
RRB

TECHNICIAN

GRADE-I SIGNAL

ENGINEERING

MATHEMATICS

Chief Editor
A.K. Mahajan

Compiled & Written By
YCT Expert Team

Computer Graphics
Balkrishna Tripathi & Ashish Giri

Editorial Office
12, Church Lane Prayagraj-211002

 9415650134

Email : yctap12@gmail.com

website : www.yctbooks.com/www.yctfastbook.com/www.yctbooksprime.com

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Syllabus

Questions will be of objective type with multiple choice answers and are likely to cover topics pertaining to the following syllabus

- **General Awareness:** Knowledge of current affairs, Indian geography, culture and history of India including freedom struggle, Indian Polity and constitution, Indian and Economy, environmental issues concerning India and the World, Sports, General scientific and technological developments, etc.
- **General Intelligence and Reasoning:** Analogies, Alphabetical and Number series, Coding and Decoding, Mathematical operations, Relationships, Syllogism, Jumbling, Venn Diagram, Data Interpretation and sufficiency, Conclusions and decision making, Similarities and differences, Analytical reasoning, Classification, Directions, Statement - Arguments and Assumptions, etc.
- **Basics of Computers and Applications:** Architecture of Computers; input and output devices: Storage devices, Networking Operating System like Windows, Unix, Linux; MS Office; Various data representation; Internet and Email; Websites & Web Browsers; Computer Virus.
- **Mathematics:** Number system, Rational and irrational numbers, BODMAS rule, Quadratic Equations, Arithmetic Progression, Similar Triangles, Pythagoras Theorem, Co-ordinate Geometry, Trigonometrical Ratios, Heights and distances, Surface area and Volume; Sets: Set and their representations, Empty set, Finite and Infinite sets, Equal sets, Subsets, Subsets of a set of real numbers, Universal set, Venn diagrams, Union and Intersection of sets, Difference of sets, Complement of a set, Properties of Complement; Statistics: Measures of Dispersion: Range, Mean deviation, variance and standard deviation of ungrouped/grouped data; probability occurrence of events, exhaustive events, mutually exclusive events.
- **Basic Science and Engineering:**
Physics' fundamentals- Units, Measurements, Mass, Weight, Density, Work Power, and Energy, Speed and Velocity, heat and Temperature;
Electricity and Magnetism-Electric Charge, Field, and intensity, Electric Potential and Potential Difference, Simple Electric Circuits, Conductors, Non-conductors/Insulators, Ohm's Law and its Limitations, Resistances in Series and Parallel of a Circuit and Specific Resistance, Relation Between Electric Potential, Energy, and Power (Wattage) Ampere's Law, Magnetic Force on Moving Charged Particle and Long Straight Conductors, Electromagnetic Induction, Faraday's Law, and Electromagnetic Flux, Magnetic Field, Magnetic Induction;
Electronics and Measurements-basic Electronics, Digital Electronics, Electronic Devices and Circuits, Microcontroller, Microprocessor, Electronic Measurements, Measuring Systems and Principles, Range Extension methods, Cathode Ray Oscilloscope, LCD LED Panel, Transducers.

Tentative Subject-wise break-up of questions and marks for CBT of Technician Gr-I Signal

Subjects	No. of Questions	Marks for Each Section
General Awareness	10	10
General Intelligence and Reasoning	15	15
Basics of Computers and Applications	20	20
Mathematics	20	20
Basic Science & Engineering	35	35
Total	100	100

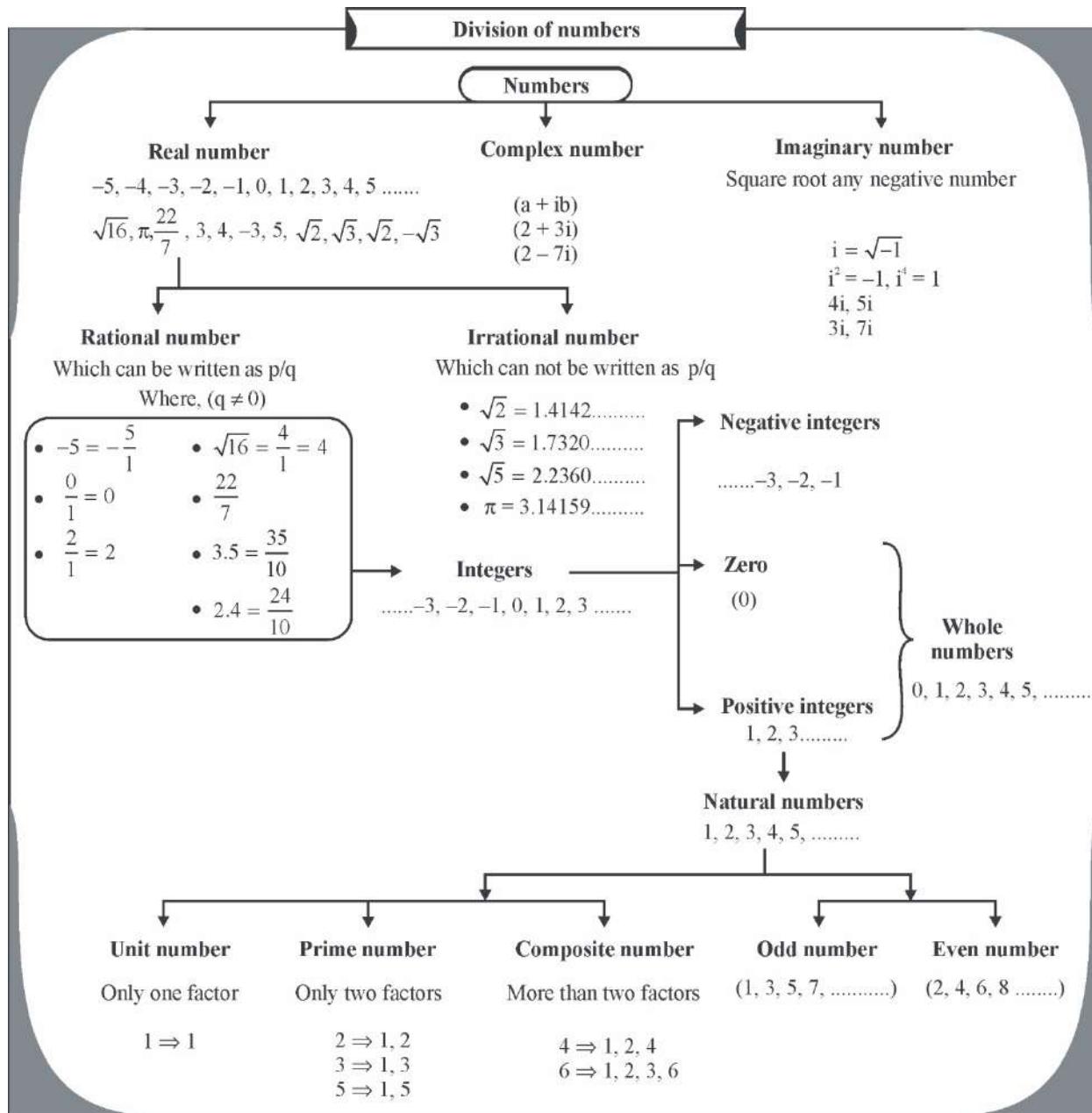
1. Duration : 90 minutes (with 30 minutes extra time for PwBD candidates using scribe).

2. The Subject-wise distribution give above is merely indicative. The question papers may vary.

01

Number system

(Rational and irrational numbers & BODMAS rule)

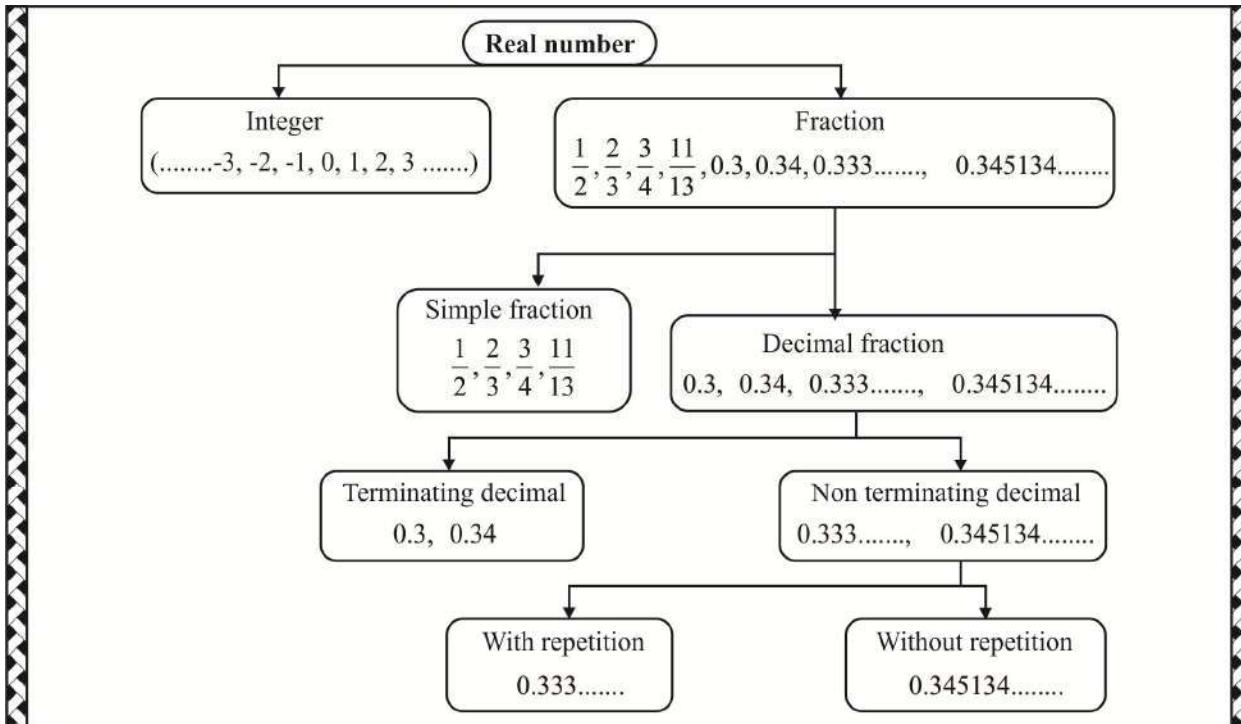


Co-prime/Relatively prime number

☞ A pair of numbers which H.C.F. (Highest common factor) is 1, is called co-prime number. Ex. (2, 3), (3, 4), (3, 5), (6, 7), (8, 11).

Twin-prime number

☞ A pair of prime numbers in which the difference is two is called twin prime number. Ex. (3, 5), (5, 7), (11, 13)



☞ Decimals with repetition can be expressed as rational numbers.

The test of prime number

■ Let a is any give number and n is the smallest number.

where, $n^2 \geq a$

Now divide the given number by ' n ' and smaller than each prime number. If ' a ' is not completely divisible by any of these numbers, then ' a ' will be a prime number otherwise not.

Ex. Test of 241:-

$$241 \Rightarrow 16^2 \geq 241$$

Prime number less than 16

$$= 2, 3, 5, 7, 11, 13$$

\therefore 241 is not divisible by any prime number less than 16

\therefore 241 is a prime number.

Ex. Test of 437:-

$$437 \Rightarrow 21^2 \geq 437$$

Prime number less than 21

$$= 2, 3, 5, 7, 11, 13, 17, 19,$$

\therefore 437 is completely divisible by 19

\therefore 437 is a composite number.

Number of prime numbers

Prime numbers between 1-10	4
Prime numbers between 1-50	15
Prime numbers between 1-100	25
Prime numbers between 1-200	46
Prime numbers between 1-1000	168

☞ First prime number = 2

☞ Each prime number can be written as $(6k \pm 1)$ form. But every $(6k \pm 1)$ from may not be necessarily prime number.

Ex. $(6 \times 2 + 1) = 13$ Prime number

$25 = (6 \times 4 + 1)$ Composite number

Divisibility Rules

Divisibility of 2, 4, 8 and 16

■ **Divisibility of 2 :-** If the digit at unit place of a number is either '0' or even number then the number is divisible by 2.

Ex. 8570, 7242, 9376

■ **Divisibility of 4 :-** If the last two digits (ten's place, units place) of a number is either '00' or divisible by 4, then the number is divisible by 4.

