞ In the given question all the numbers are odd, no number will be divisible by 2. Hence no digit of two will appear in the product of these numbers. Hence not a single zero will be obtained at the end of the product of the given question.

Ex. Multiplying the first 100 prime numbers, How many zeros will come to right side.

Sol. 2 × 3 × 5 × 7 × 11 × 13 × 17 × 19 × .... × 97

⇒ 2 × 5

⇒ $2^1 \times 5^1$

= Number of zero = 1

Ex. How many zeroes on the right end of the product of (1 × 3 × 5 × 7 × .... × 99) × 8.

Sol. (1 × 3 × 5 × 7 × .... × 99) × 8

(5 × 15 × 25 × 35 × .... × 95) × 8

{For pair of 5 and 2}

⇒ $5^{12} \times 2^3$

$5^{12} \times 2^3 \xrightarrow{\text{No. of pair}} 3 \xrightarrow{\text{No. of zero}} 3$

(Which power less)

Ex. Find the number of zeroes.

$(3^{123} - 3^{122} - 3^{121})\,(2^{121} - 2^{120} - 2^{119})$

Sol. $(3^{123} - 3^{122} - 3^{121})\,(2^{121} - 2^{120} - 2^{119})$

$3^{121}\,(3^2 - 3^1 - 3^0)\,2^{119}\,(2^2 - 2^1 - 2^0)$

3^{121} (9 – 3 – 1) 2119 (4 – 2 – 1)

$3^{121}\,(5)\,2^{119}\,(1)$

$2^{119} \times 3^{121} \times 5^1$

$2^{119} \times 5^1 \times 3^{121}$

No. of pair 1 → no. of zero = 1

Ex. If 100! divisible by 3^n then find the maximum value of n :

Sol. 100! = 100 × 99 × 98 × .... × 1

$\frac{100}{3}$ 33

$\frac{33}{3}$ 11

$\frac{11}{3}$ 3

$\frac{3}{3}$ 1

⇒ 33 + 11 + 3 + 1 = 48

Hence n = 48

Ex. If 122! is divisible by 6^n then find the maximum value of n :

Sol. $\frac{122!}{6}$ $\frac{122!}{2\ \ 3}$

To make a pair of 2 and 3, the power of 3 will be reduced.

$\frac{122}{3}$ 40

$\frac{40}{3}$ 13

$\frac{13}{3}$ 4

$\frac{4}{3}$ 1

⇒ 40 + 13 + 4 + 1 = 58

Hence n = 58

Ex. If 123! is divisible by 12^n then find the maximum value of n :

Sol. $\frac{123!}{12^n}$ $\frac{123}{3\ \ 2^2}$

$\frac{123!}{3^{59}\ \ 2^{117}}$

$\frac{123!}{3^{59}\ \ {2^2}^{58}\ \ 2^1}$

$\frac{123!}{3^{59}\ \ 4^{58}\ \ 2^1}$

Hence n = 58

$\frac{123}{3}$ 41

$\frac{41}{3}$ 13

$\frac{13}{3}$ 4

$\frac{4}{3}$ 1

Sum = 59

$\frac{123}{2}$ 61

$\frac{61}{2}$ 30

$\frac{30}{2}$ 15

$\frac{15}{2}$ 7

$\frac{7}{2}$ 3

$\frac{3}{2}$ 1

Sum = 117

Number of factors

Factors

Factors are positive integers that can divide a number exactly.

Ex. Factors of 12

1, 2, 3, 4, 6, 12

☞ Multiple of 12

12, 24, 36, 48, ...

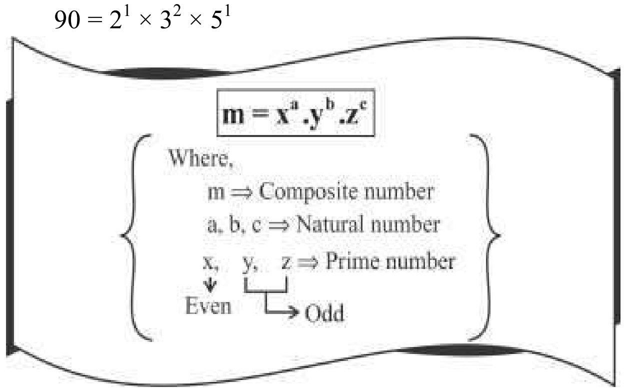
How to find factors

■ Writing any numbers as its prime factors.

Ex. $12 = 2^2 \times 3^1$

$72 = 2^3 \times 3^2$

$90 = 2^1 \times 3^2 \times 5^1$



■ The number of total factors−: **(a + 1) (b + 1) (c + 1)**

■ The number of odd factors−: **(b + 1) (c + 1)**

■ The number of even factors−: **a (b + 1) (c + 1)**

■ The sum of all factors−:
$(x^0 + x^1 + x^2 \ldots\ldots x^a) \times (y^0 + y^1 + y^2 \ldots\ldots y^b) \times (z^0 + z^1 + z^2 \ldots\ldots z^c)$

■ The sum of odd factors−: **$(y^0 + y^1 + \ldots\ldots y^b) \times (z^0 + z^1 + z^2 \ldots\ldots z^c)$**

■ The sum of even factors−: **$(x^1 + x^2 + x^3 \ldots\ldots x^a) \times (y^0 + y^1 \ldots\ldots y^b) \times (z^0 + z^1 + z^2 \ldots\ldots z^c)$**

■ The product of factors −: **$(x.y.z)^{\text{Total no. of factors}/2}$**

- Sum of reciprocal of factors of n) = $\frac{\text{sum of factors}}{n}$
- Average $\frac{\text{Sum of factors}}{\text{No. of factors}}$

For the factors of 12

$12 = 2^2 \times 3^1$

- **The number of total factors–**

$12 = 2^2 \times 3^1$

$(2 + 1) \times (1 + 1)$

$3 \times 2 = \mathbf{6}$

- **The number of odd factors–**

$12 = 2^2 \times 3^1$

$(1 + 1) = \mathbf{2}$

- **The number of even factors–**

$12 = 2^2 \times 3^1$

$2 \times (2^1 \times 3^1)$

Even $(1 + 1) \times (1 + 1)$

$(2) \times (2) = \mathbf{4}$

- **The sum of factors–**

$12 = 2^2 \times 3^1$

$= (2^0 + 2^1 + 2^2)\ (3^0 + 3^1)$

$= (1 + 2 + 4)\ (1 + 3)$

$= 7 \times 4 \Rightarrow \mathbf{28}$

- **The sum of odd factors–**

$12 = 2^2 \times 3^1$

$\Rightarrow (3^0 + 3^1)$

$1 + 3 \Rightarrow \mathbf{4}$

☞ For the sum of odd factors, leave out even factors.

- **The sum of even factors–**

$12 = 2^2 \times 3^1$

$\Rightarrow (2^1 + 2^2)\ (3^0 + 3^1)$

$\Rightarrow (2 + 4)\ (1 + 3)$

$\Rightarrow 6 \times 4$

$\Rightarrow \mathbf{24}$

☞ For sum of even factors, don't start from 2^0.

- **The product of all factors–**

$12 = 2^2 \times 3^1$

Product of all factors of N = $N^{\text{Total no. of factors/2}}$

$= 12^{\frac{6}{2}}$

$= \mathbf{12^3}$

$12 = 2^2 \times 3^1$

$(2 + 1) \times (1 + 1)$

$3 \times 2 = \mathbf{6}$

- **How many factors of 864 which are multiple of 6?**

Sol. $864 = 2^5 \times 3^3$

$864 = 2 \times 3\ [2^4 \times 3^2]$ {For the multiple of 6}

$= 6\ [2^4 \times 3^2]$

$(4 + 1)\ (2 + 1)$

$\Rightarrow 5 \times 3$

$\Rightarrow 15$

- **How many factors of $2^7 \times 3^8 \times 5^9 \times 7^{10}$ which are completely square?**

Sol. $2^7 \times 3^8 \times 5^9 \times 7^{10}$

$\Rightarrow [(2^2)^3\ 2 \times (3^2)^4 \times (5^2)^4\ 5 \times (7^2)^5]$

{For the complete square }

$= 2 \times 5\ [(2^2)^3 \times (3^2)^4 \times (5^2)^4 \times (7^2)^5]$

$\Rightarrow (3+1)\times(4+1)\times(4+1)\times(5+1)$

No. of factors = $4 \times 5 \times 5 \times 6 \Rightarrow 600$

- **How many factors of $2^6 \times 3^8 \times 5^{10} \times 7^{12}$ which are completely cube?**

Sol. $2^6 \times 3^8 \times 5^{10} \times 7^{12}$

$\Rightarrow (2^3)^2 \times (3^3)^2 \times (5^3)^3 \times 5 \times (7^3)^4$

$\Rightarrow 3^2 \times 5[(2^3)^2 \times (3^3)^2 \times (5^3)^3 \times (7^3)^4]$

$\Rightarrow (2+1)\times(2+1)\times(3+1)\times(4+1)$

$\Rightarrow 3 \times 3 \times 4 \times 5 \Rightarrow 180$

- **How many factors of $2^6 \times 3^{15} \times 5^{35} \times 7^{42}$ which are completely square as well as completely cube?**

Sol. $2^6 \times 3^{15} \times 5^{35} \times 7^{42}$

Power for square = 2

Power for cube = 3

LCM = 6

$\Rightarrow [(2^6)^1 \times (3^6)^2 \times 3^3 \times (5^6)^5 \times 5^5 \times (7^6)^7]$

$\Rightarrow 3^3 \times 5^5[(2^6)^1 \times (3^6)^2 \times (5^6)^5 \times (7^6)^7]$

$\Rightarrow (1+1)\times(2+1)\times(5+1)\times(7+1)$

$\Rightarrow [2 \times 3 \times 6 \times 8] \quad \Rightarrow [6 \times 6 \times 8]$

$\Rightarrow [36 \times 8] \quad \Rightarrow 288$

- **Find the sum of all factors of $2^5 \times 3^6 \times 5^4$ that are completely square.**

Sol. $2^5 \times 3^6 \times 5^4$

$\Rightarrow [2^0 + 2^2 + 2^4]\ [3^0 + 3^2 + 3^4 + 3^6]\ [5^0 + 5^2 + 5^4]$

$\Rightarrow [1 + 4 + 16]\ [1 + 9 + 81 + 729]\ [1 + 25 + 625]$

$\Rightarrow [21] \times [820] \times [651] \Rightarrow 11210220$

- **Find the sum of all factors of $2^5 \times 3^6 \times 5^4$ that are completely cube.**

Sol. $2^5 \times 3^6 \times 5^4$

$\Rightarrow [2^0 + 2^3]\ [3^0 + 3^3 + 3^6]\ [5^0 + 5^3]$

$\Rightarrow [1 + 8]\ [1 + 24 + 729]\ [1+125]$

$\Rightarrow [9]\ [757]\ [126] \Rightarrow 858438$

- **Find the sum of reciprocal of factors of 90.**

Sol. Sum of reciprocal of factors of n) = $\frac{\text{sum of factors}}{n}$

90 = 21 × 32 × 51

$\Rightarrow \frac{2^0 \quad 2^1 \quad 3^0 \quad 3^1 \quad 3^2 \quad 5^0 \quad 5^1}{90}$

$\Rightarrow \frac{1 \quad 2 \quad 1 \quad 3 \quad 9 \quad 1 \quad 5}{90}$

$\Rightarrow \frac{3 \quad 13 \quad 6}{90} \Rightarrow \frac{39 \quad 6}{90}$

$\Rightarrow \frac{234}{90} \Rightarrow 2.6$

■ **Find the average of all the factors of 144.**

Sol. Average $\frac{\text{Sum of factors}}{\text{No. of factors}}$

For sum of factors–

$144 = 2^4 \times 3^2$

$\Rightarrow [(2^0 + 2^1 + 2^2 + 2^3 + 2^4)(3^0 + 3^1 + 3^2)]$

$\Rightarrow [(1 + 2 + 4 + 8 + 16)(1 + 3 + 9)]$

$\Rightarrow [(31)(13)] \Rightarrow 403$

For no. of factors–

$\Rightarrow (4 + 1)(2 + 1) \Rightarrow 5 \times 3$

$\Rightarrow 15$

Average = $\frac{403}{15} \Rightarrow 26.86$

■ Only a perfect square number has odd number of factors.

or

If a number has odd number of factors that means number is a perfect square.

■ Square of a prime number has only 3 factors.

■ **The total number of 2 digit no's which have only 3 factors?**

Sol. ∵ Square of a prime number has only 3 factor.

$(5^2) = 25 \xrightarrow{\text{Factors}} 1, 5, 25$

$(7^2) = 49 \xrightarrow{\text{Factors}} 1, 7, 49$

5, 7 → Prime number

Hence, 2, two digit no. will have 3 factors.

■ **The total number of 3 digit no's which have only 3 factors?**

Sol.

$(11)^2 = 121 \xrightarrow{\text{Factors}} 1, 11, 121$

$(13)^2 = 169 \xrightarrow{\text{Factors}} 1, 13, 169$

$(17)^2 = 289 \xrightarrow{\text{Factors}} 1, 17, 289$

$(19)^2 = 361 \xrightarrow{\text{Factors}} 1, 19, 361$

$(23)^2 = 529 \xrightarrow{\text{Factors}} 1, 23, 529$

$(29)^2 = 841 \xrightarrow{\text{Factors}} 1, 29, 841$

$(31)^2 = 961 \xrightarrow{\text{Factors}} 1, 31, 961$

Hence, 7, three digit no. will have 3 factors.