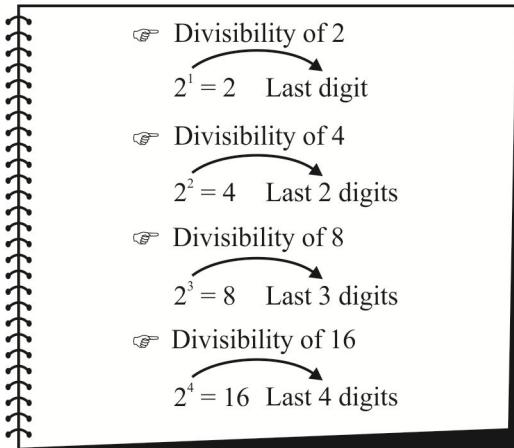
Ex. 8700, 6924, 6376

■ **Divisibility of 8 :-** If the last three digits (Hundred's place, tenth place, units place) of a number is either '000' or divisible by 8, then the number is divisible by 8.

Ex. 63000, 9248, 7464

■ **Divisibility of 16 :-** If the last three digits (Thousand's place, hundred's place, ten's place, units place) of a number is either '0000' or divisible by 16, then the number is divisible by 16.

Ex. 630000, 948464



Divisibility of 3 and 9

■ **Divisibility of 3 :-** If the sum of its all digits of a number is divisible by 3, then the number is divisible by 3.

Ex. 78141

$$\Rightarrow \frac{7+8+1+4+1}{3} = \frac{21}{3} = 7 \text{ (divisible)}$$

Hence, the number 78141 will be divisible by 3

Ex. 246753

$$\Rightarrow \frac{2+4+6+7+5+3}{3} = \frac{27}{3} = 9 \text{ (divisible)}$$

Hence, the number 246753 will be divisible by 3

■ **Divisibility of 9 :-** If the sum of its all digits of a number is divisible by 9, then the number is divisible by 9)

Ex. 764352

$$\Rightarrow \frac{7+6+4+3+5+2}{9} = \frac{27}{9} = 3 \text{ (divisible)}$$

Hence, the number 764352 will be divisible by 9

Ex. 432432

$$\Rightarrow \frac{4+3+2+4+3+2}{9} = \frac{18}{9} = 2 \text{ (divisible)}$$

Hence, the number 432432 will be divisible by 9

☞ In divisibility of 3 and 9, we can use 'digital sum' in place of sum.

Digital sum :- It is just a position of remainder when it is divided by 9. That is, the sum of the digits should be 9. If it is more than 9 then add the digits together.

Ex. 10 $\frac{\text{Digital sum}}{9} \rightarrow 1+0=1$

11 $\frac{\text{Digital sum}}{9} \rightarrow 1+1=2$

84 $\frac{\text{Digital sum}}{9} \rightarrow 8+4=12 \quad 1 \quad 2 \quad 3$

786 $\frac{\text{Digital sum}}{9} \rightarrow 7+8+6=21 \quad 2 \quad 1 \quad 3$

☞ Cut all digits whose sum is 9

☞ Digital sum of a perfect square number 0 or 9, 1, 4, 7

☞ To calculate digital sum in fraction number, then always make digital sum 1 in denominator.

Denominator	Multiply	Digital sum
4	$4 \times 7 = 28$	1
7	$7 \times 4 = 28$	1
5	$5 \times 2 = 10$	1
2	$2 \times 5 = 10$	1
8	$8 \times 8 = 64$	1

Note- If the denominator of a number is 3, 6 or 9 then 1 can not be made for the digital sum.

Divisibility of 5, 10, 25 and 100

■ **Divisibility of 5 :-** If the digit at unit place of a number is either 0 or 5 then the number is divisible by 5.

Ex. 24520, 28735

■ **Divisibility of 10 :-** If the digit at unit place of a number is 0 then the number is divisible by 10.

Ex. 570120, 4567890

■ **Divisibility of 25 :-** If the last two digits (ten's, unit's place) of a number either 25, 50, 75 or 00, then the number is divisible by 25.

Ex. 8725, 68750, 931275, 8600

■ **Divisibility of 100 :-** If the last two digits (ten's, unit's place) of a number 00, then the number is divisible by 100.

Ex. 689200

■ **Divisibility of 7 :-** If the number obtained by subtracting twice the unit digit from the remaining number excluding the unit digit, is divisible by 7, then that number will be divisible by 7. Repeat this process again and again for larger numbers.

Ex. 343

$$\begin{array}{r} 343 \\ -6 \times \\ \hline 28 \end{array} \Rightarrow \frac{28}{7} = \text{Integer}$$

Hence, 343 is divisible by 7

Ex. 383838

$$\begin{array}{r} 383838 \\ -16 \times \\ \hline 3836 \end{array} \begin{array}{r} 7 \\ -14 \times \\ \hline 14 \end{array} \begin{array}{r} 2 \\ -4 \times \\ \hline 37 \end{array} \begin{array}{r} 8 \\ -16 \times \\ \hline 21 \end{array} \Rightarrow \frac{21}{7} = 3 \text{ Integer}$$

Hence, 383838 is divisible by 7

■ **Divisibility of 11 :-** If the difference of the sum of the digits in even position and the sum of the digits in odd position is zero or multiple of 11.

Ex. 352143

Sum of even position = $4+2+3=9$

Sum of odd position = $3+1+5=9$

$$\Rightarrow |9-9|=0$$

Hence, the number 352143 is divisible by 11

Ex. 71940

Sum of even position = $4 + 1 = 5$

Sum of odd position = $0 + 9 + 7 = 16$

$$\Rightarrow \frac{|5-16|}{11} = 1 \text{ (Integer)}$$

Hence, the number 71940 is divisible by 11

Divisibility of 7, 11, 13

- Make pairs of three digits from the right side of a numbers. Find the difference between sum of pairs at even places and sum of pairs at odd places—
 - ☞ If the difference is 0, then the number will be divisible by 7, 11 and 13.
 - ☞ If the difference is divisible by any of 7, 11 and 13, then the number will also be divisible by that.

Ex. 786786

$$\underline{786} \underline{786} = |786 - 786| \Rightarrow 0$$

Hence, the number is divisible by 7, 11 and 13.

Ex. 1001

$$\underline{001} \underline{001} = |001 - 001| \Rightarrow 0$$

Hence, the number is divisible by 7, 11 and 13.

Ex. 786730

$$\underline{786} \underline{730} = |786 - 730|$$

$$\Rightarrow 56 \text{ (Divisible by 7)}$$

Hence, the number is divisible by 7

Ex. 5786

$$\underline{005} \underline{786} = |005 - 786|$$

$$\Rightarrow 781 \text{ (Divisible by 11)}$$

Hence, the number is divisible by 11

Ex. 91689

$$\underline{091} \underline{689} = |091 - 689|$$

$$\Rightarrow 598 \text{ (Divisible by 13)}$$

Hence, the number is divisible by 13

Ex. 786709

$$\underline{786} \underline{709} = |786 - 709|$$

$$\Rightarrow 77 \text{ (Divisible by 7 and 11)}$$

Hence, the number is divisible by 7 and 11.

- When a number is divisible by another number, It is also divisible by the factor of the number.

Ex. 48 is divisible by 12

Then, 48 is also divisible by factor (1, 2, 3, 4, 6, 12) of 12.

- When a number is divisible by two or more co-prime numbers, It is also divisible by their products.

Ex. 12 is divisible by 2 and 3.

$\therefore (2, 3) \rightarrow \text{Co-prime number}$

$\therefore 12, 12 \text{ is divisible by } (2 \times 3).$

- When a number is a factor of two given number It is also a factor of their sum and difference.

Ex. $\because 6$ is factor of 30 and 6 is factor of 18.

Then, 6 is factor of $\{(30 + 18) = 48\}$ and $\{(30 - 18) = 12\}$

- When a number is a factor of another number, It is also a factor of any multiple of that number.

Ex. $\because 4$ is factor of 12

Then, 4 is also factor of multiple (12, 24, 36,) of 12.

- ☞ If a number is formed by repeating a digit six times, it will be divisible by 3, 7, 11, 13, 37.

Ex. (111111), (222222), (333333)

- ☞ If a number is formed by repeating 2 digit 3 times, it will be divisible by 3, 7, 13, 37.

Ex. 383838, 171717, 595959

- ☞ If a number repeats the same digit 3, 6, 9, 12 (multiple of 3), then that number will be divisible by 3 and 37.

Ex. (111), (222222), (333333333), (444444444444)

Place value and face value

Place value :- The place value of a digit describes its place in a given number.

Ex. Place value of 7 in number 7345724—

7345724

$$\begin{array}{r} \text{Place value} \\ \downarrow \\ 7 \times 100 = 700 \\ \downarrow \\ 7 \times 1000000 = 7000000 \end{array}$$

Ex.

Number

$$\begin{array}{r} \text{Place value} \\ \downarrow \\ 2 \times 1 = 2 \\ \downarrow \\ 7 \times 10 = 70 \\ \downarrow \\ 5 \times 100 = 500 \\ \downarrow \\ 3 \times 1000 = 3000 \end{array}$$

Ex. Write 'Eleven thousand eleven hundred eleven' in digits—

11000

1100

$\begin{array}{r} + 11 \\ \hline \end{array}$

12111

Face value :- Face value is the value of the digit itself in a number. It does not depend upon its position in the number.

Ex. Face value of 7 in number 7345724—

7345724

$$\begin{array}{r} \text{Face value} \\ \downarrow \\ 7 \\ \downarrow \\ 7 \end{array}$$

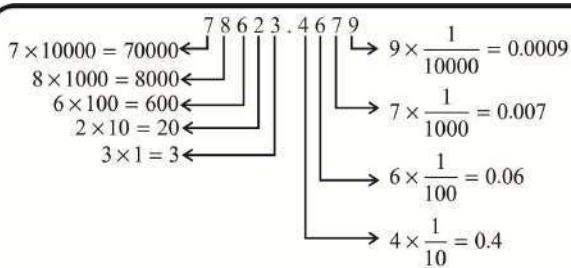
Ex.

Number

$$\begin{array}{r} \text{Face value} \\ \downarrow \\ 2 \\ \downarrow \\ 7 \\ \downarrow \\ 5 \\ \downarrow \\ 3 \end{array}$$

- ☞ The face value as well as place value of zero is always zero.

Place value of a decimal number



Significant Number

Definition: A significant number (or significant digit) is any digit in a number that contributes to its accuracy or precision.

These are the digits that carry real meaning in terms of the measurement or value, not just placeholders.

Ex.- 123.45 → 5 Significant figures

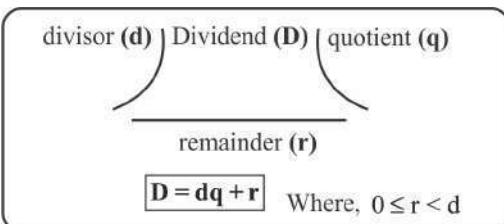
0.00789 → 3 Significant figures.

2000 → 1 Significant figure.

1.020 → 4 Significant figures

7.8900×10^{-3} = 5 Significant figure.

Division operation in numbers



Ex. Find the number in which dividing by 15 gives quotient 14 and remainder 13?

Solve- $D = dq + r$

$$D = 15 \times 14 + 13$$

$$D = 223$$

Ex. By dividing a number by 11 and 5 successively, the remainder remains 2 and 3 respectively, what will be the remainder if the number is divided by 55?

Solve- $\therefore 11 \times 5 = 55$

11 and 5 are factors of 55

$$\therefore D = 11 \times 3 + 2$$

$$D = 35$$

Ex. When two different numbers are divided by a divisor, the remainder becomes 547 and 349 respectively when the sum of both numbers is divided by the same divisor, the remainder is 211, find the divisor.

Solve-

First quotient = q_1

Second quotient = q_2

Common divisor = d

$$\therefore \text{First number} = dq_1 + 547$$

$$\text{Second number} = dq_2 + 349$$

$$\text{then, } \frac{(dq_1 + 547) + (dq_2 + 349)}{d} \xrightarrow{\text{Remainder}} 211$$

$$\therefore d = 547 + 349 - 211$$

$$d = 685$$

Ex. When a number is divided by 441, the remainder is 40. If the same number is divided by 21, the remainder will be?

Solve- $\because 21$ is the factor of 441

$$\therefore \frac{40}{21} \xrightarrow{\text{Remainder}} 19$$

Hence, the remainder will be 19.

Ex. When a number is divided by 231, the remainder is 45. If the same number is divided by 17, the remainder will be?