How to find prime factor

$m = x^a . y^b . z^c$

Where,

m ⇒ Composite number

x, y, z ⇒ Prime number

a, b, c ⇒ Natural number

Number of prime factors = a + b + c

Sum of prime factors = ax + by + cz

■ **Find the total number of prime factors of 144.**

Sol. $144 = 2^4 \times 3^2$

No. of prime factors $= 4 + 2 \Rightarrow 6$

■ **Find the total number of prime factor of $2^5 \times 3^6 \times 7^{12}$.**

Sol. $2^5 \times 3^6 \times 7^{12}$

No. of prime factors $= 5 + 6 + 12 \Rightarrow 23$

■ **Find the total number of prime factor of $6^6 \times 10^{10} \times 35^3$.**

Sol. $6^6 \times 10^{10} \times 35^3$

$\Rightarrow (2 \times 3)^6 \times (2 \times 5)^{10} \times (5 \times 7)^3$

$\Rightarrow 2^6 \times 3^6 \times 2^{10} \times 5^{10} \times 5^3 \times 7^3$

No. of prime factors

$= (6 + 6 + 10 + 10 + \ + 3)$

$\Rightarrow (12 + 20 + 6)$

$\Rightarrow (18 + 20) \Rightarrow 38$

■ **Find sum of all the prime factors of $2^3 \times 3^4 \times 5^6$.**

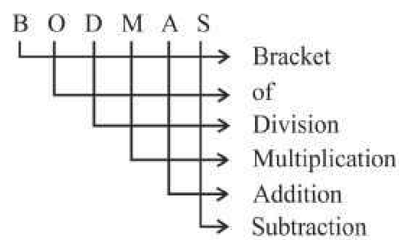
Sol. $2^3 \times 3^4 \times 5^6$

⇒ (2 + 2 + ... 3 times) + (3 + 3 +.......... 4 times) + (5 + 5 + ... 6 times)

$\Rightarrow (2 \times 3) + (3 \times 4) + (5 \times 6)$

$\Rightarrow 6 + 12 + 30 \Rightarrow 48$

BODMAS Rule



☞ Solve the brackets from inside to outside.

Types of brackets :

➢ Line/Bar bracket → ‾‾‾

➢ Circular/Small/Open bracket → **( )**

➢ Curly/Braces bracket → { }

➢ Square/Closed bracket → **[ ]**

■ **To solve :**

$$\mathbf{222 - \frac{1}{3} of \ \ 42 + \ 56 \ \ \overline{8 + 9} \ \ + 108}$$

Sol. $222 - \frac{1}{3} \text{of} \ \ 42 + \ 56 \ \ \overline{8 + 9} \ \ + 108$

$222 - \frac{1}{3} \text{of} \ \ 42 + \ 56 \ \ 17 \ \ + 108$

$\Rightarrow \ 222 \ \ \frac{1}{3} \text{of} \ \ 42 \ \ 39 \ \ 108$

$\Rightarrow \ 222 \ \ \frac{1}{3} \text{of} \ \ 81 \ \ 108$

$\Rightarrow \ 222 \ \ \frac{1}{3} \text{of} \ 81 \ \ 108$

$\Rightarrow \ 222 - [27 + 108]$

$\Rightarrow \ 222 - 135 \Rightarrow$ **87**

■ **To solve :**

$\mathbf{a \quad b \quad c \quad a \quad \overline{b \quad c}}$

Sol. $a \quad b \quad c \quad a \quad \overline{b \quad c}$

$\Rightarrow a \quad b \quad c \quad a \quad b \quad c$

$\Rightarrow a \quad b \quad c \quad a \quad b \quad c$

$\Rightarrow a \quad b \quad b \quad a$

$\Rightarrow a \quad b \quad b \quad a$

$\Rightarrow a \quad a$

$\Rightarrow$ **0**

■ **To solve :**

19170 ÷ 54 ÷ 5

Sol. 19170 ÷ 54 ÷ 5

$\Rightarrow 19170 \quad \frac{1}{54} \quad \frac{1}{5}$

$\Rightarrow \frac{355}{5}$

$\Rightarrow$ **71**

■ **To solve :**

$\mathbf{\frac{9}{13} \div \frac{18}{26} \div \frac{90}{52}}$

Sol. $\frac{9}{13} \div \frac{18}{26} \div \frac{90}{52}$

$\Rightarrow \frac{9}{13} \times \frac{26}{28} \times \frac{52}{90}$

$\Rightarrow \mathbf{\frac{26}{45}}$

■ **To solve :**

5.8 + (7.4 ÷ 3.7 × 5) – 6 × 2 ÷ 2.5

Sol. 5.8 + (7.4 ÷ 3.7 × 5) – 6 × 2 ÷ 2.5

$\Rightarrow 5.8 + (2 \times 5) - 6 \times \frac{2}{2.5}$

$\Rightarrow 5.8 + 10 - 4.8$

$\Rightarrow 15.8 - 4.8$

$\Rightarrow$ **11**

Question based on series

➢ $\frac{1}{a \times b} \quad \frac{1}{b \quad a} \quad \frac{1}{a} \quad \frac{1}{b}$

➢ $\frac{1}{a \times b \times c} \quad \frac{1}{c \quad a} \quad \frac{1}{ab} \quad \frac{1}{bc}$

➢ $\frac{1}{a \times b \times c \times d} \quad \frac{1}{d \quad a} \quad \frac{1}{abc} \quad \frac{1}{bcd}$

➢ 1 × 2 + 2 × 3 + 3 × 4 + 4 × 5 + ... + n(n + 1)

$\frac{n \quad n \quad 1 \quad n \quad 2}{3}$

■ **Find the value :**

$\mathbf{\frac{1}{20} + \frac{1}{30} + \frac{1}{42} + \frac{1}{56} + \frac{1}{72} + \frac{1}{90}}$

Sol. $\frac{1}{20} + \frac{1}{30} + \frac{1}{42} + \frac{1}{56} + \frac{1}{72} + \frac{1}{90}$

$\Rightarrow \frac{1}{4 \quad 5} \quad \frac{1}{5 \quad 6} \quad \frac{1}{6 \quad 7} \quad \frac{1}{7 \quad 8} \quad \frac{1}{8 \quad 9} \quad \frac{1}{9 \quad 10}$

$\Rightarrow \frac{1}{4} \quad \frac{1}{5} \quad \frac{1}{5} \quad \frac{1}{6} \quad \frac{1}{6} \quad \frac{1}{7} \quad \frac{1}{7} \quad \frac{1}{8} \quad \frac{1}{8} \quad \frac{1}{9} \quad \frac{1}{9} \quad \frac{1}{10}$

$= \frac{1}{4} \quad \frac{1}{10}$

$= \frac{5 \quad 2}{20} \quad \mathbf{\frac{30}{20}}$

■ **Find the value :**

$\mathbf{\frac{1}{1 \times 4} + \frac{1}{4 \times 7} + \frac{1}{7 \times 10} + \frac{1}{10 \times 13} + \frac{1}{13 \times 16} = ?}$

Sol. $\frac{1}{1 \times 4} + \frac{1}{4 \times 7} + \frac{1}{7 \times 10} + \frac{1}{10 \times 13} + \frac{1}{13 \times 16}$

$\Rightarrow \frac{1}{3} \quad \frac{3}{1 \quad 4} \quad \frac{3}{4 \quad 7} \quad \frac{3}{7 \quad 10} \quad \frac{3}{10 \quad 13} \quad \frac{3}{13 \quad 16}$

$\Rightarrow \frac{1}{3} \quad \frac{1}{4} \quad \frac{1}{4} \quad \frac{1}{4} \quad \frac{1}{7} \quad \frac{7}{7} \quad \frac{1}{10} \quad \frac{1}{10} \quad \frac{1}{13} \quad \frac{1}{13} \quad \frac{1}{16}$

$\Rightarrow \frac{1}{3} \quad \frac{1}{1} \quad \frac{1}{16}$

$\Rightarrow \frac{1}{3} \quad \frac{16 \quad 1}{16} \quad \frac{1}{3} \quad \frac{15}{16} \quad \mathbf{\frac{5}{16}}$

■ **Find the value :**

$\mathbf{\frac{2}{15} + \frac{4}{45} + \frac{7}{144} + \frac{9}{400} = ?}$

Sol. $\frac{2}{15} + \frac{4}{45} + \frac{7}{144} + \frac{9}{400}$

$\Rightarrow \frac{2}{3 \quad 5} \quad \frac{4}{5 \quad 9} \quad \frac{7}{9 \quad 16} \quad \frac{9}{16 \quad 25}$

$\Rightarrow \frac{1}{3} \quad \frac{1}{5} \quad \frac{1}{5} \quad \frac{1}{9} \quad \frac{1}{9} \quad \frac{1}{16} \quad \frac{1}{16} \quad \frac{1}{25}$

$\frac{1}{3} \quad \frac{1}{25}$

$\frac{25 \quad 3}{75} \quad \mathbf{\frac{22}{75}}$

■ **Find the value :**

$\mathbf{\frac{3}{1^2.2^2} + \frac{5}{2^2.3^2} + \frac{7}{3^2.4^2} + \frac{9}{4^2.5^2} .... \frac{19}{9^2.10^2}}$

Sol. $\frac{3}{1^2.2^2} + \frac{5}{2^2.3^2} + \frac{7}{3^2.4^2} + \frac{9}{4^2.5^2} .... \frac{19}{9^2.10^2}$

$\frac{3}{1 \quad 4} \quad \frac{5}{4 \quad 9} \quad \frac{7}{9 \quad 16} \quad \frac{9}{16 \quad 25} .... \frac{19}{81 \quad 100}$

$\Rightarrow \frac{1}{1} \quad \frac{1}{4} \quad \frac{1}{4} \quad \frac{}{9} \quad \frac{1}{9} \quad \frac{1}{16} \quad \frac{1}{16} \quad \frac{1}{25} .... \frac{1}{81} \quad \frac{1}{100}$

$\Rightarrow \frac{1}{1} \quad \frac{1}{100}$

$\Rightarrow \mathbf{\frac{99}{100}}$

■ **Find the value :**

$$1\ \frac{1}{2}\quad 1\ \frac{1}{3}\quad 1\ \frac{1}{4}\ \ldots\ldots\ldots\ldots\ 1\ \frac{1}{n} = ?$$

Sol. $1\ \frac{1}{2}\quad 1\ \frac{1}{3}\quad 1\ \frac{1}{4}\ \ldots\ldots\ldots\ldots\ 1\ \frac{1}{n}$

$$\frac{3}{2}\times\frac{4}{3}\times\frac{5}{4}\ldots\ldots\ldots\ldots\frac{(n-1)}{n}$$

$$\frac{n+1}{2}$$

■ **Find the value :**

$$1\ \frac{1}{2}\quad 1\ \frac{1}{3}\quad 1\ \frac{1}{4}\ \ldots\ldots\ldots\ldots\ 1\ \frac{1}{n} = ?$$

Sol. $1\ \frac{1}{2}\quad 1\ \frac{1}{3}\quad 1\ \frac{1}{4}\ \ldots\ldots\ldots\ldots\ 1\ \frac{1}{n}$

$$\frac{1}{2}\times\frac{2}{3}\times\frac{3}{4}\ldots\ldots\ldots\frac{(n-1)}{n}$$

$\Rightarrow \frac{1}{n}$

■ **Find the value :**

$$1\ \frac{1}{3^2}\quad 1\ \frac{1}{4^2}\quad 1\ \frac{1}{5^2}\ \ldots\ldots\ 1\ \frac{1}{11^2}\quad 1\ \frac{1}{12^2}$$

Sol. $1\ \frac{1}{3^2}\quad 1\ \frac{1}{4^2}\quad 1\ \frac{1}{5^2}\ \ldots\ldots\ 1\ \frac{1}{11^2}\quad 1\ \frac{1}{12^2}$

$a^2 - b^2 = (a + b)\ (a - b)$

$1+\frac{1}{3}\quad 1\ \frac{1}{3}\quad 1+\frac{1}{4}\quad 1\ \frac{1}{4}\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

$\ldots\ldots\ 1+\frac{1}{11}\quad 1\ \frac{1}{11}\quad 1+\frac{1}{12}\quad 1\ \frac{1}{12}$

$\Rightarrow\ 1+\frac{1}{3}\quad 1\ \frac{1}{4}\quad 1+\frac{1}{5}\ \ldots\ldots\ 1\ \frac{1}{12}\ \times$

$1\ \frac{1}{3}\quad 1\ \frac{1}{4}\quad 1\ \frac{1}{5}\ \ldots\ldots\ 1\ \frac{1}{12}$

$$\left[\frac{4}{3}\times\frac{5}{4}\times\frac{6}{5}\ldots\ldots\frac{13}{12}\right]\left[\frac{2}{3}\times\frac{3}{4}\times\ldots\ldots\frac{11}{12}\right]$$

$\Rightarrow \frac{13}{3}\quad \frac{2}{12}$

$\Rightarrow \frac{13}{3}\quad \frac{1}{6}$

$\Rightarrow \mathbf{\frac{13}{18}}$

■ **Find the value :**

$$\frac{2\times8+8\times32+18\times72+\ldots\ldots\ldots}{1+16+81+\ldots\ldots\ldots\ldots}^{\frac{1}{4}} = ?$$

Sol. $\frac{2\times8+8\times32+18\times72+\ldots\ldots\ldots}{1+16+81+\ldots\ldots\ldots\ldots}^{\frac{1}{4}}$

$\Rightarrow\ 16\ \frac{1\quad 16\quad 81\quad \ldots\ldots}{1\quad 16\quad 81\quad \ldots\ldots}^{\frac{1}{4}}$

$\Rightarrow\ 16^{\frac{1}{4}}$

$\Rightarrow\ 2^{4\ \frac{1}{4}}\quad \mathbf{2}$

■ **Find the value :**

$$\frac{1.2.4+2.4.8+3.6.12+\ldots\ldots\ldots}{1.3.9+2.6.18+3.9.27+\ldots\ldots}^{\frac{1}{3}}$$

Sol. $\frac{1.2.4+2.4.8+3.6.12+\ldots\ldots\ldots}{1.3.9+2.6.18+3.9.27+\ldots\ldots}^{\frac{1}{3}}$

$\Rightarrow\ \frac{8}{27}^{\frac{1}{3}}$

$= \mathbf{\frac{2}{3}}$

Exponential Series

$$e = 1+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\frac{1}{4!}+\ldots\ldots\quad 2.71828$$

■ **Find the value :**

$$\frac{1}{1.2.3}+\frac{1}{1.2.3.4}+\frac{1}{1.2.3.4.5}+\ldots\ldots\ldots\ldots$$

Sol. $\frac{1}{3!}\quad\frac{1}{4!}\quad\frac{1}{5!}\quad\ldots\ldots\ldots\ldots$

$1\quad \frac{1}{1!}\quad\frac{1}{2!}\quad\frac{1}{3!}\quad\frac{1}{4!}\quad\ldots\ldots\ldots\quad 1\quad\frac{1}{1!}\quad\frac{1}{2!}$

= (2.71828) – (1 + 1 + 0.5)

= **0.21828**

■ **Find the value :**

$$\frac{8!\times7!\times6!}{9!\times5!\times3!} = ?$$

Sol. $\frac{8!\quad 7\quad 6\quad 5!\quad 6\quad 5\quad 4\quad 3!}{9\quad 8!\quad 5!\quad 3!}$

$\Rightarrow 28 \times 20$

$\Rightarrow$ **560**

■ **Find the value in the form of 6! :**

8! - 7! - 6!

Sol. 8! - 7! - 6!

$\Rightarrow$ [8 × 7 × 6! – 7 × 6! – 6]

$\Rightarrow$ 6! [8 × 7 – 7 – 1]

$\Rightarrow$ 6! [56 – 8]

$\Rightarrow$ **6![48]**

■ **If a * b = 2(a + b) then find the value 1 * [2 * 3]**

Sol. 1 * [2 * 3]

$\Rightarrow$ 1 * [2 (2 + 3)]

$\Rightarrow$ 1 * [2 × 5]

$\Rightarrow$ 1 * 10

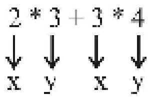
$\Rightarrow$ 2 [1 + 10]

$\Rightarrow$ 2 × 11

= **22**

■ **If x * y = 3x + 2y, then find the value 2 * 3 + 3 * 4**

Sol.



⇒ (3 × 2 + 2 × 3) + (3 × 3 + 2 × 4)

⇒ (6 + 6) + (9 + 8)

= 12 + 17 = **29**

■ **If @ is an operation such that**

$$a@b = \begin{cases} 2a \text{ यदि } > b \\ a + b \text{ यदि } a < b \\ a^2 \text{ यदि } = b \end{cases}$$

then, $\dfrac{\mathbf{5@7 \ + \ 4@4}}{\mathbf{3 \ \ 5@5 \quad 15@11 \quad 3}} = ?$

Sol. $\dfrac{5 \quad 7 \qquad 4^{\ 2}}{3\ \ 5^{\ 2} \quad 2 \quad 15 \quad 3}$

⇒ $\dfrac{12 \quad 16}{75 \quad 30 \quad 3}$

⇒ $\dfrac{28}{75 \quad 33}$

⇒ $\dfrac{28}{42} = \mathbf{\dfrac{2}{3}}$

■ **Find the value :**

$\mathbf{999\dfrac{995}{999} \times 999}$

Sol. $999\dfrac{995}{999} \times 999$

⇒ $999 + \dfrac{995}{999} \quad 999$

⇒ $1000 \quad 1 \quad \dfrac{995}{999} \quad 999$

⇒ $\dfrac{1000 \quad 1\ 999 \quad 995}{999} \qquad 999$

⇒ 999000 – 999 + 995

= 999000 – 4 = **998996**

■ **Find the value :**

$\mathbf{999\dfrac{1}{9} + 999\dfrac{2}{7} + 999\dfrac{3}{7} + 999\dfrac{4}{7} + 999\dfrac{5}{7} + 999\dfrac{6}{7}}$

Sol. $999\dfrac{1}{9} + 999\dfrac{2}{7} + 999\dfrac{3}{7} + 999\dfrac{4}{7} + 999\dfrac{5}{7} + 999\dfrac{6}{7}$

⇒ $999 \quad 6 \qquad \dfrac{1}{7} \quad \dfrac{2}{7} \quad \dfrac{3}{7} \quad \dfrac{4}{7} \quad \dfrac{5}{7} \quad \dfrac{6}{7}$

⇒ $1000 \quad 1\ \ 6 \qquad \dfrac{21}{7}$

= 6000 – 6 + 3

= 6000 – 3 = **5997**

■ **Find the value :**

$\mathbf{3\dfrac{1}{3} + 33\dfrac{1}{3} + 333\dfrac{1}{3} + 3333\dfrac{1}{3} + 33333\dfrac{1}{3}}$

Sol. $3\dfrac{1}{3} + 33\dfrac{1}{3} + 333\dfrac{1}{3} + 3333\dfrac{1}{3} + 33333\dfrac{1}{3}$

⇒ (3 + 33 + 333 + 3333 + 33333) +

$\dfrac{1}{3} \quad \dfrac{1}{3} \quad \dfrac{1}{3} \quad \dfrac{1}{3} \quad \dfrac{1}{3}$

= $37035 + \dfrac{5}{3}$ ⇒ $37035 \quad 1\dfrac{2}{3}$

= $37036 \quad \dfrac{2}{3}$ ⇒ $\mathbf{37036\dfrac{2}{3}}$

Continuous fraction

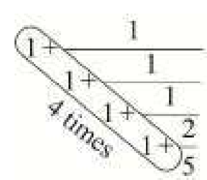
■ **To solve :**

$$\mathbf{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{2}{5}}}}}$$

Sol. Step-1 : Write the last fraction $\dfrac{2}{5}$ first

Step-2 : Write the numerator (2) first then the denominator (5).

Step-3 : Next number will appear as many times as one is given in the question and to find the next number, immediately add the previous number to that number.



⇒ $\mathbf{\dfrac{31}{19}}$

■ **To solve :**

$$\mathbf{1 \quad \cfrac{1}{1 \quad \cfrac{1}{1 \quad \cfrac{1}{1 \quad \cfrac{2}{5}}}}}$$

Sol.

Hence, the fraction = $\dfrac{3}{5} \quad \mathbf{\dfrac{3}{5}}$

- **To solve :**

$$1+\cfrac{1}{2+\cfrac{1}{3+\cfrac{1}{4}}}$$

Sol. $1,\ 4 \xrightarrow{\times 3+1} (13) \xrightarrow{\times 2+4} (30) \xrightarrow{\times 1\ \ 13} (43)$

Hence, the fraction $= \frac{43}{30}$

- **To solve :**

$$1 \quad \cfrac{1}{2 \quad \cfrac{1}{3 \quad \cfrac{1}{4}}}$$

Sol. $1,\ 4 \xrightarrow{\times 3-1} (11) \xrightarrow{\times 2-4} (18) \xrightarrow{\times 1-11} (7)$

- **Find the value a+ b + c :**

$$\cfrac{1}{a+\cfrac{1}{b+\cfrac{1}{c}}} = \frac{13}{29}$$

Sol.

13) 29 (2 – a
26
3) 13 (4 – b
12
1) 3 (3 – c
3
×

$\therefore \quad a + b + c = 2 + 4 + 3$

$a + b + c = 9$

- **Find the value a+ b + c :**

$$\cfrac{1}{a \quad \cfrac{1}{b+\cfrac{1}{c+\cfrac{1}{2}}}} = \frac{16}{23}$$

Sol.

16) 23 (1 = a
16
7) 16 (2 = b
14
2) 7 (3 = c
6
1

$\because$ Last term $= \frac{1}{2}$

$\therefore a + b + c = 1 + 2 + 3 \Rightarrow$ **6**

- **Recurring decimal :**

➢ $0.\overline{a} = \frac{a}{9}$

➢ $0.\overline{ab} = \frac{ab}{99}$

➢ $0.\overline{abc} = \frac{abc}{999}$

➢ $0.a\overline{b} = \frac{ab \quad a}{90}$

➢ $0.ab\overline{c} = \frac{abc \quad ab}{900}$

➢ $0.a\overline{bc} = \frac{abc \quad a}{990}$

- **Find the value**

$8.3\overline{1} + 0.\overline{6} + 0.00\overline{2} = ?$

Sol.

Without bar = 2

With bar = 1, 1, 1 LCM = 1

Without bar	With bar	
8.31	1	1 1 1 ....
0.66	6	6 6 6 ....
0.00	2	2 2 2 ....
8.97	9	9 9 9

$\Rightarrow$ **$8.97\overline{9}$**

- **Find the value :**

$22.\overline{4} + 11.5\overline{67} \quad 33.5\overline{9} = ?$

Sol. Without bar = 1

With bar = 1, 2, 1 LCM = 2

Without bar	With bar	
22.4	4 4	4 4 ....
+11.5	6 7	6 7 ....
–33.5	9 9	9 9 ....
0.4	1 2	1 2

Surds and Indies

Surds : $\sqrt[n]{a}$

$\sqrt{\ } \rightarrow$ Radical

$n \rightarrow$ Order of surd

$a \rightarrow$ Radicand

☞ **Entire surds :**

$\sqrt{a}, \quad (\sqrt{a}+\sqrt{b})$

☞ **Mixed surds :**

$a\sqrt{b}$

☞ **Like & Similar surds :**

$x\sqrt{b},\ y\sqrt{b},\ z\sqrt{b}$

☞ **Unlike & unsimilar surds :**

$x\sqrt{b},\ y\sqrt{c},\ z\sqrt{d}$

☞ **Conjugate surds :**

$\sqrt{7}+\sqrt{5} \xrightarrow{\text{Conjugate}} \sqrt{7}-\sqrt{5}$

$\sqrt{4}-\sqrt{3} \xrightarrow{\text{Conjugate}} \sqrt{4}+\sqrt{3}$

Product of conjugate surds is a rational number.

☞ **Quadratic surds :**

$a+\sqrt{b},\ \sqrt{a}+\sqrt{b}+c$

☞ **Equation involving surds-**

If the surds, $a \quad \sqrt{b} \quad c \quad \sqrt{d}$

then,

a	c
b	d

Hence, the rational part of one side is equal to the rational part of other side and the irrational part of one side is equal to the irrational part of other side.

Rationalization–

Surds	Rationalization factor
$\sqrt{a} \quad \sqrt{b}$	$\sqrt{a} \quad \sqrt{b}$
$\sqrt{a} \quad \sqrt{b}$	$\sqrt{a} \quad \sqrt{b}$
$a \quad \sqrt{b}$	$a \quad \sqrt{b}$
$a \quad \sqrt{b}$	$a \quad \sqrt{b}$
$a^{2/3} \quad b^{2/3} \quad a^{1/3}b^{1/3}$	$a^{1/3} \quad b^{1/3}$
$a^{2/3} \quad b^{2/3} \quad a^{1/3}b^{1/3}$	$a^{1/3} \quad b^{1/3}$

Law of surds and indices

- $a \times a \times a \times$ .... m term $= a^m$
  $a \times a \times a \times$ .... n term $= a^n$
- $(a \times a \times$ .... m term$) \times (a \times a \times$ .... n term$)$
  $= a^m \times a^n$
  $\Rightarrow \boxed{a^{m+n}}$
- $\dfrac{a \quad a \quad a \quad ....\text{m terms}}{a \quad a \quad a \quad ....\text{n terms}} \quad \dfrac{a^m}{a^n} \Rightarrow \boxed{a^{m \quad n}}$