Solve-

$\because 17$ is not the factor of 231

\therefore The remainder can not be determined

Unit digit

■ The last digit of a number is called the unit digit.

$$4364357$$

↳ Unit digit

$$\Leftrightarrow 763 + 542 \Rightarrow 1305$$

↳ Unit digit

$$\Leftrightarrow 765 + 849 \Rightarrow 1614$$

↳ Unit digit

$$\Leftrightarrow 763 - 542 \Rightarrow 221$$

↳ Unit digit

$$\Leftrightarrow 765 - 347 \Rightarrow 418$$

↳ Unit digit

$$\Leftrightarrow 765 - 947 \Rightarrow -182$$

↳ Unit digit

$$\Leftrightarrow 765 - 943 \Rightarrow -178$$

↳ Unit digit

☞ In subtraction problems, while finding the unit digit, the smaller number is subtracted from the larger number.

☞ The last digit of the answer obtained will be unit digit. The answer obtained can be positive or negative, but not the unit digit.

Finding the unit digit when number is raised to the power

- When the unit digit of a number is 0, 1, 5 and 6 and it has any power, then its unit digit will be the same digit.

$$\text{Ex. } (1530)^{999} \quad \text{Unit digit}$$

$$\text{Ex. } (761)^{789} \quad \text{Unit digit}$$

$$\text{Ex. } (765)^{897} \quad \text{Unit digit}$$

$$\text{Ex. } (786)^{547} \quad \text{Unit digit}$$

- When the unit digit of a number is 2, 3, 4, 7, 8, and 9 and it has any power, then find the unit digit-

Digit last two digits of power by 4 and find out remainder

Last two digits of power

$$\begin{array}{c} 4 \\ \hline \text{Remainder} \Rightarrow 1, 2, 3, 0 \end{array}$$

Remainder	Power
1	1
2	2
3	3
0	4

$$\text{Ex. } [172]^{4325}$$

$$\frac{25}{4} \xrightarrow{\text{Remainder}} 1 \xrightarrow{\text{Power}} 1$$

$$2^1 \Rightarrow 2 \quad \text{Unit digit}$$

$$\text{Ex. } [978]^{4798}$$

$$\frac{98}{4} \xrightarrow{\text{Remainder}} 2 \xrightarrow{\text{Power}} 2$$

$$8^2 \Rightarrow 64 \quad \text{Unit digit}$$

$$\text{Ex. } [567]^{8759}$$

$$\frac{59}{4} \xrightarrow{\text{Remainder}} 3 \xrightarrow{\text{Power}} 3$$

$$7^3 \Rightarrow 343 \quad \text{Unit digit}$$

$$\text{Ex. } [6543]^{9972}$$

$$\frac{72}{4} \xrightarrow{\text{Remainder}} 0 \xrightarrow{\text{Power}} 4$$

$$3^4 \Rightarrow 81 \quad \text{Unit digit}$$

When the number is in the form of $N!$

- When the power is in the form of $n!$ -

$$1! = 1 \quad \frac{n!}{4} \xrightarrow{\text{Remainder}} 0 \xrightarrow{\text{Power}} 4$$

$$2! = 2 \times 1 \quad \text{Where, } n! \geq 4$$

$$3! = 3 \times 2 \times 1$$

$$4! = 4 \times 3 \times 2 \times 1$$

$$5! = 5 \times 4 \times 3 \times 2 \times 1$$

$$5! = 5 \times 4!$$

$$\vdots \quad \vdots$$

$$n! = n(n-1)!$$

$$\text{Ex. } 992^{786!}$$

$$\because 786! > 4! \xrightarrow{\text{Remainder}} 0 \xrightarrow{\text{Power}} 4$$

$$\therefore 2^4 = 16$$

$$\text{Unit digit}$$

- When the number is in the form of multiplication of $n!$ -

Number	0!	1!	2!	3!	4!
Unit digit	1	1	2	6	4

- 5! and greater than 5! give unit digit 0.

Unit digit of multiplication by 5

- $5 \times \text{Odd number} \xrightarrow{\text{Unit digit}} 5$

$$\text{Ex. } 5 \times 1 = 5 \xrightarrow{\text{Unit digit}} 5$$

$$\text{Ex. } 5 \times 3 = 15 \xrightarrow{\text{Unit digit}} 5$$

- $5 \times \text{Even number} \xrightarrow{\text{Unit digit}} 0$

$$\text{Ex. } 5 \times 2 = 10 \xrightarrow{\text{Unit digit}} 0$$

$$\text{Ex. } 5 \times 4 = 20 \xrightarrow{\text{Unit digit}} 0$$

- $5 \times \text{Odd number} \times \text{Even number} \xrightarrow{\text{Unit digit}} 0$

$$\text{Ex. } 5 \times 1 \times 2 = 10 \xrightarrow{\text{Unit digit}} 0$$

$$\text{Ex. } 5 \times 3 \times 4 = 60 \xrightarrow{\text{Unit digit}} 0$$

- The unit digit of a perfect square number can be 0, 1, 4, 5, 6 or 9 but if the unit digit of a number is 0, 1, 4, 5, 6 or 9 then it is not necessary that it is a perfect square number.

Zero Place Number of trailing zeroes

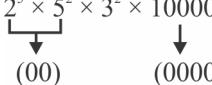
- A zero is formed by a pair of 5 and 2, i.e. by multiplying 5 and 2, we get zero
- In any question, as many pairs of five and two are formed, The same zero is formed. Therefore, to solve the question the powers of 5 and 2 are seen and whose power is less, the same zero is created.

	<p>☞ $5 \times 2 = 10$ $5^1 \times 2^1$ <u>No. of pair</u> → 1 <u>No. of zero</u> → 1</p> <p>☞ $25 \times 4 = 100$ $5^2 \times 2^2$ <u>No. of pair</u> → 2 <u>No. of zero</u> → 2</p> <p>☞ $125 \times 4 = 500$ $5^3 \times 2^2$ <u>No. of pair</u> → 2 <u>No. of zero</u> → 2 (Which power less)</p> <p>☞ $25 \times 8 = 200$ $5^2 \times 2^3$ <u>No. of pair</u> → 2 <u>No. of zero</u> → 2 (Which power less)</p> <p>☞ $125 \times 8 = 1000$ $5^3 \times 2^3$ <u>No. of pair</u> → 3 <u>No. of zero</u> → 3</p>
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Ex. Multiplying $25 \times 16 \times 2 \times 5$ will be how many zeros on the right side.

Sol. $25 \times 16 \times 2 \times 5$
 $\Rightarrow 5 \times 5 \times 2 \times 2 \times 2 \times 2 \times 2 \times 5$
 $\Rightarrow 5^3 \times 2^5$
 $5^3 \times 2^4$ No. of pair → 3 No. of zero → 3
(Which power less)

Ex. Multiplying $300 \times 400 \times 24 \times 25$ will be how many zeros on right side.

Sol. $300 \times 400 \times 24 \times 25$
 $\Rightarrow 3 \times 4 \times 24 \times 25 \times 10000$
 $\Rightarrow 3 \times 4 \times 2 \times 2 \times 2 \times 3 \times 5 \times 5 \times 10000$
 $\Rightarrow 2^5 \times 5^2 \times 3^2 \times 10000$
 $\overbrace{2^5} \times \overbrace{5^2} \times \overbrace{3^2} \times 10000$

Number of zeroes = 6

Ex. Multiplying all natural numbers from 1 to 60, how many zeros will come to the right side.

Sol. $1 \times 2 \times 3 \times \dots \times 25 \times \dots \times 50 \times \dots \times 60$
 $\frac{60}{5} = 12$
 $\frac{12}{5} = 2$ $12 + 2 = 14$ (Zeroes)

☞ In the given question it is clear that on multiplying, the power of five is less than that of 2.

☞ Stop dividing when the quotient is less than 5.

Ex. Multiplying all natural number from 1 to 100, How many zeros will come to right side.

Sol. $1 \times 2 \times 3 \times \dots \times 25 \times \dots \times 50 \times \dots \times 75 \times \dots \times 100$

$$\Rightarrow \frac{100}{5} = 20$$

$$\frac{100}{25} = 4 \quad 20 + 4 = 24 \text{ (Zeroes)}$$

Ex. Multiplying all natural numbers from 1 to 500, how many zeros will come to right side.

Sol. $1 \times 2 \times 3 \times \dots \times 25 \times \dots \times 50 \times \dots \times 100 \times \dots \times 500$

$$\frac{500}{5} = 100$$

$$\frac{100}{5} = 20$$

$$\frac{20}{5} = 4 \quad 100 + 20 + 4 = 124 \text{ (Zeroes)}$$

Ex. Multiplying all natural numbers 1 to 1000, How many zeros will come to right side.

Ex. $1 \times 2 \times 3 \times \dots \times 25 \times \dots \times 50 \times \dots \times 100 \times \dots \times 1000$

$$\frac{1000}{5} = 200$$

$$\frac{200}{5} = 40$$

$$\frac{40}{5} = 8$$

$$\frac{8}{5} = 1 \quad 200 + 40 + 8 + 1 = 249 \text{ (Zeroes)}$$

Ex. Multiplying all even numbers upto 80, How many zeros will come to right side.

Sol. $2 \times 4 \times 6 \times \dots \times 80$

$$\frac{80}{10} = 8$$

$$\frac{8}{5} = 1 \quad 8 + 1 = 9 \text{ (Zeroes)}$$

☞ In multiplication of even number, first divide by 10, then by 5

Ex. Multiplying all the numbers 51 to 100, How many zeros will come to right side.

Sol. $51 \times 52 \times 53 \dots \times 100$

$$\Rightarrow [1 \times 2 \times 3 \dots \times 100] - [1 \times 2 \times 3 \dots \times 50]$$

$$\Rightarrow \frac{100}{5} = 20 \quad \frac{50}{5} = 10$$

$$\frac{20}{5} = 4 \quad \frac{10}{5} = 2$$

$$\Rightarrow [20 + 4 = 24] \quad [10 + 2 = 12]$$

$$\Rightarrow [24] - [12] = 12 \text{ (Zeroes)}$$

Ex. On solving $96!$ how many zeros will come to right side.

Sol. $96! = 96 \times 95 \times 94 \times \dots \times 1$

$$\frac{96}{5} = 19$$

$$\frac{19}{5} = 3 \quad 19 + 3 = 22 \text{ (Zeroes)}$$

Ex. On solving $9860!$, How many zeros will come to right side.

Sol. $9860! = 9860 \times 9859 \times \dots \times 1$

$$\therefore \frac{9860}{5} = 1972$$

$$\frac{1972}{5} = 394$$

$$\frac{394}{5} = 78$$

$$\frac{78}{5} = 15$$

$$\frac{15}{5} = 3$$

$$\Rightarrow 1972 + 394 + 78 + 15 + 3 = 2462 \text{ (Zeroes)}$$

Ex. Multiplying all the odd numbers 1 to 100, how many zeros will come to right side.

Sol. $1 \times 3 \times 5 \times 7 \times 9 \times 11 \times \dots \times 99$

“Number of zeroes is zero”

☞ In the given question all the numbers are odd, no number will be divisible by 2. Hence no digit of two will appear in the product of these numbers. Hence not a single zero will be obtained at the end of the product of the given question.

Ex. Multiplying the first 100 prime numbers, How many zeros will come to right side.

Sol. $2 \times 3 \times 5 \times 7 \times 11 \times 13 \times 17 \times 19 \times \dots \times 97$

$$\Rightarrow 2 \times 5$$

$$\Rightarrow 2^1 \times 5^1$$

= Number of zero = 1

Ex. How many zeroes on the right end of the product of $(1 \times 3 \times 5 \times 7 \times \dots \times 99) \times 8$.

Sol. $(1 \times 3 \times 5 \times 7 \times \dots \times 99) \times 8$

$$(5 \times 15 \times 25 \times 35 \times \dots \times 95) \times 8 \quad \{ \text{For pair of 5 and 2} \}$$

$$\Rightarrow 5^{12} \times 2^3$$

$$5^{12} \times 2^3 \xrightarrow{\text{No. of pair}} 3 \xrightarrow{\text{No. of zero}} 3$$

(Which power less)

Ex. Find the number of zeroes.

$$(3^{123} - 3^{122} - 3^{121})(2^{121} - 2^{120} - 2^{119})$$

$$\text{Sol. } (3^{123} - 3^{122} - 3^{121})(2^{121} - 2^{120} - 2^{119})$$

$$3^{121}(3^2 - 3^1 - 3^0)2^{119}(2^2 - 2^1 - 2^0)$$

$$3^{121}(9 - 3 - 1)2^{119}(4 - 2 - 1)$$

$$3^{121}(5)2^{119}(1)$$

$$2^{119} \times 3^{121} \times 5^1$$

$$2^{119} \times 5^1 \times 3^{121}$$



No. of pair 1 → no. of zero = 1

Ex. If $100!$ divisible by 3^n then find the maximum value of n :

Sol. $100! = 100 \times 99 \times 98 \times \dots \times 1$

$$\frac{100}{3} = 33$$

$$\frac{33}{3} = 11$$

$$\frac{11}{3} = 3$$

$$\frac{3}{3} = 1$$

$$\Rightarrow 33 + 11 + 3 + 1 = 48$$

Hence $n = 48$

Ex. If $122!$ is divisible by 6^n then find the maximum value of n :

Sol. $\frac{122!}{6} = \frac{122!}{2 \times 3}$

To make a pair of 2 and 3, the power of 3 will be reduced.

$$\frac{122}{3} = 40$$

$$\frac{40}{3} = 13$$

$$\frac{13}{3} = 4$$

$$\frac{4}{3} = 1$$

$$\Rightarrow 40 + 13 + 4 + 1 = 58$$

Hence $n = 58$

Ex. If $123!$ is divisible by 12^n then find the maximum value of n :

Sol. $\frac{123!}{12^n} = \frac{123}{3 \times 2^2} \quad \frac{123}{3} = 41 \quad \frac{123}{2} = 61$

$$\frac{123!}{3^{59} \times 2^{117}} \quad \frac{41}{3} = 13 \quad \frac{61}{2} = 30$$

$$\frac{123!}{3^{59} \times (2^2)^{58} \times 2^1} \quad \frac{13}{3} = 4 \quad \frac{30}{2} = 15$$

$$\frac{123!}{3^{59} \times (4)^{58} \times 2^1} \quad \frac{4}{3} = 1 \quad \frac{15}{2} = 7$$

$$\text{Hence } n = 58 \quad \text{Sum} = 59 \quad \frac{7}{2} = 3$$

$$\frac{3}{2} = 1$$

$$\text{Sum} = 117$$

Number of factors

Factors

Factors are positive integers that can divide a number exactly.

Ex. Factors of 12

1, 2, 3, 4, 6, 12

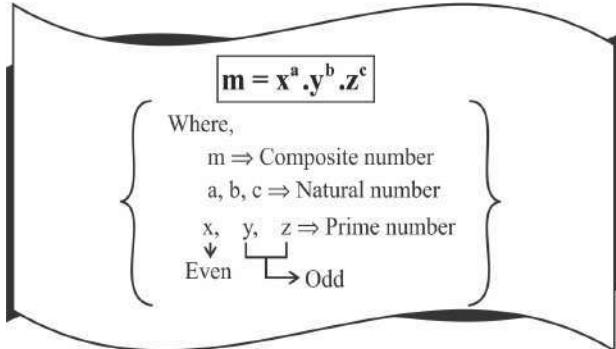
☞ Multiple of 12

12, 24, 36, 48,

How to find factors

- Writing any numbers as its prime factors.

Ex. $12 = 2^2 \times 3^1$
 $72 = 2^3 \times 3^2$
 $90 = 2^1 \times 3^2 \times 5^1$



- The number of total factors: $(a + 1)(b + 1)(c + 1)$
- The number of odd factors: $(b + 1)(c + 1)$
- The number of even factors: $a(b + 1)(c + 1)$
- The sum of all factors: $(x^0 + x^1 + x^2 + \dots + x^a) \times (y^0 + y^1 + y^2 + \dots + y^b) \times (z^0 + z^1 + z^2 + \dots + z^c)$
- The sum of odd factors: $(y^0 + y^1 + \dots + y^b) \times (z^0 + z^1 + z^2 + \dots + z^c)$
- The sum of even factors: $(x^1 + x^2 + x^3 + \dots + x^a) \times (y^0 + y^1 + \dots + y^b) \times (z^0 + z^1 + z^2 + \dots + z^c)$
- The product of factors: $(x \cdot y \cdot z)^{\frac{\text{Total no. of factors}}{2}}$
- Sum of reciprocal of factors of n: $\frac{\text{sum of factors}}{n}$
- Average: $\frac{\text{Sum of factors}}{\text{No. of factors}}$

For the factors of 12

$$12 = 2^2 \times 3^1$$

- The number of total factors-

$$12 = 2^2 \times 3^1$$

$$\downarrow \quad \downarrow$$

$$(2 + 1) \times (1 + 1)$$

$$3 \times 2 = 6$$

- The number of odd factors-

$$12 = 2^2 \times 3^1$$

$$\downarrow$$

$$(1 + 1) = 2$$

- The number of even factors-

$$12 = 2^2 \times 3^1$$

$$2 \times (2^1 \times 3^1)$$

$$\downarrow \quad \downarrow$$

$$\text{Even } (1 + 1) \times (1 + 1)$$

$$(2) \times (2) = 4$$

- The sum of factors-

$$12 = 2^2 \times 3^1$$

$$= (2^0 + 2^1 + 2^2)(3^0 + 3^1)$$

$$= (1 + 2 + 4)(1 + 3)$$

$$= 7 \times 4 \Rightarrow 28$$

- The sum of odd factors-

$$12 = 2^2 \times 3^1$$

$$\Rightarrow (3^0 + 3^1)$$

$$1 + 3 \Rightarrow 4$$

For the sum of odd factors, leave out even factors.

- The sum of even factors-

$$12 = 2^2 \times 3^1$$

$$\Rightarrow (2^1 + 2^2)(3^0 + 3^1)$$

$$\Rightarrow (2 + 4)(1 + 3)$$

$$\Rightarrow 6 \times 4$$

$$\Rightarrow 24$$

For sum of even factors, don't start from 2^0 .

- The product of all factors-

$$12 = 2^2 \times 3^1$$

$$\text{Product of all factors of } N = N^{\frac{\text{Total no. of factors}}{2}}$$

$$= 12^{\frac{6}{2}}$$

$$= 12^3$$

$$12 = 2^2 \times 3^1$$

$$\downarrow \quad \downarrow$$

$$(2 + 1) \times (1 + 1)$$

$$3 \times 2 = 6$$

- How many factors of 864 which are multiple of 6?

Sol. $864 = 2^5 \times 3^3$

$$864 = 2 \times 3 [2^4 \times 3^2] \quad \{ \text{For the multiple of 6} \}$$

$$= 6 [2^4 \times 3^2]$$

$$\downarrow \quad \downarrow$$

$$(4 + 1) \quad (2 + 1)$$

$$\Rightarrow 5 \times 3$$

$$\Rightarrow 15$$

- How many factors of $2^7 \times 3^8 \times 5^9 \times 7^{10}$ which are completely square?

Sol. $2^7 \times 3^8 \times 5^9 \times 7^{10}$

$$\Rightarrow [(2^2)^3 2 \times (3^2)^4 \times (5^2)^4 5 \times (7^2)^5]$$

{For the complete square }

$$= 2 \times 5 [(2^2)^3 \times (3^2)^4 \times (5^2)^4 \times (7^2)^5]$$

$$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$$

$$(3+1) \times (4+1) \times (4+1) \times (5+1)$$

$$\text{No. of factors} = 4 \times 5 \times 5 \times 6 \Rightarrow 600$$

- How many factors of $2^6 \times 3^8 \times 5^{10} \times 7^{12}$ which are completely cube?

Sol. $2^6 \times 3^8 \times 5^{10} \times 7^{12}$

$$\Rightarrow (2^3)^2 \times (3^3)^2 \times (5^3)^3 \times (7^3)^4$$

$$\Rightarrow 3^2 \times 5 [(2^3)^2 \times (3^3)^2 \times (5^3)^3 \times (7^3)^4]$$

$$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$$

$$(2+1) \times (2+1) \times (3+1) \times (4+1)$$

$$\Rightarrow 3 \times 3 \times 4 \times 5 \Rightarrow 180$$

- How many factors of $2^6 \times 3^{15} \times 5^{35} \times 7^{42}$ which are completely square as well as completely cube?

Sol. $2^6 \times 3^{15} \times 5^{35} \times 7^{42}$

$$\text{Power for square} = 2$$

$$\text{Power for cube} = 3$$

$$\text{LCM} = 6$$

$$\Rightarrow [(2^6)^1 \times (3^6)^2 \times (5^6)^5 \times (7^6)^7]$$

$$\begin{aligned}
 & \Rightarrow 3^3 \times 5^5 [(2^6)^1 \times (3^6)^2 \times (5^6)^5 \times (7^6)^7] \\
 & \Rightarrow (1+1) \times (2+1) \times (5+1) \times (7+1) \\
 & \Rightarrow [2 \times 3 \times 6 \times 8] \Rightarrow [6 \times 6 \times 8] \\
 & \Rightarrow [36 \times 8] \Rightarrow 288
 \end{aligned}$$

■ Find the sum of all factors of $2^5 \times 3^6 \times 5^4$ that are completely square.

$$\begin{aligned}
 \text{Sol. } & 2^5 \times 3^6 \times 5^4 \\
 & \Rightarrow [2^0 + 2^2 + 2^4] [3^0 + 3^2 + 3^4 + 3^6] [5^0 + 5^2 + 5^4] \\
 & \Rightarrow [1 + 4 + 16] [1 + 9 + 81 + 729] [1 + 25 + 625] \\
 & \Rightarrow [21] \times [820] \times [651] \Rightarrow 11210220
 \end{aligned}$$

■ Find the sum of all factors of $2^5 \times 3^6 \times 5^4$ that are completely cube.

$$\begin{aligned}
 \text{Sol. } & 2^5 \times 3^6 \times 5^4 \\
 & \Rightarrow [2^0 + 2^3] [3^0 + 3^3 + 3^6] [5^0 + 5^3] \\
 & \Rightarrow [1 + 8] [1 + 27 + 729] [1 + 125] \\
 & \Rightarrow [9] [757] [126] \Rightarrow 858438
 \end{aligned}$$

■ Find the sum of reciprocal of factors of 90.

$$\begin{aligned}
 \text{Sol. } & \text{Sum of reciprocal of factors of } n = \frac{\text{sum of factors}}{n} \\
 & 90 = 2^1 \times 3^2 \times 5^1 \\
 & \Rightarrow \frac{(2^0 + 2^1)(3^0 + 3^1 + 3^2)(5^0 + 5^1)}{90} \\
 & \Rightarrow \frac{[(1+2)(1+3+9)(1+5)]}{90} \\
 & \Rightarrow \frac{[3 \times 13 \times 6]}{90} \Rightarrow \frac{[39 \times 6]}{90} \\
 & \Rightarrow \frac{234}{90} \Rightarrow 2.6
 \end{aligned}$$

■ Find the average of all the factors of 144.

$$\text{Sol. } \text{Average} = \frac{\text{Sum of factors}}{\text{No. of factors}}$$

For sum of factors-

$$\begin{aligned}
 144 &= 2^4 \times 3^2 \\
 &\Rightarrow [(2^0 + 2^1 + 2^2 + 2^3 + 2^4)(3^0 + 3^1 + 3^2)] \\
 &\Rightarrow [(1+2+4+8+16)(1+3+9)] \\
 &\Rightarrow [(31)(13)] \Rightarrow 403
 \end{aligned}$$

For no. of factors-

$$\begin{aligned}
 &\Rightarrow (4+1)(2+1) \Rightarrow 5 \times 3 \\
 &\Rightarrow 15
 \end{aligned}$$

$$\text{Average} = \frac{403}{15} \Rightarrow 26.86$$

■ Only a perfect square number has odd number of factors.

or

If a number has odd number of factors that means number is a perfect square.

■ Square of a prime number has only 3 factors.

■ The total number of 2 digit no's which have only 3 factors?

Sol. \because Square of a prime number has only 3 factor.

$$(5^2) = 25 \xrightarrow{\text{Factors}} 1, 5, 25$$

$$(7^2) = 49 \xrightarrow{\text{Factors}} 1, 7, 49$$

5, 7 \rightarrow Prime number

Hence, 2, two digit no. will have 3 factors.

■ The total number of 3 digit no's which have only 3 factors?