■ If $a > 0$, $a \neq 1$ and m, n, p are integers then,

- $a^m \times a^n = a^{m+n}$
- $a^m \times a^n \times a^p = a^{m+n+p}$
- $a^{m \quad n} \quad a^{mn}$
- $\dfrac{a^m}{a^n} \quad a^{m \quad n}$
- $a^0 = 1$
- $a^{-m} = \dfrac{1}{a^m}$
- $a^{m^n} \quad a^{m^n}$
- $a^{m^{n^p}} \quad a^{m^{n^p}} \quad a^{m^{n^p}}$
- $ab^{\ n} \quad a^n b^n$
- $abc^{\ n} \quad a^n b^n c^n$
- If $a^n \quad y$ then $a = y^{1/n}$
  If $a^x = b^y$ then $a = b^{y/x}$
  If $a^x = b^y$ then $a^{1/y} = b^{1/x}$
- $x^n = a \Rightarrow x \quad \sqrt[n]{a}$, $(a \in R, a \geq 0)$
- If n is an odd positive integer and $a > 0$ then,
  $\sqrt[n]{\ a} \quad \sqrt[n]{a}$
  If $m, n \geq 2$, and $a, b > 0$ then–
- $\sqrt[n]{a} \quad a^{1/n}$
- $\sqrt[n]{a}^{\ m} \quad a^{m/n}$
- $\sqrt[n]{a}.\sqrt[n]{b} \quad \sqrt[n]{ab} \quad ab^{\ 1/n}$
- $\dfrac{\sqrt[n]{a}}{\sqrt[n]{b}} \quad \sqrt[n]{\dfrac{a}{b}}$
- $\sqrt[n]{\sqrt[m]{a}} \quad a^{1/m \quad 1/n} \quad a^{1/mn}$
- $\sqrt[n]{a}.\sqrt[m]{a} \quad a^{1/n}.a^{1/m}$
  $\Rightarrow a^{1/n \quad 1/m}$
  $\Rightarrow a^{\frac{m \quad n}{mn}} \quad \sqrt[mn]{a^{m \quad n}}$
- $\dfrac{\sqrt[n]{a}}{\sqrt[m]{a}} \quad \dfrac{a^{1/n}}{a^{1/m}} \quad a^{\frac{1}{n} \quad \frac{1}{m}} \quad a^{\frac{m \quad n}{mn}}$
  $\Rightarrow \sqrt[mn]{a^{m \quad n}}$
- $\sqrt[z]{\sqrt[y]{\sqrt[x]{a}^{\ p}}^{\ q}}^{\ r} \quad a^{\frac{pqr}{xyz}}$

Find square root

- $(a+b)^2 = a^2 + b^2 + 2ab$
- $\left(\sqrt{a} + \sqrt{b}\right)^2 = a + b + 2\sqrt{ab}$
- $(a-b)^2 = a^2 + b^2 - 2ab$
- $\left(\sqrt{a} - \sqrt{b}\right)^2 = a + b - 2\sqrt{ab}$
- $(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$
- $\left(\sqrt{a} + \sqrt{b} + \sqrt{c}\right)^2 = a + b + c + 2\sqrt{ab} + 2\sqrt{bc} + 2\sqrt{ca}$
- $(a-b)(a+b) = a^2 - b^2$

■ **Find the square root–**

$11 + 2\sqrt{30}$

Sol.

$\sqrt{11 + 2\sqrt{30}}$

$5 + 6 \quad 5 \times 6$

$\sqrt{\sqrt{5}^{\ 2} \quad \sqrt{6}^{\ 2} \quad 2\sqrt{5} \quad \sqrt{6}}$

$\sqrt{\sqrt{5} \quad \sqrt{6}^{\ 2}}$

$\sqrt{5} + \sqrt{6}$

■ **Find the square root–**

$13 + 2\sqrt{30}$

Sol.

$\sqrt{13 + 2\sqrt{30}}$

$10 + 3 \quad 10 \times 3$

$\sqrt{\sqrt{10} + \sqrt{3}^{\ 2}}$

$\sqrt{10} + \sqrt{3}$

■ **Find the square root–**

$17 \quad 2\sqrt{30}$

Sol.

$\sqrt{17 - 2\sqrt{30}}$

$15 + 2 \quad 15 \times 2$

$\sqrt{\sqrt{15} \quad \sqrt{2}^{\ 2}}$

$\sqrt{15} \quad \sqrt{2}$

■ **Find the square root–**

$8 \quad 2\sqrt{7}$

Sol.

$\sqrt{17-2\sqrt{30}}$

$7+1 \qquad 7\times 1$

$\sqrt{\sqrt{7} \quad \sqrt{1}\ ^2}$

$\sqrt{7} \quad \sqrt{1}$

■ **Find the square root–**

$12+\sqrt{140}$

Sol. $\sqrt{12 \quad \sqrt{140}}$

$\sqrt{12+2\sqrt{35}}$

$7+5 \qquad 7\times 5$

$\sqrt{\sqrt{7} \quad \sqrt{5}\ ^2}$

$\sqrt{7}+\sqrt{5}$

■ **Find the square root–**

$8 \quad \sqrt{60}$

Sol. $\sqrt{8 \quad \sqrt{60}}$

$\sqrt{8-2\sqrt{15}}$

$5+3 \qquad 5\times 3$

$\sqrt{\sqrt{5} \quad \sqrt{3}\ ^2}$

$\sqrt{5} \quad \sqrt{3}$

■ **Find the square root–**

$7+4\sqrt{3}$

Sol. $\sqrt{7 \qquad \sqrt{3}}$

$\sqrt{7 \qquad \sqrt{12}}$

$\sqrt{7+2\sqrt{12}}$

$4+3 \qquad 4\times 3$

$\sqrt{\sqrt{4} \quad \sqrt{3}\ ^2}$

$2+\sqrt{3}$

■ **Find the square root–**

$12 \qquad \sqrt{3}$

Sol. $\sqrt{12 \quad 6\sqrt{3}}$

$\sqrt{12 \quad 2\sqrt{27}}$

$\sqrt{12-2\sqrt{27}}$

$9+3 \qquad 9\times 3$

$\sqrt{\sqrt{9} \quad \sqrt{3}\ ^2}$

$3 \quad \sqrt{3}$

■ **Find the square root–**

$3+\sqrt{5}$

Sol. $\sqrt{3 \quad \sqrt{5}}$

$\sqrt{\frac{2}{2}\ 3 \quad \sqrt{5}}$

$\frac{1}{\sqrt{2}}\sqrt{6 \qquad \sqrt{5}}$

$\frac{1}{\sqrt{2}}\sqrt{6+2\sqrt{5}}$

$5+1 \qquad 5\times 1$

$\frac{1}{\sqrt{2}}\sqrt{\sqrt{5} \quad 1\ ^2}$

$\frac{1}{\sqrt{2}}\ \sqrt{5}+1$

■ **Find the square root–**

$4 \quad \sqrt{15}$

Sol. $\sqrt{4 \quad \sqrt{15}}$

$\sqrt{\frac{2}{2}\ 4 \quad \sqrt{15}}$

$\frac{1}{\sqrt{2}}\sqrt{8 \quad 2\sqrt{15}}$

$\frac{1}{\sqrt{2}}\sqrt{8-2\sqrt{15}}$

$5+3 \qquad 5\times 3$

$\frac{1}{\sqrt{2}}\sqrt{\sqrt{5} \quad \sqrt{3}\ ^2}$

$\frac{1}{\sqrt{2}}\ \sqrt{5} \quad \sqrt{3}$

■ **Find the square root–**

$15+\sqrt{60}+\sqrt{84}+\sqrt{140}$

Sol. $\sqrt{15 \quad \sqrt{60} \quad \sqrt{84} \quad \sqrt{140}}$

$\sqrt{15 \quad 2\sqrt{15} \quad 2\sqrt{21} \quad 2\sqrt{35}}$

$\sqrt{15 \quad 2.\sqrt{3}.\sqrt{5} \quad 2.\sqrt{5}.\sqrt{7} \quad 2.\sqrt{7}.\sqrt{3}}$

$\sqrt{\sqrt{3}\ ^2 \quad \sqrt{5}\ ^2 \quad \sqrt{7}\ ^2 \quad 2.\sqrt{3}.\sqrt{5} \quad 2.\sqrt{5}.\sqrt{7} \quad 2.\sqrt{7}.\sqrt{3}}$

$\sqrt{\sqrt{3} \quad \sqrt{5} \quad \sqrt{7}\ ^2}$

$\sqrt{3}+\sqrt{5}+\sqrt{7}$

Some important results

- If, $x = \sqrt{a\sqrt{a\sqrt{a\sqrt{a........}}}}$

  then, $\boxed{x = a}$

- If, $x = \sqrt{a\sqrt{a\sqrt{a...........n \text{ times}}}}$

  then, $\boxed{x = a^{\frac{2^n \;\; 1}{2^n}}}$

- If, $x = \sqrt[n]{a \times \sqrt[n]{a \times \sqrt[n]{a..........}}}$

  then, $\boxed{x = \sqrt[n\;\;1]{a}}$

- If, $x = \sqrt[n]{a \quad \sqrt[n]{a \quad \sqrt[n]{a \quad ....}}}$

  then, $\boxed{x = \sqrt[n\;\;1]{a}}$

- If, $x = \sqrt{a + b\sqrt{a + b\sqrt{a + ...}}}$

  then, $\boxed{x = \frac{\sqrt{4a + b^2} + b}{2}}$

- If, $x = \sqrt{a + \sqrt{a \quad \sqrt{a \quad ....}}}$

  then, $\boxed{x = \frac{\sqrt{4a + 1} + 1}{2}}$

- If, $x = \sqrt{a \quad b\sqrt{a \quad b\sqrt{a \quad ....}}}$

  then, $\boxed{x = \frac{\sqrt{4a + b^2} \quad b}{2}}$

- If, $x = \sqrt{a \quad \sqrt{a \quad \sqrt{a \quad ....}}}$

  then, $\boxed{x = \frac{\sqrt{4a + 1} \quad 1}{2}}$

- If, $x = \sqrt{a + b\sqrt{a \quad b\sqrt{a + b\sqrt{a \quad ....}}}}$

  then, $\boxed{x = \frac{\sqrt{4a \quad 3b^2} \quad b}{2}}$

- If, $x = \sqrt{a + \sqrt{a \quad \sqrt{a + \sqrt{a \quad ....}}}}$

  then, $\boxed{x = \frac{\sqrt{4a \quad 3} \quad 1}{2}}$

- If, $x \quad \sqrt{a \quad b\sqrt{a \quad b\sqrt{a \quad b\sqrt{a \quad b\sqrt{a \quad ....}}}}}$

  then, $\boxed{x = \frac{\sqrt{4a \quad 3b^2} \quad b}{2}}$

- If, $x \quad \sqrt{a \quad \sqrt{a \quad \sqrt{a \quad \sqrt{a \quad \sqrt{a \quad ....}}}}}$

  then, $\boxed{x = \frac{\sqrt{4a \quad 3} \quad 1}{2}}$

LCM and H.C.F.

Difference between multiple and factor

S. N.	Multiple	Factor
1.	The multiples are defined as the numbers obtained when multiplied by other numbers	Factors are defined as the exact divisors of the given number
2.	The number of multiples is infinite	The number of factors is finite
3.	The operation used to find the multiples is a multiplication.	The operation used to find the factors is a division
4.	The outcome of the multiples should be greater than or equal to the given number	The outcome of the factors should be less than or equal to the given number.

L.C.M.

L.C.M. : Least common multiple

- L.C.M. is the smallest number which is completely divided by two or more numbers.
- The LCM of x, y and z is completely divisible by x, y, and z.

■ **L.C.M. of 12 and 16–:**

12 Multiple – 12, 24, 36, **48**, 60, 72, 84, **96**, ...

16 Multiple – 16, 32, **48**, 64, 80, **96**, 112, 128, ...

Common multiple – 48, 96

Least common multiple – 48

L.C.M. = 48

Methods of finding L.C.M.

■ In this method, divide the given numbers by common prime number until the remainder is 1.

Ex. Finding the L.C.M. of 9, 12 and 15

Sol.

2	9, 12, 15
2	9, 6, 15
3	9, 3, 15
3	3, 1, 5
5	1, 1, 5
	1, 1, 1

(L.C.M.) $= 2 \times 2 \times 3 \times 3 \times 5$
$= \mathbf{180}$

■ **Prime Factor Method–:** First express the given numbers in the form of prime factors. The product of factors with highest power will be the L.C.M.

Ex. Finding the L.C.M. of 9, 12 and 15

Sol. 9 = 3 × 3

12 = 2 × 2 × 3

15 = 3 × 5

L.C.M. = 2 × 2 × 3 × 3 × 5

= **180**

Types of questions

➢	Find the smallest no. which is exactly divisible by x, y, z.	**L.C.M. of (x, y, z)**
➢	Find the smallest no. which when divided by x, y, z leaves remainder 'r' in each case.	**L.C.M. of (x, y, z) + r**
➢	Find the smallest no. which when divided by x, y, z leaves remainder a, b, c respectively.	**L.C.M. of (x, y, z) – k** Where, k = (x – a) = (y – b) = (z – c)

H.C.F.

H.C.F. : Highest common factor
(Greatest common divider)

☞ H.C.F is the largest number, which can divide two or more numbers completely.

☞ The HCF of x, y and z will divide x, y, and z completely.

■ **H.C.F. of 12 and 16–:**

12 Factor – **1**, **2**, 3, **4**, 6, 12   16 Factor – **1**, **2**, **4**, 8, 16

Common factor – 1, 2, 4   Highest common factor – 4

H.C.F. = 4

Methods of finding H.C.F.

■ **Division Method–**Find the H.C.F. of two number x and y. (Where, y > x)

On dividing y by x remainder is r_1. Then on dividing x by r_1 the remainder is r_2. Then r_1 is divided by r_2. This process will be repeated until the remainder becomes zero. Last divisor will be the H.C.F. of x and y.