Sol.

$$(11^2) = 121 \xrightarrow{\text{Factors}} 1, 11, 121$$

$$(13^2) = 169 \xrightarrow{\text{Factors}} 1, 13, 169$$

$$(17^2) = 289 \xrightarrow{\text{Factors}} 1, 17, 289$$

$$(19^2) = 361 \xrightarrow{\text{Factors}} 1, 19, 361$$

$$(23^2) = 529 \xrightarrow{\text{Factors}} 1, 23, 529$$

$$(29^2) = 841 \xrightarrow{\text{Factors}} 1, 29, 841$$

$$(31^2) = 961 \xrightarrow{\text{Factors}} 1, 31, 961$$

Hence, 7, three digit no. will have 3 factors.

How to find prime factor

$$m = x^a \cdot y^b \cdot z^c$$

Where,
 $m \Rightarrow$ Composite number
 $x, y, z \Rightarrow$ Prime number
 $a, b, c \Rightarrow$ Natural number
Number of prime factors = $a + b + c$
Sum of prime factors = $ax + by + cz$

■ Find the total number of prime factors of 144.

$$\text{Sol. } 144 = 2^4 \times 3^2$$

No. of prime factors = $4 + 2 \Rightarrow 6$

■ Find the total number of prime factor of $2^5 \times 3^6 \times 7^{12}$.

$$\text{Sol. } 2^5 \times 3^6 \times 7^{12}$$

No. of prime factors = $5 + 6 + 12 \Rightarrow 23$

■ Find the total number of prime factor of $6^6 \times 10^{10} \times 35^3$.

$$\text{Sol. } 6^6 \times 10^{10} \times 35^3$$

$$\Rightarrow (2 \times 3)^6 \times (2 \times 5)^{10} \times (5 \times 7)^3$$

$$\Rightarrow 2^6 \times 3^6 \times 2^{10} \times 5^{10} \times 5^3 \times 7^3$$

No. of prime factors

$$= (6 + 6 + 10 + 10 + 3 + 3)$$

$$\Rightarrow (12 + 20 + 6)$$

$$\Rightarrow (18 + 20) \Rightarrow 38$$

■ Find sum of all the prime factors of $2^3 \times 3^4 \times 5^6$.

$$\text{Sol. } 2^3 \times 3^4 \times 5^6$$

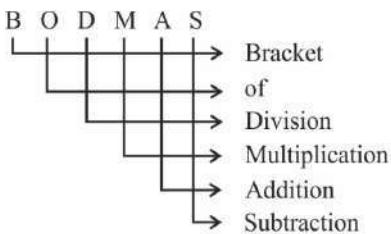
$$\Rightarrow (2 + 2 + \dots \text{ 3 times}) + (3 + 3 + \dots \text{ 4 times})$$

$$+ (5 + 5 + \dots \text{ 6 times})$$

$$\Rightarrow (2 \times 3) + (3 \times 4) + (5 \times 6)$$

$$\Rightarrow 6 + 12 + 30 \Rightarrow 48$$

BODMAS Rule



☞ Solve the brackets from inside to outside.

Types of brackets :

- Line/Bar bracket → —
- Circular/Small/Open bracket → ()
- Curly/Braces bracket → {}
- Square/Closed bracket → []

To solve :

$$222 - \frac{1}{3} \text{ of } \{42 + (56 - \overline{8+9})\} + 108$$

Sol. $222 - \frac{1}{3} \text{ of } \{42 + (56 - 17)\} + 108$

$$222 - \frac{1}{3} \text{ of } \{42 + (56 - 17)\} + 108$$

$$\Rightarrow 222 - \frac{1}{3} \text{ of } \{42 + 39\} + 108$$

$$\Rightarrow 222 - \frac{1}{3} \text{ of } \{81\} + 108$$

$$\Rightarrow 222 - \frac{1}{3} \text{ of } 81 + 108$$

$$\Rightarrow 222 - [27 + 108]$$

$$\Rightarrow 222 - 135 \Rightarrow 87$$

To solve :

$$19170 \div 54 \div 5$$

Sol. $19170 \div 54 \div 5$

$$\Rightarrow 19170 \times \frac{1}{54} \times \frac{1}{5}$$

$$\Rightarrow \frac{355}{5}$$

$$\Rightarrow 71$$

To solve :

$$\frac{9}{13} \div \frac{18}{26} \div \frac{90}{52}$$

Sol. $\frac{9}{13} \div \frac{18}{26} \div \frac{90}{52}$

$$\Rightarrow \frac{9}{13} \times \frac{26}{18} \times \frac{52}{90}$$

$$\Rightarrow \frac{26}{45}$$

To solve:

$$5.8 + (7.4 \div 3.7 \times 5) - 6 \times 2 \div 2.5$$

Sol. $5.8 + (7.4 \div 3.7 \times 5) - 6 \times 2 \div 2.5$

$$\Rightarrow 5.8 + (2 \times 5) - 6 \times \frac{2}{2.5}$$

$$\Rightarrow 5.8 + 10 - 4.8$$

$$\Rightarrow 15.8 - 4.8$$

$$\Rightarrow 11$$

Question based on series

➢ $\frac{1}{a \times b} = \frac{1}{(b-a)} \frac{1}{a} - \frac{1}{b}$

➢ $\frac{1}{a \times b \times c} = \frac{1}{(c-a)} \frac{1}{ab} - \frac{1}{bc}$

➢ $\frac{1}{a \times b \times c \times d} = \frac{1}{(d-a)} \frac{1}{abc} - \frac{1}{bcd}$

➢ $1 \times 2 + 2 \times 3 + 3 \times 4 + 4 \times 5 + \dots + n(n+1) = \frac{n(n+1)(n+2)}{3}$

Find the value :

$$\frac{1}{20} + \frac{1}{30} + \frac{1}{42} + \frac{1}{56} + \frac{1}{72} + \frac{1}{90}$$

Sol. $\frac{1}{20} + \frac{1}{30} + \frac{1}{42} + \frac{1}{56} + \frac{1}{72} + \frac{1}{90}$

$$\Rightarrow \frac{1}{4 \times 5} + \frac{1}{5 \times 6} + \frac{1}{6 \times 7} + \frac{1}{7 \times 8} + \frac{1}{8 \times 9} + \frac{1}{9 \times 10}$$

$$\Rightarrow \frac{1}{4} - \frac{1}{5} + \frac{1}{5} - \frac{1}{6} + \frac{1}{6} - \frac{1}{7} + \frac{1}{7} - \frac{1}{8} + \frac{1}{8} - \frac{1}{9} + \frac{1}{9} - \frac{1}{10}$$

$$= \frac{1}{4} - \frac{1}{10}$$

$$= \frac{5-2}{20} \Rightarrow \frac{3}{20}$$

Find the value :

$$\frac{1}{1 \times 4} + \frac{1}{4 \times 7} + \frac{1}{7 \times 10} + \frac{1}{10 \times 13} + \frac{1}{13 \times 16} = ?$$

Sol. $\frac{1}{1 \times 4} + \frac{1}{4 \times 7} + \frac{1}{7 \times 10} + \frac{1}{10 \times 13} + \frac{1}{13 \times 16}$

$$\Rightarrow \frac{1}{3} \left[\frac{3}{1 \times 4} + \frac{3}{4 \times 7} + \frac{3}{7 \times 10} + \frac{3}{10 \times 13} + \frac{3}{13 \times 16} \right]$$

$$\Rightarrow \frac{1}{3} \left[\frac{1}{1} - \frac{1}{4} + \frac{1}{4} - \frac{1}{7} + \frac{1}{7} - \frac{1}{10} + \frac{1}{10} - \frac{1}{13} + \frac{1}{13} - \frac{1}{16} \right]$$

$$\Rightarrow \frac{1}{3} \frac{1}{1} - \frac{1}{16}$$

$$\Rightarrow \frac{1}{3} \frac{16-1}{16} = \frac{1}{3} \times \frac{15}{16} = \frac{5}{16}$$

Find the value :

$$\frac{2}{15} + \frac{4}{45} + \frac{7}{144} + \frac{9}{400} = ?$$

Sol. $\frac{2}{15} + \frac{4}{45} + \frac{7}{144} + \frac{9}{400}$

$$\Rightarrow \frac{2}{3 \times 5} + \frac{4}{5 \times 9} + \frac{7}{9 \times 16} + \frac{9}{16 \times 25}$$

$$\Rightarrow \frac{1}{3} - \frac{1}{5} + \frac{1}{5} - \frac{1}{9} + \frac{1}{9} - \frac{1}{16} + \frac{1}{16} - \frac{1}{25}$$

$$= \frac{1}{3} - \frac{1}{25}$$

$$= \frac{25-3}{75} = \frac{22}{75}$$

■ Find the value :

$$\frac{3}{1^2 \cdot 2^2} + \frac{5}{2^2 \cdot 3^2} + \frac{7}{3^2 \cdot 4^2} + \frac{9}{4^2 \cdot 5^2} \dots \dots \dots \frac{19}{9^2 \cdot 10^2}$$

Sol. $\frac{3}{1^2 \cdot 2^2} + \frac{5}{2^2 \cdot 3^2} + \frac{7}{3^2 \cdot 4^2} + \frac{9}{4^2 \cdot 5^2} \dots \dots \dots \frac{19}{9^2 \cdot 10^2}$

$$\frac{3}{1 \times 4} + \frac{5}{4 \times 9} + \frac{7}{9 \times 16} + \frac{9}{16 \times 25} \dots \dots \dots \frac{19}{81 \times 100}$$

$$\Rightarrow \frac{1}{1} - \frac{1}{4} + \frac{1}{4} - \frac{1}{9} + \frac{1}{9} - \frac{1}{16} + \frac{1}{16} - \frac{1}{25} \dots \dots \frac{1}{81} - \frac{1}{100}$$

$$\Rightarrow \frac{1}{1} - \frac{1}{100}$$

$$\Rightarrow \frac{99}{100}$$

■ Find the value :

$$1 + \frac{1}{2} \quad 1 + \frac{1}{3} \quad 1 + \frac{1}{4} \quad \dots \dots \quad 1 + \frac{1}{n} = ?$$

Sol. $1 + \frac{1}{2} \quad 1 + \frac{1}{3} \quad 1 + \frac{1}{4} \quad \dots \dots \quad 1 + \frac{1}{n}$

$$\frac{3}{2} \times \frac{4}{3} \times \frac{5}{4} \dots \dots \frac{(n+1)}{n}$$

$$= \frac{(n+1)}{2}$$

■ Find the value :

$$1 - \frac{1}{2} \quad 1 - \frac{1}{3} \quad 1 - \frac{1}{4} \quad \dots \dots \quad 1 - \frac{1}{n} = ?$$

Sol. $1 - \frac{1}{2} \quad 1 - \frac{1}{3} \quad 1 - \frac{1}{4} \quad \dots \dots \quad 1 - \frac{1}{n}$

$$\frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} \dots \dots \frac{(n-1)}{n}$$

$$\Rightarrow \frac{1}{n}$$

■ Find the value :

$$1 - \frac{1}{3^2} \quad 1 - \frac{1}{4^2} \quad 1 - \frac{1}{5^2} \quad \dots \dots \quad 1 - \frac{1}{11^2} \quad 1 - \frac{1}{12^2}$$

Sol. $1 - \frac{1}{3^2} \quad 1 - \frac{1}{4^2} \quad 1 - \frac{1}{5^2} \quad \dots \dots \quad 1 - \frac{1}{11^2} \quad 1 - \frac{1}{12^2}$

$$a^2 - b^2 = (a+b)(a-b)$$

$$1 + \frac{1}{3} \quad 1 - \frac{1}{3} \quad 1 + \frac{1}{4} \quad 1 - \frac{1}{4} \quad \dots \dots \dots$$

$$\dots \dots \quad 1 + \frac{1}{11} \quad 1 - \frac{1}{11} \quad 1 + \frac{1}{12} \quad 1 - \frac{1}{12}$$

$$\Rightarrow \quad 1 + \frac{1}{3} \quad 1 + \frac{1}{4} \quad 1 + \frac{1}{5} \quad \dots \dots \quad 1 + \frac{1}{12} \quad \times$$

$$1 - \frac{1}{3} \quad 1 - \frac{1}{4} \quad 1 - \frac{1}{5} \quad \dots \dots \quad 1 - \frac{1}{12}$$

$$\left[\frac{4}{3} \times \frac{5}{4} \times \frac{6}{5} \dots \dots \frac{13}{12} \right] \left[\frac{2}{3} \times \frac{3}{4} \times \dots \dots \frac{11}{12} \right]$$

$$\Rightarrow \frac{13}{3} \times \frac{2}{12}$$

$$\Rightarrow \frac{13}{3} \times \frac{1}{6}$$

$$\Rightarrow \frac{13}{18}$$

■ Find the value :

$$\frac{2 \times 8 + 8 \times 32 + 18 \times 72 + \dots \dots \dots \frac{1}{4}}{1 + 16 + 81 + \dots \dots \dots} = ?$$