Ex. Finding the H.C.F. of 12 and 16 :

Sol. 12, 16 of H.C.F.

```
12) 16 (1
    12
    ---
    4) 12 (3
       12
       ---
        0
```

H.C.F. = 4

Ex. Finding the H.C.F. of 25, 35 and 40 :

Sol. 25, 35 and 40 of H.C.F.

```
25) 35 (1              5) 40 (8
    25                    40
    ---                   ---
    10) 25 (2              0
        20
        ---            H.C.F. = 5
        5) 10 (2
           10
           ---
            0
```

■ **Prime factor method–:** First, write each given numbers in the form of product of their prime factors. The product of common factors with least power will be the H.C.F. of given numbers.

Ex. Finding the H.C.F. of 12 and 16 :

Sol. 12, 16 of H.C.F.

$12 = 2 \times 2 \times 3 \Rightarrow 2^2 \times 3$

$16 = 2 \times 2 \times 2 \times 2 \Rightarrow 2^4$

H.C.F. = $\mathbf{2^2 \Rightarrow 4}$

Ex. Finding the H.C.F. of 25, 35 and 40 :

Sol. 25, 35 and 40 of H.C.F.

$25 = 5 \times 5 \Rightarrow 5^2$

$35 = 5 \times 7 \Rightarrow 5^1 \times 7^1$

$40 = 2 \times 2 \times 2 \times 5 \Rightarrow 2^3 \times 5^1$

H.C.F. = 5

■ **Difference method–**

Let,

H.C.F. of two numbers = h

then, numbers = hx, hy

Where, x, y → Co-prime

Difference = hx – hy

⇒ **h (x – y)**

☞ (x – y) = 1 → H.C.F. is a difference between numbers.

☞ (x – y) > 1 → H.C.F. is a factor of difference of numbers.

☞ H.C.F. of two numbers never greater than difference of these numbers.

Hence, H.C.F. can be either difference of these number or factor of difference.

Ex. Finding the H.C.F. of 30 and 45 :

Sol. 30, 45 of H.C.F.

30, 45

difference = 45 – 30 ⇒ 15

H.C.F. = 15 or factor of 15

∵ 30 and 45 are completely divisible by 15

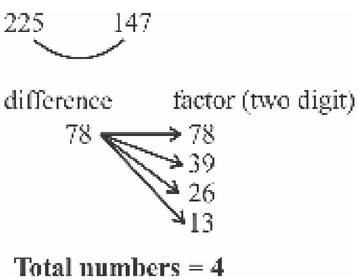
Hence, H.C.F. = 15

Types of questions

➢	Find the largest no. which can divide x, y, z. exactly	**H.C.F. of (x, y, z)**
➢	Find the largest no. which can divide x, y, z and leaves same remainder in each case.	**H.C.F. of (x– y), (y – z), (z – x)**
➢	Find the largest no. which can divide x, y, z and leaves remainder 'r' in each case.	**H.C.F. of (x– r), (y –r), (z – r)**
➢	Find the largest number which can divide x, y, z and leaves remainder a, b, c respectively.	**H.C.F. of (x– a), (y – b), (z – c)**

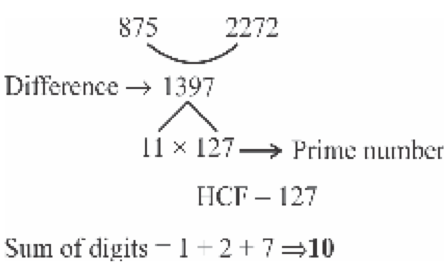
- If two numbers are divided by their difference or factors of difference then leaves same remainder.
- **A two digit number can divide 225 and 147, leaves same remainder in each case. How many such two digit numbers would be possible?**

Sol.



- **The two numbers 875 and 2272 are divided by a three digit number. Then there is same remainder left in each case what will be the sum of the digits of such three digits?**

Sol.



Relation between L.C.M. and H.C.F.

- First no. × second no. = L.C.M × H.C.F.
- ☞ If H.C.F. = h
  First no. = hx
  Second no. = hy
  then, L.C.M. = hxy

L.C.M. and H.C.F. of fraction

- $\text{L.C.M. of fraction} = \frac{\text{L.C.M. of numerator}}{\text{H.C.F. of denominator}}$
- $\text{H.C.F. of fraction} = \frac{\text{H.C.F. of numerator}}{\text{L.C.M. of denominator}}$

L.C.M. and H.C.F. of indices

- When the base of the given numbers are same, then the number with highest power will be the LCM of the given numbers.

Ex. $7^2, 7^4, 7^9$ **of L.C.M. =** 7^9

- When the base is not same and there is no common factors in the base, then the product of given numbers will be the LCM.

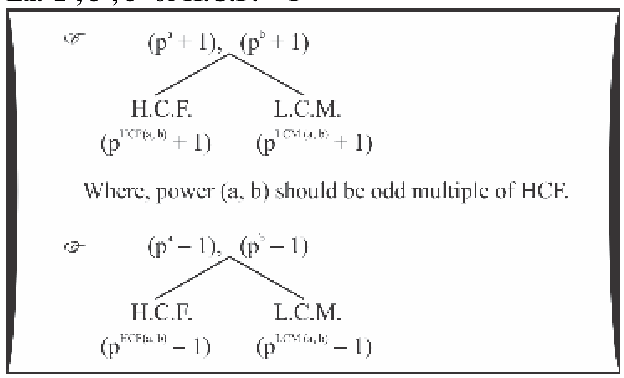
Ex. $2^2, 3^5, 5^4$ **of L.C.M. =** $2^2 \times 3^5 \times 5^4$

- When the base of the given number are same, then the number with least power will be the H.C.F. of given numbers.

Ex. $7^2, 7^4, 7^9$ **of H.C.F. =** 7^2

- When the base is not same and there is no common factor in the base, then the required H.C.F. of given numbers will be 1.

Ex. $2^2, 3^5, 5^4$ **of H.C.F. = 1**



Questions asked in previous Exams

1. Which of the following numbers is divisible completely by both 9 and 11 ?

(a) 277218 (b) 10098
(c) 12345 (d) 181998

RRB NTPC (Stage-II) 17/06/2022 (Shift-II)

Ans. (b) : Divisibility rule of 9 -
When the sum of the digits of a number is divisible by 9 then the number is also divisible by 9.
Divisibility rule of 11 -
When the difference between the sum of the digit in even and odd place of a number is 0 (zero) or a multiple of 11, then the number will also be divisible by 11.
From option (b),
1 + 0 + 0 + 9 + 8 = 18
i.e. 18 is divisible by 9
∴ Option (d) us divisible by 9.
And
10098 = (9 + 0) – (8+0+1) = 9–9 = 0
Hence option (b) 10098, is divisible by both 9 and 11.

2. Which of the following numbers is NOT divisible by 9 ?

(a) 49104 (b) 77832
(c) 35253 (d) 45390

RRB NTPC (Stage-II) –12/06/2022 (Shift-II)

Ans. (d) : Divisibility rule of 9 : A number whose sum of its digit is exactly divisible by 9 then the number is always divisible by 9.

from options -

(a) 49104 $\rightarrow$ 4 + 9 + 1 + 0 + 4 = 18, divisible by 9.

(b) 77832 $\rightarrow$ 7 + 7 + 8 + 3 + 2 = 27, divisible by 9.

(c) 35253 $\rightarrow$ 3 + 5 + 2 + 5 + 3 = 18, divisible by 9.

(d) 45390 $\rightarrow$ 4 + 5 + 3 + 9 + 0 = 21, not divisible by 9.

3. Which of the following number is NOT divisible by 8?

(a) 35792 (b) 35112
(c) 35412 (d) 35552

RRB NTPC (Stage-II) 15/06/2022 (Shift-III)

Ans. (c) : Divisibility rule of 8- If the last three digits of a number are divisible by 8, then the number is completely divisible by 8.

from the given options -

(a) 35 792

$\frac{792}{8} = 99$ (Completely divisible)

(b) 35 112

$\frac{112}{8} = 14$ (Completely divisible)

(c) 35 412

$\frac{412}{8} = 51.5$ (Not completely divisible)

(d) 35 552

$\frac{552}{8} = 69$ (Completely divisible)

Hence, option (c) is not divisible by 8.

4. If the 7 digit number 504x5y3 is divisible by 11, then one of the values of the sum of x and y is:

(a) 11 (b) 5
(c) 17 (d) 7

RRB NTPC (Stage-II) –13/06/2022 (Shift-II)

Ans. (c) : Given, 504x5y3

Divisibility rule of 11:- If the difference of the sum of digits at even place and at odd place is zero or divisible by 11 then the given number will be divisible by 11.

504x5y3

(0 + x + y) – (5 + 4 + 5 + 3)

x + y – 17 = 0

x + y = 17

Hence, Sum of x + y = 17

5. If 11-digit number 88p554085k6, k ≠ p, is divisible by 72, then what is the value of (3k + 2p)?

(a) 12 (b) 7
(c) 13 (d) 23

RRB NTPC (Stage-II) –13/06/2022 (Shift-II)

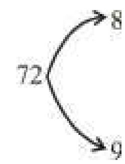
Ans. (c) : Given,

88p554085k6 Where, k ≠ p

Note- The number which is divisible by 72 is also divisible by 8 and 9.

Divisibility rule of 8– If the last three digit of the number are divisible by 8, then the number will be divisible by 8.

Divisibility rule of 9– If the sum of the all digits of a given number is divisible by 9, then number will be divisible by 9.



88p554085k6

On putting, k = 3

$\frac{536}{8} = 67$ (Completely divisible by 8)

and On putting p = 2

$\frac{8+8+2+5+5+4+0+8+5+3+6}{9}$

$= \frac{54}{9} = 6$ (Completely divisible)

Then, (3k + 2p)

= 3 × 3 + 2 × 2

= 13

6. Find the remainder, when 171 × 172 × 173 is divided by 17.

(a) 9 (b) 8
(c) 6 (d) 7

RRB Group-D 29/08/2022 (Shift-III)

Ans. (c) : According to the question,

$\frac{171\times172\times173}{17}$

$\Rightarrow \frac{(170+1)\times(170+2)(170+3)}{17}$

$\Rightarrow \frac{1\times2\times3}{17}$

$\Rightarrow \frac{6}{17}$

$\Rightarrow$ 6 (Remainder)

Hence option (c) is correct.

7. When a number is divided by a divisor, the remainder is 16. When twice the original number is divided by the same divisor, the remainder is 3. Find the value of that divisor

(a) 29 (b) 51
(c) 23 (d) 53

RRB Group-D 30/08/2022 (Shift-II)

Ans. (a) : Let, the original number be N, the divisor be d, quotient be q.

N = dq + 16

∴ 2 N = 2(dq + 16)

2 N = 2dq + 32

When (2dq + 32) is divided d then remainder is 3.

2dq is completely divisible by d, then

∴ Required number = 32 – 3 = 29

8. If the number 6484y6 is divisible by 8, then find the least value of y?

(a) 3 (b) 4
(c) 1 (d) 7

RRB Group-D 02/09/2022 (Shift-II)

Ans. (c) : Divisibility rule of 8 - If the last three digits of the given number are divisible by 8 then it will be divisible by 8.
On putting Least value of y = 1
Number = 648416
Divided by = $\frac{416}{8} = 52$

9. If the 15 digit number 4a5124356789734 is divisible by 9, then the value of "a" is ....

(a) 1 (b) 4
(c) 5 (d) 3

RRB GROUP-D – 22/09/2022 (Shift-III)

Ans. (b) : Divisibility rule of 9 - If the sum of the digits are divisible by 9, then the number is divisible by 9.
Number - 4a5124356789734
On divided by 9 -

$$\frac{4+a+5+1+2+4+3+5+6+7+8+9+7+3+4}{9}$$

$= \frac{a+68}{9}$ ⇒ On putting a = 4 ⇒ $\frac{4+68}{9} = \frac{72}{9} = 8$

Hence the value of a = 4

10. If the 8 digit number 3x5479y4 is divisible by 88 and the 8 digit number 425139z2 is divisible by 9, then find the maximum possible value of (3x + 2y – z).

(a) 33 (b) 37
(c) 25 (d) 35

RRB Group-D 09/09/2022 (Shift-III)

Ans. (a) : On dividing 3x5479y4 by 88 i.e. 8 and 11
Divisibility rule of 8 - If the last three digits of the given number are divisible by 8, then it will be divisible by 8.
Maximum possible value = 8

$\frac{984}{8} = 123$

Divisibility rule of 11 - The given number can only be completely divided by 11 if the difference of the sum of digits at odd place and sum of digits at even place in a number is 0 or multiple of 11.
3x547984⇒ (4+9+4+x) ~ (8+7+5+3)
17+x ~ 23 = 0
x = 6
On dividing 425139z2 by 9
Divisibility rule of 9 :- If the sum of the digits of a number are divisible by 9, then the number is divisible by 9.

$$\frac{4+2+5+1+3+9+z+2}{9} = \frac{26+z}{9}$$

On putting z = 1

$$\frac{26+1}{9} = \frac{27}{9} = 3$$

Hence, 3x +2y –z = 3×6+2×8–1 = 33

11. If each even digit is divided by 2 and 2 is added to each odd digit in the number 4723361, what will be the sum of the largest and the smallest digits thus formed?

(a) 12 (b) 10 (c) 11 (d) 9

RRB GROUP-D – 11/10/2022 (Shift-I)

Ans. (b) : Given, 4723361
According to the question,
New number obtained by dividing each even digit by 2 and adding 2 to each odd digit.

$$\frac{4}{2}(7+2),\left(\frac{2}{2}\right)(3+2)(3+2),\frac{6}{2}(1+2) \Rightarrow 2915533$$

Hence Sum of largest digit and smallest digit = 9 + 1
= 10

12. If 3 is added to each odd digit and 1 is subtracted from each even digit in the number 42514563, what will be difference between the highest and lowest digits thus formed?

(a) 2 (b) 7
(c) 5 (d) 8

RRB GROUP-D – 17/08/2022 (Shift-I)

Ans. (b) : Given number = 42514563
According to the question, the number obtained by adding 3 to the odd digit and subtracting 1 from the even digit of the number is = 31843856
Hence required difference = 8 – 1 = 7

13. If 3 is added to each odd digit and 2 is subtracted from each even digit in the number 6452851, what will be difference between the largest and smallest digits thus formed?

(a) 8 (b) 6
(c) 4 (d) 2

RRB GROUP-D – 27/09/2022 (Shift-I)

Ans. (a) : The number obtained by adding 3 to the odd digit and subtracting 2 from the even digit of the number is

6 4 5 2 8 5 1
–2 –2 +3 –2 –2 +3 +3
4 2 8 0 6 8 4

Hence the difference of largest and smallest digits
= 8–0
= 8

14. If 1 is subtracted from each odd digit and 1 is added to each even digit in the number 92379654, what will be the sum of the digits which are second from the left and third from the right?

(a) 6 (b) 8
(c) 10 (d) 5

RRB GROUP-D – 18/09/2022 (Shift-II)

Ans. (c) : The number obtained by adding 1 to the even digit and subtracting 1 from the odd digit of the number is 92379654

9 2 3 7 9 6 5 4
–1 +1 –1 –1 –1 +1 –1 +1
8 (3) 2 6 8 (7) 4 5

So the required sum = 3 + 7 = 10

15. The sum of the digits of a two-digit number is 12. The number obtained by interchanging its digits exceeds the given number by 18. The number is:

(a) 76 (b) 67
(c) 27 (d) 57

RRB GROUP-D – 16/09/2022 (Shift-II)

Ans. (d) : Let the two digit number be 10x + y
Number obtained by interchanging the digits = 10y + x
According to the question,
x + y = 12 -------- (i)
And, On reversing the digits,
(10y + x) – (10x + y) = 18
y – x = 2 ------- (ii)
On adding eq. (i) and (ii)

$$\begin{array}{r} x + y = 12 \\ -x + y = 2 \\ \hline 2y = 14 \end{array}$$

$y = 7$
$x = 5$
Hence, number = 10x + y = 10 × 5 + 7 = 57

16. In a five digit number, the digit in the hundred's place is 2 and the digit in the unit's place is twice the digit in the hundred's place. The digit at thousands place is zero. The digit in the ten thousand's place is the sum of the digit in the hundred's place and the digit in the unit's place. The digit in the ten's place is the digit in the ten thousand's place minus 1. The number is:

(a) 60234 (b) 60224
(c) 60254 (d) 60264

RRB NTPC 09.02.2021 (Shift-I) Stage Ist

Ans. (c) : Let us assume the number = a b c d e
As per question,
c = 2
e = 2 × c
e = 2 × 2
e = 4
b = 0
a = 2 + 4
a = 6
d = 6 – 1
d = 5
Putting all values, then the required number = 60254

17. What is the smallest four digit number formed by using the digits 3, 5, 0, 6?

(a) 3056 (b) 0356
(c) 0536 (d) 3506

RRB NTPC 08.02.2021 (Shift-I) Stage Ist

Ans. (a) : The smallest four-digit number formed by 3,5,0,6 = 3056

18. What is the smallest five-digit number formed by using the digits 2, 3, 4, 0, 5?

(a) 23045 (b) 20435
(c) 02345 (d) 20345

RRB NTPC 04.02.2021 (Shift-I) Stage Ist

Ans. (d) : Largest 5 digit number = 99999
Smallest 5 digit number = 10000
The smallest five digit number that can be formed from the digits 2, 3, 4, 0, 5 is = 20345

19. Find sum of the smallest and the largest positive numbers of 6 digits which contains only digits 0, 4, 6 and each of these digits appears at least once.

(a) 666444 (b) 604604
(c) 666666 (d) 1066646

RRB NTPC 09.02.2021 (Shift-II) Stage Ist

Ans. (d) : According to the question-
∵ Smallest 6 digit no = 400006
Greatest 6 digit no = 666640
∴ Required sum = 400006 + 666640 = 1066646

20. How many times is digit 3 comes in counting from 301 to 399?

(a) 119 (b) 11
(c) 121 (d) 21

RRB NTPC 10.01.2021 (Shift-II) Stage Ist

Ans. (a) : In Counting from 301 to 399, the digit 3 comes a total of 119 times.

21. Find the two-digit number such that the sum of its digits is 8 and the digits of the number get reversed when 36 is added to it.

(a) 71 (b) 35
(c) 62 (d) 26

RRB NTPC 15.02.2021 (Shift-II) Stage Ist

Ans. (d) : Let number = 10x+y
According to the question,
x+y=8 …(i)
(10x+y) + 36= 10y+ x
9y – 9x = 36
y – x = 4 …(ii)
On solving equation (i) and equation (ii)
x = 2
y = 6
Hence, required number = 10x + y = 10 × 2 + 6 = 26

22. Find the total number of prime numbers less than 50.

(a) 13 (b) 15
(c) 17 (d) 14

RRB Group-D 06/09/2022 (Shift-III)

Ans. (b) : Total number of prime number less than 50 is 15 which is as follows -
2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47

23. What is the positive difference between the sum of all prime numbers between 11 and 20 (both included) and the sum of all prime numbers between 30 and 50 (both included)?

(a) 139 (b) 141
(c) 137 (d) 135

RRB GROUP-D – 15/09/2022 (Shift-III)

Ans. (a) : The sum of all prime numbers between 11 and 20 (both included) = (11 + 13 + 17 + 19) = 60
The sum of all prime number between 30 and 50 (both included) = (31 + 37 + 41 + 43 + 47) = 199
$\therefore$ Required positive difference = 199 ~ 60 = 139

24. The greatest prime number less than 200 is:
(a) 199 (b) 193
(c) 197 (d) 191
RRB NTPC 21.01.2021 (Shift-II) Stage Ist

Ans. (a) : The greatest prime number less than 200 is 199.

25. Which of the following numbers is prime?
(a) 323 (b) 571
(c) 513 (d) 715
RRB NTPC 02.03.2021 (Shift-II) Stage Ist

Ans. (b) : According to option,
571 is a prime number. Whereas 323 is divisible by 17, 513 is divisible by 3 and 715 is divisible by 5.

26. Find the smallest three digit prime number?
(a) 107 (b) 109
(c) 103 (d) 101
RRB NTPC 23.07.2021 (Shift-II) Stage Ist

Ans. (d) : The smallest three-digit prime number = 101

27. Which of the following pairs of numbers are co-prime?
(a) 28, 81 (b) 12, 27
(c) 21, 56 (d) 36, 20
RRB NTPC 23.07.2021 (Shift-II) Stage Ist

Ans. (a) : Co-prime numbers are the numbers whose common factor is only 1.
Hence, in the given option (28, 81) are co-prime numbers.

28. One-third of the sum of all the prime numbers greater than 5 but less than 18 is the square of:
(a) 3 (b) 5
(c) 6 (d) 4
RRB NTPC 08.04.2021 (Shift-I) Stage Ist

Ans. (d) : Prime numbers greater than 5 but smaller than 18 = 7, 11,13, 17
According to the question-
$$=\frac{7+11+13+17}{3}=\frac{48}{3}=16=(4)^2$$
Hence, required number = 4

29. Which of the following is a prime number?
(a) 143 (b) 173
(c) 123 (d) 213
RRB NTPC 15.03.2021 (Shift-I) Stage Ist

Ans. (b) : Prime number are the numbers, which are only divisible by 1 and itself.
From the given options-
(a) 143 is divisible by 11, so it is not a prime number.
(b) 173 is divisible by 1 and itself, so it is a prime number.
(c) 123 is divisible by 3, so it is not a prime number.
(d) 213 is divisible by 3, so it is not a prime number.

30. Find the sum of prime no. between 50 and 60.
(a) 118 (b) 114
(c) 110 (d) 112
RRB NTPC 31.01.2021 (Shift-I) Stage Ist

Ans. (d) : The prime number between 50 and 60–
53 and 59
Required Sum = 53 + 59 = 112

31. Find the number of all prime numbers less than 55.
(a) 18 (b) 17
(c) 16 (d) 15
RRB NTPC 30.12.2020 (Shift-I) Stage Ist

Ans. (c) : The number of all prime numbers less than 55 is 16
i.e. $\Rightarrow$ (2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53)

32. The number of pairs of twin primes between 1 and 100 are:
(a) 7 (b) 8
(c) 10 (d) 9
RRB NTPC 26.07.2021 (Shift-I) Stage Ist

Ans. (b) : The number of pairs of twin primes between 1 and 100 are 8.
The numbers are -
$$\{(3,5),(5,7),(11,13),(17,19),(29,31),(41,43),(59,61),(71,73)\}$$
Note- Twins prime numbers are that numbers whose difference is 2.

33. If each packet contains the same number of pencils and there are 96 pencils in all in 12 packets, how many packets will one have to purchase if one requires 304 pencils?
(a) 39 (b) 38
(c) 33 (d) 36
RRB NTPC (Stage-II) –16/06/2022 (Shift-II)

Ans. (b) : $\because$ Pencils present in 12 packets = 96
$\because$ Pencils present in 1 packet $=\frac{96}{12}=8$ Pencils
Number of packets required for 304 pencils
$$=\frac{304}{8}=38 \text{ Packets.}$$

34. From 3/4 of a number P, Ramakrishna subtracts 2/3 of another number Q and obtain 5/8 as the difference. What is the answer Ramakrishna should obtain if he subtracts eight times of Q from nine times of P?
(a) $\frac{15}{2}$ (b) $\frac{25}{4}$
(c) $\frac{20}{3}$ (d) $\frac{25}{3}$
RRB NTPC (Stage-II) –12/06/2022 (Shift-II)

Ans. (a) : According to the question,

$$P\times\frac{3}{4}-Q\times\frac{2}{3}=\frac{5}{8}$$

$$\Rightarrow \frac{3P}{4}-\frac{2Q}{3}=\frac{5}{8}\Rightarrow \frac{9P-8Q}{12}=\frac{5}{8}$$

$$\Rightarrow 9P-8Q=\left(\frac{5}{8}\right)\times 12 \ \Rightarrow 9P-8Q=\frac{60}{8}$$

$$\therefore\ 9P-8Q=\frac{15}{2}$$

35. In a class of 80 students $\frac{1}{10}$ of the class likes chocolate D and $\frac{1}{20}$ of the class likes chocolate E. What is the difference between the number of students who like chocolate D and the number of students who like chocolate E ?

(a) 2 (b) 9

(c) 5 (d) 4

RRB NTPC (Stage-2) 17/06/2022 (Shift-I)

Ans. (d) : Students who likes chocolate D = $80\times\frac{1}{10}$

= 8

Students who likes chocolate E = $80\times\frac{1}{20}$ = 4

Hence, the required difference = 8–4 = 4

36. Sunita won 3/5 of the marbles that were there in the beginning of the game. Ravi won 2/3 of the remaining marbles while Sunny won the remaining 60 marbles. How many marbles did Sunita Win?

(a) 255 (b) 240

(c) 285 (d) 270

RRB NTPC (Stage-II) –12/06/2022 (Shift-I)

Ans. (d) : Let, number of marbles be x.

Won by Sunita = $\frac{3x}{5}$

Number of remaining marbles = $x-\frac{3x}{5}=\frac{2x}{5}$

Won by Ravi $=\frac{2x}{5}\times\frac{2}{3}=\frac{4x}{15}$

According to the question,

$$\frac{3x}{5}+\frac{4x}{15}+60=x$$

$$60=x-\left(\frac{3x}{5}+\frac{4x}{15}\right)$$

$$60=x-\frac{13x}{15}$$

$$\because \frac{2x}{15}=60\Rightarrow x=450$$

Number of marbles Won by Sunita $=450\times\frac{3}{5}=270$

37. The difference between two numbers is 18. If the difference between their squares is 360, find the larger number.

(a) 18 (b) 15

(c) 19 (d) 16

RRB GROUP-D – 29/09/2022 (Shift-I)

Ans. (c) : Let the smaller number = y

and larger number = x

According to the question,

x – y = 18 .... (i)

$x^2 - y^2 = 360$

(x + y) (x – y) = 360

(x + y) 18 = 360

x + y = 20 ....(ii)

On adding equation (i) and equation (ii) -

$$x+y=20$$
$$x-y=18$$
$$2x=38$$

x = 19

y = 20 – x

= 20 – 19

= 1

Hence larger number = 19 and smaller number = 1

38. A 91 cm long wire is cut into two pieces so that the length of one piece is three-fourth of the other. Find the length of the shorter piece.

(a) 36.23 m (b) 39 cm

(c) 42.17 cm (d) 38 cm

RRB Group-D 22/08/2022 (Shift-I)

Ans. (b) : Let the length of second piece = x cm

Length of first piece $=x\times\frac{3}{4}=\frac{3x}{4}$

According to the question,

$$\Rightarrow \frac{3x}{4}+x=91$$

$$\Rightarrow 7x = 91 \times 4$$

$$\Rightarrow x=\frac{91\times 4}{7}$$

length of second piece (x) = 52 cm

Length of first piece $=52\times\frac{3}{4}$

= 39 cm

Hence the length of the shorter piece = 39 cm

39. A 3 digit number is such that the ratio of its units digit, tens digit and hundreds digit is 1 : 2 : 3. The sum of this number and the reversed number obtained by reversing the order of its digits is 1332. Find the number.