Sol. $\frac{2 \times 8 + 8 \times 32 + 18 \times 72 + \dots \dots \dots \frac{1}{4}}{1 + 16 + 81 + \dots \dots \dots}$

$$\Rightarrow 16 \frac{1 + 16 + 81 + \dots \dots \dots \frac{1}{4}}{1 + 16 + 81 + \dots \dots \dots}$$

$$\Rightarrow [16]^{\frac{1}{4}}$$

$$\Rightarrow 2^{\frac{1}{4}} = 2$$

■ Find the value :

$$\frac{1.2.4 + 2.4.8 + 3.6.12 + \dots \dots \dots \frac{1}{3}}{1.3.9 + 2.6.18 + 3.9.27 + \dots \dots \dots}$$

Sol. $\frac{1.2.4 + 2.4.8 + 3.6.12 + \dots \dots \dots \frac{1}{3}}{1.3.9 + 2.6.18 + 3.9.27 + \dots \dots \dots}$

$$\Rightarrow \frac{8}{27}$$

$$= \frac{2}{3}$$

Exponential Series

$$e = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \dots \dots \dots \quad 2.71828$$

■ Find the value :

$$\frac{1}{1.2.3} + \frac{1}{1.2.3.4} + \frac{1}{1.2.3.4.5} + \dots \dots \dots$$

Sol. $\frac{1}{3!} + \frac{1}{4!} + \frac{1}{5!} + \dots$
 $1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \dots - 1 + \frac{1}{1!} + \frac{1}{2!}$
 $= (2.71828) - (1 + 1 + 0.5)$
 $= 0.21828$

■ **Find the value :**

$$\frac{8! \times 7! \times 6!}{9! \times 5! \times 3!} = ?$$

Sol. $\frac{8! \times 7 \times 6 \times 5! \times 6 \times 5 \times 4 \times 3!}{9 \times 8! \times 5! \times 3!}$
 $\Rightarrow 28 \times 20$
 $\Rightarrow 560$

■ **Find the value in the form of 6! :**
 $[8! - 7! - 6!]$

Sol. $[8! - 7! - 6!]$
 $\Rightarrow [8 \times 7 \times 6! - 7 \times 6! - 6!]$
 $\Rightarrow 6! [8 \times 7 - 7 - 1]$
 $\Rightarrow 6! [56 - 8]$
 $\Rightarrow 6! [48]$

■ **If $a * b = 2(a + b)$ then find the value $1 * [2 * 3]$**

Sol. $1 * [2 * 3]$
 $\Rightarrow 1 * [2 (2 + 3)]$
 $\Rightarrow 1 * [2 \times 5]$
 $\Rightarrow 1 * 10$
 $\Rightarrow 2 [1 + 10]$
 $\Rightarrow 2 \times 11$
 $= 22$

■ **If $x * y = 3x + 2y$, then find the value $2 * 3 + 3 * 4$**

Sol. $2 * 3 + 3 * 4$
 $\downarrow \downarrow \downarrow \downarrow$
 $x \ y \ x \ y$
 $\Rightarrow (3 \times 2 + 2 \times 3) + (3 \times 3 + 2 \times 4)$
 $\Rightarrow (6 + 6) + (9 + 8)$
 $= 12 + 17 = 29$

■ **If $@$ is an operation such that
 $2a$ यदि $> b$**

$$a @ b = a + b \text{ यदि } a < b$$

$$a^2 \text{ यदि } = b$$

then, $\frac{(5 @ 7) + (4 @ 4)}{3(5 @ 5) - (15 @ 11) - 3} = ?$

Sol. $\frac{(5 + 7) + (4)^2}{3(5)^2 - (2 \times 15) - 3}$
 $\Rightarrow \frac{12 + 16}{75 - 30 - 3}$
 $\Rightarrow \frac{28}{75 - 33}$
 $\Rightarrow \frac{28}{42} = \frac{2}{3}$

■ **Find the value:**

$$999 \frac{995}{999} \times 999$$

Sol. $999 \frac{995}{999} \times 999$

$$\Rightarrow 999 + \frac{995}{999} 999$$

$$\Rightarrow (1000 - 1) + \frac{995}{999} 999$$

$$\Rightarrow \frac{(1000 - 1)999 + 995}{999} \times 999$$

$$\Rightarrow 999000 - 999 + 995$$

$$= 999000 - 4 = 998996$$

■ **Find the value :**

$$999 \frac{1}{9} + 999 \frac{2}{7} + 999 \frac{3}{7} + 999 \frac{4}{7} + 999 \frac{5}{7} + 999 \frac{6}{7}$$

Sol. $999 \frac{1}{9} + 999 \frac{2}{7} + 999 \frac{3}{7} + 999 \frac{4}{7} + 999 \frac{5}{7} + 999 \frac{6}{7}$

$$\Rightarrow (999 \times 6) + \frac{1}{7} + \frac{2}{7} + \frac{3}{7} + \frac{4}{7} + \frac{5}{7} + \frac{6}{7}$$

$$\Rightarrow (1000 - 1)6 + \frac{21}{7}$$

$$= 6000 - 6 + 3$$

$$= 6000 - 3 = 5997$$

■ **Find the value :**

$$3 \frac{1}{3} + 33 \frac{1}{3} + 333 \frac{1}{3} + 3333 \frac{1}{3} + 33333 \frac{1}{3}$$

Sol. $3 \frac{1}{3} + 33 \frac{1}{3} + 333 \frac{1}{3} + 3333 \frac{1}{3} + 33333 \frac{1}{3}$

$$\Rightarrow (3 + 33 + 333 + 3333 + 33333) +$$

$$\frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3}$$

$$= 37035 + \frac{5}{3} \Rightarrow 37035 + 1 \frac{2}{3}$$

$$= 37036 + \frac{2}{3} \Rightarrow 37036 \frac{2}{3}$$

Continuous fraction

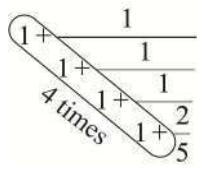
■ **To solve :**

$$1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{2}{5}}}}$$

Sol. Step-1 : Write the last fraction $\frac{2}{5}$ first

Step-2 : Write the numerator (2) first then the denominator (5).

Step-3 : Next number will appear as many times as one is given in the question and to find the next number, immediately add the previous number to that number.



$$2, 5 \xrightarrow{5+2} 7 \xrightarrow{7+5} 12 \xrightarrow{12+7} 19 \xrightarrow{19+12} 31 \\ \Rightarrow \frac{31}{19}$$

■ To solve :

$$1 - \frac{1}{1 - \frac{1}{1 - \frac{1}{1 - \frac{2}{5}}}}$$

Sol.

$$2, 5 \xrightarrow{5-2} 3 \xrightarrow{3-5} -2 \xrightarrow{-2-3} -5 \xrightarrow{-5-(-2)} -3$$

$$\text{Hence, the fraction} = \frac{-3}{-5} = \frac{3}{5}$$

■ To solve :

$$1 + \frac{1}{2 + \frac{1}{3 + \frac{1}{4}}}$$

$$\text{Sol. } 1, 4 \xrightarrow{\times 3+1} 13 \xrightarrow{\times 2+4} 30 \xrightarrow{\times 1+13} 43$$

$$\text{Hence, the fraction} = \frac{43}{30}$$

■ To solve :

$$1 - \frac{1}{2 - \frac{1}{3 - \frac{1}{4}}}$$

$$\text{Sol. } 1, 4 \xrightarrow{\times 3-1} 11 \xrightarrow{\times 2-4} 18 \xrightarrow{\times 1-11} 7$$

■ Find the value $a+b+c$:

$$\frac{1}{a + \frac{1}{b + \frac{1}{c}}} = \frac{13}{29}$$

Sol.

$$13 \overline{)29} (2 = a$$

$$\overline{26} \overline{)3} (4 = b$$

$$\overline{12} \overline{)3} (3 = c$$

$$\therefore a+b+c = 2+4+3$$

$$a+b+c = 9$$

■ Find the value $a+b+c$:

$$a + \frac{1}{b + \frac{1}{c + \frac{1}{2}}} = \frac{16}{23}$$

Sol.

$$16 \overline{)23} (1 = a$$

$$\overline{16} \overline{)7} (2 = b$$

$$\overline{14} \overline{)2} (3 = c$$

$$\therefore \text{Last term} = \frac{1}{2} \overline{)6} (1$$

$$\therefore a+b+c = 1+2+3 = 6$$

■ Recurring decimal :

$$0.\overline{a} = \frac{a}{9} \quad 0.\overline{ab} = \frac{ab}{99}$$

$$0.\overline{abc} = \frac{abc}{999} \quad 0.\overline{ab} = \frac{ab-a}{90}$$

$$0.\overline{abc} = \frac{abc-ab}{900} \quad 0.\overline{abc} = \frac{abc-a}{990}$$

■ Find the value

$$8.\overline{31} + 0.\overline{6} + 0.0\overline{02} = ?$$

Sol.

Without bar = 2

With bar = 1, 1, 1 LCM = 1

Without bar With bar

$$\begin{array}{r} \downarrow \downarrow \\ 8.31 \mid 1 \ 1 \ 1 \ \dots \dots \\ 0.66 \mid 6 \ 6 \ 6 \ \dots \dots \\ 0.00 \mid 2 \ 2 \ 2 \ \dots \dots \\ \hline 8.97 \mid 9 \ 9 \ 9 \end{array}$$

$$\Rightarrow 8.97\overline{9}$$

■ Find the value :

$$22.\overline{4} + 11.5\overline{67} - 33.5\overline{9} = ?$$

Sol. Without bar = 1

With bar = 1, 2, 1 LCM = 2

Without bar With bar

$$\begin{array}{r} \downarrow \downarrow \\ 22.4 \mid 4 \ 4 \ 4 \ \dots \dots \\ +11.5 \mid 6 \ 7 \ 6 \ 7 \ \dots \dots \\ \hline -33.5 \mid 9 \ 9 \ 9 \ \dots \dots \\ \hline 0.4 \mid 1 \ 2 \ 1 \ 2 \end{array}$$

Surds and Indices

Surds : $\sqrt[n]{a}$

$\sqrt{}$ → Radical

n → Order of surd

a → Radicand

<ul style="list-style-type: none"> Entire surds : \sqrt{a}, $(\sqrt{a} + \sqrt{b})$ Mixed surds : $a\sqrt{b}$ Like & Similar surds : $x\sqrt{b}$, $y\sqrt{b}$, $z\sqrt{b}$ Unlike & unsimilar surds : $x\sqrt{b}$, $y\sqrt{c}$, $z\sqrt{d}$ Conjugate surds : $\sqrt{7} + \sqrt{5} \xrightarrow{\text{Conjugate}} \sqrt{7} - \sqrt{5}$ $\sqrt{4} - \sqrt{3} \xrightarrow{\text{Conjugate}} \sqrt{4} + \sqrt{3}$ Product of conjugate surds is a rational number. Quadratic surds : $a + \sqrt{b}$, $\sqrt{a} + \sqrt{b} + c$

Equation involving surds-

If the surds, $a + \sqrt{b} = c + \sqrt{d}$

$$\begin{cases} a = c \\ b = d \end{cases}$$

Hence, the rational part of one side is equal to the rational part of other side and the irrational part of one side is equal to the irrational part of other side.