(a) 246 (b) 414

(c) 123 (d) 369

RRB Group-D 26/08/2022 (Shift-III)

Ans. (d) :
Let three digit number = 100 × 3x + 10 × 2x + x
= 300x + 20x + x = 321x
New number obtained by reversing the digits
= 100 × x + 10 × 2x + 3x
= 100x + 20x + 3x = 123x
According to the question,
321x + 123x = 1332
444x = 1332
X = 3
Hence number = 100×3 + 10×2×3 + 3×3
= 300 + 60 + 9 = 369

40. A man plants 21,025 mango trees in his garden in such a way that there are as many rows as there are mango trees in each row. Find the number of rows.
(a) 135 (b) 125
(c) 145 (d) 130
RRB Group-D 30/08/2022 (Shift-II)

Ans. (c) : Let the number of rows in garden = x
And number of tree in each row = x
According to the question,
x × x = 21025
$x = \sqrt{21025}$
x = 145
Hence, Number of rows in garden = 145

41. The sum of two numbers is 27. Five times one number is equal to 4 times the other. The smaller of the two numbers is :
(a) 12 (b) 11 (c) 13 (d) 15
RRB Group-D 30/08/2022 (Shift-II)

Ans. (a) : Let the numbers be x and y
According to the question :
∵ → x + y = 27 ————(i)
∵ → 5x = 4y
5x – 4y = 0 ————(ii)
On solving equation (i) and (ii) :
y = 15
x = 12
Hence, the smaller number is 12.

42. There are two consecutive natural numbers such that the sum of their squares is 313. Find smaller of these two numbers.
(a) 12 (b) 14 (c) 15 (d) 13
RRB Group-D 24/08/2022 (Shift-I)

Ans. (a) :
Let two consecutive natural numbers are x and (x + 1)
According to the question.
$x^2 + (x + 1)^2 = 313$
$x^2 + x^2 + 1 + 2x = 313$
$2x^2 + 2x = 312$
$x^2 + x = 156$
x (x + 1) = 13 × 12
x =12
Hence, smaller of these two numbers = 12

43. Find the least number which when added to 1780 makes the sum a perfect square.
(a) 46 (b) 49
(c) 69 (d) 72
RRB JE - 27/05/2019 (Shift-II)

Ans : (c) On adding 69 to the number 1780 it will be 1849, which is a perfect square number.
Thus-
1780 + 69 = 1849
1849 = 43 × 43
$(43)^2 = 1849$

44. Find the smallest integer whose cube is equal to itself.
(a) -1 (b) 2
(c) 1 (d) 0
RRB JE - 22/05/2019 (Shift-I)

Ans : (a) –1 and 1 are such integers whose cube is equal to itself.
Hence, the smallest integer = –1
$\because (-1)^3 = -1$

45. If the cube of a number is subtracted from $(153)^2$ the result gives 1457. Find the number.
(a) 18 (b) 16
(c) 28 (d) 24
RRB JE - 24/05/2019 (Shift-I)

Ans : (c) Let the number be x.
According to the question,
$(153)^2 - x^3 = 1457$
$x^3 = (153)^2 - 1457$
$x^3 = 23409 - 1457$
$x^3 = 21952$
$\therefore\ x = \sqrt[3]{21952} = \sqrt[3]{28\times28\times28} = 28$

46. Five times of a positive integer is 3 less than twice of its square. Find the integer.
(a) 3 (b) 8
(c) 2 (d) 5
RRB RPF Constable -19/01/2019 (Shift-I)

Ans : (a) Let the positive integer is x.
According to the question-
$5x = 2x^2 - 3$
$2x^2 - 5x - 3 = 0$
$2x^2 - 6x + x - 3 = 0$
2x (x–3) + 1 (x –3) = 0
(x – 3) (2x + 1) = 0
x – 3 = 0
2x + 1 = 0
x = 3 or x = –½ (Invalid)

47. Which of these square numbers cannot be expressed as the sum of two prime numbers?
(a) 81 (b) 49
(c) 121 (d) 144
RRB JE - 30/05/2019 (Shift-II)

Ans : (c)
81 → 2 + 79 (both of which are prime number)
49 → 2 + 47 (both of which are prime number)
144 → 3 + 141(both of which are prime number)
121 → 2 + 119 (but 119 is not prime number)
Hence, option (c) cannot be expressed as the sum of two prime numbers.

48. Three times the square of a number subtracting by 4 times the number is equal to 50 more than the number. Find the number.
(a) 5 (b) 4
(c) 6 (d) 10
RRB JE - 28/05/2019 (Shift-II)

Ans : (a) Let the number be = x
According to the question,
$3x^2 - x \times 4 = x + 50$
$3x^2 - 4x - x - 50 = 0$
$3x^2 - 5x - 50 = 0$
$3x^2 - 15x + 10x - 50 = 0$
$3x(x - 5) + 10(x - 5) = 0$
$(x - 5)(3x + 10) = 0$
$x - 5 = 0$
$x = 5$

49. Which of the following is not a perfect square?
(a) 2025 (b) 16641
(c) 1250 (d) 9801
RRB RPF Constable -20/01/2019 (Shift-I)

Ans : (c) From options-
$1250 = (35.36)^2$ is not a perfect square
$2025 = (45)^2$
$16641 = (129)^2$
$9801 = (99)^2$
Hence 1250 is not a perfect square, while others are perfect squares.

50. Which of these numbers is not a sum of two squares?
(a) 41 (b) 13
(c) 23 (d) 37
RRB JE - 26/06/2019 (Shift-I)

Ans : (c) From options-
(a) $41 = 5^2 + 4^2$ (b) $13 = 2^2 + 3^2$
(c) 23 (d) $37 = 6^2 + 1^2$
Hence the number 23 is not the sum of two squares.

51. Which of these is a perfect square?
(a) 9801 (b) 9887
(c) 9013 (d) 9016
RRB JE - 01/06/2019 (Shift-III)

Ans. (a) From option (a),

	99
9	98 01
9	81
189	17 01
9	17 01
	××××

Hence, 9801 is a perfect square of 99.

52. If the last digit of the square of a number is 1. Find the last digit of its cube.
(a) Only 9 (b) 1 or 9
(c) Any odd number (d) Only 1
RRB JE - 27/06/2019 (Shift-I)

Ans : (b) Let the number be 9. The last digit of whose square is 1. Which is as follows-
$9^2 = 81$
Last digit of 729 which is cube of 9 = 9
Let the number be 11. The last digit of whose square is 1. Which is as follows-
$11^2 = 121$
The last digit of the cube of 11-
$11^3 = 1331$
Hence the last digit = 1
Hence the number will be 1 or 9.

53. Find the sum of prime factors of $9^6 \times 12^4 \times 7^7$
(a) 13 (b) 12
(c) 14 (d) 11
RRB Group-D 26/08/2022 (Shift-III)

Ans. (b) : $9^6 \times 12^4 \times 7^7$
$= 3^{12} \times 3^4 \times 2^8 \times 7^7$
$= 3^{16} \times 2^8 \times 7^7$
Sum of prime factors = 3+2+7 = 12

54. For any natural number n, $6^n - 5^n$ always ends with ;
(a) 7 (b) 1
(c) 5 (d) 3
RRB NTPC 28.12.2020 (Shift-II) Stage Ist

Ans. (b) : The unit value of $6^n–5^n$ for any natural number 'n' will always be 1 because 6 can be any natural number in the power that units number in the power of 5 has its unit digit as 5.

55. What is the total number of odd and even divisors of 120, respectively?
(a) 12,4 (b) 16,0
(c) 4,12 (d) 8,8
RRB NTPC 01.02.2021 (Shift-II) Stage I

Ans. (c) : Divisors of 120–
1, 2, 3, 4, 5, 6, 8, 10, 12, 15, 20, 24
30, 40, 60, 120
Number of even divisors – 12,
Number of odd divisors – 4

56. If the sum of five consecutive multiples of 2 is 660, then find the largest number.
(a) 162 (b) 130
(c) 125 (d) 136
RRB NTPC 15.02.2021 (Shift-II) Stage Ist

Ans. (d) : Let five consecutive multiple of 2 –
2x, 2x+2, 2x+4, 2x+6, 2x+8
According to the question,
$2x + 2x + 2 + 2x + 4 + 2x + 6 + 2x + 8 = 660$
$10x+20=660$
$10x=640$
$\boxed{x = 64}$
Hence, largest number = $2x + 8 = 2\times64+8 = 128+8$
$= \boxed{136}$

57. How many factors of $2^7 \times 3^4 \times 5^3 \times 7$ are even ?
(a) 40 (b) 280
(c) 320 (d) 84
RRB NTPC 31.01.2021 (Shift-I) Stage Ist
RRB NTPC 14.03.2021 (Shift-I) Stage Ist

Ans. (b) : $2^7 \times 3^4 \times 5^3 \times 7$ Number of factors.
$= (7 + 1) (4 + 1) (3 + 1) (1 + 1)$
$= 8 \times 5 \times 4 \times 2 = 320$
$\therefore$ Number of even factors = 320 – total no. of odd factors.
$= 320 - \{(4 + 1) (3 + 1) (1 + 1)\}$
$= 320 - \{5 \times 4 \times 2\}$
$= 320 - 40 = 280$

58. Find the digit in the unit's place of $124^n + 124^{(n+1)}$, where n is any whole number.
(a) 4 (b) 8
(c) 2 (d) 0
RRB NTPC 17.02.2021 (Shift-II) Stage Ist

Ans. (d) : $124^n + 124^{(n+1)}$
On putting n =1
$= 124 + (124)^2$
For unit digit 4 + 6 = 10
Hence, It is clear that the digit come in the unit place will be '0'.

59. What is the unit digit in the following product?
91×92×93×..........×99
(a) 2 (b) 1 (c) 4 (d) 0
RRB NTPC 09.02.2021 (Shift-II) Stage Ist

Ans. (d) : $\because 91\times92\times93\times94\times95\times96\times97\times98\times99$
It is clear that multiplying by taking unit digits of all the numbers will give '0' i.e. where 2×5 comes then its unit digit is always zero.

60. Find the number of factors of 4200.
(a) 48 (b) 56 (c) 64 (d) 46
RRB NTPC 26.07.2021 (Shift-II) Stage Ist

Ans. (a) : $4200 = 2\times2\times2\times5\times5\times3\times7$
$= 2^3\times5^2\times3^1\times7^1$
The number of factors = $(3+1)\times(2+1)\times(1+1)\times(1+1)$
$= 4\times3\times2\times2$
$= 48$

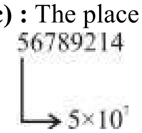
61. How many factors does the number 12288 have?
(a) 24 (b) 26
(c) 28 (d) 22
RRB NTPC 23.07.2021 (Shift-I) Stage Ist

Ans. (b) : $12288 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 = 2^{12} \times 3^1$
Hence numbers of factors = $(12 + 1) \times (1 + 1)$
$= 13 \times 2$
$= 26$

62. If a positive number N, when divided by 5 leaves a remainder 3, then the unit's place digit of N is?
(a) 0 or 5 (b) 0 or 2
(c) 3 or 8 (d) 1 or 5
RRB NTPC 25.01.2021 (Shift-I) Stage Ist

Ans. (c) : Required positive number
$= 5K+3$ ($\because$ K = 0,1,2…..)
$= 5\times0+3 = 3$ (On putting K = 0)
$= 5\times1+3 = 8$ (On putting K = 1)
Hence, unit digit of N = 3 or 8

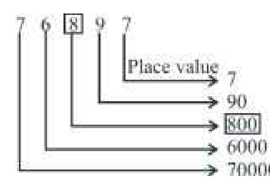
63. What is the place value of 5 in the number 56789214?
(a) 5×10^6 (b) 5×10^4
(c) 5×10^7 (d) 5×10^5
RRB NTPC 29.01.2021 (Shift-II) Stage I

Ans. (c) : The place value of 5 in 56789214 –


64. Find the sum of the place value and the face value of 7 in the number 53736.
(a) 77 (b) 707
(c) 770 (d) 777
RRB NTPC 29.01.2021 (Shift-II) Stage Ist

Ans. (b) : The place value and the face value of 7 in the number 53736.
Place value of 7 = 700
Face value of 7 = 7
Required sum = 700 + 7 = 707

65. In the number 76897, what is the place value of 8?
(a) 8 (b) 8000
(c) 800 (d) 80
RRB NTPC 09.03.2021 (Shift-II) Stage Ist

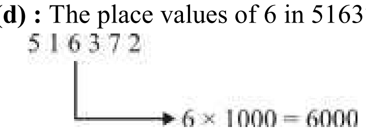
Ans. (c) :


Hence, place value of 8 in 76897 will be 800.