Rationalization-

Surds	Rationalization factor
$\sqrt{a} + \sqrt{b}$	$\sqrt{a} - \sqrt{b}$
$\sqrt{a} - \sqrt{b}$	$\sqrt{a} + \sqrt{b}$
$a + \sqrt{b}$	$a - \sqrt{b}$
$a - \sqrt{b}$	$a + \sqrt{b}$
$a^{2/3} + b^{2/3} - a^{1/3}b^{1/3}$	$(a^{1/3} + b^{1/3})$
$a^{2/3} + b^{2/3} + a^{1/3}b^{1/3}$	$(a^{1/3} - b^{1/3})$

Law of surds and indices

- $a \times a \times a \times \dots \text{m term} = a^m$
- $a \times a \times a \times \dots \text{n term} = a^n$
- $(a \times a \times \dots \text{m term}) \times (a \times a \times \dots \text{n term}) = a^m \times a^n \Rightarrow a^{m+n}$
- $\frac{a \times a \times a \times \dots \text{m terms}}{a \times a \times a \times \dots \text{n terms}} = \frac{a^m}{a^n} \Rightarrow a^{m-n}$
- If $a > 0$, $a \neq 1$ and m, n, p are integers then,
 - $a^m \times a^n = a^{m+n}$
 - $a^m \times a^n \times a^p = a^{m+n+p}$
 - $(a^m)^n = a^{mn}$
 - $\frac{a^m}{a^n} = a^{m-n}$
 - $a^0 = 1$

- $a^{-m} = \frac{1}{a^m}$
- $a^{m^n} = a^{(m^n)}$
- $a^{m^{np}} = a^{m^{(np)}} = a^{m^{(n^p)}}$
- $(ab)^n = a^n b^n$
- $(abc)^n = a^n b^n c^n$
- If $a^n = y$ then $a = y^{1/n}$
If $a^x = b^y$ then $a = b^{y/x}$
If $a^x = b^y$ then $a^{1/y} = b^{1/x}$
- $x^n = a \Rightarrow x = \sqrt[n]{a}$, ($a \in \mathbb{R}, a \geq 0$)
- If n is an odd positive integer and $a > 0$ then,
 $\sqrt[n]{-a} = \sqrt[n]{a}$
If $m, n \geq 2$, and $a, b > 0$ then–
- $\sqrt[n]{a} = a^{1/n}$
- $(\sqrt[n]{a})^m = a^{m/n}$
- $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab} = (ab)^{1/n}$
- $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$
- $\sqrt[n]{\sqrt[m]{a}} = (a^{1/m})^{1/n} = a^{1/mn}$
- $\sqrt[n]{a} \cdot \sqrt[m]{a} = a^{1/n} \cdot a^{1/m}$
 $\Rightarrow a^{1/n+1/m}$
 $\Rightarrow a^{\frac{m+n}{mn}} = \sqrt[mn]{a^{(m+n)}}$
- $\frac{\sqrt[n]{a}}{\sqrt[m]{a}} = \frac{a^{1/n}}{a^{1/m}} = a^{\frac{1}{n}-\frac{1}{m}} = a^{\frac{m-n}{mn}}$
 $\Rightarrow \sqrt[mn]{a^{(m-n)}}$
- $\sqrt[xyz]{(\sqrt[n]{a})^p} = a^{\frac{pqr}{xyz}}$

Find square root

<ul style="list-style-type: none"> $(a+b)^2 = a^2 + b^2 + 2ab$ $(\sqrt{a} + \sqrt{b})^2 = a + b + 2\sqrt{ab}$ $(a-b)^2 = a^2 + b^2 - 2ab$ $(\sqrt{a} - \sqrt{b})^2 = a + b - 2\sqrt{ab}$ $(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$ $(\sqrt{a} + \sqrt{b} + \sqrt{c})^2 = a + b + c + 2\sqrt{ab} + 2\sqrt{bc} + 2\sqrt{ca}$ $(a-b)(a+b) = a^2 - b^2$

■ Find the square root-

$$11 + 2\sqrt{30}$$

Sol.

$$\begin{array}{r} \sqrt{11 + 2\sqrt{30}} \\ \downarrow \quad \downarrow \\ 5 + 6 \quad 5 \times 6 \\ \sqrt{(\sqrt{5})^2 + (\sqrt{6})^2 + 2\sqrt{5} \times \sqrt{6}} \\ \sqrt{(\sqrt{5} + \sqrt{6})^2} \\ (\sqrt{5} + \sqrt{6}) \end{array}$$

■ Find the square root-

$$13 + 2\sqrt{30}$$

Sol.

$$\begin{array}{r} \sqrt{13 + 2\sqrt{30}} \\ \downarrow \quad \downarrow \\ 10 + 3 \quad 10 \times 3 \\ \sqrt{(\sqrt{10} + \sqrt{3})^2} \\ (\sqrt{10} + \sqrt{3}) \end{array}$$

■ Find the square root-

$$17 - 2\sqrt{30}$$

Sol.

$$\begin{array}{r} \sqrt{17 - 2\sqrt{30}} \\ \downarrow \quad \downarrow \\ 15 + 2 \quad 15 \times 2 \\ \sqrt{(\sqrt{15} - \sqrt{2})^2} \\ (\sqrt{15} - \sqrt{2}) \end{array}$$

■ Find the square root-

$$8 - 2\sqrt{7}$$

Sol.

$$\begin{array}{r} \sqrt{8 - 2\sqrt{7}} \\ \quad \quad \quad \{ \because 8 = 7 + 1, 7 = 7 \times 1 \} \\ \sqrt{(\sqrt{7} - \sqrt{1})^2} \\ (\sqrt{7} - \sqrt{1}) \end{array}$$

■ Find the square root-

$$12 + \sqrt{140}$$

Sol.

$$\begin{array}{r} \sqrt{12 + \sqrt{140}} \\ \sqrt{12 + 2\sqrt{35}} \\ \downarrow \quad \downarrow \\ 7 + 5 \quad 7 \times 5 \\ \sqrt{(\sqrt{7} + \sqrt{5})^2} \\ (\sqrt{7} + \sqrt{5}) \end{array}$$

■ Find the square root-

$$8 - \sqrt{60}$$

Sol.

$$\begin{array}{r} \sqrt{8 - \sqrt{60}} \\ \sqrt{8 - 2\sqrt{15}} \\ \downarrow \quad \downarrow \\ 5 + 3 \quad 5 \times 3 \\ \sqrt{(\sqrt{5} - \sqrt{3})^2} \\ (\sqrt{5} - \sqrt{3}) \end{array}$$

■ Find the square root-

$$7 + 4\sqrt{3}$$

Sol.

$$\begin{array}{r} \sqrt{7 + 4\sqrt{3}} \\ \sqrt{7 + 2\sqrt{12}} \\ \downarrow \quad \downarrow \\ 4 + 3 \quad 4 \times 3 \\ \sqrt{(\sqrt{4} + \sqrt{3})^2} \\ (2 + \sqrt{3}) \end{array}$$

■ Find the square root-

$$12 - 6\sqrt{3}$$

Sol.

$$\begin{array}{r} \sqrt{12 - 6\sqrt{3}} \\ \sqrt{12 - 2\sqrt{27}} \\ \downarrow \quad \downarrow \\ 9 + 3 \quad 9 \times 3 \\ \sqrt{(\sqrt{9} - \sqrt{3})^2} \\ (3 - \sqrt{3}) \end{array}$$

■ Find the square root-

$$3 + \sqrt{5}$$

Sol.

$$\begin{array}{r} \sqrt{3 + \sqrt{5}} \\ \sqrt{\frac{2}{2}(3 + \sqrt{5})} \\ \frac{1}{\sqrt{2}} \sqrt{6 + 2\sqrt{5}} \\ \frac{1}{\sqrt{2}} \sqrt{6 + 2\sqrt{5}} \\ \downarrow \quad \downarrow \quad \downarrow \\ 5 + 1 \quad 5 \times 1 \\ \frac{1}{\sqrt{2}} \sqrt{(\sqrt{5} + 1)^2} \\ \frac{1}{\sqrt{2}} (\sqrt{5} + 1) \end{array}$$

■ Find the square root-

$$4 - \sqrt{15}$$

Sol.

$$\begin{array}{r} \sqrt{4 - \sqrt{15}} \\ \sqrt{\frac{2}{2}(4 - \sqrt{15})} \\ \frac{1}{\sqrt{2}} \sqrt{8 - 2\sqrt{15}} \\ \frac{1}{\sqrt{2}} \sqrt{8 - 2\sqrt{15}} \\ \downarrow \quad \downarrow \quad \downarrow \\ 5 + 3 \quad 5 \times 3 \end{array}$$

$$\frac{1}{\sqrt{2}} \sqrt{(\sqrt{5} - \sqrt{3})^2}$$

$$\frac{1}{\sqrt{2}} (\sqrt{5} - \sqrt{3})$$

■ **Find the square root-**

$$15 + \sqrt{60} + \sqrt{84} + \sqrt{140}$$

Sol. $\sqrt{15 + \sqrt{60} + \sqrt{84} + \sqrt{140}}$

$$\sqrt{15 + 2\sqrt{15} + 2\sqrt{21} + 2\sqrt{35}}$$

$$\sqrt{15 + 2\sqrt{3}\sqrt{5} + 2\sqrt{5}\sqrt{7} + 2\sqrt{7}\sqrt{3}}$$

$$\sqrt{(\sqrt{3})^2 + (\sqrt{5})^2 + (\sqrt{7})^2 + 2\sqrt{3}\sqrt{5} + 2\sqrt{5}\sqrt{7} + 2\sqrt{7}\sqrt{3}}$$

$$\sqrt{(\sqrt{3} + \sqrt{5} + \sqrt{7})^2}$$

$$(\sqrt{3} + \sqrt{5} + \sqrt{7})$$

Some important results

➤ If, $x = \sqrt{a\sqrt{a\sqrt{a\sqrt{a\cdots\cdots\infty}}}}$

then, $x = a$

➤ If, $x = \sqrt{a\sqrt{a\sqrt{a\cdots\cdots n \text{ times}}}}$

then, $x = a^{\frac{2^n-1}{2^n}}$

➤ If, $x = \sqrt[n]{a \times \sqrt[n]{a \times \sqrt[n]{a \cdots\cdots\infty}}}$

then, $x = \sqrt[n-1]{a}$

➤ If, $x = \sqrt[n]{a \div \sqrt[n]{a \div \sqrt[n]{a \div \cdots\cdots\infty}}}$

then, $x = \sqrt[n+1]{a}$

➤ If, $x = \sqrt{a + b\sqrt{a + b\sqrt{a + \cdots\cdots\infty}}}$

then, $x = \frac{\sqrt{4a + b^2} + b}{2}$

➤ If, $x = \sqrt{a + \sqrt{a + \sqrt{a + \cdots\cdots\infty}}}$

then, $x = \frac{\sqrt{4a + 1} + 1}{2}$

➤ If, $x = \sqrt{a - b\sqrt{a - b\sqrt{a - \cdots\cdots\infty}}}$

then, $x = \frac{\sqrt{4a + b^2} - b}{2}$

➤ If, $x = \sqrt{a - \sqrt{a - \sqrt{a - \cdots\cdots\infty}}}$

then, $x = \frac{\sqrt{4a + 1} - 1}{2}$

➤ If, $x = \sqrt{a + b\sqrt{a - b\sqrt{a + b\sqrt{a - \cdots\cdots\infty}}}}$

then, $x = \frac{\sqrt{4a - 3b^2} + b}{2}$

➤ If, $x = \sqrt{a + \sqrt{a - \sqrt{a + \sqrt{a - \cdots\cdots\infty}}}}$

then, $x = \frac{\sqrt{4a - 3} + 1}{2}$

➤ If, $x = \sqrt{a - b\sqrt{a + b\sqrt{a - b\sqrt{a + b\sqrt{a - \cdots\cdots\infty}}}}$

then, $x = \frac{\sqrt{4a - 3b^2} - b}{2}$

➤ If, $x = \sqrt{a - \sqrt{a + \sqrt{a - \sqrt{a + \sqrt{a - \cdots\cdots\infty}}}}$

then, $x = \frac{\sqrt{4a - 3} - 1}{2}$

LCM and H.C.F.

Difference between multiple and factor

S. N.	Multiple	Factor
1.	The multiples are defined as the numbers obtained when multiplied by other numbers	Factors are defined as the exact divisors of the given number
2.	The number of multiples is infinite	The number of factors is finite
3.	The operation used to find the multiples is a multiplication.	The operation used to find the factors is a division
4.	The outcome of the multiples should be greater than or equal to the given number	The outcome of the factors should be less than or equal to the given number.

L.C.M.

L.C.M. : Least common multiple

☞ L.C.M. is the smallest number which is completely divided by two or more numbers.

☞ The LCM of x, y and z is completely divisible by x, y, and z.

■ L.C.M. of 12 and 16–:

12 Multiple = 12, 24, 36, **48**, 60, 72, 84, **96**,
 16 Multiple = 16, 32, **48**, 64, 80, **96**, 112, 128,
 Common multiple = 48, 96
 Least common multiple = 48
L.C.M. = 48

Methods of finding L.C.M.

- In this method, divide the given numbers by common prime number until the remainder is 1.

Ex. Finding the L.C.M. of 9, 12 and 15

Sol.

2	9, 12, 15
2	9, 6, 15
3	9, 3, 15
3	3, 1, 5
5	1, 1, 5
	1, 1, 1

$$\begin{aligned} (\text{L.C.M.}) &= 2 \times 2 \times 3 \times 3 \times 5 \\ &= 180 \end{aligned}$$

- **Prime Factor Method–:** First express the given numbers in the form of prime factors. The product of factors with highest power will be the L.C.M.

Ex. Finding the L.C.M. of 9, 12 and 15

Sol. $9 = 3 \times 3$

$$12 = 2 \times 2 \times 3$$

$$15 = 3 \times 5$$

$$\begin{aligned} \text{L.C.M.} &= 2 \times 2 \times 3 \times 3 \times 5 \\ &= 180 \end{aligned}$$

Types of questions

➤	Find the smallest no. which is exactly divisible by x, y, z.	L.C.M. of (x, y, z)
➤	Find the smallest no. which when divided by x, y, z leaves remainder 'r' in each case.	L.C.M. of (x, y, z) + r
➤	Find the smallest no. which when divided by x, y, z leaves remainder a, b, c respectively.	L.C.M. of (x, y, z) – k Where, $k = (x - a)$ $= (y - b)$ $= (z - c)$

H.C.F.

H.C.F. : Highest common factor
(Greatest common divisor)

- ☞ H.C.F is the largest number, which can divide two or more numbers completely.
- ☞ The HCF of x, y and z will divide x, y, and z completely.

■ H.C.F. of 12 and 16–:

12 Factor = **1, 2, 3, 4, 6, 12** 16 Factor = **1, 2, 4, 8, 16**
 Common factor = 1, 2, 4 Highest common factor = 4
H.C.F. = 4

Methods of finding H.C.F.

- **Division Method–:** Find the H.C.F. of two number x and y. (Where, $y > x$)

On dividing y by x remainder is r_1 . Then on dividing x by r_1 the remainder is r_2 . Then r_1 is divided by r_2 . This process will be repeated until the remainder becomes zero. Last divisor will be the H.C.F. of x and y.

Ex. Finding the H.C.F. of 12 and 16 :

Sol. 12, 16 of H.C.F.

$$\begin{array}{r} 12 \) 16 \left(1 \\ \quad 12 \right. \\ \hline \quad 4 \) 12 \left(3 \\ \quad 12 \right. \\ \hline \quad 0 \end{array}$$

$$\text{H.C.F.} = 4$$

Ex. Finding the H.C.F. of 25, 35 and 40 :

Sol. 25, 35 and 40 of H.C.F.

$$\begin{array}{r} 25 \) 35 \left(1 \\ \quad 25 \right. \\ \hline \quad 10 \) 25 \left(2 \\ \quad 20 \right. \\ \hline \quad 5 \) 10 \left(2 \\ \quad 10 \right. \\ \hline \quad 0 \end{array} \quad \begin{array}{r} 40 \) 40 \left(8 \\ \quad 40 \right. \\ \hline \quad 0 \end{array} \quad \text{H.C.F.} = 5$$

- **Prime factor method–:** First, write each given numbers in the form of product of their prime factors. The product of common factors with least power will be the H.C.F. of given numbers.

Ex. Finding the H.C.F. of 12 and 16 :

Sol. 12, 16 of H.C.F.

$$12 = 2 \times 2 \times 3 \Rightarrow 2^2 \times 3$$

$$16 = 2 \times 2 \times 2 \times 2 \Rightarrow 2^4$$

$$\text{H.C.F.} = 2^2 \Rightarrow 4$$

Ex. Finding the H.C.F. of 25, 35 and 40 :

Sol. 25, 35 and 40 of H.C.F.

$$25 = 5 \times 5 \Rightarrow 5^2$$

$$35 = 5 \times 7 \Rightarrow 5^1 \times 7^1$$

$$40 = 2 \times 2 \times 2 \times 5 \Rightarrow 2^3 \times 5^1$$

$$\text{H.C.F.} = 5$$

■ **Difference method-**

Let,

H.C.F. of two numbers = h
then, numbers = hx, hy

Where, $x, y \rightarrow$ Co-prime

Difference = $hx - hy$

$$\Rightarrow h(x - y)$$

- ☞ $(x - y) = 1 \rightarrow$ H.C.F. is a difference between numbers.
- ☞ $(x - y) > 1 \rightarrow$ H.C.F. is a factor of difference of numbers.
- ☞ H.C.F. of two numbers never greater than difference of these numbers.
Hence, H.C.F. can be either difference of these number or factor of difference.

Ex. Finding the H.C.F. of 30 and 45 :

Sol. 30, 45 of H.C.F.

30, 45

$$\text{difference} = 45 - 30 \Rightarrow 15$$

H.C.F. = 15 or factor of 15

∴ 30 and 45 are completely divisible by 15

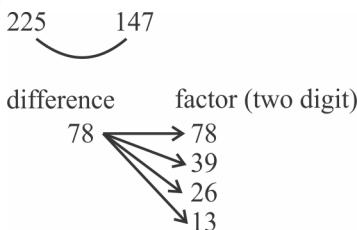
Hence, H.C.F. = 15

Types of questions

➤	Find the largest no. which can divide x, y, z exactly	H.C.F. of (x, y, z)
➤	Find the largest no. which can divide x, y, z and leaves same remainder in each case.	H.C.F. of $(x - y), (y - z), (z - x)$
➤	Find the largest no. which can divide x, y, z and leaves remainder 'r' in each case.	H.C.F. of $(x - r), (y - r), (z - r)$
➤	Find the largest number which can divide x, y, z and leaves remainder a, b, c respectively.	H.C.F. of $(x - a), (y - b), (z - c)$

- If two numbers are divided by their difference or factors of difference then leaves same remainder.
- **A two digit number can divide 225 and 147, leaves same remainder in each case. How many such two digit numbers would be possible?**

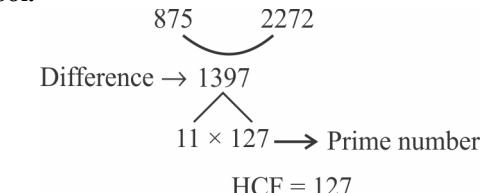
Sol.



Total numbers = 4

- **The two numbers 875 and 2272 are divided by a three digit number. Then there is same remainder left in each case what will be the sum of the digits of such three digits?**

Sol.



$$\text{Sum of digits} = 1 + 2 + 7 \Rightarrow 10$$

Relation between L.C.M. and H.C.F.

- First no. \times second no. = L.C.M. \times H.C.F.

- ☞ If H.C.F. = h

First no. = hx

Second no. = hy

then, L.C.M. = hxy

L.C.M. and H.C.F. of fraction

$$\text{■ L.C.M. of fraction} = \frac{\text{L.C.M. of numerator}}{\text{H.C.F. of denominator}}$$

$$\text{■ H.C.F. of fraction} = \frac{\text{H.C.F. of numerator}}{\text{L.C.M. of denominator}}$$

L.C.M. and H.C.F. of indices

- When the base of the given numbers are same, then the number with highest power will be the LCM of the given numbers.

Ex. $7^2, 7^4, 7^9$ of L.C.M. = 7^9

- When the base is not same and there is no common factors in the base, then the product of given numbers will be the LCM.

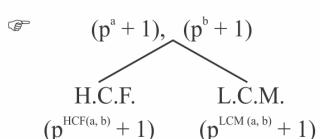
Ex. $2^2, 3^5, 5^4$ of L.C.M. = $2^2 \times 3^5 \times 5^4$

- When the base of the given number are same, then the number with least power will be the H.C.F. of given numbers.

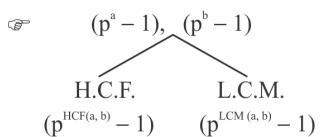
Ex. $7^2, 7^4, 7^9$ of H.C.F. = 7^2

- When the base is not same and there is no common factor in the base, then the required H.C.F. of given numbers will be 1.

Ex. $2^2, 3^5, 5^4$ of H.C.F. = 1



Where, power (a, b) should be odd multiple of HCF.



9. Evaluate: $\frac{1}{\left(\frac{5}{6}\right) + \left(\frac{7}{9}\right)} \div \frac{5}{23}$

(a) $2\frac{53}{61}$ (b) $2\frac{59}{69}$
 (c) $2\frac{5}{6}$ (d) $4\frac{1}{27}$

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Ans. (b) :
$$\begin{aligned} & \frac{1}{\left(\frac{5}{6}\right) + \left(\frac{7}{9}\right)} \div \frac{5}{23} \\ &= \frac{1}{\frac{15+14}{18}} \times \frac{23}{5} = \frac{18}{29} \times \frac{23}{5} \\ &= \frac{414}{145} = 2\frac{124}{145} \\ &= 2\frac{59 \times 2.101}{69 \times 2.101} \text{ (approximate)} = 2\frac{59}{69} \end{aligned}$$

10. Evaluate : $38 - 9 \div 6 \times 6$

(a) 28 (b) 32
 (c) 27 (d) 29

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Ans. (d) : $38 - 9 \div 6 \times 6$

$$\begin{aligned} &= 38 - \frac{9}{6} \times 6 = 38 - 9 \\ &= 29 \end{aligned}$$

11. When performing the operation $(1.23456 \times 10^3) + (1.234 \times 10^2)$, how many significant figures should be reported in the result, assuming no rounding errors?

(a) 5 (b) 2
 (c) 4 (d) 3

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Ans. (a) : $(1.23456 \times 10^3) + (1.234 \times 10^2)$

$\Rightarrow 1.23456 \times 10^3 + 0.1234 \times 10^3$

$\Rightarrow (1.23456 + 0.1234) \times 10^3$

$\Rightarrow 1.35796 \times 10^3$

\Rightarrow Round of 4 decimal place 1.35796×10^3

Now to count the significant figure

\Rightarrow 5 significant figures are in 1.3580×10^3

12. The smallest natural number which is divisible by 24, 6, 36 and 13 is:

(a) 936 (b) 1008
 (c) 1011 (d) 943

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Ans. (a) : The LCM of 24, 6, 36, 13 will be

$13 = 13^1$

$36 = 2 \times 2 \times 3 \times 3 = 2^2 \times 3^2$

$24 = 2 \times 2 \times 2 \times 3 = 2^3 \times 3^1$

$$\begin{aligned} 6 &= 2 \times 3 = 2^1 \times 3^1 \\ \text{LCM} &= 2^3 \times 3^2 \times 13^1 \\ &= 8 \times 9 \times 13 \\ &= 936 \end{aligned}$$

So, smallest natural number divisible by 24, 6, 36, 13 is '936'

13. Evaluate : $\frac{1}{\left(\frac{3}{4}\right) + \left(\frac{4}{6}\right)} \div \frac{2}{15}$

(a) $5\frac{5}{17}$ (b) $5\frac{2}{17}$
 (c) $5\frac{6}{17}$ (d) $5\frac{4}{17}$

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Ans. (a) :
$$\begin{aligned} & \frac{1}{\left(\frac{3}{4} + \frac{4}{6}\right)} \div \frac{2}{15} \\ &= \frac{1}{\left(\frac{9+8}{12}\right)} \div \frac{2}{15} \\ &= \frac{12}{17} \times \frac{15}{2} = \frac{90}{17} = 5\frac{5}{17} \end{aligned}$$

14. The value of

$$\sqrt{144} + \sqrt{0.0324} - \sqrt{6.76} =$$

(a) 8.53 (b) 14.76
 (c) 9.58 (d) 2.7

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Ans. (c) :
$$\begin{aligned} & \sqrt{144} + \sqrt{0.0324} - \sqrt{6.76} \\ &= 12 + 0.18 - 2.6 \\ &= 9.58 \end{aligned}$$

15. Which of the following numbers is divisible completely by both 9 and 11?

(a) 277218 (b) 10098
 (c) 12345 (d) 181998

RRB NTPC (Stage-II) 17/06/2022 (Shift-II)

Ans. (b) : Divisibility rule of 9 -

When the sum of the digits of a number is divisible by 9 then the number is also divisible by 9.

Divisibility rule of 11 -

When the difference between the sum of the digit in even and odd place of a number is 0 (zero) or a multiple of 11, then the number will also be divisible by 11.

From option (b),

$$1 + 0 + 0 + 9 + 8 = 18$$

i.e. 18 is divisible by 9

\therefore Option (d) is divisible by 9.

And

$$10098 = (9 + 0) - (8 + 0 + 1) = 9 - 9 = 0$$

Hence option (b) 10098, is divisible by both 9 and 11.