66. The face value of 8 in 758639 is :
(a) 8000 (b) 80
(c) 800 (d) 8
RRB NTPC 25.01.2021 (Shift-II) Stage Ist

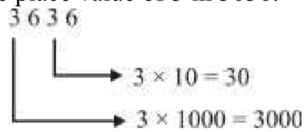
Ans. (d) : In the given number = 758639
The face value of 8 = 8

67. Find the difference of the place and face values of 6 in 516372
(a) 5998 (b) 6698
(c) 5394 (d) 5994
RRB NTPC 25.01.2021 (Shift-II) Stage Ist

Ans. (d) : The place values of 6 in 516372–


the face values of 6 = 6
Required difference = 6000 – 6 = 5994

68. **The sum of the place values of 3 in 3636 is:**
(a) 330 (b) 3030
(c) 3 (d) 3003
RRB NTPC 25.01.2021 (Shift-II) Stage Ist

Ans. (b) : The place value of 3 in 3636.


Sum of place values of 3 = 3000 + 30 = 3030

69. **The difference between the place values of 2 and 3 in the number 128935 is:**
(a) 300 (b) 19970
(c) 20000 (d) 30
RRB NTPC 02.03.2021 (Shift-I) Stage Ist

Ans. (b) :


Required difference = 20000 – 30 = 19970

70. **The sum of the place values of 9 in 96961 is:**
(a) 9000 (b) 18
(c) 9090 (d) 90900
RRB NTPC 19.01.2021 (Shift-I) Stage Ist

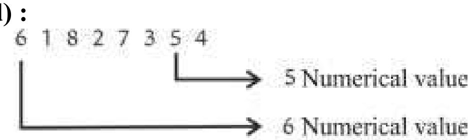
Ans. (d) : Sum of the place value of 9 in number 96961
= 90000+ 900
= 90900
Hence, option (d) is correct.

71. **Find the difference between the place values of 8 and 4 in the number 683479.**
(a) 7 (b) 80000
(c) 79600 (d) 76600
RRB NTPC 04.03.2021 (Shift-II) Stage Ist

Ans. (c) :


Hence, required difference = 80000 – 4000 = 79600

72. **Find the sum of the face values of 6 and 5 in 61827354**
(a) 60000300 (b) 30
(c) 40 (d) 11
RRB NTPC 12.01.2021 (Shift-II) Stage Ist

Ans. (d) :


Required sum = 6 + 5 = 11

73. **By how much is $\frac{1}{6}$th of 432 smaller than $\frac{3}{4}$th of 216?**
(a) –90 (b) 72
(c) 90 (d) 162
RRB NTPC 15.03.2021 (Shift-II) Stage Ist

Ans. (c) : According to the question-
$\frac{1}{6}$ part of 432 = $432\times\frac{1}{6}=72$
and $\frac{3}{4}$ part of 216 = $216\times\frac{3}{4}=162$
Required difference = 162 – 72
= 90

74. **Terry consumes 1700 mL of milk every day. How many litres of milk will she consume in 5 weeks?**
(a) 59 L (b) 60 L
(c) 58.5 L (d) 59.5 L
RRB NTPC 09.02.2021 (Shift-II) Stage I

Ans. (d) :
$\because$ Terry consumes in 1 day = 1700 mL
$\therefore$ In 5 weeks = 35 days $=\frac{1700\times35}{1000}$
$=\frac{59500}{1000}$ L
$=59.5$L

75. **Mohan earns ₹60 on first day and spends ₹50 on the second day. He again earns ₹60 on the third day and spends ₹50 on the fourth day and so on. On which day will he have ₹200 with him before spending?**
(a) 10^{th} (b) 14^{th}
(c) 28^{th} (d) 29^{th}
RRB NTPC 24.07.2021 (Shift-II) Stage Ist

Ans. (d) : Mohan earns on the first day = ₹60
and spends on the second day = ₹50
Thus, in 2 days Mohan saves = ₹10
Hence, Mohan saves in 28 days= ₹140
Mohan will earn on 29^{th} day = ₹60
So, On the 29^{th} day Mohan has = 140 + 60
= ₹200

76. **In a farmer's house, there are chickens and goats. The total number of their heads is 42 and the total number of their legs is 138. Find the number of chickens.**
(a) 15 (b) 18
(c) 20 (d) 22
RRB NTPC 01.02.2021 (Shift-I) Stage Ist

Ans. (a) : Let the number of chickens = x
Number of goats = y
According to the question,
x + y = 42 ____(i)
2x + 4y = 138 ____ (ii)
On solving the equation (i) × 4 and (ii)
$4x+4y=168$
$\underline{-2x\pm4y=-138}$
$2x = 30$
$\boxed{x=15}$
Hence, the number of chickens = x = 15

77. Two bus tickets from city P to Q and three tickets from city P to R cost ₹99, but three tickets from city P to Q and two tickets from city P to R cost ₹91. What are the respective fares from city P to Q and from city P to R.

(a) ₹23, ₹15 (b) ₹51, ₹32
(c) ₹15, ₹23 (d) ₹32, ₹51

RRB NTPC 31.01.2021 (Shift-I) Stage Ist

Ans. (c) : Let the fares from city P to Q = ₹x
and the fares from city P to R = ₹y
According to the question,

$2x + 3y = 99$ ...(i)
$3x + 2y = 91$ ...(ii)

On multiplying by 3 in equation (i) and 2 in equation (ii)

$6x + 9y = 297$ ...(iii)
$6x + 4y = 182$ ...(iv)

From equation (iii) & (iv) we have –

$5y = 115$

$\boxed{y = ₹23}$

On putting the value of y in equation (i),

$2x + 3 \times 23 = 99$
$2x + 69 = 99$
$2x = 99 - 69$
$x = \frac{30}{2}$
$\boxed{x = ₹15}$

Hence the fares from city P to Q and the fares from city P to R are ₹15, ₹23 respectively.

78. There are 40 persons in a palace. If every person shakes hands with every other person, what will be the total number of handshakes?

(a) 750 (b) 780
(c) 800 (d) 790

RRB NTPC 21.01.2021 (Shift-I) Stage Ist

Ans. (b) : Total number of handshakes $= \frac{n(n-1)}{2}$

$= \frac{40(40-1)}{2}$

$= \frac{40 \times 39}{2}$

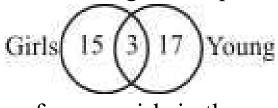
$= 20 \times 39$
$= 780$

79. In a group of 35 persons, 20 are young and 18 are girls. How many young girls are there in the group ?

(a) 1 (b) 3
(c) 18 (d) 2

RRB NTPC 17.01.2021 (Shift-II) Stage Ist

Ans. (b) : According to the question,



Number of young girls in the group = (20+18) – 35
= 38 – 35 = 3

80. Find the value of (919+9.019+0.919+9.0019)

(a) 937.3999 (b) 973.9399
(c) 937.9399 (d) 973.9939

RRB NTPC (Stage-2) 14/06/2022 (Shift-I)

Ans. (c) : 919 + 9.019 + 0.919 + 9.0019
= 919 + 18.9399
= 937.9399

81. 484.71 + 285.33 – 827.38 +73.9 = ?

(a) 19.78 (b) 36. 54
(c) 16.56 (d) 15.78

RRB NTPC (Stage-2) 17/06/2022 (Shift-I)

Ans. (c) : 484.71 + 285.33 – 827.38 + 73.9 = ?
= 484.71 + 285.33+73.9 – 827.38
= 843.94 – 827.38
= 16.56

82. Which of the following options is the closest approximate value which will come in place of question mark (?) in the following equation?
67.69 + 5.12 - 0.89 ÷ 31.88 = ?

(a) 150 (b) 35
(c) 73 (d) 48

RRB NTPC (Stage-2) 12/06/2022 (Shift-I)

Ans. (c) : 67.69 + 5.12 – 0.89 ÷ 31.88 = ?
Assuming approximately

$= 68 + 5 - \frac{1}{32}$

= 73 – 0.031 ≈ 73

83. Which of the following options is the closest approximate value which will come in place of question mark (?) in the following equation?
895.98 + 185.01 + 851.86 + 524.09 = ?

(a) 2460 (b) 1490
(c) 2010 (d) 3540

RRB NTPC (Stage-2) 16/06/2022 (Shift-III)

Ans. (a) : 895.98 + 185.01 + 851.86 + 524.09
Almost assuming
= 896 + 185 + 852 + 524
= 2457 ≈ 2460

84. 19 × 19 = 361. What will be the value of 190 × 0.0019?

(a) 0.00361 (b) 0.361
(c) 3.61 (d) 0.0361

RRB NTPC 17.02.2021 (Shift-II) Stage I

Ans. (b) : 19 × 19 = 361
⇒ 190 × 0.0019
= 0.361

85. Find the quotient of 0.5 ÷ 0.71 (correct to three decimal places)

(a) 0.706 (b) 0.714
(c) 0.705 (d) 0.704

RRB NTPC 03.02.2021 (Shift-II) Stage I

Ans. (d) : Given that,

$\frac{0.5}{0.71} = \frac{500}{710} = 0.704$

86. What will the value of the following be (correct to three decimal points)?
160.342 – 32.124

(a) 128.340 (b) 128.242
(c) 128.218 (d) 128.337

RRB NTPC 01.02.2021 (Shift-II) Stage I

Ans. (c) : Given that,
160.342 – 32.124 = 128.218

87. Simplify the following.
5 × 0.5 × 0.05 × 0.005 × 500

(a) 3125 (b) 0.3125
(c) 0.003125 (d) 31.25

RRB NTPC 28.01.2021 (Shift-I) Stage I

Ans. (b) : 5 × 0.5 × 0.05 × 0.005 × 500

$= 5\times\frac{5}{10}\times\frac{5}{100}\times\frac{5}{1000}\times500$

$= \frac{5\times5\times5\times5\times5}{10000} = \frac{3125}{10000}$

= 0.3125

88. The value of 80.6 ÷ 4030 = ?
80.6 ÷ 4030 = ?

(a) 0.2 (b) 2
(c) 0.02 (d) 20

RRB NTPC 18.01.2021 (Shift-II) Stage Ist

Ans. (c) : 80.6 ÷ 4030

$= \frac{80.6}{4030} = \frac{806}{40300}$

$= \frac{2}{100}$

= 0.02

89. How many one-thirds are in 72?

(a) 24 (b) 288
(c) 144 (d) 216

RRB NTPC 21.01.2021 (Shift-II) Stage Ist

Ans. (d) : From question,

No. of one-third in 72 = $\frac{72}{\frac{1}{3}} = 216$

90. Simplify the following expression :
(15 ÷ 3) –[{(19 – 1) ÷ 2} – {5 × 20 – (7 × 9 – (– 2))}]

(a) 21 (b) 31
(c) – 21 (d) 35

RRB NTPC (Stage-2) 16/06/2022 (Shift-I)

Ans. (b) :
(15 ÷ 3) –[{(19 – 1) ÷ 2} – {5 × 20 – (7 × 9 – (– 2))}]
= 5 – [{(19 – 1)÷ 2} – {5×20 – (7×9 – (– 2))}]
= 5 – [{18 ÷ 2} – {100 – (63 + 2) }]
= 5 – [9 – {100 – 65}]
= 5 – [9 – 35]
= 5 +26
= 31

91. Find the value of 84÷32×8–15÷8×(19–35)

(a) 38 (b) 45
(c) 51 (d) 42

RRB NTPC (Stage-2) 14/06/2022 (Shift-I)

Ans. (c) : 84 ÷ 32 × 8 – 15 ÷ 8 × (19 – 35)
= 84 ÷ 32 × 8 – 15 ÷ 8 × (–16)

$= \frac{84}{32}\times8-\frac{15}{8}\times(-16) = 21 + 30 = 51$

92. Find the value of 72 ÷ 4 × { 8 × 4 – (14 – 19)}

(a) 666 (b) 444
(c) 222 (d) 1296

RRB NTPC (Stage-2) 14/06/2022 (Shift-I)

Ans. (a) : 72 ÷ 4 × { 8 × 4 – (14 – 19)}
= 72÷ 4{8×4–(–5)}
= 72÷ 4{8×4 + 5}
= 72 ÷4{32 + 5}
= 72 ÷4 × 37
= 18 × 37 = 666

93. Find the value of 529 ÷ 23 × 61 – 1403

(a) 0 (b) 2
(c) 3 (d) 1

RRB Group-D 01/09/2022 (Shift-III)

Ans. (a) : 529 ÷ 23 ×61 – 1403
= 23 × 61 – 1403
= 1403 – 1403 = 0

94. Simplify the given expression using BODMAS :

$\mathbf{\frac{4}{11}\times\frac{121}{16}\times24(75^2-55^2)\times\frac{1}{100}}$

(a) 1736 (b) 1726
(c) 1746 (d) 1716

RRB NTPC 30.01.2021 (Shift-I) Stage Ist

Ans. (d) : $\frac{4}{11}\times\frac{121}{16}\times24\left(75^2-55^2\right)\frac{1}{100}$

From BODMAS,

$= \frac{11}{4}\times24\left[(75+55)(75-55)\right]\times\frac{1}{100}$

We know that, $\left[\because a^2-b^2=(a+b)(a-b)\right]$

$= 66\times(130\times20)\times\frac{1}{100}$

$= 66\times2600\times\frac{1}{100}$

= 1716

95. The value of 3 + [3 × {3 – (3 + 3) ÷ 6}] is:

(a) 3 (b) 9
(c) 6 (d) -3

RRB NTPC 13.03.2021 (Shift-I) Stage I

Ans. (b) : The value of 3 + [3 × {3 – (3 + 3) ÷ 6}]
= 3 + [3 × {3 – 6 ÷ 6}]
= 3 + [3 × {3 – 1}]
= 3 + [3 × 2]
= 3 + 6 = 9